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Energy dependence of hadron polarization in $e^+e^- \to hX$ at high energies

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The longitudinal polarization of a hyperon in e^+e^- annihilation at high energies depends on the longitudinal polarization of the quark produced at the e^+e^- annihilation vertex, whereas the spin alignment of vector mesons is independent of it. They exhibit very different energy dependences. We use the longitudinal polarization of the Lambda hyperon and the spin alignment of K^* as representative examples to present numerical results of energy dependences and demonstrate such distinct differences. We present the results at the leading twist with perturbative QCD evolutions of fragmentation functions at the leading order.

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I. INTRODUCTION

The spin dependence of fragmentation functions (FFs) has attracted much attention since it provides not only important information on the hadronization mechanism but also an important place to study properties of quantum chromodynamics (QCD). High energy e^+e^- annihilation is the cleanest place to study FFs. Among different aspects of the spin dependence, vector polarizations of hyperons and tensor polarizations of vector mesons are two topics that attracted special attention because both of them can be measured by the angular distributions of the decay products. Hyperon polarizations can be determined by the angular distribution of the decay products of the spin self-analyzing parity violating weak decay. Different components of the tensor polarization of vector mesons can also be determined by the angular distribution of decay products of the strong decay into two spin zero hadrons. Measurements have been carried out, e.g., many years ago at the Large Electron-Positron Collider (LEP) for the longitudinal polarization of a Λ hyperon [1,2] and for spin alignments of vector mesons [3–5] in the inclusive production process $e^+e^- \to hX$, and sizable effects have been observed. These data have attracted many phenomenological studies, and different approaches have been proposed to describe them [6–24].

In the theoretical framework of the QCD parton model, hadron polarizations are expressed in terms of different FFs [25–31]. These FFs are defined via quark-quark and/or quark-gluon-quark correlators. The results of the complete decomposition of a quark-quark correlator as well as those for that of a quark-gluon-quark at twist-3 for spin-1 hadrons has been presented, e.g., in [31–33]. A general framework for $e^+e^- \to V\pi X$ has been constructed [31], and QCD parton model results for hadron polarizations in terms of FFs have been presented up to twist-3 at the leading order (LO) of perturbative QCD.

With these results, we can make predictions on the energy dependence of hadron polarization within the theoretical framework of QCD if we have the results at a given energy and the scale dependence of FFs. In fact, from the results presented in [25–31], we see one distinct feature for hadron polarizations in e^+e^- annihilations at high energies, i.e., at the leading twist, polarizations of hadrons are divided into two categories. In one of them, the polarization of hadrons depends on the initial longitudinal polarization P_q of the quark (or antiquark) produced at the e^+e^--vertex and is parity violated. In the second category, the polarization is independent of P_q and is parity conserved. The most well-known example in the inclusive process $e^+e^- \to hX$ is the longitudinal polarization of hyperons, such as Λ, Σ, and Ξ, while spin alignments of vector mesons, such as ρ and K^* are representatives of the second category. The longitudinal polarization P_q is a result of a weak interaction and is completely determined by the electroweak process at the parton level. It takes the maximum for e^+e^- annihilation at the Z pole and changes very fast with energy. Hence, we expect that the polarization in the first category has a strong energy dependence. The energy dependence for hadron polarizations in the second category comes mainly from the scale dependence of the corresponding FFs and/or higher twist contributions. We expect that they change quite slowly with energy compared with that in the first category. We should see very much different behaviors in energy dependence.

The scale dependence of FFs are determined by the QCD evolution equation. For the leading twist one dimensional FFs, the evolutions are well established and are determined by corresponding Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations [34–37] with timelike splitting functions [38–43]. For the spin-dependent FFs, we have, in particular, a global fit to those for the production of a Λ hyperon already available many years ago with next-to-leading order (NLO) QCD evolutions [43]. However, there is still no similar study for vector mesons available yet. Clearly, the energy dependence provides not only a good place to study the spin dependence of FF but also a good place to study the QCD evolution of the spin-dependent FF and higher twist contributions. In view that there are some data available from experiments at LEP [1–5] and that new measurements can be carried out in experiments at very much different energies such as BES III and BELLE [44] and possibly at the future facilities planned and/or discussed [45], it is very interesting to present some numerical results at different energies to guide experiment and test models.

In this paper, after a brief summary of hadron polarizations in terms of FFs in $e^+e^- \to hX$, we take the longitudinal polarization of Λ and the spin alignment of K^* as two representative examples for the two categories and calculate the energy dependence. We take them as examples because we have data from LEP for both of them. We follow the same way as in [43], make a simple working parametrization for the corresponding FFs at an initial scale by fitting the LEP data [1–5], and evolve them to other energies. We present the results numerically that can be used as a rough guide for future experiments.

The rest of the paper is organized as follows. After this Introduction, we summarize the results of FFs defined via quark-quark correlator, those for hadron polarizations in terms of FFs in $e^+e^- \to hX$ and QCD evolution equations for FFs in Sec. II. In Sec. III, we present a working parametrization of the corresponding FFs, show the numerical results of QCD evolution at the leading order, and present the energy dependence of the two representative examples. We make a short summary and an outlook in Sec. IV.

II. HADRON POLARIZATIONS IN $e^+e^- \to hX$ IN TERMS OF FFS

High energy $e^+e^- \to hX$ is the best place to study FFs in different connections. The results for hadron polarizations expressed in terms of FFs up to twist-3 in leading order in pQCD are given in different papers such as [25–31]. Here, we make a short summary of these results and present, in particular, the formulas that will be used in the numerical estimations.

A. FFs defined via quark-quark correlator

The polarization of a hadron produced in a high energy reaction is described by the spin density matrix. For spin-1/2 hadrons, the polarization is described by a 2×2 spin density matrix that is usually decomposed as $\rho = (1+\vec{S}\cdot\vec{\sigma})/2$, where $\vec{\sigma}$ is the Pauli matrix, and $\vec{S}$ is the polarization vector which is represented by the helicity λ and the transverse polarization vector S_T^μ, i.e.,

$$S^\mu = \lambda \frac{p^+}{M}\bar{n}^\mu + S_T^\mu - \lambda \frac{M}{2p^+} n^\mu, \tag{1}$$

where n and $\bar{n}$ are the two unit vectors in light cone coordinates. For spin-1 hadrons, the polarization is described by a 3×3 density matrix, which, in the rest frame of the hadron, is usually decomposed as [46]

$$\rho = \frac{1}{3}\left(\mathbf{1} + \frac{3}{2}S^i\Sigma^i + 3T^{ij}\Sigma^{ij}\right), \tag{2}$$

where Σ^i is the spin operator of a spin-1 particle, and $\Sigma^{ij} = \frac{1}{2}(\Sigma^i\Sigma^j + \Sigma^j\Sigma^i) - \frac{2}{3}\mathbf{1}\delta^{ij}$. The spin polarization tensor $T^{ij} = \mathrm{Tr}(\rho\Sigma^{ij})$ and is parametrized as

$$\mathbf{T} = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^{x} \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^{y} \\ S_{LT}^{x} & S_{LT}^{y} & \frac{4}{3}S_{LL} \end{pmatrix}. \tag{3}$$

The tensor polarization part has five independent components that are given by a Lorentz scalar S_{LL}, a Lorentz vector $S_{LT}^\mu = (0, S_{LT}^x, S_{LT}^y, 0)$, and a Lorentz tensor $S_{TT}^{\mu\nu}$ that has two nonzero independent components $S_{TT}^{xx} = -S_{TT}^{yy}$ and $S_{TT}^{xy} = S_{TT}^{yx}$.

For the fragmentation of the quark (or antiquark), the FFs are defined via the quark-quark and/or the quark-gluon-quark correlators. The quark-quark or quark-gluon-quark correlator can, in general, be expressed as a sum of a spin-independent part, a vector polarization dependent part, and a tensor polarization dependent part. To describe the production of spin zero hadrons, we need only the spin-independent part. For spin-1/2 hadrons, the vector-polarization dependent part is involved, and for spin-1 hadrons, the tensor polarization dependent part is also needed. FFs are obtained by making Lorentz decompositions of the corresponding part in terms of 4-momenta and variables describing the polarization. Hence, formally, the spin independent part is exactly the same for hadrons with different spins, the vector polarization dependent part is also the same for spin-1/2 and spin-1 hadrons.

The results for the complete decomposition of a quark-quark correlator are summarized, e.g., in [31]. At the leading twist, there are totally 18 TMD FFs that are summarized in Table II of [31]. From the table, we see that 5 of these 18 leading twist TMD FFs describe fragmentation of unpolarized, 4 of them describe longitudinally polarized, and 9 of them describe transversely polarized quark. For those describing an unpolarized quark fragmentation, we have the well-known $D_1(z, k_\perp)$ describing the number density of hadrons produced in the fragmentation and the other 4 describing the induced polarizations. Similarly, for FFs of the longitudinally and transversely polarized quark, we have the direct spin transfer G_{1L} and H_{1T}, respectively, and others describing the number density and/or "worm-gear effects".

After integrating over the transverse momentum, we obtain the results in the one-dimensional case. In this case, we have only five FFs left at the leading twist, i.e., the number density $D_1(z)$, the induced $D_{1LL}(z)$, the direct spin transfers in the longitudinally polarized case $G_{1L}(z)$, and in the transversely polarized case, $H_{1T}(z)$ and $H_{1LT}(z)$.

We emphasize that one-dimensional FFs are needed to describe inclusive processes, such as $e^+e^- \to hX$, while three-dimensional FFs are needed for semi-inclusive processes, such as $e^+e^- \to h_1h_2X$. They can be studied in the corresponding processes, respectively. Also, to study those FFs for unpolarized, transversely polarized or longitudinally polarized quarks, one needs to create quarks in the corresponding polarization states and know the polarizations of them before the fragmentation.

B. Quark polarization in $e^+e^- \to q\bar{q}$

It is well-known that the quark or antiquark from $e^+e^- \to Z \to q\bar{q}$ is longitudinally polarized. The polarization is given by

$$P_q^{\rm Zpole}(\theta) = -\frac{c_1^e c_3^q(1+\cos^2\theta) + 2c_3^e c_1^q \cos\theta}{c_1^e c_1^q(1+\cos^2\theta) + 2c_3^e c_3^q\cos\theta}, \quad (4)$$

where θ is the angle between the incident electron and the produced quark, $c_1^e = (c_V^e)^2 + (c_A^e)^2$, $c_3^e = 2c_V^e c_A^e$, c_V^e and c_A^e are defined in the weak interaction current $\bar{\psi}\gamma^\mu(c_V^e - c_A^e\gamma^5)\psi$, and the superscript denotes that they are for the electron, and similarly, for different flavors of quarks.

Although the quark (antiquark) is not transversely polarized, their transverse spin components are correlated. This is described by the transverse spin correlation function c_{nn}^q defined as

$$c_{nn}^q \equiv \frac{|\hat{m}_{n++}|^2 + |\hat{m}_{n--}|^2 - |\hat{m}_{n+-}|^2 - |\hat{m}_{n-+}|^2}{|\hat{m}_{n++}|^2 + |\hat{m}_{n--}|^2 + |\hat{m}_{n+-}|^2 + |\hat{m}_{n-+}|^2}, \quad (5)$$

where $\hat{m}$ is the scattering amplitude and $+$ or $-$ denotes that the quark or antiquark is in $s_n = 1/2$ or $-1/2$ state. If we take $\vec{n}$ as the normal of the production plane, we obtain

$$c_{nn}^{q,\rm Zpole}(\theta) = \frac{c_1^e c_2^q \sin^2\theta}{c_1^e c_1^q(1+\cos^2\theta) + 2c_3^e c_3^q\cos\theta}, \quad (6)$$

where $c_2^q = (c_V^q)^2 - (c_A^q)^2$. Define $y = l_2 \cdot p_q / q \cdot p_q \approx (1+\cos\theta)/2$ (l_1 and l_2 are the 4-momenta of the incident e^- and e^+, $q = l_1 + l_2$ is that of the Z boson, and p_q is that of the produced quark), we can express P_q and c_{nn}^q in terms of y, i.e.,

$$P_q^{\rm Zpole}(y) = T_1^q(y)/T_0^q(y), \quad (7)$$

$$c_{nn}^{q,\rm Zpole}(y) = c_1^e c_2^q C(y)/2T_0^q(y), \quad (8)$$

$$T_0^q(y) = c_1^e c_1^q A(y) - c_3^e c_3^q B(y), \quad (9)$$

$$T_1^q(y) = -c_1^e c_3^q A(y) + c_3^e c_1^q B(y). \quad (10)$$

Here, we denote as usual $A(y) = (1-y)^2 + y^2 \approx (1+\cos^2\theta)/2$, $B(y) = 1-2y \approx -\cos\theta$, and $C(y) = 4y(1-y) \approx \sin^2\theta$.

Experimental studies are often carried out irrespective of θ or y. The obtained results just correspond to the results integrated over θ or y. For the polarization and correlation of quark given above, if we integrate over θ or y, we obtain

$$\bar{P}_q^{\rm Zpole} = -c_3^q/c_1^q, \quad (11)$$

$$\bar{c}_{nn}^{q,\rm Zpole} = c_2^q/2c_1^q. \quad (12)$$

We see that, the quark is negatively polarized in the longitudinal direction. Also $c_2 < 0$ since c_V^2 is smaller than c_A^2, so we have a negative c_{nn}^q at the Z pole.

In general, for $e^+e^- \to q\bar{q}$, we need to consider contributions from $e^+e^- \to Z \to q\bar{q}$, those from $e^+e^- \to \gamma^* \to q\bar{q}$, and the interference terms. In this case, we have

$$P_q(y) = \Delta w_q(y)/w_q(y), \quad (13)$$

$$c_{nn}^q(y) = 2y(1-y)(e_q^2 + \chi c_1^e c_2^q + \chi_{\rm int}^q c_V^e c_V^q)/w_q(y). \quad (14)$$

Here, e_q is the electric charge of q, and $w_q(y)$ and $\Delta w_q(y)$ are given by

$$w_q(y) = \chi T_0^q(y) + e_q^2 A(y) + \chi_{\rm int}^q I_0^q(y), \quad (15)$$

$$\Delta w_q(y) = \chi T_1^q(y) + \chi_{\rm int}^q I_1^q(y), \quad (16)$$

$$I_0^q(y) = c_V^e c_V^q A(y) - c_A^e c_A^q B(y), \quad (17)$$

$$I_1^q(y) = -c_V^e c_A^q A(y) + c_A^e c_V^q B(y), \quad (18)$$

$$\chi = s^2/[(s-M_Z^2)^2 + \Gamma_Z^2 M_Z^2]\sin^4 2\theta_W, \quad (19)$$

$$\chi_{\rm int}^q = -2e_q s(s-M_Z^2)/[(s-M_Z^2)^2 + \Gamma_Z^2 M_Z^2]\sin^2 2\theta_W, \quad (20)$$

where M_Z and Γ_Z are the mass and decay width of Z, θ_W is the Weinberg angle, and $s = q^2 = Q^2$.

After integrating over y, we obtain

$$\bar{P}_q = \Delta W_q/W_q, \quad (21)$$

$$\bar{c}_{nn}^q = (e_q^2 + \chi c_1^e c_2^q + \chi_{\rm int}^q c_V^e c_V^q)/3W_q, \quad (22)$$

where ΔW_q and W_q are the results of $\Delta w_q(y)$ and $w_q(y)$ after integration over y, and they are given by

$$\Delta W_q = -\frac{2}{3}(\chi c_1^e c_3^q + \chi_{\rm int}^q c_V^e c_A^q), \quad (23)$$

$$W_q = \frac{2}{3}(e_q^2 + \chi c_1^e c_1^q + \chi_{\rm int}^q c_V^e c_V^q). \quad (24)$$

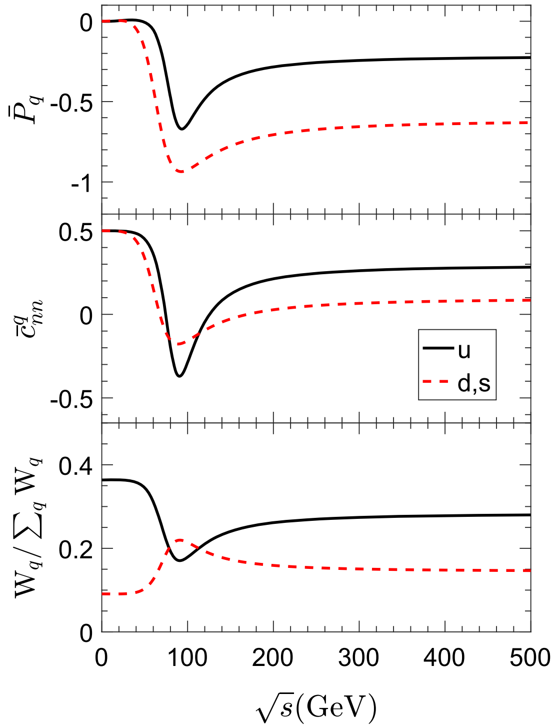

FIG. 1. Energy dependence of the longitudinal polarization $\bar{P}_q$, the transverse quark-antiquark spin correlation $\bar{c}^q_{nn}$, and the normalized weight $W_q/\sum_q W_q$ of a different flavor q of quark produced in e^+e^- annihilation.

We see that both $\bar{P}_q$ and $\bar{c}^q_{nn}$ depend on the energy $\sqrt{s}$, and behave quite differently in the energy dependence. For comparison, we plot them in Fig. 1 together with the normalized weight $W_q/\sum_q W_q$. We see clearly that, in the energy region $\sqrt{s} \leq M_Z$, as $\sqrt{s}$ decreases, the electromagnetic interaction becomes dominate, the longitudinal polarization of quark $\bar{P}_q$ goes to zero rapidly, but the correlation $\bar{c}^q_{nn}$ goes from negative to positive and reaches the maximum $1/2$ rapidly. For $\sqrt{s} \geq M_Z$, we have contributions from both weak and electromagnetic interactions, and they combine together to give rise to a negative P_q but positive c^q_{nn}. The correlation between the transverse spin components of the quark and antiquark is strong and positive.

From these results, we see, in particular, the following. In e^+e^--annihilation at high energies, we have possibilities to study FFs of unpolarized, longitudinally polarized as well as transversely polarized quarks. First, we can study FFs of unpolarized or longitudinally polarized quarks by studying singly polarized reactions, i.e., by measuring only the polarization of one hadron in the final state. More precisely, we can study one-dimensional FFs of unpolarized or longitudinally polarized quarks in $e^+e^- \to hX$ by measuring the corresponding components of polarizations of h in the final state. By studying the semi-inclusive process $e^+e^- \to h_1h_2X$ and measuring the polarization of h_1, we can study the corresponding three-dimensional FFs. Second, FFs of the transversely polarized quark can also be studied in e^+e^--annihilation at high energies. But in this case, we need at least to measure polarizations or other spin dependent asymmetries of two hadrons in the final states since the nonzero quantity at the parton level is the transverse spin correlation between the initial quark and antiquark but not the transverse polarization of the quark or antiquark. In this paper, we start with the simplest case, i.e., $e^+e^- \to hX$, where only one-dimensional FFs for the unpolarized or longitudinally polarized quark can be studied.

C. Hadron polarizations at the Z pole

Hadron polarizations in e^+e^--annihilations at high energies are given e.g., in [29–31] in terms of FFs. For $e^+e^- \to Z \to VX$ at the leading order in pQCD and up to twist-3, for the longitudinal components, we obtain

$$\langle\lambda\rangle(z,y) = \frac{2}{2S+1}\frac{\sum_q P_q(y)T^q_0(y)G_{1L}(z)}{\sum_q T^q_0(y)D_1(z)}, \tag{25}$$

$$\langle S_{LL}\rangle(z,y) = \frac{3}{2(2S+1)}\frac{\sum_q T^q_0(y)D_{1LL}(z)}{\sum_q T^q_0(y)D_1(z)}. \tag{26}$$

Here, for brevity and clarity, we omit the superscript $q \to V$ in the fragmentation functions, e.g., $D_1(z) = D_1^{q\to V}(z)$; and S is the spin of hadron h. The factor $(2S+1)$ appears here because, in the conventions used in [31] in defining FFs via a quark-quark correlator and/or a quark-gluon-quark correlator, $D_1(z)$ is the number density for the produced h averaging over rather than summing over the spin of h. We write this factor explicitly so that the corresponding expressions eventually take the same form for spin-$1/2$ as well as spin-1 hadrons. For the transverse components, we have

$$\langle S^x_T\rangle(z,y) = -\frac{8MD(y)}{(2S+1)zQ}\frac{\sum_q T^q_3(y)G_T(z)}{\sum_q T^q_0(y)D_1(z)}, \tag{27}$$

$$\langle S^y_T\rangle(z,y) = \frac{8MD(y)}{(2S+1)zQ}\frac{\sum_q T^q_2(y)D_T(z)}{\sum_q T^q_0(y)D_1(z)}, \tag{28}$$

$$\langle S^x_{LT}\rangle(z,y) = -\frac{8MD(y)}{(2S+1)zQ}\frac{\sum_q T^q_2(y)D_{LT}(z)}{\sum_q T^q_0(y)D_1(z)}, \tag{29}$$

$$\langle S^y_{LT}\rangle(z,y) = \frac{8MD(y)}{(2S+1)zQ}\frac{\sum_q T^q_3(y)G_{LT}(z)}{\sum_q T^q_0(y)D_1(z)}, \tag{30}$$

where $D(y) = \sqrt{y(1-y)}$, and we also define,

$$T^q_2(y) = -c^e_3c^q_3 + c^e_1c^q_1B(y), \tag{31}$$

$$T_3^q(y) = c_3^e c_1^q - c_1^e c_3^q B(y). \tag{32}$$

We recall that $\langle S_T^y \rangle$ is P even and naive T odd, $\langle S_T^x \rangle$ is P odd and naive T even, and $\langle S_{LT}^y \rangle$ is P odd and naive T odd. We emphasize that formally vector polarization components, such as $\langle \lambda \rangle$, $\langle S_T^x \rangle$ and $\langle S_T^y \rangle$, have exactly the same expressions in terms of FFs for spin-1/2 hadrons or vector mesons. This means that Eqs. (25) and (27)–(28) are the same for hyperons and for vector mesons. They are just given by the corresponding FFs for specified hadrons.

The spin alignment of the vector meson is measured by the 00 component ρ_{00} of the spin density matrix ρ in the helicity basis. It is directly related to $\langle S_{LL} \rangle$ by $\rho_{00} = (1 - 2\langle S_{LL} \rangle)/3$, which means

$$\rho_{00}(z,y) = \frac{1}{3} - \frac{1}{3} \frac{\sum_q T_0^q(y) D_{1LL}(z)}{\sum_q T_0^q(y) D_1(z)}. \tag{33}$$

We consider the case of integrated over θ or y, and we have

$$\bar{\lambda}(z) = -\frac{2}{2S+1} \frac{\sum_q c_3^q G_{1L}(z)}{\sum_q c_1^q D_1(z)}, \tag{34}$$

$$\bar{\rho}_{00}(z) = \frac{1}{3} - \frac{1}{3} \frac{\sum_q c_1^q D_{1LL}(z)}{\sum_q c_1^q D_1(z)}, \tag{35}$$

$$\bar{S}_T^x(z) = -\frac{3\pi M}{2(2S+1)zQ} \frac{\sum_q c_3^e c_1^q G_T(z)}{\sum_q c_1^e c_1^q D_1(z)}, \tag{36}$$

$$\bar{S}_T^y(z) = -\frac{3\pi M}{2(2S+1)zQ} \frac{\sum_q c_3^e c_3^q D_T(z)}{\sum_q c_1^e c_1^q D_1(z)}, \tag{37}$$

$$\bar{S}_{LT}^x(z) = \frac{3\pi M}{2(2S+1)zQ} \frac{\sum_q c_3^e c_3^q D_{LT}(z)}{\sum_q c_1^e c_1^q D_1(z)}, \tag{38}$$

$$\bar{S}_{LT}^y(z) = \frac{3\pi M}{2(2S+1)zQ} \frac{\sum_q c_3^e c_1^q G_{LT}(z)}{\sum_q c_1^e c_1^q D_1(z)}. \tag{39}$$

We see that at the leading twist, we have only two nonzero components, i.e., the longitudinal polarization $P_{Lh} = \langle \lambda \rangle$ and $\rho_{00} = (1 - 2\langle S_{LL} \rangle)/3$. The transverse polarization exists at twist-3, i.e., it is power suppressed. We also note that there is no twist-3 contribution to $\langle \lambda \rangle$ or $\langle S_{LL} \rangle$. The higher twist corrections to these two components come only from twist-4 or even higher twists [29].

D. Hadron polarizations at different energies

At different energies, we need to consider contributions from $e^+e^- \to Z \to VX$, those from $e^+e^- \to \gamma^* \to VX$, and those from the interference terms. For the longitudinal components, we have

$$\langle \lambda \rangle(z,y) = \frac{2}{2S+1} \frac{\sum_q P_q(y) w_q(y) G_{1L}(z)}{\sum_q w_q(y) D_1(z)}, \tag{40}$$

$$\langle S_{LL} \rangle(z,y) = \frac{3}{2(2S+1)} \frac{\sum_q w_q(y) D_{1LL}(z)}{\sum_q w_q(y) D_1(z)}, \tag{41}$$

and for the transverse components

$$\langle S_T^x \rangle(z,y) = -\frac{8MD(y)}{(2S+1)zQ} \frac{\sum_q \Delta_x w_q(y) G_T(z)}{\sum_q w_q(y) D_1(z)}, \tag{42}$$

$$\langle S_T^y \rangle(z,y) = \frac{8MD(y)}{(2S+1)zQ} \frac{\sum_q \Delta_y w_q(y) D_T(z)}{\sum_q w_q(y) D_1(z)}, \tag{43}$$

$$\langle S_{LT}^x \rangle(z,y) = -\frac{8MD(y)}{(2S+1)zQ} \frac{\sum_q \Delta_y w_q(y) D_{LT}(z)}{\sum_q w_q(y) D_1(z)}, \tag{44}$$

$$\langle S_{LT}^y \rangle(z,y) = \frac{8MD(y)}{(2S+1)zQ} \frac{\sum_q \Delta_x w_q(y) G_{LT}(z)}{\sum_q w_q(y) D_1(z)}, \tag{45}$$

where $w_q(y)$ is given by Eq. (15), and $\Delta_x w_q(y)$ and $\Delta_y w_q(y)$ are given by

$$\Delta_x w_q(y) = \chi T_3^q(y) + \chi_{\rm int}^q I_3^q(y), \tag{46}$$

$$\Delta_y w_q(y) = \chi T_2^q(y) + \chi_{\rm int}^q I_2^q(y), \tag{47}$$

$$I_2^q(y) = -c_A^e c_A^q + c_V^e c_V^q B(y), \tag{48}$$

$$I_3^q(y) = c_A^e c_V^q - c_V^e c_A^q B(y). \tag{49}$$

After integrating over y, we obtain

$$\bar{\lambda}(z) = \frac{2}{(2S+1)} \frac{\sum_q \bar{P}_q W_q G_{1L}(z)}{\sum_q W_q D_1(z)}, \tag{50}$$

$$\bar{\rho}_{00}(z) = \frac{1}{3} - \frac{1}{3} \frac{\sum_q W_q D_{1LL}^{q\to h}(z)}{\sum_q W_q D_1^{q\to h}(z)}, \tag{51}$$

$$\bar{S}_T^x(z) = -\frac{8M}{(2S+1)zQ} \frac{\sum_q \Delta_x W_q G_T(z)}{\sum_q W_q D_1(z)}, \tag{52}$$

$$\bar{S}_T^y(z) = \frac{8M}{(2S+1)zQ} \frac{\sum_q \Delta_y W_q D_T(z)}{\sum_q W_q D_1(z)}, \tag{53}$$

$$\bar{S}_{LT}^x(z) = -\frac{8M}{(2S+1)zQ} \frac{\sum_q \Delta_y W_q D_{LT}(z)}{\sum_q W_q D_1(z)}, \tag{54}$$

$$\bar{S}_{LT}^y(z) = \frac{8M}{(2S+1)zQ} \frac{\sum_q \Delta_x W_q G_{LT}(z)}{\sum_q W_q D_1(z)}, \tag{55}$$

where $\Delta_x W_q = \pi(\chi c_3^e c_1^q + \chi_{\rm int}^q c_A^e c_V^q)/8$, and $\Delta_y W_q = -\pi(\chi c_3^e c_3^q + \chi_{\rm int}^q c_A^e c_A^q)/8$. We see again that there exist twist-3 transverse polarizations that can be used to study higher twist effects, in particular, the corresponding higher twist FFs. However, we should also note that at lower energies where electromagnetic interactions dominate, such twist-3 contributions are nonzero only at a given y but vanish after the integration over y or θ in the entire region. This is consistent with the data available [47]. One can, however, study such effects by measuring transverse polarizations integrated in a given region of θ or y, such as in the forward or backward hemisphere.

E. QCD evolution equations for G_{1L} and D_{1LL}

QCD evolutions for leading twist one dimensional FFs have been well established and are determined by corresponding DGLAP equations [34–37] with timelike splitting functions [38–43]. We just give the equations that will be used in our numerical estimations in the following. The evolution of the spin transfer G_{1L} is given by DGLAP in the longitudinally polarized case, while that for the S_{LL}-dependent FF D_{1LL} is the same as that for unpolarized FF D_1. They are given by

$$\frac{\partial}{\partial \ln Q^2} G_{1L}^{i\to h}(z,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\sum_j \int_z^1 \frac{dy}{y} G_{1L}^{j\to h}\left(\frac{z}{y},Q^2\right)\Delta P_{ji}(y,\alpha_s), \quad (56)$$

$$\frac{\partial}{\partial \ln Q^2} D_{1LL}^{i\to h}(z,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\sum_j \int_z^1 \frac{dy}{y} D_{1LL}^{j\to h}\left(\frac{z}{y},Q^2\right) P_{ji}(y,\alpha_s), \quad (57)$$

where i or j denotes different types of partons, such as different flavors of quarks, antiquarks, and gluons. At the leading order (LO) in pQCD, the polarized splitting functions are given by [41,42]

$$\Delta P_{qq}(y) = C_F\left[\frac{1+y^2}{(1-y)_+} + \frac{3}{2}\delta(1-y)\right], \quad (58)$$

$$\Delta P_{gq}(y) = C_F\frac{1-(1-y)^2}{y}, \quad (59)$$

$$\Delta P_{qg}(y) = [y^2-(1-y)^2]/2, \quad (60)$$

$$\Delta P_{gg}(y) = N_c\left[(1+y^4)\left(\frac{1}{y}+\frac{1}{(1-y)_+}\right) - \frac{(1-y)^3}{y}\right] + \frac{11N_c - 2N_f}{6}\delta(1-y), \quad (61)$$

where $N_c = 3$ and $C_F = (N_c^2-1)/2N_c$. The unpolarized splitting functions are given by

$$P_{qq}(y) = \Delta P_{qq}(y), \quad (62)$$

$$P_{gq}(y) = C_F\frac{1+(1-y)^2}{y}, \quad (63)$$

$$P_{qg}(y) = [y^2+(1-y)^2]/2, \quad (64)$$

$$P_{gg}(y) = N_c\left[\frac{2y}{(1-y)_+} - 2\left(y^2 - y - \frac{1}{y} + 1\right)\right] + \frac{11N_c - 2N_f}{6}\delta(1-y). \quad (65)$$

The NLO results for these splitting functions have also been obtained, and a global fit for the spin-dependent FF $G_{1L}^{q\to\Lambda}(z,Q)$ has been given in [43]. However, the data available are still too far to make such detailed analysis for other hadrons. Even for $G_{1L}^{q\to\Lambda}(z,Q)$, we still far away from a reliable parameterization of different contributions [43]. The purpose of our studies in this paper is not to make a global fit for polarized FFs but to demonstrate the two distinctly different behaviors in the energy dependence of hadron polarization in e^+e^--annihilation. We therefore limit ourselves to the next-to-leading order in pQCD, where only leading order splitting functions given above are used.

III. NUMERICAL RESULTS FOR $P_{L\Lambda}$ AND $\bar{\rho}_{00}^{K^*}$

As has already been emphasized in Sec. II, from the results given by Eqs. (25)–(30) and (40)–(45), we see clearly that at the leading twist there exist only two components of the polarization, $P_{Lh} = \bar{\lambda}$ and $\bar{S}_{LL}$ or $\bar{\rho}_{00}$, and that there is a distinct difference between them: the former depends on the initial longitudinal polarization $\bar{P}_q$ of the quark and describes the longitudinal spin transfer in the fragmentation of the quark. It is parity violating in $e^+e^- \to hX$ and is caused by the weak channel $e^+e^- \to Z \to hX$ and its interference to the electromagnetic channel $e^+e^- \to \gamma^* \to hX$. The latter is independent of $\bar{P}_q$ and is an induced polarization in the quark fragmentation. It is parity conserved and exists even in the fragmentation of the unpolarized quark. Clearly, such distinct differences should manifest themselves in different high energy reactions. One of the consequences in e^+e^--annihilation is the different behavior in the energy dependence.

As can be seen from Eq. (50), the energy dependence of P_{Lh} comes from three factors: the quark polarization P_q, the relative weight W_q of different flavors, and the scale dependence of FFs. The first two factors are determined by the electroweak interactions and can easily be calculated. From the results (see, e.g., Fig. 1), we see clearly that the energy dependence of $\bar{P}_q$ is very strong, in particular, in the region for $\sqrt{s}$, a bit larger than M_Z down to few GeV, say, $5 < \sqrt{s} < 200$ GeV. That of the normalized W_q in this energy region is also quite obvious but quite smoother than that of $\bar{P}_q$. Furthermore, the influence of W_q on the polarization of a hadron can only be transferred via the flavor dependence of the corresponding FFs and can be weakened in kinematic regions, where the flavor dependence of FF is not strong. The scale dependence of FF is determined by QCD evolution and is perhaps the smoothest among the three factors [43] in the above-mentioned energy region. Hence, without detailed calculations, we can already expect that the longitudinal polarization of the hyperon in e^+e^--annihilation changes fast with energy, and the behavior is dominated by that of $\bar{P}_q$. In contrast, for $\bar{S}_{LL}$, the energy dependence comes only from that of W_q and FFs. There should be a much smoother energy dependence for S_{LL} in the energy region mentioned above. This is a clear prediction based on the general features of the QCD quark-parton picture and is independent of the detailed behaviors of FFs that can be tested by experiments.

Currently, our knowledge on the precise forms of FFs is still very limited due to limitations of data available, in particular, in the polarized case. It is quite fortunate that there are some data available from LEP [1–5] on the longitudinal polarization $P_{L\Lambda}$ of Λ and the spin alignment $\bar{\rho}_{00}^{K^*}$ of K^*. Although they are still very far from sufficient for a detailed analysis, we can use them to initialize such a study to demonstrate the essential features and guide future experimental studies.

A. Parametrizations and QCD evolutions of G_{1L} and D_{1LL}

Because of decay contributions, polarization of a Λ hyperon is much more involved than other hyperons and/or vector mesons. In general, the leading twist FF for a quark to a baryon, $q \to B_i$, can be written as the sum of a direct fragmentation and a decay contribution part, i.e.,

$$D_1^{q\to B_i}(z) = D_{1\mathrm{dir}}^{q\to B_i}(z) + \sum_{j\neq i} R_D^{ji} \int dz' K_D^{ji}(z,z') D_1^{q\to B_j}(z'), \tag{66}$$

$$G_{1L}^{q\to B_i}(z) = G_{1L,\mathrm{dir}}^{q\to B_i}(z) + \sum_{j\neq i} R_D^{ji} \times \int dz' K_D^{ji}(z,z') t_D^{ji}(z,z') G_{1L}^{q\to B_j}(z'), \tag{67}$$

where R_D^{ji} is the decay branch ratio of $B_j \to B_i + X$, $K_D^{ji}(z,z')$ is the probability for a B_j with z' to decay into a B_i with z, and $t_D^{ji}(z,z')$ is the spin transfer factor in the decay process.

Numerical results show that, for the Λ hyperon, the contributions from Σ^0 and $\Xi^{0,-}$ are sizable [7,8]. However, since there is no suitable data for Σ^0 or Ξ polarization in e^+e^- available yet, it is impossible to make such a detailed analysis. On the other hand, the energy dependences of the hadron polarizations that we will study in this paper come mainly from the QCD evolution of FFs and the energy dependence of the polarization of the quark produced at the e^+e^--annihilation vertex. We would expect that the influences from the decay contributions on the energy dependence are not very large. Hence, in this paper, as a rough estimation, we simply parametrize the final $D_1^{q\to\Lambda}(z)$ and $G_{1L}^{q\to\Lambda}(z)$, and evolve them to other energies using DGLAP equations given by Eq. (56).

Currently, for the unpolarized FF $D_1(z)$, there exist already a number of parametrizations in the literature for the production of hadrons such as pion, Kaon, proton, and Λ [48]. We can just take the most recent parametrizations AKK08 given in [49] for Λ.

For the polarized FF $G_{1L}^{q\to\Lambda}(z)$, a global fit and detailed analysis have already been given in 1998 in [43] by de Florian, Stratmann, and Vogelsang (DSV98) to the NLO in QCD evolution. However, as have already been pointed out in [43], the data available are far from sufficient to fix all different contributions. They had to make some assumptions such as that the heavy-flavor contributions are neglected, that the u and d fragmentations are taken as equal, and that the polarized unfavored and gluon FFs are taken as zero at the initial scale etc. in order to carry out the calculations. They presented also the results at different scenarios. The results already show explicitly that, compared to the drastic change of $\bar{P}_q$ with energy shown in Fig. 1, the scale dependence of FF is smooth and the difference between LO and NLO results is not very large.

There is not much progress on parametrizations of polarized FFs since DSV98 [43] besides some improvements for unpolarized FFs. There are, however, many phenomenological studies [6–19,50–52] using different models on hyperon polarization in different high energy reactions. We note, in particular, a series of analysis with the aids of the Monte Carlo event generator such as PYTHIA based on Lund string fragmentation model [53,54] and another series [12–18] based on the Gribov relation [55] between PDFs and FFs. With a Monte Carlo event generator, one can analyze in detail the influence of different contributions. From these different phenomenological studies [6–19,50–52], we see that although there are distinct differences in different models, some features are common. Consistent with the data available [1–5], all models seem to suggest that polarization dependent FFs are important only in the large z region. In the language of the Feynman-Field type of recursive cascade fragmentation picture [56–59], polarized FFs are dominated by the contributions of the "first rank" hadrons, i.e., those containing the fragmenting quarks and/or antiquarks. It implies also that the main features are determined by the "favored" fragmentations. The contributions from the "unfavored" and gluon fragmentations are from "higher rank" hadrons. They are, in general, small and have negligible dependence on the flavor of the initial quark (antiquark) or gluon. This is also consistent with the assumptions made by DSV in [43].

Since the purpose of the calculations in this section is to demonstrate the main feature of the energy dependence of hyperon polarization, we do not intend to make a best fit for the data available. Instead, we would like to pick up the most influential parts to demonstrate the main features expected. In view that the differences between LO and NLO QCD evolution results are not very large [43] and the accuracy that we can reach at this stage is not very high, we choose to do only a LO QCD evolution. We take a parametrization in the same form as DSV98 [43] in the second scenario with the same assumptions and/or approximations in connection with the heavy flavor contributions, u and d flavor dependence, unfavored, and gluon fragmentation at the initial scale. We will use the most recent parametrization for the corresponding unpolarized FFs and readjust the parameters to get a better fit to the LEP data [1,2]. More precisely, for the s-quark fragmentation, we take

$$G_{1L}^{s\to\Lambda}(z) = z^a D_1^{s\to\Lambda}(z), \tag{68}$$

while for u and d quark, we take

$$G_{1L}^{q\to\Lambda}(z) = N z^a D_1^{q\to\Lambda}(z), \tag{69}$$

where $q = u$ or d, and limit the parameters $a > 0$ and $|N| \leq 1$ so that the positivity bounds are satisfied [43]. We further limit N to be negative and small to be consistent with the expectation from the Gribov relation [55] and polarized PDFs [60]. We choose an initial scale at $Q = 5$ GeV and evolve the FFs to larger Q's and fix the parameters by the LEP data for Λ polarization [1,2]. In this way, we fix the parameters as $a = 0.5$ and $N = -0.1$. The result of the fit for Λ polarization is shown by the solid line in Fig. 2. The obtained $G_{1L}^{q\to\Lambda}(z, Q^2)$ at different Q for $q = u$, d, and s are shown in Fig. 3. We see that in general $G_{1L}^{s\to\Lambda}(z, Q^2)$ is positive and much larger than $G_{1L}^{u\to\Lambda}(z, Q^2)$ or $G_{1L}^{d\to\Lambda}(z, Q^2)$. The small difference between $G_{1L}^{u\to\Lambda}(z, Q^2)$ and $G_{1L}^{d\to\Lambda}(z, Q^2)$ comes from that between $D_1^{u\to\Lambda}(z, Q^2)$ and $D_1^{d\to\Lambda}(z, Q^2)$ from AKK08 [49].

For vector mesons such as K^*, the decay contributions are negligible. So we need to simply parametrize and evolve the corresponding D_1 and D_{1LL} to obtain $\bar{S}_{LL}$ at different energies. However, our knowledge about parametrizations of the corresponding FFs is even more limited than that for Λ. There is even no parametrization of FF in the unpolarized case available yet. As a start, we make a simple parametrization by using those for $K^\pm$ from AKK08 [49] and by parametrizing the data for the ratio of K^* to K

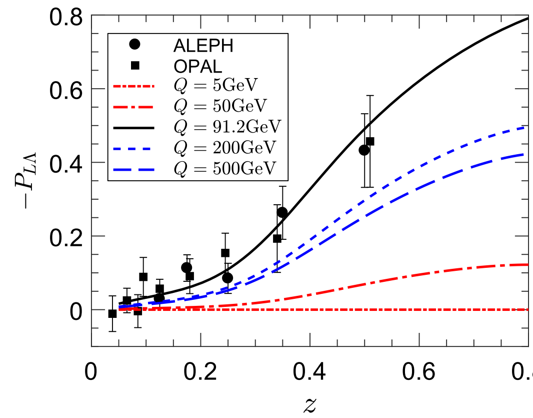

FIG. 2. Longitudinal polarization of Λ in $e^+e^- \to \Lambda X$ at high energies. The LEP data are taken from [1,2]. The solid line is the fit described in the text while those at other energies are calculated results using DGLAP for FFs and energy dependence of P_q.

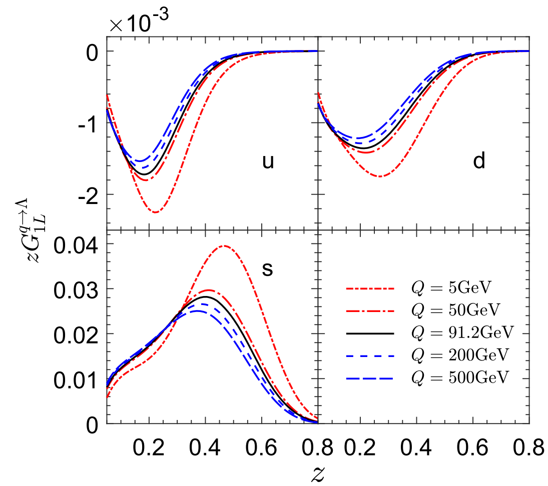

FIG. 3. The longitudinal spin transfer fragmentation function $G_{1L}(z, Q^2)$ for $q \to \Lambda$ as a function of z for different flavors of q at different values of Q. The solid lines are obtained by fitting data for $P_{L\Lambda}$ at $Q = M_Z$, and the others are obtained using DGLAP with leading order splitting functions described in the text.

as given in [61]. The z dependence for the ratio is taken as linear, i.e., $D_1^{K^*}(z)/3D_1^{K^+}(z) = 0.2z + 0.1$, and is assumed to be the same for different flavors.

For the S_{LL}-dependent FFs, $D_{1LL}^{q\to K^*}(z)$, we carry out the calculations at the same level as that given above for $G_{1L}^{q\to\Lambda}(z)$. Inspired by the almost linear z dependence of data of ρ_{00} [3], we parametrize $D_{1LL}^{q\to K^*}(z)$ as

$$D_{1LL}^{q\to K^*}(z) = c_1 D_1^{q\to K^*}(z), \tag{70}$$

for unfavored fragmentations and

$$D_{1LL}^{q\to K^*}(z) = (c_1 + c_2 z) D_1^{q\to K^*}(z), \tag{71}$$

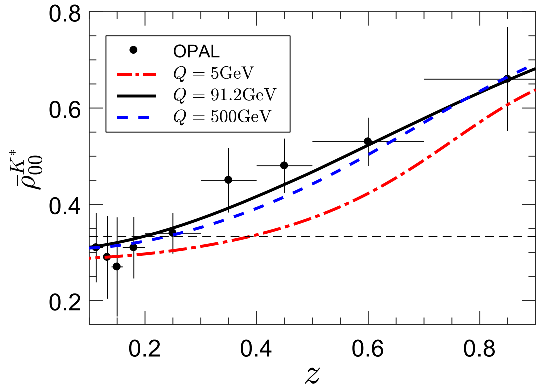

FIG. 4. Spin alignment of K^* as a function of z. The solid line is the fit described in the text while those at other energies are calculated results using DGLAP for FFs. The data points are from OPAL at LEP and are taken from [3].

for favored fragmentations. We limit $-3/2 < c_1 < 3$ and $-3/2 < c_1 + c_2 z < 3$ for $0 < z < 1$ to satisfy the positivity bound. By fitting the available z-dependence data on the spin alignment of K^* from OPAL [3], we fix the parameters as $c_1 = 0.15$ and $c_2 = -1.2$ at $Q = 5$ GeV. The fitted curve is presented in Fig. 4. The obtained $D_{1LL}^{q\to K^{*0}}(z, Q^2)$ at different Q for $q = u$, d, or s is given by the solid line in Fig. 5.

Because the data (see Fig. 4) for $\rho_{00}^{K^*}$ are larger than $1/3$ in the large z region, the S_{LL}-dependent FF $D_{1LL}(z, Q^2)$ should be negative in the corresponding z region. At small z, ρ_{00} is smaller than $1/3$, which implies a positive $D_{1LL}(z, Q^2)$. These features are shown clearly in Fig. 5, where we can see that for favored fragmentations, $D_{1LL}(z, Q^2)$ are negative at larger z while those for unfavored fragmentations also play an important role in the small z region and are positive.

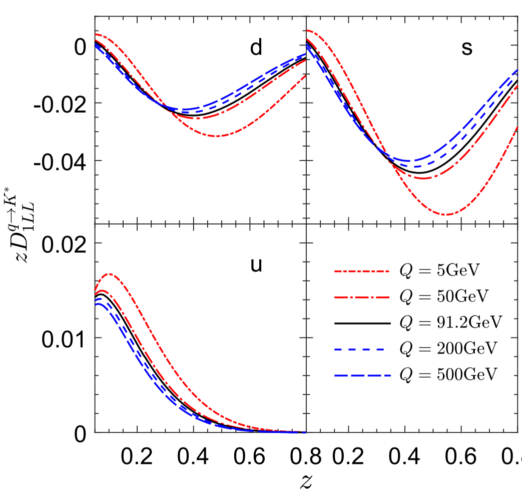

FIG. 5. The S_{LL} dependent fragmentation function $D_{1LL}(z, Q^2)$ for $q \to K^*$ as a function of z for different flavors of q at different values of Q. The solid lines are obtained by fitting data for $\rho_{00}^{K^*}$ at $Q = M_Z$, the others are obtained using DGLAP with leading order splitting functions described in the text.

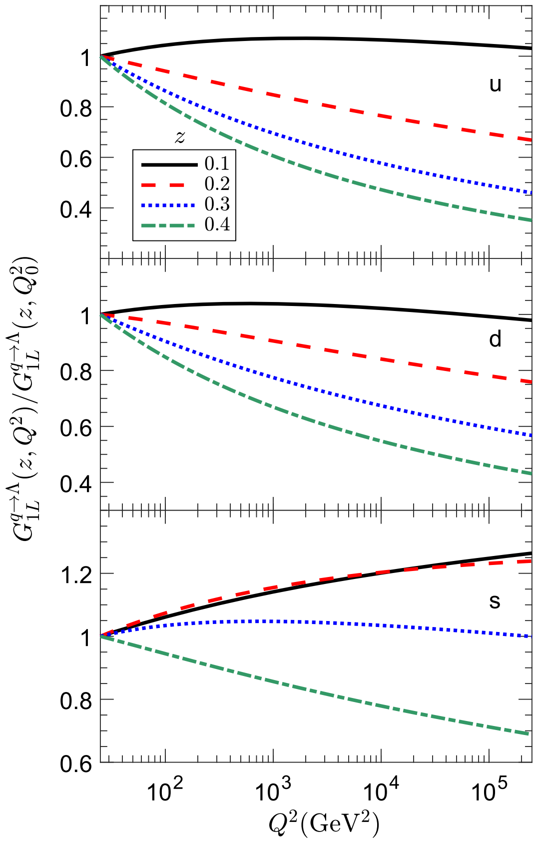

FIG. 6. QCD evolved $G_{1L}(z, Q^2)$ for $q \to \Lambda$ as a function of Q at a different z divided by the corresponding value at $Q_0 = 5$ GeV.

From Fig. 3, we see that the peaks for $zG_{1L}(z, Q^2)$ shift towards smaller z for larger Q. In the large z region, $G_{1L}(z, Q^2)$ decreases with increasing Q, while for small z, it increases with increasing Q. This former is shown more obviously in Fig. 6, where $G_{1L}^{q\to\Lambda}(z, Q^2)$ as a function of Q at different values of z is shown. Similarly, in the large z region, we see the same tendency for $D_{1LL}(z, Q^2)$ as a function of Q from Figs. 5 and 7, i.e., the magnitude of $D_{1LL}(z, Q^2)$ also decreases with increasing Q for large z values. The relative rapid changes for the corresponding $D_{1LL}^{q\to K^*}(z, Q^2)$ at $z = 0.1$ for $q = d$ or s is due to the crossover with zero at $z \sim 0.1-0.2$. We see clearly that the magnitudes of these FFs do not change with Q as drastically as P_q does (see Fig. 1). We therefore expect that the energy dependence of $P_{L\Lambda}$ should be dominated by that of P_q and that of $\rho_{00}^{K^*}$ is smooth.

B. Energy dependence of P_Λ and $\bar{\rho}_{00}^{K^*}$

By inserting these results for FFs at different Q, we obtain $P_{L\Lambda}$ and $\bar{\rho}_{00}^{K^*}$ at different energies $\sqrt{s} = Q$. We plot the results in Figs. 2 and 4, respectively.

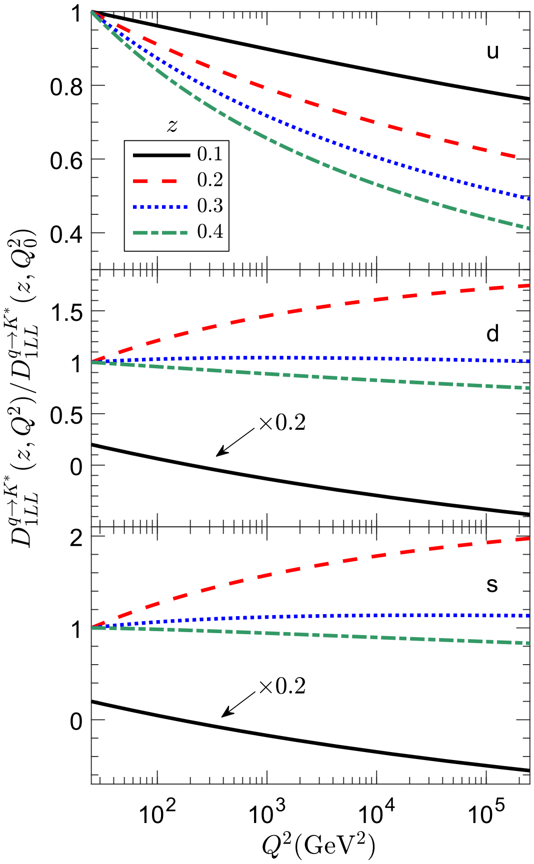

FIG. 7. QCD evolved $D_{1LL}(z, Q^2)$ for $q \to K^*$ as function of Q at a different z divided by the corresponding value at $Q_0 = 5$ GeV.

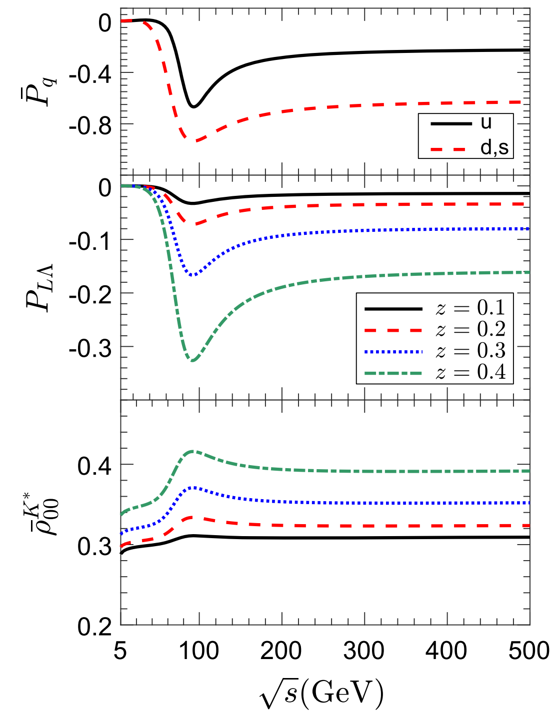

FIG. 8. Energy dependence of the longitudinal Λ polarization in e^+e^- annihilation.

From Figs. 2 and 4, we see clearly that there is a strong energy dependence for $P_{L\Lambda}$, whereas that for $\bar{\rho}_{00}^{K^*}$ is quite weak. The former comes mainly from the energy dependence of P_q, while the latter comes mainly from QCD evolution of FFs. To show this more explicitly, we plot $P_{L\Lambda}$ at a given z as a function of Q in the same figure as $\bar{P}_q$ in Fig. 8. For comparison, we also plot $\bar{\rho}_{00}^{K^*}$ in the third panel of the same figure.

From Fig. 8, we explicitly see that $P_{L\Lambda}$ behaves in very much the same way as P_q as functions of Q. This shows clearly that the energy dependence of $P_{L\Lambda}$ is dominated by that of P_q. We see in particular that just like $\bar{P}_q$, $\bar{P}_{L\Lambda}$ changes very fast with energy and goes to zero when Q deviates from $Q = M_Z$ for $Q < M_Z$. This is because at smaller Q, electromagnetic interaction becomes dominant and weak interaction via exchange of Z boson becomes negligible rapidly. Whereas at large Q, although smaller than that at the Z pole, it is still sizable and becomes quite flat with increasing Q. The results show, in particular, that at BES or BELLE energies, $P_{L\Lambda}$ should be negligibly small. Furthermore, from the results presented in Sec. II D such as Eqs. (40)–(45), we see that there is no twist-3 contribution to $P_{L\Lambda}$ but there can be a twist-3 contribution to the transverse components. Higher twist contributions to $P_{L\Lambda}$ come only from a twist-4 or even higher twist [29]. This implies that at BES energies, the transverse components could even become larger than the longitudinal component for a given region of θ or y.

In contrast to $P_{L\Lambda}$, $\bar{\rho}_{00}$ changes with Q quite weakly and remains sizable even at BES energies. The relatively rapid change in the energy region around M_Z comes from the influence of W_q. This is a clear prediction that can be tested by future experiments [62].

At the end of this section, we would like to emphasize once more the following. Since the energy dependence of $P_{L\Lambda}$ is dominated by P_q, the influence from other effects such as heavy flavor contribution, u and d flavor dependence, "unfavored", and gluon fragmentation etc. contribute only to the fine structure of the results shown in Fig. 8. Lacking data and other related information, we simply neglected them at the initial scale in obtaining the results in Fig. 8. However, they are definitely worthwhile for experimental and theoretical studies in the future. Furthermore, since they are addenda to the contribution from P_q in the case of $P_{L\Lambda}$, it might be more difficult to separate them from each other. On the other hand, there is no contribution from P_q for a vector meson spin alignment ρ_{00}. This means that such effects should play more important roles and manifest themselves more explicitly in different properties of ρ_{00}. It could be much easier to study them in detail by studying ρ_{00}. In this sense, vector meson spin alignment could be a much better place to study different contributions in detail. Furthermore, since it is independent of initial quark polarization, it is also foreseeable that the effect of tensor polarization determined by ρ_{00} can also be studied in other high energy reactions independent of whether the initial hadron is polarized.

IV. SUMMARY AND OUTLOOK

Using the longitudinal polarization $P_{L\Lambda}$ of a Λ hyperon and the spin alignment $\rho_{00}^{K^*}$ of K^{*0} as representative examples, we demonstrate the two very different behaviors in energy dependences of hadron polarizations in e^+e^- annihilations. The results show clearly that $P_{L\Lambda}$ has a very strong energy dependence due to its direct dependence on the initial longitudinal polarization P_q of the quark q, while $\rho_{00}^{K^*}$ has a rather weak energy dependence since it is independent of P_q. The former is dominated by the energy dependence of P_q, while the latter comes mainly from the QCD evolutions of the FFs. We have presented the results at the leading twist with pQCD evolution at the leading order. In view that the measurements of both $P_{L\Lambda}$ and $\rho_{00}^{K^*}$ can, in principle, be easily carried out in experiments at BES or BELLE, we think that this provides a good place to test QCD evolutions of FFs and/or to check whether higher twist effects are important.

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