Hints of quantum gravity from the horizon fluid

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For many years, researchers have tried to glean hints about quantum gravity from black hole thermodynamics. However, black hole thermodynamics suffers from the problem of universality—at leading order, several approaches with different microscopic degrees of freedom lead to Bekenstein-Hawking entropy. We attempt to bypass this issue by using a minimal statistical mechanical model for the horizon fluid based on the Damour-Navier-Stokes (DNS) equation. For stationary asymptotically flat black hole spacetimes in general relativity, we show explicitly that, at equilibrium, the entropy of the horizon fluid is the Bekenstein-Hawking entropy. Further, we show that, for the bulk viscosity of the fluctuations of the horizon fluid to be identical to Damour, a confinement scale exists for these fluctuations, implying quantization of the horizon area. The implications and possible mechanisms from the fluid point of view are discussed.

Key questions in any theory of quantum gravity concern which operators to quantize and what is their spectrum?. Such a problem is prohibitively complicated for a full theory of quantum gravity. However, it is hoped that one can make progress when we restrict the spacetimes under consideration. Black holes are the most promising contenders to make such progress, as they are the some of the simplest spacetimes.

One promising inroad into this problem has been through black hole thermodynamics [1], which has shown that the horizon area is associated to the entropy [2–4] and suggested that this area could be the appropriate variable to quantize [5–7]. However, black hole thermodynamics now has the problem of universality [8]; at leading order, several approaches using completely different microscopic degrees of freedom lead to Bekenstein-Hawking entropy [9]. Currently, it is not possible to identify which are the true degrees of freedom that are responsible for the black hole entropy [10].

Alternatively, fluid and gravity correspondence—projecting the Einstein equations onto the black hole horizon leads to an equation similar in style to Navier-Stokes [11–21]—can provide a way to understand these black hole micro-states from the microscopic degrees of freedom of the horizon fluid. This is more interesting as we have a better understanding about the microscopic degrees of freedom of most fluid systems than gravity. More specifically, given a horizon fluid equation of state, it is possible to constrain the microscopic degrees of freedom and, hence, the problem of universality encountered in black hole thermodynamics can be curtailed [22–26].

Importantly, it was shown recently that modeling of the horizon fluid as a Bose-Einstein condensate (BEC) one may recover Bekenstein-Hawking entropy for the Schwarzschild, AdS, and Boulware-Deser black holes [26,27]. Using the theory of fluctuations [28], two of the current authors constructed [29] a statistical-mechanical description of the (number) fluctuations of the horizon fluid and explicitly showed that the coefficient of bulk viscosity for the horizon fluid corresponding to the four-dimensional Schwarzschild black hole matches exactly with the value found from the equations of motion for the horizon fluid [11]. It is important to note that while the entropy calculation [27] requires a model of the horizon fluid (i.e. BEC), the fluctuation dissipation analysis of the horizon fluid leading to the negative bulk viscosity [29] is independent of the details of the horizon fluid.

The Schwarzschild black hole is the simple obvious first test case [27,29]. Since the Schwarzschild solution depends on the one parameter (the black hole mass), it does not answer some of the important questions, such as when and why does the statistical mechanical picture of the horizon fluid work? Does the statistical mechanical picture of the horizon fluid and its fluctuations provide any condition on the microscopic structures of the fluid horizon? The purpose of this work is to answer these questions by extending the analyses of [27,29] to the *D*-dimensional Schwarzschild and Kerr-Newman black holes.

Using an appropriate quasilocal energy for the black hole [30–32], and constraint equations between the thermodynamic variables leads to the correct value of entropy of the fluid in the above spacetimes. We explicitly evaluate the number density fluctuations of the horizon fluid in the corotating frame in these spacetimes and show that in order for the bulk viscosity of the horizon fluid to match Damour's result [11] there must be a confinement scale i.e. fluctuations exist as bound states, as in diverse condensed matter models [33–35]. One of the important implications of the

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confinement scale is that the scale implies the quantization of black hole area as conjectured by Bekenstein [5–7]. To our knowledge, this is the first evidence that the fluid-gravity correspondence provides concrete evidence for the quantization of horizon area.

Horizon fluid constraint equations: Generalizing Damour's calculation [11] to D-dimensional spacetime (see Appendix A), and projecting onto the $(D-2)$-dimensional horizon [36], one recovers an equation similar to the Navier-Stokes equation describing a viscous fluid. This construction can be carried out for arbitrary null surfaces but the correspondence to the normal Navier-Stokes equation is exact for the stationary spacetimes considered here. The fluid has pressure (P),

$$P = \frac{\kappa}{8\pi} = \frac{k_B T}{4}, \tag{1}$$

where T is the Hawking temperature of the black hole. In this work, we set $c = G = \hbar = 1$ and therefore must keep k_B explicit, and assume that $T > 0$ corresponding to nonextremal black holes. The horizon fluid is special in that besides the above constraint (1), it further obeys an extra constraint relating the fluid energy and temperature [25].

The fluid has a naturally defined energy density, which we can integrate over the horizon area to find the total energy. This procedure bypasses the issue of choosing amongst notions of quasilocal energy [37–39], and naturally leads to a constraint equation relating E, T, and A. This energy is identical to one commonly used notion of quasilocal mass in the literature [30–32]. See Appendix B.

For the D-dimensional Schwarzschild [40], we have

$$E = M = \left(\frac{D-2}{D-3}\right)\frac{Ak_B T}{4}, \tag{2}$$

while the Kerr-Newman black hole, the quasilocal mass evaluated on the outer horizon, obeys [31]

$$E = \sqrt{M^2 - Q^2 - a^2} = \frac{1}{2}Ak_B T. \tag{3}$$

Note that for $D = 4$, Eq. (2) leads to Eq. (3).

Beside the constraints, the bulk viscosity of the horizon fluid is negative [11]. The coefficient of bulk (ζ) and shear viscosity (η), respectively, are given by

$$\zeta = -\left(\frac{D-3}{D-2}\right)\frac{1}{8\pi}; \qquad \eta = \frac{1}{16\pi}. \tag{4}$$

As mentioned earlier, two of the current authors [29] have systematically obtained negative bulk viscosity from the fluctuations of the horizon fluid. More importantly, it was shown that negative sign is due to the teleological nature of the horizon [12,13], as it leads to an anticausal response to infalling matter. Indeed, using a local trapping horizon

instead of teleological notion of horizon one recovers a positive bulk viscosity [41,42].

The above discussion clearly points that the horizon fluid corresponding to any stationary black hole is an odd system; the macroscopic parameters P, T, E and A (volume) are not independent. More importantly, the constraint equations (1), (2), (3) imply that the physical mechanism that drives the horizon fluid from an initial configuration, say, P_1, T_1, E_1, A_1 to final configuration P_2, T_2, E_2, A_2, cannot be arbitrary and, hence, the fluctuations of these macroscopic quantities are also constrained. As we will show, the constraint equations play a crucial role in the derivation of the entropy and the bulk viscosity. Setup: We list below the key ingredients in evaluating the physical quantities from the horizon fluid:

(1) The macroscopic properties of the horizon fluid satisfy the black hole constraints (1) and (2) [or (3)]. These macroscopic properties are specified by the N microscopic degrees of freedom.

(2) Following the results of [25,43], we assume that there exists some temperature T_c at which all N microscopic degrees of freedom are in the ground state, and that the system remains close to the critical point T_c [22,24,27].

(3) The total energy of the fluid of N particles is [25]

$$E \propto N\varepsilon \propto N/r_H = N\alpha k_B T, \tag{5}$$

where α is constant.

(4) The above energy should satisfy the constraint between energy E, A and T. Following (2) and (3), we have

$$E = Ak_B T/\gamma, \tag{6}$$

and γ is a dimension-dependent constant. From (5), we get

$$N = E/(\alpha k_B T) = A/(\gamma \alpha). \tag{7}$$

Before we proceed with the main calculations, we would like to stress the following points: First, our modeling of the horizon fluid is minimal. At equilibrium, N contains all the information about the horizon fluid and all the physical variables are related to N. As we will show, the entropy of the horizon fluid is proportional to N which leads to the correct Bekenstein-Hawking entropy. Second, to determine the transport coefficients of the horizon fluid we need to go beyond the equilibrium description i.e. study the fluctuations of the horizon fluid (δN) [29]. As we will show, using the theory of fluctuations [28], it is possible to relate the bulk viscosity to the auto-correlation function of δN. Horizon fluid entropy: An immediate consequence of the above setup is the analytical prediction of the entropy of the horizon fluid using Mean field theory [44]. Defining the order parameter (η) of the homogeneous fluid as

$$\eta = \sqrt{KN} \quad \text{K a positive constant}, \tag{8}$$

the thermodynamic potential is given by [44]

$$\Phi = \Phi_0 + a(P)(T - T_c)\eta^2 + B(P)\eta^4, \qquad (9)$$

where $a(P)$ and $B(P)$ are to be determined. Splitting into temperature-dependent and -independent parts and matching the potential $\Phi = -PA$ [45] as in [44], we relate P to T via (1) and equate the terms of order TA to get

$$a = -\frac{\gamma \alpha k_B}{4K}. \qquad (10)$$

Unusually compared to most phase transitions, the negative value of a (as K, α, and γ must all be positive) means that the system is in the ordered phase for $T > T_c$ an the disordered phase for $T < T_c$ [44]. In the ordered phase, Φ is a minimum for

$$\eta^2 = KN = \frac{a(T - T_c)}{2B}. \qquad (11)$$

Now, if we write the entropy in the disordered and ordered phase as S_0 and $S_0 + \Delta S$ respectively, one may argue that S_0 is generically small as in [27] (see Appendix D) and

$$\Delta S \equiv -\frac{\partial \Phi}{\partial T} = -aKN = k_B \frac{A}{4}. \qquad (12)$$

This is the first key result of this work and validates the modeling of the horizon fluid as a critical system. Further, we like to stress the following points regarding this result: (i) This is a generic result for any D-dimensional stationary (spherical or axisymmetric) black hole in general relativity, and (ii) one of the key results from black hole thermodynamics is that black holes in general relativity have an entropy $S = k_B A/4$. This is a key test for modeling the horizon fluid, which has proven difficult for several prior models [22–24]; the fact that this holds true for a large class of black holes is encouraging for the success of our model of horizon fluid. Bulk viscosity of horizon fluid: For the horizon fluid, the equilibrium state corresponds to the minimum of the thermodynamic potential (Φ). Transport phenomena in a fluid can be viewed as nonequilibrium processes occurring within the fluid, where the deviation from equilibrium is small. The advantage of this approach is that it is minimalistic—indeed we may henceforth discard the assumptions about the existence of a phase transition (See Appendix E) Using the theory of fluctuations [28], transport coefficients can be related to autocorrelation function of number density fluctuations (δN). Following Kubo [28], the coefficient of bulk viscosity is given by

$$\zeta = \left(\frac{1}{n}\right)\frac{1}{Ak_BT}\int_{-\infty}^{\infty}dt\sum_a\sum_b\langle J^{aa}(0)J^{bb}(t)\rangle, \qquad (13)$$

where, $n = \text{Tr}(\delta_{ab})$, a, b run from $1, \cdots (D-2)$ and the current J^{ab} is

$$J^{ab} = \delta_{ab}\delta(PV) = V\delta P\delta_{ab}. \qquad (14)$$

We can write this as an entropic force—moving the system back to equilibrium—which, for the horizon fluid, takes the form

$$F_{\text{Th}} = PA\delta A\theta(-t), \qquad (15)$$

where $\theta(t)$ is the theta function, and enforces the anticausal, teleological nature of the horizon (for a detailed discussion, see [29]). The bulk viscosity can therefore be rewritten as

$$\begin{aligned}\zeta &= \frac{1}{Ak_BT}\int_{-\infty}^{\infty}dt\langle F_{Th}(t)F_{Th}(0)\rangle \\ &= \frac{(\alpha\gamma)^2 k_BT}{16A}\int_{-\infty}^{\infty}dt\langle\delta N(t)\delta N(0)\rangle\theta(-t),\end{aligned} \qquad (16)$$

using (15), (1) and (5). Since our interest lies in the long wavelength (fluid) limit, we may evaluate the viscosity from the linear response of the horizon fluid [28]:

$$\zeta = \lim_{\epsilon\to 0}\Im\left[\frac{(\alpha\gamma)^2 k_BT\langle\delta N^2(0)\rangle}{16A}\int_{-\infty}^{\infty}dt\exp[i(\omega - i\epsilon)]\theta(-t)\right], \qquad (17)$$

leading to

$$\zeta = -\frac{(\alpha\gamma)^2 k_BT}{16A}\frac{\langle\delta N^2(0)\rangle}{\omega}, \qquad (18)$$

where ω corresponds to the lowest energy mode of the fluctuations that the horizon fluid support and sustain. Assuming that the fluctuations satisfy Maxwell-Boltzmann statistics, we get

$$\langle\delta N^2(0)\rangle = \frac{4A}{(\gamma\alpha)^2}, \qquad (19)$$

and the bulk viscosity simplifies to

$$\zeta = -\frac{k_BT}{4\omega}. \qquad (20)$$

As this is the second main result of this work we would like to stress the following points: (i) Bulk viscosity is independent of the constants α, γ; these can only be determined with the knowledge of the microscopic theory. This is consistent as the fluid description does not require complete knowledge of the microscopic degrees of freedom. (ii) Bulk viscosity depends on the horizon fluid temperature and the lowest energy mode of the fluctuations. To go about determining the lowest energy mode, it may be important to get a physical insight. Let us consider fluctuations that cause a change in the horizon fluid area from A to $A + dA$. For the horizon fluid corresponding to Schwarzschild black hole, the change

in area is only due to the change in the quasilocal energy. In Ref. [29], this was done by picking the largest wavelength to be the circumference of the black hole. However, for generic black holes like the Kerr-Newman, the change in the area can also be due to the change in the electrostatic energy or quasilocal energy $\kappa \times dA$ [39]. Using the fact that the lowest energy modes are adiabatic (slowly evolving), and that the energy and area are strongly constrained, the minimum energy change can be related to a minimum change in area through (5) and (7):

$$\Delta E = (\Delta N)\alpha T \propto (\Delta A)T, \qquad (21)$$

and, thus, using (3) the minimum energy mode for any stationary black hole is

$$\omega = \Delta E = \left(\frac{D-2}{D-3}\right)\Delta A \frac{T}{4}. \qquad (22)$$

This brings us to the key result of this work and indicates that the existence of the lowest energy mode of fluctuation (δN) of the horizon fluid relates to the minimum change or quantization of the horizon area. We will now make several comments about the result and embed them in a broader context.

First, the area quantization has long been a key result of both horizon thermodynamics and various quantum gravity proposals. The first attempt goes back to Bekenstein [3], who uses the fact that particles entering the black hole cannot have zero size, to find the minimum area increase when one is absorbed by a Kerr-Newman black hole. This result is very general [25], and in D dimensions leads to

$$\Delta A_{\min} = 8\pi \ell_P^{D-2}, \qquad (23)$$

where we have briefly reinstituted an explicit Planck length (ℓ_P) for clarity. Substituting Bekenstein's minimum area in (22), we get

$$\omega = 2\pi \left(\frac{D-2}{D-3}\right) k_B T. \qquad (24)$$

Substituting the above form of minimum energy mode in (20), we get

$$\zeta = -\left(\frac{D-2}{D-3}\right)\frac{1}{8\pi}. \qquad (25)$$

It is important to note that this is the expression for bulk viscosity for all asymptotically flat spacetimes in all dimensions and matches exactly the expression from the DNS equation (see Appendix A). This is a *nontrivial* result, using fluctuation-dissipation [28] and the fact that the horizon is anticausal [29], so it is not possible to obtain the known bulk viscosity without invoking area quantization.

Second, so far, it has not been necessary to place a value on α. However, if we take $\Delta N_{\min} = 1$ using (23),

$$\alpha = 8\pi, \qquad (26)$$

corresponding to [3]. Naturally, there have been several alternative proposals from counting black hole microstates or Bohr's principle with quasinormal modes [46,47], and arguments from loop quantum gravity [48], for various values of α. Note that all these proposals give answers of the same order of magnitude (see [49,50]).

Third, why should the existence of the long wavelength mode imply quantization of the horizon area? In other words, why does the IR cutoff (lowest energy mode) have anything to do with the Planck-scale physics? One of the main assumptions in our work is that the horizon fluid is described by N microscopic degrees of freedom. While we do not have any information about these microscopic structures, existence of the largest length scale of the fluctuation λ leads to the fact that $\lambda^{D-2} \simeq N\ell_P^{D-2}$ where ℓ_P physically refers to the separation of the microscopic structures or the size of the structures themselves. In the language of Wheeler's "It from Bit" [51], bits are the N microscopic structures of the horizon fluid.

Fourth, why should the minimum area condition give a minimum ω from the horizon fluid? Consider (5), with α as given in (26). Adding one extra particle to the ground state of the condensate is the minimum energy we can add to the system, and corresponds exactly to adding a minimum of extra area. As this energy is larger than or equal to the energy of the mode spanning the circumference of the horizon, the adding of one extra horizon fluid particle is the minimum energy. This physically motivates picking the proportionality factor as unity between ω and ΔE in Eq. (22).

Fifth, what does the minimum energy mode physically correspond to in the horizon fluid? The existence of a minimum energy mode implies a confinement scale; i.e. density fluctuations exist as bound states [33–35]. In other words, this implies that to add one microscopic constituent to the horizon fluid will cost that much energy. In the language of statistical mechanics, the minimal energy mode can be treated as a chemical potential corresponding to the excitations in the fluid. In Appendix C, we have constructed an explicit model that shows $\mu \propto T$.

Sixth, the status of the third law is unclear in black hole thermodynamics: while the process version of the third law seems to hold, the Planck version, stating that the entropy goes to zero as the temperature does, clearly does not hold for black holes [9]. It naturally follows from the existence of a positive chemical potential (see Appendix C) that the system retains a residual entropy, analogous to that of other condensed matter systems, and probably implying a degeneracy in the ground state of the system. The large entropy of black holes at low temperature can possibly be viewed as residual entropy.

Finally, the fact that the horizon fluid has a confinement scale signals the occurrence of a mass gap. There has been a

long history of possible connection between the general relativity and Yang-Mills theory, in which mass gap is known to occur. Specifically, it has been shown that four-graviton amplitudes with one loop in Einstein gravity are similar to products of integrands appearing in gauge theory [52–54]. We suggest here that it might be possible to understand the connection to Yang-Mills through the presence of a mass gap for the horizon fluid.

There are a number of ways in which one could extend and generalize this result: These calculations were carried out using the natural split between space and time given to us by the fluid picture, but one can make alternative choices, and it may be elucidating to check what changes in the analysis when performed in a different frame. Possible extensions to other theories of gravity, including supersymmetric theories where we have a better understanding of microscopic degrees of freedom could be elucidating. Further exploration into the nature of the fluid and the mechanism generating the IR cutoff may provide further avenues for such insights. The statistical mechanical picture of the horizon fluid developed over the past few years can provide a new and exciting window into the nature of gravity, and hints towards quantum gravity.

Coming back to the problem of universality, it is clear that reproducing the black hole entropy is only one test of a microscopic model of black hole physics, and that the viscosities in the DNS equation can help us choose between the multitude of different scenarios.

ACKNOWLEDGMENTS

APPENDIX A: DAMOUR-NAVIER-STOKES EQUATION IN ARBITRARY DIMENSIONS

Closely following the procedure adopted in [17], take a D-dimensional spacetime. We will use small latin letters for indices running over all D dimensions, greek letters for $(D-1)$-dimensional null surface, and capital latin for $(D-2)$ spacelike dimensions.

Start with the Einstein equations,

$$R_{ab} - \frac{1}{2}g_{ab} = 8\pi T_{ab}, \tag{A1}$$

and consider a null surface, which is therefore traced out by null geodesics. These are described by the geodesic equation (in this case with a nonaffine parametrization),

$$l^a \nabla_a l_b = \kappa l_b. \tag{A2}$$

We can, without loss of generality, choose coordinates such that

$$l = \partial_t + v^A \partial_A; \qquad l^a = (1, v^A, 0). \tag{A3}$$

We can also, for convenience, construct another null vector, k, such that $k \cdot l = -1$.

The metric on the $(D-2)$-dimensional surface, denoted by q_{AB}, for which

$$ds^2 = q_{AB}(dx^A - v^A dt)(dx^B - v^B dt),$$
$$q_{ab} = g_{ab} + l_a k_b + l_b k_a. \tag{A4}$$

It can explicitly be seen that

$$q_{ab} l^b = q_{ab} k^b = 0. \tag{A5}$$

The Damour-Navier-Stokes (DNS) equation is a consequence of the contracted Codazzi equation formed with l and q_{AB},

$$R_{mn} l^m q_a^n \equiv R_{mA} l^m = \left(\frac{1}{2}g_{ab} + 8\pi T_{ab}\right) l^m q_a^n$$
$$= 8\pi T_{ab} l^m q_a^n. \tag{A6}$$

Here the last equality is through (A5). We rewrite the lhs of (A6) as

$$R_{mA} l^m = R_{\mu A} = \nabla_\mu (\nabla_A l^\mu) - \partial_A (\nabla_\mu l^\mu). \tag{A7}$$

We expand both terms of the rhs of (A7) in terms of expansion, shear and the velocity v_A.

Define

$$\chi_\alpha^\beta = \nabla_\alpha l^\beta \tag{A8}$$

and

$$\omega_\alpha = \chi_\alpha^0, \tag{A9}$$

which is the energy-momentum vector of the horizon fluid.

As in equation (21) of [17] we expand out the second term of the rhs,

$$\partial_A(\nabla_\mu l^\mu) = \nabla_A l^A + \nabla_0 l^0 = \theta + \omega_A v^A + \omega_0 = \theta + \kappa, \tag{A10}$$

and is independent of D.

The other term of equation (A7),

$$\nabla_\mu(\nabla_A l^\mu) = \nabla_\mu \chi_A^\mu, \tag{A11}$$

can be evaluated by taking a frame where we neglect Christoffel symbols (see discussion in [17]), so that

$$\nabla_\mu \chi_A^\mu = \partial_\mu \chi_A^\mu = \partial_0 \omega_A + \partial_B \chi_A^B. \tag{A12}$$

Now use the fact that

$$\chi_{AB} = \Theta_{AB} + \omega_A v_B, \tag{A13}$$

and we can further split Θ_{AB} into trace and traceless parts

$$\Theta_{AB} = \sigma_{AB} + \left(\frac{1}{D-2}\right)\theta\delta_B^A. \tag{A14}$$

Note the dimensional-dependent prefactor, ensuring $\mathrm{tr}(\Theta) = \theta$.

Putting all this together into Eq. (A7),

$$R_{mA}l^m = (\partial_0 + v^B\partial_B)\omega_A - \partial_A(\kappa + \theta)$$
$$+ \partial_B\left(\sigma_A^B + \frac{1}{D-2}\theta\delta_A^B\right), \tag{A15}$$

so that

$$8\pi T_{mA}l^m = (\partial_0 + v^B\partial_B)\omega_A + \partial_B\sigma_A^B - \partial_A\kappa - \frac{D-3}{D-2}\partial_A\theta \tag{A16}$$

or, equivalently,

$$-\frac{(\partial_0 + v^B\partial_B)\omega_A}{8\pi} = -\frac{\partial}{\partial x^A}\left(\frac{\kappa}{8\pi}\right) + 2\frac{1}{16\pi}\sigma_{A|B}^B - l^a T_{aA}$$
$$- \left(\frac{D-3}{D-2}\right)\frac{1}{8\pi}\frac{\partial\theta}{\partial x^A}, \tag{A17}$$

where $-\omega_A/(8\pi)$ is the momentum density of the fluid.

We, thus, arrive at a Navier-Stokes equation with pressure

$$P = \frac{\kappa}{8\pi} = \frac{k_B T}{4}, \tag{A18}$$

shear viscosity

$$\eta = \frac{1}{16\pi}, \tag{A19}$$

and bulk viscosity

$$\xi = -\left(\frac{D-3}{D-2}\right)\frac{1}{8\pi}. \tag{A20}$$

As noted in [17], there are some differences between the DNS and the usual Navier-Stokes (NS) equation in the case of a general null surface. Firstly, the shear vector is not constructed solely from the velocity of the fluid, having an additional term. In the case of a stationary horizon, the extra term, $\partial_0 q_{AB}$ goes to zero and the shear tensor has the usual form. Secondly, in the DNS equation there is a Lie derivative rather than a convective derivative in the normal fluid case, consisting of an additional term of the form $\omega_B D_A v^B$. Again, for stationary spacetimes, this term goes to zero as the velocity is a constant.

In deriving (A17), we have worked with a particular coordinate system. One may wonder how the equation is affected when changing to another coordinate system, in particular changing to a different time coordinate. Examining (A17), one may see that the difference is that the first term $(\partial_0 + v^B\partial_B)\omega_A$ will change. As long as we work with the class of transformations such that we do not introduce an explicit time dependence in the definitions of the spacelike coordinates, v^A will remain constant, and the $\omega_B D_A v^B$ term will not reappear. The DNS equation will transform in the same way as changing coordinates in the NS equation, in a similar manner to looking at a normal laboratory fluid in a boosted frame.

We could consider this from a different angle, and ask, for a general null surface, is our analysis of the bulk viscosity coefficient independent of the boost frame chosen? We answer here the question in the affirmative and sketch an argument below why. For processes concerning only the bulk viscosity, we need only consider the part of the free energy that comes from its volume. Thus the part of the free energy that is dependent on the velocity of the horizon fluid does not play any role in the processes solely involving changes in the bulk of the fluid. The extra term that comes due to going over to a boosted inertial frame, is for a general null surface, $\omega_B D_A v^B$, but this term does not affect calculations relating to the bulk viscosity. The change in the volume of the cross section of the horizon does affect ω. This can be seen from the fact that the rate of the change of the volume, the scalar θ is given by the divergence of the null normal to the horizon. This means as we boost the frame, ω can change but θ would remain the same. Therefore the term in DNS equation, which makes it different from the standard Navier-Stokes equation need not be considered when looking at processes involving change in the bulk of the fluid only. Thus whichever frame we do the calculation in, it would give us the same result as when we calculated the free energy in the frame comoving with the horizon fluid. So our analysis does not depend on the boost.

APPENDIX B: CONSTRAINT EQUATIONS AND HORIZON FLUID ENERGY

To understand thermodynamic relations for the horizon fluids, we must first define the appropriate notion of the fluid energy. There are several definitions of energy in general relativity [37–39]. Here we want a quasilocal notion to associate to the black hole horizon. Various competing definitions exist, but our task is simplified by the fact that we have a naturally defined fluid energy density on horizon, given by $\frac{\omega_0}{8\pi}$, where

$$\omega_0 \equiv \nabla_0 l^0 = \kappa \tag{B1}$$

and the equality follows from (A10), noting that as the $v_A\omega_B$ is antisymmetric, its trace is automatically zero.

The total energy of the horizon is now simply

$$E = \frac{1}{8\pi}\int \kappa \mathrm{d}A, \tag{B2}$$

where both κ and A are functions of all the black hole variables (M, Q, a). This gives the natural value of M in the D-dimensional Schwarzschild.

In fact, from Eqs. (12.5.33)–(12.5.37) in [39], one can see that this is exactly

$$E = \frac{1}{8\pi}\int \kappa \mathrm{d}A = \frac{1}{8\pi}\oint_S \mathrm{d}S_{\mu\nu}\nabla^\mu l^\nu; \qquad l = \partial_t + v^A \partial_A, \tag{B3}$$

when v_A is the rotational velocity at the horizon, making l the standard combination considered when evaluating, e.g. the surface gravity of black holes. This corresponds to a quasilocal mass frequently used in the literature [30,31], evaluated on the horizon. This is almost equal to the well-known Komar mass, the difference being the replacement $\partial_t \to l$. Physically this corresponds to a quasilocal energy in the corotating frame [32], the appropriate choice for the fluid that co-rotates with the horizon, as ours does.

Explicitly, for the Kerr-Newman we find

$$E = \frac{r_+ - r_-}{2}. = \sqrt{M^2 - Q^2 - a^2} \tag{B4}$$

The black hole is an odd system in many ways, as P, T, E and A are not independent. Instead they obey $P = T/4$ and an extra constraint equation. The form of this equation varies depending on the class of black holes. We derive this relation for the D-dimensional Schwarzschild black hole and the four-dimensional Kerr-Newman black hole.

1. *D*-dimensional Schwarzschild

The D-dimensional Schwarzschild, also known as a the Schwarzschild-Tangherlini black hole [40], has the form

$$\mathrm{d}s^2 = -\left(1 - \left(\frac{r_H}{r}\right)^{D-3}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1 - (\frac{r_H}{r})^{D-3}} + r^2 \mathrm{d}\Omega_{D-2}^2, \tag{B5}$$

where

$$\Omega_{D-2} = \frac{2\pi^{\frac{D-1}{2}}}{\Gamma(\frac{D-1}{2})}. \tag{B6}$$

The horizon "area" is now given by

$$A = \Omega_{D-2} r_H^{D-2}, \tag{B7}$$

and the standard temperature is

$$T = \frac{D-3}{4\pi k_B r_H}. \tag{B8}$$

Here the horizon radius is related to the mass of the black hole by

$$r_H = \left(\frac{16\pi M}{(D-2)\Omega_{D-2}^2}\right)^{\frac{1}{D-3}}. \tag{B9}$$

This M is the energy of the horizon fluid, so (B7)–(B9) combine to give a constraint equation

$$E = \left(\frac{D-2}{D-3}\right)\frac{k_B A T}{4}. \tag{B10}$$

2. Kerr-Newman

The charged, rotating black hole,

$$ds^2 = -(\mathrm{d}t - a\sin^2\theta \mathrm{d}\phi)^2 \frac{\Delta}{\rho^2} - \left(\frac{\mathrm{d}r^2}{\Delta} + \mathrm{d}\theta^2\right)\rho^2 + ((r^2 + a^2)\mathrm{d}\phi - a\mathrm{d}t)^2 \frac{\sin^2\theta}{\rho^2} \tag{B11}$$

where

$$\rho^2 = r^2 + a^2\cos^2\theta; \qquad \Delta = r^2 - 2Mr + Q^2 + a^2. \tag{B12}$$

has inner and outer horizons given by

$$r_\pm = M + \sqrt{M^2 - Q^2 - a^2} \tag{B13}$$

We will only be concerned by the physically relevant outer horizon.

The horizon area and black hole temperature can be easily seen to be

$$A = 4\pi(r_+^2 + a^2); \qquad T = \frac{1}{2\pi}\frac{r_+ - r_-}{2k_B(r_+^2 + a^2)}. \tag{B14}$$

And the energy on horizon is [31]

$$E = \frac{r_+ - r_-}{2}. \tag{B15}$$

Combining these results we can see that

$$E = \frac{1}{2}k_B A T. \tag{B16}$$

Note that this energy is associated here first and foremost to the horizon fluid, rather than the energy contained within the horizons, and therefore different to the similar relation in [36]. This is identical to the expression for the

four-dimensional expression for the D-dimensional black hole, though the forms of A and T are very different. These constraint equations play a crucial role in the derivation of the entropy and the bulk viscosity.

APPENDIX C: CHEMICAL POTENTIAL AND IR CUTOFF FOR THE DENSITY WAVES IN THE HORIZON FLUID

Here we shall point out that there is another way to view the physical condition imposed by the IR cutoff in the frequency of the perturbations. Consider density wave excitations, which are bosons. The presence of such a cutoff signals the existence of a positive chemical potential, μ for these excitations. In fact the presence of a positive chemical potential ensures that there can be no excitation with frequency Ω such that $\Omega < \mu$. This can be seen by considering the expression for the occupation number of such excitations with frequency Ω. Let us denote the occupation number by n_Ω. Then, for degeneracy g_Ω,

$$n_\Omega = \frac{g_\Omega}{\exp(\mu + \beta\Omega) - 1}. \tag{C1}$$

It is seen from (C1) that $n_\Omega > 0$ only if $\Omega > \mu$. In our case, $\mu = \Omega_{IR}$, hence density waves have a frequency minimum given by that value.

There are two important physical implications of the fact $\mu \propto T$ for the horizon fluid.

(i) The fluctuations in the area of the horizon fluid are quantized [44]. To see how this comes about, let us note that from thermodynamic relations, one can write

$$\mu\delta n = -T\delta S. \tag{C2}$$

If $\mu = CT$, then we have, $\delta S = C\delta n$. Since, $\delta S = \frac{1}{4}\delta A$, it follows that

$$\delta A = 4C\delta n, \tag{C3}$$

which tells us that area is quantized in terms of an unit, whose value is given by $4C$. Here, C is positive. From (C2), we see that this makes $\delta S < 0$.

The change in entropy is negative because we are dealing with the system moving towards equilibrium, the higher entropy state, only obtaining equilibrium in the future. So if there are fluctuations in the system, i.e. it is excited, then it has less entropy. This shows us that the area quantization can also be viewed as a consequence of $\Omega_{IR} \propto T$.

(ii) The entropy density of the perturbations of the horizon fluid has a constant term. Like the previous result, this one is also a direct consequence of $\mu \propto T$. A similar case is that of liquid helium He^3 close to zero temperature (see chapter 21, in [55]). This

constant entropy term is referred to as the residual entropy in the literature [56]. Typically it occurs because at very low temperatures, the systems have degeneracy at the configurational level [57,58]. Well-known systems with this property are the carbon monoxide gas, where the residual entropy is present due to a degeneracy in the molecular arrangement [56], and the liquid helium He^3, where the residual entropy results due to the disordered orientation of the nuclear spins of He atoms [55]. In analogy with the known cases, residual entropy can be thought of as due to some unknown configurational degeneracy in the horizon fluid. These configurations correspond to unknown physics presumably corresponding to some kind of underlying discrete structure of black holes.

APPENDIX D: ENTROPY OF THE DISORDERED PHASE

This discussion will follow very closely that of [27]. The horizon fluid has a low energy in the disordered phase, as in this phase $T < T_c$. For simplicity, we assume that the system can be described by a scalar field, ϕ, but this is not a key feature of the derivation.

In the disordered phase, the partition function can be expressed by

$$\begin{aligned} Z &= \int_{p_{Low}} D\phi \exp\left[-\frac{\beta}{2}\int d^2x\phi(x)(-\nabla^2 + m^2)\phi(x)\right] \\ &= \text{Det}[\beta(-\nabla^2 + m^2)]^{-1/2}, \end{aligned} \tag{D1}$$

where p_{Low} indicates we are only integrating over low-momentum modes. Standard calculations lead to

$$\ln Z = A\int_{p_{Low}} \frac{d^2p}{(2\pi)^2}\ln[\beta(-\nabla^2 + m^2)]. \tag{D2}$$

If p_{Low} is zero, this entropy would also be zero. However, this will not be zero, due to the presence of the minimum area change. Then

$$\ln Z = A\ln(m^2\beta)\frac{p_{min}^2}{(2\pi)^2} \approx A\ln(\beta)\frac{p_{min}^2}{(2\pi)^2}. \tag{D3}$$

Now, using $p_{min} = \omega$

$$\ln Z = A(k_BT)^2\left(\frac{D-2}{D-3}\right)^2\ln\left(\frac{1}{k_BT}\right), \tag{D4}$$

which is small. In the case of Kerr, for instance, when $a \ll M$, $\ln Z \sim \ln(r_+)$ as in the Schwarzschild case, while for $a \approx M$, T^2 dominates over $\ln(1/T)$ and S_0 goes to zero.

APPENDIX E: MODEL-INDEPENDENT DERIVATION OF COEFFICIENT OF BULK VISCOSITY

In the main body of the paper, we have explicitly used $N \propto A$, which is a natural consequence of the horizon fluid being in its ground state. Here we will show that the fluctuation-dissipation analysis of the bulk viscosity of the horizon fluid is independent of such details, and of the microscopic the structure of the fluid, as had been noted in [29].

The horizon fluid has two constraints, between P and T; and relating E, A and T. We see explicitly that the volume of the horizon fluid contributes to its free energy. For the processes corresponding only to the changes in the bulk, only this part of the free energy need be considered, so only this part will concern us. Thus, we can make the choice to go over to the rest frame of the fluid and write down the free energy relevant for the bulk viscosity. It is important to note that the analysis can be performed where the 'kinetic' energy of the fluid constituents also contributes to the total energy E (as discussed in section I), as we only need the relevant part of the energy.

For the highly constrained system of the horizon-fluid, it turns out that (P, E, T, A) are all related to each other to the extent that the free energy can be expressed in terms of the variation of a single variable. This univariability of the horizon fluid was seen for the Schwarzschild black hole in [29] and has now been found to persist in the generalizations made here. The variable A is a good physical choice and the free energy of the horizon fluid can be expressed in its terms. For example, this is what we have done in the specific model for the fluid, and can be seen from the relations

$$\Phi = \Phi_0 + a(P)(T - T_c)\eta^2 + B(P)\eta^4; \qquad \text{(E1)}$$

$$\eta = \sqrt{\frac{k}{2\alpha}}\sqrt{A}. \qquad \text{(E2)}$$

Here a, B are parameters as defined in the main body of the paper.

To describe transport processes, we need to consider the fluid at a state slightly away from equilibrium, the minimum value of the free energy. We can express the fluctuation in free energy as

$$\delta\Phi = \frac{1}{8}(T - T_c)\frac{\delta A^2}{A}. \qquad \text{(E3)}$$

Here we note that whichever variable we choose to express the free energy in, the lowest order fluctuation in the free energy of the horizon fluid has to be quadratic in that variable. This quadratic dependence, $\delta\Phi \propto \delta X^2$ is the key relation we need and does not depend on the specific relation of (E2), merely coming from the fact that the free energy is proportional to the convenient variable we are working with. Now any fluid is composed of microscopic dof, whose total number, N however, can be treated as a macroscopic variable. So it is possible to express the free energy in terms of N as well. Hence the change in the free energy, $\delta\Phi$ is quadratic in δN, the fluctuation in the variable N. In that case, it follows that $\delta N \propto \delta A$. It is to be emphasized here that we obtain this result here without assuming any proportionality between N and A as in the case of the BEC model of the horizon fluid.

One can now proceed in the same way as shown in the paper and evaluate the coefficient of bulk viscosity as shown in the main body of the paper.

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