Hidden-charm pentaquarks and their hidden-bottom and B_c-like partner states

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In the framework of the color-magnetic interaction, we have systematically studied the mass splittings of the possible hidden-charm pentaquarks $qqqc\bar{c}$ ($q = u, d, s$) where the three light quarks are in a color-octet state. We find that (i) the LHCb P_c states fall in the mass region of the studied system, (ii) most pentaquarks should be broad states since their S-wave open-charm decays are allowed while the lowest state is the $J^P = \frac{1}{2}^-$ Λ-like pentaquark with probably the suppressed $\eta_c \Lambda$ decay mode only, and (iii) the $J^P = \frac{5}{2}^-$ states do not decay through the S wave and their widths are not so broad. The masses and widths of the two LHCb P_c baryons are compatible with such pentaquark states. We also explore the hidden-bottom and B_c-like partners of the hidden-charm states and find the possible existence of the pentaquarks which are lower than the relevant hadronic molecules.

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I. INTRODUCTION

In 2015, the LHCb Collaboration [1] reported two pentaquarklike resonances $P_c(4380)$ and $P_c(4450)$ in the process $\Lambda_b^0 \to J/\psi K^- p$ with the same decay mode $J/\psi p$. The decay channel indicates that their minimal quark content is $nnnc\bar{c}$ ($n = u, d$). The resonance parameters are $M_{P_c(4380)} = 4380 \pm 8 \pm 29$, $\Gamma_{P_c(4380)} = 205 \pm 18 \pm 86$, and $M_{P_c(4450)} = 4449.8 \pm 1.7 \pm 2.5$, $\Gamma_{P_c(4450)} = 39 \pm 5 \pm 19$ MeV. The preferred angular momenta are $\frac{3}{2}$ and $\frac{5}{2}$, respectively, and their P parities are opposite. Later, the $P_c(4380)$ and $P_c(4450)$ were confirmed by the reanalysis with a model-independent method [2]. Recently, these two P_c states were also observed in the $\Lambda_b^0 \to J/\psi p \pi^-$ decay [3].

In fact, the theoretical exploration of the hidden-charm pentaquarks was performed before the observation of two P_c states by LHCb. In Refs. [4,5], the authors predicted two $N^*_{c\bar{c}}$ states and four $\Lambda^*_{c\bar{c}}$ states, where their masses, decay behaviors and production properties were given in a coupled-channel unitary approach. Possible molecular states composed of a charmed baryon and an anticharmed meson were systematically studied with the one-boson-exchange (OBE) model in Ref. [6] and the chiral quark model in Ref. [7]. More investigations can be found in Refs. [8–15]. In addition, Li and Liu indicated the existence of hidden-charm pentaquarks by the analysis of a global group structure [16].

After the announcement of the $P_c(4380)$ and $P_c(4450)$ states by LHCb, these two P_c states were interpreted as $\Sigma_c \bar{D}^*$, $\Sigma_c^* \bar{D}$, or $\Sigma_c^* \bar{D}^*$ molecules [17–28], bound states, or resonances of charmonium and nucleon [29–32], diquark-diquark-antiquark states [33–38], diquark-triquark states [39,40], compact pentaquark states [41–43], kinematical

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effects due to $\chi_{c1}p$ rescattering [44], due to triangle singularity [45–47], or due to a $\bar{D}$ soliton [48], or a bound state of the colored baryon and meson [49]. Their decay and production properties were studied in Refs. [50–66].

The observation of these P_c resonances also stimulated the arguments for more possible pentaquarks [67–71]. Productions of another N^* and $\Lambda^*_{c\bar{c}}$ were discussed in Ref. [72] and Refs. [73–75], respectively. For the detailed overview on the hidden-charm pentaquarks, the readers may refer to Refs. [76,77].

The dynamical calculations of the bound states are relatively easier if one treats the system as two clusters. The investigation at the quark level is also simplified when one assumes the existence of substructures in a five-body system. If a hidden-charm pentaquark really exists, its spin partners with the same flavor content should also exist. The existence of substructures certainly results in less pentaquarks. Needless to say, configurations with various substructures (baryon-meson, diquark-diquark-antiquark, or diquark-triquark) lead to different results. From symmetry consideration, a physical pentaquark state should be a mixture of all these configurations with various color structures. Here we would like to explore a pentaquark structure without the assumption of its substructure.

The masses of the P_c's are both above the threshold of $J/\psi p$. Because the interaction between the J/ψ and nucleon is very weak, the scattering resonances in this channel are not appropriate interpretations for the observed P_c states. We focus on the possible pentaquark configurations where either the three light quark qqq or the $c\bar{c}$ pair is a color octet state. We investigate whether the lower P_c state can be assigned as a tightly bound five-quark state and explore its possible partner states. Recently, there appeared a preliminary quark model study on the hidden color-octet uud baryons [43].

In principle, a dynamical calculation for a five-body problem is needed in order to calculate their masses. In this work, we calculate their mass splittings with a simple color-magnetic interaction from the one-gluon-exchange (OGE) potential. For example, the Δ^+ baryon and the proton have the same quark content and color structure and their mass difference mainly arises from the color-magnetic interaction. With the calculated mass splittings and a reference threshold, one can estimate the pentaquark masses roughly.

This paper is organized as follows. In Sec. II, we construct the $flavor \otimes color \otimes spin$ wave functions of the hidden-charm pentaquark states and calculate the matrix elements for the color magnetic interaction in the symmetric limit. Then we consider the flavor-breaking case in Sec. III and give numerical results in Sec. IV. In Sec. V, we explore the heavier pentaquarks. We discuss our results and summarize in the final section.

II. WAVE FUNCTIONS AND COLOR-MAGNETIC INTERACTION

The color-magnetic Hamiltonian reads

$$H = \sum_i m_i + H_{CM},$$
$$H_{CM} = -\sum_{i<j} C_{ij}\lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j, \tag{1}$$

where the ith Gell-Mann matrix λ_i should be replaced with $-\lambda_i^*$ for an antiquark. In the Hamiltonian, m_i is the effective mass of the ith quark and $C_{ij} \sim \langle\delta(r_{ij})\rangle/(m_i m_j)$ is the effective coupling constant between the ith quark and the jth quark. The values of the parameters for light quarks and those for heavy quarks are different, and they will be extracted from the known hadron masses. For the hidden-charm systems, we have four types of coupling parameters C_{qq}, C_{qc}, $C_{q\bar{c}}$, and $C_{c\bar{c}}$ with $q = u, d, s$. More parameters need to be determined for the other $qqqQ\bar{Q}$ pentaquarks ($Q = b, c$). In Ref. [78], we have estimated the mass of another not-yet-observed but plausible exotic meson T_{cc} with this simple model. In Refs. [79–81], we discussed the mass splittings for the $QQ\bar{Q}\,\bar{Q}$, $cs\bar{c}\bar{s}$, and $QQ\bar{Q}\,\bar{q}$ systems, respectively.

To calculate the required matrix elements of the color-magnetic interaction (CMI), we here construct the flavor-color-spin wave functions of the ground state pentaquark systems. These wave functions will also be useful in the study of other properties of the pentaquak states in quark models. In Ref. [8], a study with the color-magnetic interaction is also involved but the wave functions are constructed with flavor $SU(4)$ symmetry. Now we consider flavor $SU(3)$ symmetry and treat the heavy (anti)quark as a flavor singlet state.

Because the three light quarks qqq must obey the Pauli principle, it is convenient to discuss the constraint with flavor-spin $SU(6)_{fs}$ symmetry. The three-quark colorless ground baryons belong to the symmetric $[3] = 56$ representation. Its $SU(3)_f \otimes SU(2)_s$ decomposition gives $(10,4) + (8,2)$ and, therefore, the flavor singlet baryon in $3 \otimes 3 \otimes 3 = 10 + 8 + 8 + 1$ is forbidden. Now the color-octet qqq must belong to the mixed $[21] = 70$ $SU(6)_{fs}$ representation. The $SU(3)_f \otimes SU(2)_s$ decomposition gives $(10,2) + (8,4) + (8,2) + (1,2)$. So the flavor singlet pentaquark is allowed, and we have two flavor octets with different spins. There is no symmetry constraint for the heavy quark pair, and one finally gets three pentaquark decuplets with $J = 1/2$, $1/2$, and $3/2$, three octets with $J = 1/2$, $1/2$, and $3/2$, four octets with $J = 3/2$, $1/2$, $3/2$, and $5/2$, and three singlets with $J = 1/2$, $1/2$, and $3/2$.

To get a totally antisymmetric qqq wave function, we need the components presented in Table I and we need to make appropriate combinations. The notation MS (MA) means that the first two quarks are symmetric

TABLE I. Flavor multiplets and wave functions of the colored qqq in different spaces. Young diagrams for the color-spin $SU(6)_{cs}$ are also given in the first column.

Multiplet	Space	Wave function		Wave function	
10_f	Color	ϕ^{MS}		ϕ^{MA}	
	Spin	χ^{MA}		χ^{MS}	
$[111]_{cs}$	Flavor	F^S		F^S	
1_f	Color	ϕ^{MS}		ϕ^{MA}	
	Spin	χ^{MS}		χ^{MA}	
$[3]_{cs}$	Flavor	F^A		F^A	
$8_f(1)$	Color	ϕ^{MS}		ϕ^{MA}	
	Spin	χ^S		χ^S	
$[21]_{cs}$	Flavor	F^{MA}		F^{MS}	
$8_f(2)$	Color	ϕ^{MS}		ϕ^{MA}	
	Spin	χ^{MS}	χ^{MA}	χ^{MS}	χ^{MA}
$[21]_{cs}$	Flavor	F^{MA}	F^{MS}	F^{MS}	F^{MA}

(antisymmetric) when they are exchanged in corresponding space and the superscript S (A) means that the wave function is totally symmetric (antisymmetric). When one combines the color and spin (or flavor and color, or flavor and spin), a wave function with a required symmetry is determined by the relative sign between different components. For example, $(\phi^{MS}\otimes\chi^{MS}+\phi^{MA}\otimes\chi^{MA})$ is symmetric for the exchange of color and spin indices simultaneously for any two quarks. If one uses a minus sign, the wave function is symmetric only for the first two quarks, i.e. a mixed MS-type color-spin wave function. One has to adopt a self-consistent convention for the SU(3) Clebsch-Gordan (C.G.) coefficients when constructing the wave functions. We here take the convention convenient for use [82,83]. For clarity, we present the color wave functions of qqq in Fig. 1 and those of $c\bar{c}$ in Fig. 2. The flavor octet wave functions are easy to obtain with the replacements $r\to u$, $g\to d$, and $b\to s$. The spin wave functions for the spin-half case are $\chi_\uparrow^{MS}=-\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\uparrow-2\uparrow\uparrow\downarrow]$, $\chi_\downarrow^{MS}=\frac{1}{\sqrt{6}}[(\uparrow\downarrow+\downarrow\uparrow)\downarrow-2\downarrow\downarrow\uparrow]$, $\chi_\uparrow^{MA}=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\uparrow$, and $\chi_\downarrow^{MA}=\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\downarrow$. There is no confusion for the totally symmetric flavor and spin wave functions and

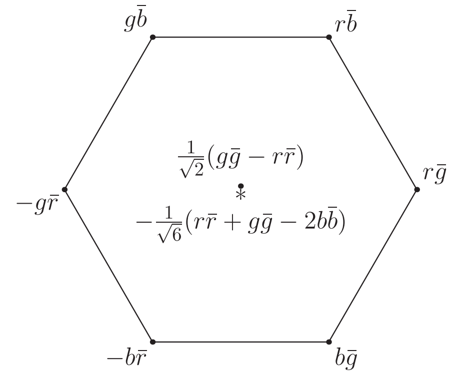

FIG. 2. Color wave functions of the heavy quark pair.

we do not show them explicitly. With explicit calculation, we find the totally antisymmetric wave functions of the colored qqq states and show them in Table II.

With the above wave functions and the C.G. coefficients of SU(3) [82,83] and SU(2), one can construct the pentaquark wave functions. We here only show the color part,

$$\begin{aligned}\phi_{\text{penta}}^{MS,MA}=\frac{1}{2\sqrt{2}}\Big[&-p_C^{MS,MA}(b\bar{r})-n_C^{MS,MA}(b\bar{g})-\Sigma_C^{+MS,MA}(g\bar{r})\\&+\Sigma_C^{-MS,MA}(r\bar{g})-\Xi_C^{0MS,MA}(g\bar{b})+\Xi_C^{-MS,MA}(r\bar{b})\\&-\frac{1}{\sqrt{2}}\Sigma_C^{0MS,MA}(g\bar{g}-r\bar{r})\\&+\frac{1}{\sqrt{6}}\Lambda_C^{MS,MA}(r\bar{r}+g\bar{g}-2b\bar{b})\Big],\end{aligned}\tag{2}$$

where the baryon symbols with the subscript "C" are borrowed from the flavor octet and represent the color wave functions in Fig. 1. The structure of the full wave functions is the same as that in Table II by adding a subscript "penta" to each wave function.

Since the color-spin interaction is the same for baryons in the same flavor multiplet in the SU(3) limit, it is enough to consider only pentaquarks with flavor content $uuuc\bar{c}$ in the decuplet, $uudc\bar{c}$ in the octet, and $udsc\bar{c}$ in the singlet. After some calculations, we get the results for the $\langle H_{CM}\rangle$ as follows,

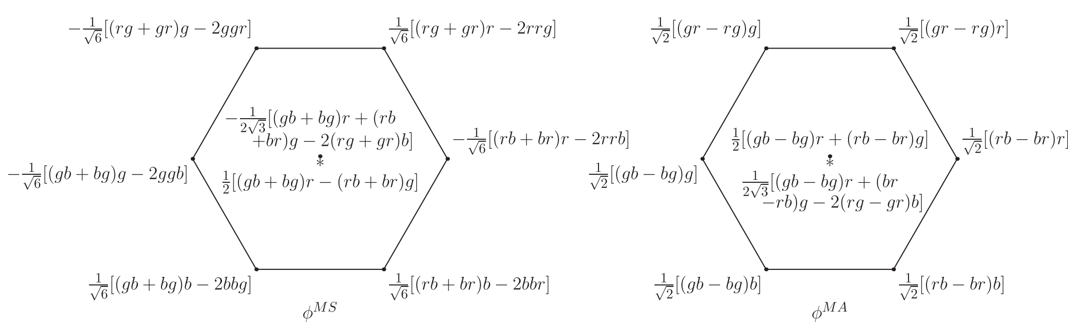

FIG. 1. Color wave functions of the light quarks.

$$10_f:\ \langle H_{CM}\rangle = 10C_{qq} + 2C_{c\bar{c}}, \quad \text{for}\ \left(S_{c\bar{c}} = 0, J = \frac{1}{2}\right)$$
$$\langle H_{CM}\rangle = 10C_{qq} - \frac{2}{3}C_{c\bar{c}} - \frac{20}{3}(C_{qc} - C_{q\bar{c}}), \quad \text{for}\ \left(S_{c\bar{c}} = 1, J = \frac{1}{2}\right)$$
$$\langle H_{CM}\rangle = 10C_{qq} - \frac{2}{3}C_{c\bar{c}} + \frac{10}{3}(C_{qc} - C_{q\bar{c}}), \quad \text{for}\ \left(S_{c\bar{c}} = 1, J = \frac{3}{2}\right), \tag{3}$$

$$1_f:\langle H_{CM}\rangle = -14C_{qq} + 2C_{c\bar{c}}, \quad \text{for}\ \left(S_{c\bar{c}} = 0, J = \frac{1}{2}\right)$$
$$\langle H_{CM}\rangle = -14C_{qq} - \frac{2}{3}C_{c\bar{c}} - \frac{4}{3}(C_{qc} + 11C_{q\bar{c}}), \quad \text{for}\ \left(S_{c\bar{c}} = 1, J = \frac{1}{2}\right)$$
$$\langle H_{CM}\rangle = -14C_{qq} - \frac{2}{3}C_{c\bar{c}} + \frac{2}{3}(C_{qc} + 11C_{q\bar{c}}), \quad \text{for}\ \left(S_{c\bar{c}} = 1, J = \frac{3}{2}\right), \tag{4}$$

$$8_f(1):\ \langle H_{CM}\rangle = 2C_{qq} + 2C_{c\bar{c}}, \quad \text{for}\ \left(S_{c\bar{c}} = 0, J = \frac{3}{2}\right)$$
$$\langle H_{CM}\rangle = 2C_{qq} - \frac{2}{3}C_{c\bar{c}} - 10(C_{qc} + C_{q\bar{c}}), \quad \text{for}\ \left(S_{c\bar{c}} = 1, J = \frac{1}{2}\right)$$
$$\langle H_{CM}\rangle = 2C_{qq} - \frac{2}{3}C_{c\bar{c}} - 4(C_{qc} + C_{q\bar{c}}), \quad \text{for}\ \left(S_{c\bar{c}} = 1, J = \frac{3}{2}\right)$$
$$\langle H_{CM}\rangle = 2C_{qq} - \frac{2}{3}C_{c\bar{c}} + 6(C_{qc} + C_{q\bar{c}}), \quad \text{for}\ \left(S_{c\bar{c}} = 1, J = \frac{5}{2}\right), \tag{5}$$

$$8_f(2):\ \langle H_{CM}\rangle = -2C_{qq} + 2C_{c\bar{c}}, \quad \text{for}\ \left(S_{c\bar{c}} = 0, J = \frac{1}{2}\right)$$
$$\langle H_{CM}\rangle = -2C_{qq} - \frac{2}{3}C_{c\bar{c}} - 4(C_{qc} + C_{q\bar{c}}), \quad \text{for}\ \left(S_{c\bar{c}} = 1, J = \frac{1}{2}\right)$$
$$\langle H_{CM}\rangle = -2C_{qq} - \frac{2}{3}C_{c\bar{c}} + 2(C_{qc} + C_{q\bar{c}}), \quad \text{for}\ \left(S_{c\bar{c}} = 1, J = \frac{3}{2}\right). \tag{6}$$

One may confirm the part for C_{qq} with the formula [84,85]

$$\left\langle \sum_{i<j}(\lambda_i\cdot\lambda_j)(\sigma_i\cdot\sigma_j)\right\rangle = -\left[8N + \frac{4}{3}S(S+1) + 2C_2[SU(3)_c] - 4C_2[SU(6)_{cs}]\right], \tag{7}$$

where $N = 3$, $S = \frac{1}{2}$ or $\frac{3}{2}$, and $C_2[SU(g)]$ is the quadratic Casimir operator specified by the Young diagram $[f_1, \ldots, f_g]$

$$C_2[SU(g)] = \frac{1}{2}\left[\sum_i f_i(f_i - 2i + g + 1) - \frac{N^2}{g}\right]. \tag{8}$$

The Young diagram for color symmetry is $[21]_c$ and those for color-spin $SU(6)_{cs}$ symmetry can be found in Table I. The part for $C_{c\bar{c}}$ can be verified with the formula

$$\langle(\lambda_4\cdot\lambda_5)(\sigma_4\cdot\sigma_5)\rangle = 4\left[C_2[SU(3)_c] - \frac{8}{3}\right]\left[S_{c\bar{c}}(S_{c\bar{c}}+1) - \frac{3}{2}\right]. \tag{9}$$

TABLE II. Antisymmetric wave function for a color-octet qqq state.

Multiplet	Flavor-color-spin wave function
10_f	$\frac{1}{\sqrt{2}}[F^S\otimes(\phi^{MS}\otimes\chi^{MA} - \phi^{MA}\otimes\chi^{MS})]$
1_f	$\frac{1}{\sqrt{2}}[F^A\otimes(\phi^{MS}\otimes\chi^{MS} + \phi^{MA}\otimes\chi^{MA})]$
$8_f(1)$	$\frac{1}{\sqrt{2}}[(F^{MS}\otimes\phi^{MA} - F^{MA}\otimes\phi^{MS})\otimes\chi^S]$
$8_f(2)$	$\frac{1}{2}[(F^{MS}\otimes\chi^{MA} + F^{MA}\otimes\chi^{MS})\otimes\phi^{MS}$ $+(F^{MS}\otimes\chi^{MS} - F^{MA}\otimes\chi^{MA})\otimes\phi^{MA}]$

However, it is problematic to discuss mass splittings for pentaquarks with Eqs. (3)–(6) directly because of violations of the heavy quark spin symmetry (HQSS) and the flavor SU(3) symmetry.

The heavy quark symmetry is strict in the limit $m_c \to \infty$, which leads to the irrelevance of the heavy quark spin (and flavor) for the interaction between a heavy quark and a light quark. In this limit, the interaction within the heavy quark pair is also irrelevant with their spin (but not the flavor) and the spin-flip between the $S_{c\bar{c}} = 0$ case and the $S_{c\bar{c}} = 1$ case is suppressed. The color-magnetic interaction obviously violates HQSS. This means that only C_{qq} terms in Eqs. (3)–(6) are important in the heavy quark limit and there are four degenerate multiplets with the mass ordering: 10_f, $8_f(1)$, $8_f(2)$, and 1_f from high to low.

After the heavy quark mass correction is included, all the terms involving m_c in Eqs. (3)–(6) contribute. Since now the spin-flip between the $S_{c\bar{c}} = 0$ case and the $S_{c\bar{c}} = 1$ case is considered, the mixing between states with the same J occurs, which results from the term proportional to $1/m_c m_q$. Then one should determine the final $\langle H_{CM}\rangle$'s for mixed states by diagonalizing the specified matrix.

Usually, the flavor mixing between different multiplet representations occurs once the symmetry breaking is considered. In the present case, even in the $SU(3)_f$ limit, the mixing between the two octets is nonvanishing, which complicates the color magnetic interactions. To be convenient, we now collect the averages of the CMI in a matrix form for the $nnnc\bar{c}$ ($n = u$, d) and $sssc\bar{c}$ cases. For the other cases containing the s quark, we show results in the next section.

For $I = \frac{3}{2}$ $nnnc\bar{c}$ states (3 baryons in 10_f),

$$\begin{aligned}
\langle H_{CM}\rangle_{J=\frac{3}{2}} &= 10C_{nn} + \frac{10}{3}(C_{nc} - C_{n\bar{c}}) - \frac{2}{3}C_{c\bar{c}},\\
\langle H_{CM}\rangle_{J=\frac{1}{2}} &= \begin{pmatrix} 10C_{nn} - \frac{20}{3}(C_{nc} - C_{n\bar{c}}) - \frac{2}{3}C_{c\bar{c}} & \frac{10}{\sqrt{3}}(C_{nc} + C_{n\bar{c}}) \\ & 10C_{nn} + 2C_{c\bar{c}} \end{pmatrix}.
\end{aligned} \tag{10}$$

One gets similar expressions for the $I = 0$ $sssc\bar{c}$ states (3 baryons in 10_f) by replacing n with s.

For $I = 1/2$ $nnnc\bar{c}$ states (7 baryons in 8_f), the results read

$$\begin{aligned}
\langle H_{CM}\rangle_{J=\frac{5}{2}} &= 2C_{nn} + 6(C_{nc} + C_{n\bar{c}}) - \frac{2}{3}C_{c\bar{c}},\\
\langle H_{CM}\rangle_{J=\frac{3}{2}} &= \begin{pmatrix} 2C_{nn} - 4(C_{nc} + C_{n\bar{c}}) - \frac{2}{3}C_{c\bar{c}} & 2\sqrt{15}(C_{nc} - C_{n\bar{c}}) & -\frac{2\sqrt{10}}{3}(C_{nc} - 4C_{n\bar{c}}) \\ & 2(C_{nn} + C_{c\bar{c}}) & \frac{2\sqrt{6}}{3}(C_{nc} + 4C_{n\bar{c}}) \\ & & -2C_{nn} + 2(C_{nc} + C_{n\bar{c}}) - \frac{2}{3}C_{c\bar{c}} \end{pmatrix},\\
\langle H_{CM}\rangle_{J=\frac{1}{2}} &= \begin{pmatrix} 2C_{nn} - 10(C_{nc} + C_{n\bar{c}}) - \frac{2}{3}C_{c\bar{c}} & -\frac{4}{\sqrt{3}}(C_{nc} + 4C_{n\bar{c}}) & -\frac{4}{3}(C_{nc} - 4C_{n\bar{c}}) \\ & 2(-C_{nn} + C_{c\bar{c}}) & 2\sqrt{3}(C_{nc} - C_{n\bar{c}}) \\ & & -2C_{nn} - 4(C_{nc} + C_{n\bar{c}}) - \frac{2}{3}C_{c\bar{c}} \end{pmatrix},
\end{aligned} \tag{11}$$

where the bases for $J = \frac{3}{2}$ and $J = \frac{1}{2}$ are $(8_f(1)^{[S_{c\bar{c}}=1]}, 8_f(1)^{[S_{c\bar{c}}=0]}, 8_f(2)^{[S_{c\bar{c}}=1]})^T$ and $(8_f(1)^{[S_{c\bar{c}}=1]}, 8_f(2)^{[S_{c\bar{c}}=0]}, 8_f(2)^{[S_{c\bar{c}}=1]})^T$, respectively.

III. $SU(3)_f$ BREAKING

Up to now, we have not considered the $SU(3)_f$ breaking. Once the mass difference between the strange quark and the u, d quarks is included, the general mixing between flavor multiplets appears. Such an effect is included in the color-magnetic interaction and we now discuss this case. The systems we need to consider additionally are $nnsc\bar{c}$ and $ssnc\bar{c}$. They are classified into two categories according to the symmetry for the first two quarks in flavor space: symmetric $nnsc\bar{c}$ ($I = 1$) and $ssnc\bar{c}$ ($I = 0$) and antisymmetric $nnsc\bar{c}$ ($I = 0$). In the following, we use the symbol like $[(qqq')^{MA}_{MS}(c\bar{c})^0_8]^J$ to denote the base states. In this example, the subscript MS means that the color representation for the (qqq') is 8^{MS} and the color wave function is ϕ^{MS}. The subscript 8 is the color representation for the $(c\bar{c})$. The superscript MA means that the spin wave function for the (qqq') is χ^{MA} and the spin is $1/2$. The superscript 0 (J) indicates the spin of the $(c\bar{c})$ (pentaquark).

The calculation method can be found in Refs. [86,87]. We first give the results for the symmetric category. For the case $J = \frac{5}{2}$,

$$\langle H_{CM}\rangle_{J=\frac{5}{2}} = \frac{2}{3}(4C_{12} - C_{13} + 7C_{14} + 2C_{15} + 2C_{34} + 7C_{35} - C_{45}). \tag{12}$$

There is only one base state $[(qqq')^S_{MA}(c\bar{c})^1_8]^{\frac{5}{2}}$. For the case $J=\frac{3}{2}$, we use the base vector $([(qqq')^S_{MA}(c\bar{c})^1_8]^{\frac{3}{2}}$, $[(qqq')^S_{MA}(c\bar{c})^0_8]^{\frac{3}{2}}$, $[(qqq')^{MS}_{MA}(c\bar{c})^1_8]^{\frac{3}{2}}$, $[(qqq')^{MA}_{MS}(c\bar{c})^0_8]^{\frac{3}{2}})^T$ and get

$$\langle H_{CM}\rangle_{J=\frac{3}{2}} = \begin{pmatrix} \frac{2}{9}(3\mu - 2\alpha - 2\gamma) & \frac{2}{9}\sqrt{15}(\beta+\delta) & -\frac{2\sqrt{5}}{9}(\alpha - 2\gamma) & \frac{\sqrt{5}}{21}(13\alpha - 15\beta) \\ & \frac{2}{3}(8\lambda - 4\nu - 7\mu) & \frac{2\sqrt{3}}{9}(\beta - 2\delta) & \frac{\sqrt{3}}{21}(15\alpha - 13\beta) \\ & & \frac{2}{9}(3\nu + 2\alpha - \gamma) & \frac{1}{21}(42\mu - 42\nu + 13\alpha - 15\beta) \\ & & & \frac{1}{21}(14\lambda + 13\gamma + 15\delta) \end{pmatrix}, \tag{13}$$

where $\alpha = 7C_{14} + 2C_{15}$, $\beta = 7C_{14} - 2C_{15}$, $\gamma = 2C_{34} + 7C_{35}$, $\delta = 2C_{34} - 7C_{35}$, $\mu = 4C_{12} - C_{13} - C_{45}$, $\nu = 4C_{12} + 2C_{13} - C_{45}$, and $\lambda = 6C_{12} - C_{45}$. For the case $J=\frac{1}{2}$, we have

$$\langle H_{CM}\rangle_{J=\frac{1}{2}} = \begin{pmatrix} \frac{2}{9}(3\mu - 5\alpha - 5\gamma) & \frac{2\sqrt{2}}{9}(-\alpha + 2\gamma) & \frac{2\sqrt{6}}{9}(-\beta + 2\delta) & \frac{\sqrt{2}}{21}(13\alpha - 15\beta) & -\frac{\sqrt{6}}{21}(15\alpha - 13\beta) \\ & \frac{2}{9}(3\nu - 4\alpha + 2\gamma) & \frac{2\sqrt{3}}{9}(2\beta - \delta) & \frac{2}{21}\begin{pmatrix} 21\mu - 21\nu \\ -13\alpha + 15\beta \end{pmatrix} & -\frac{\sqrt{3}}{21}(15\alpha - 13\beta) \\ & & \frac{2}{3}(8\lambda - 8\mu - 3\nu) & -\frac{\sqrt{3}}{21}(15\alpha - 13\beta) & 2(\mu - \nu) \\ & & & \frac{2}{21}(7\lambda - 13\gamma - 15\delta) & \frac{\sqrt{3}}{21}(15\gamma + 13\delta) \\ & & & & 2(2C_{12} + C_{45}) \end{pmatrix} \tag{14}$$

with the base vector $([(qqq')^S_{MA}(c\bar{c})^1_8]^{\frac{1}{2}}$, $[(qqq')^{MS}_{MA}(c\bar{c})^1_8]^{\frac{1}{2}}$, $[(qqq')^{MS}_{MA}(c\bar{c})^0_8]^{\frac{1}{2}}$, $[(qqq')^{MA}_{MS}(c\bar{c})^1_8]^{\frac{1}{2}}$, $[(qqq')^{MA}_{MS}(c\bar{c})^0_8]^{\frac{3}{2}})^T$.

Now we present the results for the antisymmetric category. For the case $J=\frac{5}{2}$, the base state is $[(qqq')^S_{MS}(c\bar{c})^1_8]^{\frac{5}{2}}$ and the matrix element is

$$\langle H_{CM}\rangle_{J=\frac{5}{2}} = \frac{2}{3}(-2C_{12} + 5C_{13} + 5C_{14} + 10C_{15} + 4C_{34} - C_{35} - C_{45}). \tag{15}$$

For the $J=\frac{3}{2}$ case, we use the base vector $([(qqq')^S_{MS}(c\bar{c})^1_8]^{\frac{3}{2}}$, $[(qqq')^S_{MS}(c\bar{c})^0_8]^{\frac{3}{2}}$, $[(qqq')^{MS}_{MS}(c\bar{c})^1_8]^{\frac{3}{2}}$, $[(qqq')^{MA}_{MA}(c\bar{c})^0_8]^{\frac{3}{2}})^T$ and the obtained matrix is

$$\langle H_{CM}\rangle_{J=\frac{3}{2}} = \begin{pmatrix} -\frac{2}{9}(3\mu' + 10\alpha' + 2\delta') & \frac{2\sqrt{15}}{9}(5\beta' + \gamma') & \frac{2\sqrt{5}}{9}(-5\alpha' + 2\delta') & \frac{\sqrt{5}}{3}(\alpha' - 3\beta') \\ & \frac{2}{15}(11\mu' + 8\nu' - 4\lambda') & \frac{2\sqrt{3}}{9}(5\beta' - 2\gamma') & \frac{\sqrt{3}}{3}(3\alpha' - \beta') \\ & & \frac{2}{9}(10\alpha' - \delta' - 3\nu') & \frac{1}{15}(6\mu' - 6\nu' + 5\alpha' - 15\beta') \\ & & & -\frac{1}{6}(4\lambda' - 15\gamma' + 13\delta') \end{pmatrix}, \tag{16}$$

where $\alpha' = C_{14} + 2C_{15}$, $\beta' = C_{14} - 2C_{15}$, $\gamma' = 4C_{34} + C_{35}$, $\delta' = 4C_{34} - C_{35}$, $\mu' = 2C_{12} - 5C_{13} + C_{45}$, $\nu' = 2C_{12} + 10C_{13} + C_{45}$, and $\lambda' = 12C_{12} + C_{45}$. For the $J=\frac{1}{2}$ case, we have

$$\langle H_{CM}\rangle_{J=\frac{1}{2}}=-\frac{2}{9}\times\begin{pmatrix}(3\mu'+25\alpha'+5\delta') & \sqrt{2}(5\alpha'-2\delta') & \sqrt{6}(5\beta'-2\gamma') & \frac{3}{\sqrt{2}}(3\beta'-\alpha') & \frac{3\sqrt{6}}{2}(3\alpha'-\beta') \\ & (20\alpha'-2\delta'+3\nu') & -\sqrt{3}(10\beta'-\gamma') & \frac{3}{5}\begin{pmatrix}3\nu'-3\mu'\\+5\alpha'-15\beta'\end{pmatrix} & \frac{3\sqrt{3}}{2}(3\alpha'-\beta') \\ & & \frac{3}{5}(4\lambda'-16\mu'-3\nu') & \frac{3\sqrt{3}}{2}(3\alpha'-\beta') & \frac{9}{5}(\nu'-\mu') \\ & & & \frac{3}{2}(2\lambda'+15\gamma'-13\delta') & \frac{3\sqrt{3}}{4}(13\gamma'-15\delta') \\ & & & & \frac{3}{5}(9\lambda'-16\mu'-8\nu')\end{pmatrix} \quad (17)$$

with the base vector $([(qqq')^S_{MS}(c\bar{c})^1_8]^{\frac{1}{2}}, [(qqq')^{MS}_{MS}(c\bar{c})^1_8]^{\frac{1}{2}}, [(qqq')^{MS}_{MS}(c\bar{c})^0_8]^{\frac{1}{2}}, [(qqq')^{MA}_{MA}(c\bar{c})^1_8]^{\frac{1}{2}}, [(qqq')^{MA}_{MA}(c\bar{c})^0_8]^{\frac{3}{2}})^T$.

One may use the matrices in this section to numerically reproduce the $\langle H_{CM}\rangle$'s for the $nnnc\bar{c}$ systems after diagonalization. For the $J=\frac{5}{2}$ case, both Eq. (12) and Eq. (15) give the same formula and thus the same result when $q'=q=n$. For the case $J=\frac{3}{2}$, Eq. (13) and Eq. (16) result in different eigenvalues by assuming $q'=q=n$. However, one finds that the common numbers of the two sets of eigenvalues are just the results for the $I=\frac{1}{2}$ $nnnc\bar{c}$ systems. The remaining value given by Eq. (13) is the result for the $I=\frac{3}{2}$ $nnnc\bar{c}$ system while that given by Eq. (16) can be thought as a forbidden number because of the Pauli principle. The $J=\frac{1}{2}$ case has similar features with the $J=\frac{3}{2}$ case. Probably these features can be used to simplify the calculation for multiquark systems.

IV. NUMERICAL RESULTS FOR THE HIDDEN-CHARM SYSTEMS

By calculating the CMI matrix elements for ground state baryons and mesons [88], we extract the effective coupling parameters presented in Table III. In determining C_{cn}, one may also use the mass difference between Λ_c and Σ_c. The resulting pentaquark masses would be around 10 MeV lower, which is a not a large number in the present method of estimation. Since we also discuss hidden-bottom and B_c-like pentaquark states, Table III displays relevant parameters, too. Because there is no experimental data for the B_c^* meson, we determine the value of $C_{b\bar{c}}$ to be 3.3 MeV from a quark model calculation $m_{B_c^*}-m_{B_c}=70$ MeV [89].

In the simple model used in the present study, the mass splittings of the pentaquark states mainly rely on the coupling parameters. The masses can be roughly estimated with the formula $M=\sum_i m_i+\langle H_{CM}\rangle$ or from a reference mass $M=M_{ref}-\langle H_{CM}\rangle_{ref}+\langle H_{CM}\rangle$. In the latter scheme, the reference system has the same quark content with the pentaquark system and the dependence on the effective quark masses is partially canceled. We will show results in both schemes. Here,

TABLE III. The effective coupling parameters extracted from the mass differences between ground hadrons.

Hadron	CMI	Hadron	CMI	Parameter(MeV)
N	$-8C_{nn}$	Δ	$8C_{nn}$	$C_{nn}=18.4$
Σ	$\frac{8}{3}C_{nn}-\frac{32}{3}C_{ns}$	Σ^*	$\frac{8}{3}C_{nn}+\frac{16}{3}C_{ns}$	$C_{ns}=12.4$
Ξ^0	$\frac{8}{3}(C_{ss}-4C_{ns})$	Ξ^{*0}	$\frac{8}{3}(C_{ss}+C_{ns})$	
Ω	$8C_{ss}$			$C_{ss}=6.5$
Λ	$-8C_{nn}$			
D	$-16C_{c\bar{n}}$	D^*	$\frac{16}{3}C_{c\bar{n}}$	$C_{c\bar{n}}=6.7$
D_s	$-16C_{c\bar{s}}$	D_s^*	$\frac{16}{3}C_{c\bar{s}}$	$C_{c\bar{s}}=6.7$
B	$-16C_{b\bar{n}}$	B^*	$\frac{16}{3}C_{b\bar{n}}$	$C_{b\bar{n}}=2.1$
B_s	$-16C_{b\bar{s}}$	B^*	$\frac{16}{3}C_{b\bar{s}}$	$C_{b\bar{s}}=2.3$
η_c	$-16C_{c\bar{c}}$	J/ψ	$\frac{16}{3}C_{c\bar{c}}$	$C_{c\bar{c}}=5.3$
η_b	$-16C_{b\bar{b}}$	Υ	$\frac{16}{3}C_{b\bar{b}}$	$C_{b\bar{b}}=2.9$
Σ_c	$\frac{8}{3}C_{nn}-\frac{32}{3}C_{cn}$	Σ_c^*	$\frac{8}{3}C_{nn}+\frac{16}{3}C_{cn}$	$C_{cn}=4.0$
Ξ_c'	$\frac{8}{3}C_{ns}-\frac{16}{3}C_{cn}-\frac{16}{3}C_{cs}$	Ξ_c^*	$\frac{8}{3}C_{ns}+\frac{8}{3}C_{cn}+\frac{8}{3}C_{cs}$	$C_{cs}=4.8$
Σ_b	$\frac{8}{3}C_{nn}-\frac{32}{3}C_{bn}$	Σ_b^*	$\frac{8}{3}C_{nn}+\frac{16}{3}C_{bn}$	$C_{bn}=1.3$
Ξ_b'	$\frac{8}{3}C_{ns}-\frac{16}{3}C_{bn}-\frac{16}{3}C_{bs}$	Ξ_b^*	$\frac{8}{3}C_{ns}+\frac{8}{3}C_{bn}+\frac{8}{3}C_{bs}$	$C_{bs}=1.2$

the effective quark masses are $m_n = 361.8$, $m_s = 540.4$, $m_c = 1724.8$, and $m_b = 5052.9$ MeV, which are also extracted from the ground hadrons. In Ref. [80], it is illustrated that these effective quark masses result in overestimated hadron masses and one may treat the values as theoretical upper limits. Then we mainly focus on the second scheme and use various meson-baryon thresholds as reference masses.

A. The $nnnc\bar{c}$ system

In this case, there are two types of thresholds we may use: (charmonium) + (light baryon) and (charmed baryon) + (charmed meson). However, the CMI matrix elements $\langle H_{CM}\rangle_{(J/\psi p)} = -119$ MeV and $\langle H_{CM}\rangle_{(\Sigma_c\bar{D})} = -102$ MeV are not consistent with the thresholds (4035 for $J/\psi p$ and 4320 MeV for $\Sigma_c\bar{D}$). This indicates that one cannot eliminate completely the quark mass effects with the reference mass scheme. The reason is that the model does not involve dynamics and the contributions from the other terms in the potential model are related with the system structure. For example, the additional kinetic energy can probably shift the estimated mass to a more physical value [90]. To understand which threshold is more reasonable, one needs detailed calculation in a future work. Here, we estimate pentaquark masses with both types of thresholds. The numerical results are given in Table IV. The use of the $J/\psi N$, $J/\psi\Delta$, $\eta_c N$, and $\eta_c\Delta$ thresholds gives similar values and that of $\Sigma_c\bar{D}$, $\Sigma_c\bar{D}^*$, $\Sigma_c^*\bar{D}$, and $\Sigma_c^*\bar{D}^*$ gives similar values. The reference threshold $\Lambda_c\bar{D}$ or $\Lambda_c\bar{D}^*$ results in around 10 MeV lower masses than $\Sigma_c\bar{D}$ does. We do not present these results in the table.

Figure 3(a) shows relative positions for these pentaquarks when one adopts the threshold of $(\Sigma_c\bar{D})$ as a reference. We also plot all the thresholds of the related rearrangement decay patterns, i.e., $J/\psi N$, $J/\psi\Delta$, $\eta_c N$, $\eta_c\Delta$, $\Sigma_c\bar{D}$, $\Sigma_c\bar{D}^*$, $\Sigma_c^*\bar{D}$, $\Sigma_c^*\bar{D}^*$, $\Lambda_c\bar{D}$, and $\Lambda_c\bar{D}^*$. The decays may occur through the S- or D-wave interactions and each pentaquark with $J^P = \frac{1}{2}^-$, $\frac{3}{2}^-$, or $\frac{5}{2}^-$ can decay to these channels from the parity conservation and the angular momentum conservation. The isospin conservation reduces the number of decay channels and we label the isospin, for convenience, in the subscripts of the meson-baryon states. Once the considered state is an initial pentaquark plotted with dashed (solid) line, it can decay into meson-baryon channels having the subscript $\frac{3}{2}$ ($\frac{1}{2}$). Of course, whether the decay can happen or not is also kinematically constrained by the pentaquark mass, which depends on models. Contrary to the light quark case, the decay for the hidden-charm pentaquarks may also get constraints from the heavy quark symmetry. For the states with $(I,J) = (\frac{3}{2},\frac{3}{2})$ and $(I,J) = (\frac{1}{2},\frac{5}{2})$, the $c\bar{c}$ spin is always 1. Their decays into $\eta_c\Delta$ and $\eta_c N$, respectively, involve the heavy quark spin-flip and are suppressed. In fact, all the hidden-charm decay channels of the studied pentaquarks are probably suppressed because the transition from a colored $c\bar{c}$ to a colorless $c\bar{c}$ is a high order correction of $1/m_c$. With these considerations in mind, it is easy to judge which channels can be used to search for such unobserved pentaquark states.

From the results in Fig. 3(a), it is obvious that the heaviest state is a decuplet baryon with $J = \frac{1}{2}$. The lightest state belonging to the flavor octet also has the spin $J = \frac{1}{2}$. The observed $P_c(4380)$ is just below the threshold of $\Sigma_c^*\bar{D}$ and falls in the mass range of the studied system. Thus, if the estimated masses are reasonable, the interpretation for the $P_c(4380)$ as a tightly bound pentaquark with colored $c\bar{c}$ is not excluded, although the present study cannot give a preferred spin. The partner states decaying into $J/\psi p$ in the mass region ($4280 \sim 4520$ MeV) are also possible. Above the 4520 MeV, the observation of pentaquarklike baryons

TABLE IV. Calculated CMIs and estimated pentaquark masses of the $nnnc\bar{c}$ systems in units of MeV. The masses in the forth column are calculated with the effective quark masses and are theoretical upper limits.

$nnnc\bar{c}$ $(I=\frac{3}{2})$					
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(J/\psi\Delta)$	$(\Sigma_c\bar{D})$
$\frac{3}{2}^-$	171.5	171.5	4706.5	4325.5	4591.1
$\frac{1}{2}^-$	$\begin{pmatrix} 198.5 & 61.8 \\ 61.8 & 194.6 \end{pmatrix}$	$\begin{pmatrix} 258.3 \\ 134.7 \end{pmatrix}$	$\begin{pmatrix} 4793.3 \\ 4669.7 \end{pmatrix}$	$\begin{pmatrix} 4412.4 \\ 4288.8 \end{pmatrix}$	$\begin{pmatrix} 4677.9 \\ 4554.3 \end{pmatrix}$
$nnnc\bar{c}$ $(I=\frac{1}{2})$					
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(J/\psi N)$	$(\Sigma_c\bar{D})$
$\frac{5}{2}^-$	97.5	97.5	4632.5	4251.6	4517.1
$\frac{3}{2}^-$	$\begin{pmatrix} -9.5 & -20.9 & 48.1 \\ -20.9 & 47.4 & 50.3 \\ 48.1 & 50.3 & -18.9 \end{pmatrix}$	$\begin{pmatrix} -83.1 \\ 74.8 \\ 27.3 \end{pmatrix}$	$\begin{pmatrix} 4451.9 \\ 4609.8 \\ 4562.3 \end{pmatrix}$	$\begin{pmatrix} 4071.0 \\ 4228.9 \\ 4181.4 \end{pmatrix}$	$\begin{pmatrix} 4336.5 \\ 4494.4 \\ 4446.9 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} -73.7 & -71.1 & 30.4 \\ -71.1 & -26.2 & -9.4 \\ 30.4 & -9.4 & -83.1 \end{pmatrix}$	$\begin{pmatrix} -133.0 \\ -80.8 \\ 30.8 \end{pmatrix}$	$\begin{pmatrix} 4402.0 \\ 4454.2 \\ 4565.8 \end{pmatrix}$	$\begin{pmatrix} 4021.1 \\ 4073.3 \\ 4184.9 \end{pmatrix}$	$\begin{pmatrix} 4286.6 \\ 4338.8 \\ 4450.4 \end{pmatrix}$

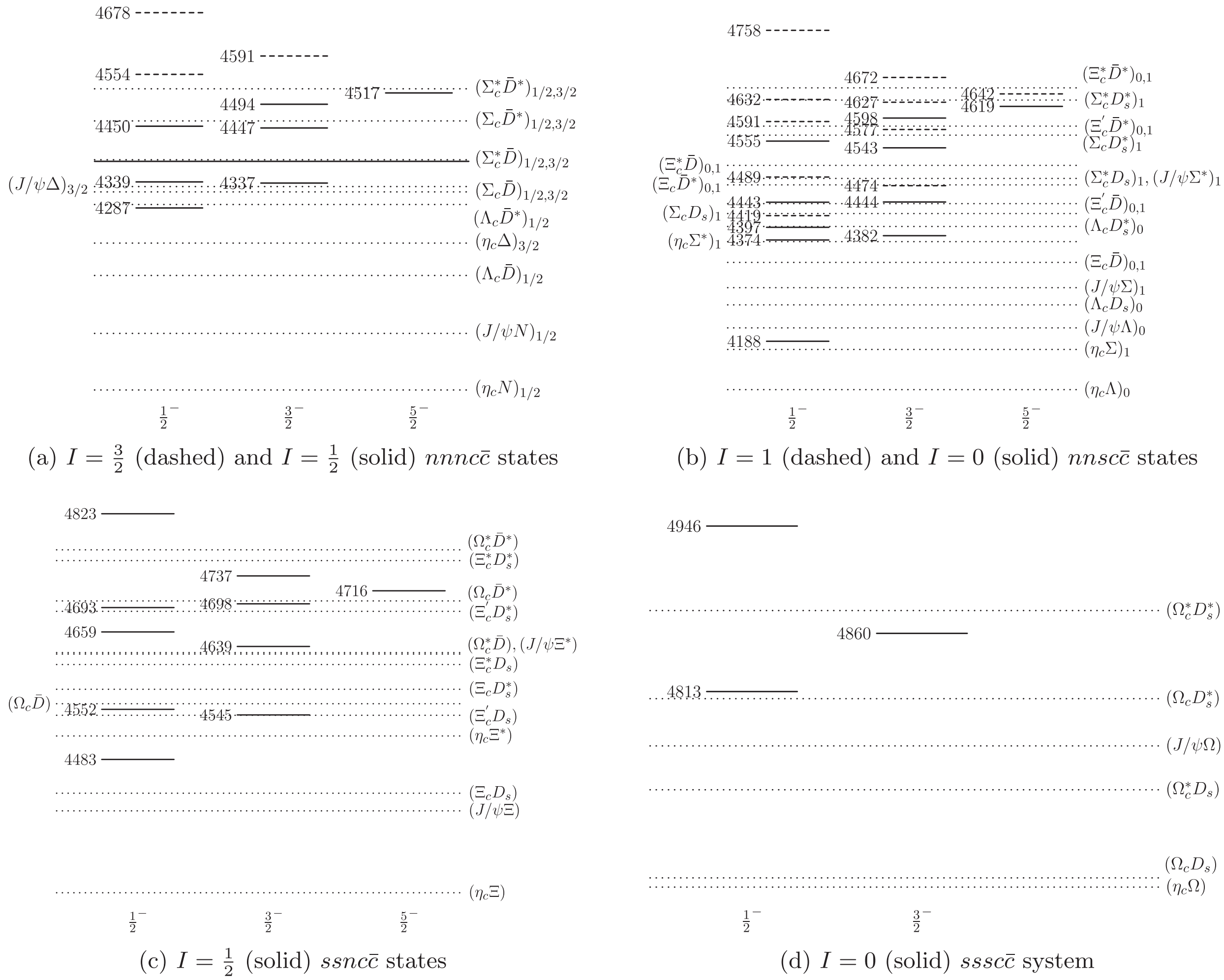

FIG. 3. Relative positions for the obtained $qqqc\bar{c}$ pentaquark states. The dotted lines indicate various meson-baryon thresholds and the long solid line in (a) indicates the observed isospin-half $P_c(4380)$. When a number in the subscript of a meson-baryon state is equal to the isospin of an initial state, the decay for the initial state into that meson-baryon channel through the S or D wave is allowed. We adopt the masses estimated with the reference thresholds of $\Sigma_c\bar{D}$ (a), $\Xi_c\bar{D}$ (b), $\Xi_c D_s$ (c), and $\Omega_c D_s$ (d). The masses are all in units of MeV.

in the invariant mass of $J/\psi\Delta$ is possible, too. All these pentaquarks have open-charm decay channels and are probably broad states. However, their decays into hidden-charm channels should have a small fraction. The feature of the broad width does not contradict with the observed $P_c(4380)$. The branching ratios for various decay channels will be crucial information to understand its nature. Note that the decays of the $J^P = \frac{5}{2}^-$ state (slightly below the $\Sigma_c^*\bar{D}^*$ threshold) into the channels in the figure are all through the D wave. The width of this state should not be so broad. If our result is overestimated, both the mass and the width seem not to be contradicted with those of the $P_c(4450)$, although the parity is opposite to the preferred $P = +$. In the literature, various calculations also find a $J^P = \frac{5}{2}^-$ state below the $\Sigma_c^*\bar{D}^*$ threshold. If the parity of the $P_c(4450)$ is really $+$, search for such a negative parity state is also strongly called for. It is difficult to understand the nature of the $P_c(4450)$ without further investigations.

Now we discuss the masses estimated with the $J/\psi N$ threshold. As mentioned in Ref. [80], the obtained masses seem to be underestimated and can be treated as lower limits in this simple model. The argument is based on the formulas $M = \sum_i m_i + \langle H_{CM}\rangle$ and $M = M_{\rm ref} - \langle H_{CM}\rangle_{\rm ref} + \langle H_{CM}\rangle$, where the effective quark masses are assumed to be equal for various hadrons. Of course this assumption leads to uncertainty for the hadron mass estimation. We illustrate this uncertainty with the $J/\psi N$ threshold. Because the used $m_c = 1724.8$ MeV gives overestimated $m_{J/\psi}$, it is not surprising that the former formula results in overestimated masses, where a charm-anticharm pair exists. When using the latter formula, in principle, one should have $M_{\rm ref}^{th} = 2m_c + 3m_n + \langle H_{CM}\rangle_{\rm ref}$ in order to cancel the quark mass dependence in the former formula. Because we have adopted $M_{\rm ref} = m_{J/\psi} + m_N = 2m_c^\psi + 3m_n^N + \langle H_{CM}\rangle_{\rm ref}$ but $m_c^\psi < m_c$ and $m_n^N = m_n = 361.8$ MeV, the resulting masses are underestimated.

TABLE V. Calculated CMIs and estimated pentaquark masses of the $nnsc\bar{c}$ systems in units of MeV. The masses in the fourth column are calculated with the effective quark masses and are theoretical upper limits.

$nnsc\bar{c}$ $(I=1)$						
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(J/\psi\Sigma)$	$(\Sigma_c D_s)$	$(\Xi_c\bar{D})$
$\frac{5}{2}^-$	102.5	102.5	4816.1	4443.8	4625.6	4641.8
$\frac{3}{2}^-$	$\begin{pmatrix} -6.2 & -19.5 & 35.6 & 34.0 \\ -19.5 & 51.4 & 34.3 & 35.6 \\ 35.6 & 34.3 & 67.9 & -59.2 \\ 34.0 & 35.6 & -59.2 & 78.4 \end{pmatrix}$	$\begin{pmatrix} 132.8 \\ 87.3 \\ -65.7 \\ 37.2 \end{pmatrix}$	$\begin{pmatrix} 4846.4 \\ 4800.9 \\ 4647.9 \\ 4750.8 \end{pmatrix}$	$\begin{pmatrix} 4474.0 \\ 4428.5 \\ 4275.5 \\ 4378.4 \end{pmatrix}$	$\begin{pmatrix} 4655.9 \\ 4610.4 \\ 4457.4 \\ 4560.3 \end{pmatrix}$	$\begin{pmatrix} 4672.0 \\ 4626.5 \\ 4473.5 \\ 4576.5 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} -71.5 & 22.5 & -48.6 & 21.5 & -50.3 \\ 22.5 & 50.4 & 25.6 & -104.8 & -35.6 \\ -48.6 & 25.6 & 76.2 & -35.6 & -74.4 \\ 21.5 & -104.8 & -35.6 & 53.4 & 29.9 \\ -50.3 & -35.6 & -74.4 & 29.9 & 84.2 \end{pmatrix}$	$\begin{pmatrix} 219.1 \\ -120.8 \\ 92.4 \\ 51.9 \\ -49.9 \end{pmatrix}$	$\begin{pmatrix} 4932.7 \\ 4592.8 \\ 4806.0 \\ 4765.5 \\ 4663.7 \end{pmatrix}$	$\begin{pmatrix} 4560.3 \\ 4220.5 \\ 4433.6 \\ 4393.1 \\ 4291.3 \end{pmatrix}$	$\begin{pmatrix} 4742.2 \\ 4402.3 \\ 4615.5 \\ 4575.0 \\ 4473.2 \end{pmatrix}$	$\begin{pmatrix} 4758.4 \\ 4418.5 \\ 4631.6 \\ 4591.1 \\ 4489.3 \end{pmatrix}$
$nnsc\bar{c}$ $(I=0)$						
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(J/\psi\Lambda)$	$(\Lambda_c D_s)$	$(\Xi_c\bar{D})$
$\frac{5}{2}^-$	79.6	79.6	4793.2	4411.1	4588.8	4618.9
$\frac{3}{2}^-$	$\begin{pmatrix} -31.0 & -18.2 & -30.8 & 34.0 \\ -18.2 & 27.4 & -38.0 & 35.6 \\ -30.8 & -38.0 & -74.8 & -59.2 \\ 34.0 & 35.6 & -59.2 & -113.1 \end{pmatrix}$	$\begin{pmatrix} -157.5 \\ -95.8 \\ 58.2 \\ 3.6 \end{pmatrix}$	$\begin{pmatrix} 4556.1 \\ 4617.8 \\ 4771.8 \\ 4717.2 \end{pmatrix}$	$\begin{pmatrix} 4174.0 \\ 4235.8 \\ 4389.7 \\ 4335.2 \end{pmatrix}$	$\begin{pmatrix} 4351.7 \\ 4413.4 \\ 4567.4 \\ 4512.8 \end{pmatrix}$	$\begin{pmatrix} 4381.7 \\ 4443.5 \\ 4597.5 \\ 4542.9 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} -97.3 & -19.5 & 53.8 & 21.5 & -50.3 \\ -19.5 & -182.5 & -46.1 & -104.8 & -35.6 \\ 53.8 & -46.1 & -96.6 & -35.6 & -74.4 \\ 21.5 & -104.8 & -35.6 & -226.1 & -43.1 \\ -50.3 & -35.6 & -74.4 & -43.1 & -136.6 \end{pmatrix}$	$\begin{pmatrix} -351.3 \\ -165.4 \\ -142.2 \\ -96.2 \\ 16.0 \end{pmatrix}$	$\begin{pmatrix} 4362.3 \\ 4548.2 \\ 4571.4 \\ 4617.4 \\ 4729.6 \end{pmatrix}$	$\begin{pmatrix} 3980.3 \\ 4166.1 \\ 4189.4 \\ 4235.4 \\ 4347.5 \end{pmatrix}$	$\begin{pmatrix} 4157.9 \\ 4343.8 \\ 4367.0 \\ 4413.0 \\ 4525.2 \end{pmatrix}$	$\begin{pmatrix} 4188.0 \\ 4373.8 \\ 4397.1 \\ 4443.1 \\ 4555.2 \end{pmatrix}$

Even in this low mass limit, the $I=\frac{1}{2}$ $(I=\frac{3}{2})$ states above the threshold of $\Lambda_c\bar{D}$ $(\Sigma_c\bar{D})$ should be broad.

From the above arguments, probably the masses estimated with the $\Sigma_c\bar{D}$ threshold are more reasonable. If this is the case, more hidden-charm pentaquarks that can decay into $J/\psi\Delta$ or $J/\psi p$ are allowed. After a dynamical calculation with more potential terms is performed in a future work, one can get more information for the masses of these hidden-charm pentaquarks. From the obtained masses, various thresholds in Fig. 3(a), and the above

TABLE VI. Calculated CMIs and estimated pentaquark masses of the $ssnc\bar{c}$ and $sssc\bar{c}$ systems in units of MeV. The masses in the fourth column are calculated with the effective quark masses and are theoretical upper limits.

$ssnc\bar{c}$ $(I=\frac{1}{2})$						
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(J/\psi\Xi)$	$(\Omega_c\bar{D})$	$(\Xi_c D_s)$
$\frac{5}{2}^-$	73.5	73.5	4965.7	4571.9	4774.5	4716.2
$\frac{3}{2}^-$	$\begin{pmatrix} -39.8 & -16.1 & 31.2 & 32.8 \\ -16.1 & 19.7 & 37.7 & 36.5 \\ 31.2 & 37.7 & 39.0 & -59.7 \\ 32.8 & 36.5 & -59.7 & 28.7 \end{pmatrix}$	$\begin{pmatrix} -98.2 \\ 94.0 \\ 55.4 \\ -3.6 \end{pmatrix}$	$\begin{pmatrix} 4794.0 \\ 4986.2 \\ 4947.6 \\ 4888.6 \end{pmatrix}$	$\begin{pmatrix} 4400.2 \\ 4592.5 \\ 4553.9 \\ 4494.9 \end{pmatrix}$	$\begin{pmatrix} 4602.8 \\ 4795.1 \\ 4756.5 \\ 4697.5 \end{pmatrix}$	$\begin{pmatrix} 4544.5 \\ 4736.8 \\ 4698.2 \\ 4639.2 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} -107.7 & 19.7 & -53.3 & 20.7 & -51.6 \\ 19.7 & 13.0 & 30.5 & -103.7 & -36.5 \\ -53.3 & 30.5 & 44.5 & -36.5 & -74.4 \\ 20.7 & -103.7 & -36.5 & 10.1 & 26.2 \\ -51.6 & -36.5 & -74.4 & 26.2 & 36.6 \end{pmatrix}$	$\begin{pmatrix} 180.1 \\ -159.9 \\ -90.6 \\ 50.3 \\ 16.5 \end{pmatrix}$	$\begin{pmatrix} 5072.3 \\ 4732.3 \\ 4801.6 \\ 4942.5 \\ 4908.7 \end{pmatrix}$	$\begin{pmatrix} 4678.5 \\ 4338.6 \\ 4407.9 \\ 4548.7 \\ 4515.0 \end{pmatrix}$	$\begin{pmatrix} 4881.1 \\ 4541.2 \\ 4610.5 \\ 4751.3 \\ 4717.6 \end{pmatrix}$	$\begin{pmatrix} 4822.8 \\ 4482.9 \\ 4552.2 \\ 4693.0 \\ 4659.3 \end{pmatrix}$
$sssc\bar{c}$ $(I=0)$						
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(J/\psi\Omega)$	$(\Omega_c D_s)$	
$\frac{3}{2}^-$	55.1	55.1	5125.9	4744.3	4859.7	
$\frac{1}{2}^-$	$\begin{pmatrix} 74.1 & 66.4 \\ 66.4 & 75.6 \end{pmatrix}$	$\begin{pmatrix} 141.3 \\ 8.5 \end{pmatrix}$	$\begin{pmatrix} 5212.1 \\ 5079.3 \end{pmatrix}$	$\begin{pmatrix} 4830.4 \\ 4697.6 \end{pmatrix}$	$\begin{pmatrix} 4945.8 \\ 4813.0 \end{pmatrix}$	

TABLE VII. Calculated CMIs and estimated pentaquark masses of the $nnnb\bar{b}$ systems in units of MeV. The masses in the fourth column are calculated with the effective quark masses and are theoretical upper limits.

$nnnb\bar{b}$ $(I=\frac{3}{2})$					
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(\Upsilon\Delta)$	$(\Sigma_b B)$
$\frac{3}{2}^-$	179.4	179.4	11370.6	10709.6	11268.5
$\frac{1}{2}^-$	$\begin{pmatrix} 187.4 & 19.6 \\ 19.6 & 189.8 \end{pmatrix}$	$\begin{pmatrix} 208.3 \\ 168.9 \end{pmatrix}$	$\begin{pmatrix} 11399.5 \\ 11360.1 \end{pmatrix}$	$\begin{pmatrix} 10738.5 \\ 10699.2 \end{pmatrix}$	$\begin{pmatrix} 11297.4 \\ 11258.0 \end{pmatrix}$
$nnnb\bar{b}$ $(I=\frac{1}{2})$					
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	(ΥN)	$(\Sigma_b B)$
$\frac{5}{2}^-$	55.3	55.3	11246.5	10585.6	11144.4
$\frac{3}{2}^-$	$\begin{pmatrix} 21.3 & -6.2 & 15.0 \\ -6.2 & 42.6 & 15.8 \\ 15.0 & 15.8 & -31.9 \end{pmatrix}$	$\begin{pmatrix} 46.3 \\ -39.3 \\ 25.0 \end{pmatrix}$	$\begin{pmatrix} 11237.5 \\ 11151.9 \\ 11216.2 \end{pmatrix}$	$\begin{pmatrix} 10576.6 \\ 10491.0 \\ 10555.3 \end{pmatrix}$	$\begin{pmatrix} 11135.4 \\ 11049.8 \\ 11114.1 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} 0.9 & -22.4 & 9.5 \\ -22.4 & -31.0 & -2.8 \\ 9.5 & -2.8 & -52.3 \end{pmatrix}$	$\begin{pmatrix} -54.1 \\ -42.3 \\ 13.8 \end{pmatrix}$	$\begin{pmatrix} 11137.1 \\ 11148.9 \\ 11205.0 \end{pmatrix}$	$\begin{pmatrix} 10476.3 \\ 10488.1 \\ 10544.2 \end{pmatrix}$	$\begin{pmatrix} 11035.0 \\ 11046.8 \\ 11102.9 \end{pmatrix}$

arguments, most pentaquarks are probably broad states with a small fraction for the hidden-charm decays. This feature should be different from the molecule picture, where the hidden-charm decays through rearrangement mechanisms are probably not suppressed. The exceptional case is for the $J^P=\frac{5}{2}^-$ state. Its decays are through D wave and its width is probably not so broad. If the LHCb hidden-charm pentaquark states are confirmed, models of rearrangement decays need to be constructed on the one hand, and the measurement of various branching ratios and the search for more proposed states, on the other hand, are strongly called for. Of course, the interference effects around the mass region would make the experimental analysis more difficult.

TABLE VIII. Calculated CMIs and estimated pentaquark masses of the $nnsb\bar{b}$ systems in units of MeV. The masses in the forth column are calculated with the effective quark masses and are theoretical upper limits.

$nnsb\bar{b}$ $(I=1)$						
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(\Upsilon\Sigma)$	$(\Sigma_b B_s)$	$(\Xi_b B)$
$\frac{5}{2}^-$	60.1	60.1	11429.9	10777.5	11239.8	11264.1
$\frac{3}{2}^-$	$\begin{pmatrix} 24.7 & -7.6 & 11.8 & 10.6 \\ -7.6 & 46.6 & 12.4 & 11.2 \\ 11.8 & 12.4 & 65.5 & -69.7 \\ 10.6 & 11.2 & -69.7 & 73.3 \end{pmatrix}$	$\begin{pmatrix} 139.2 \\ 52.2 \\ 32.4 \\ -13.6 \end{pmatrix}$	$\begin{pmatrix} 11509.0 \\ 11422.0 \\ 11402.2 \\ 11356.2 \end{pmatrix}$	$\begin{pmatrix} 10856.6 \\ 10769.6 \\ 10749.8 \\ 10703.8 \end{pmatrix}$	$\begin{pmatrix} 11318.9 \\ 11231.9 \\ 11212.1 \\ 11166.1 \end{pmatrix}$	$\begin{pmatrix} 11343.2 \\ 11256.2 \\ 11236.4 \\ 11190.4 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} 3.5 & 7.4 & -17.6 & 6.7 & -15.8 \\ 7.4 & 60.1 & 9.0 & -83.9 & -11.2 \\ -17.6 & 9.0 & 71.4 & -11.2 & -74.4 \\ 6.7 & -83.9 & -11.2 & 68.3 & 8.2 \\ -15.8 & -11.2 & -74.4 & 8.2 & 79.4 \end{pmatrix}$	$\begin{pmatrix} 168.8 \\ 129.2 \\ 27.6 \\ -26.4 \\ -16.6 \end{pmatrix}$	$\begin{pmatrix} 11538.6 \\ 11499.0 \\ 11397.4 \\ 11343.4 \\ 11353.2 \end{pmatrix}$	$\begin{pmatrix} 10886.3 \\ 10846.7 \\ 10745.1 \\ 10691.0 \\ 10700.9 \end{pmatrix}$	$\begin{pmatrix} 11348.5 \\ 11308.9 \\ 11207.3 \\ 11153.3 \\ 11163.1 \end{pmatrix}$	$\begin{pmatrix} 11372.9 \\ 11333.2 \\ 11231.6 \\ 11177.6 \\ 11187.4 \end{pmatrix}$
$nnsb\bar{b}$ $(I=0)$						
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(\Upsilon\Lambda)$	$(\Lambda_b B_s)$	$(\Xi_b B)$
$\frac{5}{2}^-$	34.9	34.9	11404.7	10742.6	11205.2	11238.9
$\frac{3}{2}^-$	$\begin{pmatrix} 1.5 & -6.4 & -11.2 & 10.6 \\ -6.4 & 22.6 & -11.0 & 11.2 \\ -11.2 & -11.0 & -97.5 & -69.7 \\ 10.6 & 11.2 & -69.7 & -136.8 \end{pmatrix}$	$\begin{pmatrix} -189.6 \\ -53.0 \\ 26.4 \\ 6.0 \end{pmatrix}$	$\begin{pmatrix} 11180.2 \\ 11316.8 \\ 11396.2 \\ 11375.8 \end{pmatrix}$	$\begin{pmatrix} 10518.2 \\ 10654.7 \\ 10734.2 \\ 10713.8 \end{pmatrix}$	$\begin{pmatrix} 10980.7 \\ 11117.3 \\ 11196.7 \\ 11176.3 \end{pmatrix}$	$\begin{pmatrix} 11014.4 \\ 11151.0 \\ 11230.4 \\ 11210.0 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} -18.5 & -7.1 & 15.6 & 6.7 & -15.8 \\ -7.1 & -132.5 & -13.9 & -83.9 & -11.2 \\ 15.6 & -13.9 & -101.4 & -11.2 & -74.4 \\ 6.7 & -83.9 & -11.2 & -173.8 & -15.8 \\ -15.8 & -11.2 & -74.4 & -15.8 & -141.4 \end{pmatrix}$	$\begin{pmatrix} -252.1 \\ -186.0 \\ -68.6 \\ -56.7 \\ -4.1 \end{pmatrix}$	$\begin{pmatrix} 11117.7 \\ 11183.8 \\ 11301.2 \\ 11313.1 \\ 11365.7 \end{pmatrix}$	$\begin{pmatrix} 10455.7 \\ 10521.7 \\ 10639.1 \\ 10651.0 \\ 10703.6 \end{pmatrix}$	$\begin{pmatrix} 10918.2 \\ 10984.3 \\ 11101.7 \\ 11113.6 \\ 11166.2 \end{pmatrix}$	$\begin{pmatrix} 10951.9 \\ 11018.0 \\ 11135.4 \\ 11147.3 \\ 11199.9 \end{pmatrix}$

B. The $nnsc\bar{c}$ system

This case is related with the Σ-like or Λ-like baryons with an excited charm-anticharm pair. In Refs. [4,5], the Λ-like molecules above 4.2 GeV were predicted. Higher states around 4.6 GeV are also proposed in Refs. [73,75]. From a calculation with the one-meson-exchange model [71], several (charmed baryon)-(charmed strange meson) and (charmed strange baryon)-(charmed meson) type molecules above 4.5 GeV are possible. Now we discuss the mass spectrum of a compact structure.

For the compact pentaquarks with colored $c\bar{c}$, the masses of the isovector (isoscalar) states result from the mixing of the 10_f, $8_f(1)$, and $8_f(2)$ ($8_f(1)$, $8_f(2)$, and 1_f) induced by the color-magnetic interaction. In the present scheme of mass estimation, there are three types of reference thresholds: $(\text{charmonium}) + (nns\text{baryon})$, $(nnc\text{baryon}) + (\bar{c}s\text{meson})$, and $(nsc\text{baryon}) + (\bar{c}n\text{meson})$, which give three sets of masses. The first threshold gives the lower limits of the masses and the masses estimated with the other thresholds are slightly different. We present the numerical results in Table V.

In Fig. 3(b), we show the masses estimated with the threshold of $\Xi_c\bar{D}$ and various thresholds of relevant meson-baryon states into which the pentaquarks may decay. The rearrangement decays involve 18 channels in total. Similar to the $nnnc\bar{c}$ case, these channels are not forbidden by parity and angular momentum conservations, and the decays constrained by the isospin conservation and kinematics are easy to identify from the figure. As for the constraint from the heavy quark symmetry, the hidden-charm decay channels are suppressed by the color transition from octet to singlet. For the decays of the $J = \frac{5}{2}$ states (both $I = 0$ and $I = 0$) into the η_c channels, the suppression is also a heavy quark spin-flip type.

Compared with the $nnnc\bar{c}$ case, the number of pentaquark masses and that of rearrangement decay patterns are both larger. It is obvious that most states have open-charm decay channels except the lightest one with $(I, J) = (0, \frac{1}{2})$. If the existence of open-charm channels means broad widths, the spectrum indicates that most states are broad and that a narrow pentaquark is possible in the $nnsc\bar{c}$ system. The dominant channel for this narrow baryon is $\eta_c\Lambda$. Even if the mass is underestimated about 100 MeV, its open-charm decay channel is still not opened and its narrow nature does not change. In the extreme case that the mass approaches its upper limit (4362 MeV), two open-charm channels $\Xi_c\bar{D}$ and $\Lambda_c D_s$ are opened but it should be still narrower than other states. The masses estimated with the $\Sigma_c D_s$ ($\Lambda_c D_s$) threshold are 16 (30) MeV lower than those with the $\Xi_c\bar{D}$ threshold. Since this number is not large, the main features do not change. Considering the lower limit in the present model, we may refine the mass region for this state to be $3980 \sim 4360$ MeV. Note that the predicted Λ-like states in the molecule picture in Refs. [4,5] are slightly above this pentaquark. The search for such a state in the $\eta_c\Lambda$ channel or $J/\psi\Lambda$ channel is strongly called for.

The CMI in the $SU_f(3)$ symmetric case [Eqs. (3)–(6)] may give us a hint why the low mass and thus narrow Λ-type pentaquark is possible. There are two matrix elements with obviously negative values, one in 1_f ($S_{c\bar{c}} = 1$, $J = \frac{1}{2}$) and the other in $8_f(2)$ ($S_{c\bar{c}} = 1$, $J = \frac{1}{2}$). Relevant states can both be isoscalar spin-half states. Two more states with the

TABLE IX. Calculated CMIs and estimated pentaquark masses of the $ssnb\bar{b}$ and $sssb\bar{b}$ systems in units of MeV. The masses in the fourth column are calculated with the effective quark masses and are theoretical upper limits.

$ssnb\bar{b}$ ($I = \frac{1}{2}$)						
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(\Upsilon\Xi)$	$(\Omega_b B)$	$(\Xi_b B_s)$
$\frac{5}{2}^-$	27.3	27.3	11575.7	10902.0	11383.8	11321.9
$\frac{3}{2}^-$	$\begin{pmatrix} -6.3 & -7.1 & 10.7 & 11.9 \\ -7.1 & 14.9 & 10.8 & 12.0 \\ 10.7 & 10.8 & 33.9 & -69.1 \\ 11.9 & 12.0 & -69.1 & 26.1 \end{pmatrix}$	$\begin{pmatrix} 99.2 \\ -50.5 \\ 19.6 \\ 0.2 \end{pmatrix}$	$\begin{pmatrix} 11647.6 \\ 11497.9 \\ 11568.0 \\ 11548.6 \end{pmatrix}$	$\begin{pmatrix} 10973.8 \\ 10824.2 \\ 10894.3 \\ 10874.9 \end{pmatrix}$	$\begin{pmatrix} 11455.6 \\ 11306.0 \\ 11376.1 \\ 11356.7 \end{pmatrix}$	$\begin{pmatrix} 11393.8 \\ 11244.1 \\ 11314.2 \\ 11294.8 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} -26.5 & 6.8 & -15.2 & 7.5 & -17.0 \\ 6.8 & 28.1 & 7.6 & -85.1 & -12.0 \\ -15.2 & 7.6 & 39.7 & -12.0 & -74.4 \\ 7.5 & -85.1 & -12.0 & 19.9 & 8.4 \\ -17.0 & -12.0 & -74.4 & 8.4 & 31.8 \end{pmatrix}$	$\begin{pmatrix} 129.7 \\ 89.7 \\ -64.3 \\ -55.2 \\ -7.0 \end{pmatrix}$	$\begin{pmatrix} 11678.1 \\ 11638.1 \\ 11484.1 \\ 11493.2 \\ 11541.4 \end{pmatrix}$	$\begin{pmatrix} 11004.4 \\ 10964.4 \\ 10810.4 \\ 10819.5 \\ 10867.7 \end{pmatrix}$	$\begin{pmatrix} 11486.2 \\ 11446.2 \\ 11292.2 \\ 11301.3 \\ 11349.5 \end{pmatrix}$	$\begin{pmatrix} 11424.3 \\ 11384.3 \\ 11230.3 \\ 11239.4 \\ 11287.6 \end{pmatrix}$
$sssb\bar{b}$ ($I = 0$)						
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(\Upsilon\Omega)$	$(\Omega_b B_s)$	
$\frac{3}{2}^-$	59.4	59.4	11786.4	11124.7	11506.5	
$\frac{1}{2}^-$	$\begin{pmatrix} 70.4 & 20.2 \\ 20.2 & 70.8 \end{pmatrix}$	$\begin{pmatrix} 90.8 \\ 50.4 \end{pmatrix}$	$\begin{pmatrix} 11817.8 \\ 11777.4 \end{pmatrix}$	$\begin{pmatrix} 11156.1 \\ 11115.7 \end{pmatrix}$	$\begin{pmatrix} 11537.9 \\ 11497.5 \end{pmatrix}$	

same (I, J), one in 1_f ($S_{c\bar{c}} = 0$, $J = \frac{1}{2}$) and the other in $8_f(2)$ ($S_{c\bar{c}} = 0$, $J = \frac{1}{2}$), have also negative matrix elements because of the relation $C_{c\bar{c}} < C_{qq}$. The mixing between these states (and a state in $8_f(1)$) will effectively provide additional attraction for the lowest pentaquark. Therefore, it is not surprising that the compact $J^P = \frac{1}{2}^-$ Λ-like pentaquark with colored $c\bar{c}$ has a low mass. Numerically, the attractive nature is illustrated in Table V, where the smallest diagonal CMI matrix elements (-182.5, -226.1, and -136.6 MeV) all appear in the case $(I, J) = (0, \frac{1}{2})$. The structure mixing effect further induces the lower value -351.3 MeV, which corresponds to the lowest $nnsc\bar{c}$ state.

In addition to this narrow compact pentaquark with $J^P = \frac{1}{2}^-$, the $J^P = \frac{5}{2}^-$ Λ-like state around 4.6 GeV is probably not a broad one because its dominant open-charm decays are through the D wave. Experimentally, it can be searched for in the $J/\psi\Lambda$ channel. Since the Σ-like state with $J^P = \frac{5}{2}^-$ is around the threshold of $\Sigma_c^* D_s^*$ and the present model only gives a rough estimation of its mass, whether it has a broad width or not needs further study. A possible channel to search for it is in the $J/\psi\Sigma$ invariant mass distribution.

C. The $ssnc\bar{c}$ and the $sssc\bar{c}$ systems

Similar to the Σ-like pentaquarks, all these Ξ-like $ssnc\bar{c}$ states result from the mixing between the 10_f, $8_f(1)$, and $8_f(2)$ multiplets. To estimate the masses of these pentaquark states, we also have three types of reference thresholds to use: (charmonium) + (ssn baryon), (ssc baryon) + ($\bar{c}n$ meson), and (nsc baryon) + ($\bar{c}s$ meson). The first threshold gives the lower limits of masses and the other two thresholds result in two sets of masses with their difference being 60 MeV. The numerical results are shown in Table VI.

To understand the properties further, we plot in Fig. 3(c) various thresholds and the relative positions of these states with the masses estimated with the threshold of $\Xi_c D_s$. Unlike the $nnnc\bar{c}$ and $nnsc\bar{c}$ cases, it is not necessary to label the isospin in the meson-baryon states since all the pentaquarks have isospin 1/2. As before, all the given channels in the figure are not forbidden by parity or angular momentum conservations but the heavy quark symmetry suppresses the hidden-charm decays. The remaining constraint comes only from the kinematics. Probably all these states are broad except the $J^P = \frac{5}{2}^-$ one, similar to the $nnnc\bar{c}$ case, because all the states have open-charm decay channels but the decays

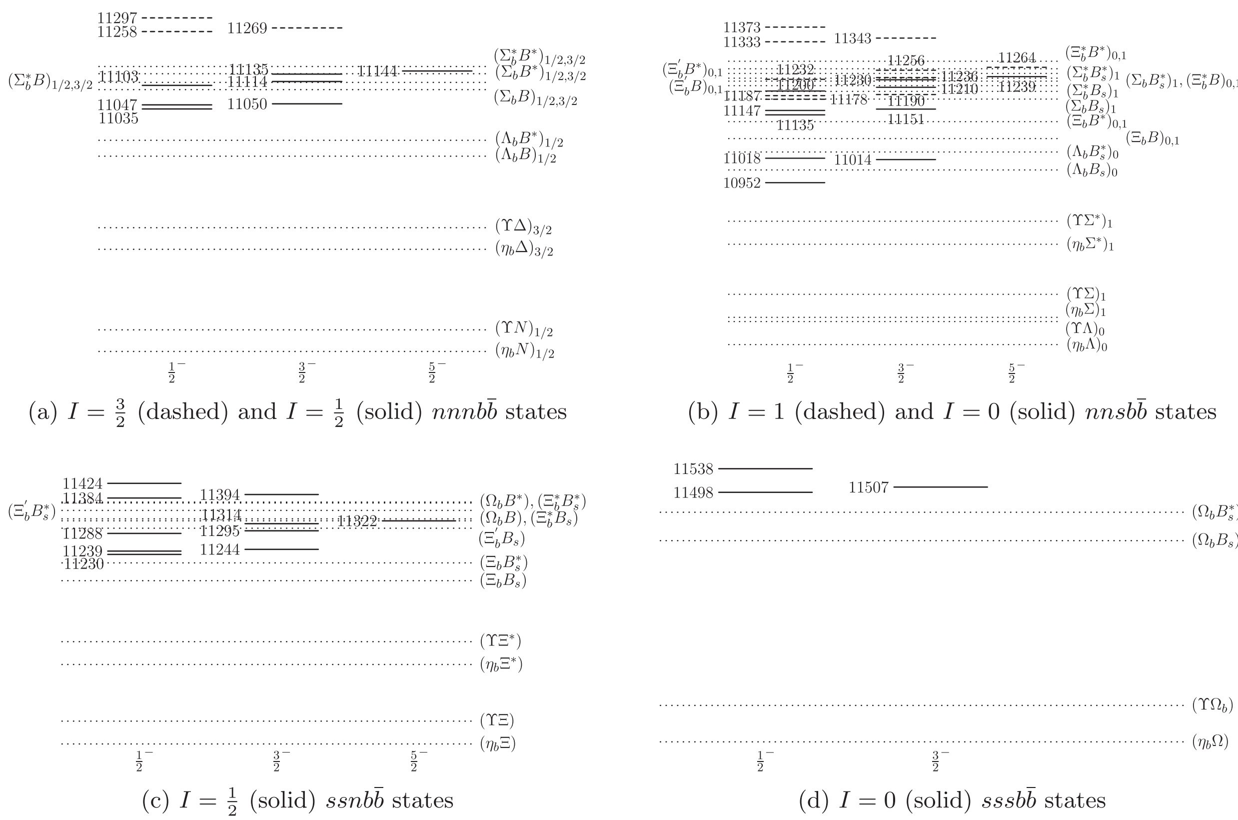

FIG. 4. Relative positions for the obtained $qqqb\bar{b}$ pentaquark states. The dotted lines indicate various meson-baryon thresholds. When a number in the subscript of a meson-baryon state is equal to the isospin of an initial state, the decay for the initial state into that meson-baryon channel through the S or D wave is allowed. We adopt the masses estimated with the reference thresholds of $\Sigma_b B$ (a), $\Xi_b B$ (b), $\Xi_b B_s$ (c), and $\Omega_b B_s$ (d). The masses are all in units of MeV.

TABLE X. Calculated CMIs and estimated pentaquark masses of the $nnnb\bar{c}$ systems in units of MeV. The masses in the fourth column are calculated with the effective quark masses and are theoretical upper limits.

$nnnb\bar{c}$ $(I=\frac{3}{2})$					
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(B_c\Delta)$	$(\Sigma_b\bar{D})$
$\frac{3}{2}^-$	163.8	163.8	8026.9	7577.1	7911.9
$\frac{1}{2}^-$	$\begin{pmatrix} 217.8 & 46.2 \\ 46.2 & 190.6 \end{pmatrix}$	$\begin{pmatrix} 252.3 \\ 156.1 \end{pmatrix}$	$\begin{pmatrix} 8115.4 \\ 8019.2 \end{pmatrix}$	$\begin{pmatrix} 7665.6 \\ 7569.4 \end{pmatrix}$	$\begin{pmatrix} 8000.4 \\ 7904.2 \end{pmatrix}$
$nnnb\bar{c}$ $(I=\frac{1}{2})$					
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	(B_cN)	$(\Sigma_b\bar{D})$
$\frac{5}{2}^-$	82.6	82.6	7945.7	7496.0	7830.7
$\frac{3}{2}^-$	$\begin{pmatrix} 2.6 & -41.8 & 53.8 \\ -41.8 & 43.4 & 45.9 \\ 53.8 & 45.9 & -23.0 \end{pmatrix}$	$\begin{pmatrix} -93.5 \\ 72.0 \\ 44.5 \end{pmatrix}$	$\begin{pmatrix} 7769.6 \\ 7935.1 \\ 7907.6 \end{pmatrix}$	$\begin{pmatrix} 7319.9 \\ 7485.4 \\ 7457.9 \end{pmatrix}$	$\begin{pmatrix} 7654.6 \\ 7820.1 \\ 7792.6 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} -45.4 & -64.9 & 34.0 \\ -64.9 & -30.2 & -18.7 \\ 34.0 & -18.7 & -71.0 \end{pmatrix}$	$\begin{pmatrix} -109.1 \\ -77.3 \\ 39.7 \end{pmatrix}$	$\begin{pmatrix} 7754.0 \\ 7785.8 \\ 7902.8 \end{pmatrix}$	$\begin{pmatrix} 7304.3 \\ 7336.1 \\ 7453.1 \end{pmatrix}$	$\begin{pmatrix} 7639.0 \\ 7670.8 \\ 7787.8 \end{pmatrix}$

of the $J^P=\frac{5}{2}^-$ state are through the D wave. The estimation with the threshold of $\Omega_c\bar{D}$ does not change the main decay nature of these pentaquarks.

The properties of the Ω-like hidden-charm pentaquarks $sssc\bar{c}$ are similar to those of the $I=\frac{3}{2}$ $nnnc\bar{c}$ states. We also present the numerical results in Table VI. The relative positions plotted with the masses estimated with the threshold of Ω_cD_s and the decay properties can be found in Fig. 3(d). They should all be broad states.

V. HIDDEN-BOTTOM AND B_c-LIKE PENTAQUARKS

The formulas can be easily applied to much heavier hidden-bottom pentaquarks. If hidden-charm pentaquarks really exist, their bottom partners are more likely to form because of the less kinetic energy in the Hamiltonian. Before the observation of the exotic P_c baryons, such hidden-bottom states had been investigated in Refs. [6,91,92]. For recent studies in different

TABLE XI. Calculated CMIs and estimated pentaquark masses of the $nnsb\bar{c}$ systems in units of MeV. The masses in the fourth column are calculated with the effective quark masses and are theoretical upper limits.

$nnsb\bar{c}$ $(I=1)$						
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(B_c\Sigma)$	(Σ_bD_s)	$(\Xi_b\bar{D})$
$\frac{5}{2}^-$	86.5	86.5	8128.2	7687.0	7938.1	7949.5
$\frac{3}{2}^-$	$\begin{pmatrix} 6.7 & -42.0 & 37.8 & 38.0 \\ -42.0 & 47.4 & 32.6 & 32.4 \\ 37.8 & 32.6 & 62.4 & -57.4 \\ 38.0 & 32.4 & -57.4 & 70.1 \end{pmatrix}$	$\begin{pmatrix} 123.9 \\ 79.0 \\ -77.0 \\ 60.8 \end{pmatrix}$	$\begin{pmatrix} 8165.6 \\ 8120.7 \\ 7964.7 \\ 8102.5 \end{pmatrix}$	$\begin{pmatrix} 7724.4 \\ 7679.5 \\ 7523.5 \\ 7661.3 \end{pmatrix}$	$\begin{pmatrix} 7975.5 \\ 7930.6 \\ 7774.6 \\ 7912.4 \end{pmatrix}$	$\begin{pmatrix} 7986.9 \\ 7942.0 \\ 7786.0 \\ 7923.8 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} -41.2 & 23.9 & -46.1 & 24.0 & -45.9 \\ 23.9 & 65.3 & 13.8 & -108.4 & -32.4 \\ -46.1 & 13.8 & 72.2 & -32.4 & -74.4 \\ 24.0 & -108.4 & -32.4 & 73.9 & 13.3 \\ -45.9 & -32.4 & -74.4 & 13.3 & 80.2 \end{pmatrix}$	$\begin{pmatrix} 212.4 \\ 116.6 \\ -94.0 \\ 61.2 \\ -45.6 \end{pmatrix}$	$\begin{pmatrix} 8254.1 \\ 8158.3 \\ 7947.7 \\ 8102.9 \\ 7996.1 \end{pmatrix}$	$\begin{pmatrix} 7812.9 \\ 7717.1 \\ 7506.5 \\ 7661.7 \\ 7554.9 \end{pmatrix}$	$\begin{pmatrix} 8064.0 \\ 7968.2 \\ 7757.6 \\ 7912.8 \\ 7806.0 \end{pmatrix}$	$\begin{pmatrix} 8075.4 \\ 7979.6 \\ 7769.0 \\ 7924.2 \\ 7817.4 \end{pmatrix}$
$nnsb\bar{c}$ $(I=0)$						
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(B_c\Lambda)$	(Λ_bD_s)	$(\Xi_b\bar{D})$
$\frac{5}{2}^-$	62.3	62.3	8104.0	7653.1	7904.5	7925.3
$\frac{3}{2}^-$	$\begin{pmatrix} -17.2 & -42.2 & -38.4 & 38.0 \\ -42.2 & 23.4 & -32.1 & 32.4 \\ -38.4 & -32.1 & -76.3 & -57.4 \\ 38.0 & 32.4 & -57.4 & -116.5 \end{pmatrix}$	$\begin{pmatrix} -159.0 \\ -107.7 \\ 52.4 \\ 27.6 \end{pmatrix}$	$\begin{pmatrix} 7882.7 \\ 7934.0 \\ 8094.1 \\ 8069.3 \end{pmatrix}$	$\begin{pmatrix} 7431.8 \\ 7483.1 \\ 7643.2 \\ 7618.4 \end{pmatrix}$	$\begin{pmatrix} 7683.2 \\ 7734.5 \\ 7894.6 \\ 7869.8 \end{pmatrix}$	$\begin{pmatrix} 7704.0 \\ 7755.3 \\ 7915.4 \\ 7890.6 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} -65.0 & -24.3 & 45.5 & 24.0 & -45.9 \\ -24.3 & -175.6 & -51.0 & -108.4 & -32.4 \\ 45.5 & -51.0 & -100.6 & -32.4 & -74.4 \\ 24.0 & -108.4 & -32.4 & -215.1 & -51.4 \\ -45.9 & -32.4 & -74.4 & -51.4 & -140.6 \end{pmatrix}$	$\begin{pmatrix} -350.7 \\ -156.5 \\ -121.4 \\ -91.7 \\ 23.4 \end{pmatrix}$	$\begin{pmatrix} 7691.0 \\ 7885.2 \\ 7920.3 \\ 7950.0 \\ 8065.1 \end{pmatrix}$	$\begin{pmatrix} 7240.1 \\ 7434.3 \\ 7469.4 \\ 7499.1 \\ 7614.2 \end{pmatrix}$	$\begin{pmatrix} 7491.5 \\ 7685.7 \\ 7720.8 \\ 7750.5 \\ 7865.6 \end{pmatrix}$	$\begin{pmatrix} 7512.3 \\ 7706.5 \\ 7741.6 \\ 7771.3 \\ 7886.4 \end{pmatrix}$

TABLE XII. Calculated CMIs and estimated pentaquark masses of the $ssnb\bar{c}$ and $sssb\bar{c}$ systems in units of MeV. The masses in the fourth column are calculated with the effective quark masses and are theoretical upper limits.

$ssnb\bar{c}$ $(I=\frac{1}{2})$						
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(B_c\Xi)$	$(\Omega_b\bar{D})$	$(\Xi_b D_s)$
$\frac{5}{2}^-$	54.4	54.4	8274.7	7812.1	8069.9	8020.9
$\frac{3}{2}^-$	$\begin{pmatrix} -24.8 & -42.4 & 38.4 & 38.2 \\ -42.4 & 15.7 & 32.2 & 32.3 \\ 38.4 & 32.2 & 30.4 & -57.3 \\ 38.2 & 32.3 & -57.3 & 22.8 \end{pmatrix}$	$\begin{pmatrix} -112.3 \\ 84.1 \\ 46.0 \\ 26.3 \end{pmatrix}$	$\begin{pmatrix} 8108.0 \\ 8304.4 \\ 8266.3 \\ 8246.6 \end{pmatrix}$	$\begin{pmatrix} 7645.4 \\ 7841.8 \\ 7803.7 \\ 7784.0 \end{pmatrix}$	$\begin{pmatrix} 7903.1 \\ 8099.6 \\ 8061.4 \\ 8041.7 \end{pmatrix}$	$\begin{pmatrix} 7854.2 \\ 8050.6 \\ 8012.5 \\ 7992.8 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} -72.4 & 24.3 & -45.5 & 24.1 & -45.7 \\ 24.3 & 34.3 & 13.2 & -108.5 & -32.3 \\ -45.5 & 13.2 & 40.5 & -32.3 & -74.4 \\ 24.1 & -108.5 & -32.3 & 25.8 & 13.7 \\ -45.7 & -32.3 & -74.4 & 13.7 & 32.6 \end{pmatrix}$	$\begin{pmatrix} 172.7 \\ -128.0 \\ -85.1 \\ 77.2 \\ 24.1 \end{pmatrix}$	$\begin{pmatrix} 8393.0 \\ 8092.3 \\ 8135.2 \\ 8297.5 \\ 8244.4 \end{pmatrix}$	$\begin{pmatrix} 7930.4 \\ 7629.7 \\ 7672.6 \\ 7834.9 \\ 7781.9 \end{pmatrix}$	$\begin{pmatrix} 8188.1 \\ 7887.5 \\ 7930.3 \\ 8092.6 \\ 8039.6 \end{pmatrix}$	$\begin{pmatrix} 8139.2 \\ 7838.5 \\ 7881.4 \\ 8043.7 \\ 7990.6 \end{pmatrix}$
$sssb\bar{c}$ $(I=0)$						
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(B_c\Omega)$	$(\Omega_b D_s)$	
$\frac{3}{2}^-$	44.5	44.5	8443.4	7992.9	8163.4	
$\frac{1}{2}^-$	$\begin{pmatrix} 99.5 & 45.6 \\ 45.6 & 71.6 \end{pmatrix}$	$\begin{pmatrix} 133.2 \\ 37.8 \end{pmatrix}$	$\begin{pmatrix} 8532.1 \\ 8436.7 \end{pmatrix}$	$\begin{pmatrix} 8081.6 \\ 7986.2 \end{pmatrix}$	$\begin{pmatrix} 8252.2 \\ 8156.8 \end{pmatrix}$	

scenarios, one may consult Refs. [76,77,93] for an overview.

When estimating the hidden-bottom pentaquark masses, the reference thresholds we use are $\Upsilon\Delta$, ΥN, and $\Sigma_b B$ for the $nnnb\bar{b}$ states, $\Upsilon\Sigma$, $\Upsilon\Lambda$, $\Sigma_b B_s$, $\Lambda_b B_s$, and $\Xi_b B$ for the $nnsb\bar{b}$ states, $\Upsilon\Xi$, $\Omega_b B$, and $\Xi_b B_s$ for the $ssnb\bar{b}$ states, and $\Upsilon\Omega$ and $\Omega_b B_s$ for the $sssb\bar{b}$ states. We present estimations for the masses of these hidden-bottom pentaquarks in Tables VII, VIII, and IX, respectively. The relative positions for the $nnnb\bar{b}$, $nnsb\bar{b}$, $ssnb\bar{b}$, and $sssb\bar{b}$ states are shown in Fig. 4 with the masses estimated from the thresholds of $\Sigma_b B$, $\Xi_b B$, $\Xi_b B_s$, and $\Omega_b B_s$, respectively. The following discussions are based on the assumption that these estimated masses are reasonable.

The basic features for the mass spectrum and the decay properties of the $nnnb\bar{b}$ system are similar to the hidden charm case: (i) All the nonstrange pentaquarks can decay into Υ (or η_b) plus p (or Δ) and all of them have also open-bottom decay channels, (ii) maybe the width for the $J^P = 5/2^-$ state is not so broad while others have broad widths, (iii) the hidden bottom decay channels should have smaller branching ratios than the molecules do. If we compare results with those in Refs. [6,91,92], where the S-wave $\Sigma_b\bar{B}$ hadronic molecule was proposed, one can conclude here that lower hidden-bottom pentaquarks than the $\Sigma_b\bar{B}$ threshold are possible, too.

The Σ-like or Λ-like hidden bottom system is more interesting than the hidden charm case. Now, the states

TABLE XIII. Calculated CMIs and estimated pentaquark masses of the $nnnc\bar{b}$ systems in units of MeV. The masses in the fourth column are calculated with the effective quark masses and are theoretical upper limits.

$nnnc\bar{b}$ $(I=\frac{3}{2})$					
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(B_c\Delta)$	$(\Sigma_c B)$
$\frac{3}{2}^-$	188.1	188.1	8051.2	7601.4	7948.7
$\frac{1}{2}^-$	$\begin{pmatrix} 169.1 & 35.2 \\ 35.2 & 190.6 \end{pmatrix}$	$\begin{pmatrix} 216.7 \\ 143.0 \end{pmatrix}$	$\begin{pmatrix} 8079.8 \\ 8006.1 \end{pmatrix}$	$\begin{pmatrix} 7630.0 \\ 7556.3 \end{pmatrix}$	$\begin{pmatrix} 7977.3 \\ 7903.6 \end{pmatrix}$
$nnnc\bar{b}$ $(I=\frac{1}{2})$					
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(B_c N)$	$(\Sigma_c B)$
$\frac{5}{2}^-$	71.2	71.2	7934.3	7484.6	7831.8
$\frac{3}{2}^-$	$\begin{pmatrix} 10.2 & 14.7 & 9.3 \\ 14.7 & 43.4 & 20.2 \\ 9.3 & 20.2 & -26.8 \end{pmatrix}$	$\begin{pmatrix} 55.0 \\ -32.9 \\ 4.7 \end{pmatrix}$	$\begin{pmatrix} 7918.1 \\ 7830.2 \\ 7867.8 \end{pmatrix}$	$\begin{pmatrix} 7468.4 \\ 7380.5 \\ 7418.1 \end{pmatrix}$	$\begin{pmatrix} 7815.6 \\ 7727.7 \\ 7765.3 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} -26.4 & -28.6 & 5.9 \\ -28.6 & -30.2 & 6.6 \\ 5.9 & 6.6 & -63.4 \end{pmatrix}$	$\begin{pmatrix} -69.6 \\ -50.8 \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 7793.5 \\ 7812.3 \\ 7863.5 \end{pmatrix}$	$\begin{pmatrix} 7343.8 \\ 7362.6 \\ 7413.8 \end{pmatrix}$	$\begin{pmatrix} 7691.0 \\ 7709.8 \\ 7761.0 \end{pmatrix}$

TABLE XIV. Calculated CMIs and estimated pentaquark masses of the $nnsc\bar{b}$ systems in units of MeV. The masses in the fourth column are calculated with the effective quark masses and are theoretical upper limits.

J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(B_c\Sigma)$	(Σ_cB_s)	(Ξ_cB)
$nnsc\bar{b}$ $(I=1)$						
$\frac{5}{2}^-$	77.2	77.2	8118.9	7677.7	7928.4	7957.5
$\frac{3}{2}^-$	$\begin{pmatrix} 12.9 & 14.9 & 9.5 & 6.6 \\ 14.9 & 47.4 & 14.2 & 14.3 \\ 9.5 & 14.2 & 72.0 & -71.5 \\ 6.6 & 14.3 & -71.5 & 82.7 \end{pmatrix}$	$\begin{pmatrix} 149.0 \\ 62.2 \\ 8.3 \\ -4.6 \end{pmatrix}$	$\begin{pmatrix} 8190.7 \\ 8103.9 \\ 8050.0 \\ 8037.1 \end{pmatrix}$	$\begin{pmatrix} 7749.5 \\ 7662.7 \\ 7608.8 \\ 7595.9 \end{pmatrix}$	$\begin{pmatrix} 8000.2 \\ 7913.4 \\ 7859.5 \\ 7846.6 \end{pmatrix}$	$\begin{pmatrix} 8029.3 \\ 7942.5 \\ 7888.5 \\ 7875.7 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} -25.7 & 6.0 & -20.0 & 4.1 & -20.2 \\ 6.0 & 46.2 & 20.8 & -80.3 & -14.3 \\ -20.0 & 20.8 & 72.2 & -14.3 & -74.4 \\ 4.1 & -80.3 & -14.3 & 48.9 & 24.8 \\ -20.2 & -14.3 & -74.4 & 24.8 & 80.2 \end{pmatrix}$	$\begin{pmatrix} 178.2 \\ 100.4 \\ -50.1 \\ -26.5 \\ 19.8 \end{pmatrix}$	$\begin{pmatrix} 8219.9 \\ 8142.1 \\ 7991.6 \\ 8015.2 \\ 8061.5 \end{pmatrix}$	$\begin{pmatrix} 7778.7 \\ 7700.9 \\ 7550.4 \\ 7574.0 \\ 7620.3 \end{pmatrix}$	$\begin{pmatrix} 8029.4 \\ 7951.6 \\ 7801.1 \\ 7824.7 \\ 7871.0 \end{pmatrix}$	$\begin{pmatrix} 8058.5 \\ 7980.6 \\ 7830.1 \\ 7853.8 \\ 7900.0 \end{pmatrix}$
$nnsc\bar{b}$ $(I=0)$						
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(B_c\Lambda)$	(Λ_cB_s)	(Ξ_cB)
$\frac{5}{2}^-$	53.2	53.2	8094.9	7644.0	7890.5	7933.5
$\frac{3}{2}^-$	$\begin{pmatrix} -11.1 & 17.6 & -3.6 & 6.6 \\ 17.6 & 23.4 & -16.9 & 14.3 \\ -3.6 & -16.9 & -94.9 & -71.5 \\ 6.6 & 14.3 & -71.5 & -132.3 \end{pmatrix}$	$\begin{pmatrix} -187.5 \\ -46.8 \\ 37.7 \\ -18.4 \end{pmatrix}$	$\begin{pmatrix} 7854.2 \\ 7994.9 \\ 8079.4 \\ 8023.3 \end{pmatrix}$	$\begin{pmatrix} 7403.3 \\ 7544.0 \\ 7628.5 \\ 7572.4 \end{pmatrix}$	$\begin{pmatrix} 7649.8 \\ 7790.5 \\ 7875.0 \\ 7818.9 \end{pmatrix}$	$\begin{pmatrix} 7692.7 \\ 7833.5 \\ 7918.0 \\ 7861.9 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} -49.7 & -2.3 & 24.0 & 4.1 & -20.2 \\ -2.3 & -138.3 & -9.0 & -80.3 & -14.3 \\ 24.0 & -9.0 & -100.6 & -14.3 & -74.4 \\ 4.1 & -80.3 & -14.3 & -183.7 & -7.5 \\ -20.2 & -14.3 & -74.4 & -7.5 & -140.6 \end{pmatrix}$	$\begin{pmatrix} -253.2 \\ -189.2 \\ -84.4 \\ -71.0 \\ -15.1 \end{pmatrix}$	$\begin{pmatrix} 7788.5 \\ 7852.5 \\ 7957.3 \\ 7970.7 \\ 8026.6 \end{pmatrix}$	$\begin{pmatrix} 7337.6 \\ 7401.6 \\ 7506.4 \\ 7519.8 \\ 7575.7 \end{pmatrix}$	$\begin{pmatrix} 7584.1 \\ 7648.1 \\ 7752.9 \\ 7766.3 \\ 7822.2 \end{pmatrix}$	$\begin{pmatrix} 7627.1 \\ 7691.0 \\ 7795.9 \\ 7809.3 \\ 7865.1 \end{pmatrix}$

concentrated around the threshold of Σ_bB_s are more than those around Σ_cD_s. There are probably three narrow Λ-like states with $J^P=\frac{1}{2}^-$ or $\frac{3}{2}^-$ and two $J^P=\frac{5}{2}^-$ states having relatively narrow widths, which is sensitive to the mass values. If the states shown in Fig. 4(b) are underestimated, say 60 MeV, all the states would have open-bottom decay channels and broader widths are expected. If they are overestimated, say 60 MeV, the lowest three states will not have open-bottom decay channels and should have narrow widths. The S-wave decay channels $\Sigma_b^*B_s^*$ and $\Xi_b^*B^*$ are

TABLE XV. Calculated CMIs and estimated pentaquark masses of the $ssnc\bar{b}$ and $sssc\bar{b}$ systems in units of MeV. The masses in the fourth column are calculated with the effective quark masses and are theoretical upper limits.

J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(B_c\Xi)$	(Ω_cB)	(Ξ_cB_s)
$ssnc\bar{b}$ $(I=\frac{1}{2})$						
$\frac{5}{2}^-$	47.5	47.5	8267.8	7805.2	8089.5	8018.3
$\frac{3}{2}^-$	$\begin{pmatrix} -20.2 & 19.2 & 3.6 & 6.6 \\ 19.2 & 15.7 & 16.3 & 16.2 \\ 3.6 & 16.3 & 43.6 & -71.5 \\ 6.6 & 16.2 & -71.5 & 33.1 \end{pmatrix}$	$\begin{pmatrix} 110.0 \\ -42.4 \\ 32.7 \\ -28.2 \end{pmatrix}$	$\begin{pmatrix} 8330.3 \\ 8177.9 \\ 8253.0 \\ 8192.1 \end{pmatrix}$	$\begin{pmatrix} 7867.8 \\ 7715.3 \\ 7790.4 \\ 7729.6 \end{pmatrix}$	$\begin{pmatrix} 8152.1 \\ 7999.7 \\ 8074.8 \\ 8013.9 \end{pmatrix}$	$\begin{pmatrix} 8080.9 \\ 7928.5 \\ 8003.6 \\ 7942.7 \end{pmatrix}$
$\frac{1}{2}^-$	$\begin{pmatrix} -60.8 & 2.3 & -23.1 & 4.1 & -22.9 \\ 2.3 & 7.8 & 24.9 & -80.3 & -16.2 \\ -23.1 & 24.9 & 40.5 & -16.2 & -74.4 \\ 4.1 & -80.3 & -16.2 & 5.3 & 20.9 \\ -22.9 & -16.2 & -74.4 & 20.9 & 32.6 \end{pmatrix}$	$\begin{pmatrix} 139.9 \\ -87.9 \\ -69.8 \\ 58.0 \\ -14.9 \end{pmatrix}$	$\begin{pmatrix} 8360.2 \\ 8132.4 \\ 8150.5 \\ 8278.3 \\ 8205.4 \end{pmatrix}$	$\begin{pmatrix} 7897.6 \\ 7669.8 \\ 7687.9 \\ 7815.7 \\ 7742.9 \end{pmatrix}$	$\begin{pmatrix} 8182.0 \\ 7954.2 \\ 7972.3 \\ 8100.1 \\ 8027.2 \end{pmatrix}$	$\begin{pmatrix} 8110.8 \\ 7883.0 \\ 7901.0 \\ 8028.8 \\ 7956.0 \end{pmatrix}$
$sssc\bar{b}$ $(I=0)$						
J^P	$\langle H_{CM}\rangle$	Eigenvalue	Mass	$(B_c\Omega)$	(Ω_cB_s)	
$\frac{3}{2}^-$	71.1	71.1	8470.0	8019.5	8203.8	
$\frac{1}{2}^-$	$\begin{pmatrix} 46.1 & 41.0 \\ 41.0 & 71.6 \end{pmatrix}$	$\begin{pmatrix} 101.8 \\ 15.9 \end{pmatrix}$	$\begin{pmatrix} 8500.7 \\ 8414.8 \end{pmatrix}$	$\begin{pmatrix} 8050.2 \\ 7964.3 \end{pmatrix}$	$\begin{pmatrix} 8234.5 \\ 8148.6 \end{pmatrix}$	

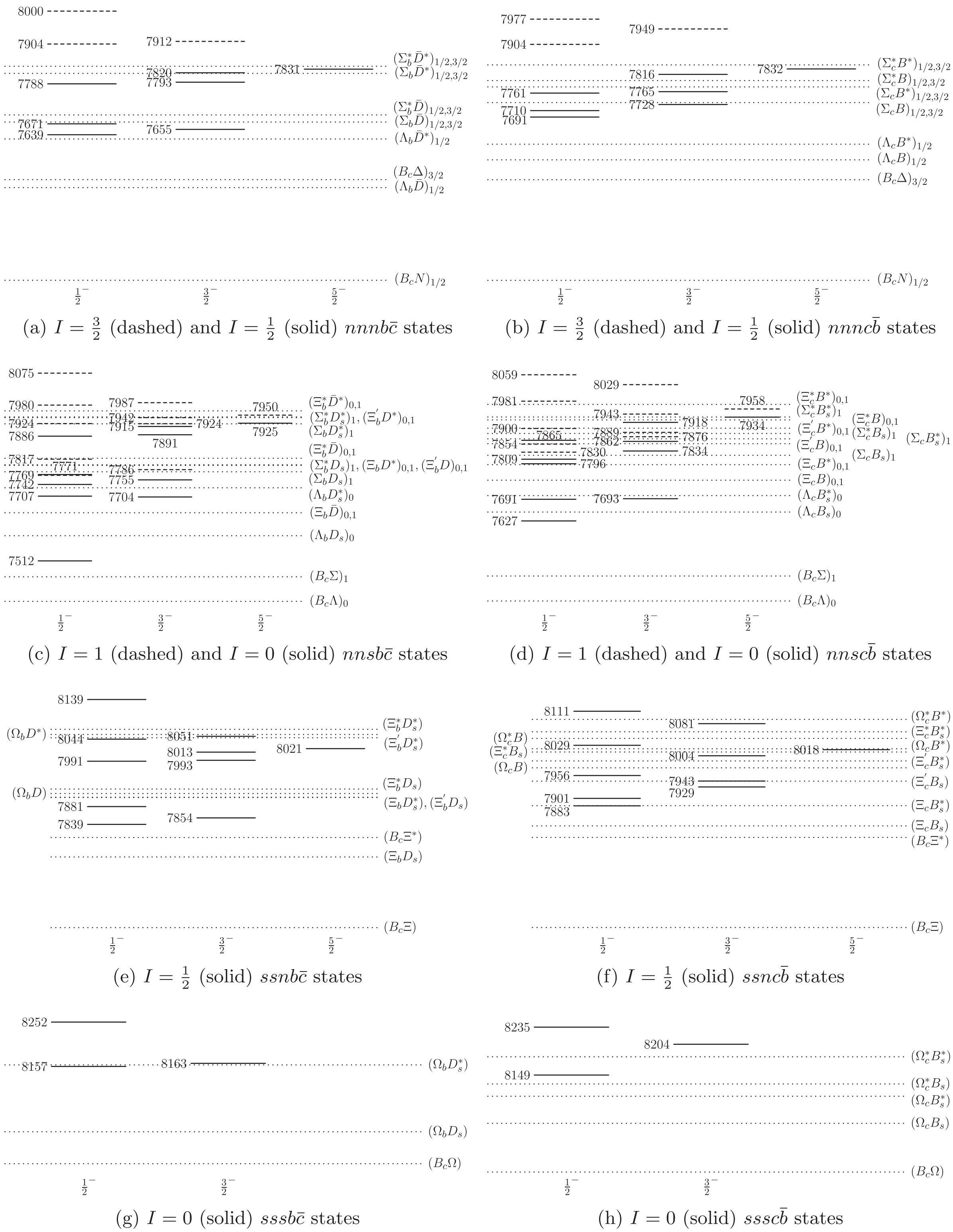

FIG. 5. Relative positions for the obtained $qqqb\bar{c}$ and $qqqc\bar{b}$ pentaquark states. The dotted lines indicate various meson-baryon thresholds. When a number in the subscript of a meson-baryon state is equal to the isospin of an initial state, the decay for the initial state into that meson-baryon channel through the S or D wave is allowed. We adopt the masses estimated with the reference thresholds of $\Sigma_b\bar{D}$ (a), $\Sigma_c B$ (b), $\Xi_b\bar{D}$ (c), $\Xi_c B$ (d), $\Xi_b D_s$ (e), $\Xi_c B_s$ (f), $\Omega_b D_s$ (g), and $\Omega_c B_s$ (h). The masses are all in units of MeV.

both closed for the $J^P = \frac{5}{2}^-$ states, too. Anyway, the Λ-like states below 11100 MeV are worthwhile study [91,92]. The searching for such states in the $\Upsilon\Lambda$ channel will give more information. The study for the isovector pentaquarks in the $\Upsilon\Sigma$ channel is also proposed.

For the Ξ-like and Ω-like hidden-bottom pentaquark systems, there is only one candidate having relatively narrow width, the $J^P = \frac{5}{2}^-$ $ssnb\bar{b}$ state. If its mass is below the threshold of $\Xi_b^* B_s^*$, all the two-body open-bottom decays are through the D wave. Although the decay into $\Upsilon\Xi^*$ may be through the S wave, its contribution should be suppressed. One may search for this state in the $\Upsilon\Xi$ channel.

Now we move on to the more exotic B_c-like pentaquarks. In Ref. [17], possible B_c-like pentaquarks are explored. Such baryons are lighter than the hidden-bottom states but heavier than the hidden-charm partners. If their existence is possible, we here give an estimation for the mass spectra of the $qqqb\bar{c}$ and $qqqc\bar{b}$ systems with colored qqq. These two types of pentaquarks have slightly different masses. We present the numerical results for the $nnnb\bar{c}$, $nnsb\bar{c}$, and $ssnb\bar{c}$ (also $sssb\bar{c}$) systems in Tables X, XI, and XII, respectively. Those for the $nnnc\bar{b}$, $nnsc\bar{b}$, and $ssnc\bar{b}$ (also $sssc\bar{b}$) systems in Tables XIII, XIV, and XV, respectively. For comparison, we show the relative positions for these two types of B_c-like pentaquarks in the same Fig. 5. For the two-body strong decays, all the channels involve flavored hadrons. Here, when we say "open-flavored channel," for convenience, it means that each final hadron contains a heavy (anti)quark.

For the $nnnb\bar{c}$ system, the lowest open-flavored decay channel is $\Lambda_b\bar{D}$ ($\Sigma_b\bar{D}$) for the $I = \frac{1}{2}$ ($I = \frac{3}{2}$) states. All the pentaquarks are above the thresholds and are probably not narrow states. The decays of the $J^P = \frac{5}{2}^-$ state are again through the D wave. The $nnnc\bar{b}$ system has similar features. In Ref. [17], the lowest hadronic molecule is found to be $\Sigma_b\bar{D}$ ($\Sigma_c\bar{B}^*$) in the case that the quark content is $nnnb\bar{c}$ ($nnnc\bar{b}$). Then we here may conclude that more pentaquarks below the molecules are possible. To distinguish a molecule from a compact pentaquark, one needs both further theoretical study and information from measured masses and decay branching ratios. Experimentally, the search can be performed in the $B_c^\pm N$ channels.

For the $nnsb\bar{c}$ and $nnsc\bar{b}$ systems, two $J^P = \frac{1}{2}^-$ narrow states decaying into $B_c^\pm\Lambda$ are possible. If the masses are overestimated, one finds more narrow states. To search for them, the invariant mass distributions in the $B_c^\pm\Lambda$ channels should be studied. One may also search for relatively narrow pentaquarks in the $B_c^\pm\Sigma$ channels.

For the $ssnb\bar{c}$ and $ssnc\bar{b}$ systems, all the $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$ pentaquarks seem to be broad states unless the masses are overestimated. Those with $J^P = \frac{5}{2}^-$ are probably not-so-broad states. The search in the $B_c^\pm\Xi$ channels may provide useful information.

All the pentaquarks in the $sssb\bar{c}$ and $sssc\bar{b}$ systems are probably broad states. It seems not easy to study them experimentally.

In the hidden-charm and hidden-bottom cases, the colored heavy quark pair may be generated from a gluon. However, in the B_c case, the two heavy quarks are produced from different gluons, which indicates that the masses of the pentaquarks with colored $b\bar{c}$ or $c\bar{b}$ are probably affected importantly by the colorless B_c channels. The effects could be studied in a future work.

VI. DISCUSSIONS AND SUMMARY

Recently, the existence of heavy quark states with four quark constituents has been confirmed. Heavy quark pentaquarks should also exist from various theoretical calculations, where the less kinetic energy is helpful to their formation. The observed hidden-charm resonances $P_c(4380)$ and $P_c(4450)$ give an evidence for the existence of pentaquarks, although the states still need further confirmation. For a heavy quark many-body system, the number of the spin partner states becomes large with the increasing number of quarks and the mass splittings between these states should not be large. The identification of such states will help us to understand the formation of multiquark states in QCD. As a preliminary study, we have here investigated the ground state compact pentaquarks $qqqQ\bar{Q}$ ($q = u$, d, s and $Q = b$, c) with colored $Q\bar{Q}$ in a simple model.

We have constructed the flavor-color-spin wave functions for the $nnnc\bar{c}$ ($n = u$, d) pentaquark states from the SU(3) symmetry and calculated the color-spin interactions for the systems. Their masses are estimated with different reference thresholds. Although the study is not a dynamical calculation, the obtained mass spectrum gives a basic feature for the possible pentaquarks. The constructed wave functions are also helpful to study hadron decays in quark models. After including the $SU(3)$-breaking effects and extending the calculation to other heavy quark cases, we obtain systematic results for various pentaquark systems.

There is a feature for the states we considered: the transition from the colored $Q\bar{Q}$ into the colorless $Q\bar{Q}$ should be suppressed and thus the branching ratios for the hidden-flavor decays are small. Our results indicate that most pentaquarks have open-flavor decay channels and should be broad states. The high spin states with $J^P = \frac{5}{2}^-$ probably have relatively narrow widths because the D-wave decays are suppressed. In these pentaquark states, an interesting observation is that the $nnsQ\bar{Q}$ systems contain the low mass and thus narrow Λ-like states. To search for them, the invariant mass distributions in the $J/\psi(\text{or}\eta_c, \Upsilon, B_c) + \Lambda$ channel are strongly called for. In the literature, many investigations were performed in the molecule models. Here we find that pentaquark states below the molecules are also possible. Compared with possible baryon-meson molecules, the compact pentaquarks with the colored qqq cluster should have smaller branching ratios for the hidden-flavor decay channels.

In the present study, we have used the effective coupling constants derived from the conventional hadrons. The couplings are related with the hadron wave functions. But the wave functions in the conventional hadrons and those in the multiquark states should be different, one needs further investigations to answer whether this extension is appropriate or not.

In short summary, we have estimated the masses of the hidden-charm pentaquarks $qqqc\bar{c}$ with a color-magnetic interaction and several reference thresholds, where the qqq cluster is always a color-octet state. Their hidden-bottom and B_c-like partners are also investigated. In each case, we find that pentaquarks with lower masses than hadronic molecules are possible. In the obtained baryon states, the lowest ones seem to be the $J^P = \frac{1}{2}^-$ Λ-like pentaquarks with suppressed decay channels, and the $J^P = \frac{5}{2}^-$ ones decaying through the D wave seem to be relatively narrow, while more states have S-wave open-flavored decay channels and should be broad. The masses and widths of the LHCb $P_c(4380)$ and $P_c(4450)$ are compatible with these features.

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