Natural broadening in the quantum emission spectra of higher-dimensional Schwarzschild black holes

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Following an intriguing heuristic argument of Bekenstein, many researches have suggested during the last four decades that quantized black holes may be characterized by discrete radiation spectra. Bekenstein and Mukhanov (BM) have further argued that the emission spectra of quantized (3 + 1)-dimensional Schwarzschild black holes are expected to be sharp in the sense that the characteristic natural broadening $\delta \omega$ of the black-hole radiation lines, as deduced from the quantum time-energy uncertainty principle, is expected to be much smaller than the characteristic frequency spacing $\Delta \omega = O(T_{\rm BH}/\hbar)$ between adjacent black-hole quantum emission lines. It is of considerable physical interest to test the general validity of the interesting conclusion reached by BM regarding the sharpness of the Schwarzschild black-hole quantum radiation spectra. To this end, in the present paper we explore the physical properties of the expected radiation spectra of quantized (D + 1)-dimensional Schwarzschild black holes. In particular, we analyze the functional dependence of the characteristic dimensionless ratio $\zeta(D) \equiv \delta \omega / \Delta \omega$ on the number D + 1of spacetime dimensions. Interestingly, it is proved that the dimensionless physical parameter $\zeta(D)$, which characterizes the sharpness of the black-hole quantum emission spectra, is an increasing function of D. In particular, we prove that the quantum emission lines of (D + 1)-dimensional Schwarzschild black holes in the regime $D \gtrsim 10$ are characterized by the dimensionless ratio $\zeta(D) \gtrsim 1$ and are therefore effectively blended together. The results presented in this paper thus suggest that, even if the underlying energy spectra of quantized (D + 1)-dimensional Schwarzschild black holes are fundamentally discrete, as argued by many authors, the quantum phenomenon of natural broadening is expected to smear the characteristic emission spectra of these higher-dimensional black holes into a continuum.

DOI: 10.1103/PhysRevD.95.024012

I. INTRODUCTION

While studying the interaction of fundamental fields with curved black-hole spacetimes, Hawking [1] has reached the remarkable conclusion that, due to quantum effects, black holes are actually not completely black. In particular, Hawking's seminal analysis has revealed the intriguing fact that semiclassical black holes, like mundane black-body emitters, are characterized by continuous emission spectra with well-defined thermal properties [1,2].

Hawking's interesting conclusion regarding the thermal nature of quantum fields in curved black-hole spacetimes has attracted the attention of both physicists and mathematicians over the last four decades and is certainly one of the most important predictions of fundamental theoretical physics. One should bear in mind, however, that the ground-breaking analysis presented in [1] has a fundamentally asymmetric nature; while the fundamental fields living in the curved black-hole spacetime are properly treated at the quantum level, the black hole itself (and, in particular, its horizon) is treated in [1] as a fixed classical entity.

One should therefore regard the continuous black-hole radiation spectrum derived by Hawking [1] as an important prediction of semiclassical general relativity in which quantized fundamental fields interact with the classical curved spacetime of a black hole. Taking cognizance of the fundamental limitations of the semiclassical theory of general relativity, it is quite natural to expect that some modifications to the continuous black-hole emission spectrum predicted by Hawking [1] may arise within the framework of a self-consistent quantum theory of gravity, a theory in which the black-hole spacetime itself (and not just the matter fields) is properly treated as a quantum physical entity [3].

One of the most intriguing quantization schemes for the surface area (or equivalently, for the energy spectra) of black holes was presented by Bekenstein more than four decades ago [3]. Following the interesting physical observation that the surface area of a nonextremal black hole behaves as a fundamental adiabatic invariant quantity [3,4], Bekenstein has argued, using the Ehrenfest principle [5], that the surface area of a quantized black hole should be characterized by a uniformly spaced discrete spectrum of the form [3,6]

$$A_n = 4\gamma\hbar \cdot n; \quad n \in \mathbb{Z},\tag{1}$$

where γ is a dimensionless constant of order unity. The three most commonly used values of the parameter γ which appear in the physics literature are $\gamma = 2\pi$ [3], $\gamma = \ln 2$

[7,8], and $\gamma = \ln 3$ [9]. Interestingly, the remarkably compact formula (1) suggested by Bekenstein [3] for the discrete surface area of (3 + 1)-dimensional quantized black holes has been rederived by several authors who have used different physically motivated quantization schemes (see [7–28] and references therein).

Bekenstein has further argued [3] that the uniformly spaced area spectrum (1) should be associated with a discrete mass (energy) spectrum $\{M_n\}$ for quantized black holes. In particular, using the simple mass-area relation

$$A = 16\pi M^2 \tag{2}$$

for (3 + 1)-dimensional Schwarzschild black holes, one immediately deduces from (1) a discrete mass spectrum of the form

$$M_n = \sqrt{\frac{\gamma\hbar}{4\pi} \cdot n}; \quad n \in \mathbb{Z}$$
 (3)

for these spherically symmetric black holes. Taking cognizance of the discrete energy spectrum (3), Bekenstein and Mukhanov (BM) [3,7] have raised the intriguing idea that, within the framework of a quantum theory of gravity, quantized Schwarzschild black holes may be characterized by discrete radiation spectra.

In particular, as stressed by BM [3,7], the decay of a macroscopic [29] (3 + 1)-dimensional quantized Schwarzschild black hole of mass M_n into lower energy levels is expected to be accompanied by the emission of discrete field quanta whose characteristic frequencies are given by [30]

$$\omega_k = \frac{M_n - M_{n-k}}{\hbar} = k \cdot \varpi; \quad k = 1, 2, 3, ...,$$
(4)

where the fundamental (*smallest* possible) frequency ϖ which characterizes the quantized black-hole emission spectrum is given by the simple dimensionless relation [see Eq. (3)]

$$M\varpi = \frac{\gamma}{8\pi}.$$
 (5)

Interestingly, the quantized radiation spectrum (4) advocated by BM [3,7] for macroscopic [29] (3 + 1)-dimensional Schwarzschild black holes is characterized by the remarkably simple constant spacing [31]

$$\Delta \omega = \varpi \tag{6}$$

between the corresponding frequencies of adjacent blackhole emission lines.

II. NATURAL BROADENING OF THE QUANTIZED BLACK-HOLE EMISSION LINES

In order to establish the discrete nature of the proposed radiation spectrum (4) of a quantized (3 + 1)-dimensional

Schwarzschild black hole, Bekenstein and Mukhanov [7] have analyzed the influence of the quantum phenomenon of natural broadening [5] on the widths of the black-hole emission lines. In particular, using the time-energy quantum uncertainty principle [5], BM [7] (see also [32,33]) have related the natural frequency broadening $\delta \omega$ of the black-hole quantum emission lines to the reciprocal of the characteristic finite lifetime τ of the black-hole *n*th energy (mass) level [5,34],

$$\delta \omega = \frac{1}{\tau}.\tag{7}$$

In the spirit of the Bohr correspondence principle [5], Bekenstein and Mukhanov [7] have further suggested to relate the characteristic lifetime τ of the *n*th energy (mass) level of a macroscopic [29] quantized (3 + 1)-dimensional Schwarzschild black hole to the reciprocal of the corresponding semiclassical emission rate which characterizes the black hole [1,35–38],

$$\tau = \left(\frac{dN}{dt}\right)^{-1}.$$
(8)

The sharpness of the black-hole emission spectrum (4) can be characterized by the dimensionless ratio $\delta\omega/\Delta\omega$ between the natural frequency width of the spectral lines [see Eq. (7)] and the characteristic frequency spacing between adjacent radiation lines [see Eqs. (5) and (6)]. In particular, discrete emission spectra are characterized by the strong inequality $\delta\omega/\Delta\omega \ll 1$, whereas emission spectra which are effectively continuous are characterized by the strong inequality $\delta\omega/\Delta\omega \gg 1$.

Interestingly, one finds [7,33] that the emission spectrum of a quantized (3 + 1)-dimensional Schwarzschild black hole is characterized by the relation [see Eq. (20) below]

$$\zeta(D=3) \equiv \frac{\delta\omega}{\Delta\omega} \ll 1. \tag{9}$$

As emphasized by BM [7], the small ratio (9) found for the dimensionless physical parameter $\zeta(D = 3)$ implies that the discrete eigenfrequencies (4), which according to Bekenstein [3] (see also [7–28]) are expected to characterize the radiation spectra of quantized (3 + 1)-dimensional Schwarzschild black holes, are unlikely to overlap.

III. THE SPECTRAL EMISSION LINES OF (D + 1)-DIMENSIONAL SCHWARZSCHILD BLACK HOLES AND THEIR NATURAL QUANTUM BROADENING

It is of physical interest to test the general validity of the intriguing conclusion reached by BM [7], according to which the quantum phenomenon of natural broadening has a negligible effect on the suggested discrete emission spectra (4) of quantized (3 + 1)-dimensional Schwarzschild black holes. In particular, one naturally wonders whether the strong inequality $\delta \omega \ll \Delta \omega$ [see Eq. (9)], which characterizes the emission spectra of quantized (3 + 1)-dimensional Schwarzschild black holes, is a generic property of *all* (*D* + 1)-dimensional quantized Schwarzschild black holes.

In order to address this physically interesting question, in the present paper we shall analyze the functional dependence of the dimensionless physical ratio

$$\zeta(D) \equiv \frac{\delta\omega(D)}{\Delta\omega(D)},\tag{10}$$

which quantifies the sharpness [39] of the black-hole quantum emission spectra, on the number D + 1 of spacetime dimensions.

It is worth emphasizing that, as extensively discussed in the literature (see [40-43] and references therein), higherdimensional physical theories with extra spatial dimensions provide intriguing candidates for self-consistent physical theories unifying the fundamental forces of nature. Interestingly, the suggested higher-dimensional physical theories [40–43] may provide an elegant resolution for the hierarchy problem observed in our Universe. Moreover, physical theories with extra spatial dimensions predict the formation of mini black holes in future high-energy accelerators [40-43]. Interestingly, these higher-dimensional mini black holes are expected to be characterized by quantum emission spectra. It is therefore of physical interest to explore the physical properties (and, in particular, the characteristic quantum emission spectra) of these predicted higher-dimensional black holes that will hopefully be observed in future man-made high-energy scattering experiments.

A. The emission spectra of quantized (D + 1)-dimensional Schwarzschild black holes

Interestingly, the quantization schemes presented in [3,9,44] suggest that higher-dimensional Schwarzschild black holes, like their (3 + 1)-dimensional counterparts, are expected to be characterized by evenly spaced discrete emission spectra of the form

$$\omega_k = k \cdot \boldsymbol{\varpi}(D); \quad k = 1, 2, 3, \dots, \tag{11}$$

where the fundamental radiation frequency of a quantized (D + 1)-dimensional Schwarzschild black hole is given by the compact physical expression [44,45]

$$\varpi(D) = \frac{\gamma T_{\rm BH}(D)}{\hbar}.$$
 (12)

Here [46,47]

$$T_{\rm BH}(D) = \frac{(D-2)\hbar}{4\pi r_{\rm H}} \tag{13}$$

is the characteristic Bekenstein-Hawking temperature of a (D + 1)-dimensional Schwarzschild black hole of horizon radius $r_{\rm H}$.

Note that the quantized radiation spectrum (11), suggested for macroscopic [29] (D + 1)-dimensional Schwarzschild black holes by the quantization schemes of [3,9,44], is characterized by the constant frequency spacing [31]

$$\Delta \omega(D) = \varpi(D) \tag{14}$$

between adjacent black-hole spectral lines.

B. Natural broadening and radiation fluxes of (D + 1)-dimensional Schwarzschild black holes

Following the physical procedure suggested by Bekenstein and Mukhanov [7] (see also [32,33]), we shall determine the natural frequency broadening $\delta\omega(D)$ of the spectral lines (11), which are expected to characterize the emission spectra of macroscopic (D + 1)-dimensional quantized Schwarzschild black holes, from the characteristic relation [see Eqs. (7) and (8)]

$$\delta\omega(D) = \frac{dN}{dt},\tag{15}$$

where dN/dt is the corresponding (D + 1)-dimensional semiclassical emission rate (that is, the number of quanta emitted per unit of time) of the higher-dimensional Schwarzschild black hole [1,35–38,48].

Below we shall consider the emission of massless gravitons and photons [1,35,49,50]. In particular, in the present analysis we shall assume that the radiating (D+1)dimensional Schwarzschild black holes are macroscopic in the sense that the emission of massive particles in the regime $\mu \cdot r_{\rm H} \gg D^2 \hbar$ is exponentially suppressed [35,49]. [Note, in particular, that an emitted massive particle can reach spatial infinity only if its proper energy satisfies the inequality $\hbar \omega \geq \mu$. The exponential factor $\omega^{D-1}/(e^{\hbar \omega/T_{\rm BH}}-1)$ that governs the radiation flux of a (D+1)-dimensional Schwarzschild black hole [see Eq. (16) below] implies that the corresponding radiation rate of massive quanta in the regime $\hbar \omega \ge \mu \gg D^2 \hbar / r_{\rm H}$ is exponentially suppressed]. Furthermore, it should be noted that had we considered an extended set of emitted particles (which includes massive particles along with massless fields), we would have found shorter lifetimes for the metastable (radiating) black-hole states. Thus, extending the family of emitted field modes would merely strengthen our final conclusion [see Eqs. (21) and (23) below] that, in the large-D regime, the quantum phenomenon of natural broadening [5] is expected to smear the characteristic emission spectra of radiating (D + 1)dimensional Schwarzschild black holes into a continuum.

The semiclassical radiation flux out of a (D + 1)dimensional Schwarzschild black hole for one massless bosonic degree of freedom is given by the expression [1,35,49,50]

$$\frac{dN}{dt} = \frac{1}{2^{D-1}\pi^{D/2}\Gamma(D/2)} \sum_{j} \int_{0}^{\infty} \Gamma \frac{\omega^{D-1}}{e^{\hbar\omega/T_{\rm BH}} - 1} d\omega, \quad (16)$$

where *j* denotes the angular harmonic parameters of the emitted field quanta. The absorption probabilities $\Gamma = \Gamma(\omega; j, D)$ (known as the black-hole-field greybody factors) [35] that appear in the integral relation (16) quantify the linearized interaction of the emitted field modes with the effective gravitational potential of the curved (D + 1)-dimensional Schwarzschild black-hole spacetime. These characteristic black-hole-field reflection coefficients are determined by solving a standard problem of wave scattering in the curved black-hole spacetime. In particular, the linearized interaction (scattering) of the emitted field modes with the curved black-hole spacetime is governed by the generalized (D + 1)-dimensional Regge-Wheeler equation [1,35,49,50]

$$\left(\frac{d^2}{dx^2} + \omega^2 - V\right)\phi = 0, \tag{17}$$

where the radial coordinate x is related to the Schwarzschild coordinate r by the differential relation $dx/dr = [1 - (r_{\rm H}/r)^{D-3}]^{-1}$. For a massless perturbation field of harmonic index l, the effective (D + 1)-dimensional black-hole curvature potential in (17) is given by the cumbersome expression [50]

$$V(r; D) = \left[1 - \left(\frac{r_{\rm H}}{r}\right)^{D-3}\right] \times \left[\frac{l(l+D-2) + (D-1)(D-3)/4}{r^2} + \frac{(1-p^2)(D-1)^2 r_{\rm H}^{D-2}}{4r^D}\right].$$
 (18)

Here one should take the values $p = \{0, 2, 2/(D-1), 2(D-2)/(D-1)\}$ for the distinct cases of gravitational tensor fields, gravitational vector fields, electromagnetic vector fields, and electromagnetic scalar fields, respectively [50] [It should be noted that the effective radial potential in the generalized (D + 1)-dimensional Regge-Wheeler equation (17) for the case of gravitational scalar perturbation fields is characterized by a rather complicated expression which is given in [50]].

IV. SHARPNESS OF THE (D + 1)-DIMENSIONAL SCHWARZSCHILD BLACK-HOLE EMISSION SPECTRA: NUMERICAL AND ANALYTICAL RESULTS

In the present section we shall explore the functional dependence of the dimensionless physical parameter $\zeta(D) \equiv \delta\omega(D)/\Delta\omega(D)$ [see Eqs. (14) and (15)], which

characterizes the sharpness [39] of the black-hole quantum emission spectra, on the number D + 1 of spacetime dimensions.

A. The (3+1)-dimensional Schwarzschild black hole

The total emission rate of massless gravitons and photons from a macroscopic (3 + 1)-dimensional Schwarzschild black hole is given by [35] $dN/dt \simeq 1.6 \times 10^{-4} M^{-1}$. Using the characteristic relation (15), one finds

$$\delta\omega(D=3) \simeq 1.6 \times 10^{-4} M^{-1}$$
 (19)

for the natural frequency broadening which characterizes the emission lines of the quantized (3 + 1)-dimensional Schwarzschild black holes.

Taking cognizance of Eqs. (5), (6), and (19), one finds that the emission spectra of quantized (3 + 1)-dimensional Schwarzschild black holes are characterized by the remarkably small dimensionless ratio [51,52]

$$\zeta(D=3) \equiv \frac{\delta\omega(D=3)}{\Delta\omega(D=3)} \simeq 4 \times 10^{-3} \ll 1.$$
 (20)

The extremely small value (20) found for the dimensionless physical parameter $\zeta(D = 3)$ implies that the quantum phenomenon of natural broadening has a negligible effect on the expected emission spectra of quantized (3 + 1)dimensional Schwarzschild black holes. In particular, as emphasized by BM [7], the characteristic strong inequality $\delta\omega(D = 3) \ll \Delta\omega(D = 3)$ implies that the discrete emission frequencies (4), which according to Bekenstein [3] (see also [7–28]) are expected to characterize the radiation spectra of quantized (3 + 1)-dimensional Schwarzschild black holes, are unlikely to overlap.

B. (D + 1)-dimensional Schwarzschild black holes: Intermediate D values

In the previous subsection we have seen that (3 + 1)dimensional quantized Schwarzschild black holes are characterized by a remarkably small value of the physical parameter $\zeta(D = 3)$. In the present subsection we shall explicitly prove that the fundamental dimensionless ratio $\zeta(D)$, which characterizes the sharpness [39] of the blackhole quantum emission spectra, is an increasing function of the spacetime dimension D + 1.

The emission rates of massless gravitons and photons from (D + 1)-dimensional Schwarzschild black holes were computed numerically in [50]. In Table I we display, for intermediate values of the black-hole spacetime dimension D + 1, the numerically computed values of the dimensionless physical parameter $\zeta(D) \equiv \delta \omega / \Delta \omega$ [see Eqs. (12)–(15)] [51]. The data presented in Table I reveal the intriguing fact that the physical parameter $\zeta(D)$, which quantifies the sharpness of the quantized (D + 1)-dimensional

TABLE I. The dimensionless physical parameter $\zeta(D) \equiv \delta\omega/\Delta\omega$ which quantifies the sharpness of the (D + 1)-dimensional Schwarzschild black-hole quantum emission spectra. Here $\delta\omega$ is the natural broadening of the black-hole quantum emission lines [see Eq. (15)], and $\Delta\omega$ [51] is the characteristic frequency spacing between adjacent emission lines of the quantized (D + 1)-dimensional Schwarzschild black holes [see Eqs. (12)–(14)]. One finds that the dimensionless physical parameter $\zeta(D)$ is an increasing function of the number D + 1 of spacetime dimensions. In particular, we find that higher-dimensional Schwarzschild black holes in the regime D = O(10) are characterized by the relation $\delta\omega/\Delta\omega = O(1)$ [53].

D + 1	5	6	7	8	9	10	11
$\zeta(D)\equiv\delta\omega/\Delta\omega$	0.046	0.151	0.310	0.645	1.036	2.153	3.639

Schwarzschild black-hole emission spectra, is an increasing function of the number D + 1 of spacetime dimensions.

In particular, one finds from Table I that higherdimensional Schwarzschild black holes in the regime $D \gtrsim 10$ are characterized by the inequality [53]

$$\delta \omega \gtrsim \Delta \omega \quad \text{for } D \gtrsim 10.$$
 (21)

The relation (21) strongly suggests that, due to the quantum phenomenon of natural broadening [5], the characteristic spectral lines (11) of quantized higher-dimensional Schwarzschild black holes are expected to be effectively blended together in the regime $D \gtrsim 10$.

C. (D + 1)-dimensional Schwarzschild black holes: The large-D regime

In the previous subsection we have used numerical data to reveal the interesting fact that the dimensionless physical parameter $\zeta = \zeta(D)$, which quantifies the sharpness of the black-hole quantum emission lines, is an increasing function of the number D + 1 of spacetime dimensions. In the present subsection we shall use analytical results in order to prove that this fundamental physical parameter is characterized by the asymptotic behavior $\zeta(D \gg 1) \gg 1$.

It has recently been demonstrated explicitly [54] that the semiclassical radiation spectra [1,35-38] of (D + 1)dimensional Schwarzschild black holes in the asymptotic large-*D* regime are described remarkably well by the eikonal (short-wavelength) approximation. In particular, using the geometric-optics approximation, one finds [54] the remarkably compact analytical formula

$$\frac{dN}{dt} \times r_{\rm H} = \frac{(4\pi)^2}{e} \left(\frac{D}{4\pi}\right)^{D+3} \quad \text{for } D \gg 1 \qquad (22)$$

for the dimensionless semiclassical radiation flux of a (D + 1)-dimensional Schwarzschild black hole in the asymptotic large-*D* regime.

Taking cognizance of Eqs. (12), (13), (14), (15), and (22), one deduces that the emission spectra of quantized

higher-dimensional Schwarzschild black holes in the large-*D* regime are expected to be characterized by the dimensionless asymptotic relation [55,56]

$$\zeta(D) = \frac{(4\pi)^2}{\gamma e} \left(\frac{D}{4\pi}\right)^{D+2} \quad \text{for } D \gg 1.$$
 (23)

The characteristic strong inequality $\delta \omega \gg \Delta \omega$ found in the large-*D* regime [see Eqs. (10) and (23)] strongly suggests that the quantum phenomenon of natural broadening [5] would effectively smear the corresponding radiation lines of these higher-dimensional Schwarzschild black holes into a continuum.

V. SUMMARY AND DISCUSSION

Following the highly influential work of Bekenstein [3], many researches (see [7–28] and references therein) have argued during the last four decades that, within the framework of a self-consistent quantum theory of gravity [57], black holes should be characterized by discrete radiation spectra with evenly spaced spectral lines [see Eq. (4)].

Furthermore, using the quantum time-energy uncertainty principle [5], Bekenstein and Mukhanov [7] have reached the important physical conclusion that the characteristic radiation spectra of quantized (3 + 1)-dimensional Schwarzschild black holes are expected to be sharp in the sense that the characteristic natural broadening $\delta \omega$ [see Eqs. (7) and (8)] of the black-hole quantum emission lines is much smaller than the characteristic spacing $\Delta \omega = O(T_{\rm BH}/\hbar)$ [see Eqs. (5) and (6)] between adjacent emission lines of the quantized black holes. It was therefore concluded by BM [7] that, for quantized (3 + 1)-dimensional Schwarzschild black holes, the characteristic discrete spectral lines (4), as predicted in [3,7–28], are unlikely to overlap.

One naturally wonders whether the strong inequality $\delta\omega(D=3) \ll \Delta\omega(D=3)$ [7], which characterizes the expected spectral lines (4) of quantized (3+1)dimensional Schwarzschild black holes, is a generic property of all quantized (D + 1)-dimensional Schwarzschild black holes? In order to address this physically interesting question, in the present paper we have studied the characteristic radiation spectra of (D+1)-dimensional quantized Schwarzschild black holes. In particular, we have analyzed the functional dependence of the characteristic dimensionless ratio $\zeta(D) \equiv \delta \omega / \Delta \omega$ on the spacetime dimension D + 1 of the quantized black hole. Interestingly, we have explicitly proved that the dimensionless physical parameter $\zeta(D)$, which quantifies the natural broadening (the sharpness) of the black-hole quantum emission lines, is an increasing function of the number D + 1 of spacetime dimensions (see Table I).

In particular, we have shown that the quantum emission lines of (D + 1)-dimensional Schwarzschild black holes in the regime $D \gtrsim 10$ are characterized by the dimensionless

ratio $\zeta(D) \gtrsim 1$ [see Eq. (21)]. Moreover, we have proved that the emission spectra of quantized (D + 1)-dimensional Schwarzschild black holes are characterized by the large-*D* asymptotic behavior $\zeta(D \to \infty) \to \infty$ [see Eq. (23)] [55]. These intriguing findings imply, in particular, that the characteristic emission lines of these higher-dimensional quantized black holes are effectively blended together.

The results presented in the present paper therefore suggest that, even if the underlying energy spectra of quantized (D + 1)-dimensional Schwarzschild black holes

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are fundamentally discrete, as argued by many authors [3,7–28], the quantum phenomenon of natural broadening [5] is expected to smear the characteristic emission spectra of these higher-dimensional black holes into a continuum.

ACKNOWLEDGMENTS

This research is supported by the Carmel Science Foundation. I thank Yael Oren, Arbel M. Ongo, Ayelet B. Lata, and Alona B. Tea for stimulating discussions.

relations $1 \le k \ll n$ [note that $n \gg 1$ for macroscopic black holes, see Eq. (1) and [29]].

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- [48] It should be noted that, in order to estimate the characteristic emission rate of a (3 + 1)-dimensional Schwarzschild black hole, BM [7] have used an approximated approach in which the radiating black hole was modeled as a perfect black-body emitter. In the present paper we shall consider the exact radiation fluxes of (D + 1)-dimensional Schwarzschild black holes, taking into account the partial reflection of the emitted field modes by the gravitational potential [see Eq. (18) below] of the curved black-hole spacetime.
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- [52] See [33] for the analysis of the natural broadening phenomenon which is expected to characterize the emission spectra of quantized rapidly rotating (3 + 1)-dimensional Kerr black holes.
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- [55] It is worth emphasizing the fact that, from the analytically derived expression (23), one finds the asymptotic behavior $\zeta(D \to \infty) \to \infty$ for the fundamental physical parameter ζ . Note, in particular, that this characteristic asymptotic behavior of the dimensionless ratio $\delta\omega/\Delta\omega$ is *independent* of the value of the dimensionless constant γ .
- [56] It is worth noting that the analytically derived asymptotic formula (23), which is based on the geometric-optics (eikonal) approximation in the asymptotic large-*D* regime, provides the correct order of magnitude for the dimensionless physical parameter $\zeta(D)$ already for D = 10. Specifically, one finds from (23) $\zeta^{\text{analytical}}(D = 10) = 3.746$ [51], which is a factor ~1.03 larger than the corresponding numerically computed value $\zeta^{\text{numerical}}(D = 10) \approx 3.639$ (see Table I).
- [57] The term "quantum theory of gravity" is used here to describe a symmetric self-consistent theory in which *both* the black hole and the fundamental fields are properly quantized. This should be contrasted with the fundamentally asymmetric nature of the seminal semiclassical analysis of Hawking [1], in which the fundamental fields that live in the curved black-hole spacetime are properly analyzed at the *quantum* level, but the black hole itself (and, in particular, its horizon) is treated as a fixed *classical* entity.