$QQ\bar{Q}\,\bar{Q}$ states: Masses, production, and decays

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The question of whether there exist bound states of two heavy quarks $Q = (c, b)$ and antiquarks $\bar{Q} = (\bar{c}, \bar{b})$, distinct from a pair of quark-antiquark mesons, has been debated for more than forty years. We estimate masses of $Q_1 Q_2 \bar{Q}_3 \bar{Q}_4$ resonant states $X_{Q_1 Q_2 \bar{Q}_3 \bar{Q}_4}$ and suggest a means of producing and observing them. We concentrate on the $cc\bar{c}\,\bar{c}$ channel which is most easily produced and the $bb\bar{b}\,\bar{b}$ channel which has a better chance of being relatively narrow. We obtain $M(X_{cc\bar{c}\,\bar{c}}) = 6,192 \pm 25$ MeV and $M(X_{bb\bar{b}\,\bar{b}}) = 18,826 \pm 25$ MeV, for the $J^{PC} = 0^{++}$ states involving charmed and bottom tetraquarks, respectively. An experimental search for these states in the predicted mass range is highly desirable.

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I. INTRODUCTION

The understanding of hadrons as bound states of colored quarks could accommodate mesons as $q\bar{q}$ and baryons as qqq states, but has remained mute about the possible existence of more complicated color-singlet combinations such as $qq\bar{q}\,\bar{q}$ (tetraquarks) or $q^4\bar{q}$ (pentaquarks). In the past dozen years or so, evidence has accumulated for such combinations, but it has not been clear whether they are genuine bound states, with equal roles for all constituents, or loosely bound "molecules" of two mesons or a meson and a baryon, with quarks mainly belonging to one hadron or the other.

A frequent agent for binding hadrons into molecules has been pion exchange ([1] and references therein) and, in the case in which nonstrange quarks are absent but strange quarks are present, possibly η exchange [2]. A situation in which neither is possible is a multiquark state in which all the constituents are heavy (c or b), such as $cc\bar{c}\,\bar{c}$. In comparison with states with two heavy and two light quarks, a state such as $cb\bar{c}\,\bar{b}$ has a clear advantage in binding, as the kinetic energy of its constituent quarks, scaling as the inverse of their masses, is less. The same is true for $cc\bar{c}\,\bar{c}$, but not all configurations are allowed by the Pauli principle, so the situation is less clear. Starting more than 40 years ago [3], suggestions were made for producing and observing $cc\bar{c}\,\bar{c}$ states, but there was no unanimity in whether these were above or below the lowest threshold, $2M(\eta_c) = 5967.2$ MeV, for a pair of $c\bar{c}$ mesons. (See, for example, Refs. [4–19].) We will present our own mass estimates, noting experimental strategies that might be particularly appropriate for present-day and near-future searches. We shall first discuss the lightest "heavy tetraquarks," $cc\bar{c}\,\bar{c}$, to be denoted generically as $X_{cc\bar{c}\,\bar{c}}$, as they are the easiest to produce. We will then present remarks on states $X_{bb\bar{b}\,\bar{b}}$ containing b quarks, which have a better chance of being narrow, and will briefly mention mixed states $X_{bc\bar{b}\,\bar{c}}$.

Ingredients in estimating the mass of the lightest $X_{cc\bar{c}\,\bar{c}}$ state include the charmed quark mass, the color-electric force, and the color-magnetic interaction leading to hyperfine splitting. We discuss the problems in evaluating each of them in Sec. II. In contrast to previous semiempirical approaches (e.g., [20–22] and references therein), we utilize a relation between meson and baryon masses which allows us to extrapolate to $QQ\bar{Q}\,\bar{Q}$ systems. We discuss $cc\bar{c}\,\bar{c}$ production in Sec. III and decay in Sec. IV. Section V treats states containing b quarks. Section VI contains remarks on tetraquarks with both b and c quarks, while Sec. VII summarizes.

II. ESTIMATING THE GROUND-STATE $cc\bar{c}\,\bar{c}$ MASS

A. Charmed quark mass

In estimating the masses of baryons containing two heavy quarks [22], we found the effective mass of the charmed quark in mesons to be 1663.3 MeV, while in baryons it was found to be 1710.5 MeV. The difference has been known for some time [23] and is mirrored in a similar difference in constituent-quark masses in mesons and baryons containing the light quarks u, d, and s. It was noted by Lipkin [24] that these effective masses differed by approximately the same amount for strange and nonstrange quarks. To see this in a current context, we perform a least-squares fit to five ground-state mesons and eight ground-state baryons. The results, shown in Table I, imply mass differences $m_q^b - m_q^m = 55.1$ MeV and $m_s^b - m_s^m = 54.5$ MeV. The square root of the average mean-squared

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TABLE I. Quark model description of ground-state hadrons containing u, d, s. A least-squares fit to mesons gives $m_u^m = m_d^m \equiv m_q^m = 308.6$, $m_s^m = 481.8$, $b/(m_q^m)^2 = 78.7$ MeV, while a fit to baryons gives $m_u^b = m_d^b \equiv m_q^b = 363.7$, $m_s^b = 536.3$, and hyperfine interaction term $a/(m_q^b)^2 = 49.3$ MeV.

State (mass in MeV)	Spin	Expression for mass [23]	Predicted mass (MeV)
$\pi(138)$	0	$2m_q^m - 6b/(m_q^m)^2$	145.3
$\rho(775)$, $\omega(782)$	1	$2m_q^m + 2b/(m_q^m)^2$	774.6
$K(496)$	0	$m_q^m + m_s^m - 6b/(m_q^m m_s^m)$	488.1
$K^*(894)$	1	$m_q^m + m_s^m + 2b/(m_q^m m_s^m)$	891.2
$\phi(1019)$	1	$2m_s^m + 2b/(m_s^m)^2$	1028.1
$N(939)$	1/2	$3m_q^b - 3a/(m_q^b)^2$	943.3
$\Delta(1232)$	3/2	$3m_q^b + 3a/(m_q^b)^2$	1239.0
$\Lambda(1116)$	1/2	$2m_q^b + m_s^b - 3a/(m_q^b)^2$	1115.9
$\Sigma(1193)$	1/2	$2m_q^b + m_s^b + a/(m_q^b)^2 - 4a/m_q^b m_s^b$	1179.4
$\Sigma(1385)$	3/2	$2m_q^b + m_s^b + a/(m_q^b)^2 + 2a/m_q^b m_s^b$	1379.9
$\Xi(1318)$	1/2	$2m_s^b + m_q^b + a/(m_s^b)^2 - 4a/m_q^b m_s^b$	1325.4
$\Xi(1530)$	3/2	$2m_s^b + m_q^b + a/(m_s^b)^2 + 2a/m_q^b m_s^b$	1525.9
$\Omega(1672)$	3/2	$3m_s^b + 3a/(m_s^b)^2$	1677.0

error in the fit is 6.72 MeV, for a six-parameter fit to thirteen data points.

The near equality of nonstrange and strange quark mass differences between mesons and baryons suggests a simpler fit with universal quark masses for mesons and baryons but with a constant S added to baryon masses. The results of this fit are shown in Table II. The quality of this fit is nearly identical to that of the fit with separate quark masses for mesons and baryons. The square root of the average mean-squared error is 6.73 for a five-parameter fit to thirteen data points.

One can motivate the addition of a universal constant for baryon masses in a QCD-string-junction picture [25]. A quark-antiquark meson contains a single QCD string connecting a color triplet with an antitriplet. A three-quark baryon contains three triplet strings, each leading to the same junction. Thus, the added term S may be thought of as representing the contribution of two additional QCD strings and one junction. (Fig. 1.)

Now consider the baryonium (tetraquark) state consisting of two quarks and two antiquarks, illustrated in Fig. 1(c). It contains five QCD strings and two junctions, so one would expect an additional additive contribution of S with respect to a baryon or $2S$ with respect to a meson. There will be additional contributions from binding effects and spin-dependent interactions, like those considered in Ref. [22].

We estimate the charmed quark mass using $M(\Lambda_c) = S + 2m_q + m_c - 3a/m_q^2 = 2286.5$ MeV, obtaining $m_c = 1655.6$ MeV. This is only slightly different from the value

TABLE II. Quark model description of ground-state mesons and baryons containing u, d, s, with universal quark masses for mesons and baryons but a constant term $S = 165.1$ MeV added to baryon masses. A least-squares fit gives $m_u^m = m_d^m \equiv m_q = 308.5$, $m_s = 482.2$, $a/m_q^2 = 50.4$, $b/m_q^2 = 78.8$ MeV.

State (mass in MeV)	Spin	Expression for mass	Predicted mass (MeV)
$\pi(138)$	0	$2m_q - 6b/(m_q)^2$	144.0
$\rho(775)$, $\omega(782)$	1	$2m_q + 2b/(m_q)^2$	774.8
$K(496)$	0	$m_q + m_s - 6b/(m_q m_s)$	488.0
$K^*(894)$	1	$m_q + m_s + 2b/(m_q m_s)$	891.6
$\phi(1019)$	1	$2m_s + 2b/(m_s)^2$	1028.9
$N(939)$	1/2	$S + 3m_q - 3a/(m_q)^2$	939.4
$\Delta(1232)$	3/2	$S + 3m_q + 3a/(m_q)^2$	1242.1
$\Lambda(1116)$	1/2	$S + 2m_q + m_s - 3a/m_q^2$	1113.1
$\Sigma(1193)$	1/2	$S + 2m_q + m_s + a/m_q^2 - 4a/m_q m_s$	1185.7
$\Sigma(1385)$	3/2	$S + 2m_q + m_s + a/m_q^2 + 2a/m_q m_s$	1379.4
$\Xi(1318)$	1/2	$S + 2m_s + m_q + a/m_s^2 - 4a/m_q m_s$	1329.5
$\Xi(1530)$	3/2	$S + 2m_s + m_q + a/m_s^2 + 2a/m_q m_s$	1523.2
$\Omega(1672)$	3/2	$S + 3m_s + 3a/m_s^2$	1673.6

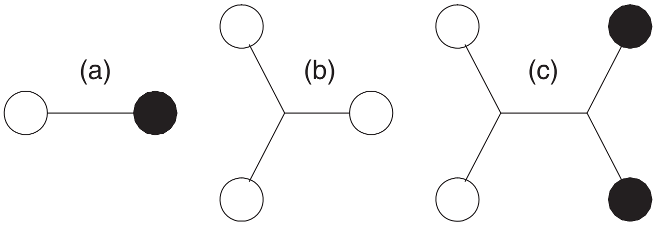

FIG. 1. QCD strings connecting quarks (open circles) and antiquarks (filled circles). (a) Quark-antiquark meson with one string and no junctions; (b) Three-quark baryon with three strings and one junction; (c) Baryonium (tetraquark) with five strings and two junctions.

obtained from mesons and will be used henceforth. As a cross-check, we calculate the mass of $\Sigma_c(2454)$ [26] to be

$$M(\Sigma_c(2454)) = S + 2m_q + m_c + \frac{a}{m_q^2} - \frac{4a}{m_q m_c} = 2450.5\ \text{MeV}, \quad (1)$$

to be compared with 2444 MeV in Ref. [22].

B. Effects of interactions

In this subsection, we investigate the mass of a $cc\bar{c}\,\bar{c}$ state in which the cc ($\bar{c}\,\bar{c}$) forms an S-wave color 3^* (3), necessarily with spin 1 by the Pauli principle. We follow a discussion parallel to that in Ref. [22]. There we needed to evaluate the mass of a QQ color-3^* spin-1 diquark. We noted that the binding energy for a QQ color-3^*, by QCD, was half that of a $Q\bar{Q}$ color singlet. (The picture of a diquark-antidiquark system $Qq\bar{Q}\,\bar{q}$ involving two heavy quarks Q and two light quarks q has been used to describe exotic states such as $X(3872)$, e.g., in Refs. [27,28].) The spin-averaged $1S$ charmonium mass, updating inputs based on [26], is [22]

$$\bar{M}(c\bar{c}; 1S) = [3M(J/\psi) + M(\eta_c)]/4 = 3068.5\ \text{MeV}, \quad (2)$$

so the $c\bar{c}$ (spin-averaged) binding energy in a color singlet is

$$B(c\bar{c}, 1) = [3068.6 - 2(1655.6)]\ \text{MeV} = -242.7\ \text{MeV}, \quad (3)$$

and the cc (spin-averaged) binding energy in a color 3^* is half that, or -121.3 MeV. The hyperfine interaction between two c quarks in an S-wave spin-1 color 3^* was estimated in Ref. [22] to be $a_{cc}/m_c^2 = 14.2$ MeV. The effective mass of a cc spin-1 color-3^* diquark is then

$$M(cc, 3^*) = [2(1655.6) - 121.3 + 14.2]\ \text{MeV} = 3204.1\ \text{MeV}. \quad (4)$$

We next seek the binding energy of a cc color-antitriplet diquark with a $\bar{c}\,\bar{c}$ color-triplet antidiquark. For this, we will interpolate between the 1S binding energies of $c\bar{c}$ and $b\bar{b}$, implicitly assuming that the doubly heavy diquarks are almost pointlike, as is the case for $m_Q/\Lambda_{\text{QCD}} \to \infty$. This approximation, while not perfect, provides a concrete physically-motivated prescription for estimating the strength of the binding between two diquarks.

We have already evaluated the 1S binding energy of $c\bar{c}$ to be -242.6 MeV. To perform a comparable calculation for $b\bar{b}$, we retrace steps in Ref. [22]. We first need an estimate for the b quark mass. We use $M(\Lambda_b) = S + 2m_q + m_b - 3a/m_q^2 = 5619.5$ MeV, obtaining $m_b = 4988.6$ MeV. This is only slightly less than the value of 5003.8 MeV obtained from mesons in Ref. [22]. It gives $M(\Sigma_b(5813))$ [26] $= S + 2m_q + m_b + a/m_q^2 - 4a/(m_q m_b) = 5808.6$ MeV, to be compared with 5805 MeV obtained in Ref. [22]. With

$$\bar{M}(b\bar{b}; 1S) = [3M(\Upsilon(1S)) + M(\eta_b(1S))]/4 = 9445.0\ \text{MeV}, \quad (5)$$

the $b\bar{b}$ (spin-averaged) binding energy in a color singlet is

$$B(b\bar{b}, 1) = [9445.0 - 2(4988.6)]\ \text{MeV} = -532.2\ \text{MeV}. \quad (6)$$

We can interpolate between this binding energy and that for the charmonium to find the binding energy B for an antitriplet and triplet, each of mass 3204.1 MeV. Assuming a power-law dependence of binding energy B with constituent mass M, $B_1/B_2 = (M_1/M_2)^p$, using $M_1 = 1655.6$, $M_2 = 4988.6$, $B_1 = -242.7$, and $B_2 = -532.2$ MeV, we find $p = 0.7120$ and $B_3 = B_{(cc)(\bar{c}\,\bar{c})} = -388.3$ MeV for $M_3 = M(cc, 3^*) = 3204.1$ MeV.

Finally, the spin-spin force between the spin-1 diquark cc and the spin-1 antidiquark $\bar{c}\,\bar{c}$ may be estimated by interpolation between the hyperfine splittings ΔM for $c\bar{c}$ and $b\bar{b}$ S-wave ground states. We assume $\Delta M_1/\Delta M_2 = (M_1/M_2)^q$, using $M_1 = 1655.6$, $M_2 = 4988.6$, $\Delta M_1 = 113.5$, and $\Delta M_2 = 62.3$ MeV. The power law is found to be $q = -0.5438$. Then for a pair of spin-1/2 quarks each of mass $M_3 = 3204.1$ MeV, we would find $\Delta M_3 = 79.3$ MeV.

We assume the spin-dependent hyperfine splitting is of the form $A\langle S_1 \cdot S_2\rangle$, where

$$\langle S_1 \cdot S_2\rangle = \frac{1}{2}[S(S+1) - S_1(S_1+1) - S_2(S_2+1)], \quad (7)$$

where S is the total spin and $S_{1,2}$ are the spins of the constituents. For $S_1 = S_2 = 1/2$, the splitting between

$S = 1$ and $S = 0$ states is A, which we identify as the term $\Delta M_3 = 79.3$ MeV found above. For $S_1 = S_2 = 1$, the lowest-mass state, with $S = 0$, lies $2A = 158.5$ MeV below the value without hyperfine interaction.

Putting the terms together, we find the mass of the lowest-lying $cc\bar{c}\,\bar{c}$ state in this configuration (with $J^{PC} = 0^{++}$) to be

$$\begin{aligned} M(X_{cc\bar{c}\,\bar{c}}[0^{++}]) &= 2S + 2M_{cc} + B_{(cc)(\bar{c}\,\bar{c})} + \Delta M_{HF} \\ &= [2(165.1) + 2(3204.1) - 388.3 \\ &\quad - 158.5]\ \text{MeV} \\ &= 6191.5\ \text{MeV}. \end{aligned} \tag{8}$$

This lies just below the $J/\psi J/\psi$ threshold (6193.8 MeV) and cannot decay to $J/\psi\eta_c$ (threshold 6080.5 MeV) by virtue of angular momentum and parity conservation. However, it can decay to $\eta_c\eta_c$ (threshold 5966.8 MeV) and thus is unlikely to be narrow. We assign an error of ± 25 MeV to this estimate, multiplying by two the error [22] expected in estimation of QQq masses.

C. Color-spin calculation

The preceding analysis, based on the string-junction physical picture, suggests that the $cc\bar{c}\,\bar{c}$ tetraquark is likely to be above the $\eta_c\eta_c$ threshold. Since this is the crucial issue here, it is useful to do a cross-check with the help of another approach, namely color-spin $SU(6)$.

The dynamics of exotic combinations of quarks and antiquarks was examined by combining the color $SU(3)$ and spin $SU(2)$ groups into a color-spin $SU(6)$ [29,30]. (Particular attention was paid to the $qq\bar{q}\,\bar{q}$ baryon-antibaryon resonances [31], as proposed in [32].)

Since the total chromoelectric interaction should not depend on the individual color groupings of the constituents [33,34], color-spin may be employed to compare the binding energies of various $QQ\bar{Q}\,\bar{Q}$ states, where Q is a heavy quark which will be taken to be c in the following.

Neglecting effects in which QQ and $Q\bar{Q}$ have different relative wave functions, the spin-dependent force Δ may be expressed in terms of Pauli spin matrices $\vec{\sigma}$ and $SU(3)$ generators λ^a $(a = 1, \ldots, 8)$ as

$$\begin{aligned} \Delta &= -\sum_a^8 \sum_{i>j} \vec{\sigma}_i \cdot \vec{\sigma}_j \lambda_i^a \lambda_j^a \\ &= 8N + \frac{1}{2}C_6(\text{tot}) - \frac{4}{3}S_{\text{tot}}(S_{\text{tot}}+1) \\ &\quad + C_3(Q) + \frac{8}{3}S_Q(S_Q+1) - C_6(Q) + C_3(\bar{Q}) \\ &\quad + \frac{8}{3}S_{\bar{Q}}(S_{\bar{Q}}+1) - C_6(\bar{Q}), \end{aligned} \tag{9}$$

where N is the total number of quarks, and C_3 and C_6 are quadratic Casimir operators of $SU(3)$ and $SU(6)$, whose relevant values are given in Tables III and IV. (We use the normalization of Ref. [30].)

TABLE III. Quadratic Casimir operators for $SU(3)$ representations.

$SU(3)$ rep	C_3
1	0
3	16/3
6	40/3
8	12

We first calculate Δ for the η_c, as we will be looking for a configuration which is more deeply bound than two η_cs. Here, $N = 2$, while the deepest binding is achieved in an $SU(6)$ singlet with $C_6(1) = 0$ and $S_{\text{tot}} = 0$. The terms describing individual quarks are

$$C_3(3) + \frac{8}{3}S_c(S_c+1) - C_6(c) = \frac{16}{3} + 2 - \frac{70}{3} = -16 \tag{10}$$

with a similar term for $\bar{c}$, so

$$\Delta(\eta_c) = 8(2) - 2(16) = -16; \quad \Delta(2\eta_c) = -32. \tag{11}$$

A corresponding calculation may be made in which cc $(\bar{c}\,\bar{c})$ are first combined into diquarks (antidiquarks). The color-spin of cc in the ground state must be antisymmetric by Fermi statistics, so the $SU(6)$ representation of the cc ground state must be $15 = (6 \times 6)_A$. The allowed $SU(6)$ representations are then $(15 \times \overline{15}) = 1 + 35 + 189$. Here, as before, the deepest binding is achieved with $C_6(\text{tot}) = S_{\text{tot}} = 0$, while the terms for individual quarks depend on which $SU(3)$, $SU(2)$ representations of the $SU(6)$ 15-plet are chosen:

$$15 = (3^*, S = 1) + (6, S = 0). \tag{12}$$

For $(3^*, S = 1)$, we have

$$C_3(3^*) + \frac{8}{3}S(S+1) - C_6(15) = \frac{16}{3} + \frac{16}{3} - \frac{112}{3} = -\frac{80}{3}, \tag{13}$$

TABLE IV. Quadratic Casimir operators for $SU(6)$ representations.

$SU(6)$ rep	C_6
1	0
6	70/3
15	112/3
21	160/3
35	48
189	80

with a similar term for antiquarks, while for $(6, S = 0)$, we have

$$C_3(6) + \frac{8}{3} S(S+1) - C_6(15) = \frac{40}{3} - \frac{112}{3}, \quad (14)$$

which is less negative and, hence, disfavored.[1]

The final result for this configuration is

$$\Delta = 8(4) - 2\frac{80}{3} = -\frac{64}{3}, \quad (15)$$

which is less deeply bound than two η_cs. This supports our previous estimate.

As a caveat, one should note that the color-spin approach ignores the distance between diquarks; everything depends only on the color-spin algebra. From comparison of the $\bar{c}c$ and $\bar{b}b$ quarkonia, we know that this is an oversimplification. In fact, the radii and the binding energies of these states exhibit significant dependence on the quark mass, as utilized in Sec. II B above. So the color-spin approach should be viewed as qualitative, while the numbers coming from the spin-junction approach are likely to be more reliable.

D. Configuration mixing

As noted above, the total chromoelectric interaction should not depend on the individual color groupings of the constituents. Thus, we may count ways of coupling two color triplets and two antitriplets in several ways, but should end up with the same result. In the previous subsection, we coupled cc to a color 3^* and $\bar{c}\,\bar{c}$ to a color 3, then forming an overall singlet in the product $3^* \times 3 = 1 + 8$. We could also have coupled cc to a color 6 and $\bar{c}\,\bar{c}$ to a color 6^*, then forming an overall singlet in the product $6 \times 6^* = 1 + 8 + 27$. An explicit calculation using Casimir operators verifies that the chromoelectric interaction is the same for these two groupings. Residual interactions may split these two configurations.

A different grouping is obtained by combining each c with a $\bar{c}$. One can form an overall singlet again in two ways. Combining each c with a $\bar{c}$ in a color singlet, it is clear the final $(c\bar{c})(c\bar{c})$ state is a singlet. Two η_cs represent the lowest-lying $cc\bar{c}\,\bar{c}$ state in this configuration. Combining each c with a $\bar{c}$ in a color octet, one can form another overall singlet in the product $8 \times 8 = 1 + \ldots$.

One can represent the twofold nature of couplings to an overall singlet by placing c and $\bar{c}$ quarks at alternating vertices of a square, as shown in Fig. 2. One can draw QCD strings either (a) vertically or (b) horizontally. The incorporation of spins is simplest for the case in which all spins are pointed in the same direction. This represents two

[1]This confirms the assumption used in Sec. II B that the diquarks are antitriplets of color and have spin 1.

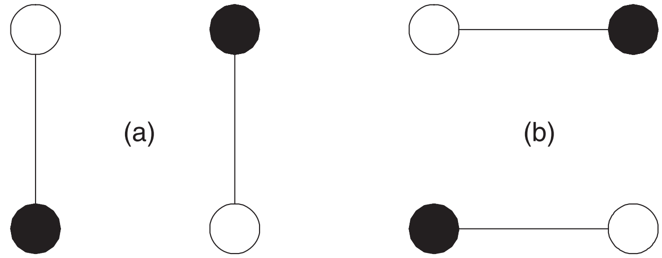

FIG. 2. Configurations of two quarks (open circles) and two antiquarks (filled circles) at alternate vertices of a square. QCD strings may run either (a) vertically or (b) horizontally.

parallel J/ψ states coupled up to a total spin 2. Tunneling between the two configurations then will ensure mixing such that one eigenstate has a mass greater than $2M(J/\psi)$, while the other has a mass less than $2M(J/\psi)$. This is not enough to ensure a small decay width for the lighter state as it may still be heavier than $2M(\eta_c)$, but its $J^{PC} = 2^{++}$ will force the $\eta_c\eta_c$ decay to be D-wave and, thus, suppressed. Its decay to $\eta_c J/\psi$ will be forbidden by charge conjugation invariance. One should bear in mind that the 2^{++} state might well not turn out to be the lowest-mass $cc\bar{c}\,\bar{c}$ resonance; we have focused on it just for the sake of simplicity.

If Fig. 2(a) represents a pair of η_cs, (b) will contain admixtures of other states, so the effect of tunneling between the two configurations is not as easy to evaluate. Lattice gauge theory may be of some help here.

A configuration related to that in Fig. 2 is possible in the binding of two positronium atoms to one another. It was first proposed by Wheeler [35] and verified by a variational calculation in Ref. [36]. Subsequent calculations ([37] and references therein) zeroed in on a binding energy of 0.435 eV, which is ~6% of the binding energy of positronium [0.68 eV = (1/2)Ry]. Finally, this state was indeed produced by Cassidy and Mills [38]; Ref. [39] discusses its excitation and contains further references.

The analogous situation for two quarkonium states is worth considering. In the limit of very heavy quarks, the binding is dominated by the chromoelectric Coulomb force. The existence of "dipositronium" thus implies that an analog diquarkonium state exists even though it need not have the specific color network structure assumed for tetraquarks. Charmed quarks are probably not heavy enough for this argument to hold, but we shall explore it for bottom quarks in a subsequent section.

III. PRODUCTION OF $cc\bar{c}\,\bar{c}$ STATES

A. States X accompanying J/ψ in $e^+e^- \to J/\psi X$

The strong production of a pair of heavy quarks Q occurs at some cost, depending on the process. As an example, consider the reaction $e^+e^- \to c\bar{c}$, whose cross section far above threshold is 4/3 that for muon pair production: at a center-of-mass energy $\sqrt{s}$,

$$\sigma(e^+e^- \to c\bar{c}) = \frac{4}{3}\left(\frac{4\pi\alpha^2}{3s}\right)\left(1-\frac{4m_c^2}{s}\right)^{1/2}\left(1+\frac{2m_c^2}{s}\right) \simeq 1\ \text{nb} \quad \text{at}\ \sqrt{s} = 10.6\ \text{GeV}. \tag{16}$$

In $e^+e^- \to J/\psi X$, the mass $M(X)$ shows peaks at states with $J^{PC} = 0^{\pm +}$: notably $\eta_c(2984)$, $\chi_{c0}(3415)$, $\eta_c(3639)$, and $X(3940)$ [40–42], as well as a continuum above $D\bar{D}$ threshold. The inclusive cross section for $e^+e^- \to J/\psi c\bar{c}$ at $\sqrt{s} = 10.6$ GeV is about 0.9 pb [40,41] and dominates the inclusive J/ψ production cross section [41]:

$$\frac{\sigma(e^+e^- \to J/\psi c\bar{c})}{\sigma(e^+e^- \to J/\psi X)} = 0.59^{+0.15}_{-0.13} \pm 0.12 \tag{17}$$

Thus, very roughly, the probability for producing a $c\bar{c}$ pair when one is already present is about 10^{-3}. We may use this figure in comparing, say, production of the tetraquark $c\bar{c}c\bar{c}$ with that of a typical quarkonium state.

The somewhat counterintuitively large ratio on the right-hand side of Eq. (17) can be understood as follows. If J/ψ is produced at high e^+e^- energy, its c and $\bar{c}$ are unlikely to have come from the same primary photon, so there tends to be another $c\bar{c}$ pair around. A smaller probability is associated with a final c and $\bar{c}$ both connected to the initial photon, with light hadrons coupling to the c and/or $\bar{c}$ by gluons.

B. Inclusive double charm production at the LHC

The LHCb Collaboration has measured prompt charm production at the CERN Large Hadron Collider (LHC) [43]. In the kinematic range studied, $2.0 < y < 4.5$ and $1 < p_T < 8$ GeV/c, they report a total cross section for charm production of about 1 mb at $\sqrt{s} = 5$ TeV (see their Fig. 10) and about twice that at $\sqrt{s} = 13$ TeV. Doubling these values for the contribution of $-4.5 < y < -2.0$ and accounting for central production contributions of similar order, one estimates $\sigma(pp \to c\bar{c}X) \simeq 5$–10 mb at $\sqrt{s} = 13$ TeV. Now one applies the estimate of the previous subsection, that an additional charm pair appears with a probability of about 10^{-3}, to estimate

$$\sigma(pp \to cc\bar{c}\,\bar{c}) \simeq 5\text{–}10\ \mu\text{b} \quad \text{at}\ \sqrt{s} = 13\ \text{TeV}. \tag{18}$$

These four quarks would form a tetraquark state with low probability. If produced by an initial gluon, each $c\bar{c}$ pair has a low effective mass, typically not more than several (e.g., 4) times m_c, whereas, if correlated in a tetraquark state, the relative effective mass of each pair should be within $\Lambda_{\text{QCD}} \sim 200$ MeV of $2m_c$. Thus, we estimate a suppression factor of $(\sim 6/0.2)^2 \sim 10^3$ from demanding these correlations. In addition, the two $c\bar{c}$ pairs must be close to one another in rapidity space to be accommodated in a resonant state. Let us assume this costs another suppression factor of at least 10. Then we would obtain a cross section for tetraquark production of no more than 1 nb at $\sqrt{s} = 13$ TeV, and less at lower energies. One might expect this estimate to be accurate, give or take a factor of 3.

This crude estimate can be checked by noting the cross section for double charmonium production, which has been measured by ATLAS [44], CMS [45], and LHCb [46]. These results are summarized in Table V. Very roughly, one may quadruple the LHCb result to account for the full rapidity range $-4.5 < y < 4.5$ to estimate

$$\sigma(pp \to J/\psi J/\psi X) \simeq 20\ \text{nb} \quad \text{at}\ \sqrt{s} = 7\ \text{TeV}. \tag{19}$$

Now one may use an estimate (see Sec. III B of Ref. [15]) that the ratio of tetraquark to J/ψ pair production by two gluons is 3.5% to conclude that the tetraquark production cross section in proton-proton collisions at $\sqrt{s} = 7$ TeV is about 0.7 nb. This is consistent with our very rough estimate above.

IV. DECAYS OF $cc\bar{c}\,\bar{c}$ STATES

A number of final states are accessible to decays of a $cc\bar{c}\,\bar{c}$ resonance. Some of these are summarized in Table VI. Here ℓ stands for any charged lepton (e, μ, τ), h stands for any hadron, and g stands for a gluon. Invariances under spin, parity, and charge conjugation may suppress certain final states.

If the lowest $c\bar{c}c\bar{c}$ tetraquark mass exceeds $2M(\eta_c) = (5967.2 \pm 1.4)$ MeV, such a state will decay primarily into the decay products of any open charmonium pair channel. Thus, for example, a 6000 MeV $c\bar{c}c\bar{c}$ tetraquark with

TABLE V. Double J/ψ production at the LHC.

Experiment	$\sqrt{s}$	y range	p_T range	σ
ATLAS [44]	8 TeV	$\lvert y\rvert < 2.1$	>8.5 GeV/c	$160 \pm 12 \pm 14 \pm 2 \pm 3$ pb
CMS [45]	7 TeV	$\lvert y\rvert < 1.2$	>6.5 GeV/c	
	7 TeV	$1.2 < \lvert y\rvert < 1.43$	(a)	
	7 TeV	$1.43 < \lvert y\rvert$	>4.5 GeV/c	$1.49 \pm 0.07 \pm 0.13$ nb
LHCb [46]	7 TeV	$2.0 < y < 4.5$	<10 GeV/c	$5.1 \pm 1.0 \pm 1.1$ nb

(a) p_T scaled linearly from 6.5 to 4.5 GeV/c.

TABLE VI. Some final states accessible to decays of a $cc\bar{c}\,\bar{c}$ resonance $X_{cc\bar{c}\,\bar{c}}$.

Subprocess	Resulting final state	Maximum kinetic energy available
$2(c\bar{c}\to\gamma\gamma)$	$\gamma\gamma$	$M(X_{cc\bar{c}\,\bar{c}})$
$c_1\bar{c}_1\to\gamma(\gamma)$, $c_2\bar{c}_2\to\ell^+\ell^-$	$\gamma(\gamma)\ell^+\ell^-$	$M(X_{cc\bar{c}\,\bar{c}})-2M(\ell)$
$c_1\bar{c}_1\to\ell_1^+\ell_1^-$, $c_2\bar{c}_2\to\ell_2^+\ell_2^-$	$\ell_1^+\ell_1^-\ell_2^+\ell_2^-$	$M(X_{cc\bar{c}\,\bar{c}})-2M(\ell_1)-2M(\ell_2)$
$c_1\bar{c}_1\to\gamma(\gamma)$, $c_2\bar{c}_2\to h^+h^-$	$\gamma(\gamma)h^+h^-$	$M(X_{cc\bar{c}\,\bar{c}})-2M(h)$
$2(c\bar{c})\to gg$	Light hadrons	$M(X_{cc\bar{c}\,\bar{c}})-2M(\pi)$
$c\bar{c}\to\gamma$	$\eta_c\gamma$ or $J/\psi\gamma$	$M(X_{cc\bar{c}\,\bar{c}})-M(\eta_c)$ or $M(X_{cc\bar{c}\,\bar{c}})-M(J/\psi)$
$c\bar{c}\to\gamma$	$D\bar{D}\gamma$	$M(X_{cc\bar{c}\,\bar{c}})-2M(D)$
$c\bar{c}\to\ell^+\ell^-$	$\ell^+\ell^-D\bar{D}$	$M(X_{cc\bar{c}\,\bar{c}})-2M(\ell)-2M(D)$
$c\bar{c}\to q\bar{q}$	$D\bar{D}+$ anything	$M(X_{cc\bar{c}\,\bar{c}})-2M(D)$
Rearrangement	$2\eta_c$	$M(X_{cc\bar{c}\,\bar{c}})-2M(\eta_c)$

$J^{PC}=0^{++}$ may be expected to have primarily the decay products of two η_c mesons.

If $M(X_{cc\bar{c}\,\bar{c}}[0^{++}])$ is less than $2M(\eta_c)$, the main decay products will involve the subprocess $c\bar{c}\to g^*\to q\bar{q}$, illustrated in Fig. 3. All other processes are higher order in the strong interactions or involve at least one electromagnetic interaction.

The rate for the process illustrated in Fig. 3 may be crudely estimated by comparing it with the rate for leptonic decay of the J/ψ [26]:

$$\Gamma(J/\psi\to e^+e^-)=(5.55\pm0.24\pm0.02)\ \text{keV}. \qquad (20)$$

Leaving aside group-theoretic factors of order 1, and assuming the wave function at the origin in the tetraquark for $c\bar{c}\to g^*\to q\bar{q}$ is about the same as for $c\bar{c}\to\gamma^*\to e^+e^-$, the rate for the process of Fig. 3 is approximately $(\alpha_s/\alpha)^2$ times that of the leptonic decay process (20). Taking $\alpha_s=0.35$ at a scale m_c (cf. Ref. [47] for a measurement at m_τ), we have $(\alpha_s/\alpha)^2\simeq2300$ or

$$\Gamma(X_{cc\bar{c}\,\bar{c}})\simeq13\ \text{MeV}. \qquad (21)$$

Other less significant modes include

$$c\bar{c}c\bar{c}\to gg\to \text{light hadrons}\quad\text{or}\quad c\bar{c}c\bar{c}\to\gamma^*\gamma^*, \qquad (22)$$

where the virtual photons will materialize into lepton or hadron pairs. The relative branching fractions to gluon or virtual photon pairs will depend on details of color-spin groupings: A color-octet $c\bar{c}$ pair with $J=1$ will decay to a gluon, while a color-singlet $c\bar{c}$ pair with $J=1$ will decay to a virtual photon. Decays of $J=0$ $c\bar{c}$ pairs will involve more than one gluon and/or virtual photon.

Although the expected branching fractions are likely to be small, the channels in which both virtual photons materialize as lepton pairs are worth investigating. One should see final states of $2\tau^+2\tau^-$, $\tau^+\tau^-\mu^+\mu^-$, $\tau^+\tau^-e^+e^-$, $2\mu^+2\mu^-$, $\mu^+\mu^-e^+e^-$, and $2e^+2e^-$ in the ratio 1:2:2:1:2:1. In that case, there will also be channels in which one or both virtual photons materialize as hadrons containing u, d, and s quarks, with well-defined branching ratios.

A crude estimate of the 4-lepton branching fraction of a $c_1c_2\bar{c}_1\bar{c}_2$ tetraquark may be made as follows. The partial width for the $c_1\bar{c}_1$ pair in a color singlet $^3S_1$ ground state to decay to an e^+e^- pair is just $\Gamma(J/\psi\to e^+e^-)=5.5$ keV [Eq. (20)]. In a tetraquark with $J^{PC}=0^{++}$ (expected to be lightest), this leaves the remaining $c_2\bar{c}_2$ pair also in a color singlet $^3S_1$ state. If $c_1\bar{c}_1$ is sufficiently off-shell, there will remain enough phase space for $c_2\bar{c}_2$ to decay not only to a lepton pair [with partial width (20)] but also a pair of charmed mesons (if the effective mass of $c_2\bar{c}_2$ is above $2M(D)=3.73$ GeV). Comparing these two channels, we see that the charmed meson pair decay is likely to have a partial width of order tens of MeV, or about 10^3 that of the decay to a pair of charged leptons, unless the penalty for $c_1\bar{c}_1$ being off-shell is very great. In that case, the factor of 10^3 might be replaced by a quantity as small as unity. Taking account of the total width estimate (21), one then estimates

$$\begin{aligned}\mathcal{B}(X_{cc\bar{c}\,\bar{c}}[0^{++}]&\to\ell_1^+\ell_1^-\ell_2^+\ell_2^-)\\&=(1\text{ to }10^{-3})(11\ \text{keV})/(13\ \text{MeV})\\&=\mathcal{O}(10^{-3}\text{ to }10^{-6}).\end{aligned} \qquad (23)$$

The higher branching ratio would very likely involve at least one lepton pair with mass J/ψ. If there is a $cc\bar{c}\,\bar{c}$ tetraquark below $2M(\eta_c)$ (unlikely in our opinion), the cross section for its production and observation in the four-lepton mode at the LHC (13 TeV) is estimated to lie in the range of 1 fb–1 pb.

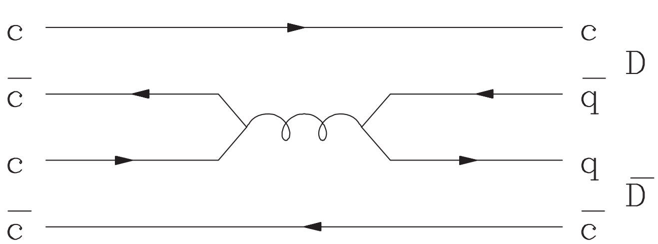

FIG. 3. Lowest-order process governing decay of a $cc\bar{c}\,\bar{c}$ resonance whose mass is below $2M(\eta_c)$.

V. STATES CONTAINING BOTTOM QUARKS

A. Ground state mass estimate

The threshold for a "fall-apart" decay of a $bb\bar{b}\,\bar{b}$ tetraquark with $J^{PC} = 0^{++}$ or 2^{++} is $2M(\eta_b) = (18,798 \pm 4.6)$ MeV. For one estimate of the mass of the lowest $bb\bar{b}\,\bar{b}$ state, we can repeat the $cc\bar{c}\,\bar{c}$ calculation which envisioned cc diquarks interacting with $\bar{c}\,\bar{c}$ antidiquarks. We already estimated $\bar{M}(b\bar{b};1S) = 9445.0$ MeV and $B(b\bar{b};1) = -532.2$ MeV, so $B(bb,3^*) = -266.1$ MeV. Taking into account a small hyperfine contribution [22] of $a_{bb}/m_b^2 = 7.8$ MeV, this implies $M(bb,3^*) = 9718.9$ MeV. Using the power-law relation $B_3/B_2 = (M_3/M_2)^{0.712}$ employed previously, for $B_2 = -532.2$, $M_2 = 4988.6$, and $M_3 = 9718.9$ MeV, we obtain $B_3 = -855.7$ MeV for the binding energy between the bb diquark and the $\bar{b}\,\bar{b}$ antidiquark.

The evaluation of the hyperfine interaction is similarly straightforward. Using $\Delta M_3/\Delta M_2 = (M_3/M_2)^{-0.5438}$ and $\Delta M_2 = 62.3$ MeV, we find $A = 43.35$ and $-2A = -86.7$ MeV. The final calculation gives

$$\begin{aligned} M(X_{bb\bar{b}\bar{b}}[0^{++}]) &= 2S + 2M(bb,3^*) + B_{(bb)(\bar{b}\bar{b})} + \Delta M_{HF} \\ &= [2(165.1) + 2(9718.9) - 855.7 \\ &\quad -86.7]\,\text{MeV} \\ &= 18{,}825.6\,\text{MeV}. \end{aligned} \tag{24}$$

As in Sec. II B, we assign an error of ± 25 MeV to this estimate, corresponding to twice the error assigned in Ref. [22] to estimates of QQq masses. This lies 95.0 below $2M(\Upsilon(1S))$, 33.7 below $M(\Upsilon(1S)) + M(\eta_b)$, and 27.6 MeV above $2M(\eta_b)$. This is to be compared with the estimate in Sec. II B of $M(X_{cc\bar{c}\,\bar{c}}[0^{++}]) = 6191.5$ MeV, 224.3 MeV above $2M(\eta_c) = 5967.2$ MeV. Thus, there is a chance that the lowest $bb\bar{b}\,\bar{b}$ state is narrow enough to be visible in a mode other than those coming from the decays of individual η_b components.

The example of dipositronium discussed in the previous section can be applied to the case of the bottom quark. With $m_b \approx 5$ GeV and an effective value of $\alpha_s = 0.35$ for the QCD Coulombic interaction, the binding energy for two spin-triplet states (neglecting Casimir operators of order unity) will be

$$B[\Upsilon(1S)\Upsilon(1S)] = (1/4) m_b \alpha_s^2 \cdot 0.06 \simeq 9\ \text{MeV}. \tag{25}$$

This state is above $2M(\eta_b)$, so that will be its dominant decay, but the discussion confirms the earlier estimate that two $\Upsilon(1S)$ can form a molecule, even if only weakly bound.

B. Production of the lowest $bb\bar{b}\,\bar{b}$ state

Recently, the CMS Collaboration [48] has observed 38 ± 7 events of $\Upsilon(1S)$ pairs produced with an integrated luminosity of 20.7 fb^{-1} at $\sqrt{s} = 8$ TeV, each decaying to μ pairs. The reported fiducial cross section, with each $\Upsilon(1S)$ required to have rapidity $|y| < 2$, is $68.8 \pm 12.7(\text{stat}) \pm 7.4(\text{syst}) \pm 2.8(\mathcal{B})$ pb. It is estimated in one theoretical calculation [49] that about 30% of this value is due to double-parton scattering and another 12 pb is due to feed-down from $\Upsilon(2S)\Upsilon(1S)$ production, leaving 36 pb for $\Upsilon(1S)$ pair production without feed-down. In analogy with our discussion of the relation between J/ψ pair and $cc\bar{c}\,\bar{c}$ tetraquark production, we expect the latter to be a few percent of the former, implying (at 8 TeV)

$$\sigma(pp \to X_{bb\bar{b}\,\bar{b}}) \simeq 1\ \text{pb}, \tag{26}$$

or about twice that at 13 TeV. (The LHCb Collaboration [50] has found that the rate for single-b production roughly doubles from 7 to 13 TeV.)

C. Decays of the lowest $bb\bar{b}\,\bar{b}$ state

The predicted mass of the lowest $bb\bar{b}\,\bar{b}$ state is only about 28 ± 25 MeV above $2M(\eta_b)$. This suggests that a hadronic decay into two η_b mesons, followed by their individual decays, may not be the only decay mode of $X_{bb\bar{b}\,\bar{b}}$. If its mass is actually below $2M(\eta_b)$, decay occurs when each b annihilates a $\bar{b}$, or when one b annihilates a $\bar{b}$ and the other $b\bar{b}$ pair emerges as a pair of B-flavored mesons in the manner akin to Fig. 3. In analogy with our calculation for charm, we can compare the expected rate for this process with the leptonic width [26]

$$\Gamma(\Upsilon(1S) \to e^+e^-) = (1.340 \pm 0.018)\ \text{keV}. \tag{27}$$

Taking $\alpha_s(m_b) = 0.22$ [26], we have $(\alpha_s/\alpha)^2 \simeq 900$ or

$$\Gamma(X_{bb\bar{b}\,\bar{b}}) \simeq 1.2\ \text{MeV}. \tag{28}$$

Assuming that the lowest $b_1 b_2 \bar{b}_1 \bar{b}_2$ tetraquark decays with $b_1\bar{b}_1 \to \ell_1^+\ell_1^-$ with partial width approximately equal to (27), and partial width for $b_2\bar{b}_2$ decay ranging from (27) to tens of MeV, one predicts

$$\begin{aligned} \mathcal{B}(X_{bb\bar{b}\,\bar{b}}[0^{++}] &\to \ell_1^+\ell_1^-\ell_2^+\ell_2^-) \\ &= (1\ \text{to}\ 10^{-4})(2.7\ \text{keV})/(1.2\ \text{MeV}) \\ &= \mathcal{O}(2\times 10^{-3}\ \text{to}\ 2\times 10^{-7}). \end{aligned} \tag{29}$$

This implies a cross section for the four-lepton observation of a $bb\bar{b}\,\bar{b}$ tetraquark,

$$\sigma(pp \to X_{bb\bar{b}\bar{b}}[0^{++}] \to \ell_1^+\ell_1^-\ell_2^+\ell_2^-) \leq 4 \text{ fb} \quad (\text{LHC}, 13 \text{ TeV}), \tag{30}$$

where the upper limit is attained only if there is not significant competition from the decay mode

$$X_{bb\bar{b}\bar{b}}[0^{++}] \to \ell^+\ell^- B\bar{B}X. \tag{31}$$

At 7 or 8 TeV, one would expect about half this, or 2 fb.

VI. REMARKS ON MIXED STATES

If heavy quarks in a tetraquark are produced in quark-antiquark pairs, one might expect tetraquarks of the form $b\bar{b}c\bar{c}$ to be much more abundant than $bb\bar{c}\bar{c}$ or $cc\bar{b}\bar{b}$ tetraquarks. The following remarks, thus, apply only to $b\bar{b}c\bar{c}$ states. One would expect their production cross section in a hadronic reaction to be intermediate between that of $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$. One would probably not expect the lowest-mass $b\bar{b}c\bar{c}$ state to lie below $M(\eta_c) + M(\eta_b)$. In that unlikely case, however, there are fewer opportunities for heavy-quark annihilation than for $cc\bar{c}\bar{c}$ or $bb\bar{b}\bar{b}$, as each quark has only one antiquark with which to annihilate.

The dominant decay will then be annihilation of a single heavy quark pair into a light quark pair (analogous to the process in Fig. 3), leading to a total width of several MeV. Production cross sections at the LHC would be several tens of pb. Expected branching fractions to a four-lepton final state could be as large as 10^{-3} but could be several orders of magnitude smaller if the decays

$$X_{bc\bar{b}\bar{c}}[0^{++}] \to \ell^+\ell^- D\bar{D}X, \quad \ell^+\ell^- B\bar{B}X \tag{32}$$

played a dominant role.

VII. CONCLUSIONS

We have estimated the mass of the lowest-lying $c\bar{c}c\bar{c}$ tetraquark and find it unlikely to be less than twice the mass of the lowest charmonium state η_c. In that unlikely case, however, the decay may proceed by annihilation of each $c\bar{c}$ pair as long as each is in a $J = 1$ state. In that case, one expects final states of hadrons from pairs of intermediate gluons, and of hadrons or leptons from pairs of intermediate virtual photons. Similar arguments apply to the heavier tetraquarks $b\bar{b}c\bar{c}$ and $b\bar{b}b\bar{b}$. The predicted masses of the lowest-lying states are $M(X_{cc\bar{c}\bar{c}}[0^{++}]) = 6,192 \pm 25$ MeV and $M(X_{bb\bar{b}\bar{b}}[0^{++}]) = 18,826 \pm 25$ MeV, for the charmed and bottom tetraquarks, respectively. The proximity of the predicted $(bb)(\bar{b}\bar{b})$ mass to $2M(\eta_b)$ suggests that if we have overestimated it by an amount comparable to our uncertainty, its decays to a pair of real or virtual photons or a pair of gluons may stand a chance of being observable. Other estimates of resonant $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ masses, summarized in Table VII, give mixed signals as to whether the lightest state is above or below the mass of the lightest quarkonium pair.

TABLE VII. Predictions for the mass of the $QQ\bar{Q}\bar{Q}$ tetraquark.

Reference	$M(X_{cc\bar{c}\bar{c}})$ (MeV)	$M(X_{bb\bar{b}\bar{b}})$ (MeV)
This work	$6,192 \pm 25(0^{++})$	$18,826 \pm 25(0^{++})$
[3]	~6,200	–
[11]	6,908	–
[13]	6,038	–
[15]	$5,966(0^{++})$	$18,754(0^{++})$
[15]	$6,051(1^{+-})$	$18,808(1^{+-})$
[15]	$6,223(2^{++})$	$18,916(2^{++})$
[16]	$5,300 \pm 500$	–
[17]	5,617–6,254	18,462–18,955
[18]	$6,440 \pm 150$	$18,450 \pm 150$
[19][a]	–	$18,690 \pm 30$

[a] Appeared after the first version of the current work.

Searches in the four-lepton and $\ell^+\ell^- B\bar{B}$ final states have been performed at the LHC [51,52]. These are devoted to the search for the standard-model Higgs boson decaying into two light pseudoscalars a, which then decay to such final states as $\mu^+\mu^-$, $\tau^+\tau^-$, and $b\bar{b}$. These are ideal samples for the searches advocated here.

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[1] M. Karliner and J. L. Rosner, New Exotic Meson and Baryon Resonances from Doubly-Heavy Hadronic Molecules, Phys. Rev. Lett. **115**, 122001 (2015).
[2] M. Karliner and J. L. Rosner, Exotic resonances due to η exchange, Nucl. Phys. **A954**, 365 (2016).
[3] Y. Iwasaki, A possible model for new resonances—exotics and hidden charm, Prog. Theor. Phys. **54**, 492 (1975).
[4] K. T. Chao, The $cc\bar{c}\,\bar{c}$ (diquark–anti-diquark) states in e^+e^- annihilation, Z. Phys. C **7**, 317 (1981).
[5] J. P. Ader, J. M. Richard, and P. Taxil, Do narrow heavy multi-quark states exist?, Phys. Rev. D **25**, 2370 (1982).
[6] S. Zouzou, B. Silvestre-Brac, C. Gignoux, and J. M. Richard, Four quark bound states, Z. Phys. C **30**, 457 (1986).
[7] L. Heller and J. A. Tjon, On the existence of stable dimesons, Phys. Rev. D **35**, 969 (1987).
[8] B. Silvestre-Brac, Systematics of $Q^2(\bar{Q}^2)$ systems with a chromomagnetic interaction, Phys. Rev. D **46**, 2179 (1992).
[9] B. Silvestre-Brac and C. Semay, Systematics of $L=0$ $q^2\bar{q}^2$ systems, Z. Phys. C **57**, 273 (1993); Spectrum and decay properties of diquonia, Z. Phys. C **59**, 457 (1993).
[10] C. Semay and B. Silvestre-Brac, Diquonia and potential models, Z. Phys. C **61**, 271 (1994).
[11] R. J. Lloyd and J. P. Vary, All charm tetraquarks, Phys. Rev. D **70**, 014009 (2004).
[12] T.-W. Chiu and T.-H. Hsieh (TWQCD Collaboration), Y(4260) on the lattice, Phys. Rev. D **73**, 094510 (2006).
[13] N. Barnea, J. Vijande, and A. Valcarce, Four-quark spectroscopy within the hyperspherical formalism, Phys. Rev. D **73**, 054004 (2006).
[14] A. V. Berezhnoy, A. K. Likhoded, A. V. Luchinsky, and A. A. Novoselov, Double J/ψ-meson Production at LHC and 4c-tetraquark state, Phys. Rev. D **84**, 094023 (2011).
[15] A. V. Berezhnoy, A. V. Luchinsky, and A. A. Novoselov, Tetraquarks composed of 4 heavy quarks, Phys. Rev. D **86**, 034004 (2012).
[16] W. Heupel, G. Eichmann, and C. S. Fischer, Tetraquark bound states in a Bethe-Salpeter approach, Phys. Lett. B **718**, 545 (2012).
[17] J. Wu, Y. R. Liu, K. Chen, X. Liu, and S. L. Zhu, Heavy-flavored tetraquark states with the $QQ\bar{Q}\,\bar{Q}$ configuration, arXiv:1605.01134.
[18] W. Chen, H. X. Chen, X. Liu, T. G. Steele, and S. L. Zhu, Hunting for exotic doubly hidden-charm/bottom tetraquark states, arXiv:1605.01647.
[19] Y. Bai, S. Lu, and J. Osborne, Beauty-full tetraquarks, arXiv:1612.00012.
[20] M. Karliner, B. Keren-Zur, H. J. Lipkin, and J. L. Rosner, The quark model and b baryons, Ann. Phys. (Berlin) **324**, 2 (2009).
[21] M. Karliner, H. J. Lipkin, and N. A. Törnqvist, New states with heavy quarks, Eur. Phys. J. Web Conf. **70**, 00024 (2014).
[22] M. Karliner and J. L. Rosner, Baryons with two heavy quarks: Masses, production, decays, and detection, Phys. Rev. D **90**, 094007 (2014).
[23] S. Gasiorowicz and J. L. Rosner, Hadron spectra and quarks, Am. J. Phys. **49**, 954 (1981).
[24] H. J. Lipkin, A unified description of mesons, baryons and baryonium, Phys. Lett. B **74**, 399 (1978).
[25] G. Rossi and G. Veneziano, The string-junction picture of multiquark states: An update, J. High Energy Phys. 06 (2016) 041.
[26] C. Patrignani *et al.* (Particle Data Group Collaboration), Review of particle physics, Chin. Phys. C **40**, 100001 (2016).
[27] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Diquark-antidiquarks with hidden or open charm and the nature of X(3872), Phys. Rev. D **71**, 014028 (2005).
[28] S. J. Brodsky, D. S. Hwang, and R. F. Lebed, Dynamical Picture for the Formation and Decay of the Exotic XYZ Mesons, Phys. Rev. Lett. **113**, 112001 (2014).
[29] R. L. Jaffe, Multi-quark hadrons. 1. The phenomenology of $Q^2\bar{Q}^2$ mesons, Phys. Rev. D **15**, 267 (1977).
[30] R. L. Jaffe, Multi-quark hadrons. 2. Methods, Phys. Rev. D **15**, 281 (1977).
[31] R. L. Jaffe, $Q^2\bar{Q}^2$ resonances in the baryon-antibaryon system, Phys. Rev. D **17**, 1444 (1978).
[32] J. L. Rosner, Possibility of Baryon-Antibaryon Enhancements with Unusual Quantum Numbers, Phys. Rev. Lett. **21**, 950 (1968).
[33] M. Karliner and H. J. Lipkin, Diquarks and antiquarks in exotics: A Menage a trois and a menage a quatre, Phys. Lett. B **638**, 221 (2006).
[34] H. J. Lipkin, Triality, exotics and the dynamical basis of the quark model, Phys. Lett. B **45**, 267 (1973).
[35] J. Wheeler, Polyelectrons, Ann. N.Y. Acad. Sci. **48**, 219 (1946).
[36] E. A. Hylleraas and A. Ore, Binding energy of the positronium molecule, Phys. Rev. **71**, 493 (1947).
[37] P. M. Kozlowski and L. Adamowicz, Lifetime of positronium molecule, J. Phys. Chem. **100**, 6266 (1996).
[38] D. B. Cassidy and A. P. Mills, Jr., The production of molecular positronium, Nature (London) **449**, 195 (2007).
[39] M. Puchalski and A. Czarnecki, Dipole Excitation of Dipositronium, Phys. Rev. Lett. **101**, 183001 (2008).
[40] K. Abe *et al.* (Belle Collaboration), Production of Prompt Charmonia in e^+e^- Annihilation at $\sqrt{s}\approx 10.6$ GeV, Phys. Rev. Lett. **88**, 052001 (2002).
[41] K. Abe *et al.* (Belle Collaboration), Observation of Double $c\bar{c}$ Production in e^+e^- Annihilation at $\sqrt{s}\approx 10.6$ GeV, Phys. Rev. Lett. **89**, 142001 (2002).
[42] K. Abe *et al.* (Belle Collaboration), Observation of a New Charmonium State in Double Charmonium Production in e^+e^- Annihilation at $\sqrt{s}\approx 10.6$ GeV, Phys. Rev. Lett. **98**, 082001 (2007).
[43] R. Aaij *et al.* (LHCb Collaboration), Measurements of prompt charm production cross-sections in pp collisions at $\sqrt{s}=5$ TeV, arXiv:1610.02230.
[44] M. Aaboud *et al.* (ATLAS Collaboration), Measurement of the prompt J/ψ pair production cross-section in pp collisions at $\sqrt{s}=8$ TeV with the ATLAS detector, arXiv:1612.02950.

[45] V. Khachatryan *et al.* (CMS Collaboration), Measurement of prompt J/ψ pair production in pp collisions at $\sqrt{s} = 7$ TeV, J. High Energy Phys. 09 (2014) 094.

[46] R. Aaij *et al.* (LHCb Collaboration), Observation of J/ψ pair production in pp collisions at $\sqrt{s} = 7$ TeV, Phys. Lett. B **707**, 52 (2012).

[47] K. Ackerstaff *et al.* (OPAL Collaboration), Measurement of the strong coupling constant α_s and the vector and axial vector spectral functions in hadronic τ decays, Eur. Phys. J. C **7**, 571 (1999).

[48] V. Khachatryan *et al.* (CMS Collaboration), Observation of $\Upsilon(1S)$ pair production in proton-proton collisions at $\sqrt{s} = 8$ TeV, arXiv:1610.07095.

[49] A. V. Berezhnoy, A. K. Likhoded, and A. A. Novoselov, Υ-meson pair production at LHC, Phys. Rev. D **87**, 054023 (2013).

[50] R. Aaij *et al.* (LHCb Collaboration), Measurement of the b-quark production cross-section in 7 and 13 TeV pp collisions, arXiv:1612.05140.

[51] G. Aad *et al.* (ATLAS Collaboration), Search for Higgs bosons decaying to aa in the $\mu\mu\tau\tau$ final state in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS experiment, Phys. Rev. D **92**, 052002 (2015).

[52] V. Khachatryan *et al.* (CMS Collaboration), Search for light bosons in decays of the 125 GeV Higgs boson in proton-proton collisions at $\sqrt{s} = 8$ TeV, arXiv:1701.02032.