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Van der Waals like behavior of topological AdS black holes in massive gravity

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Motivated by recent developments in black hole thermodynamics, we investigate van der Waals phase transitions of charged black holes in massive gravity. We find that massive gravity theories can exhibit strikingly different thermodynamic behavior compared to that of Einstein gravity, and that the mass of the graviton can generate a range of new phase transitions for topological black holes that are otherwise forbidden.

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I. INTRODUCTION

Understanding the quantum behavior of gravity could be related to the possible mass of the graviton. The consistency of including this in the context of extending general relativity has been a long-standing basic physical question of classical field theory. Although the primitive linear theory of massive gravity [1] contains Boulware-Deser ghost modes [2], a nonlinear generalization that is ghost-free to all orders has recently been constructed [3] by de Rham, Gabadadze, and Tolley (dRGT massive theory). Note that dRGT is stable, enjoys the absence of a Boulware-Deser ghost [4], and has yielded a number of intriguing results in terms of its cosmological behavior and black hole solutions [5]. The mass terms are produced by consideration of a reference metric, which plays a crucial role in the construction of dRGT [6]. Motivated by applications of gauge-gravity duality, Vegh proposed a new reference metric in which the graviton behaves like a lattice excitation and exhibits a Drude peak [7]. This theory is also ghost-free and stable [8], and $d-$dimensional ($d \geq 3$) black hole solutions in the presence of linear and nonlinear electrodynamics with van der Waals-like behavior have been obtained [9]. Higher curvature generalizations have also been constructed [10]. Although some classes of nonlinear massive gravity theories are Lorentz violating and bear a close relation to Horava-Lifshitz gravity [11], it was shown that there are Lorentz-invariant versions of nonlinear massive gravity as well [3]. Massive gravity is also motivated by observation. Obtaining an empirical upper limit on the mass of graviton an outstanding challenge (for more details see [12]), one that should soon be attainable once recent Laser Interferometer Gravitational Wave Observatory (LIGO) results [13] are improved and expanded. In this regard, one may use the results of Refs. [14] and [15] to obtain a bound on the energy flux emitted from a binary pulsar and on the propagation speed of the graviton.

Here, we consider a class of dRGT theories, which we regard as the minimal modification of general relativity that yields a massive graviton [3]. We demonstrate that black holes of nonspherical topology can exhibit van der Waals phase transitions in dRGT-like gravity. Such transitions are forbidden in standard Einstein gravity and also higher curvature theories such as Lovelock gravity.

The study of black hole thermodynamics began with the pioneering work of Hawking and Page [16] that indicated anti–de Sitter (AdS) black holes can undergo phase transitions. Asymptotically AdS black holes have been of special interest since they admit a gauge-gravity duality description, and their thermodynamics plays a crucial role in constructing a consistent theory of quantum gravity [17]. Indeed, this duality can be applied to a qualitative study of the behavior of various condensed matter phenomena [18]. Substantial progress was recently made when van der Waals behavior of asymptotically AdS charged black holes was observed [19]. Based on the canonical ensemble, a small-large AdS black hole phase transition analogous to the liquid-gas phase transition in a thermodynamic system was discovered. A number of significant results were subsequently obtained, including the existence of triple points [20], reentrant phase transitions [21], and analogous Carnot-cycle heat engines [22]. These properties established a connection between black hole thermodynamics and everyday "chemical" thermodynamics, known as "black hole chemistry." This likewise triggered investigations into the implications for gauge-gravity duality, such as holographic superconductors [23], the Kerr/CFT correspondence [24], and holographic entanglement entropy [25] (see [26] for a review).

However, van der Waals behavior and its applications in Einstein gravity are seen only for AdS black holes with spherical horizon topology [19]; no such behavior takes place for AdS black holes with flat or hyperbolic horizons (no real critical point). Such reports motivate one to look

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for an extension of Einstein gravity to a modified version, in which van der Waals behavior is seen not only for spherical AdS black holes, but for AdS black holes with different horizon curvature. In what follows, we demonstrate that black holes in massive gravity exhibit van der Waals behavior independent of the choice of horizon curvature. This relaxes the constraint on the topological structure of the solutions, allowing the possibility of conducting studies in the context of black hole chemistry, the AdS/CFT correspondence, and the nonrelativistic AdS/CMT correspondence, regardless of horizon geometry.

II. BASIC SETUP

The d-dimensional action of Einstein-Λ-massive gravity with a $U(1)$ gauge field is

$$\mathcal{I} = \frac{-1}{16\pi}\int d^dx\sqrt{-g}\left(\mathcal{R} - 2\Lambda - \mathcal{F} + m^2\sum_i^4 c_i\mathcal{U}_i(g,f)\right), \tag{1}$$

in which $\mathcal{R}$ is the scalar curvature of the metric $g_{\mu\nu}$, Λ is the negative cosmological constant and $\mathcal{F} = F_{\mu\nu}F^{\mu\nu}$ is the Maxwell invariant, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic tensor with gauge potential A_μ, m is the mass parameter, and $f_{\mu\nu}$ is a fixed symmetric tensor. The c_i's are constants and the $\mathcal{U}_i$'s are symmetric polynomials of the eigenvalues of the $d\times d$ matrix $\mathcal{K}^\mu_\nu = \sqrt{g^{\mu\alpha}f_{\alpha\nu}}$, where

$$\mathcal{U}_1 = [\mathcal{K}], \qquad \mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2],$$
$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3],$$
$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4].$$

Variation of the action (1) with respect to the metric tensor $g_{\mu\nu}$ and the gauge potential A_μ leads to

$$G_{\mu\nu} + \Lambda g_{\mu\nu} + m^2\chi_{\mu\nu} = -2\left(F_{\mu\rho}F^\rho_\nu - \frac{1}{4}g_{\mu\nu}\mathcal{F}\right), \tag{2}$$

$$\partial_\mu(\sqrt{-g}F^{\mu\nu}) = 0, \tag{3}$$

in which $G_{\mu\nu}$ is the Einstein tensor and $\chi_{\mu\nu}$ is

$$\begin{aligned}\chi_{\mu\nu} = {}& \frac{c_1}{2}(\mathcal{K}_{\mu\nu} - \mathcal{U}_1 g_{\mu\nu}) - \frac{c_2}{2}(\mathcal{U}_2 g_{\mu\nu} - 2\mathcal{U}_1\mathcal{K}_{\mu\nu} + 2\mathcal{K}^2_{\mu\nu}) \\ & - \frac{c_3}{2}(\mathcal{U}_3 g_{\mu\nu} - 3\mathcal{U}_2\mathcal{K}_{\mu\nu} + 6\mathcal{U}_1\mathcal{K}^2_{\mu\nu} - 6\mathcal{K}^3_{\mu\nu}) \\ & - \frac{c_4}{2}(\mathcal{U}_4 g_{\mu\nu} - 4\mathcal{U}_3\mathcal{K}_{\mu\nu} + 12\mathcal{U}_2\mathcal{K}^2_{\mu\nu} \\ & - 24\mathcal{U}_1\mathcal{K}^3_{\mu\nu} + 24\mathcal{K}^4_{\mu\nu}).\end{aligned} \tag{4}$$

In order to obtain AdS topological static charged black holes in massive gravity, we consider the metric of

TABLE I. Roots of metric function ($\psi(r) = 0$) for $\Lambda = -3$, $d = 4$, $q = 1.67$, $c = 1.2$, $c_1 = -2$, $c_2 = 2.2$, $c_3 = 1.5$ and $m_0 = 34$.

k	m	Roots
1	2.1600	3.8955, 9.2446
0	2.1300	3.1305, 9.3969
−1	2.1600	2.6165, 10.2691
1	2.1059	0.51778, 3.8325, 8.4374
0	2.1291	0.5432, 3.1271, 9.3857
−1	2.1530	0.5756, 2.5755, 10.1809
1	2.1000	0.4381, 0.6198, 3.8237, 8.3482
0	2.1000	0.3753, 0.8565, 2.9959, 9.0022
−1	2.1000	0.3449, 1.2545, 2.1028, 9.5275

$d = (n+2)$-dimensional spacetime in the following form:

$$ds^2 = -\psi(r)dt^2 + \frac{dr^2}{\psi(r)} + r^2h_{ij}dx_idx_j, \tag{5}$$

where $i, j = 1, 2, 3, \ldots, n$ and $h_{ij}dx_idx_j$ is a spatial metric of constant curvature d_2d_3k and volume $\mathcal{V}_{d_2}$, where $d_i = d - i$. The reference metric $f_{\mu\nu}$ is related to the spatial components h_{ij} of the spacetime line element. Accordingly, we employ the ansatz $f_{\mu\nu} = \mathrm{diag}(0, 0, c^2h_{ij})$, yielding

$$\mathcal{U}_j = \frac{c^j}{r^j}\Pi^{j+1}_{k=2}d_k, \tag{6}$$

where c is a positive constant [27]. This choice of reference metric preserves general covariance in the temporal and radial coordinates but not in the transverse spatial coordinates [7], so the graviton mass terms will have a Lorentz-breaking property.

Setting $d = 4$, the gauge potential ansatz $A_\mu = h(r)\delta^0_\mu$ yields, from (3), $F_{tr} = \frac{q}{r^2}$ as the only nonzero component of electromagnetic field tensor, in which q is an integration constant and is related to the electrical charge.

The field equations then yield

$$\psi(r) = k - \frac{m_0}{r} - \frac{\Lambda r^2}{3} + \frac{q^2}{r^2} + m^2\mathcal{A}, \tag{7}$$

where $\mathcal{A} = \frac{cc_1}{2}r + c^2c_2 + \frac{c^3c_3}{r}$. The quantity m_0 is an integration constant that is related to the total mass of this black hole. We note that for zero-graviton mass ($m = 0$), the solution (7) reduces to the Reissner-Nordström black hole in four dimensions. Calculations of the Ricci and Kretschmann scalars indicate a divergence at the origin ($\lim_{r\to 0}R = \infty$ and $\lim_{r\to 0}R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} \to \infty$); as $r \to \infty$ we find $R_{\alpha\beta\gamma\delta} \to \frac{\Lambda}{3}(g_{\alpha\gamma}g_{\beta\delta} - g_{\beta\gamma}g_{\alpha\delta})$, confirming that the solution is asymptotically AdS.

To study the effects of the massive terms on our solutions, we can investigate the roots of the metric

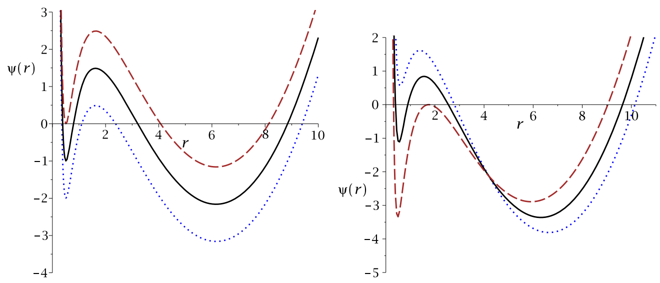

FIG. 1. $\psi(r)$ versus r for $\Lambda = -1$, $d = 4$, $q = 1.7$, $c = 1.00$, $c_1 = -2.00$, $c_2 = 3.18$, $c_3 = 4.00$ and $m_0 = 30$. Left panel: $m = 2.1$, $k = -1$ (dotted line), $k = 0$ (continuous line) and $k = 1$ (dashed line). Right panel: $k = -1$, $m = 2.16$ (dotted line), $m = 2.12$ (continuous line) and $m = 2.07$ (dashed line).

function $[\psi(r) = 0]$. In massive gravity, it is possible for there to be as many as four real roots in all horizon topologies: spherical ($k = 1$), flat ($k = 0$) and hyperbolic ($k = -1$); we illustrate sample results in Table I (see also Fig. 1). The existence of more than two roots for the metric function is due to the presence of the massive terms. Such multihorizon solutions have been of interest in understanding anti-evaporation processes [28]. We postpone a discussion of the causal and geodesic structures of this class of solutions for future work, concentrating on their thermodynamic behavior.

III. THERMODYNAMICS IN THE EXTENDED PHASE SPACE

In extended phase space, the cosmological constant is regarded as a thermodynamic variable corresponding to pressure, with $P = -\frac{\Lambda}{8\pi}$. This postulate leads to an interpretation of the black hole mass as enthalpy [29]. Using Gauss's law and counterterm methods, we compute the various conserved and thermodynamic quantities of these solutions, obtaining

$$T = \frac{k}{4\pi r_+} - \frac{r_+\Lambda}{4\pi} - \frac{q^2}{4\pi r_+^3} + \frac{m^2}{4\pi r_+}(cc_1 r_+ + c_2 c^2), \tag{8}$$

$$S = \frac{\mathcal{V}_2 r_+^2}{4}, \qquad Q = \frac{\mathcal{V}_2 q}{4\pi}, \qquad M = \frac{\mathcal{V}_2 m_0}{8\pi}, \tag{9}$$

where $\mathcal{V}_2$ is the area of a unit volume of constant (t, r) space (4π for $k = 0$). Also, the electric potential is

$$\Phi = A_\mu \chi^\mu|_{r\to\infty} - A_\mu \chi^\mu|_{r\to r_+} = \frac{q}{r_+}. \tag{10}$$

With these relations, we find that the solutions obey the first law of black hole thermodynamics in an extended phase space (including massive variables):

$$dM = TdS + \Phi dQ + VdP + \mathcal{C}_1 dc_1 + \mathcal{C}_2 dc_2 + \mathcal{C}_3 dc_3, \tag{11}$$

where the conjugate quantities associated with the intensive parameters S, Q, P, c_i's are

$$T = \left(\frac{\partial M}{\partial S}\right)_{Q,P,c_i}, \tag{12}$$

$$\Phi = \left(\frac{\partial M}{\partial Q}\right)_{S,P,c_i}, \tag{13}$$

$$V = \left(\frac{\partial M}{\partial P}\right)_{S,Q,c_i} = \frac{\mathcal{V}_2 r_+^3}{3}, \tag{14}$$

$$\mathcal{C}_1 = \left(\frac{\partial M}{\partial c_1}\right)_{S,Q,P,c_2,c_3} = \frac{\mathcal{V}_2 c m^2 r_+^2}{16\pi}, \tag{15}$$

$$\mathcal{C}_2 = \left(\frac{\partial M}{\partial c_2}\right)_{S,Q,P,c_1,c_3} = \frac{\mathcal{V}_2 c^2 m^2 r_+}{8\pi}, \tag{16}$$

$$\mathcal{C}_3 = \left(\frac{\partial M}{\partial c_3}\right)_{S,Q,P,c_1,c_2} = \frac{\mathcal{V}_2 c^3 m^2}{8\pi}, \tag{17}$$

with T and Φ given in Eqs. (8) and (10). In addition, the corresponding Smarr relation can be derived by a scaling (dimensional) argument as

$$M = 2TS - 2PV + \Phi Q + \mathcal{C}_1 c_1 - \mathcal{C}_3 c_3, \tag{18}$$

where c_2 does not appear since it has scaling weight 0. Since the c_2 term in the metric function is a constant term in four dimensions with no thermodynamical contribution, we set $dc_2 = 0$.

We note that the thermodynamic volume (14) does not depend on the graviton mass. This in turn implies that the isoperimetric ratio

$$\mathcal{R} = \left(\frac{3V}{\mathcal{V}_2}\right)^{\frac{1}{3}} \left(\frac{\mathcal{V}_2}{A}\right)^{\frac{1}{2}} = 1, \tag{19}$$

and so the reverse isoperimetric inequality ($\mathcal{R} \geq 1$) [30] can be satisfied.

To study critical phenomena and van der Waals behavior, we compute the equation of state and the Gibbs free energy,

$$P = \frac{4\pi T - m^2 c_1 c}{8\pi r_+} - \frac{k + m^2 c_2 c^2}{8\pi r_+^2} + \frac{q^2}{8\pi r_+^4}, \tag{20}$$

$$G = H - TS = M - TS$$
$$= -\frac{Pr_+^3}{6} + \frac{m^2 r_+ (c_2 c^2 r_+ + 2c_3 c^3) + (kr_+^2 + 3q^2)}{16\pi r_+}, \quad (21)$$

using Eqs. (8) and (9). Computing the inflection point $\left(\frac{\partial P}{\partial r_+}\right)_T = \left(\frac{\partial^2 P}{\partial r_+^2}\right)_T = 0$ of the equation of state, we find

$$kr_{+c}^2 - 6q^2 + m^2 c_2 c^2 r_{+c}^2 = 0, \quad (22)$$

where r_{+c} yields the critical volume V_c via Eq. (14). This leads to the following respective critical horizon radius, temperature, and pressure:

$$r_{+c} = \frac{\sqrt{6}|q|}{\sqrt{k + m^2 c_2 c^2}}, \quad (23)$$

$$T_c = \frac{(k + m^2 c_2 c^2)^{3/2}}{3\sqrt{6}\pi q} + \frac{m^2 c_1 c}{4\pi}, \quad (24)$$

$$P_c = \frac{(k + m^2 c_2 c^2)^2}{96\pi q^2}, \quad (25)$$

and we see that for all values of k, critical behavior is possible provided the constraint

$$k + m^2 c_2 c^2 \geq 0 \quad (26)$$

is obeyed. Moreover, taking into account Eq. (20), the pressure is positive for large volume provided

$$T > \frac{m^2 c_1 c}{4\pi}, \quad (27)$$

a relationship that is automatically satisfied if (26) holds. Previous investigations of the critical behavior of black holes in Einstein gravity have indicated that only spherical topology ($k = 1$) admits van der Waals-like behavior, with the $k = 0$ case behaving like an ideal gas. The graviton mass significantly modifies this behavior, opening up new possibilities: topological black holes ($k \neq 1$) can exhibit second-order phase transitions and van der Waals-like behavior (see Fig. 2). This admits new prospects for investigating critical behavior of black holes in the context of classical gravity, the AdS/CFT correspondence, holographic interpretation of black holes, and duality with superconductivity.

The existence of van der Waals-like behavior for non-spherical black holes provides another reason for considering modified versions of Einstein gravity to include massive terms. In other words, a nonzero m admits the possibility of critical behavior for $k \neq 1$. Furthermore, the massive coefficient c_3 in the Gibbs free energy (absent in pressure and temperature) makes it possible to modify the energy of

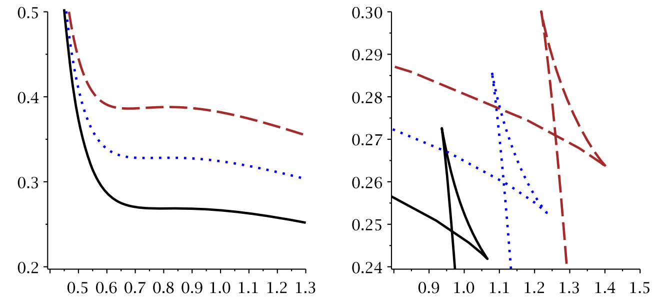

FIG. 2. $P - r_+$ (left panel) and $G - T$ (right panel) diagrams for $d = 4$, $q = m = c = c_1 = c_3 = 1$ and $c_2 = 10$. Left panel $T = T_c$, right panel $P = 0.5P_c$: $k = -1$ (continuous line), $k = 0$ (dotted line) and $k = 1$ (dashed line).

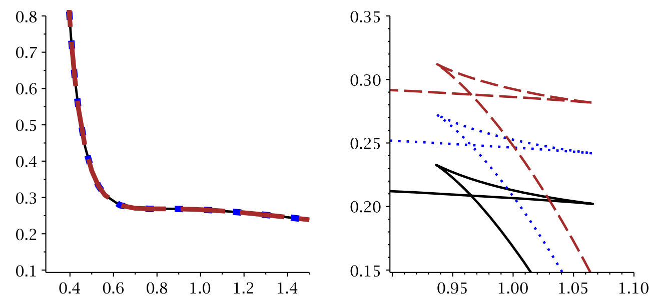

FIG. 3. $P - r_+$ (left panel) and $G - T$ (right panel) diagrams for $d = 4$, $q = m = c = c_1 = 1$, $k = -1$ and $c_2 = 10$. Left panel $T = T_c$, right panel $P = 0.5P_c$: $c_3 = 0$ (continuous line), $c_3 = 1$ (dotted line) and $c_3 = 2$ (dashed line).

different phases, without any modification in critical values and their corresponding diagrams [see Eq. (21) and Fig. 3]. For example, the formation of a second-order phase transition can take place at the same critical temperature, pressure, and horizon radius but at differing energies of the various phases. Such interesting behaviors of Gibbs free energy and critical values could introduce new phenomenology for the phase structure of black holes.

IV. CLOSING REMARKS

In this paper we have demonstrated that topological black holes in dRGT can exhibit van der Waals behavior and critical phenomena, in striking contrast to their counterparts in Einstein gravity. For $k = 0$, it is sufficient to have any nonzero value of the graviton mass parameter m, whereas for $k = -1$ black holes, this parameter must be sufficiently large. However, too large a value of m will destabilize the pressure, causing it to become negative for sufficiently large volume.

Recent progress in gauge-gravity duality in extended phase space [31] suggests that massive gravity will open up avenues of investigation in black hole thermodynamics. Since such theories admit critical behavior of black holes of any horizon curvature, a range of new phenomena in entanglement entropy [32], holographic ferromagnetism [33], quasinormal modes [34], and confinement-deconfinement phase transitions for heavy quarks [35] can now be explored.

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