PHYSICAL REVIEW D **95**, 026003 (2017)

Fermions on the antibrane: Higher order interactions and spontaneously broken supersymmetry

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(Received 9 June 2016; published 4 January 2017)

It has been recently argued that inserting a probe $\overline{\text{D3}}$-brane in a flux background breaks supersymmetry spontaneously instead of explicitly, as previously thought. In this paper we argue that such spontaneous breaking of supersymmetry persists even when the probe $\overline{\text{D3}}$-brane is kept in a curved background with an internal space that does not have to be a Calabi-Yau manifold. To show this we take a specific curved background generated by fractional 3-branes and fluxes on a non-Kähler resolved conifold where supersymmetry breaking appears directly from certain worldvolume fermions becoming massive. In fact this turns out to be a generic property even if we change the dimensionality of the antibrane, or allow higher-order fermionic interactions on the antibrane. We argue for the former by taking a probe $\overline{\text{D7}}$-brane in a flux background and demonstrate the spontaneous breaking of supersymmetry using worldvolume fermions. We argue for the latter by constructing an all-order fermionic action for the $\overline{\text{D3}}$-brane from which the spontaneous nature of supersymmetry breaking can be demonstrated by bringing it to a κ-symmetric form.

DOI: 10.1103/PhysRevD.95.026003

I. INTRODUCTION

It has recently been shown [1,2] that a probe $\overline{\text{D3}}$-brane in a flux background breaks supersymmetry spontaneously, and furthermore, if the $\overline{\text{D3}}$ is placed on an orientifold plane, the only low-energy field content is a single massless fermion.[1] The implications of this are twofold: 1) that supersymmetry (SUSY) breaking is spontaneous, as opposed to explicit, indicates that there is no perturbative instability in the D3-$\overline{\text{D3}}$ system famously used to construct the Kachru-Kallosh-Linde-Trivedi (KKLT) de Sitter solution [6], and 2) as the only four-dimensional field content is a single massless fermion, which can be expressed in the $d = 4$ $\mathcal{N} = 1$ supergravity theory as the spinor component of a nilpotent multiplet, this provides a natural starting point for a string theory embedding of the inflation models proposed in Refs. [7–9] and other works.

This result, and the connection to string cosmology, provides impetus to further investigate $\overline{\text{Dp}}$-brane systems in order to populate the landscape of stable nonsupersymmetric compactifications with $\overline{\text{Dp}}$-branes, to better understand supersymmetry breaking in these models, and to perhaps stumble upon new string theory settings where de Sitter space and inflation naturally arise. It is with these goals in mind that we present three interconnected analyses, which generalize and build upon the work of Refs. [1–3].

A. Spontaneous vs. explicit supersymmetry breaking with antibranes

Before we proceed with our analysis, let us start with a discussion of spontaneous supersymmetry breaking.

Spontaneous supersymmetry breaking is a crucial element of string theory model building. This is because a consistent study of four-dimensional physics requires that all or almost all moduli be stabilized, and all known mechanisms of moduli stabilization[2] are understood in terms of a supersymmetric four-dimensional theory, e.g. the complex structure moduli are fixed via the flux-induced superpotential as in Ref. [11]. Without an underlying supersymmetric theory, i.e. in the case that supersymmetry is explicitly broken, it is not clear to what extent the known methods of moduli stabilization are applicable.

Spontaneous symmetry breaking occurs when the ground state of a theory does not respect the symmetries of the action. This is an essential part of model building in particle physics, supergravity, and string theory, as it gives theoretical control over corrections to the action. The situation in string theory is slightly more complicated than in particle physics, since proposed de Sitter solutions in string theory (for example KKLT [6]) rarely exist as the ground state of the theory, but rather as metastable minima. Given this, we will drop the phrase "ground state" from our

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[1] See also Refs. [3,5], and especially the key papers in Ref. [4], that motivated the research on spontaneous SUSY breaking in the presence of a $\overline{\text{D3}}$-brane.

[2] With the exception of "string gas" moduli stabilization; see e.g. Ref. [10].

definition, and instead refer to nonsupersymmetric states in a supersymmetric theory as spontaneously breaking the supersymmetry.

In simple cases, for example Ref. [2], there is a smoking gun of spontaneous supersymmetry breaking by antibranes: a worldvolume fermion remains massless, which one can identify with the Goldstino of SUSY breaking. However, as discussed in Ref. [3], it will not in general be true that a worldvolume fermion remains massless. Instead, the Goldstino of SUSY breaking will be some combination of open- and closed-string modes. Thus a more general diagnostic of spontaneous breaking is needed, which we will now develop. We will see that even in the absence of a massless fermion on the brane, supersymmetry breaking can still be shown to be spontaneous.

Our diagnostic for spontaneous supersymmetry breaking by a probe $\overline{\rm Dp}$-brane is the following: a solution breaks supersymmetry spontaneously if it is a solution of the theory with the action

$$S = S_{\rm IIB} + S_{\bar{\rm Dp}}, \tag{1}$$

where $S_{\rm IIB}$ is action of type IIB supergravity. The above action is explicitly supersymmetric, since an antibrane is $1/2$ BPS, and thus negates the requirement to "find" the Goldstino in order to deduce that supersymmetry breaking is spontaneous. A probe $\overline{\rm D3}$ in a noncompact flux background without sources can be studied in this way. This reasoning applies directly to our second example: a $\overline{\rm D7}$ in a warped bosonic background without sources, which we will study in Sec. III.

However, this diagnostic is limited in its applicability, as many interesting backgrounds have explicit brane or orientifold content in addition to the probe $\overline{\rm Dp}$. Fortunately, the condition (1) can in fact be extended to apply to a subset of these cases, by making use of string dualities to relate a flux background with branes to a background without branes. Again, this makes no recourse to the Goldstino being a pure open-string mode, i.e. a worldvolume fermion.

Our first example in this paper, a $\overline{\rm D3}$ in a resolved conifold background with wrapped 5-branes, which we study in Sec. II, is an example where dualities must be used to make sense of Eq. (1). One way to arrive at the resolved conifold with wrapped 5-branes background is as a solution to $S = S_{\rm IIB} + S_{\rm D5}$, in which case the addition of a $\overline{\rm D3}$ would break supersymmetry explicitly, since the D5 and $\overline{\rm D3}$ are invariant under different κ symmetries. However, the resolved conifold background can alternatively be found as the dual to the deformed conifold with fluxes and no branes[3]; see for example Refs. [13,14]. In this dual frame the underlying action is source-free, and the addition of a $\overline{\rm D3}$ (again in the dual deformed conifold) will break SUSY spontaneously. The deformed conifold with $\overline{\rm D3}$ can then be dualized back to a resolved conifold with a wrapped D5 along with a $\overline{\rm D3}$, but the spontaneous (as opposed to explicit) nature of SUSY breaking is only manifest in the dual frame.

As we will see, backreaction of the $\overline{\rm D3}$ on the resolved conifold induces masses for all the fermions, so there is no obvious candidate for the Goldstino; this further indicates that the resolved conifold with a wrapped D5 and a $\overline{\rm D3}$ system exhibits explicit breaking of supersymmetry. This is consistent with our discussion above: the spontaneous nature of SUSY breaking is only manifest in the dual deformed conifold description. In terms of moduli stabilization, a dual description in terms of spontaneous breaking allows one to consistently define a superpotential for both the Kähler and complex structure moduli, which is precisely the feature of "spontaneous breaking" that is useful for studying $4d$ physics from string theory.

B. Outline of the paper

Our first analysis, studied in Sec. II, considers a probe $\overline{\rm D3}$-brane, not in a Calabi-Yau background [11,15] as studied in Ref. [2], but in a non-Kähler resolved conifold background with integer and fractional 3-branes. We will construct a supersymmetric deformation to the Calabi-Yau resolved conifold that converts it to a non-Kähler resolved conifold, provides a nonzero curvature to the internal space, and which induces a nonzero amount of imaginary self-dual (ISD) fluxes. Once a probe $\overline{\rm D3}$ is introduced, supersymmetry is spontaneously broken by the coupling of ISD fluxes to the worldvolume fermions, giving masses to the worldvolume fermions. This breaking is in fact "soft" as the fluxes and fermion masses are set by the non-Kählerity of the internal space, which is in turn a tunable parameter. The picture is somewhat similar to the case with Calabi-Yau internal space as studied in Ref. [2] but the analysis differs in terms of fluxes and backreaction. In particular, the analysis in the probe approximation now yields *two* massless fermions, as opposed to one in Ref. [2]. This result is modified upon considering backreaction of the $\overline{\rm D3}$ on the bulk fluxes, which generates both (2, 1) and (1, 2) 3-form fluxes, inducing masses for *all* the worldvolume fermions, i.e. there are *zero* massless fermions remaining in the spectrum. We also study certain aspects of de Sitter vacua from our analysis. It interesting to note that a curved internal space appears to be a requirement for de Sitter solutions in string theory, at least in many contexts, especially negatively curved internal spaces (see for example Ref. [16] and references therein). With this in mind, we consider moduli stabilization in this background, and the connection to de Sitter space in this model.

The physics discussed above remains largely unchanged even if we change the dimensionality of the antibrane. In Sec. III, we consider a second application of antibrane

[3]The dual is succinctly described in supergravity when the number of wrapped D5-branes is very large [12,13].

fermionic actions and take a probe[4] $\overline{\mathrm{D7}}$-brane, this time working with a Calabi-Yau background. Supersymmetry is again broken spontaneously via flux-induced fermion masses, and the masses are proportional to the piece of the 3-form flux which is ISD in the space transverse to the brane. In the $\overline{\mathrm{D3}}$ case, where the transverse space is the entire internal space, this flux is precisely the flux of the Giddings-Kachru-Polchinski (GKP) background.[5] However, in the $\overline{\mathrm{D7}}$ case, the fermion masses are now sourced by the subset of these fluxes which are ISD in the two directions transverse to the brane. In other words, the fermion masses are now determined solely by fluxes that have two legs on the brane, and one leg off. We show that for a special class of flux background there can be many massless fermions in the low-energy spectrum, while in a general flux background there may be none. This provides yet another instance of a string theory realization of nilpotent Goldstinos,[6] and a possible starting point for inflation and de Sitter solutions.

Our final application is actually closer to a derivation; we study the fermionic $\overline{\mathrm{D3}}$ action at *all* orders in the fermionic expansion. To do this, we promote the bosonic fields to superfields, and discuss the physics at the self-dual point. At the self-dual point we can use U-dualities to relate various pieces of the multiplet and consequently determine the fermionic completions of the different fields. Once we move away from the self-dual point, we can determine the fermionic completions of all the bosonic fields in a compact form. As an added bonus, we find that the all-order fermionic action can be written in a manifestly κ-symmetric form, even without precise details of the form of the terms in the action. The orientifolding action can then be easily incorporated in the action. This indicates that the spontaneous nature of supersymmetry breaking by antibranes, both in the presence and in the absence of an orientifold plane, is not a leading-order effect, but in fact continues to be true to all orders. This puts the conclusions of Refs. [1,2], and its implications for KKLT, on solid footing.

We conclude with a short discussion of the implications of our work and directions for future research.

II. $\overline{\mathrm{D3}}$-BRANE IN A RESOLVED CONIFOLD BACKGROUND: SOFT (AND SPONTANEOUS) BREAKING OF SUPERSYMMETRY

The breaking of supersymmetry by a probe $\overline{\mathrm{D3}}$-brane in a warped bosonic background was studied recently in Ref. [2]. They studied a $\overline{\mathrm{D3}}$-brane in a GKP background, and found that supersymmetry was spontaneously broken by the coupling of ISD fluxes to the worldvolume fermions. In this section we perform a similar analysis, focusing instead on a probe $\overline{\mathrm{D3}}$-brane in a resolved conifold background. We will consider a deformation to the Calabi-Yau resolved conifold which maintains supersymmetry but provides a nonzero curvature to the internal space, and which induces ISD 3-form fluxes from a set of integer and fractional D3-branes. Once a probe $\overline{\mathrm{D3}}$ is introduced, supersymmetry is again spontaneously (and softly) broken by the coupling of ISD fluxes to the worldvolume fermions, and the fermion masses can be straightforwardly computed. As we will see, the "soft" nature of supersymmetry breaking is due to the tunable nature of the non-Kählerity of the internal manifold.

The key details of the fermionic action for a $\overline{\mathrm{D3}}$-brane in a warped bosonic background are given in Ref. [2]. These will be the starting point of our analysis, so here we merely quote them. The worldvolume action is given, in a convenient κ-symmetry gauge, by

$$\mathcal{L}_f^{\overline{\mathrm{D3}}} = T_3 e^{4A_0}\bar{\theta}^1\left[2e^{-\phi}\Gamma^\mu\nabla_\mu - \frac{i}{12}(\mathcal{G}^{\mathrm{ISD}}_{mnp} - \overline{\mathcal{G}}^{\mathrm{ISD}}_{mnp})\Gamma^{mnp}\right]\theta^1 \tag{2}$$

where θ^1 is a 16-component[7] $10d$ Majorana-Weyl spinor,[8] and we have defined the 3-form flux $\mathcal{G}_3$ as $\mathcal{G}_{(3)} = F_{(3)} - \tau H_{(3)}$. The 16-component spinor θ^1 can be decomposed into four $4d$ Dirac spinors λ^0, λ^i with $i = 1, 2, 3$. On a Calabi-Yau manifold, the λ^0 is a singlet under the SU(3) holonomy group of the internal Calabi-Yau manifold while the λ^i transform as a triplet.

We can now rewrite the $\overline{\mathrm{D3}}$-brane action (2) using the $4d$ decomposition of the θ^1 spinor in the following way:

$$\begin{aligned}\mathcal{L}_f^{\overline{\mathrm{D3}}} = 2T_3 e^{4A_0-\phi}\Big[&\bar{\lambda}^{\bar{0}}_-\gamma^\mu\nabla_\mu\lambda^0_+ + \bar{\lambda}^{\bar{j}}_-\gamma^\mu\nabla_\mu\lambda^i_+\delta_{i\bar{j}}\\ &+\frac{1}{2}m_0\bar{\lambda}^0_+\lambda^0_+ + \frac{1}{2}\overline{m}_0\bar{\lambda}^{\bar{0}}_-\lambda^{\bar{0}}_- + m_i\bar{\lambda}^0_+\lambda^i_+ + \overline{m}_{\bar{i}}\bar{\lambda}^{\bar{0}}_-\lambda^{\bar{i}}_-\\ &+\frac{1}{2}m_{ij}\bar{\lambda}^i_+\lambda^j_+ + \frac{1}{2}\overline{m}_{\bar{i}\bar{j}}\bar{\lambda}^{\bar{i}}_-\lambda^{\bar{j}}_-\Big],\end{aligned} \tag{3}$$

where we use $\pm$ subscripts to denote $4d$ Dirac spinors that satisfy $\lambda_\pm = \frac{1}{2}(1 \pm i\tilde{\Gamma}_{0123})\lambda$, and the masses are defined as

$$m_0 = \frac{\sqrt{2}}{12}\mathrm{i}e^\phi\overline{\Omega}^{uvw}\overline{\mathcal{G}}^{\mathrm{ISD}}_{uvw}, \qquad \text{from } (0,3) \text{ flux}, \tag{4}$$

$$m_i = -\frac{\sqrt{2}}{4}e^\phi e^u_i\overline{\mathcal{G}}^{\mathrm{ISD}}_{uv\bar{w}}J^{v\bar{w}}, \quad \text{from nonprimitive } (1,2) \text{ flux}, \tag{5}$$

[4]By assuming such a heavy object as a probe simply means that the logarithmic backreactions of the $\overline{\mathrm{D7}}$-brane on geometry and fluxes are suppressed by powers of g_s.

[5]Henceforth by GKP background we will always mean the background proposed in Refs. [11,15].

[6]See Refs. [17–19] for even more examples.

[7]16 complex components, or 32 real components.

[8]We have already fixed κ symmetry.

$$m_{ij} = \frac{\sqrt{2}}{8} \mathrm{i} e^{\phi} (e_i^w e_j^t + e_j^w e_i^t) \Omega_{uvw} g^{u\bar{u}} g^{v\bar{v}} \mathcal{G}^{\mathrm{ISD}}_{\bar{t}\bar{u}\bar{v}}, \quad \text{from primitive } (2,1) \text{ flux}, \tag{6}$$

where J and Ω are the Kähler form and holomorphic 3-form respectively.

We are interested in a more general background, where the SU(3) holonomy will be broken by a perturbation to the geometry. Compactifications on manifolds with SU(3) structure but not SU(3) holonomy have been studied in, for example, Refs. [20] and [21]. These are non-Kähler manifolds, which in general may or may not have an integrable complex structure, and are classified by five torsion classes $\mathcal{W}_i$ [22–24]. The simplest case, where all five torsion classes vanish, is a Calabi-Yau manifold that supports no fluxes. We are looking for the case with fluxes, so that we can make use of Eqs. (4), (6), and (5), and therefore some of the torsion classes must be nonzero.

Moreover, the non-Kähler manifold that we need has to be a complex manifold, otherwise the flux decomposition in terms of (2, 1), (1, 2) or (0, 3) forms would not make any sense. In addition, the manifold should be noncompact, so as to avoid any tension with Gauss' law. The simplest internal manifold that satisfies our requirements is the resolved conifold with a non-Kähler metric which allows an integrable complex structure (and by definition does not have a conifold singularity).

The goal of this section will be to study the action (2) or (3) in a resolved conifold with an arbitrary amount of D3-branes and delocalized five branes (see Refs. [25] and [26] for more details on delocalized sources). More precisely, we will put a $\overline{\mathrm{D3}}$-brane in a supersymmetric background with metric given by

$$ds^2 = \frac{1}{e^{2\phi/3}\sqrt{e^{2\phi/3}+\Delta}} ds^2_{0123} + e^{2\phi/3}\sqrt{e^{2\phi/3}+\Delta}\, ds^2_6, \tag{7}$$

where e^{ϕ} is related to the type IIB dilaton e^{ϕ}_B as $\phi_B = -\phi$ and the factor Δ encodes the backreaction of the 3-branes. It is defined using a parameter β as

$$\Delta = \sinh^2\beta (e^{2\phi/3} - e^{-4\phi/3}). \tag{8}$$

The other piece appearing in Eq. (7) is ds^2_6, which is the metric of the internal six-dimensional non-Kähler resolved conifold. This is expressed in terms of the coordinates (r, ψ, θ_i, ϕ_i) in the following way:

$$ds^2_6 = F_1 dr^2 + F_2 (d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2 + \sum_{i=1}^{2} F_{2+i} (d\theta_i^2 + \sin^2\theta_i d\phi_i^2), \tag{9}$$

where the resolution parameter is proportional to $F_3 - F_4$.

We will start by making an ansatz for the warp factors $F_i(r)$ appearing in Eq. (9) which will allow us to see how to go from a Ricci-flat Calabi-Yau metric to a non-Kähler metric on a resolved conifold. A more generic class of solutions for the warp factors exists and has been discussed in Ref. [26], but we will only consider a subset given by

$$F_1 = \frac{1}{F} + \delta F, \qquad F_2 = \frac{r^2 F}{9}, \qquad F_3 = \frac{r^2}{6} + a_1^2(r), \qquad F_4 = \frac{r^2}{6} + a_2^2(r), \qquad \phi = \phi(r), \tag{10}$$

where F, $\delta F(r)$, $a_1(r)$, and $a_2(r)$, are functions of the radial coordinate only. From the above ansatz, it is easy to see where the Calabi-Yau (CY) case fits in. It is given by

$$F(r) \equiv F_{CY} = \left(\frac{r^2 + 9a^2}{r^2 + 6a^2}\right), \qquad \delta F(r) = 0, \qquad a_1(r) = a, \qquad a_2(r) = 0, \qquad \phi = 0. \tag{11}$$

The Calabi-Yau case is fluxless (with the vanishing of the flux enforced by supersymmetry), and has a constant dilaton. Once we switch on fluxes, we can no longer assume that the other pieces of the warp factors appearing in Eq. (10) vanish.

As a cautionary tale, let us first consider whether we can perturb away from the Calabi-Yau resolved conifold simply by allowing for a small perturbation in $F(r)$ and $\phi(r)$. We will see that this in fact does not lead to useful results, and thus we will need to be more careful in constructing our geometry. Nonetheless, it is useful for establishing an algorithm for constructing solutions.

Consider a small perturbation to Eq. (11) of the form

$$F(r) = F_{CY} + \sigma f(r), \qquad \delta F(r) = 0, \qquad a_1(r) = a e^{-\phi}, \qquad a_2(r) = 0, \tag{12}$$

where σ is a dimensionless expansion parameter, that satisfies the EOMs and takes the solution from the Calabi-Yau resolved conifold to the non-Kähler resolved conifold. We can narrow down our perturbation scheme by allowing the dilaton field to behave in the following way:

$$\phi(r) = \log\left(\frac{1}{r^{\sigma}}\right), \tag{13}$$

which would guarantee the existence of a small parameter σ that, while preserving supersymmetry, would be responsible in taking us away from the Calabi-Yau case. In the limit $\sigma \to 0$, we go back to the fluxless Calabi-Yau case. This geometry is of course singular in the $r \to \infty$ limit, but we will assume for this discussion that the geometry is capped off at some sufficiently large r. In any case, this issue will not be important, as this perturbation fails for other reasons.

A way to construct such a background has already been discussed in Ref. [26], and therefore we will simply quote some of the steps. The best and probably the easiest way to analyze such a background is by using the torsion classes. For us the relevant torsion classes are $\mathcal{W}_4$ and $\mathcal{W}_5$. They can be expressed in terms of the warp factors $F_i(r)$ and the dilaton $\phi(r)$ in the following way:

$$\mathcal{W}_4 = \frac{F_{3r} - \sqrt{F_1F_2}}{4F_3} + \frac{F_{4r} - \sqrt{F_1F_2}}{4F_4} + \phi_r,$$
$$\mathrm{Re}\mathcal{W}_5 = \frac{F_{3r}}{12F_3} + \frac{F_{4r}}{12F_4} + \frac{F_{2r} - 2\sqrt{F_1F_2}}{12F_2} + \frac{\phi_r}{2}. \quad (14)$$

The other torsion classes take specific values, with $\mathcal{W}_3$ determining the torsion. This solution is generated by following the duality chain described in Ref. [26], which generates both the Ramond-Ramond (RR) and the Neveu-Schwartz (NS) 3-forms $\mathcal{F}_3$ and $\mathcal{H}_3$ respectively.

Our aim then is to use these torsion classes to determine the functional form for the warp factors F_i using the specific variation of the ansatz (10) i.e. Eqs. (12) and (13). The key relation, that allows us to find the connection between $F(r)$ and the dilaton $\phi(r)$, is the supersymmetry condition:

$$2\mathcal{W}_4 + \mathrm{Re}\mathcal{W}_5 = 0. \quad (15)$$

Plugging in the ansatz (12) and Eq. (13) into Eq. (15) will allow us to determine $f(r)$ completely in terms of the radial coordinate r and the resolution parameter a^2. The functional form for $f(r)$ turns out to be a nontrivial function of r:

$$f(r) = \frac{2}{(6a^2+r^2)}\left\{27a^2(6a^2+r^2)\left[\sum_{i=1}^{3}\Phi_i(r;a^2) + r^2\log r\right] - (9a^2+r^2)(6a^2+r^2)\left[3\log\left(\frac{r^2}{6a^2}+1\right) + 2 - \frac{r^2\log r}{6a^2+r^2}\right]\right\}, \quad (16)$$

which is defined for $a^2 > 0$. For vanishing a^2 the functional form for $f(r)$ simplifies and has been studied earlier in Ref. [27]. The other variables appearing in Eq. (16) are defined in the following way:

$$\Phi_1(r;a^2) = {}_2F_1^{(0,0,1,0)}\left(-1, 2, 3, -\frac{r^2}{6a^2}\right),$$
$$\Phi_2(r;a^2) = {}_2F_1^{(0,1,0,0)}\left(-1, 2, 3, -\frac{r^2}{6a^2}\right),$$
$$\Phi_3(r;a^2) = {}_2F_1^{(1,0,0,0)}\left(-1, 2, 3, -\frac{r^2}{6a^2}\right), \quad (17)$$

where the notation ${}_2F_1^{(0,1,0,0)}$ refers to $\partial_{y_2}F_1[x;y;z;w]$, and similarly for ${}_2F_1^{(1,0,0,0)}$ and ${}_2F_1^{(0,0,1,0)}$. This perturbation to $F(r)$ corresponds to introducing a small Ricci scalar on the internal space. This could be computed using the torsion classes [28], or computed directly using standard general relativity (GR) techniques. Using GR techniques, we find that a simple expression emerges for a small resolution parameter a^2 and a small value for the parameter σ:

$$\delta R_6 = -\frac{72\sigma}{r^2}\left[3 - 2\log\left(\frac{6a^2}{r^2}\right)\right], \quad (18)$$

which is negative for $r \geq 1.2a$. Furthermore one can check that for *general* a, i.e. not small a, while the expression for δR_6 is no longer simple, it is negative definite. It is interesting to note that negatively curved internal spaces have been widely studied as a mechanism for finding de Sitter solutions in string theory; see the discussion and references in Ref. [16].

The above analysis, although interesting because of the control we can have on the non-Kählerity of the internal manifold, is ultimately *not* useful for finding the masses of the $\overline{\mathrm{D3}}$ worldvolume fermions, as it in fact renders the internal manifold with a nonintegrable complex structure. Thus, there exists an almost complex structure but the manifold itself may not be complex.[9] This means we cannot decompose our $\mathcal{G}_3$ flux in terms of (1, 2), (2, 1) or (0, 3) forms in a global sense, making the fermionic mass decompositions given in Eqs. (6), (5) and (4), not very practical in analyzing the fermions on the probe $\overline{\mathrm{D3}}$. This of course does not mean that we cannot study the spontaneous SUSY breaking; we can, but the analysis will not be so straightforward as it was with the complex decomposition of the 3-form fluxes.

The question then is: can we have a *complex* non-Kähler resolved conifold satisfying a more generic ansatz like Eq. (10) where we can use Eqs. (4), (6), and (5), to study spontaneous SUSY breaking with a probe $\overline{\mathrm{D3}}$? The answer turns out to be in the affirmative, and in the following section we elaborate the story.[10]

[9]There might exist a nontrivial *integrable* complex structure, but we have not been able to find one.

[10]Note that there is some subtlety with the mapping to Ref. [29] at this stage, for example the possibility of a non-Kähler special Hermitian solution with a constant dilaton that we get here demanding supersymmetry as opposed to a Calabi-Yau resolved conifold with a constant dilaton studied in Ref. [29]. This has been discussed in detail in Ref. [26] so we will not dwell on this any further.

A. A SUSY perturbation of the resolved conifold

Let us start with a simple example of a D3-brane located at a point in an internal manifold specified by the metric ds_6^2 where ds_6^2 is given by

$$ds_6^2 = dr^2 + g_{mn}dy^m dy^n, \tag{19}$$

where $(r,\ y^m)$ are the coordinates of the internal six-dimensional space. To avoid contradiction with Gauss' law, the internal manifold has to be noncompact, although a compact example could be constructed by either inserting orientifold planes, or antibranes. Details of this will be discussed later. The backreaction of the D3-brane converts the vacuum manifold

$$ds^2_{\rm vac} = ds^2_{0123} + ds_6^2, \tag{20}$$

with ds^2_{0123} being the Minkowski metric along the space-time directions, to the following:

$$ds^2_{10} = \frac{1}{\sqrt{h}}ds^2_{0123} + \sqrt{h}ds_6^2, \tag{21}$$

where h is the warp factor. The five-form flux in the background (21) is now given as

$$\mathcal{F}_5 = \frac{1}{g_s}(1 + *_{10})dh^{-1} \wedge dx^4. \tag{22}$$

The above analysis is generic, but it is highly nontrivial to actually compute the warp factor h. For a complicated internal space, the equation for h typically becomes an involved second-order partial differential equation. Furthermore, in the presence of other type IIB fluxes, for example the 3-form fluxes $\mathcal{H}_3$ and $\mathcal{F}_3$, the metric is more complicated than Eq. (21). Additionally, the string coupling constant generically will not be constant.

There is, however, a way out of the above conundrum if we analyze the picture from a more general setting. We can use the powerful machinery of torsional analysis [23,24,30] to write the background of a D5-brane wrapped on some two-cycle, parametrized by (θ_1, ϕ_1), of a generic six-dimensional internal space. Assuming that the size of the wrapped cycle is smaller than some chosen scale, any fluctuations along the (θ_1, ϕ_1) will take very high energy to excite. This means at low energies the theory will be of an effective D3-brane[11] and the source charge of the wrapped D5-brane C_6 will decompose as

$$C_6(\vec{\mathbf{x}}, \theta_1, \phi_1) = C_4(\vec{\mathbf{x}}) \wedge \left(\frac{e_{\theta_1} \wedge e_{\phi_1}}{\sqrt{V}}\right), \tag{23}$$

where V is the volume of the two-cycle on which we have the wrapped D5-brane. Therefore using the criteria (23), the supergravity background for the configuration of the effective D3-brane is given by

$$\begin{aligned} ds^2 &= e^{-\phi}ds^2_{0123} + e^{\phi}ds_6^2,\\ \mathcal{F}_3 &= e^{2\phi} *_6 d(e^{-2\phi}J), \end{aligned} \tag{24}$$

where ϕ is the dilaton and the Hodge star and the fundamental form J are with respect to the dilaton deformed metric $e^{2\phi}ds_2^6$. The 5-brane charge in Eq. (24) decomposes as Eq. (23) once we express it as a seven-form $\mathcal{F}_7 = *_{10}\mathcal{F}_3$. The metric ds_6^2 is in general a noncompact non-Kähler metric that may not even have an integrable complex structure.

If we allow for background 3-forms $\mathcal{F}_3$ and $\mathcal{H}_3$, the above background (24) changes. One way to see the change would be to work out the precise EOMs. However there exists another way, using a series of duality transformations, to study the background in the presence of the 3-form fluxes. The steps have been elaborated in Refs. [26,33,34]. The solutions we will study here are specific realizations of the general solutions found and analyzed in Ref. [26], where supersymmetry of the final "dualized" solution was explicitly confirmed.[12] The idea is the following:

(1) Compactify the spatial coordinates $x^{1,2,3}$ and T-dualize three times along these directions. The resulting picture will now be in type IIA theory.
(2) Lift the type IIA configuration to M-theory and make a boost along the eleventh direction using a boost parameter β. This boosting will create the necessary gauge charges.
(3) Reduce this down to type IIA and T-dualize three times along the spatial coordinates to go to type IIB theory. The IIB background now automatically has the 3-form fluxes, as well as a five-form flux.

The result of this duality procedure is that the type IIB background (24) now converts to exactly what we expect in Eq. (21), namely[13]

[11] Also known as a fractional D3-brane. There is yet another way to generate a fractional D3-brane which we do not explore here. For example if we take wrapped D5-$\overline{\rm D5}$-branes with (n_1, n_2) amount of gauge fluxes on each of them, then we can have bound D3-branes with charges n_1 and n_2 respectively. If n_i are fractional, these give fractional 3-branes with vanishing global 5-brane charges. See Refs. [31,32] for more details.

[12] In addition, the fact that the T-duality transformations lead to solutions that solve explicitly the supergravity EOMs has been shown earlier in Refs. [35–37]. In Refs. [21] and [26], this was confirmed using torsion classes. The subtlety that such transformations *do not* lead to nontrivial Jacobians follows from the fact that the supergravity fields have no dependence on the T-duality directions. If the supergravity fields start to depend on the T-duality directions, there will arise nontrivial Jacobians as discussed in some detail in Ref. [38]. We thank the referee for raising this question.

[13] There is some subtlety in interpreting the final background with fluxes or with sources. This has been discussed in Ref. [34] which the readers may refer to for details.

$$ds^2 = \frac{1}{\sqrt{h}} ds^2_{0123} + \sqrt{h} ds^2_6$$
$$= \frac{1}{e^{2\phi/3}\sqrt{e^{2\phi/3}+\Delta}} ds^2_{0123} + e^{2\phi/3}\sqrt{e^{2\phi/3}+\Delta}\, ds^2_6, \tag{25}$$

confirming the low-energy effective D3-brane behavior, and the following background for the three- and the five-form fluxes:

$$\mathcal{F}_3 = \cosh\beta e^{2\phi} *_6 d(e^{-2\phi}J), \qquad \mathcal{H}_3 = -\sinh\beta d(e^{-2\phi}J),$$
$$d\tilde{\mathcal{F}}_5 = -\sinh\beta\cosh\beta e^{2\phi} d(e^{-2\phi}J) \wedge *_6 d(e^{-2\phi}J), \tag{26}$$

with the type IIB dilaton $e^{\phi_B} = e^{-\phi}$. One may verify that Eqs. (25) and (26) together solve the type IIB EOMs.

We will concentrate on a specific background given by a (generically non-Kähler) singular, resolved or deformed conifold. The typical internal metric ds^2_6 in this class is given by a variant of Eq. (9) as

$$ds^2_6 = F_1 dr^2 + F_2(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2$$
$$+ \sum_{i=1}^{2} F_{2+i}(d\theta_i^2 + \sin^2\theta_i d\phi^2)$$
$$+ F_5 \sin\psi(d\phi_1 d\theta_2 \sin\theta_1 + d\phi_2 d\theta_1 \sin\theta_2)$$
$$+ F_6 \cos\psi(d\theta_1 d\theta_2 - d\phi_1 d\phi_2 \sin\theta_1 \sin\theta_2), \tag{27}$$

where $F_i(r)$ are warp factors that are functions of the radial coordinate r only[14] and in the following, unless mentioned otherwise, we will only consider the resolved conifold, i.e we take $F_5 = F_6 = 0$ henceforth. The above background (27) can be easily converted to a background with both $\mathcal{H}_3$ and $\mathcal{F}_3$ fluxes by the series of duality specified above. Using Eq. (25), our background becomes

$$ds^2 = \frac{1}{e^{2\phi/3}\sqrt{e^{2\phi/3}+\Delta}} ds^2_{0123} + e^{2\phi/3}\sqrt{e^{2\phi/3}+\Delta}\, ds^2_6,$$
$$\mathcal{F}_3 = -e^{2\phi}\cosh\beta\sqrt{\frac{F_2}{F_1}}(g_1 e_\psi \wedge e_{\theta_1} \wedge e_{\phi_1} + g_2 e_\psi \wedge e_{\theta_2} \wedge e_{\phi_2}),$$
$$\tilde{\mathcal{F}}_5 = -\sinh\beta\cosh\beta(1 + *_{10})\mathcal{C}_5(r) d\psi \wedge \prod_{i=1}^{2} \sin\theta_i d\theta_i \wedge d\phi_i,$$
$$\mathcal{H}_3 = \sinh\beta[(\sqrt{F_1F_2} - F_{3r}) e_r \wedge e_{\theta_1} \wedge e_{\phi_1}$$
$$+ (\sqrt{F_1F_2} - F_{4r}) e_r \wedge e_{\theta_2} \wedge e_{\phi_2}], \tag{28}$$

with a dilaton $e^{\phi_B} = e^{-\phi}$ and with Δ defined as in Eq. (8),

$$\Delta = \sinh^2\beta(e^{2\phi/3} - e^{-4\phi/3}), \tag{29}$$

and β is the boost parameter discussed above while the others, namely $(g_1, g_2, \mathcal{C}_5)$ are given by

$$g_1(r) = F_3\left(\frac{\sqrt{F_1F_2} - F_{4r}}{F_4}\right),$$
$$g_2(r) = F_4\left(\frac{\sqrt{F_1F_2} - F_{3r}}{F_3}\right),$$
$$\mathcal{C}_5(r) = \int^r \frac{e^{2\phi}F_3F_4\sqrt{F_1F_2}}{F_1}\left[\left(\frac{\sqrt{F_1F_2} - F_{3r}}{F_3}\right)^2\right.$$
$$\left. + \left(\frac{\sqrt{F_1F_2} - F_{4r}}{F_4}\right)^2\right] dr. \tag{30}$$

The above background for the D3-brane is consistent as long as the energy is less than the inverse size of the sphere parametrized by (θ_1, ϕ_1). For vanishing size of the sphere, which would happen for a singular conifold, our analysis continues to hold to arbitrary energies.

Equation (28) contains all the information that we need, so now the relevant question is to find appropriate warp factors that allow us to have a non-Kähler resolved conifold with an integrable complex structure. A simple analysis of the fluxes along the lines of Ref. [26] will tell us that an integrable complex structure is possible when the dilaton has no profile in the internal direction. This means we can take, without any loss of generality, a vanishing dilaton inducing the following complex structure on the internal space:

$$\tau_k \equiv (i\coth\beta, i, i). \tag{31}$$

The metric on the internal space now is not too hard to find if one takes care of all the subtleties pointed out in Ref. [26]. The subtleties are generically related to flux quantization and integrability conditions. Once the dust settles the metric becomes

$$ds^2 = 4F^2_{2r}\left(\frac{1-G}{2+F_2}\right)dr^2$$
$$+ F_2(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2$$
$$+ G(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2)$$
$$+ G(1-G)\left(\frac{F_2}{2+F_2}\right)(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2), \tag{32}$$

where $F_2(r)$ is taken to be dimensionless. This means all terms of the metric are dimensionless, and thus if r has a dimension of length, the warp factor should have the dimension of inverse length. This works out fine because

[14]One may generalize this to make the warp factors F_i functions of all coordinates except (θ_1, ϕ_1), i.e the directions of the wrapped brane. We will not discuss the generalization here.

the coefficient of dr^2 is indeed the derivative of F_2. We could also rewrite the metric with dimensionful warp factors but this would not change any of the physics. Note also that $G(r)$ appearing in Eq. (32) is not an independent function, but depends on F_2 in the following way:

$$(1-G)^3 = \frac{(2+F_2)^3}{F_2(3+2F_2)^2}, \tag{33}$$

and therefore an appropriate choice of F_2 will fix the functional form for G. Furthermore, the resolution parameter for the resolved conifold is no longer a constant, but a function of the radial coordinate r that takes the following form:

$$a^2(r) \equiv \frac{(2+GF_2)G}{2+F_2}, \tag{34}$$

which is by construction a positive-definite function provided G remains positive definite everywhere. It is definitely a well-behaved function at any point in r since $F_2 > 0$ and if F_2 is chosen to be a well-behaved function of r. Positivity of G implies that at any point in r, F_2 should satisfy

$$F_2^3 + 2F_2^2 - F_2 > \frac{8}{3}, \tag{35}$$

which is not hard to satisfy. This also implies $G < 1$ at any point in r. A simple choice of $F_2(r)$ would be to consider the following functional form that should make all the warp factors positive definite:

$$F_2(r) = 1.1022 + \tilde{F}_2^2(r), \tag{36}$$

assuming $\tilde{F}_2(r)$ never hits zero at any point in r. We can also bring our metric (32) to the form (10) by appropriately defining δF, $a_1(r)$ and $a_2(r)$.

It is now time to determine the fluxes that preserve the background supersymmetry. As is well known, the fluxes should be ISD and primitive, so the appropriate choice is to take (2, 1) forms. This can be easily worked out from Eq. (28), and once we fix the complex structure to be Eq. (31), and with the above warp factors and dilaton, the 3-form flux takes a particularly simple form[15]:

$$\begin{aligned}\mathcal{G}_3 &= \frac{\sinh\beta}{4\sqrt{H}\sqrt{F_1\sqrt{H}}}\left[\left(\frac{\sqrt{F_1F_2}-F_{3r}}{F_3}\right)-\left(\frac{\sqrt{F_1F_2}-F_{4r}}{F_4}\right)\right](E_1\wedge E_3\wedge\bar{E}_3 - E_1\wedge E_2\wedge\bar{E}_2),\\ &= \frac{\sqrt{F}(2-F\delta F)}{8\mathrm{cosech}\beta}\left[\left(\frac{rF\delta F-12a_1a_{1r}}{r^2+6a_1^2}\right)-\left(\frac{rF\delta F-12a_2a_{2r}}{r^2+6a_2^2}\right)\right](E_1\wedge E_3\wedge\bar{E}_3 - E_1\wedge E_2\wedge\bar{E}_2),\end{aligned} \tag{37}$$

which is ISD, primitive, and a (2,1) form. In the second line we have used the ansatz (10) with a vanishing dilaton. Note also that the three functions δF, a_1, and a_2 are constrained by supersymmetry, via Eq. (15), which is a first-order ordinary differential equation. The SUSY condition also forces the (1, 2) components of $\mathcal{G}_3$ to vanish identically.

One can see that the boost parameter β, which counts the units of $\mathcal{F}_3$ flux, or equivalently the number of delocalized [25] 5-branes, in the resolved conifold background, controls the amount of ISD flux. Naively, if we take $\beta \to 0$, the flux vanishes. However the complex structure (31) also blows up in this limit, so the vanishing β case has to be studied differently. This is indeed the case because, in the language of Ref. [26], taking $\beta \to 0$ takes us to the "before duality" picture where only RR 3-form fluxes are present. Therefore the way we derived our background, we can take β arbitrarily small but not zero.

This completes our analysis of the supersymmetric fluxes on a non-Kähler resolved conifold background that allows an integrable complex structure. In the following section we will insert a $\overline{\text{D3}}$-brane in this background and study the fluxes and the corresponding supersymmetry-breaking scenario using the worldvolume action. We start with the bosonic action for a $\overline{\text{D3}}$-brane in this background.

B. Bosonic action for a $\overline{\text{D3}}$-brane

Before considering a $\overline{\text{D3}}$, let us consider a D3. In the previous section we saw how to incorporate the backreaction of a single (or generically N) *effective* D3-brane in a flux background. We can compute the bosonic action of

[15]Where the E_i are defined as

$$E_1 = e_1 + i\coth\beta e_2, \qquad E_2 = e_3 + ie_4, \qquad E_3 = e_5 + ie_6,$$

with

$$\begin{aligned}e_1 &= \sqrt{F_1\sqrt{H}}e_r,\\ e_2 &= \sqrt{F_2\sqrt{H}}(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2) = \sqrt{F_2\sqrt{H}}e_\psi,\\ e_3 &= \sqrt{F_3\sqrt{H}}\left(-\sin\frac{\psi}{2}e_{\phi_1} + \cos\frac{\psi}{2}e_{\theta_1}\right),\\ e_4 &= \sqrt{F_3\sqrt{H}}\left(\cos\frac{\psi}{2}e_{\phi_1} + \sin\frac{\psi}{2}e_{\theta_1}\right),\\ e_5 &= \sqrt{F_4\sqrt{H}}\left(-\sin\frac{\psi}{2}e_{\phi_2} + \cos\frac{\psi}{2}e_{\theta_2}\right),\\ e_6 &= \sqrt{F_4\sqrt{H}}\left(\cos\frac{\psi}{2}e_{\phi_2} + \sin\frac{\psi}{2}e_{\theta_2}\right).\end{aligned}$$

the D3-brane in this background, not as a probe, but as an actual backreacted object. This is *different* from what has been done earlier in Refs. [3,35,39–43] where the D3-brane has been considered as a probe in a GKP background [11,15] of the form

$$ds^2 = e^{2A} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A} g_{mn} dy^m dy^n,$$
$$\mathcal{G}_3 = \mathcal{F}_3 + \tau \mathcal{H}_3, \quad \mathcal{F}_5 = (1 + *_{10}) d\alpha \wedge d\mathrm{vol}_{R^{3,1}}, \qquad (38)$$

where $\tau = C_0 + i e^{-\phi_B}$ and $\alpha = e^{4A}$. For our case, with the backreaction of the D3-branes taken into account, we can define the following quantities:

$$e^{2A} = \sqrt{\alpha} = \frac{1}{e^{2\phi/3}\sqrt{e^{2\phi/3} + \Delta}}, \qquad g_{\mu\nu} = \eta_{\mu\nu},$$
$$\Phi_+ = \frac{2}{e^{2\phi}\cosh^2\beta - \sinh^2\beta}, \qquad \Phi_- = 0. \qquad (39)$$

The above equation implies that the scalar fields on a D3-brane are completely massless (as the masses of the scalar fields are determined by Φ_- [3]). Other details regarding the action can be worked out from Refs. [3,35,39–43].

Let us now consider a $\overline{\mathrm{D3}}$ in this background. We will take this as a probe so that the backreaction of the antibrane will not be felt strongly in Eq. (28). Details of this will be discussed in the next section. For the time being we shall assume that a small profile for the dilaton is now switched on, along with small changes in the 3-form fluxes. Furthermore, the tachyonic instability of the antibrane will not be visible in the probe limit. The worldvolume multiplet on the antibrane will have the usual vector field A_μ and six scalars φ^m associated with the six internal directions of the resolved conifold (27). The bosonic action in the Einstein frame is then given by

$$S_{\overline{\mathrm{D3}}} = -\tau_{D3} l_s^4 \int d^4x \left(\frac{\pi}{2g_s^2} f_{\mu\nu} f^{\mu\nu} + \frac{\pi}{g_s} g_{mn} \mathcal{D}_\mu \varphi^m \mathcal{D}^\mu \varphi^n + \frac{\pi}{g_s} \partial_m \partial_n \Phi_+ \varphi^m \varphi^n + \mathcal{L}_{\mathrm{int}} \right), \qquad (40)$$

where the interaction Lagrangian $\mathcal{L}_{\mathrm{int}}$ is given by the following expression:

$$\mathcal{L}_{\mathrm{int}} = \frac{2\pi}{l_s^2 g_s} \partial_m \Phi_+ \varphi^m + \frac{i\pi}{12} \Phi_+ (\mathrm{Re} G_+)_{mnp} \varphi^m \varphi^n \varphi^p + \frac{\pi}{l_s^4 g_s} \Phi_+, \qquad (41)$$

where g_{mn} is the metric of the internal non-Kähler resolved conifold (27) and $G_+ = (*_6 + i) G_3$ where $*_6$ is the Hodge star with respect to the warped metric (38).

For a conifold background, there are five compact scalars, namely (φ^{θ_1}, φ^{ϕ_1}, φ^{θ_2}, φ^{ϕ_2}, φ^{ψ}), and one noncompact scalar φ^r. The compact scalars are all massless, and the mass of the noncompact scalar is given by

$$m_{\varphi^r}^2 = \frac{\pi}{g_s}\left(\frac{\partial^2 \Phi_+}{\partial r^2}\right) = \frac{8\pi e^{2\phi}\cosh^2\beta}{g_s(e^{2\phi}\cosh^2\beta - \sinh^2\beta)^2} \times \left[\left(\frac{e^{2\phi}\cosh^2\beta + \sinh^2\beta}{e^{2\phi}\cosh^2\beta - \sinh^2\beta}\right)\left(\frac{\partial\phi}{\partial r}\right)^2 - \frac{1}{2}\frac{\partial^2\phi}{\partial r^2}\right], \qquad (42)$$

where due to the presence of the linear interaction in Eq. (40), the noncompact scalar is shifted from its original value φ^r to the following:

$$\tilde{\varphi}^r \equiv \varphi^r + \frac{1}{l_s^2}\left[\frac{\partial}{\partial r}\log\left(\frac{\partial \Phi_+}{\partial r}\right)\right]^{-1}. \qquad (43)$$

In a generic setting, where the warp factors and the dilaton ϕ are functions of all the internal coordinates, all six scalars would be massive and the antibrane will be fixed at a point in the internal space where the mass matrix is extremized.

However, the background we have constructed has a *constant* dilaton, and thus Φ_+ is constant and φ^r is massless. If one allows for a small dilaton profile, for example by perturbing beyond the probe limit, a mass is generated for φ^r. In the limit where β is small, this happens at the point where the dilaton satisfies the following differential equation:

$$\frac{\partial^3\phi}{\partial r^3} - 6\left[\frac{\partial^2\phi}{\partial r^2} - \frac{2}{3}\left(\frac{\partial\phi}{\partial r}\right)^2\right]\frac{\partial\phi}{\partial r} + \mathcal{O}(\beta) = 0. \qquad (44)$$

For the solution discussed above, and allowing for some $\overline{\mathrm{D3}}$ backreaction in the form of a small profile for the dilaton, Φ_+ takes the following simple form (for arbitrary values of β):

$$\Phi_+ = 2 - 4\phi(r)\cosh^2\beta + \mathcal{O}(\phi^2). \qquad (45)$$

This form of Φ_+ will fix φ^r to be 0. The remaining scalars can be stabilized along the lines of Ref. [44]; the angular moduli receive masses upon “gluing” the noncompact throat geometry onto a compact Calabi-Yau. Alternatively, one can place the $\overline{\mathrm{D3}}$ directly on an orientifold plane, as in Ref. [17], which fixes all the scalars and gauge fields.[16]

[16]For more details on orientifolding conifolds see Refs. [45,46], and for the consistency of placing antibranes on orientifolds of conifolds see Ref. [17].

C. SUSY breaking and the fermionic action for a $\overline{\text{D3}}$

Now let us return to the fermionic action, which we gave in Eq. (3). The masses of the fermions are dictated by the ISD 3-form flux $\mathcal{G}_3$ given in Eq. (37), which is valid strictly in the probe approximation. The backreaction of the $\overline{\text{D3}}$ induces corrections to the flux, which we will come back to shortly.

Staying within the probe approximation, the flux is given by Eq. (37),

$$\mathcal{G}_3 = \frac{\sqrt{F}(2-F\delta F)}{8\mathrm{cosech}\beta}\left[\left(\frac{rF\delta F - 12a_1a_{1r}}{r^2+6a_1^2}\right) - \left(\frac{rF\delta F - 12a_2a_{2r}}{r^2+6a_2^2}\right)\right](E_1\wedge E_3\wedge \overline{E}_3 - E_1\wedge E_2\wedge\overline{E}_2).$$

Clearly the masses m_0 and m_i will be zero (since $\mathcal{G}_3$ is ISD and primitive). The breaking of supersymmetry is done purely through the mass matrix m_{ij}, defined in Eq. (6). Evaluating these masses explicitly, we find

$$m_{23} = m_{32} = \frac{\sqrt{2}}{8}i|\mathcal{G}_3|, \quad m_{12} = m_{21} = m_{13} = m_{31} = 0, \quad (46)$$

where $|\mathcal{G}_3|$ is

$$|\mathcal{G}_3| = \frac{\sinh\beta}{8}\sqrt{F}(2-F\delta F)\left[\left(\frac{rF\delta F - 12a_1a_{1r}}{r^2+6a_1^2}\right) - \left(\frac{rF\delta F - 12a_2a_{2r}}{r^2+6a_2^2}\right)\right]. \quad (47)$$

From this we see that the λ^2 and λ^3 fermions will have a mass induced by $\mathcal{G}_3$, which spontaneously breaks the $\mathcal{N}=1$ supersymmetry of the resolved conifold. This leaves *two* massless fermions, λ^0 and λ^1, as the low-energy field content. This is in contrast to a $\overline{\text{D3}}$ in a GKP background, studied in Ref. [2], where there was only a single massless fermion. Interestingly, the scale of SUSY breaking is controlled by $\delta F(r)$, a_{1r}, and a_{2r}, and thus we can easily allow for soft breaking of supersymmetry.

D. Perturbing away from the probe limit

Let us now consider perturbing away from the probe limit, which corresponds to taking the $\overline{\text{D3}}$ to be a large yet finite distance away from the D5-brane (fractional D3). We will neglect subtleties regarding boundary conditions, which can lead to divergences in the fluxes when a stack of $\overline{\text{D3}}$'s is considered (see e.g. Ref. [47] and more recently Ref. [48]), and also continue to study only a single $\overline{\text{D3}}$. As we will see, even with this issue neglected, backreaction changes the story considerably. In the presence of a probe $\overline{\text{D3}}$, the background changes from what we have thus far studied. The question then is to compute the changes in the background metric and fluxes to account for the fermionic masses on the antibrane worldvolume. We will *not* attempt to find an exact backreacted solution with a $\overline{\text{D3}}$, but rather take on a simpler task; we can compute the leading corrections to the fluxes and thus fermion masses by perturbing away from the probe limit.

The situation is not as hard as it sounds. Due to the (perturbatively) probe nature of the $\overline{\text{D3}}$, and as we hinted before, the tachyonic degree of freedom will not be visible at the supergravity level. Furthermore the backreaction of the $\overline{\text{D3}}$-brane will appear from its energy-momentum tensor that comes solely from the Born-Infeld part (the Chern-Simons piece, that can distinguish between a brane and an antibrane, does not contribute to the energy-momentum tensor). This is good because then at the supergravity level we are effectively inserting a 3-brane in a wrapped D5-brane background. To compensate for this new source of the energy-momentum tensor the warp factors change slightly as

$$F_i \to F_i + \delta F_i, \quad (48)$$

where this change is over and above the δF change in Eq. (10) that was there in the absence of a $\overline{\text{D3}}$-brane.[17] The dilaton ϕ also changes from zero to $\delta\phi$, but, as a first trial, we keep the complex structure of the non-Kähler resolved conifold fixed to Eq. (31) (as we shall see, this will have to be changed). Note that for a supersymmetric perturbation, the complex structure would have also changed exactly in a way so as to remove any (1,2) fluxes. Taking this into account, the ISD primitive (2,1) flux (37) now changes to the following additional piece:

$$\delta\mathcal{G}_3^{(1)} = \frac{\sinh\beta}{4\sqrt{H}\sqrt{F_1\sqrt{H}}}\left(1+\frac{\delta F_1}{2F_1}+\frac{3}{4}\frac{\delta H}{H}\right)\times\left[\frac{\sqrt{F_1F_2}}{2}\left(\frac{\delta F_1}{F_1}+\frac{\delta F_2}{F_2}\right)\left(\frac{1}{F_3}-\frac{1}{F_4}\right)+\left(\frac{\delta F_{4r}}{F_4}-\frac{\delta F_{3r}}{F_3}\right) + \left(\frac{\sqrt{F_1F_2}-F_{4r}}{F_4}\right)\left(\frac{\delta F_4}{F_4}-\delta\phi\right)-\left(\frac{\sqrt{F_1F_2}-F_{3r}}{F_3}\right)\times\left(\frac{\delta F_3}{F_3}-\delta\phi\right)\right](E_1\wedge E_3\wedge\overline{E}_3 - E_1\wedge E_2\wedge\overline{E}_2), \quad (49)$$

which is again a primitive (2,1) form. When combined with the primitive (2,1) piece that we had in Eq. (37), this would enter the mass formula given in Eq. (6) to give masses to the corresponding fermions. Note that, the E_i's appearing above are the *original* vielbein used earlier to write the (2, 1) flux (37), but could be replaced by the modified vielbein under Eq. (48), i.e.

$$E_i \to E_i + \delta E_i, \quad (50)$$

[17] Note that due to the probe nature, $\delta F_5 = \delta F_6 = 0$ along with vanishing (F_5, F_6), so that the *form* of the metric remains (27) and the topology does not change.

without changing any physics. This will also be the case for all other (2, 1) and (1, 2) perturbations that we shall discuss below: we will express them in terms of old vielbeins although we could also use Eq. (50). Using the old vielbeins E_i, we do however develop an *additional* contribution to the (2, 1) flux, other than Eqs. (37) and (49), that typically takes the following form:

$$\delta\mathcal{G}_3^{(2)} = (\alpha_1 \delta F + \alpha_2 a_{1r} + \alpha_3 a_{3r}) \times (E_i \wedge \delta E_j \wedge \overline{E}_k \pm \sigma E_i \wedge E_j \wedge \delta\overline{E}_k), \tag{51}$$

where $\alpha_i(r)$ and $\sigma(r)$ are certain well-defined functions of r that could be derived from our flux formulas discussed above. We cannot simply ignore this term as it is of the same order as the second line in Eq. (49) above, but we can absorb this in Eq. (37) by resorting to the modified vielbein (50). The conclusion then remains unchanged: all $\delta\mathcal{G}_3^{(k)}$ will be expressed in terms of E_i, but the original (2, 1) flux (37) will now be expressed in terms of Eq. (50) under perturbative backreaction of the $\overline{\rm D3}$-brane.

Coming back to our analysis, the primitive (2,1) pieces are involved in determining the masses, but we do also get another (2, 1) piece that is *neither* primitive nor ISD. This appears because we have not changed our complex structure, and it is given by the following form:

$$\delta\mathcal{G}_3^{(3)} = \mathcal{G}_0^{(\delta\phi)}\left[\left(\frac{\sqrt{F_1F_2} - F_{4r}}{F_4}\right) E_1 \wedge E_2 \wedge \overline{E}_2 + \left(\frac{\sqrt{F_1F_2} - F_{3r}}{F_3}\right) E_1 \wedge E_3 \wedge \overline{E}_3\right], \tag{52}$$

which becomes an ISD primitive form when the sum of the coefficients of the two terms vanish. This is no surprise because it is exactly the supersymmetry condition that we had in Ref. [26]. We have also defined the coefficient $\mathcal{G}(r)$ in terms of the warp factors H and F_1 in the following way:

$$\mathcal{G}_0^{(\delta\phi)} \equiv -\frac{\delta\phi \sinh\beta}{4\sqrt{H}\sqrt{F_1\sqrt{H}}}\left(1 + \frac{\delta F_1}{2F_1} + \frac{3}{4}\frac{\delta H}{H}\right). \tag{53}$$

Additionally, under supersymmetry $\delta\phi$ vanishes, so this term never shows up. For the present case, clearly we cannot impose the supersymmetry conditions. However if we change the complex structure (31) a bit as

$$\delta\tau_k = (i\delta\phi \coth\beta, 0, 0), \tag{54}$$

instead of keeping it completely rigid as we discussed above, we can make this term vanish. Note that some care is required to interpret this result. As mentioned earlier, we can change the complex structure to absorb any appearance of (1, 2) forms so that supersymmetry is restored. This case should then be interpreted differently. As we shall see below, we do get (1, 2) forms and they will be nonzero for the shifted complex structure (54) as well as for the original complex structure (31).

The (1, 2) piece is given by the following form:

$$\begin{aligned}\delta\mathcal{G}_3^{(4)} = {}& \frac{\sinh\beta}{4\sqrt{H}\sqrt{F_1\sqrt{H}}}\left(1 + \frac{\delta F_1}{2F_1} + \frac{3}{4}\frac{\delta H}{H}\right) \\ & \times \left[\frac{\sqrt{F_1F_2}}{2}\left(\frac{\delta F_1}{F_1} + \frac{\delta F_2}{F_2}\right)\left(\frac{1}{F_3} + \frac{1}{F_4}\right) \right. \\ & - \left(\frac{\delta F_{4r}}{F_4} + \frac{\delta F_{3r}}{F_3}\right) - \left(\frac{\sqrt{F_1F_2} - F_{4r}}{F_4}\right)\frac{\delta F_4}{F_4} \\ & \left. - \left(\frac{\sqrt{F_1F_2} - F_{3r}}{F_3}\right)\frac{\delta F_3}{F_3}\right] \\ & \times (E_2 \wedge \overline{E}_1 \wedge \overline{E}_2 + E_3 \wedge \overline{E}_1 \wedge \overline{E}_3),\end{aligned} \tag{55}$$

which is an ISD but nonprimitive form, and therefore breaks supersymmetry. As before, we have ignored terms of the form $\delta F_i \delta F_j$ and $\delta F_i \delta\phi$, as we are assuming the perturbations to be small. When the perturbations are not small we need to use more exact expressions which can be derived with some effort, but we will not do this here. The above (1, 2) form (55) enters the mass formula (5), inducing a nonzero $m_{\overline{1}}$. This acts as an interaction between $\lambda^{\overline{1}}$ and $\lambda^{\overline{0}}$. Similarly, $\overline{\delta\mathcal{G}_3^{(3)}}$ induces an interaction $m_1\lambda^0\lambda^1$. This is given by

$$m_1 = \frac{1}{\sqrt{2}} e^{\delta\phi} |\overline{\delta\mathcal{G}_3^{(3)}}|, \tag{56}$$

where $|\overline{\delta\mathcal{G}_3^{(3)}}|$ is the coefficient of $(E_2 \wedge \overline{E}_1 \wedge \overline{E}_2 + E_3 \wedge \overline{E}_1 \wedge \overline{E}_3)$ in Eq. (55).

Note that in deriving the perturbations to our background we did not find any (0, 3) or imaginary anti-self-dual (IASD) forms. This is expected from the probe nature of our analysis. On the other hand the (1, 2) form that we got above in Eq. (55) cannot be absorbed by the change in the complex structure (54). However one might ask if a more generic analysis could be performed. In other words, is it possible to find the most generic (2, 1) and (1, 2) perturbations in the non-Kähler resolved conifold background?

The way to answer this question would be to first find the complete basis for the (2, 1) and (1, 2) forms in the resolved conifold background. This has been studied in Ref. [49], and we reproduce it here for completeness. The bases for the (2, 1) forms are

$$\begin{aligned} u_1 &\equiv E_1 \wedge E_2 \wedge \overline{E}_2 - E_1 \wedge E_3 \wedge \overline{E}_3, \\ u_2 &\equiv E_1 \wedge E_2 \wedge \overline{E}_3 - E_1 \wedge E_3 \wedge \overline{E}_2, \\ u_3 &\equiv E_1 \wedge E_2 \wedge \overline{E}_1 + E_2 \wedge E_3 \wedge \overline{E}_3, \\ u_4 &\equiv E_1 \wedge E_3 \wedge \overline{E}_1 - E_2 \wedge E_3 \wedge \overline{E}_2, \\ u_5 &\equiv E_2 \wedge E_3 \wedge \overline{E}_1, \end{aligned} \tag{57}$$

where all of them are ISD and primitive. The first basis, u_1, was used earlier to write both the original and the perturbed (2, 1) forms. The bases $(u_2, \ldots, u_5)$ are useful when the $\overline{\rm D3}$ backreaction is not as simple as Eq. (48). Thus a generic (2, 1) perturbation can be of the form

$$\delta\mathcal{G}_3^{(2,1)} = \sum_{n=1}^{5} a_n u_n, \tag{58}$$

where a_n could be functions of all the coordinates of the internal non-Kähler resolved conifold. We can then use Eq. (58) in Eq. (6) to expresses the masses of the relevant fermions on the $\overline{\rm D3}$-brane. Most importantly, it will in general no longer be the case that λ^1 is massless, since more general (2,1) fluxes induce nonzero masses, i.e. we will now have

$$m_{12} \neq 0, \qquad m_{13} \neq 0. \tag{59}$$

One may similarly construct the complete basis for the (1, 2) forms for the resolved conifold background. We will again require our basis forms to be ISD to solve the background EOMs. For a (1, 2) form this is possible only if it is proportional to the fundamental form J, thus restricting the number of such forms to be just three. They are given by [49]

$$\begin{aligned} w_1 &\equiv E_1 \wedge \overline{E}_1 \wedge \overline{E}_3 + E_2 \wedge \overline{E}_2 \wedge \overline{E}_3, \\ w_2 &\equiv E_1 \wedge \overline{E}_1 \wedge \overline{E}_2 - E_3 \wedge \overline{E}_2 \wedge \overline{E}_3, \\ w_3 &\equiv E_2 \wedge \overline{E}_1 \wedge \overline{E}_2 + E_3 \wedge \overline{E}_1 \wedge \overline{E}_3, \end{aligned} \tag{60}$$

where one may check that they are ISD but not primitive. We had used w_3 earlier to express the (1, 2) perturbation in Eq. (55). Thus a more generic nonsupersymmetric perturbation in the presence of a $\overline{\rm D3}$-brane can be expressed by the following (1, 2) form:

$$\delta\mathcal{G}_3^{(1,2)} = \sum_{n=1}^{3} b_n w_n, \tag{61}$$

where b_n, as for a_n above, could be generic functions of all the coordinates of the internal non-Kähler resolved conifold. This could now be inserted into Eq. (5) to determine the mixing between the $\lambda_\pm^0$ and $\lambda_\pm^i$ fermions, i.e.

$$m_1 \neq 0. \tag{62}$$

The consequence of this is that the backreaction-induced fluxes give a mass to λ^0 and λ^1, and hence there are *no massless fermions left in the spectrum*. This is a striking difference to the probe approximation, where there were two massless fermions.

Let us take a moment to consider why this is the case. From the supergravity perspective, a $\overline{\rm D3}$ is equivalent to a D3. The background we are considering has a wrapped D5-brane, and since a D3-D5 system is nonsupersymmetric, the induced fluxes will include supersymmetry-breaking fluxes. It is these fluxes which give a mass to the would-be massless fermions on the $\overline{\rm D3}$ worldvolume. In the GKP analysis of Ref. [2], there was no D5-brane, and thus this issue will not arise when considering backreaction.

This completes our discussion of spontaneous supersymmetry breaking via massive fermions on the $\overline{\rm D3}$-brane worldvolume. In the following section we will briefly dwell on certain aspects of moduli stabilization and de Sitter space.

E. Moduli stabilization and de Sitter vacua

In order to construct a concrete phenomenological model, the resolved conifold geometry we have studied should be *glued* onto a compact, non-Kähler space. As discussed in Ref. [44], and also Ref. [50], this gluing induces corrections to the $\overline{\rm D3}$ scalar moduli masses.

In addition to this, a compact space requires charge cancellation. Since charge cancellation is a global requirement, the necessary fluxes can be placed far from the resolved conifold which contains the $\overline{\rm D3}$, so as not to disrupt the local dynamics we have studied. In other words, for the case that we study here, the internal six-dimensional manifold (27) should be thought of as extending to a fixed radius $r = r_0$, and beyond which a compact manifold is attached. The boundary condition implies that at $r = r_0$, the compact manifold should have a topology of $\mathbf{S}^2 \times \mathbf{S}^3$. The compact manifold is equipped with the right amount of fluxes etc. that is necessary for global charge cancellation.

Finally, we note that moduli stabilization should be included in this picture. We need to consider two sets of moduli: the Kähler and the complex structure moduli of our non-Kähler space. The moduli of compactifications on non-Kähler manifolds was discussed in Ref. [51], and reviewed in Ref. [52]. An interesting feature of these models is that the radial modulus and the complex structure moduli can be stabilized at tree level whereas the other Kähler moduli, including the axio-dilaton need additional nonperturbative effects for stabilization. There are also other moduli, namely the moduli of the $\overline{\rm D3}$-brane, fractional 3-branes and possible 7-branes (that we did not discuss here, but are nonetheless important).

From the point of view of the Einstein equations, the existence of de Sitter vacua is rather nontrivial to see. Switching on Eqs. (58) and (61) gives masses to worldvolume fermions and simultaneously fixes the complex structure moduli (including the radial modulus) of our non-Kähler space. However the potential generated by the SUSY-breaking flux (61)

$$V = \frac{1}{2\kappa_{10}^2} \int \frac{\delta\mathcal{G}_3^{(1,2)} \wedge *\delta\overline{\mathcal{G}}_3^{(1,2)}}{\mathrm{Im}\tau}, \tag{63}$$

where τ is the axio-dilaton, vanishes identically. This means the presence of a $\overline{\rm D3}$-brane takes a supersymmetric anti–de Sitter space to a nonsupersymmetric one, and therefore does not contribute any positive vacuum energy to the system. This conclusion is not new and is another manifestation of the no-go condition of Gibbons-Maldacena-Nunez [53,54], recently updated in Ref. [55]. This means that to allow for a positive cosmological solution in the four space-time directions, the no-go condition should be averted.[18]

This then brings us to the recent study done in Ref. [55] from an uplift in M-theory. Quantum corrections play an important role, and a positive cosmological constant is only achieved in four space-time directions if the following condition is satisfied:

$$\langle \mathcal{T}^{\mu}_{\mu} \rangle_q > \langle \mathcal{T}^{m}_{m} \rangle_q, \tag{64}$$

which is a generalization of the classical condition studied in Ref. [53,54]. Here $\mathcal{T}_{mn}$ is the energy-momentum tensor and the subscript q denotes the quantum part of it. For more details, and the derivation of this, the readers may want to refer to Ref. [55].

This indicates that a concrete realization of de Sitter vacua in this context, and a precise connection to KKLT [6], would thus require including at least a subset of the above corrections (similar to "Kähler uplifting" [56]). Note that our setup would not involve the Kachru-Pearson-Verlinde (KPV) process [57], whereby a stack of $\overline{\mathrm{D3}}$'s polarize into an NS5, as we are only considering a single antibrane.

III. PROBE $\overline{\mathrm{D7}}$ IN A GKP BACKGROUND

In the previous section we generalized the work of Refs. [1,2] to a more general background, and found several interesting features. We now consider a different generalization: we turn our attention to a $\overline{\mathrm{D7}}$ brane in a GKP background. Similar to the $\overline{\mathrm{D3}}$ case, the $\overline{\mathrm{D7}}$-brane differs from the D7-brane only in the sign of the κ-symmetry projector, and the charge under the RR fields. The embedding of D7-branes into flux compactifications has been the focus of many works (for example Refs. [58], [59], [49], and [60]). In particular, many details of the D7 and $\overline{\mathrm{D7}}$ fermionic action were worked out in Refs. [61] and [62].

Placing a $\overline{\mathrm{D7}}$ in a warped $\mathcal{N} = 1$ background will spontaneously break supersymmetry. The breaking of supersymmetry manifests itself in the fermionic action via a mass for the fermions (see Ref. [61] for details), and the spontaneous nature of SUSY breaking can be deduced via the condition discussed in Sec. I A. Furthermore, for general background fluxes, all the $\overline{\mathrm{D7}}$ worldvolume fermions are massive. Only under special circumstances will there remain a massless fermion in the low-energy spectrum; demonstrating this will be the focus of this section. We will find that, under suitable conditions, we have not only one massless fermion, but many. This is similar to the $\overline{\mathrm{D3}}$ in a resolved conifold case studied in Sec. II, where (in the probe approximation) we found not one but two massless fermions.

A. The fermionic action for a $\overline{\mathrm{D7}}$ in a flux background

The quadratic fermionic action for a single Dp-brane (in the string frame) was detailed in Ref. [43]: we will follow their conventions in what follows. The only difference for an antibrane is in the κ-symmetry projector, which changes sign relative to the brane case. For the case of $p = 7$ this reads

$$S_f^{\overline{\mathrm{D7}}} = -\frac{1}{2} T_7 (2\pi\alpha')^2 \int \mathrm{d}^8\xi e^{\phi} \sqrt{-\det(G + \mathcal{F})} \bar{\theta} \times [1 - \Gamma_{\overline{\mathrm{D7}}}(\mathcal{F})](\not{\mathcal{D}} - \Delta)\theta, \tag{65}$$

where we scaled our action by an overall factor of $(2\pi\alpha)'^2$ (to match with the convention of writing the gauge field as $2\pi\alpha' F_{\mu\nu}$). As before, the spinor θ is a ten-dimensional 64 (32) real (complex) component Majorana spinor, which is a doublet of ten-dimensional (left-handed) 32 (16) real (complex) component Majorana-Weyl spinors.

The factor $[1 - \Gamma_{\overline{\mathrm{D7}}}(\mathcal{F})]$ is the κ-symmetry projector, which depends on the worldvolume flux $\mathcal{F}$, and we have defined

$$\Gamma_{\overline{\mathrm{D7}}} = -i\sigma_2 \frac{1}{\sqrt{-g}} \Gamma_{01234567} + \mathcal{O}(\mathcal{F}), \tag{66}$$

and we take the brane to be along the $x^0, \ldots, x^7$ coordinate directions. The covariant derivative $\tilde{\mathcal{D}}$ on the brane is defined as

$$\not{\mathcal{D}} = (M^{-1})^{\alpha\beta} \Gamma_\beta \tilde{D}_\alpha, \tag{67}$$

where M_{ab} is defined using $\mathcal{F}_{ab}$ and the pull-back of the metric g_{ab} as

$$M_{ab} = g_{ab} + \mathcal{F}_{ab}, \tag{68}$$

with $\mathcal{F} = P[\mathcal{B}_{(2)}] + 2\pi\alpha' F_2$. We have also defined $\tilde{D}_\alpha$ as a shifted covariant derivative,

$$\tilde{D}_m = D_m \mathbb{I}_2 + \sigma_1 W_m, \tag{69}$$

which we shall define in more detail momentarily. It is important to note that the contraction $\not{D} = \Gamma^m D_m$ sums

[18]All the energy-momentum tensors are computed using both the bosonic and the fermionic terms on the branes and the planes. Note that the no-go conditions in Refs. [53–55] were derived exclusively using the bosonic terms on the branes and the planes. However if we use Eq. (144) (see Sec. IV) to define the pullbacks of the type IIB fields on the branes and the planes, we can easily see that the conclusions of Refs. [53–55] remain unchanged in the presence of the fermionic terms.

only over the indices on the brane-worldvolume, and as mentioned above, we will take the brane to be oriented along the $(x^0, x^1, \ldots x^7)$ directions. In contrast to this, the contractions appearing in Δ will sum over *all* indices[19]; for example Δ contains the term $\Gamma^{MNP}\mathcal{H}_{MNP}$ where $M, N, P = 0.9$. We can further decompose $\mathcal{H}_{MNP}$ into pieces with 0, 1, and 2, indices along the transverse two-dimensional space parametrized by (x^8, x^9) coordinates.

In a general GKP background the worldvolume flux $\mathcal{F}$ will be nonzero, and this cannot be gauged away. To make our analysis simple, we will focus on a class of backgrounds with the property that $\mathcal{B}_2$ is constant along the brane worldvolume, i.e. $\mathcal{B}_2 = \mathcal{B}_2(x^8, x^9)$, and there is an equal and opposite Dirac-Born-Infeld gauge F_2, such that $\mathcal{F} = 0$. This allows us to take the M_{ab} appearing in Eq. (68) as simply g_{ab}, and $\Gamma_{\overline{\mathrm{D7}}}$ to be $-i\sigma_2 \frac{1}{\sqrt{-g}}\Gamma_{01234567}$. Recall that a GKP background also comes equipped with a self-dual five-form flux $\tilde{\mathcal{F}}_5$, given by

$$\tilde{\mathcal{F}}_5 = (1 + *)(\mathrm{d}\alpha \wedge \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3), \tag{70}$$

where the function α depends on the coordinates of the internal space, and is responsible for setting the profile of the warp factor, i.e. $\alpha = e^{4A}$. We will see that $\tilde{\mathcal{F}}_5$ generically contributes to the fermion masses, unless $\alpha = \alpha(x^8, x^9)$, i.e. α is independent of the brane coordinates.

Let us consider an explicit choice of background flux which realizes this. We again define in the standard way $\mathcal{G}_3 = \mathcal{F}_3 - \tau\mathcal{H}_3$. A choice of $\mathcal{G}_3$ which meets the above criteria is

$$\mathcal{G}_3 = N E_1 \wedge E_2 \wedge \overline{E}_3, \tag{71}$$

where N is a constant and we take the complex structure $J = (i, i, i)$, i.e. $z^1 = x^4 + ix^5$ and so on. One can easily check that this is ISD and primitive.[20] The corresponding $\mathcal{B}_2$ and $\mathcal{C}_2$ which generate this $\mathcal{G}_3$ are

$$\begin{aligned} \mathcal{C}_2 &= N(x^4 dx^6 \wedge dx^8 - x^4 dx^7 \wedge dx^9 - x^5 dx^6 \wedge dx^9 \\ &\quad - x^5 dx^7 \wedge dx^8) \\ \mathcal{B}_2 &= N e^{\phi_0}(x^9 dx^4 \wedge dx^6 + x^8 dx^4 \wedge dx^7 + x^8 dx^5 \wedge dx^6 \\ &\quad + x^9 dx^5 \wedge dx^7), \end{aligned} \tag{72}$$

where we take the dilaton to be constant $\phi = \phi_0$. With the above example in mind, we will proceed in our analysis with a general $\mathcal{G}_3$, but with the assumption that $F_2 = -P[\mathcal{B}_2]$ and hence $\mathcal{F} = 0$.

As mentioned above, the IIB spinor θ is actually a doublet of 16-component left-handed (i.e. same chirality) Majorana-Weyl spinors; this "doublet" is a 32-component Majorana spinor (note that it is *not* Weyl). The gamma matrices in the 64-component representation are related to the 16-component representation by

$$\Gamma_m^{\mathrm{doublet}} = \Gamma_m \otimes \mathbb{I}_2, \tag{73}$$

[see e.g. below Eq. (85) in Ref. [43]].

We gauge fix κ symmetry by enforcing the κ-symmetry projection to satisfy the following condition:

$$\overline{\theta}(1 + \Gamma_{\overline{\mathrm{D7}}}) = 0. \tag{74}$$

This enforces a relation between $\theta_{1,2}$ components of the doublet θ, given by

$$\theta_2 = \Gamma_{012\ldots7}\theta_1. \tag{75}$$

This choice of gauge fixing was used in recent papers by Kallosh *et al.* (for example Refs. [1,2]), as it is consistent with an orientifold projection. Alternatively, one could use a condition $\theta_2 = 0$, as was used in papers by Martucci *et al.* (e.g. Refs. [43] and [41,42]). Here, we will only use the condition above, namely, $\theta_2 = \Gamma_{012\ldots7}\theta_1$.

Last, we note that the operators W_m and Δ appearing in Eq. (65) are given by (see for example Ref. [2])

$$\begin{aligned} \Delta &= -\frac{1}{2}\Gamma^M\partial_M\phi - \frac{1}{24}(\mathcal{H}_{MNP}\sigma_3 - e^\phi\mathcal{F}_{MNP}\sigma_1)\Gamma^{MNP}, \\ W_m &= -\frac{1}{4}e^\phi(i\sigma_2)\mathcal{F}_m + \frac{1}{8}(\mathcal{H}_{mNP}\sigma_3 - e^\phi\mathcal{F}_{mNP}\sigma_1)\Gamma^{NP} \\ &\quad - \frac{1}{8\cdot 4!}(i\sigma_2)e^\phi\mathcal{F}_{NPQRS}\Gamma^{NPQRS}\Gamma_m, \end{aligned} \tag{76}$$

where $m = 4, 5, 6, 7$, and $M, N = 0, 1, \ldots, 9$. Additionally any quantity not appearing with a σ_i is implicitly a tensor product with the 2×2 identity matrix.

We can now expand our action (65), using the operators (76) and the κ-symmetry fixing condition (75). We use the fact that the fluxes are only in the internal space, and that the only nonvanishing bilinears for $10d$ Majorana-Weyl spinors have three or seven gamma matrices. The action can be written in terms of θ_1 as

$$S_f^{\overline{\mathrm{D7}}} = -\frac{1}{2}T_7(2\pi\alpha')^2\int \mathrm{d}^8\xi e^\phi\sqrt{-\det G}\mathcal{L}_\theta, \tag{77}$$

where G is the warped metric, and $\mathcal{L}_\theta$ is given purely in terms of θ_1 as

[19] We take our 3-form fluxes to be only in the internal space.

[20] To avoid clutter we are using the same symbol E_i to denote the vielbeins as before although now the definitions of the vielbeins are very different. Furthermore since the background is no longer a non-Kähler resolved conifold we are not restricted to the basis (60) to express the 3-form $\mathcal{G}_3$.

$$\mathcal{L}_\theta = 2\bar{\theta}_1\left[\Gamma^m D_m - \frac{3}{16}e^{\phi}\Gamma^{mna}(\mathcal{F}_{mna}\Gamma_{012\ldots7} + e^{-\phi}\mathcal{H}_{mna}) - \frac{5}{16}e^{\phi}\Gamma^{mnp}\mathcal{F}^q_{0123}\epsilon_{mnpq}\right]\theta_1, \qquad (78)$$

with the indices running as m, n, p, $q = 4$, 5, 6, 7 and $a = 8$, 9. Note the interesting feature that the only 3-form fluxes which contribute to the action are those with two-legs along the brane, and one leg transverse to the brane. The other contributions—1) three legs along the brane, zero transverse and 2) one leg along the brane, two transverse—cancel out of the action. As we see, there is a possible contribution from the 5-form flux when all legs of the flux lie along the brane. This can be made to vanish if we impose that α depends only on the transverse directions to the brane. This is different from the $\overline{\text{D3}}$ case, where the $\tilde{\mathcal{F}}_5$ term simply did not contribute, regardless of the choice of α. We will return to this point in Sec. III D; for the moment we will take $\alpha = \alpha(x^8, x^9)$ and hence $\tilde{\mathcal{F}}_5$ will not contribute to the masses. There can generally also be a contribution from the 1-form flux, but a GKP background does not have these, due to the lack of 1-cycles on a CY manifold[21].

The action (77) can be simplified further by using $\Gamma_{0\ldots9}\theta_1 = \theta_1$, which implies that $\mathcal{F}_{mna}\Gamma_{012\ldots7}\theta_1 = (*_2\mathcal{F}_3)_{mna}\theta_1$, where $*_2$ is the Hodge duality in the (x^8, x^9) directions. We can also write this in terms of the familiar $\mathcal{G}_3 = \mathcal{F}_3 - ie^{-\phi}\mathcal{H}_3$ along with the following nomenclatures: ISD2 is the "imaginary self-dual" along the transverse two-cycle and IASD2 is the "imaginary anti-self-dual" again along the transverse two-cycle pieces of $\mathcal{G}_3$ as

$$\mathcal{G}_3 = \mathcal{G}_3^{\mathrm{IASD2}} + \mathcal{G}_3^{\mathrm{ISD2}}, \qquad \mathcal{G}_3^{\mathrm{ISD2}} = \frac{1}{2}(\mathcal{G}_3 - i *_2 \mathcal{G}_3),$$
$$\mathcal{G}_3^{\mathrm{IASD2}} = \frac{1}{2}(\mathcal{G}_3 + i *_2 \mathcal{G}_3), \qquad (79)$$

which is equivalent to the decomposition

$$\mathcal{H}_3 = \frac{i}{2}e^{\phi}(\mathcal{G}_3 - \bar{\mathcal{G}}_3), \qquad \mathcal{F}_3 = \frac{1}{2}(\mathcal{G}_3 + \bar{\mathcal{G}}_3). \qquad (80)$$

With these definitions the action becomes

$$\mathcal{L}_\theta = 2\bar{\theta}_1\left[\Gamma^m D_m - \frac{3i}{32}e^{\phi}\Gamma^{mna}(\mathcal{G}_3^{\mathrm{ISD2}} - \bar{\mathcal{G}}_3^{\mathrm{ISD2}})_{mna}\right]\theta_1;$$
$$(m, n) = 4, 5, 6, 7; \qquad a = 8, 9. \qquad (81)$$

Thus the worldvolume fermions on the $\overline{\text{D7}}$-brane will have masses determined by the ISD2 $\mathcal{G}_3$ flux, where the "dual" in ISD2 refers to the *transverse to the brane* (and not the full internal space). For our example $\mathcal{G}_3$ given in Eq. (71), the flux is purely ISD2 and thus will contribute to the masses. These masses spontaneously break the background $\mathcal{N} = 1$ supersymmetry.

We could also include a flux which is ISD—and thus solves the equations of motion for a GKP background—but which is *not* ISD2, and hence will not contribute to the fermion masses. An example of such a flux is

$$\mathcal{G}_3 = M(E_1 \wedge \bar{E}_1 - E_2 \wedge \bar{E}_2) \wedge E_3 \qquad (82)$$

which is purely IASD2, and thus will not enter Eq. (81). Such a flux would come from a $\mathcal{B}_2$ of the form

$$\mathcal{B}_2 = -Me^{\phi_0}x^9 \cdot (\mathrm{d}x^4 \wedge \mathrm{d}x^5 - \mathrm{d}x^6 \wedge \mathrm{d}x^7), \qquad (83)$$

and a similar form for $\mathcal{C}_2$.

B. Fermions in 4*d* and spontaneous SUSY breaking in a GKP background

We can already see that supersymmetry will be spontaneously broken by the $\overline{\text{D7}}$ in the presence of 3-form fluxes. What remains to be checked is if there remains a massless fermion in the four-dimensional effective theory.

In the absence of the $\mathcal{G}_3$ flux, the massless fermions in the 4*d* theory are those whose dependence on the coordinates of the internal 4-cycle wrapped by the brane is harmonic. The exact spectrum of effective 4*d* fermions is therefore given by the cohomology classes of the wrapped cycle. On the other hand the coupling of the $\mathcal{G}_3$ flux to the fermions is governed by the structure of the spinors, so we do not need to know the full details of the topology of the wrapped cycle to know whether some of these fermions remain massless. Indeed, most of our calculation proceeds in the same fashion and certainly in the same spirit as the $\overline{\text{D3}}$ case.[22]

The 16-component spinor θ_1 can decomposed into two eight-component spinors θ_{1+} and θ_{1-} where the $\pm$ denotes the chirality in the transverse space, i.e. under $SO(2)$. In terms of Γ matrices, $\Gamma^3\theta_{1+} = \theta_{1-}$ and $\Gamma^{\bar{3}}\theta_{1-} = \theta_{1+}$. The four-dimensional fermions can be obtained via dimensional reduction of θ_{1+} and θ_{1-}, according to the cohomology classes of the cycle wrapped by the brane, as depicted below:

[21]Note that we are putting a $\overline{\text{D7}}$ in a GKP background with a constant dilaton and zero axion. The backreacted axionic source of the $\overline{\text{D7}}$ is suppressed by g_s and to this order we are not taking this to backreact on the $\overline{\text{D7}}$ worldvolume [the axion will only be along the (x^8, x^9) directions]. This differs slightly in spirit from the previous section where due to the nonsupersymmetric nature of the D3-D5 system, it was essential to take the perturbative backreactions into account, otherwise certain aspects of the physics would not have been visible.

[22]Without the (1, 2) perturbations of course.

$$\theta_{1+} = \sum_a \psi^a_{\pm\pm+} \otimes \chi^a_{\pm\pm+},$$
$$\theta_{1-} = \sum_a \psi^a_{\pm\pm-} \otimes \chi^a_{\pm\pm-}, \tag{84}$$

where the ψ^a are $4d$ spinors while the χ^a are internal spinors; the index a simply counts the number of $4d$ spinors. The unspecified $\pm\pm$ indices correspond in their chirality under SU(2), i.e. corresponding to their behavior under the action of Γ^1 and Γ^2. This allows us to group all the fields precisely as done in Refs. [1,2]. We define

$$\lambda^0 = \sum \psi^a_{---}, \qquad \lambda^{\bar{0}} = \sum \psi^a_{+++},$$
$$\lambda^1 = \sum \psi^a_{+--}, \qquad \lambda^{\bar{1}} = \sum \psi^a_{-++},$$
$$\lambda^2 = \sum \psi^a_{-+-}, \qquad \lambda^{\bar{2}} = \sum \psi^a_{+-+},$$
$$\lambda^3 = \sum \psi^a_{--+}, \qquad \lambda^{\bar{3}} = \sum \psi^a_{++-}. \tag{85}$$

We can now perform the fermion decomposition exactly as in Refs. [1,2], except now the fermions λ actually refer to the *set* of fermions which transform according the corresponding chirality. We have

$$\begin{aligned}\frac{\sqrt{2}}{12}\bar{\theta}^1\Gamma^{MNP}\hat{\mathcal{G}}_{MNP}\theta^1 &= \bar{\lambda}^0_+\lambda^0_+\hat{\mathcal{G}}_{123} + \bar{\lambda}^{\bar{0}}_-\lambda^{\bar{0}}_-\hat{\mathcal{G}}_{\bar{1}\bar{2}\bar{3}} \\ &\quad + (\bar{\lambda}^0_+\lambda^i_+\hat{\mathcal{G}}_{ij\bar{j}} - \bar{\lambda}^{\bar{0}}_-\lambda^{\bar{i}}_-\hat{\mathcal{G}}_{\bar{i}j\bar{j}})\delta^{j\bar{j}} \\ &\quad + \frac{1}{2}(\bar{\lambda}^i_+\lambda^j_+\epsilon_{jk\ell}\hat{\mathcal{G}}_{i\bar{k}\bar{\ell}} \\ &\quad + \bar{\lambda}^{\bar{i}}_-\lambda^{\bar{j}}_-\epsilon_{\bar{j}\bar{k}\bar{\ell}}\hat{\mathcal{G}}_{\bar{i}k\ell})\delta^{k\bar{k}}\delta^{\ell\bar{\ell}},\end{aligned} \tag{86}$$

where in our case $\hat{\mathcal{G}}_{MNP} \equiv (\mathcal{G}_3^{\mathrm{ISD2}} - \bar{\mathcal{G}}_3^{\mathrm{ISD2}})_{MNP}$, and with an abuse of notation, we now use M, N, P to refer to the internal space, $M = 4, 5, \ldots, 9$.

The $\mathcal{G}_3$ flux must be (2, 1) and primitive, since we only want supersymmetry to be broken by the presence of the brane. This on its own immediately implies that λ^0 remains massless and that the mass cross terms with λ^i vanish as well, as in the $\overline{\mathrm{D3}}$ case. The additional feature that the flux which couples to the fermions is "ISD2" further reduces the allowed components to only those that have a $\bar{3}$ index, and hence the only nonvanishing mass terms are

$$m_3 = m_{\bar{3}} \propto (\mathcal{G}_3^{\mathrm{ISD2}})_{12\bar{3}}, \tag{87}$$

where λ^3 gets its mass from $\mathcal{G}_3^{\mathrm{ISD2}}$ while $\lambda^{\bar{3}}$ gets its mass from $\bar{\mathcal{G}}_3^{\mathrm{ISD2}}$. The other fermions remain massless, i.e.

$$m_0 = m_i = m_{0i} = m_{ij} = 0; \qquad i, j = 1, 2, \tag{88}$$

and similarly for barred indices.

Thus the resulting four-dimensional massless fermionic field content consists of λ^0, λ^1 and λ^2. We emphasize that the λ's refer to *sets* of $4d$ fermions, the precise details of which can be found via dimensional reduction. Thus there are *many* massless fermions in this case, in contrast to the $\overline{\mathrm{D3}}$ in a GKP background, which has only one [2]. However, both examples illustrate how supersymmetry is broken spontaneously by a probe antibrane. Finally, we note that the bosonic field content on the brane can be taken care of as in the $\overline{\mathrm{D3}}$ case, by placing the $\overline{\mathrm{D7}}$ on an O7 plane.

C. Inclusion of $\mathcal{F}$

There is good reason to study nonzero $\mathcal{F}$: worldvolume fluxes on D7-branes generate D-terms and F-terms in the $4d$ theory [63], and may even allow for de Sitter solutions along the lines of Ref. [64]. With this in mind, let us see what happens on the antibrane side of this story, i.e. what happens when we allow worldvolume fluxes on a $\overline{\mathrm{D7}}$. Nonzero $\mathcal{F}$ modifies our previous analysis in two ways. First, it modifies the kinetic term via the matrix M_{ab} defined earlier in Eq. (68) and second, it also modifies the κ-symmetry projector, which in turn induces new mass terms.

The equations of motion require $\mathcal{F}$ to be anti-self-dual on the cycle wrapped by the antibrane, which we take to be in the (x^4, x^5, x^6, x^7) directions, with $\epsilon_{4567} = -1$ to be consistent with our conventions in the previous section. A judicious choice of vielbeins along the cycle can put the flux into the simple form,

$$\mathcal{F} = f(e_4 \wedge e_5 + e_6 \wedge e_7). \tag{89}$$

Note that in this approach we first choose a worldvolume flux, which then guides our choice of vielbeins and complex structure. This of course also affects the spacetime Γ matrices and the definitions of the fermions in the SU(3) triplet. At the end of the day, this amounts to an SU(3) transformation and does not affect the number of massless fermions, which is what we are ultimately interested in, nor does it affect the masses of the massive ones.

The modified kinetic term can be recast as a canonical kinetic term plus a generalized electromagnetic coupling by a (generally nonisotropic) rescaling of the vielbeins, as described in Ref. [43]. For our above choice of $\mathcal{F}$, the rescaling of the vielbeins to obtain a canonical kinetic term is simple. The matrix $M = g + \mathcal{F}$ now has off-diagonal terms, and in the vielbein basis its inverse is given by,

$$M^{-1} = \frac{1}{1+f^2}\begin{pmatrix} 1 & -f & 0 & 0 \\ f & 1 & 0 & 0 \\ 0 & 0 & 1 & -f \\ 0 & 0 & f & 1 \end{pmatrix}. \tag{90}$$

By defining rescaled vielbeins,

$$\hat{e}^m = \frac{1}{\sqrt{1+f^2}} e^a \qquad m = 4, 5, 6, 7, \tag{91}$$

the kinetic term becomes

$$\bar{\theta}\Gamma_m \mathcal{D}_n M^{mn}\theta = \bar{\theta}(\hat{g}^{mn} + \hat{\mathcal{F}}^{mn})\Gamma_m \mathcal{D}_n \theta, \tag{92}$$

where the "hatted" quantities are expressed in terms of the rescaled vielbeins, e.g. $\hat{g}^{mn} = \eta^{jk}\hat{e}^m_j \hat{e}^n_k$. We see that the kinetic term splits into a canonical kinetic term and a derivative coupling of the fermions to the worldvolume flux.

This derivative coupling complicates the dimensional reduction of θ_1. The underlying SU(3) structure guarantees that there are solutions to $g^{mn}\Gamma_m D_n \chi_6 = 0$, i.e. there exist zero modes of the Dirac operator on the internal space; however it will generically *not* be true that there are solutions to $(g^{mn} + \mathcal{F}^{mn})\Gamma_m D_n \chi_6 = 0$, particularly for nonsmall $\mathcal{F}$. If no zero modes exist for this "modified Dirac operator" then there will be no massless degrees of freedom. Thus the effect of the modified kinetic terms is to give mass to some, if not all, of the fermions.

We still have yet to consider the modification of the couplings to $\mathcal{G}_3$. Before doing so, we must incorporate the rescaling of the vielbeins that we performed. This is simply done by putting a factor of $\sqrt{1+f^2}$ for every lower index along the brane directions in all the quantities. To avoid notation clutter, we will assume for the remainder of this section that the spacetime fluxes are implicitly "hatted" and contractions are made using the rescaled metric. This rescaling ultimately does not affect the tensor structure of the fluxes, and therefore will not affect which fermions acquire masses.

The inclusion of $\mathcal{F}$ also modifies the κ-symmetry projector,

$$\begin{aligned}\Gamma_{\overline{D7}} &= \frac{1}{\sqrt{|g+\mathcal{F}|}}[-i\sigma_2\Gamma_{01234567} + \sigma_3 i\sigma_2(\Gamma_{012345}\mathcal{F}_{67} \\ &\quad - \Gamma_{012367}\mathcal{F}_{45}) - i\sigma_2\Gamma_{0123}\mathcal{F}_{45}\mathcal{F}_{67}] \\ &= \frac{1}{\sqrt{|g+\mathcal{F}|}}[-i\sigma_2\Gamma_{01234567} + \hat{f}\sigma_3 i\sigma_2(\Gamma_{012345} \\ &\quad - \Gamma_{012367}) - i\sigma_2\Gamma_{0123}\hat{f}^2],\end{aligned} \tag{93}$$

which in turn modifies the relation between $\theta_{1,2}$ imposed by the gauge-fixing condition $\bar{\theta}(1+\Gamma_{\overline{D7}}) = 0$, in the following way:

$$\theta_2 = [\Gamma_{01234567} + \hat{f}(\Gamma_{012345} - \Gamma_{012367}) + \hat{f}^2\Gamma_{0123}]\theta_1. \tag{94}$$

The outcome of all these changes is that now new couplings arise as

$$\begin{aligned}&\bar{\theta}_1 e^{-\phi}[\Gamma^{mna}(\mathcal{F}_{mna}\Gamma_{0\ldots7} + e^{-\phi}\mathcal{H}_{mna}) \\ &\quad + \hat{f}\Gamma^{mab}(\mathcal{F}_{mab}(\Gamma_{012345} - \Gamma_{012367}) + e^{-\phi}\mathcal{H}_{mab}) \\ &\quad + \hat{f}^2\Gamma^{mnl}(\mathcal{F}_{mnl}\Gamma_{0123} + e^{-\phi}\mathcal{H}_{mnl})]\theta_1,\end{aligned} \tag{95}$$

where the indices (m, n, l) now take values 4,5,6,7 and (a, b) as before take values (8,9).

These new terms include fluxes that have one leg or all three legs along the brane, which were not present for $\mathcal{F} = 0$. In fact, the last term is the coupling we get for an $\overline{\text{D3}}$-brane. This is to be expected, since worldvolume fluxes induce a lower-dimensional brane charge. The term linear in $\hat{f}$ is the coupling due to the induced 5-brane charge and is similar to what we would obtain if we studied a $\overline{\text{D5}}$ in a GKP background. It produces couplings to fluxes which obey a self-duality condition in the directions transverse to the cycles threaded by the flux. As in the pure $\overline{\text{D7}}$ case, this simply restricts which subset of fermions get masses and produces no new unexpected couplings. The presence of the $\overline{\text{D3}}$-like coupling means that the SU(3) triplet fermions will generically all acquire a mass (in addition to any mass they receive from the modified kinetic term), though some may remain massless due to the specific form of the flux as we saw in the previous section. The singlet fermions, however, receive no new $\mathcal{G}_3$ induced mass, for the same reason as before: its mass term does not arise from primitive (2, 1) fluxes, which we require by construction. However, as mentioned already, the singlet *does* in general receive a mass from the modified kinetic term, and hence there will generically remain no massless degrees of freedom.

D. Effect of a more general $\mathcal{F}_5$

Before we close this section we wish to comment on how the scenario changes once we allow for a more general $\mathcal{F}_5$. The combination

$$\tilde{\mathcal{F}}_5 = \mathcal{F}_5 + \mathcal{B}_2 \wedge \mathcal{F}_3 + \mathcal{C}_2 \wedge \mathcal{H}_3, \tag{96}$$

needs to be self-dual in the full 10d space. If we demand that the 3-form fluxes have only *one* leg transverse to the brane, which is necessary for them to give fermion masses, then the 5-form flux must have a leg off the brane as well and therefore will not generate a mass for the fermions! Conversely, if $\mathcal{F}_5$ is entirely along the brane directions, the corresponding 3-forms will not be of the appropriate form to generate masses. It is therefore possible to consider embeddings of the $\overline{\text{D7}}$ such that

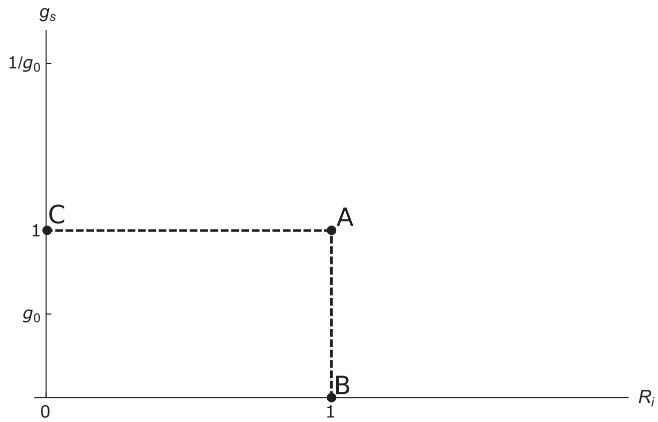

FIG. 1. The type IIB moduli space with the self-dual point denoted by A. The point B is for all $R_i = 1$ and the point C is for $g_s = 1$. Our duality mappings are defined for the point A. Going away from the point A in any direction in the moduli space will imply switching on nontrivial values for the axio-dilaton.

only one or the other type of mass contribution is present or a combination of both.

Let us consider a nonzero $\mathcal{F}_{0123m}$ component, where m is along the brane worldvolume. The fermion decomposition analysis is very similar to before. The contribution to the action is of the form

$$\begin{aligned}&\frac{\sqrt{2}}{12}\bar{\theta}^1\Gamma^{MNP}\varepsilon_{MNPQ}\mathcal{F}_{0123}{}^Q\theta^1\\&\quad=(\bar{\lambda}^0_+\lambda^i_+\varepsilon_{ij\bar{j}\bar{k}}\mathcal{F}_{0123}{}^{\bar{k}}-\bar{\lambda}^{\bar{0}}_-\lambda^{\bar{i}}_-\varepsilon_{\bar{i}j\bar{j}k}\mathcal{F}_{0123}{}^k)\delta^{j\bar{j}}\\&\qquad+\frac{1}{2}(\bar{\lambda}^i_+\lambda^a_+\varepsilon_{ak\ell}\varepsilon_{i\bar{k}\bar{\ell}m}\mathcal{F}_{0123}{}^m\\&\qquad+\bar{\lambda}^{\bar{i}}_-\lambda^{\bar{a}}_-\varepsilon_{\bar{a}\bar{k}\bar{\ell}}\varepsilon_{\bar{i}k\ell\bar{m}}\mathcal{F}_{0123}{}^{\bar{m}})\delta^{k\bar{k}}\delta^{\ell\bar{\ell}},\end{aligned}\tag{97}$$

where the $i, j, k\ldots$ indices are restricted to lie along the brane (but a has no such restriction). This results in nonzero m_{13}, m_{23} and even more notably m_{01}, m_{02}. Note that m_{11}, m_{12}, m_{22} remain vanishing, so even when both the 3-form and the 5-form fluxes contribute mass terms, there is still a massless degree of freedom remaining.

Finally, the modification of the κ projector in the scenario with worldvolume fluxes does not introduce new contributions from the 5-form flux. Indeed, the second term in $\Gamma_{\overline{\mathrm{D7}}}$, which gives the coupling to the induced 5-brane charge, can only conspire to give three or seven gamma matrices inside the resulting fermion bilinear if $\mathcal{F}_5$ has two legs in the internal space, but it must have four legs along the spacetime directions. Similarly, the third term necessarily results in a single gamma matrix, yielding a vanishing bilinear, exactly as in the $\overline{\mathrm{D3}}$ case. Note however, that combining both worldvolume fluxes and an $\mathcal{F}_5$ without transverse legs results in all the fermions acquiring a mass.

Let us also note that if we had taken the internal space to be a non-Kähler resolved conifold with fractional branes, and then inserted a $\overline{\mathrm{D7}}$-brane wrapping a four-cycle inside the non-Kähler space, the background fluxes and also the physics would have been quite different. We will however not explore this further here, but instead go to another interesting aspect of our analysis: the all-order fermionic action on a $\overline{\mathrm{D3}}$-brane.

IV. TOWARDS THE κ-SYMMETRIC ALL-ORDER FERMIONIC ACTION FOR A $\overline{\mathrm{D3}}$-BRANE

The previous two sections detailed the spontaneous breaking of supersymmetry by probe antibranes in otherwise supersymmetric compactifications. The starting point of both of these analyses has been the fermionic brane action at lowest order in θ, which takes a manifestly κ-symmetric form.

We would now like to see if this result continues to hold at higher orders in θ. As we will see, the answer to this question is in the affirmative, and to show this we need only minimal knowledge of brane actions.[23] In particular, we can use string dualities to deduce the structure of the all-order fermionic action, without needing precise information as to the form of the higher-order operators. To do so, we will define a (completely general) fermionic completion of the $\overline{\mathrm{D3}}$-brane action, as was done at lowest order in θ in Refs. [41,42], and use certain duality tricks to generate the higher-order fermionic counterparts of the bosonic fields. Note that under renormalization group (RG) flow the higher-order terms are generically irrelevant, but they are nevertheless needed to realize the full κ symmetry.

The bosonic components of the NS and RR sectors are connected by the type IIB equations of motion, and therefore once a certain set of field components are known, others can be generated from the corresponding EOMs. On the other hand, for the fermionic components no additional work is needed: knowing the fermionic fields $(\theta, \bar{\theta})$ and the bosonic fields, one should be able to predict the fermionic completions of the bosonic fields to all orders in θ and $\bar{\theta}$. This means the fermionic completions of higher p-form fields should *at least* be related to the lower p-form field (including the graviton, antisymmetric tensor and dilaton) by certain U-duality transformations at the self-dual points $g_s = 1$ and $R_i = 1$ for $i = 1, \ldots, 2k$ with R_i being the radii of the compact directions. To see why this is the case, let us study two corners of type IIB moduli space.

(1) We can go to the $g_s = e^\phi = 1$ point where we should be able to exchange $\mathbf{B}^{(1)}_{mn}$ with $\mathbf{B}^{(2)}_{mn}$, as shown by point C in Fig. 1.

(2) We can go to self-dual radii of the compact target space $R_i = 1$ where we should be able to exchange

[23] See Refs. [65], [66], and [67], for more recent related works on the Volkov-Akulov actions.

the p-form fields with $(p+2k)$-form fields, as shown by point B in Fig. 1.

This is only possible if at least a subset of the fermionic counterparts of the $(p+2k)$-form fields are the ones obtained via U-duality transformations. This trick could then be used to generate all the fermionic counterparts of the higher form fields at least at the self-dual corner $g_s = R_i = 1$ of type IIB moduli space, i.e. around the point A in Fig. 1. Once we move away from the self-dual point, we can study the fermionic counterparts of the bosonic fields at generic points in the type IIB moduli space.

On the other hand the scenario is subtle in the presence of branes. It is known that the D3- or the $\overline{\rm D3}$-branes are S-duality neutral although the worldvolume degrees of freedom differ. However they are not T-duality neutral. The other D-branes (or NS-branes) are neither S- nor T-duality neutrals. So, to effectively use the duality trick, no branes should be present. This is good because now we can determine the fermionic completion of the background without worrying about the backreactions from the branes, and then insert D-branes to study the worldvolume theory.

A. Towards an all-order θ expansion from dualities

Let us now proceed with our analysis. We start by redefining the all-order fermionic completion of type IIB scalar fields in the following way:

$$\begin{aligned}\Phi^{(i)} &= \varphi^{(i)} + \bar{\theta}\Delta^{(i)}\theta \\ &\equiv \varphi^{(i)} + \sum_j \prod_{k=1}^{j} \bar{\theta}\Delta^{(i)jk}\theta \\ &= \varphi^{(i)} + \bar{\theta}\Delta^{(11i)}\theta + \bar{\theta}\Delta^{(21i)}_{m\ldots p}\theta\bar{\theta}\Delta^{(22i)}_{q\ldots n}\theta g^{pq}\ldots g^{mn} \\ &\quad + \bar{\theta}\Delta^{(31i)}_{m\ldots p}\theta\bar{\theta}\Delta^{(32i)}_{q\ldots l}\theta\bar{\theta}\Delta^{(33i)}_{s\ldots n}\theta g^{pq}\ldots g^{ls}\ldots g^{mn} + \cdots \end{aligned} \tag{98}$$

where $\Phi^{(1)} = \phi_B$ and $\Phi^{(2)} = C^{(0)}$ are the dilaton and the axion respectively, and the dotted terms are of $\mathcal{O}(\theta^8)$. The fermion products in Eq. (98) are defined in terms of components in the following way:

$$\bar{\theta}\Delta^{(21i)}_{m\ldots p}\theta\bar{\theta}\Delta^{(22i)}_{q\ldots n}\theta \equiv \bar{\theta}^{\alpha}\Delta^{(21i)}_{m\ldots p\alpha\beta}\theta^{\beta}\bar{\theta}^{\delta}\Delta^{(22i)}_{q\ldots n\delta\gamma}\theta^{\gamma}, \tag{99}$$

where the greek indices span the 32 (complex) component[24] fermions θ. The IIB spinor θ is a doublet of 16 (complex) component Majorana spinors of the same chirality, i.e. this doublet is a 32-component Majorana spinor but is *not* Weyl. We decompose θ into the two 16 (complex) component fermions θ_1 and θ_2 as

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}, \tag{100}$$

with θ_2 generically nonvanishing. The $\Delta^{(abi)}$ are all operators that can be represented in the matrix form in the following way:

$$\Delta^{(i)} \equiv \begin{pmatrix} \Delta^{(11i)} & \Delta^{(12i)} & \Delta^{(13i)} & \ldots \\ \Delta^{(21i)} & \Delta^{(22i)} & \Delta^{(23i)} & \ldots \\ \Delta^{(31i)} & \Delta^{(32i)} & \Delta^{(33i)} & \ldots \\ \ldots & & & \end{pmatrix}, \tag{101}$$

where every element of the matrix should be viewed as an operator with its own matrix representation in some appropriate Hilbert space. The complete form of the matrix (101) is not known, but a few elements have been worked out in the literature [35,41–43]. For example it is known that

$$\Delta^{(111)}\theta = -\frac{i}{2}\bar{\delta}\lambda\theta, \qquad \Delta^{(112)}\theta = \frac{1}{2}e^{-\phi}\sigma_2\bar{\delta}\lambda\theta, \tag{102}$$

where $\bar{\delta}\lambda$ is the supersymmetric variation of the type IIB spinor λ in the presence of a $\overline{\rm D3}$ and σ_2 is the second Pauli matrix that acts on the $\theta_{1,2}$ components of Eq. (100). It should also be clear, from the way we constructed the matrix, that

$$\Delta^{(abi)} = \Delta^{(bai)}. \tag{103}$$

Additionally, in the ensuing analysis we will resort to the following simplification: instead of considering the $\Delta^{(abi)}$ operators to have an arbitrary rank q as $\Delta^{(abi)}_{m_1 m_2\ldots m_q}$ for $a \geq 2$, we will only take them to have a maximal rank 2. As will be clear from the context, this simplification will not change any of the physics, and one may easily switch to arbitrary rank $\Delta^{(abi)}$ operators without loss of generality. On the other hand, this simplification will avoid unnecessary cluttering of indices. Henceforth unless mentioned otherwise, we will take only this simplified version.

With this in mind, let us now consider the type IIB metric g_{mn}. We can expand the all-order fermionic completion in a way analogous to the scalar field:

$$\begin{aligned}\mathbf{G}_{mn} &= g_{mn} + \bar{\theta}M_{(mn)}\theta \\ &= g_{mn} + \bar{\theta}M^{(11)}_{(mn)}\theta + g^{pq}\bar{\theta}M^{(21)}_{(m|p}\theta\bar{\theta}M^{(22)}_{q|n)}\theta \\ &\quad + g^{pq}g^{ls}\bar{\theta}M^{(31)}_{(m|p}\theta\bar{\theta}M^{(32)}_{ql}\theta\bar{\theta}M^{(33)}_{s|n)}\theta + \mathcal{O}(\theta^8), \end{aligned} \tag{104}$$

which is again a sum over products of contractions of the fermions with matrix elements of the operator M_{mn}. The four-component M operator can be written using two bosonic and two fermionic components as

[24]Or 64 real components. Note that the series in Eq. (98) and in the following, terminate at some finite number of terms because of the finite number of fermionic components as well as because of the Grassmannian nature of the fermions.

$$M_{(mn)\alpha\beta} = M^{(11)}_{(mn)\alpha\beta} + M^{(21)p}_{(m|\alpha\gamma}\theta^\gamma\bar{\theta}^\delta M^{(22)}_{p|n)\delta\beta} + M^{(31)p}_{(m|\alpha\gamma}\theta^\gamma\bar{\theta}^\delta M^{(32)}_{ps\delta\sigma}\theta^\sigma\bar{\theta}^\rho M^{(33)s}_{n)\rho\beta} + \cdots \quad (105)$$

where the first term in the above expansion is well known in terms of the supersymmetric variation of the Rarita-Schwinger fermion ψ_m [35,41–43]:

$$M^{(11)}_{\alpha\beta(mn)} = -i\Gamma_{\alpha\beta(m}D_{n)} \equiv -i\Gamma_{\alpha\beta(m}\bar{\delta}\psi_{n)}. \quad (106)$$

The antisymmetric rank two tensor can also be expanded in terms of the fermionic components like the symmetric tensor (104). We can define $\mathbf{B}^{(i)}_{mn}$ as the generalized antisymmetric tensors, where $B^{(1)}_{mn} = B_{mn}$ and $B^{(2)}_{mn} = C^{(2)}_{mn}$ are the NS and RR 2-forms respectively, using a certain antisymmetric tensor $N^{(i)}_{[mn]}$ in the following way:

$$\begin{aligned}\mathbf{B}^{(i)}_{mn} &= B^{(i)}_{mn} + \bar{\theta}N^{(i)}_{[mn]}\theta \\ &= B^{(i)}_{mn} + \bar{\theta}N^{(11i)}_{[mn]}\theta + g^{pq}\bar{\theta}N^{(21i)}_{[m|p}\theta\bar{\theta}N^{(22i)}_{q|n]}\theta \\ &\quad + g^{pq}g^{ls}\bar{\theta}N^{(31i)}_{[m|p}\theta\bar{\theta}N^{(32i)}_{ql}\theta\bar{\theta}N^{(33i)}_{s|n]}\theta + \mathcal{O}(\theta^8).\end{aligned} \quad (107)$$

To see the connection between $M_{(mn)}$ and $N^{(i)}_{[mn]}$ operators let us revisit the T-duality rules of Refs. [36,68]. The powerful thing about the fermionic completion is that the T-duality rules follow *exactly* the formula laid out for the bosonic fields, except now all the fields are replaced by their fermionic completions. This can be illustrated as[25]

$$\tilde{\Phi}^{(1)} = \Phi^{(1)} - \frac{1}{2}\ln\mathbf{G}_{xx}, \quad \tilde{\mathbf{G}}_{xx} = \frac{1}{\mathbf{G}_{xx}},$$
$$\tilde{\mathbf{G}}_{mn} = \mathbf{G}_{mn} - \frac{\mathbf{G}_{mx}\mathbf{G}_{nx} - \mathbf{B}^{(1)}_{mx}\mathbf{B}^{(1)}_{nx}}{\mathbf{G}_{xx}}, \quad \tilde{\mathbf{G}}_{mx} = \frac{\mathbf{B}^{(1)}_{mx}}{\mathbf{G}_{xx}},$$
$$\tilde{\mathbf{B}}^{(1)}_{mn} = \mathbf{B}^{(1)}_{mn} - \frac{\mathbf{B}^{(1)}_{mx}\mathbf{G}_{nx} - \mathbf{G}_{mx}\mathbf{B}^{(1)}_{nx}}{\mathbf{G}_{xx}}, \quad \tilde{\mathbf{B}}^{(1)}_{mx} = \frac{\mathbf{G}_{mx}}{\mathbf{G}_{xx}} \quad (108)$$

where x is the T-duality direction. From the T-duality rule we see that, in the presence of cross terms of $\mathbf{G}$ in type IIA, $\mathbf{B}^{(1)}$ could be generated in type IIB using Eq. (108). Since both the IIA and IIB metrics use $M_{(mn)}$, this is possible if

$$\bar{\theta}N^{(1)}_{[mx]}\theta \equiv \bar{\theta}c\sigma_3^p M_{[mx]}\theta, \quad (109)$$

where the operator $M_{[mx]}$ is now expressed with respect to the T-dual fields, i.e. the IIB bosonic fields. We have also inserted the third Pauli matrix σ_3 into Eq. (109), with $p = 1$ or 2, to take care of certain subtleties that will be explained later,[26] and c is a constant matrix. The only constant matrices for our case, that do not change the chirality, are the identity and the chirality matrix Γ^{10}, so we will choose $c = \Gamma^{10}$. Since we can make T-duality along any direction, x appearing in Eq. (109) could span all directions. This means we can generalize Eq. (109) to the following:

$$\bar{\theta}N^{(1)}_{[mn]}\theta \equiv \bar{\theta}\sigma_3^p \otimes \Gamma^{10}M_{[mn]}\theta, \quad (110)$$

implying that the symmetric matrix $M_{(mn)}$ determines the generalized metric $\mathbf{G}_{mn}$, whereas the antisymmetric matrix $M_{[mn]}$ determines the generalized B-field $\mathbf{B}^{(1)}_{mn}$. In terms of components, we expect

$$N^{(111)}_{\alpha\beta[mn]} = -i\sigma_3 \otimes \Gamma^{10}\Gamma_{\alpha\beta[m}\delta\psi_{n]}, \quad (111)$$

which is consistent with the results in Refs. [35,41–43]. However the relation (110) predicts the form of *all* the operators appearing in Eq. (107) once all the corresponding operators appearing in Eq. (104) are known, not just the component given above.

To find the form of $\mathbf{B}^{(2)}_{mn}$, or the operator $N^{(2)}_{[nm]}$, we will use the T-duality trick discussed above, assuming that the T-duality rules go for the RR fields with fermionic completions exactly as their bosonic counterparts [41,42]. To proceed we will need $\Phi^{(2)}$ and $\mathbf{B}^{(1)}_{mn}$ from Eqs. (98) and (107), rewritten as

[25]There seems to be two ways of analyzing the T-duality transformations in the literature. One, is to assume that Buscher's rules are *exact* to all orders in α' and only the supergravity fields receive α' corrections. This way, Buscher's rules could be used to study supergravity field transformations order by order in α'. The other is to assume that both the T-duality transformations *and* the supergravity fields receive α' corrections. There is some confusion regarding which one should be considered, but in our opinion the more conservative picture is the latter one where *both*, the T-duality rules as well as the supergravity fields, receive α' corrections. Since T-duality transformations preserve supersymmetry, the α' corrections to the T-duality transformations would imply α' corrections to the supersymmetry transformations—a result consistent with the known facts. See for example Refs. [37,69] for the lowest-order corrections, where somewhat similar arguments have appeared, and Ref. [70] for more recent discussions. However as we will see soon, our results will not be very sensitive to this.

[26]See discussions after Eq. (126).

$$\Phi^{(2)} = C^{(0)} + \bar{\theta}\sigma_2\tilde{\Delta}^{(2)}\theta,$$
$$\mathbf{B}^{(1)}_{mn} = B_{mn} + \bar{\theta}\sigma_3 \otimes \Gamma^{10} M_{[mn]}\theta, \qquad (112)$$

where we have extracted a Pauli matrix σ_2 in defining $\Delta^{(2)} = \sigma_2\tilde{\Delta}^{(2)}$. The other components appearing in Eq. (112) are the corresponding bosonic backgrounds. The T-duality rules for the RR fields are given as[27]

$$\tilde{\mathbf{C}}^{(n)}_{xm_2\ldots m_n} = \mathbf{C}^{(n-1)}_{m_2\ldots m_n} - (n-1)\tilde{\mathbf{B}}^{(1)}_{x[m_2}\mathbf{C}^{(n-1)}_{|x|m_3\ldots m_n]},$$
$$\tilde{\mathbf{C}}^{(n)}_{m_1\ldots m_n} = \mathbf{C}^{(n+1)}_{xm_1\ldots m_n} - n\mathbf{B}^{(1)}_{x[m_1}\tilde{\mathbf{C}}^{(n)}_{|x|m_2\ldots m_n]}. \qquad (113)$$

There are now two possible ways to get the fermionic part of $\mathbf{B}^{(2)}_{mn}$: we can T-dualize the scalar $\Phi^{(2)}$ twice using the T-duality rule (113), and we can S-dualize $\mathbf{B}^{(1)}_{mn}$. Let us start by discussing the first possibility, namely the T-duality way of getting part of $\mathbf{B}^{(2)}_{mn}$. By T-dualizing once we get a vector field in type IIA as

$$\tilde{\mathbf{C}}^{(1)}_x = \Phi^{(2)}, \qquad (114)$$

and then another T-duality will give us the required RR 2-form field in the following way:

$$\hat{\mathbf{C}}^{(2)}_{yx} = \tilde{\mathbf{C}}^{(1)}_x - \hat{\mathbf{B}}^{(1)}_{yx}\tilde{\mathbf{C}}^{(1)}_y = \Phi^{(2)} - \hat{\mathbf{B}}^{(1)}_{yx}\tilde{\mathbf{C}}^{(1)}_y = \Phi^{(2)} \qquad (115)$$

because $\tilde{\mathbf{C}}^{(1)}_y = 0$ according to Eq. (114), and therefore the field $\Phi^{(2)}$ should determine the required 2-form. However before proceeding we should determine how the 32-component Majorana fermion (100) changes under the two T-dualities. It is easy to show that

$$\theta \to \Sigma_1\theta, \qquad \bar{\theta} \to \bar{\theta}\Sigma_2, \qquad (116)$$

where Σ_i are two 32×32 component matrices, i.e. they act on the doublet basis, given in terms of the 16-component gamma matrices[28] Γ_x and Γ_y by

$$\Sigma_1 = \begin{pmatrix} \mathbf{I}_{16} & 0 \\ 0 & \Gamma_x\Gamma_y \end{pmatrix}, \quad \Sigma_2 = \begin{pmatrix} \mathbf{I}_{16} & 0 \\ 0 & \Gamma_y\Gamma_x \end{pmatrix}, \qquad (117)$$

and leading to the following set of algebras that will be useful soon:

$$\Sigma_2(\sigma_2 \otimes \mathbf{I}_{16})\Sigma_1 = \sigma_3\sigma_2 \otimes \Gamma_x\Gamma_y, \quad \Sigma_2 \cdot \Sigma_1 = \mathbf{I}_{32},$$
$$\Sigma_2\begin{pmatrix} 0 & \mp i\mathbf{C} \\ \pm i\mathbf{C} & 0 \end{pmatrix}\Sigma_1 = \begin{pmatrix} \pm\mathbf{C} & 0 \\ 0 & \pm\mathbf{C} \end{pmatrix}(\sigma_3\sigma_2 \otimes \Gamma_x\Gamma_y),$$
$$\Sigma_2\begin{pmatrix} \pm\mathbf{C} & 0 \\ 0 & \pm\mathbf{C} \end{pmatrix}\Sigma_1 = \begin{pmatrix} \pm\mathbf{C} & 0 \\ 0 & \pm\mathbf{C} \end{pmatrix}, \quad (\sigma_3\sigma_2)^2 = -\mathbf{I}_2,$$
$$\begin{pmatrix} \mathbf{1} & 0 \\ 0 & \Gamma_b\Gamma_a \end{pmatrix}(\sigma_3\sigma_2 \otimes \Gamma_x\Gamma_y)\begin{pmatrix} \mathbf{1} & 0 \\ 0 & \Gamma_a\Gamma_b \end{pmatrix}$$
$$= \sigma_2 \otimes \Gamma_x\Gamma_y\Gamma_a\Gamma_b. \qquad (118)$$

Therefore using Eqs. (115) and (116) with the algebras (118), we can get one part of the 2-form $\mathbf{B}^{(2)}_{mn}$ in the following way:

$$\hat{\mathbf{C}}^{(2)}_{mn} = \bar{\theta}c\sigma_3\sigma_2 \otimes \Gamma_{mn}\tilde{\Delta}^{(2)}\theta, \qquad (119)$$

where, as before, we can take $c = \Gamma^{10}$ i.e. the chirality matrix, and $\tilde{\Delta}^{(2)}$ can either be expressed in terms of the T-dual fields or the original fields.

In deriving Eq. (119) we have not actually looked at the form of $\tilde{\Delta}^{(2)}$. Depending on the representation of gamma matrices in the definition of $\tilde{\Delta}^{(2)}$, our simple expression (119) could in principle change to a more involved one. The scenario is subtle so let us tread carefully here. We start by rewriting the RR scalar (98) field as

$$\Phi^{(2)} = C^{(0)} + (\bar{\theta}\sigma_2)^\alpha\tilde{\Delta}^{(2)}_{\alpha\beta}\theta^\beta$$
$$= C^{(0)} + (\bar{\theta}\sigma_2)^\alpha\tilde{\Delta}^{(112)}_{\alpha\beta}\theta^\beta + (\bar{\theta}\sigma_2)^\alpha\tilde{\Delta}^{(212)p}_{\alpha\chi m}\theta^\chi\bar{\theta}^\sigma\tilde{\Delta}^{(222)m}_{\sigma\beta p}\theta^\beta$$
$$+ (\bar{\theta}\sigma_2)^\alpha\tilde{\Delta}^{(312)p}_{\alpha\gamma m}\theta^\gamma\bar{\theta}^\sigma\tilde{\Delta}^{(322)l}_{\sigma\chi p}\theta^\chi\bar{\theta}^\delta\tilde{\Delta}^{(332)m}_{\delta\beta l}\theta^\beta + \mathcal{O}(\theta^8),$$
$$(120)$$

where we have assumed that the generic operator $\tilde{\Delta}^{(ab2)m}_{\alpha\beta n}$ is constructed from the products of 16-dimensional gamma matrices, the type IIB bosonic fields and covariant derivatives $\mathbf{A}_{16\times16}$ as

$$(\bar{\theta}\sigma_2^p)_\alpha\tilde{\Delta}^{(ab2)}_{mn\alpha\beta}\theta_\beta \equiv \bar{\theta}\sigma_2^p\begin{pmatrix} \mathbf{A}^{(ab)}_{16\times16} & 0 \\ 0 & \mathbf{A}^{(ab)}_{16\times16} \end{pmatrix}_{mn}\theta, \qquad (121)$$

where p can be 0 or 1 depending on what fermion combination we are looking at in Eq. (120). Using our T-duality ideas, and using the gamma matrix algebras (118), it is easy to see that the 2-form (119) appears naturally with an overall $\Gamma_m\Gamma_n$ matrix provided we impose

$$[\mathbf{A}, \Gamma_x\Gamma_y] = 0, \qquad (122)$$

without loss of generalities as transformations with an even number of gamma matrices will not change any results. The puzzle however is if Eq. (121) takes the following form:

[27]As before, we expect the T-duality rules for the RR fields to also receive α' corrections. We will discuss the consequence of this on our analysis soon.

[28]We are using the flat-space Γ matrices.

$$(\bar{\theta}\sigma_2^p)^\alpha \tilde{\Delta}^{(ab2)}_{mn\alpha\beta}\theta^\beta \equiv \bar{\theta}(\sigma_2^p \otimes \mathbf{I}_{16})(\mathbf{I}_2 \otimes \mathbf{A}^{(ab)} + \sigma_1 \otimes \mathbf{C}^{(ab)})_{mn}\theta = \bar{\theta}(\sigma_2^p \otimes \mathbf{I}_{16})\begin{pmatrix} \mathbf{A}^{(ab)}_{16\times16} & \mathbf{C}^{(ab)}_{16\times16} \\ \mathbf{C}^{(ab)}_{16\times16} & \mathbf{A}^{(ab)}_{16\times16} \end{pmatrix}_{mn}\theta, \tag{123}$$

where $(\sigma_1, \mathbf{I}_2)$ are the first Pauli matrix and two-dimensional identity matrix respectively, and $\mathbf{C}_{16\times16}$ is another 16-dimensional matrix constructed out of gamma matrices, IIB fields and covariant derivatives.

To understand the consequence of the above-mentioned representations of the operators, let us discuss a few additional gamma matrix algebras under our T-duality transformations:

$$\begin{aligned} \Sigma_2(\sigma_2\sigma_1 \otimes \mathbf{C})\Sigma_1 &= -i\sigma_3 \otimes \mathbf{C}, \\ \Sigma_2(\sigma_1 \otimes \mathbf{C})\Sigma_1 &= i(\sigma_2 \otimes \mathbf{C}\Gamma_x\Gamma_y), \\ \Sigma_2(\sigma_2 \otimes \mathbf{I}_{16})(\mathbf{I}_2 \otimes \mathbf{A})\Sigma_1 &= \sigma_3\sigma_2 \otimes \mathbf{A}\Gamma_x\Gamma_y. \end{aligned} \tag{124}$$

Using these algebras, it is now easy to see that under T-dualities the operators (121) and (123) transform in the following way:

$$\begin{aligned} &\bar{\theta}(\mathbf{I}_2 \otimes \mathbf{A} + \sigma_1 \otimes \mathbf{C})\theta \to \bar{\theta}(\mathbf{I}_2 \otimes \mathbf{A} + i\sigma_2 \otimes \mathbf{C}\Gamma_x\Gamma_y)\theta, \\ &\bar{\theta}\sigma_2(\mathbf{I}_2 \otimes \mathbf{A} + \sigma_1 \otimes \mathbf{C})\theta \to \bar{\theta}(\sigma_3\sigma_2 \otimes \mathbf{A}\Gamma_x\Gamma_y - i\sigma_3 \otimes \mathbf{C})\theta, \end{aligned} \tag{125}$$

from where we see that the first terms in Eq. (125) are clearly consistent with the duality rules that lead us to the result (119). However it is the second term in the two expressions above in Eq. (125) which would *not* fit with the generic result (119). Clearly when $\mathbf{C} = 0$ this problem does not arise.

A way out of this conundrum is in fact clear from the transformations themselves. The existence of $\mathbf{C}_{16\times16}$ in Eq. (123) would imply that this piece is T-duality neutral, and *does not* transform as a rank-2 tensor under T-duality. Thus this piece cannot be part of a RR axionic scalar whose T-duality transformations are well known. In fact its neutrality to the T-duality transformation hints that $\mathbf{C}_{16\times16}$ could be a part of the NS scalar i.e. the dilaton, unless of course we can use $\bar{\psi} \equiv \bar{\theta}\sigma_1$ to transform

$$\bar{\psi}\mathbf{C} \otimes \sigma_1\theta \to \bar{\theta}\mathbf{C}\theta, \tag{126}$$

under two T-dualities. This way the issues raised in Eq. (125) will not arise and the generic result (119) will continue to hold to arbitrary orders in the θ expansion.

Let us now come to the second possibility of getting the fermionic part of $\mathbf{B}^{(2)}_{mn}$ namely, S-dualizing $\mathbf{B}^{(1)}_{mn}$ i.e. the NS part of the 2-form (with its fermionic completion). In light of our earlier discussion, this would be like moving the type IIB coupling up, at fixed self-dual radii of the compact spaces, so as to reach the $g_s \to 1^-$ point. In other words, we are moving from region B to region A in Fig. 1.

We will however start by first fulfilling the promise that we made earlier, namely to discuss the appearance of σ_3, the third Pauli matrix, in Eq. (109) for the NS B-field $\mathbf{B}^{(1)}_{mn}$. Recall that our argument was to motivate the result from T-dualizing the metric component with cross terms from type IIA to type IIB theory. Under T-duality the 32-component type IIA chiral fermion θ_A transforms as

$$\begin{aligned} \theta_A &= \begin{pmatrix} \theta_+ \\ \theta_- \end{pmatrix} \to \begin{pmatrix} 1 & 0 \\ 0 & -\Gamma^{10}\Gamma_x \end{pmatrix}\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \equiv \tilde{\Sigma}_1\theta, \\ \bar{\theta}_A &= \begin{pmatrix} \bar{\theta}_+ & \bar{\theta}_- \end{pmatrix} \to \begin{pmatrix} \bar{\theta}_1 & \bar{\theta}_2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & \Gamma^{10}\Gamma_x \end{pmatrix} \equiv \bar{\theta}\tilde{\Sigma}_2, \end{aligned} \tag{127}$$

where the T-duality is performed along direction x to go from IIA to IIB. The above transformations immediately implies the following algebra, similar to the algebras that we discussed earlier in Eq. (118):

$$\begin{aligned} &\tilde{\Sigma}_2 \otimes \begin{pmatrix} \mathbf{C}_{16\times16} & 0 \\ 0 & \mathbf{C}_{16\times16} \end{pmatrix} \otimes \tilde{\Sigma}_1 \\ &= \begin{pmatrix} \mathbf{C}_{16\times16} & 0 \\ 0 & -\Gamma^{10}\Gamma_x\mathbf{C}_{16\times16}\Gamma^{10}\Gamma_x \end{pmatrix} = \sigma_3^p \otimes \mathbf{C}_{16\times16}, \end{aligned} \tag{128}$$

where σ_3 is the third Pauli matrix with $p = 1$ or 2 depending on the specific representation of the 16-dimensional $\mathbf{C}$ matrix. To fix the value of p, we can go to our self-dual point such that the transformation (127) becomes an intermediate transformation at $R_x = R_\perp = 1$, where $R_\perp$ is the radius of an orthogonal circle. We can choose the $\mathbf{C}$ matrix to be of the form $\mathbf{C}_{xm} \equiv \Gamma_x\mathcal{O}_m$, with $\mathcal{O}_m$ being a combination of type IIB fields and covariant derivatives with *even* or *odd* numbers of gamma matrices. In that case $p = 1$ in Eq. (128). Even when the intermediate matrix, in the θ expansion, is of the form $\mathbf{C} + \sigma_1 \otimes \tilde{\mathbf{C}}$, the result of the form (128) will continue to hold because we can absorb σ_1 in the transformation matrices as in Eq. (126). Therefore, combining the results together, and assuming $p = 1$, we can express the fermionic part of the NS B-field $\mathbf{B}^{(1f)}_{mn}$ as

$$\mathbf{B}^{(1f)}_{mn} = \bar{\theta}\sigma_3 \otimes \Gamma^{10}M_{[mn]}\theta. \tag{129}$$

As discussed earlier, we can now go to a corner of type IIB moduli space where the string coupling is strong i.e. $g_s \to 1$. Here we expect the RR B-field $\mathbf{B}^{(2)}_{mn}$ to be given at least by the S-dual of $\mathbf{B}^{(1)}_{mn}$. The S-duality matrix that concerns us here is

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{130}$$

which squares to $-\mathbf{I}_2$. This is the perturbative piece of the duality that keeps the string coupling unchanged, but changes the signs of the 2-form fields. To incorporate S-duality in our fermionic part of the NS B-field $\mathbf{B}^{(1)}_{mn}$ one only needs to insert $-i\sigma_2$ in Eq. (129) to get the following fermionic piece[29]:

$$\hat{\mathbf{D}}^{(2)}_{mn} = -i\bar{\theta}\sigma_3\sigma_2 \otimes \Gamma^{10} M_{[mn]}\theta, \tag{131}$$

such that S-dualizing twice will yield $(-i\sigma_2)^2 = -\mathbf{I}_2$. This way we will get back the same result as Eq. (130) after two S-dualities that allow for a $\mathbf{Z}_2$ phase factor. Combining Eqs. (119) and (131) together we get our final expression for the RR 2-form field along with its fermionic completion as

$$\mathbf{B}^{(2)}_{mn} = C^{(2)}_{mn} - i\bar{\theta}\sigma_3\sigma_2 \otimes \Gamma^{10}(M_{[mn]} + i\Gamma_{mn}\tilde{\Delta}^{(2)})\theta. \tag{132}$$

From the above expression we expect the fermionic terms to be suppressed by powers of the string coupling *away* from the self-dual points, so that at the self-dual point (the region A in Fig. 1) we can exchange $\mathbf{B}^{(2)}$ and $\mathbf{B}^{(1)}$ and simultaneously perform two T-dualities. To see whether this is indeed true, we need to expand Eq. (132) to higher orders in θ. This can be easily worked out using earlier expressions for $M_{[mn]}$ and $\tilde{\Delta}$ in Eqs. (105) and (120) respectively, and the result is given by

$$\begin{aligned}\mathbf{B}^{(2)}_{mn} = C^{(2)}_{mn} &- i\bar{\theta}e^{-\phi}\sigma_3\sigma_2 \otimes \Gamma^{10}(M^{(11)}_{[mn]} + i\Gamma_{mn}\tilde{\Delta}^{(112)})\theta \\ &- i\bar{\theta}e^{-\phi}\sigma_3\sigma_2 \otimes \Gamma^{10}(M^{(21)}_{[m|p}\theta\bar{\theta}M^{(22)}_{q|n]}g^{pq} \\ &+ i\Gamma_{mn}\tilde{\Delta}^{(212)}_{rp}\theta\bar{\theta}\tilde{\Delta}^{(222)}_{qs}g^{pq}g^{rs})\theta + \mathcal{O}(\theta^8),\end{aligned} \tag{133}$$

where to $\mathcal{O}(\theta^2)$ the coefficients can be read off from Eqs. (106) and (111) as (see also Refs. [41,42] for more details)

$$M^{(11)}_{[mn]} = -\Gamma_{[m}\bar{\delta}\psi_{n]}, \qquad \tilde{\Delta}^{(112)} = \frac{1}{2}\bar{\delta}\lambda. \tag{134}$$

We can see that the string coupling appears correctly in Eq. (133) so as to allow for the right behavior of the form fields in the full IIB moduli space. The fermion variations $(\bar{\delta}\psi_m, \bar{\delta}\lambda)$ are with respect to either the original type IIB variables or the T-dual type IIB variables in our transformation scheme. Note that once we know the functional form of $\hat{\Delta}^{(ab2)}_{mn}$ for generic values of (a, b), we will know the θ expansion of Eq. (133) to arbitrary orders. This is of course a challenging exercise which we will not perform here. Instead we will use our results for $\mathbf{B}^{(1)}_{mn}$ and $\mathbf{B}^{(2)}_{mn}$ etc. to determine the fermionic structure of the 4-form $\mathbf{C}_{\rm mnpq}$ around the self-dual point.

The fermionic structure of the 4-form can be determined using a similar trick as before by scanning the IIB moduli space. There are two different points in the moduli space that would give us the 4-form. First, at weak string coupling, we can go to the small compactification radii (or more appropriately the self-dual radii) where the 4-form can get contributions from the T-dual of $\mathbf{B}^{(2)}_{mn}$. Second, at strong string coupling i.e. $g_s \to 1$, we can again go to self-dual radii where the 4-form can now get contributions from the U-dual of $\mathbf{B}^{(1)}_{mn}$. For the first case, we can T-dualize the RR field $\mathbf{B}^{(2)}_{mn}$ twice along directions (a, b); and for the second case, we can S-dualize the $\mathbf{B}^{(1)}_{mn}$ field and then T-dualize twice along directions (a, b). The gamma matrix algebras useful for us are now the following:

$$\begin{aligned}&\begin{pmatrix} 1 & 0 \\ 0 & \Gamma_b\Gamma_a \end{pmatrix}(\sigma_3\sigma_2 \otimes \Gamma^{10})\begin{pmatrix} 1 & 0 \\ 0 & \Gamma_a\Gamma_b \end{pmatrix} \\ &= \sigma_2 \otimes \Gamma^{10}\Gamma_a\Gamma_b, \\ &\begin{pmatrix} 1 & 0 \\ 0 & \Gamma_b\Gamma_a \end{pmatrix}(\sigma_3\sigma_2 \otimes \Gamma^{10}\Gamma_x\Gamma_y)\begin{pmatrix} 1 & 0 \\ 0 & \Gamma_a\Gamma_b \end{pmatrix} \\ &= \sigma_2 \otimes \Gamma^{10}\Gamma_a\Gamma_b\Gamma_x\Gamma_y.\end{aligned} \tag{135}$$

Using these algebras, which are basically the T-duality rules, for both strong and weak string couplings will immediately provide us with the contributions to the 4-form from the two sources mentioned above around $g_s = R_a = R_b = 1$. The result is 7

$$\begin{aligned}\mathbf{C}_{mnpq} &= C^{(4)}_{mnpq} - i\bar{\theta}\sigma_2 \otimes \Gamma^{10}(2\Gamma_{[mn}M_{pq]} + i\Gamma_{mnpq}\tilde{\Delta}^{(2)})\theta \\ &= C^{(4)}_{mnpq} - i\bar{\theta}\sigma_2 \otimes \Gamma^{10}(2\Gamma_{[mn}M^{(11)}_{pq]} \\ &\quad + i\Gamma_{mnpq}\tilde{\Delta}^{(112)})\theta + \mathcal{O}(\theta^4) \\ &= C^{(4)}_{mnpq} - \frac{1}{2}\bar{\theta}\sigma_2 \otimes \Gamma^{10}(4\Gamma_{[mnp}\bar{\delta}\psi_{q]} \\ &\quad - \Gamma_{mnpq}\bar{\delta}\lambda)\theta + \mathcal{O}(\theta^4),\end{aligned} \tag{136}$$

where the factor of 2 signifies the contributions from the U-dual of the two B-fields, and we have determined the results up to $\mathcal{O}(\theta^2)$. One may verify with Refs. [3,35,41,42] that the result quoted above matches well with the literature at the self-dual point. It is interesting that to this order the match is exact, and therefore other possible corners of the type IIB moduli space do not contribute anything else to the fermionic parts of the bosonic RR and NS fields. At higher orders in θ there could be contributions that we cannot determine using our U-duality trick. Nevertheless, the U-duality transformations are powerful enough to

[29]The sign is chosen for later convenience.

extract out the fermionic contributions from various corners of the moduli space.

So far however we have not discussed the connection between $\Delta^{(1)}$ appearing in the dilaton and $\tilde{\Delta}^{(2)}$ appearing in the axion, as in Eq. (144). The fact that they are related can be seen from M-theory on a torus $\mathbf{T}^2$ in the limit when the torus size is shrunk to zero. Of course the scenario that we have envisioned here at the self-dual point cannot be uplifted to M-theory because we are not allowed to shrink the M-theory torus to zero size (as $g_s = 1$). However away from the self-dual point we *can* lift our configuration to M-theory, so let us discuss this point briefly. In M-theory we expect the metric to take a form similar to Eq. (104) or Eq. (144), i.e.

$$\hat{\mathbf{G}}_{mn} = G^{(11)}_{mn} + \bar{\theta}\hat{M}_{mn}\theta, \tag{137}$$

where the superscript denotes the bosonic part of the metric, and θ is the corresponding fermionic variable. If we parametrize the torus direction by (x^3, x^a) where x^a denotes the eleventh direction, then it is easy to see that in the limit of vanishing size of the torus, the type IIB axion and dilaton, with their fermionic completions, are related via

$$\exp\,[-2\Phi^{(1)}] + [\Phi^{(2)}]^2 = \frac{\hat{\mathbf{G}}_{33}}{\hat{\mathbf{G}}_{aa}}, \tag{138}$$

implying the connection between $\Delta^{(1)}$ and $\tilde{\Delta}^{(2)}$ *away from the self-dual point*. Using this one should be able to derive the $\mathcal{O}(\theta^2)$ result similar to Eq. (145) but away from the self-dual point, as also given in Refs. [41,42].

What happens at the self-dual point? The self-dual point is defined for $C^{(0)} = \phi = 0$, and therefore we should at least assume that this continues to be the case for the fermionic completions of the dilaton and axion too. In other words we should expect

$$\tau \equiv \Phi^{(2)} + ie^{-\Phi^{(1)}} = i \quad \text{(at the self-dual point)}, \tag{139}$$

to all orders in $(\theta, \bar{\theta})$. Interestingly the condition $|\tau|^2 = 1$ is similar to the M-theory condition (138) in the limit $\hat{\mathbf{G}}_{33} = \hat{\mathbf{G}}_{aa}$. To lowest order in $\theta, \bar{\theta}$ it is easy to see that Eq. (139) reduces to the following condition:

$$\bar{\theta}^\alpha \Delta^{(111)}_{\alpha\beta}\theta^\beta = -i\bar{\theta}^\alpha(\sigma_2)^\gamma_\alpha \tilde{\Delta}^{(112)}_{\gamma\beta}\theta^\beta \quad \text{(at the self-dual point)}. \tag{140}$$

In general, to all orders in $(\theta, \bar{\theta})$, the relation between $\Delta^{(1)}$ and $\tilde{\Delta}^{(2)}$ at the self-dual point can be directly seen from Eq. (139) as

$$\bar{\theta}\Delta^{(1)}\theta = -\log\,(1 + i\bar{\theta}\sigma_2\tilde{\Delta}^{(2)}\theta)\text{(at the self-dual point)}. \tag{141}$$

We expect Eqs. (140) and (141) to reproduce the condition (102) or (145) discussed in Refs. [35,41–43] at the self-dual point also. To this effect we will start by defining

$$\Delta^{(1)} = -i\tilde{\Delta}^{(2)} + \hat{\Delta}, \tag{142}$$

generically, both *at* and *away* from the self-dual point. Plugging Eq. (142) into Eq. (141), and taking into account the lowest-order results in Refs. [35,41–43], we expect $\hat{\Delta}$ to vanish to lowest order in $(\bar{\theta}, \theta)$ and use the following constraint on the fermionic coordinate:

$$\bar{\theta}(1 - \sigma_2) = 0 \quad \text{(at the self-dual point)}, \tag{143}$$

which would naturally explain the invariance under U-dualities in region A in Fig. 1. Of course away from the self-dual point we do not expect Eqs. (141) and (143) to hold, although Eq. (138) should continue to hold.

We now conclude this section by collecting together all of our results. The fermionic completions of the type IIB fields, away from the self-dual point, can be expressed in the following compact notations:

$$\begin{aligned}
&\Phi^{(1)} = \phi + \bar{\theta}\Delta^{(1)}\theta, \quad \Phi^{(2)} = C^{(0)} + \bar{\theta}e^{-\phi}\sigma_2\tilde{\Delta}^{(2)}\theta,\\
&\mathbf{B}^{(1)}_{mn} = B_{mn} + \bar{\theta}\sigma_3 \otimes \Gamma^{10} M_{[mn]}\theta, \quad \mathbf{G}_{mn} = g_{mn} + \bar{\theta}M_{(mn)}\theta,\\
&\mathbf{B}^{(2)}_{mn} = C^{(2)}_{mn} - i\bar{\theta}e^{-\phi}\sigma_3\sigma_2 \otimes \Gamma^{10}(M_{[mn]} + i\Gamma_{mn}\tilde{\Delta}^{(2)})\theta,\\
&\mathbf{C}_{\rm mnpq} = C^{(4)}_{\rm mnpq} - i\bar{\theta}e^{-\phi}\sigma_2 \otimes \Gamma^{10}(2\Gamma_{[mn}M_{pq]} + i\Gamma_{\rm mnpq}\tilde{\Delta}^{(2)})\theta,
\end{aligned} \tag{144}$$

where the θ expansion for $\Delta^{(1)}$ is given by Eq. (98), that for $\tilde{\Delta}^{(2)}$ is given by Eq. (120) and those for $M_{(mn)}$ and $M_{[mn]}$ are given by Eq. (104). We will take $(C^{(0)}, \phi) \to 0$, such that $g_s = e^\phi \to 1$ at the self-dual point. Knowing these series expansions we can in principle determine the type IIB fields to arbitrary orders in θ (provided of course there are no additional terms other than the ones obtained via U-duality transformations). In the presence of a $\overline{\rm D3}$, the functional forms for $\Delta^{(1)}$, $\tilde{\Delta}^{(2)}$ and M_{mn} become fixed. Henceforth this is the choice that we will consider, unless mentioned otherwise.[30] For example, to $\mathcal{O}(\theta^2)$, $\Delta^{(1)}, \tilde{\Delta}^{(2)}$ and M_{mn} are known to be

[30] For simplicity we will only concentrate on the integer $\overline{\rm D3}$-brane (including the D3-brane), and not discuss the fractional branes as we did for the resolved conifold case. Although with our formalism it is easy to extend to any D-brane, integer or fractional, one needs to be careful when fractional branes are present along with integer D3- or $\overline{\rm D3}$-branes. However in the presence of only fractional branes, but no integer branes, the story proceeds in exactly the same way as discussed here as long as we are below the energy scale proportional to the inverse size of the two-sphere on which we have our wrapped branes.

$$\Delta^{(1)} = -\frac{i}{2}\bar{\delta}\psi, \quad \tilde{\Delta}^{(2)} = \frac{1}{2}\bar{\delta}\psi, \quad M_{mn} = -i\Gamma_m\bar{\delta}\psi_n, \tag{145}$$

and therefore plugging them into Eq. (144) will determine the type IIB fields to $\mathcal{O}(\theta^2)$ in the presence of a $\overline{\rm D3}$-brane. The above values should be understood as operators acting on θ, and therefore to higher orders in θ one would need to express them in terms of components:

$$(\Delta^{(111)}_{\alpha\beta}, \Delta^{(ab1)}_{mn\alpha\beta}), \qquad (\tilde{\Delta}^{(112)}_{\alpha\beta}, \tilde{\Delta}^{(ab2)}_{mn\alpha\beta}), \qquad M^{(ab)}_{mn\alpha\beta}, \tag{146}$$

as elucidated in Eqs. (98), (120) and (104) to properly write the higher-order terms. Also, in Eq. (146) (m, n) are Lorentz indices, and (α, β) are spinor indices. One may easily check that these results match with the ones known in the literature [35,39–43] for $e^\phi = 1$. The interesting thing about Eq. (146) is that, knowing these coefficients, one might be able to go to higher orders in θ as discussed above.

B. κ symmetry at all orders in θ

In the previous section we managed to get the full fermionic action for the $\overline{\rm D3}$-branes using certain U-duality transformations at the self-dual point in the type IIB moduli space. The result is extendable to the D3-brane also, modulo certain subtleties that we want to elaborate here. Our answer is given in Eq. (156) which is derived for the special case of $\mathcal{F}_{mn} = 0$. The most generic case, given as Eq. (147), could also be worked out using the representations (144) for the type IIB fields, but we will not do so here.

Another issue that we briefly talked about earlier is the behavior of these higher-order terms under renormalization group flow. Under RG flow we expect these terms to be irrelevant. However as we will discuss momentarily, to argue for the full κ symmetry, all the higher-order terms are essential. Therefore for our purpose it may be useful to work with the *exact* renormalization group equations [71] to keep track of the irrelevant operators. In the following however we will not discuss the quantum behavior and concentrate only on the classical action (156) with all the higher-order terms.

The question that we want to answer here is the following: under what condition will the action (156) take the κ-symmetric form, i.e. a form like $\mathcal{L} \sim \bar{\theta}(1 - \mathbf{\Gamma}^{\pm}_{\rm D3})[\ldots]\theta$, where $\Gamma^{\pm}_{\rm D3}$ is the κ-symmetry operator?[31] The condition, as we shall see, turns out to be rather subtle so we will have to tread carefully. Therefore as a start we will take the worldvolume action, for a single D3 or $\overline{\rm D3}$, in the presence of the fermionic terms, to be given by

$$S = -T_3 \int d^4\zeta e^{-\Phi^{(1)}} \sqrt{-\det\left(\mathbf{G}_{ab} + \mathbf{B}^{(1)}_{ab} + \alpha'\mathbf{F}_{ab}\right)} \pm T_3 \int \mathbf{C} \wedge e^{\mathbf{B}+\alpha'\mathbf{F}}, \tag{147}$$

where the first term is the Born-Infeld (BI) piece and the second one is the Chern-Simons (CS) piece. The only difference now is that they both include the fermionic completions that we developed earlier which are in general different for D3- and $\overline{\rm D3}$-branes.[32] We can choose the gauge field $\mathbf{F}_{ab}$ in such a way as to cancel the fermionic contributions of the NS B-field $\mathbf{B}^{(1)}_{ab}$. This way we can write a bosonic combination $\mathcal{F}_{ab} \equiv \mathbf{B}^{(1)}_{ab} + \alpha'\mathbf{F}_{ab}$ to represent the gauge field. We can also define a matrix A in the following way:

$$A_{mn} \equiv [(g + \mathcal{F})^{-1}]^p_m \bar{\theta}^\alpha M_{pn\alpha\beta}\theta^\beta, \tag{148}$$

where the matrix M_{mn} was defined earlier in Eq. (144) to study the fermionic parts of the metric and the NS B-field. With this definition, the BI part of the antibrane action takes the following form:

$$S_{\rm BI} = -T_3 \int d^4\zeta e^{-\phi} \sqrt{-\det(g + \mathcal{F})} \times \exp\left[\frac{1}{2}{\rm tr}\log(\mathbf{I} + A) - \bar{\theta}\Delta^{(1)}\theta\right], \tag{149}$$

where $\mathbf{I}$ is the identity matrix in four dimensions, and A is the same matrix defined earlier in Eq. (148). As usual, at the self-dual point we put $\phi = 0$ to be consistent with our U-dualities. Moving away from the self-dual points, as exemplified in Eqs. (133), (134) and (144), the action has the necessary dilaton piece.

We now come to the Chern-Simons part of the brane action for both the D3 and $\overline{\rm D3}$ using the fermionic completions developed above. The action can be written as

$$S_{\rm CS} = T_3 \int d^4\zeta \epsilon^{mnpq}\left(\mathbf{C}^{\pm}_{mnpq} + \mathbf{B}^{(2\pm)}_{mn}\mathcal{F}_{pq} + \frac{1}{2}\Phi^{(2\pm)}\mathcal{F}_{mn}\mathcal{F}_{pq}\right), \tag{150}$$

where the superscripts represent D3 and $\overline{\rm D3}$ respectively, and $\mathbf{C}^-_{mnpq} \equiv \mathbf{C}_{mnpq}$, $\mathbf{B}^{(2-)}_{mn} \equiv \mathbf{B}^{(2)}_{mn}$ and $\Phi^{(2-)} \equiv \Phi^{(2)}$ for a

[31] See Eq. (152) for the definition of $\Gamma^{\pm}_{\rm D3}$.

[32] We have used three kinds of matrices, namely M_{mn}, $\Delta^{(1)}$ and $\tilde{\Delta}^{(2)}$ to express the fermionic pieces in the presence of a $\overline{\rm D3}$-brane. One may choose similar matrices to express the fermionic pieces in the presence of a D3-brane. For example we will use M^+_{mn}, $\Delta^{(1+)}$ and $\tilde{\Delta}^{(2+)}$ as the corresponding matrices for a D3-brane to represent the fermionic parts, whereas $M^-_{mn} = M_{mn}$, $\Delta^{(1-)} = \Delta^{(1)}$ and $\tilde{\Delta}^{(2-)} = \tilde{\Delta}^{(2)}$ will be reserved for the $\overline{\rm D3}$-brane to avoid clutter.

$\overline{\text{D3}}$ as we developed here. We have assumed that the background is flat along spacetime directions so that the curvature terms do not appear above. In general, for a curved background, the curvature terms with their fermionic completions (from the metric) should also appear. For our case this should only change the last term in the above action (150).

We can simplify the action (150) further by assuming $\mathcal{F}_{mn} = 0$. This would also imply that A_{mn} in Eq. (148) simplifies. This is the case we will consider here. A more generic scenario with $\mathcal{F}_{mn}$, or even with the fermionic pieces of $\mathcal{F}_{mn}$ (that we canceled here) can be studied. This will make the system more involved but will not change the physics. Therefore, for this special case we have

$$S_{\text{CS}} = T_3 \int d^4\zeta \epsilon^{mnpq} C^{(4)}_{mnpq} + T_3 \int d^4\zeta e^{-\phi} \sqrt{-\det g}\bar{\theta}\Gamma^{\pm}_{\text{D3}} \left(\frac{1}{2}\Gamma^{ba} M^{\pm}_{ab} + i\tilde{\Delta}^{(2\pm)}\right)\theta, \tag{151}$$

where, as before, $M^{-}_{ab} \equiv M_{ab}$ and $\tilde{\Delta}^{(2-)} \equiv \tilde{\Delta}^{(2)}$ represent the corresponding matrices for a $\overline{\text{D3}}$, and $\mathbf{\Gamma}^{\pm}_{\text{D3}}$ is defined as

$$\mathbf{\Gamma}^{\pm}_{\text{D3}} = \pm \frac{i\sigma_2 \otimes \Gamma^{10}\Gamma_{mnpq}\epsilon^{mnpq}}{4!\sqrt{-\det g}}. \tag{152}$$

Let us now come back to the BI piece of the action (149). To analyze this we will use the well-known expansion for log as

$$\begin{aligned} \text{tr} \log (\mathbf{I} + A) &= \text{tr}A - \frac{1}{2}\text{tr}A^2 + \frac{1}{3}\text{tr}A^3 + \cdots \\ &= \sum_{k=1}^{k_{\max}} \frac{(-1)^{k+1}\text{tr}A^k}{k}, \end{aligned} \tag{153}$$

where $k_{\max}$ is determined by the rank of the matrix. Plugging this into the BI action (149) and rearranging the action appropriately, we get for a $\overline{\text{D3}}$

$$\begin{aligned} S_{\text{BI}} &= -T_3 \int d^4\zeta e^{-\phi}\sqrt{-\det g}\left[1 + \sum_{k=1}^{k_{\max}}\left(\frac{1}{2}\text{tr}A - \bar{\theta}\Delta^{(1)}\theta - \frac{1}{2}\sum_{l=1}^{l_{\max}}\frac{\text{tr}(-A)^{l+1}}{l}\right)^k \cdot \frac{1}{k!}\right] \\ &= -T_3 \int d^4\zeta e^{-\phi}\sqrt{-\det g}\left[1 + \sum_{k=1}^{k_{\max}}\frac{(\frac{1}{2}\text{tr}A + i\bar{\theta}\tilde{\Delta}^{(2)}\theta + \mathcal{S}(A,\hat{\Delta}))^k}{k!}\right] \end{aligned} \tag{154}$$

where the first term is the standard BI term for the bosonic piece and the second term is the fermionic extension. We have also used Eq. (142) to replace $\Delta^{(1)}$ by $\tilde{\Delta}^{(2)}$ and defined the other variable appearing above in the following way:

$$\mathcal{S}(A,\hat{\Delta}) = -\frac{1}{2}\sum_{l=1}^{l_{\max}}\frac{\text{tr}(-A)^{l+1}}{l} - \bar{\theta}\,\hat{\Delta}\,\theta. \tag{155}$$

Combining the Chern-Simons and the Born-Infeld parts, i.e. Eqs. (151) and (154) respectively, we can extract the fermionic completions of the brane and antibrane actions. The result is given by

$$\begin{aligned} S^f_{\pm} &= -T_3 \int d^4\zeta e^{-\phi}\sqrt{-\det g}\mathcal{L}_{\pm} \\ \mathcal{L}_{\pm} &\equiv \left[\sum_{k=1}^{k_{\max}}\frac{(\frac{1}{2}\text{tr}A^{\pm} + i\bar{\theta}\tilde{\Delta}^{(2\pm)}\theta + \mathcal{S}^{\pm}(A,\hat{\Delta}))^k}{k!} - \bar{\theta}\Gamma^{\pm}_{\text{D3}}\left(\frac{1}{2}\Gamma^{ba}M^{\pm}_{ab} + i\tilde{\Delta}^{(2\pm)}\right)\theta\right], \end{aligned} \tag{156}$$

where the $\pm$ subscripts denote the D3-brane and $\overline{\text{D3}}$ respectively and $A^{-}_{mn} \equiv A_{mn}$ as in Eq. (148). The bosonic parts of the action for the brane and the antibrane remain the same as the standard ones, as one can easily verify. It is also easy to see that

$$\frac{1}{2}\text{tr}A^{\pm} = \frac{1}{2}\bar{\theta}\Gamma^{ba}M^{\pm}_{ab}\theta \equiv \bar{\theta}(\mathbf{N}_{\pm} - i\tilde{\Delta}^{(2\pm)})\theta, \tag{157}$$

where $\mathbf{N}_{\pm}$ is defined in such a way that the fermionic action (156) takes the following form:

$$S^f_{\pm} = -T_3 \int d^4\zeta e^{-\phi}\sqrt{-\det g}(e^{\bar{\theta}\mathbf{N}_{\pm}\theta + \mathcal{O}(\mathbf{N}^2_{\pm})} - 1 - \bar{\theta}\Gamma^{\pm}_{\text{D3}}\mathbf{N}_{\pm}\theta). \tag{158}$$

In the absence of any other information about the series $\mathbf{N}_\pm$, the above action for the fermionic terms for the D3 or the $\overline{\rm D3}$ is probably the best we can say at this stage. Simplification can occur when $\mathbf{N}_\pm$ remains small to all orders in $(\theta, \bar\theta)$, which in turn would guarantee the smallness of the $\mathcal{O}(\mathbf{N}_\pm^2)$ terms in the exponential, as well as the exponential itself. If this is the case then

$$\begin{aligned} S_\pm^f &= -T_3 \int d^4\zeta e^{-\phi}\sqrt{-\det g}\,\bar\theta(1+\mathbf{\Gamma}_{\rm D3}^\pm)\mathbf{N}_\pm\theta \\ &= -T_3 \int d^4\zeta e^{-\phi}\sqrt{-\det g}\,\bar\theta(1+\mathbf{\Gamma}_{\rm D3}^\pm)\left(\frac{1}{2}\Gamma^{ba}M_{ab}^\pm + i\tilde\Delta^{(2\pm)}\right)\theta, \end{aligned} \tag{159}$$

which would provide a strong confirmation of the recent work of Ref. [1], which was originally done to $\mathcal{O}(\theta^2)$. For our case we can use the θ expansions for $M_{ab}^- = M_{ab}$ and $\tilde\Delta^{(2-)} = \tilde\Delta^{(2)}$ for a $\overline{\rm D3}$ to express

$$\begin{aligned} \bar\theta\left(\frac{1}{2}\Gamma^{ba}M_{ab} + i\tilde\Delta^{(2)}\right)\theta &= \bar\theta^\alpha\left(\frac{1}{2}\Gamma_\alpha^{ba\gamma}M_{ab\gamma\beta}^{(11)} + i\tilde\Delta_{\alpha\beta}^{(112)}\right)\theta^\beta \\ &\quad + \bar\theta^\alpha\left(\frac{1}{2}\Gamma_\alpha^{ba\gamma}M_{ac\gamma\delta}^{(21)}\theta^\delta\bar\theta^\sigma M_{b\sigma\beta}^{(22)c} + i\tilde\Delta_{\alpha\delta m}^{(212)}\theta^\delta\bar\theta^\sigma\tilde\Delta_{\sigma\beta}^{(222)m}\right)\theta^\beta + \mathcal{O}(\theta^6) \\ &= -\frac{1}{2}i\bar\theta(\Gamma^a\bar\delta\psi_a - \bar\delta\lambda)\theta + \mathcal{O}(\theta^4), \end{aligned} \tag{160}$$

which is consistent with what we know to $\mathcal{O}(\theta^2)$ from the literature [3,35,41,42]. Now if we define $\mathbf{\Gamma}_{\rm D3}^- = \mathbf{\Gamma}_{\rm D3}$ and $\mathbf{\Gamma}_{\rm D3}^+ = -\mathbf{\Gamma}_{\rm D3}$ from Eq. (152) and $\delta^+ = \delta$ and $\delta^- = \bar\delta$ from Ref. [1], and use the fermionic actions (156) or (159) for the D3- and $\overline{\rm D3}$-branes, then to $\mathcal{O}(\theta^2)$ we can easily reproduce the expected result in a κ-symmetric form:

$$S_\pm = \frac{1}{2}T_3\int d^4\zeta e^{-\phi}\sqrt{-\det g}\,i\bar\theta(1\mp\mathbf{\Gamma}_{\rm D3})(\Gamma^a\delta^\pm\psi_a - \delta^\pm\lambda)\theta + \mathcal{O}(\theta^4). \tag{161}$$

At the orientifold point, if we assume that the action is given by Eq. (159), then to all orders in θ the fermionic coordinate satisfies $\bar\theta(1-\mathbf{\Gamma}_{\rm D3}) = 0$. This way S_+ vanishes identically and S_- remains nonzero. This result seems to be valid only if the fermionic action takes the form (159), but is not obvious from the fermionic action (156) that this will continue to be the case. In fact the action (156) has many terms, coming from the log and from the exponential pieces, that do not in any obvious way give us $S_+ = 0$ at the orientifold point. In the following we will try to see how we can adjust the background, for example Eq. (144), to get the required form of the action.

Clearly adjusting the background should affect the definition of the type IIB fields (144). From the way we derived Eq. (144), we cannot arbitrarily change the field definitions since they are related by certain U-duality transformations at a self-dual point. Thus for example, knowing $\mathbf{B}_{mn}^{(1)}$, $\Phi^{(1)}$ and $\Phi^{(2)}$, we pretty much derived the rest of the RR fields using U-dualities. All the fields and their corresponding fermionic completions depend on three sets of functional forms: M_{mn}, $\Delta^{(1)}$ and $\tilde\Delta^{(2)}$. In fact the antisymmetric part of the operator M_{mn}, namely $M_{[mn]}$, is essential to describe the fermionic completions of the p-form fields in type IIB. The symmetric part, $M_{(mn)}$, on the other hand is reserved for the fermionic completion of the metric. At the self-dual radii, $M_{(mn)}$ and $M_{[mn]}$, could be related by T-dualities along one parallel and one orthogonal spatial direction. The temporal directions however are not connected via simple T-dualities. This distinction may help us to construct the κ-symmetric form of the action from Eq. (156). To this end, we start by redefining the temporal components of the metric $\mathbf{G}_{0\mu}$ in the following way:

$$\begin{aligned} \mathbf{G}_{00} &\equiv (g_{00} + \bar\theta M_{00}\theta)\exp(2\bar\theta\Omega\theta), \\ \mathbf{G}_{0i} &\equiv (g_{0i} + \bar\theta M_{0i}\theta)\exp\left(\frac{1}{5}\bar\theta\Omega\theta\right), \end{aligned} \tag{162}$$

keeping $\mathbf{G}_{ij}$ and all other type IIB fields exactly as in Eq. (144). The $\Omega(\theta, \bar\theta)$ appearing above is again a series defined by powers of $(\theta, \bar\theta)$ as

$$\bar\theta\Omega\theta = \bar\theta^\alpha\Omega_{\alpha\beta}^{(11)}\theta^\beta + \bar\theta^\alpha\Omega_{m\ldots q\alpha\gamma}^{(21)}\theta^\gamma\bar\theta^\delta\Omega_{p\ldots n\delta\beta}^{(22)}\theta^\beta g^{qp}\ldots g^{mn} + \mathcal{O}(\theta^6) \tag{163}$$

where the coefficients can be defined in a similar way as the variables appearing in Eq. (144). As before, we could resort to rank-2 tensor representations for $\Omega^{(21)}$ and $\Omega^{(22)}$ etc., without losing much of the physics here.

Let us now revisit the Born-Infeld part of the action (147). Taking Eqs. (162) and (144) into account, it is easy to see that the BI action now takes the following form:

$$\begin{aligned} S_{BI} &= -T_3 \int d^4\zeta e^{-\Phi^{(1)}} \sqrt{-\det(\mathbf{G}_{ab} + \mathbf{B}^{(1)}_{ab} + \alpha' \mathbf{F}_{ab})}|_{\mathbf{B}^{(1)}_{ab}+\alpha'\mathbf{F}_{ab}\equiv 0} \\ &= -T_3 \int d^4\zeta e^{-\phi}\sqrt{-\det g} \exp\left[\frac{1}{2}\mathrm{tr}\log(\mathbf{I}+A) + i\bar{\theta}\tilde{\Delta}^{(2)}\theta - \bar{\theta}\hat{\Delta}\theta + \bar{\theta}\Omega\theta\right] \\ &= -T_3 \int d^4\zeta e^{-\phi}\sqrt{-\det g} \exp\left[\frac{1}{2}\mathrm{tr}A + i\bar{\theta}\tilde{\Delta}^{(2)}\theta - \left(\frac{1}{2}\sum_{k=2}^{k_{\max}}\frac{(-1)^k \mathrm{tr}A^k}{k} + \bar{\theta}\hat{\Delta}\theta\right) + \bar{\theta}\Omega\theta\right] \\ &\equiv -T_3 \int d^4\zeta e^{-\phi}\sqrt{-\det g} \exp\left[\sum_{k=1}^{k_{\max}}\frac{(-1)^{k+1}}{k}\left(\frac{1}{2}\mathrm{tr}A + i\bar{\theta}\tilde{\Delta}^{(2)}\theta\right)^k + \bar{\theta}(\Theta+\Omega)\theta\right] \end{aligned} \tag{164}$$

where going from the second-to-last to the last line of Eq. (164), we have used the mathematical identity

$$\frac{1}{2}\sum_{k=2}^{k_{\max}}\frac{(-1)^k \mathrm{tr}A^k}{k} + \bar{\theta}\hat{\Delta}\theta \equiv \sum_{k=2}^{k_{\max}}\frac{(-1)^k}{k}\left(\frac{1}{2}\mathrm{tr}A + i\bar{\theta}\tilde{\Delta}^{(2)}\theta\right)^k + \bar{\theta}\Theta\theta, \tag{165}$$

implying that the functional forms of Θ and $\hat{\Delta}$ can be used to express all $\mathrm{tr}A^k$ in terms of $(\mathrm{tr}A)^k$ to allow for Eq. (165). Additionally, since Ω in Eq. (162) is arbitrary, we can also absorb Θ in the definition of Ω to give us

$$\bar{\theta}(\Theta + \Omega)\theta = 0. \tag{166}$$

The above two conditions (165) and (166) are essential for expressing the $\overline{\mathrm{D3}}$-brane action in the κ-symmetric form. Putting Eqs. (165) and (166) into Eq. (164), we get

$$\begin{aligned} S_{BI} &= -T_3 \int d^4\zeta e^{-\Phi^{(1)}}\sqrt{-\det \mathbf{G}_{ab}} \\ &= -T_3 \int d^4\zeta e^{-\phi}\sqrt{-\det g} - T_3 \int d^4\zeta e^{-\phi}\sqrt{-\det g}\bar{\theta}\left(\frac{1}{2}\Gamma^{ba}M^{\pm}_{ab} + i\tilde{\Delta}^{(2)}\right)\theta, \end{aligned} \tag{167}$$

which is precisely the condition that is required for the BI action to take the κ-symmetric form when combined with the Chern-Simons part of the action (151). Thus putting Eqs. (167) and (151) together, we get our final expression for the $\overline{\mathrm{D3}}$-brane action

$$\begin{aligned} S_{\overline{\mathrm{D3}}} &= -T_3 \int d^4\zeta e^{-\phi}\sqrt{-\det g} - T_3 \int d^4\zeta \epsilon^{mnpq} C^{(4)}_{mnpq} \\ &\quad - T_3 \int d^4\zeta e^{-\phi}\sqrt{-\det g}\bar{\theta}(1 - \mathbf{\Gamma}^{-}_{\overline{\mathrm{D3}}})\left(\frac{1}{2}\Gamma^{ba}M^{-}_{ab} + i\tilde{\Delta}^{(2)}\right)\theta, \end{aligned} \tag{168}$$

in a manifestly κ-symmetric form. Equivalently, the above action indicates that the $\overline{\mathrm{D3}}$ κ-symmetry projector

$$(1 - \mathbf{\Gamma}^{-}_{\overline{\mathrm{D3}}}), \tag{169}$$

continues to be the κ-symmetry projector at all orders in θ. Recall that the κ-symmetry variation of $\bar{\theta}$ is given by

$$\delta_\kappa \bar{\theta} = \bar{\kappa}(1 + \mathbf{\Gamma}^{-}_{\overline{\mathrm{D3}}}). \tag{170}$$

It follows from this that the $\overline{\mathrm{D3}}$ action is manifestly κ symmetric at *all orders* in θ.

In deriving our result we have relied on the fact that at the self-dual point we do not have extra fermionic operators other than the ones given by our U-duality transformations. This seems to be the case in any given background; otherwise we will end up with extra fermionic condensates which would appear to violate equations of motion. On the other hand, the U-duality rules that we used here also have α' corrections [37,69,70] so one might worry that this could change our result. A careful thought will tell us that this is not the case, as in deriving our results we have only used generic properties of T-duality. To see this in more detail, let us investigate the two key relations where some aspects of the T-duality rules have been used, namely Eqs. (109) and (115). The first relation i.e. Eq. (109) relates $N^{(1)}_{[mx]}$ with $M_{[mx]}$ under one T-duality along direction x. This is one of Buscher's rules derived for the limit $\alpha' \to 0$, so one would ask what happens under α' corrections. Before we go about discussing α' corrections to this, let us ask what it means to have a relation like Eq. (109). Since the piece M_{mn} comes from the metric and the piece N_{mn} comes from the NS B-field, the relation, or at least the bosonic part of it, implies the connection between the momentum and the winding modes under one T-duality. Thus, this is in the spirit of charge conservation: momentum charges are exchanged with winding charges or vice versa and we can take this to be the defining property of T-duality. Since Eq. (109) implies the fermionic version of this, we will assume that Eq. (109) does not have any additional α' pieces.

A similar argument unfortunately cannot be given for Eq. (115), where the RR 2-form appears from the axion under two T-dualities, because unlike the previous argument—where momentum and winding modes appear automatically—we do not have the advantage of invoking charge conservation *a priori*. We do however notice that there is a possible *alternative* way of expressing the fermionic parts of the background fields, namely that the background fields are functions of $(\theta, \bar{\theta})$ with the tensorial parts being specified by certain functions of the spacetime coordinates. In this language the T-duality rules are simply given by the way $(\theta, \bar{\theta})$ change, i.e. the transformation rules given in Eq. (116). This way we do not have to worry about the explicit α' dependences appearing from the T-duality transformations, and the all-order result (144) should be exact with the α' dependences now appearing from the order-by-order expansions of the $(\theta, \bar{\theta})$ terms for every component of the type IIB fields in Eq. (144).

V. CONCLUSION AND DISCUSSION

In this work we have studied the interplay of $\mathcal{N} = 1$ supersymmetric backgrounds and antibranes. We found two new examples where supersymmetry is spontaneously broken by a probe antibrane: a $\overline{\text{D3}}$ in a resolved conifold, and a $\overline{\text{D7}}$ in a GKP background. In the first case, the low-energy spectrum in the probe approximation has two massless fermions. However, once backreaction of the $\overline{\text{D3}}$ on bulk fluxes is taken into account (perturbatively), the would-be massless fermions in fact become massive; this is a consequence of having a wrapped 5-brane in the background (an issue which does not arise when studying GKP-type backgrounds). In the second case, we found there can in fact be *many* massless fermions, and the precise number depends on the Hodge numbers of the 4-cycle wrapped by the $\overline{\text{D7}}$, although we did not extend the analysis to include backreaction. We also studied the effect of worldvolume fluxes, which provide extra mass terms. It is possible that for the most general worldvolume fluxes background there are no $\overline{\text{D7}}$ fermions which remain massless.

As a step towards a more complete understanding of antibranes and supersymmetry breaking, we studied the brane fermionic action at all orders in the fermionic expansion. In other words, we studied the all-order α' expansion of the fermionic action, while working at *leading order* in the bosonic α' expansion. This allowed us to neglect curvature corrections to the action, as well as purely bosonic α' corrections to the string duality transformations. Our result is that the all-order fermionic action can be written in a manifestly κ-symmetric form, which implies that our previous two analyses (and the results of Refs. [1,2]) are not simply a leading-order effect. In this analysis we neglected the effect of worldvolume flux, and while we do not expect this to qualitatively change the result (see, for example, Ref. [43]), it would be interesting to see the precise details of how this changes the all-order fermionic calculation.

There are many directions for future work. It would be interesting to see what types of inflationary scenarios can be built from the two examples we have studied, and if the interaction of the fermions with worldvolume fluxes can lead to a modification of the inflationary dynamics. In a totally different direction, we would like to see how the all-order fermionic action can be expressed in a Volkov-Akulov form, which should in principle be possible given the recent results of Ref. [66]. We plan to study all these effects in future works.

ACKNOWLEDGMENTS

It is our great pleasure to thank Eric Bergshoeff, Renata Kallosh and Antoine Van Proeyen for helpful discussions, and Timm Wrase for initial collaborations and many illuminating discussions. K. D. would like to thank the Stanford Physics department for hospitality during his sabbatical visit, where this work was started. The work of K. D. is supported in part by the National Science and Engineering Research Council of Canada and in part by the Simons Research Grant. The work of E. M. is supported by the National Science and Engineering Research Council of Canada via a post-graduate scholar doctoral fellowship.

[1] R. Kallosh and T. Wrase, Emergence of spontaneously broken supersymmetry on an anti-D3-Brane in KKLT dS vacua, J. High Energy Phys. 12 (2014) 117.
[2] E. A. Bergshoeff, K. Dasgupta, R. Kallosh, A. Van Proeyen, and T. Wrase, $\overline{\mathrm{D3}}$ and dS, J. High Energy Phys. 05 (2015) 058.
[3] P. McGuirk, G. Shiu, and F. Ye, Soft branes in supersymmetry-breaking backgrounds, J. High Energy Phys. 07 (2012) 188.
[4] D. V. Volkov and V. P. Akulov, Possible universal neutrino interaction, Pis'ma Zh. Eksp. Teor. Fiz. **16**, 621 (1972) [JETP Lett. **16**, 438 (1972)].Is the neutrino a Goldstone particle?, Phys. Lett. B **46**, 109 (1973).
[5] M. Cederwall, A. von Gussich, B. E. W. Nilsson, and A. Westerberg, The Dirichlet super three-brane in ten-dimensional type IIB supergravity, Nucl. Phys. **B490**, 163 (1997); M. Cederwall, A. von Gussich, B. E. W. Nilsson, P. Sundell, and A. Westerberg, The Dirichlet super p-branes in ten-dimensional type IIA and IIB supergravity, Nucl. Phys. **B490**, 179 (1997); E. Bergshoeff and P. K. Townsend, Super D-branes, Nucl. Phys. **B490**, 145 (1997); E. Bergshoeff, R. Kallosh, T. Ortin, and G. Papadopoulos, Kappa symmetry, supersymmetry and intersecting branes, Nucl. Phys. **B502**, 149 (1997); R. Kallosh, Volkov-Akulov theory, and D-branes, Supersymmetry and quantum field theory, Lect. Notes Phys. **509**, 49 (1998); E. Bergshoeff, F. Coomans, R. Kallosh, C. S. Shahbazi, and A. Van Proeyen, Dirac-Born-Infeld-Volkov-Akulov and deformation of supersymmetry, J. High Energy Phys. 08 (2013) 100; E. Bergshoeff, M. de Roo, B. Janssen, and T. Ortin, The Super D9-brane and its truncations, Nucl. Phys. **B550**, 289 (1999); F. Riccioni, Truncations of the D9-brane action and type I strings, Phys. Lett. B **560**, 223 (2003).
[6] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi, De Sitter vacua in string theory, Phys. Rev. D **68**, 046005 (2003).
[7] S. Ferrara, R. Kallosh, and A. Linde, Cosmology with nilpotent superfields, J. High Energy Phys. 10 (2014) 143.
[8] R. Kallosh and A. Linde, Inflation and uplifting with nilpotent superfields, J. Cosmol. Astropart. Phys. 01 (2015) 025.
[9] E. McDonough and M. Scalisi, Inflation from nilpotent Kähler corrections, J. Cosmol. Astropart. Phys. 11 (2016) 028.
[10] R. H. Brandenberger, Moduli stabilization in string gas cosmology, Prog. Theor. Phys. Suppl. **163**, 358 (2006).
[11] S. B. Giddings, S. Kachru, and J. Polchinski, Hierarchies from fluxes in string compactifications, Phys. Rev. D **66**, 106006 (2002).
[12] C. Vafa, Superstrings and topological strings at large N, J. Math. Phys. (N.Y.) **42**, 2798 (2001).
[13] M. Becker, K. Dasgupta, A. Knauf, and R. Tatar, Geometric transitions, flops and non-Kähler manifolds. I, Nucl. Phys. **B702**, 207 (2004); M. Becker, K. Dasgupta, S. H. Katz, A. Knauf, and R. Tatar, Geometric transitions, flops and non-Kähler manifolds. II, Nucl. Phys. **B738**, 124 (2006).
[14] R. Gwyn and A. Knauf, Conifolds and geometric transitions, Rev. Mod. Phys. **80**, 1419 (2008).
[15] K. Dasgupta, G. Rajesh, and S. Sethi, M theory, orientifolds and G-flux, J. High Energy Phys. 08 (1999) 023.
[16] U. H. Danielsson, S. S. Haque, P. Koerber, G. Shiu, T. Van Riet, and T. Wrase, De Sitter hunting in a classical landscape, Fortschr. Phys. **59**, 897 (2011).
[17] R. Kallosh, F. Quevedo, and A. M. Uranga, String theory realizations of the nilpotent Goldstino, J. High Energy Phys. 12 (2015) 039.
[18] M. Bertolini, D. Musso, I. Papadimitriou, and H. Raj, A Goldstino at the bottom of the cascade, J. High Energy Phys. 11 (2015) 184.
[19] I. Garcia-Etxebarria, F. Quevedo, and R. Valandro, Global string embeddings for the nilpotent Goldstino, J. High Energy Phys. 02 (2016) 148.
[20] I. Benmachiche, J. Louis, and D. Martinez-Pedrera, The effective action of the heterotic string compactified on manifolds with SU(3) structure, Classical Quantum Gravity **25**, 135006 (2008).
[21] S. Gurrieri, A. Lukas, and A. Micu, Heterotic string compactifications on half-flat manifolds. II., J. High Energy Phys. 12 (2007) 081.
[22] S. Chiossi and S. Salamon, The intrinsic torsion of SU(3) and G(2) structures, arXiv:math/0202282.
[23] G. Lopes Cardoso, G. Curio, G. Dall'Agata, D. Lust, P. Manousselis, and G. Zoupanos, Non-Kähler string backgrounds and their five torsion classes, Nucl. Phys. **B652**, 5 (2003).
[24] S. Gurrieri, J. Louis, A. Micu, and D. Waldram, Mirror symmetry in generalized Calabi-Yau compactifications, Nucl. Phys. **B654**, 61 (2003).
[25] J. Gaillard and J. Schmude, On the geometry of string duals with backreacting flavors, J. High Energy Phys. 01 (2009) 079.
[26] K. Dasgupta, M. Emelin, and E. McDonough, Non-Kähler resolved conifold, localized fluxes in M-theory and supersymmetry, J. High Energy Phys. 02 (2015) 179.
[27] F. Chen, K. Dasgupta, J. M. Lapan, J. Seo, and R. Tatar, Gauge/gravity duality in heterotic string theory, Phys. Rev. D **88**, 066003 (2013).
[28] L. Bedulli and L. Vezzoni, The Ricci tensor of SU(3)-manifolds, J. Geom. Phys. **57**, 1125 (2007).
[29] L. A. Pando Zayas and A. A. Tseytlin, 3-branes on resolved conifold, J. High Energy Phys. 11 (2000) 028.
[30] J. P. Gauntlett, D. Martelli, and D. Waldram, Superstrings with intrinsic torsion, Phys. Rev. D **69**, 086002 (2004).
[31] K. Dasgupta, J. Seo, and A. Wissanji, F-Theory, Seiberg-Witten Curves and $\mathrm{N} = 2$ Dualities, J. High Energy Phys. 02 (2012) 146.
[32] K. Dasgupta and S. Mukhi, Brane constructions, fractional branes and anti–de Sitter domain walls, J. High Energy Phys. 07 (1999) 008.
[33] J. Maldacena and D. Martelli, The Unwarped, resolved, deformed conifold: Fivebranes and the baryonic branch of the Klebanov-Strassler theory, J. High Energy Phys. 01 (2010) 104.
[34] F. Chen, K. Dasgupta, P. Franche, S. Katz, and R. Tatar, Supersymmetric configurations, geometric transitions and new non-Kähler manifolds, Nucl. Phys. **B852**, 553 (2011).
[35] E. Bergshoeff, R. Kallosh, A. K. Kashani-Poor, D. Sorokin, and A. Tomasiello, An index for the Dirac operator on D3-branes with background fluxes, J. High Energy Phys. 10 (2005) 102.

[36] E. Bergshoeff, C. M. Hull, and T. Ortin, Duality in the type II superstring effective action, Nucl. Phys. **B451**, 547 (1995).
[37] E. Bergshoeff, I. Entrop, and R. Kallosh, Exact duality in string effective action, Phys. Rev. D **49**, 6663 (1994).
[38] R. Gregory, J. A. Harvey, and G. W. Moore, Unwinding strings and T-duality of Kaluza-Klein and H-monopoles, Adv. Theor. Math. Phys. **1**, 283 (1997).
[39] M. Grana, D3-brane action in a supergravity background: The fermionic story, Phys. Rev. D **66**, 045014 (2002).
[40] P. K. Tripathy and S. P. Trivedi, D3-brane action and fermion zero modes in presence of background flux, J. High Energy Phys. 06 (2005) 066.
[41] D. Marolf, L. Martucci, and P. J. Silva, Actions and fermionic symmetries for D-branes in bosonic backgrounds, J. High Energy Phys. 07 (2003) 019.
[42] D. Marolf, L. Martucci, and P. J. Silva, Fermions, T-duality and effective actions for D-branes in bosonic backgrounds, J. High Energy Phys. 04 (2003) 051.
[43] L. Martucci, J. Rosseel, D. Van den Bleeken, and A. Van Proeyen, Dirac actions for D-branes on backgrounds with fluxes, Classical Quantum Gravity **22**, 2745 (2005).
[44] O. Aharony, Y. E. Antebi, and M. Berkooz, Open string moduli in KKLT compactifications, Phys. Rev. D **72**, 106009 (2005).
[45] J. Park, R. Rabadan, and A. M. Uranga, Orientifolding the conifold, Nucl. Phys. **B570**, 38 (2000).
[46] A. Retolaza and A. Uranga, Orientifolds of warped throats from toric Calabi-Yau singularities, J. High Energy Phys. 07 (2016) 135.
[47] I. Bena, M. Grana, S. Kuperstein, and S. Massai, Anti-D3 Branes: Singular to the bitter end, Phys. Rev. D **87**, 106010 (2013).
[48] D. Cohen-Maldonado, J. Diaz, T. van Riet, and B. Vercnocke, Observations on fluxes near antibranes, J. High Energy Phys. 01 (2016) 126.
[49] K. Dasgupta, P. Franche, A. Knauf, and J. Sully, D-terms on the resolved conifold, J. High Energy Phys. 04 (2009) 027.
[50] Z. Kenton and S. Thomas, D-brane potentials in the warped resolved conifold and natural inflation, J. High Energy Phys. 02 (2015) 127.
[51] K. Becker, M. Becker, K. Dasgupta, and P. S. Green, Compactifications of heterotic theory on non-Kähler complex manifolds: I, J. High Energy Phys. 04 (2003) 007; K. Becker, M. Becker, K. Dasgupta, and S. Prokushkin, Properties of heterotic vacua from superpotentials, Nucl. Phys. **B666**, 144 (2003); K. Becker, M. Becker, P. S. Green, K. Dasgupta, and E. Sharpe, Compactifications of heterotic strings on non-Kähler complex manifolds. 2., Nucl. Phys. **B678**, 19 (2004).
[52] M. Becker and K. Dasgupta, Kähler versus non-Kähler compactifications, arXiv:hep-th/0312221.
[53] G. W. Gibbons, *Aspects of supergravity theories, in Supersymmetry, Supergravity and Related Topics* edited by F. Del Aguila *et al.* (World Scientific, Singapore, 1985), p. 123; F. Del Aguila *et al.*Thoughts on tachyon cosmology, *Classical Quantum Gravity* ***20**, S321 (2003).*
[54] J. M. Maldacena and C. Nunez, Supergravity description of field theories on curved manifolds and a no go theorem, Int. J. Mod. Phys. A **16**, 822 (2001).
[55] K. Dasgupta, R. Gwyn, E. McDonough, M. Mia, and R. Tatar, de Sitter vacua in type IIB string theory: Classical solutions and quantum corrections, J. High Energy Phys. 07 (2014) 054.
[56] A. Westphal, de Sitter string vacua from Kähler uplifting, J. High Energy Phys. 03 (2007) 102.
[57] S. Kachru, J. Pearson, and H. L. Verlinde, Brane/flux annihilation and the string dual of a nonsupersymmetric field theory, J. High Energy Phys. 06 (2002) 021.
[58] P. Ouyang, Holomorphic D7-branes and flavored $N = 1$ gauge theories, Nucl. Phys. **B699**, 207 (2004).
[59] H. Y. Chen, P. Ouyang, and G. Shiu, On supersymmetric D7-branes in the warped deformed conifold, J. High Energy Phys. 01 (2010) 028.
[60] A. Dymarsky, S. Kuperstein, and J. Sonnenschein, Chiral symmetry breaking with non-SUSY D7-branes in ISD backgrounds, J. High Energy Phys. 08 (2009) 005.
[61] D. Lüst, F. Marchesano, L. Martucci, and D. Tsimpis, Generalized non-supersymmetric flux vacua, J. High Energy Phys. 11 (2008) 021.
[62] I. Bandos and D. Sorokin, Aspects of D-brane dynamics in supergravity backgrounds with fluxes, kappa-symmetry and equations of motion: Part IIB, Nucl. Phys. **B759**, 399 (2006).
[63] H. Jockers and J. Louis, D-terms and F-terms from D7-brane fluxes, Nucl. Phys. **B718**, 203 (2005).
[64] C. P. Burgess, R. Kallosh, and F. Quevedo, De Sitter string vacua from supersymmetric D terms, J. High Energy Phys. 10 (2003) 056.
[65] Z. Komargodski and N. Seiberg, From linear SUSY to constrained superfields, J. High Energy Phys. 09 (2009) 066.
[66] S. M. Kuzenko and S. J. Tyler, Relating the Komargodski-Seiberg and Akulov-Volkov actions: Exact nonlinear field redefinition, Phys. Lett. B **698**, 319 (2011); On the Goldstino actions and their symmetries, J. High Energy Phys. 05 (2011) 055.
[67] I. Bandos, L. Martucci, D. Sorokin, and M. Tonin, Brane induced supersymmetry breaking and de Sitter supergravity, J. High Energy Phys. 02 (2016) 080.
[68] S. F. Hassan, T duality, space-time spinors and RR fields in curved backgrounds, Nucl. Phys. **B568**, 145 (2000).
[69] E. Bergshoeff, B. Janssen, and T. Ortin, Solution generating transformations and the string effective action, Classical Quantum Gravity **13**, 321 (1996).
[70] O. Hohm and B. Zwiebach, Double Metric, Generalized metric and α'-geometry, arXiv:1509.02930; T-duality constraints on higher derivatives revisited, J. High Energy Phys. 04 (2016) 101.
[71] J. Polchinski, Renormalization and effective Lagrangians, Nucl. Phys. **B231**, 269 (1984).