Supersymmetry across the light and heavy-light hadronic spectrum. II.

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We extend our analysis of the implications of hadronic supersymmetry for heavy-light hadrons in light-front holographic QCD. Although conformal symmetry is strongly broken by the heavy quark mass, supersymmetry and the holographic embedding of semiclassical light-front dynamics derived from five-dimensional anti-de Sitter space nevertheless determine the form of the confining potential in the light-front Hamiltonian to be harmonic. The resulting light-front bound-state equations lead to a heavy-light Regge-like spectrum for both mesons and baryons. The confinement hadron mass scale and their Regge slopes depend, however, on the mass of the heavy quark in the meson or baryon as expected from heavy quark effective theory. This procedure reproduces the observed spectra of heavy-light hadrons with good precision and makes predictions for yet unobserved states.

I. INTRODUCTION

In a series of recent articles [1–4], we have shown that superconformal algebra allows the construction of relativistic light-front (LF) semiclassical bound-state equations in physical spacetime which can be embedded in a higher-dimensional classical gravitational theory. This new approach to hadron physics incorporates basic nonperturbative properties which are not apparent from the chiral QCD Lagrangian; it includes the emergence of a mass scale and confinement out of a classically scale-invariant theory, the occurrence of a zero-mass bound state, universal Regge trajectories for both mesons and baryons, and the breaking of chiral symmetry in the hadron spectrum. This holographic approach to hadronic physics gives remarkable connections between the light meson and nucleon spectra [2], as well as specific relations which can be derived for heavy-light hadrons. Remarkably, even though heavy quark masses break conformal invariance, an underlying dynamical supersymmetry still holds [3].

Our analysis is based on a procedure developed by de Alfaro, Fubini and Furlan, and Fubini and Rabinovici [1,2,5–7]. In our approach, it leads to the natural emergence of a mass scale into the Hamiltonian of a theory while retaining essential elements of both conformal invariance and supersymmetry. In the case of superconformal (graded) algebra, a generalized Hamiltonian can be constructed as a linear superposition of superconformal generators which carry different dimensions; the Hamiltonian thus remains within the superconformal algebraic structure. This procedure determines a unique form of a quark confinement potential in the light-front Hamiltonian for light mesons and baryons, and it reproduces quite well significant features of the hadron spectrum and dynamics. The resulting bound-state equations depend explicitly on orbital angular momentum, and thus, chiral symmetry is broken from the outset in the Regge excitation spectra: The ρ meson and the nucleon have no chiral partners. A striking feature of the formalism is that the supermultiplets consist of a meson wave function with internal LF angular momentum L_M and a corresponding baryon wave function with angular momentum $L_B = L_M - 1$ and identical mass. The lightest meson state with $L_M = 0$ and total quark spin zero is massless in the chiral limit and is identified with the pion; it has no supersymmetric partner.

It is not known why the effective theory based on superconformal quantum mechanics and its light-front holographic embedding captures so well essential aspects of the confinement dynamics of QCD. However, underlying aspects of the superconformal holographic construction, conformal symmetry and supersymmetry, as well as the LF cluster decomposition required by the holographic embedding, could help us understand fundamental features of QCD in its nonperturbative domain.

As it is the case for conformal quantum mechanics [5], where the action remains invariant under conformal transformations, classical QCD in the limit of massless quarks has no mass scale, but confinement and a mass gap can emerge from its quantum embodiment. The cluster decomposition of the constituents of baryons corresponding to a quark-diquark structure is necessary in order to describe baryons in light-front holographic QCD (LFHQCD) since there is only a single holographic variable [8]. The required LF clustering follows from the mapping of anti-de Sitter (AdS) equations to QCD bound-state equations in light-front physics [9], where one identifies the holographic

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variable z in the AdS classical gravity theory with the boost-invariant transverse separation ζ between constituents in the light-front quantization scheme [10,11]. In the case of mesons, $\zeta^2 = b_\perp^2 x(1-x)$ is conjugate to the invariant mass of the $q\bar{q}$ in the LF wave function; it is the invariant variable of the LF Hamiltonian theory [12]. The resulting symmetry between mesons and baryons is consistent with an essential feature of color $SU(N_C)$: A cluster of $N_C - 1$ constituents can be in the same color representation as the anti-constituent; for $SU(3)$, this means $\bar{\mathbf{3}} \in \mathbf{3} \times \mathbf{3}$ and $\mathbf{3} \in \bar{\mathbf{3}} \times \bar{\mathbf{3}}$. Thus, emerging hadronic supersymmetry can be rooted in the dynamics of color $SU(3)$ [13,14].

Our basic model describes the confinement of massless quarks [1,2,4]. Indeed, for light quark masses it makes sense to apply superconformal dynamics and to treat the quark masses as perturbations: The dynamics is then not significantly changed for nonzero quark mass, and the resulting confinement scale remains universal for the resulting hadronic bound states [4]. In contrast, in the case of heavy quark masses, we cannot rely on conclusions drawn from conformal symmetry; however, the presence of a heavy mass need not also break supersymmetry since it can stem from the dynamics of color confinement [15]. Indeed, as we have shown in Ref. [3], supersymmetric relations between the meson and baryon masses still hold to a good approximation even for heavy-light, i.e., charm and bottom, hadrons.

In addition to the constraints imposed by supersymmetry, we use additional features imposed by the holographic embedding in order to constrain the specific form of the confinement potential in the heavy-light sector. We also use the heavy-quark flavor symmetry of QCD [16] to determine the dependence of the confinement scale on the heavy quark mass in the heavy mass limit since this symmetry is compatible with the light-front holographic approach [17]. Other holographic approaches to the heavy-light sector, including the recent holographic approach given in Ref. [18], which includes chiral and heavy quark symmetry, have been proposed in Refs. [19–24].

Light quark masses are not only essential for approximate conformal symmetry, but they also guarantee the decoupling of transverse degrees of freedom—expressed through the LF variable ζ in the hadron LF wave function—from the longitudinal degrees of freedom which depends on the longitudinal LF momentum fraction x [25]. The holographic mapping derived from the geometry of AdS space encodes the kinematics in $3 + 1$ physical spacetime, and the modification of the AdS action—usually described for mesons in terms of a dilaton profile $\varphi(z)$—generates the confining LF potential $U(z)$ in the light-front bound-state equations [26].

Since light constituents are present in the heavy-light bound states of mesons or baryons, the system is still ultrarelativistic; thus the heavy-light bound states need to be described by relativistic LF bound-state equations. This means that the heavy-light system has properties common to both the chiral and the heavy-quark flavor sectors [16,18]. It also suggests that we can holographically connect the supersymmetric theory to a modified AdS space; this will be possible if the separation of the dynamical and kinematical variables also persist, at least to a good approximation, in the heavy-light domain. As we show, we can again derive a unique confinement potential for both mesons and baryons in the heavy-light sector, even when conformal symmetry is broken by a heavy quark mass. The resulting embedding leads to a LF harmonic confinement potential for the heavy-light hadrons and thus to Regge trajectories; however, as we show, the confinement scale and Regge slope depends on the mass of the heavy quark. We investigate this dependence using heavy quark effective theory (HQET) [16]. The procedure discussed in this article not only reproduces the observed data to a reasonable accuracy, but it also allows us to make predictions for yet unobserved states.

This article is organized as follows: In Sec. II we briefly review the construction of the LF Hamiltonian from supersymmetric quantum mechanics [27] using the methods developed in Refs. [1,2,6]. In Sec. III we extend our approach to systems containing a heavy, charm or bottom, quark. Notably, we discuss the constraints imposed by the holographic embedding on the supersymmetric potential, which in turn determine the form of the light-front potential. We compare our predictions with experiment in Sec. IV, and in Sec. V we discuss the constraints on the confinement scale imposed by HQET. Some final comments are given in Sec. VI. In the Appendix we give expressions for the LF wave functions and hadron distribution amplitudes which are compatible with our general approach. This article is the continuation of Ref. [3].

II. THE SUPERSYMMETRIC LIGHT-FRONT HAMILTONIAN

The light-front Hamiltonian derived in the framework of supersymmetric quantum mechanics [27,28] contains two fermionic generators, the supercharges, Q and $Q^\dagger$, with the anticommutation relations

$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0, \tag{1}$$

and the Hamiltonian H

$$H = \{Q, Q^\dagger\}, \tag{2}$$

which commutes with the fermionic generators $[Q, H] = [Q^\dagger, H] = 0$, closing the graded Lie algebra. Since the Hamiltonian H commutes with $Q^\dagger$, it follows that the states $|\phi\rangle$ and $Q^\dagger|\phi\rangle$ have identical nonvanishing eigenvalues. In addition, if $|\phi_0\rangle$ is an eigenstate of Q with zero eigenvalue, it is annihilated by the operator

$Q^\dagger: Q^\dagger|\phi_0\rangle = 0$. This implies that the lowest mesonic state on a given trajectory has no supersymmetric baryon partner [2]. This shows the special role of the pion in the supersymmetric approach to hadronic physics as a unique state of zero mass in the chiral limit.

In matrix notation

$$Q = \begin{pmatrix} 0 & q \\ 0 & 0 \end{pmatrix}, \qquad Q^\dagger = \begin{pmatrix} 0 & 0 \\ q^\dagger & 0 \end{pmatrix}, \qquad (3)$$

and

$$H = \begin{pmatrix} qq^\dagger & 0 \\ 0 & q^\dagger q \end{pmatrix}, \qquad (4)$$

with

$$q = -\frac{d}{d\zeta} + \frac{f}{\zeta} + V(\zeta), \qquad (5)$$

$$q^\dagger = \frac{d}{d\zeta} + \frac{f}{\zeta} + V(\zeta), \qquad (6)$$

where ζ is the LF invariant transverse variable and f is a dimensionless constant. One can add to the Hamiltonian (2) a constant term proportional to the unit matrix $\mu^2 I$

$$H_\mu = \{Q, Q^\dagger\} + \mu^2 I, \qquad (7)$$

where the constant μ has the dimension of a mass; thus, we obtain the general supersymmetric light-front Hamiltonian derived in Ref. [3]

$$H_\mu = \begin{pmatrix} -\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + U_M(\zeta) & 0 \\ 0 & -\frac{d^2}{d\zeta^2} + \frac{4L_B^2 - 1}{4\zeta^2} + U_B(\zeta) \end{pmatrix}$$
$$+ \mu^2 \mathbf{I}, \qquad (8)$$

where $L_B + \frac{1}{2} = L_M - \frac{1}{2} = f$ and U_M and U_B are, respectively, the meson and baryon LF confinement potentials:

$$U_M(\zeta) = V^2(\zeta) - V'(\zeta) + \frac{2L_M - 1}{\zeta} V(\zeta), \qquad (9)$$

$$U_B(\zeta) = V^2(\zeta) + V'(\zeta) + \frac{2L_B + 1}{\zeta} V(\zeta). \qquad (10)$$

The superpotential V is only constrained by the requirement that it is regular at the origin. For the special case $V = 0$, the Hamiltonian is also invariant under conformal transformations, and one can extend the supersymmetric algebra to a superconformal algebra [6,29]. In fact, the use of this procedure in supersymmetric quantum mechanics determines a unique form for the superconformal potential in (5): It is given by $V = \sqrt{\lambda}\zeta$ [1,2]. Thus, in the conformal limit $\mu^2 \to 0$, and we have

$$U_M(\zeta) \to \lambda_M^2 \zeta^2 + 2\lambda_M(L_M - 1), \qquad (11)$$

$$U_B(\zeta) \to \lambda_B^2 \zeta^2 + 2\lambda_B(L_B + 1), \qquad (12)$$

with $\lambda_M = \lambda_B = \lambda$. The Hamiltonian (8) acts on the spinor

$$|\phi\rangle = \begin{pmatrix} \phi_M \\ \phi_B \end{pmatrix}, \qquad (13)$$

where the upper component ϕ_M corresponds to a meson wave function with angular momentum L_M and a lower component ϕ_B, which corresponds to the leading-twist positive chirality component of a baryon ψ^+ [1,8] with angular momentum $L_B = L_M - 1$. The supersymmetric framework described here also incorporates a doublet consisting of the nonleading twist minus-chirality component ψ^- of a baryon which has angular momentum $L_B + 1$ and its partner tetraquark with angular momentum $L_T = L_B$ [4]. The tetraquark sector is discussed in more detail in Ref. [4].

III. EXTENSION TO THE HEAVY-LIGHT HADRON SECTOR

In LF holographic QCD the confinement potential for mesons U_M (9) follows from the dilaton term $e^{\varphi(x)}$ in the AdS$_5$ action following Ref. [30]. It is given by [31]

$$U_{\text{dil}}(\zeta) = \frac{1}{4}(\varphi'(\zeta))^2 + \frac{1}{2}\varphi''(\zeta) + \frac{2L_M - 3}{2\zeta}\varphi'(\zeta), \quad (14)$$

for $J_M = L_M$. In the conformal limit a quadratic dilaton profile, $\varphi = \lambda\zeta^2$ leads to the potential (11).

The dilaton φ is not constrained by the superconformal algebraic structure in the presence of heavy quark masses, and thus, its form and the form of the superpotential V are unknown *a priori*. Additional constraints do appear, however, by the holographic embedding which can be derived by equating the potential (14), given in terms of the dilaton profile φ, with the meson potential (9) written in terms of the superpotential V. We have

$$\frac{1}{4}(\varphi')^2 + \frac{1}{2}\varphi'' + \frac{2L - 1}{2\zeta}\varphi' = V^2 - V' + \frac{2L + 1}{\zeta}V, \quad (15)$$

where $L = L_M - 1$.

We make the ansatz

$$\varphi'(\zeta) = 2\lambda\zeta\alpha(\zeta), \qquad (16)$$

$$V(\zeta) = \lambda\zeta\beta(\zeta). \qquad (17)$$

Then, we obtain

$$U_{\text{dil}} = \lambda^2\zeta^2\alpha^2 + 2L\lambda\alpha + \lambda\zeta\alpha', \qquad (18)$$

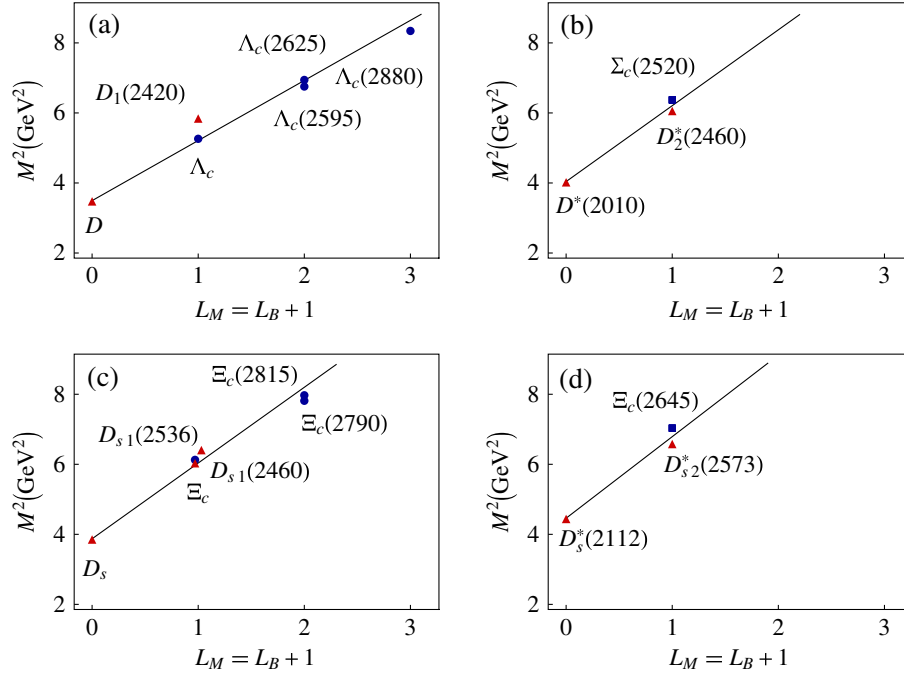$$U_{\text{susy}} = \lambda^2\zeta^2\beta^2 + 2L\lambda\beta - \lambda\zeta\beta', \qquad (19)$$

FIG. 1. Heavy-light mesons and baryons with one charm quark: $D = q\bar{c}$, $D_s = s\bar{c}$, $\Lambda_c = udc$, $\Sigma_c = qqc$, $\Xi_c = usc$. In (a) and (c) $s = 0$ and in (b) and (d) $s = 1$, where s is the total quark spin in the mesons or the spin of the quark cluster in the baryons. The data is from Ref. [32].
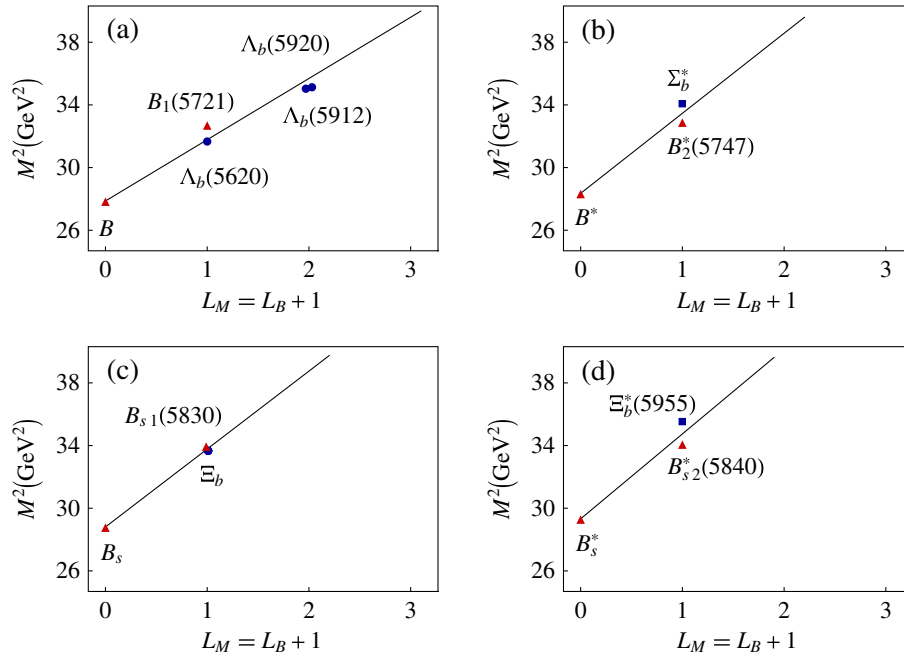

FIG. 2. Heavy-light mesons and baryons with one bottom quark: $B = q\bar{b}$, $B_s = s\bar{b}$, $\Lambda_c = udb$, $\Sigma_b = qqb$, $\Xi_c = usb$. In (a) and (c) $s = 0$ and in (b) and (d) $s = 1$, where s is the total quark spin in the mesons or the spin of the diquark cluster in the baryons. The data is from Ref. [32].

and therefore,

$$\lambda^2\zeta^2(\alpha^2 - \beta^2) + 2L\lambda(\alpha - \beta) + \lambda\zeta(\alpha' + \beta') = 0. \quad (20)$$

Introducing the linear combination

$$\sigma(\zeta) = \alpha(\zeta) + \beta(\zeta),$$
$$\delta(\zeta) = \alpha(\zeta) - \beta(\zeta), \quad (21)$$
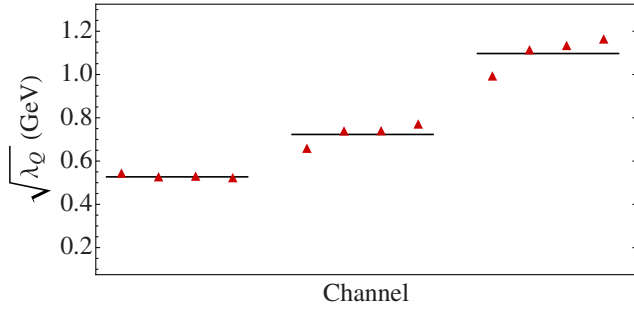
it follows that

FIG. 3. The fitted value of $\sqrt{\lambda_Q}$ for different meson-baryon trajectories, indicated by the lowest meson state on that trajectory.

$$\lambda^2\zeta^2\sigma(\zeta)\delta(\zeta) + 2L\lambda\delta(\zeta) + \lambda\zeta\sigma'(\zeta) = 0. \qquad (22)$$

This yields

$$\delta(\zeta) = -\frac{\lambda\zeta\sigma'(\zeta)}{\lambda^2\zeta^2\sigma(\zeta) + 2L\lambda}, \qquad (23)$$

and therefore,

$$\alpha(\zeta) = \frac{1}{2}\left(\sigma(\zeta) - \frac{\lambda\zeta\sigma'(\zeta)}{\lambda^2\zeta^2\sigma(\zeta) + 2L\lambda}\right), \qquad (24)$$

$$\beta(\zeta) = \frac{1}{2}\left(\sigma(\zeta) + \frac{\lambda\zeta\sigma'(\zeta)}{\lambda^2\zeta^2\sigma(\zeta) + 2L\lambda}\right). \qquad (25)$$

Using (16) and (24) we obtain upon integration the dilaton profile for a meson with angular momentum $L_M = L + 1$

$$\varphi(\zeta) = \int d\zeta\left(\lambda\zeta\sigma(\zeta) - \frac{\lambda^2\zeta^2\sigma'(\zeta)}{\lambda^2\zeta^2\sigma(\zeta) + 2(L_M - 1)\lambda}\right). \qquad (26)$$

On the other hand, from (17) and (25) it follows that this profile for arbitrary $\sigma(\zeta)$ is compatible with the SUSY potential

$$V(\zeta) = \frac{1}{2}\left(\lambda\zeta\sigma(\zeta) + \frac{\lambda^2\zeta^2\sigma'(\zeta)}{\lambda^2\zeta^2\sigma(\zeta) + 2(L_M - 1)\lambda}\right). \qquad (27)$$

The baryon equations give no further constraints.

In LFHQCD the AdS geometry fixes the nontrivial aspects of the kinematics, whereas additional deformations of AdS space encodes the dynamical features of the theory [26]. In particular, the dilaton, which describes the dynamics of confinement for mesons in holographic QCD, must be free of kinematical quantities and thus must be independent of the angular momentum L_M. This is only possible if the derivative $\sigma'(\zeta) = 0$ in (26) and (27); thus, $\sigma(\zeta) = A$ with A an arbitrary constant. From (26) and (27) it follows that

$$\varphi(\zeta) = \frac{1}{2}\lambda A\zeta^2 + B, \qquad V(\zeta) = \frac{1}{2}\lambda A\zeta. \qquad (28)$$

This result implies that the LF potential in the heavy-light sector, even for strongly broken conformal invariance, has the same quadratic form as the one dictated by the conformal algebra. The constant A, however, is arbitrary, so the strength of the potential is not determined. Notice that the interaction potential (14) is unchanged by adding a constant to the dilaton profile; thus, we can set $B = 0$ in (28) without modifying the equations of motion.

The LF eigenvalue equation $H|\phi\rangle = M^2|\phi\rangle$ from the supersymmetric Hamiltonian (8) leads to the hadronic spectrum:

$$\text{Mesons}: M^2 = 4\lambda_Q(n + L) + \mu^2,$$

$$\text{Baryons}: M^2 = 4\lambda_Q(n + L + 1) + \mu^2, \qquad (29)$$

where, as we see below, the slope constant $\lambda_Q = \frac{1}{2}\lambda A$ can depend on the mass of the heavy quark. The constant term μ contains the effects of spin coupling and quark masses. This term has been derived for light hadrons in Ref. [4], yielding very satisfactory results, as well as giving clear evidence for the universality of the confinement scale λ for light quarks. More generally, we can allow for a small breaking of the supersymmetry due to the different light quark masses in the meson or nucleon, $\mu_M^2 \simeq \mu_B^2 \simeq \mu^2$. We discuss a possible extension for heavy quarks in the Appendix, but we initially treat their masses as unconstrained constants in a fit to all the heavy-light trajectories.

IV. COMPARISONS WITH DATA

In Figs. 1 and 2 we display confirmed data for the heavy-light mesons and baryons containing one charm or one bottom quark together with the trajectory fit from (29). The internal spin s in these figures refers to the total quark spin in the mesons or the spin of the diquark cluster in the baryons [4]. The results presented in Figs. 1 and 2 constitute a test of the linearity of the trajectories predicted by the SUSY holographic embedding, and it allows us to determine the dependence of the slope λ_Q on the heavy quark mass scale. The trajectory intercepts are fixed by the lowest state in each trajectory, but are determined later by the model in the Appendix. Unfortunately, the data for heavy-light hadrons are sparse, compared with those for light hadrons. Only the D/Λ_c trajectory, Fig. 1(a) provides an independent test for the predicted harmonic potential. Thus, future data on heavy-light hadrons will be essential to test the assumptions stated in Sec. I for the light-front holographic model described here.

In Fig. 3 the fitted values for $\sqrt{\lambda_Q}$ are presented for the different trajectories. In the abscissa we indicate the lowest mass meson for that meson-baryon trajectory. The triangles indicate the fitted values, and the horizontal lines show the

TABLE I. Charmed hadrons. The quark spin s is the total quark spin of the meson or the diquark cluster, λ_Q is the fitted value for the trajectory and ΔM is the difference between the observed and the theoretical value according to (29). The lowest lying meson mass determines de value of μ^2 in (29) for each trajectory. We have added predictions for unobserved states, marked with a question mark in the column ΔM, if only one superpartner has been observed and for $L_M \leq 2$, $L_B \leq 1$.

Status	Particle	$I(J^P)$	Quark content	Spin	n, L	$\sqrt{\lambda_Q}$ [GeV]	ΔM [MeV]
obs	$D(1869)$	$\frac{1}{2}(0^-)$	$c\bar{q}$	0	0, 0	0.655	0
obs	$D_1(2400)$	$\frac{1}{2}(1^+)$	$c\bar{q}$	0	0, 1	0.655	139
obs	$\Lambda_c(2286)$	$0(\frac{1}{2}^+)$	cqq	0	0, 0	0.655	4
obs	$\Lambda_c(2595)$	$0(\frac{1}{2}^-)$	cqq	0	0, 1	0.655	−36
obs	$\Lambda_c(2625)$	$0(\frac{3}{2}^-)$	cqq	0	0, 1	0.655	−6
obs	$\Lambda_c(2880)$	$0(\frac{5}{2}^+)$	cqq	0	0, 2	0.655	−59
pred	$D_2(2630)$	$\frac{1}{2}(2^-)$	$c\bar{q}$	0	0, 2	0.655	?
pred	$D_2(2940)$	$\frac{1}{2}(3^+)$	$c\bar{q}$	0	0, 3	0.655	?
obs	$D^*(2007)$	$\frac{1}{2}(1^-)$	$c\bar{q}$	1	0, 0	0.736	0
obs	$D_2^*(2460)$	$\frac{1}{2}(2^+)$	$c\bar{q}$	1	0, 1	0.736	−29
obs	$\Sigma_c(2520)$	$1(\frac{3}{2}^+)$	cqq	1	0, 0	0.736	28
pred	$D_3^*(2890)$	$\frac{1}{2}(3^-)$	$c\bar{q}$	1	0, 2	0.736	?
pred	$\Sigma_c(2890)$	$1(\frac{5}{2}^-)$	cqq	1	0, 1	0.736	?
pred	$\Sigma_c(2890)$	$1(\frac{3}{2}^-)$	cqq	1	0, 1	0.736	?
pred	$\Sigma_c(2890)$	$1(\frac{1}{2}^-)$	cqq	1	0, 1	0.736	?
obs	$D_s(1958)$	$0(0^-)$	$c\bar{s}$	0	0, 0	0.735	0
obs	$D_{s1}(2460)$	$0(1^+)$	$c\bar{s}$	0	0, 1	0.735	23
obs	$D_{s1}(2536)$	$0(1^+)$	$c\bar{s}$	0	0, 1	0.735	73
obs	$\Xi_c(2467)$	$\frac{1}{2}(\frac{1}{2}^+)$	csq	0	0, 0	0.735	31
obs	$\Xi_c(2575)$	$\frac{1}{2}(\frac{1}{2}^+)$	csq	0	0, 0	0.735	113
obs	$\Xi_c(2790)$	$\frac{1}{2}(\frac{1}{2}^-)$	csq	0	0, 1	0.735	−67
obs	$\Xi_c(2815)$	$\frac{1}{2}(\frac{3}{2}^-)$	csq	0	0, 1	0.735	−41
pred	$D_{s2}(2856)$	$0(2^-)$	$c\bar{s}$	0	0, 2	0.735	?
obs	$D_s^*(2112)$	$0(1^-)?$	$c\bar{s}$	1	0, 0	0.766	0
obs	$D_{s2}^*(2573)$	$0(2^+)?$	$c\bar{s}$	1	0, 1	0.766	−29
obs	$\Xi_c(2646)$	$\frac{1}{2}(\frac{3}{2}^+)$	csq	1	0, 0	0.766	28
obs	$D_{s3}^*(3030)$	$0(3^-)?$	$c\bar{s}$	1	0, 2	0.766	0
pred	$\Xi_c(3030)$	$\frac{1}{2}(\frac{5}{2}^-)$	csq	1	0, 1	0.766	?
pred	$\Xi_c(3030)$	$\frac{1}{2}(\frac{3}{2}^-)$	csq	1	0, 1	0.766	?
pred	$\Xi_c(3030)$	$\frac{1}{2}(\frac{1}{2}^-)$	csq	1	0, 1	0.766	?

mean over all channels of hadrons containing the same heavy-light meson. For comparison, we also give the corresponding values for a fit to the much more abundant data for light hadrons [4]. It is obvious that the dispersion of the data is significantly smaller for the case where the model is approximately constrained by conformal symmetry, as compared to the case where it is strongly broken by heavy quark masses, and only supersymmetry remains as a constraint.

All of the results for the charmed hadrons are collected in Table I; the predictions for bottom hadrons are summarized in Table II. The slopes for charm hadrons are definitely larger than those for the light hadrons, but they agree within $\pm 10\%$ for all charm hadrons. The agreement of the data with the theoretical predictions from (29) is of the same order as for light hadrons. The average deviation is 55 MeV, but the data are rather sparse. The model, however, makes predictions for higher orbital (and radial) excitations with

TABLE II. Bottom hadrons. The notation is the same as for Table I.

Status	Particle	$I(J^P)$	Quark content	Spin	n, L	$\sqrt{\lambda_Q}$ [GeV]	ΔM [MeV]
obs	$B(5279)$	$\frac{1}{2}(0^-)$	$b\bar{q}$	0	0, 0	0.963	0
obs	$B_1(5721)$	$\frac{1}{2}(1^+)$	$b\bar{q}$	0	0, 1	0.963	101
obs	$\Lambda_b(5620)$	$0(\frac{1}{2}^+)$	bqq	0	0, 0	0.963	1
obs	$\Lambda_b(5912)$	$0(\frac{1}{2}^-)$	bqq	0	0, 1	0.963	-28
obs	$\Lambda_c(5920)$	$0(\frac{3}{2}^-)$	bqq	0	0, 1	0.963	-20
pred	$B_2(5940)$	$\frac{1}{2}(2^-)$	$c\bar{q}$	0	0, 2	0.963	?
obs	$B^*(5325)$	$\frac{1}{2}(1^-)$	$b\bar{q}$	1	0, 0	1.13	0
obs	$B_2^*(5747)$	$\frac{1}{2}(2^+)$	$b\bar{q}$	1	0, 1	1.13	-45
obs	$\Sigma_b^*(5833)$	$1(\frac{3}{2}^+)$	bqq	1	0, 0	1.13	44
pred	$B_3^*(6216)$	$\frac{1}{2}(3^-)$	$c\bar{q}$	1	0, 2	1.13	?
pred	$\Sigma_b(6216)$	$1(\frac{5}{2}^-)$	cqq	1	0, 1	1.13	?
pred	$\Sigma_b(6216)$	$1(\frac{3}{2}^-)$	cqq	1	0, 1	1.13	?
pred	$\Sigma_b(6216)$	$1(\frac{1}{2}^-)$	cqq	1	0, 1	1.13	?
obs	$B_s(5367)$	$0(0^-)$	$b\bar{s}$	0	0, 0	1.11	0
obs	$B_{s1}(5830)$	$0(1^+)$	$b\bar{s}$	0	0, 1	1.11	16
obs	$\Xi_b(5795)$	$\frac{1}{2}(\frac{1}{2}^+)$	bsq	0	0, 0	1.11	-16
pred	$B_{s2}(6224)$	$0(2^-)$	$b\bar{s}$	0	0, 2	1.11	?
pred	$\Xi_b(6224)$	$\frac{1}{2}(\frac{1}{2}^-)$	bsq	0	0, 1	1.11	?
pred	$\Xi_b(6224)$	$\frac{1}{2}(\frac{3}{2}^-)$	bsq	0	0, 1	1.11	?
obs	$B_s^*(5415)$	$0(1^-)?$	$b\bar{s}$	1	0, 0	1.16	0
obs	$B_{s2}^*(5840)$	$0(2^+)?$	$b\bar{s}$	1	0, 1	1.16	-55
obs	$\Xi_b(5945)$	$\frac{1}{2}(\frac{3}{2}^+)$	bsq	1	0, 0	1.16	55
pred	$B_{s3}^*(6337)$	$0(3^-)?$	$b\bar{s}$	1	0, 2	1.16	?
pred	$\Xi_b(6337)$	$\frac{1}{2}(\frac{5}{2}^-)$	bsq	1	0, 1	1.16	?
pred	$\Xi_b(6337)$	$\frac{1}{2}(\frac{3}{2}^-)$	bsq	1	0, 1	1.16	?
pred	$\Xi_b(6337)$	$\frac{1}{2}(\frac{1}{2}^-)$	bsq	1	0, 1	1.16	?

an accuracy of approximately ± 100 MeV. The values for the mean of the modulus of deviation between theoretical and experimental values is 55 MeV, the standard error is 72 MeV; this deviation is comparable to that obtained for light hadrons [3,4]. We have added in Table I the predicted missing superpartners and all mesons with angular momentum $L_M \leq 2$ and baryons with $L_B \leq 1$.

We have omitted the Σ_c and the Σ_b baryons from the figures and the tables since it is not clear whether they should be included in the same trajectories with the pseudoscalar or the vector meson, as is discussed in more detail at the end of the Appendix.

V. THE SCALE DEPENDENCE OF λ_Q FROM HEAVY QUARK EFFECTIVE THEORY (HQET)

It has been known for a long time [33], and has been formally proved in HQET [16], that in the case of masses of

heavy mesons M_M, the product $\sqrt{M_M} f_M$ approaches, up to logarithmic terms, a finite value
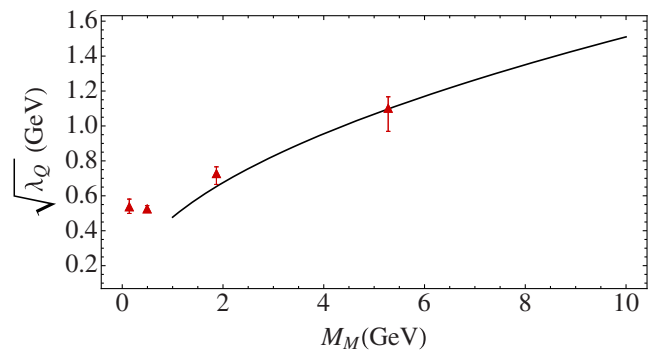
$$\sqrt{M_M} f_M \to C, \tag{30}$$

FIG. 4. The fitted value of λ_Q vs the meson mass M_M. The solid line is the square root dependence (36) predicted by HQET.

TABLE III. Leptonic decay constants. Second row: the phenomenological values; third row: theoretical values obtained from (A9) with the unmodified wave function (A4) and the fitted heavy quark masses $m_c = 1547$, $m_b = 4922$ MeV; last row: theoretical values obtained with the modified wave function with the scale factor $\alpha = \frac{1}{2}$ in (A10). The fitted masses are $m_c = 1327$, $m_b = 4572$ MeV.

Decay const. [MeV]	f_K	f_D	f_{Ds}	f_B	f_{B_s}	$\frac{f_{D_s}}{f_D}$	$\frac{f_{B_s}}{f_B}$
Phenomenology	155	212	249	187	227	1.17	1.22
Unmodified w.f.	152	127	159	81	117	1.25	1.44
Modified w.f.	$\cdots$	199	216	194	229	1.09	1.18

a relation which can also be derived using the light-front holographic approach [17]. In the present holographic framework this means that the confinement scale λ_Q has to increase with increasing quark mass. Indeed, using the results of the Appendix, we can write the decay constant f_M (A7) expressed through the wave function (A4)

$$f_M = \frac{1}{\sqrt{\int_0^1 dx \, e^{-m_Q^2/\lambda(1-x)}}} \frac{\sqrt{2N_C\lambda}}{\pi}$$
$$\times \int_0^1 dx \, e^{-m_Q^2/2\lambda(1-x)} \sqrt{x(1-x)}, \qquad (31)$$

where, for simplicity, we consider the case where $m_1 = 0$; the heavy quark mass is $m_2 = m_Q$.

We introduce $\nu^2 \equiv m_Q^2/\lambda$ and use the saddle-point method to evaluate the integral of the numerator for large values of ν^2. One expands the numerator around the value $x_0 = \frac{1}{\nu^2} + O(\frac{1}{\nu^4})$, where the integrand is maximal and obtains:

$$e^{-\frac{1}{2}\nu^2/(1-x)}\sqrt{x(1-x)}$$
$$= e^{-\nu^2/2 - \log\nu - 1/2 + O(\frac{1}{\nu})} e^{\frac{1}{4}(x-x_0)^2(m^4 + O(\nu^2))}. \qquad (32)$$

This Gaussian integral yields:

$$\int_0^1 dx \, e^{-\frac{1}{2}\nu^2/(1-x)}\sqrt{x(1-x)} = \frac{e^{-\nu^2/2}}{\sqrt{e\nu^2}} \frac{\pi}{\nu^2}\left(1 + \mathrm{erf}\left(\frac{1}{2}\right)\right). \qquad (33)$$

The integral in the denominator of (31) can be performed analytically

$$\int_0^1 dx \, e^{-\nu^2/(1-x)} = \int_1^\infty \frac{dy}{y^2} e^{-\nu^2 y}$$
$$= e^{-\nu^2} - \nu^2\Gamma\left(0, \frac{1}{\nu^2}\right)$$
$$= e^{-\nu^2}\left(\frac{1}{\nu^2} + O\left(\frac{1}{\nu^4}\right)\right). \qquad (34)$$

Thus in the large m_Q limit:

$$f_M = \sqrt{\frac{6}{e}}\left(1 + \mathrm{erf}\left(\frac{1}{2}\right)\right)\frac{\lambda^{3/2}}{m_Q^2}. \qquad (35)$$

In the limit of heavy quarks the meson mass equals the quark mass. From the HQET relation (30) it follows that

$$\lambda_Q = \mathrm{const}\, m_Q, \qquad (36)$$

where the constant in (36) has the dimension of mass. This corroborates our statement that the increase of λ_Q with increasing quark mass is dynamically necessary. In Fig. 4 we show the value of λ_Q for the π, K, D, and B mesons as function of the meson mass M_M. From the difference of the values of $\sqrt{M_M}f_M$ for the D and B mesons (see Appendix A, Table III) we must conclude that, in this region, we are still far away from the heavy quark regime. It is nevertheless remarkable that the simple functional dependence (36) derived in the heavy quark limit predicts for the c quark a value $\sqrt{\lambda_c} = 0.653$ GeV–after fixing the proportionality constant in (36) at the B meson mass, which is indeed at the lower edge of the values obtained from the fit to the trajectories (0.655 to 0.766 GeV). It makes no sense to apply HQET below the mass of the M_D. Indeed, there is no sign of an increase of $\sqrt{\lambda}$ between the π and K mass.

VI. SUMMARY AND CONCLUSIONS

In this article we have extended light-front holographic QCD to heavy-light hadrons by using the embedding of supersymmetric quantum mechanics in a modified higher dimensional space asymptotic to AdS. Remarkably, this embedding not only yields supersymmetric relations between mesons and baryons, but it also determines the superconformal potential and thus the effective potential in light-front holographic QCD. If one introduces for mesons the breaking of the maximal symmetry of AdS_5 by a dilaton term, as it is usually done, one finds that only a quadratic dilaton profile is compatible with the supersymmetric potential; thus, a harmonic LF potential again emerges, as is the case for light quark hadrons. This implies linear trajectories not only for light hadrons, but also for the heavy-light mesons and baryons. Although the experimental data are sparse, the existing data are not in contradiction with this linearity; however, future data on heavy-light

hadrons will be critical to test the dynamical assumptions described here.

In our approach the heavy quark influences the transverse degrees of freedom only indirectly by modifying the strength of the harmonic potential; this modification cannot be determined from supersymmetry. However, the dependence of the confinement scale on the heavy quark mass can be calculated in HQET, and it is in agreement with the observed increase. Indeed, HQET is compatible with the light-front holographic approach to hadron physics [17].

APPENDIX: WAVE FUNCTIONS AND DISTRIBUTION AMPLITUDES

As mentioned above, the additional term μ^2 in Eq. (29) for light hadrons was given in [4] in terms of the internal spin and the quark masses of the constituents. The spin interaction term has the simple form $2\lambda s$, where s is the quark spin of the meson or the quark spin of the diquark cluster in the baryon, respectively. There is, however a problem with the cluster spin assignment of the Σ_c and Σ_b, as explained at the end of this appendix.

In order to estimate the influence of the quark masses and also to evaluate the decay constants f_M, which play a crucial role in Sec. V, we need to have a good description of the wave functions of the hadrons. We found for a hadron with LF angular momentum L and radial excitation number n [8]

$$\psi_{n,L}^{(0)} = \frac{1}{N}\sqrt{x(1-x)}\zeta^L L_n^L(|\lambda|\zeta^2)e^{-|\lambda|\zeta^2/2}, \quad \text{(A1)}$$

with normalization

$$N = \sqrt{\frac{\pi(n+L)!}{n!}}|\lambda|^{-(L+1)/2}. \quad \text{(A2)}$$

Here, L_n^L are the associated Laguerre Polynomials, and $\zeta = \sqrt{x(1-x)}|b_\perp|$.

LFHQCD gives us no hints on the longitudinal dynamics, so we have constructed the wave function for hadrons with light quarks of mass m_i by the principle that the wave function is determined by the invariant mass of the constituents

$$\sum_{i=1}^n \frac{k_{\perp i}^2 + m_i^2}{x_i}, \quad \text{(A3)}$$

where $k_{\perp i}$ is the transverse momentum of the constituent i. This leads to the effective wave function for hadrons with small quark masses:

$$\psi_{n,L}^{(m)} = \frac{1}{N_m}e^{-\frac{1}{2\lambda}\Delta m^2}\psi_{n,L}^{(0)}, \quad \text{(A4)}$$

with

$$\Delta m^2 = \sum_{i=1}^n \frac{m_i^2}{x_i}. \quad \text{(A5)}$$

It is certainly not realistic to assume that these wave functions, derived under the assumption of small quark masses, can be simply extrapolated to heavy-light hadrons. But on the other hand, the embedding of the supersymmetric theory into modified AdS demands that the quark masses enter only indirectly through the confining (transverse) dynamics, namely by a change of the confinement scale λ. We therefore apply, in an exploratory way, the procedure developed for light quarks [8] to determine also the masses of hadrons containing a heavy quark.

According to [4] the set of constants μ^2 in (29) are given in first approximation by

$$\mu^2 = 2\lambda s + \Delta M^2[m_1, …, m_n], \quad \text{(A6)}$$

where the first term is the spin term discussed above and

$$\Delta M^2[m_1, …, m_n] = \frac{\lambda^2}{F}\frac{dF}{d\lambda}$$

with $F[\lambda] = \int_0^1 … \int x_1…x_n e^{-\frac{1}{\lambda}\Delta m^2}\delta(\sum_{i=1}^n x_i - 1)$, is the effective quadratic mass correction for hadrons used in Refs. [4,8], with $n = 2$ for mesons and $n = 3$ for baryons.

Since λ_Q has been determined in the fit to the trajectories and the light quark masses are known from the fits to light hadrons [8], the only free parameter in these formulas is the effective heavy quark mass, m_Q. For hadrons containing a charm quark, the best fit to the eight ground states of the trajectories yields $m_c = 1547$ MeV, for the bottom quark mass one obtains correspondingly $m_b = 4922$ MeV. The quality of the fit is worse than that to the trajectories; the standard deviation is 95 MeV.

A more severe test for the adequacy of the wave functions are the leptonic decay constants. The leptonic decay constant of a pseudoscalar meson M samples the light-front wave function at small distances and is a very sensitive test for the wave function. Its exact computation is given in terms of the valence light-front wave function [34,35]

$$f_M = 2\sqrt{2N_C}\int_0^1 dx\,\phi(x), \quad \text{(A7)}$$

where

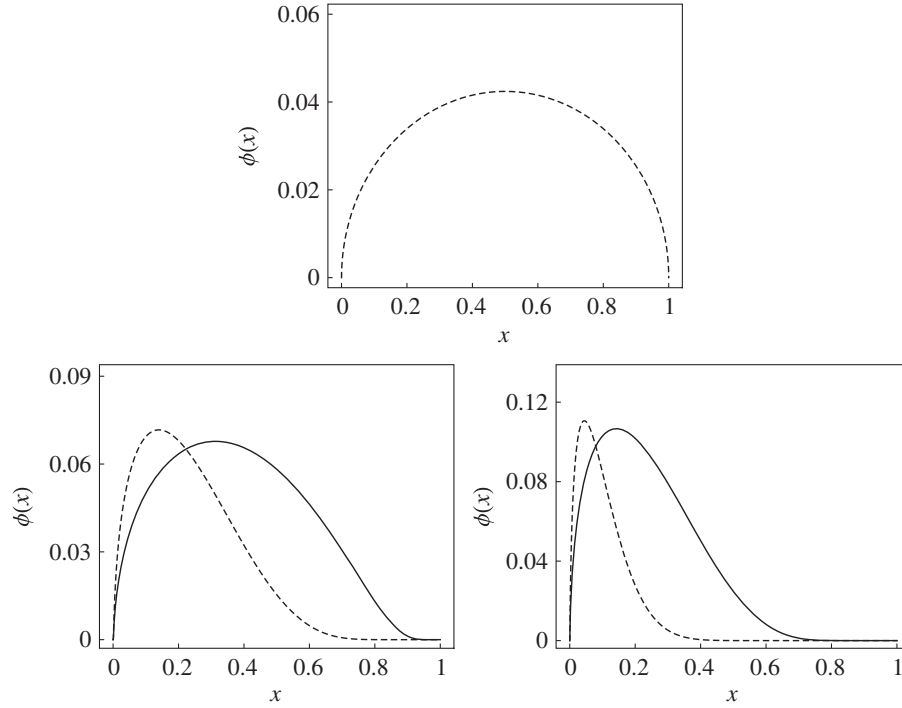$$\phi(x) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\psi(x, \mathbf{k}_\perp), \quad \text{(A8)}$$

FIG. 5. Distribution amplitudes for pseudoscalar mesons. From top to bottom: Chiral case, D meson and B meson. The dotted line for the D and B mesons is obtained with the unmodified wave function (A4), the solid line with the modified wave function with the scale factor $\alpha = \frac{1}{2}$ in (A10).

is the distribution amplitude (DA). Since $\phi(x) = \psi(x, \mathbf{b}_\perp = 0)/\sqrt{4\pi}$, we can write f_M in terms of the LFWF at zero transverse impact distance:

$$f_M = \sqrt{\frac{2N_C}{\pi}} \int_0^1 dx\, \psi(x, \mathbf{b}_\perp = 0), \qquad (A9)$$

which is identical with the result first obtained by van Royen and Weisskopf [36].

The decay constants f_M of the heavy-light mesons are not directly observable since the leptonic decay rates also depend on the matrix elements of the weak decay of heavy quarks. There are, however, many phenomenological results, notably from QCD sum rules and lattice calculations, which give a fairly consistent picture. We present in Table III, second row, the results from [32], *Leptonic decays of charged pseudoscalar mesons*. For completeness we have also included the K meson.

The results for the decay constants obtained from (A9) with the wave function (A4) are displayed in Table III, third row, "Unmodified w.f.". Though qualitative features are reproduced, the magnitude of the decay constants is grossly underestimated with increasing heavy quark mass. This is due to the fact that the heavy quark carries most of the longitudinal momentum, as it is formally expressed through the x_i dependent exponent Δm^2 (A5) in (A4). If the heavy quark mass m_2 increases, then x is pushed to very small values; this suppresses the decay constant f_M. Since this

suppression is evidently too strong, an easy remedy is to multiply the heavy quark mass in the exponential (A5) of the wave function (A4) by a factor $\alpha < 1$; thus, we modify

$$e^{-\frac{1}{2\lambda} \frac{m_Q^2}{x_Q}} \to e^{-\frac{\alpha^2}{2\lambda} \frac{m_Q^2}{x_Q}}, \qquad (A10)$$

in the LF wave function for the heavy quark with mass m_Q and longitudinal momentum x_Q.

The result for $\alpha = \frac{1}{2}$ is shown in Table III, last row, "Modified w.f.". The improvement from errors between 40% and 60% to errors between 3% and 8% is dramatic, and, most important, there is no sign of an increasing discrepancy with increasing quark mass. Since the quantity α is mass independent, it does not affect the conclusions from HQET, drawn in Sec. V, notably the relation (36); only the value of m_Q in (35) has to be multiplied by $\alpha = \frac{1}{2}$. The values for the quark masses, obtained from a fit to the data with this modified wave function, are $m_c = 1.327$ GeV and $m_b = 4.572$ GeV. The fit is slightly worse than that with the unmodified wave function (A4); the standard deviation is 125 MeV.

In Fig. 5 we show the distribution amplitude (A8) for the chiral case and for the heavy pseudoscalar mesons; the dotted lines for the heavy mesons correspond to the unmodified wave function (A4), and the solid ones are obtained from the modified wave function with the scale factor $\alpha = \frac{1}{2}$ in (A10).

The increasing discrepancy between the longitudinal momentum of the light constituents and that of the heavy quark, with increasing quark mass, could provide a plausible explanation of why the Σ_c and Σ_b do not fit on the trajectories for a pseudoscalar meson. In this case a scalar diquark cluster can be formed only by the heavy and a light quark, whereas the cluster formed of two light quarks has isopin 1 and hence quark spin 1. The trajectories for the pseudoscalar mesons are characterized by $s = 0$;

hence, they are matched to baryons of scalar diquarks. Due to the increasing difference between the longitudinal momenta, the formation of a heavy-light cluster becomes less and less probable with increasing heavy quark mass. This is also observed: The mass difference δ_M between the Σ_b^*, which must contain a spin 1 cluster, and the Σ_b is $\delta_M = 20$ MeV; in contrast, the $\Sigma_c^*(2520)$, which must contain a spin 1 cluster, and the $\Sigma_c(2455)$ is $\delta_M = 65$ MeV.

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