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Flavor changing neutral current transition of B to a_1 with light-cone sum rules

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The $B \to a_1 \ell^+ \ell^-$ decays occur by the electroweak penguin and box diagrams, which can be performed through the flavor changing neutral current (FCNC). We calculate the form factors of the FCNC $B \to a_1$ transitions in the light-cone sum rules approach, up to twist-4 distribution amplitudes of the axial vector meson a_1. Forward-backward asymmetry, as well as branching ratios of $B \to a_1 \ell^+ \ell^-$, and $B \to a_1 \gamma$ decays are considered. A comparison is also made between our results and the predictions of other methods.

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I. INTRODUCTION

The semileptonic B meson decays are helpful tools for exploring the Cabibbo, Kabayashi, and Maskawa (CKM) matrix elements and CP violations. These decays usually occur by two various diagrams: (1) Simple tree diagrams that can be performed via the weak interaction and (2) electroweak penguin and box diagrams that can be fulfilled through the flavor changing neutral current (FCNC) transitions in the standard model (SM). Future study of the FCNC decays can improve our information about the following.

(i) CP violation, T violation, and polarization asymmetries in penguin diagrams.
(ii) Exact values for the CKM matrix elements in the weak interactions.
(iii) New operators or operators that follow the SM.
(iv) Development of new physics (NP) and flavor physics beyond the SM.

The FCNC decays of the B meson are sensitive to NP contributions to penguin operators. So, to estimate the SM predictions for FCNC decays and compare these results to the corresponding experimental values, we can check the SM and search NP.

There is a growing demand for more accurate and reliable calculations of heavy to light transition form factors in QCD [1–8]. The transition of heavy B meson to light meson a_1 is one of the decays that has attracted much attention of authors. The form factors of the transition $B \to a_1 \ell \nu$ have been calculated via such different approaches as the QCD sum rules [9], the covariant quark model [10], the constituent quark-meson model (CQM) [11], and the light-cone sum rules (LCSR) [12,13]. Also, the $B \to a_1$ decay, as a FCNC process, has been studied in the perturbative QCD [14], and three-point QCD sum rules (3PSR) [15].

In this paper, the FCNC $B \to a_1 \ell^+ \ell^-$ decays are considered with the LCSR. The LCSR is one of the most effective tools used to determine nonperturbative parameters of hadronic states. In this approach, the operator product expansion is performed near the light cone $x^2 \approx 0$, while the nonperturbative hadronic matrix elements are described by the light-cone distribution amplitudes (LCDAs) of increasing twist instead of the vacuum condensates [16–20]. The main purpose of this paper is to calculate the form factors of the FCNC $B \to a_1$ transitions up to twist-4 distribution amplitudes of the axial vector meson a_1 and to compare the results of these form factors with those of other approaches.

The paper is organized as follows: In Sec. II, by using the LCSR, the form factors of $B \to a_1 \ell^+ \ell^-$ decays are derived. In Sec. III, we present the numerical analysis of the LCSR for the form factors and determine the branching ratio values of the $B \to a_1 \gamma$ and $B \to a_1 \ell^+ \ell^-$ decays. Also, the forward-backward asymmetry of these decays is considered. For a better analysis, a comparison is made between our results and the predictions of other methods.

II. TRANSITION FORM FACTORS IN THE LCSR

The $b \to d \ell^+ \ell^-$ transition in quark level is explained by the effective Hamiltonian in the SM as

$$\mathcal{H}_{\text{eff}}^{b\to d} = -\frac{G_F}{\sqrt{2}} \Bigg(V_{ub} V_{ud}^* \sum_{i=1}^{2} C_i(\mu) O_i^u(\mu) + V_{cb} V_{cd}^* \sum_{i=1}^{2} C_i(\mu) O_i^c(\mu) - V_{tb} V_{td}^* \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \Bigg),$$

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where V_{jk} are the CKM matrix elements, and $C_i(\mu)$ and $O_i(\mu)$ are the Wilson coefficients and the local operators, respectively. The local operators are current-current operators $O^{u,c}_{1,2}$, QCD penguin operators O_{3-6}, magnetic penguin operators $O_{7,8}$, and semileptonic electroweak penguin operators $O_{9,10}$. The explicit expressions of these operators can be found in [21]. The operators O_7 and $O_{9,10}$ are responsible for the short distance (SD) effects in the FCNC $b \to d$ transition, while the current-current and QCD penguin operators, O_{1-6}, involve both SD and long distance (LD) contributions in this transition. These SD and LD contributions have the same form factor dependence as C_9 in the naive factorization approximation and can therefore be absorbed into an effective Wilson coefficient $C_9^{\rm eff}$. To be more specific, we can decompose $C_9^{\rm eff}$ into the following three parts as

$$C_9^{\rm eff} = C_9 + Y_{\rm SD}(q^2) + Y_{\rm LD}(q^2), \tag{1}$$

where $Y_{\rm SD}(q^2)$ describes the SD contributions from four-quark operators far away from the resonance regions, which can be calculated reliably in perturbative theory as [21,22]

$$\begin{aligned} Y_{\rm SD}(q^2) = {}& 0.138\omega(s) + h(\hat{m}_c, s)C_0 \\ & - \frac{1}{2}h(1,s)(4C_3 + 4C_4 + 3C_5 + C_6) \\ & - \frac{1}{2}h(0,s)(2\lambda_u[3C_1 + C_2] + C_3 + 4C_4) \\ & + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6), \end{aligned}$$

where $s = q^2/m_b^2$, $\hat{m}_c = m_c/m_b$, $C_0 = -\lambda_c(3C_1 + C_2) + 3C_3 + C_4 + 3C_5 + C_6$, $\lambda_c = \frac{V_{cb}V^*_{cd}}{V_{tb}V^*_{td}}$, $\lambda_u = \frac{V_{ub}V^*_{ud}}{V_{tb}V^*_{td}}$, and

$$\begin{aligned} \omega(s) = {}& -\frac{2}{9}\pi^2 - \frac{4}{3}\mathrm{Li}_2(s) - \frac{2}{3}\ln(s)\ln(1-s) \\ & - \frac{5+4s}{3(1+2s)}\ln(1-s) - \frac{2s(1+s)(1-2s)}{3(1-s)^2(1+2s)}\ln(s) \\ & + \frac{5+9s-6s^2}{3(1-s)(1+2s)}, \end{aligned}$$

represents the $\mathcal{O}(\alpha_s)$ correction coming from one-gluon exchange in the matrix element of the operator O_9 [23], while $h(\hat{m}_c, s)$ and $h(0,s)$ represent one-loop corrections to the four-quark operators O_{1-6} [24]. The functional form of the $h(\hat{m}_c, s)$ and $h(0,s)$ is as

$$\begin{aligned} h(\hat{m}_c, s) = {}& -\frac{8}{9}\ln\frac{m_b}{\mu} - \frac{8}{9}\ln\hat{m}_c + \frac{8}{27} + \frac{4}{9}x \\ & - \frac{2}{9}(2+x)|1-x|^{1/2} \\ & \times \begin{cases} \left(\ln\left|\frac{\sqrt{1-x}+1}{\sqrt{1-x}-1}\right| - i\pi\right), & \text{for } x \equiv \frac{4\hat{m}_c^2}{s} < 1 \\ 2\arctan\frac{1}{\sqrt{x-1}}, & \text{for } x \equiv \frac{4\hat{m}_c^2}{s} > 1, \end{cases} \\ h(0,s) = {}& \frac{8}{27} - \frac{8}{9}\ln\frac{m_b}{\mu} - \frac{4}{9}\ln s + \frac{4}{9}i\pi. \end{aligned}$$

The LD contributions, $Y_{\rm LD}(q^2)$, from four-quark operators near the $u\bar{u}$, $d\bar{d}$, and $c\bar{c}$ resonances cannot be calculated from the first principles of QCD and are usually parametrized in the form of a phenomenological Breit-Wigner formula as [21,22]

$$\begin{aligned} Y_{LD}(q^2) = \frac{3\pi}{\alpha^2}\Bigg\{ & \sum_{V_i=\psi(1s),\psi(2s)} \frac{\Gamma(V_i \to l^+l^-)m_{V_i}}{m_{V_i}^2 - q^2 - i m_{V_i}\Gamma_{V_i}} \\ & - \lambda_u h(0,s)(3C_1 + C_2) \\ & \times \sum_{V_i=\rho,\omega} \frac{\Gamma(V_i \to l^+l^-)m_{V_i}}{m_{V_i}^2 - q^2 - i m_{V_i}\Gamma_{V_i}} \Bigg\}. \end{aligned}$$

To calculate the form factors of the FCNC $B \to a_1$ transition within the LCSR method, two correlation functions are written as

$$\begin{aligned} \Pi_\mu^{V,A} = {}& i\int d^4x e^{iqx}\langle a_1(p',\varepsilon)| \\ & \times \mathcal{T}\{\bar{d}(x)\gamma_\mu(1-\gamma_5)b(x)j_B^\dagger(0)\}|0\rangle, \\ \Pi_\mu^T = {}& i\int d^4x e^{iqx}\langle a_1(p',\varepsilon)| \\ & \times \mathcal{T}\{\bar{d}(x)\sigma_{\mu\nu}q^\nu(1+\gamma_5)b(x)j_B^\dagger(0)\}|0\rangle, \end{aligned} \tag{2}$$

where $j_B = i\bar{u}\gamma_5 b$ is the interpolating current for the B meson. The transition currents $\bar{d}\gamma_\mu(1-\gamma_5)b$ and $\bar{d}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b$ are derived from $O_{9,10}$ and O_7 operators, respectively. It should be noted that the rest operators are not contained in the definition of the transition form factors. According to the general philosophy of the LCSR, the above correlation functions should be calculated in two different ways. In phenomenological or physical representation, it is investigated in terms of hadronic parameters. In QCD or the theoretical side, it is obtained in terms of distribution amplitudes and QCD degrees of freedom. Physical quantities like form factors are found to equate the coefficient of the same structures from both representations of the correlation functions through dispersion relation and apply Borel transformation to suppress the contributions of the higher states and continuum.

A. Phenomenological side

By considering phenomenological representation, a complete set of hadrons with the same quantum numbers as the interpolating current operator j_B is inserted in the correlation functions. After isolating the pole mass term of the B meson and applying Fourier transformation as well as the dispersion relation, we obtain

$$\Pi_\mu^{V,A}(p',p) = \frac{\langle a_1(p',\varepsilon)|\bar{d}\gamma_\mu(1-\gamma_5)b|B(p)\rangle\langle B(p)|\bar{b}i\gamma_5 u|0\rangle}{m_B^2-p^2} + \frac{1}{\pi}\int_{s_0}^\infty \frac{\rho_\mu^{h(V,A)}(s)}{s-p^2}ds,$$
$$\Pi_\mu^{T}(p',p) = \frac{\langle a_1(p',\varepsilon)|\bar{d}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B(p)\rangle\langle B(p)|\bar{b}i\gamma_5 u|0\rangle}{m_B^2-p^2} + \frac{1}{\pi}\int_{s_0}^\infty \frac{\rho_\mu^{h(T)}(s)}{s-p^2}ds, \tag{3}$$

where ρ_μ^h shows the spectral density of the higher resonances and the continuum states in the hadronic representation. These spectral densities are approximated by evoking the quark-hadron duality assumption,

$$\rho_\mu^h(s) = \rho_\mu^{\mathrm{QCD}}(s)\theta(s-s_0), \tag{4}$$

where $\rho_\mu^{\mathrm{QCD}}(s)$ is the perturbative QCD spectral density investigated from the theoretical side of the correlation function. The threshold s_0 is chosen near the squared mass of the lowest B meson state.

The matrix elements $\langle a_1(p',\varepsilon)|\bar{d}\gamma_\mu(1-\gamma_5)b|B(p)\rangle$ and $\langle a_1(p',\varepsilon)|\bar{d}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B(p)\rangle$ are parametrized in terms of the form factors as follows:

$$\begin{aligned}\langle a_1(p',\varepsilon)|\bar{d}\gamma_\mu(1-\gamma_5)b|B(p)\rangle &= i\frac{2A(q^2)}{m_B-m_{a_1}}\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^\alpha p'^\beta - V_1(q^2)\varepsilon_\mu^*(m_B-m_{a_1})\\ &\quad - \frac{V_2(q^2)}{m_B-m_{a_1}}(\varepsilon^*.q)(p+p')_\mu + 2m_{a_1}\frac{(\varepsilon^*.q)}{q^2}q_\mu[V_3(q^2)-V_0(q^2)],\\ \langle a_1(p',\varepsilon)|\bar{d}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B(p)\rangle &= 2T_1(q^2)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^\alpha p'^\beta + iT_2(q^2)[(m_B^2-m_{a_1}^2)\varepsilon_\mu^* - (\varepsilon^*.q)(p+p')_\mu]\\ &\quad - iT_3(q^2)(\varepsilon^*.q)\left[q_\mu - \frac{q^2}{m_B^2-m_{a_1}^2}(p+p')_\mu\right],\end{aligned} \tag{5}$$

where $q = p - p'$ is the momentum transfer of the Z boson (photon), and $\varepsilon^{*\nu}$ is the polarization vector of the axial vector meson a_1. It should be noted that $V_0(0) = V_3(0)$. On the other hand, the identity $\sigma_{\mu\nu}\gamma_5 = -\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}$ implies that $T_1(0) = T_2(0)$ [25]. Also, V_3 can be written as a linear combination of V_1 and V_2,

$$V_3(q^2) = \frac{m_B-m_{a_1}}{2m_{a_1}}V_1(q^2) - \frac{m_B+m_{a_1}}{2m_{a_1}}V_2(q^2). \tag{6}$$

Taking into account the second matrix element in Eq. (3) as $\langle B(p_B)|\bar{b}i\gamma_5 u|0\rangle = \frac{f_B m_B^2}{m_b}$, where f_B is the B meson decay constant and m_b is the b quark mass, we can obtain these hadronic representations for $\Pi_\mu^{A,V}$ and Π_μ^T as

$$\begin{aligned}\Pi_\mu^{A,V} &= -\frac{f_B m_B^2}{m_b}\frac{1}{p^2-m_B^2}\left\{i\frac{2A(q^2)}{m_B-m_{a_1}}\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^\alpha p'^\beta - V_1(q^2)\varepsilon_\mu^*(m_B-m_{a_1})\right.\\ &\quad\left. - \frac{V_2(q^2)}{m_B-m_{a_1}}(\varepsilon^*.q)(p+p')_\mu + 2m_{a_1}\frac{(\varepsilon^*.q)}{q^2}q_\mu[V_3(q^2)-V_0(q^2)]\right\} + \frac{1}{\pi}\int_{s_0}^\infty\frac{\rho_\mu^{h(A,V)}(s)}{s-p^2}ds,\\ \Pi_\mu^T &= -\frac{f_B m_B^2}{m_b}\frac{1}{p^2-m_B^2}\left\{2T_1(q^2)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p^\alpha p'^\beta + iT_2(q^2)[(m_B^2-m_{a_1}^2)\varepsilon_\mu^* - (\varepsilon^*.q)(p+p')_\mu]\right.\\ &\quad\left. - iT_3(q^2)(\varepsilon^*.q)\left[q_\mu - \frac{q^2}{m_B^2-m_{a_1}^2}(p+p')_\mu\right]\right\} + \frac{1}{\pi}\int_{s_0}^\infty\frac{\rho_\mu^{h(T)}(s)}{s-p^2}ds.\end{aligned} \tag{7}$$

B. Theoretical side

Now, the QCD or the theoretical part of the correlation functions should be calculated. The calculation for the defined correlators in the region of large spacelike momenta is based on the expansion of the $\mathcal{T}$-product of the currents near the light cone $x^2 = 0$. After contracting the b quark field, we get

$$\begin{aligned}\Pi_\mu^{A,V} &= \int d^4x e^{iqx}\langle a_1(p',\varepsilon)|\bar{d}(x)\gamma_\mu(1-\gamma_5)S^b(x,0)\gamma_5 u(0)|0\rangle,\\ \Pi_\mu^{T} &= \int d^4x e^{iqx}\langle a_1(p',\varepsilon)|\bar{d}(x)\sigma_{\mu\nu}q^\nu(1+\gamma_5)S^b(x,0)\gamma_5 u(0)|0\rangle,\end{aligned} \tag{8}$$

where $S^b(x,0)$ is the full propagator of the b quark in the presence of the background gluon field as

$$S^b(x) = \int \frac{d^4k}{(2\pi)^4}e^{-ikx}\frac{\not k + m_b}{k^2 - m_b^2} - g_s\int \frac{d^4k}{(2\pi)^4}e^{-ikx}\int_0^1 du\left[\frac{1}{2}\frac{\not k + m_b}{(m_b^2-k^2)^2}G_{\mu\nu}(ux)\sigma^{\mu\nu} + \frac{1}{m_b^2-k^2}ux_\mu G^{\mu\nu}(ux)\gamma_\nu\right], \tag{9}$$

where $G_{\mu\nu}$ is the gluon field strength tensor and g_s is the strong coupling constant. In the present work, contributions with two gluons as well as four quark operators are neglected because their contributions are small. Using the Fierz rearrangement formula, Eq. (8) can be rewritten as

$$\begin{aligned}\Pi_\mu^{A,V} &= -\frac{i}{4}\int d^4x e^{iqx}[\mathrm{Tr}\{\gamma_\mu(1-\gamma_5)S^b(x)\gamma_5\Gamma_\alpha\}]\langle a_1|\bar{d}(x)\Gamma^\alpha u(0)|0\rangle,\\ \Pi_\mu^{T} &= -\frac{i}{4}\int d^4x e^{iqx}[\mathrm{Tr}\{\sigma_{\mu\nu}(1+\gamma_5)S^b(x)\gamma_5\Gamma_\alpha\}]q^\nu\langle a_1|\bar{d}(x)\Gamma^\alpha u(0)|0\rangle,\end{aligned} \tag{10}$$

where Γ_α is the full set of the Dirac matrices, $\Gamma_\alpha = (I, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu})$. In order to calculate the theoretical part of the correlator functions in Eq. (10), the matrix elements of the nonlocal operators between a_1 meson and vacuum states are needed. Two-particle distribution amplitude up to twist 4 for the axial vector meson a_1 is given in [12]

$$\begin{aligned}\langle a_1(p',\varepsilon)|\bar{d}_\alpha(x)u_\delta(0)|0\rangle = -\frac{i}{4}\int_0^1 du e^{iup'.x}\Big\{ & f_{a_1}m_{a_1}\Big[\not p'\gamma_5\frac{\varepsilon^*.x}{p'.x}\Phi_\parallel(u) + \Big(\not\varepsilon^* - \not p'\frac{\varepsilon^*.x}{p'.x}\Big)\gamma_5 g_\perp^{(a)}(u)\\ & - \not x\gamma_5\frac{\varepsilon^*.x}{2(p'.x)^2}m_{a_1}^2\bar{g}_3(u) + \epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p'^\rho x^\sigma\gamma^\mu\frac{g_\perp^{(v)}(u)}{4}\Big] + f_{a_1}^\perp\Big[\frac{1}{2}(\not p'\not\varepsilon^* - \not\varepsilon^*\not p')\gamma_5\Phi_\perp(u)\\ & - \frac{1}{2}(\not p'\not x - \not x\not p')\gamma_5\frac{\epsilon^*.x}{(p'.x)^2}m_{a_1}^2\bar{h}_\parallel^{(t)}(u) + i(\epsilon^*.x)m_{a_1}^2\gamma_5\frac{h_\parallel^{(p)}(u)}{2}\Big]\Big\}_{\delta\alpha},\end{aligned} \tag{11}$$

where for $x^2 \neq 0$, we have

$$\begin{aligned}\bar{g}_3(u) &= g_3(u) + \Phi_\parallel - 2g_\perp^{(a)}(u),\\ \bar{h}_\parallel^{(t)} &= h_\parallel^{(t)} - \frac{1}{2}\Phi_\perp(u).\end{aligned}$$

In Eq. (11), $\Phi_\parallel$, $\Phi_\perp$ are the twist-2 functions, $g_\perp^{(a)}$, $g_\perp^{(v)}$, $h_\parallel^{(t)}$, and $h_\parallel^{(p)}$ are the twist-3 functions, and g_3 is the twist-4 function. The definitions for $\Phi_\parallel$, $\Phi_\perp$, $g_\perp^{(a)}$, $g_\perp^{(v)}$, $h_\parallel^{(t)}$, $h_\parallel^{(p)}$, and g_3 are given in Appendix A.

Two-particle chiral-even distribution amplitudes are given by [12]

$$\langle a_1(p',\varepsilon)|\bar{d}(x)\gamma_\mu\gamma_5 u(0)|0\rangle = if_{a_1}m_{a_1}\int_0^1 du e^{iup'.x}\left\{p'_\mu\frac{\varepsilon^*.x}{p'.x}\Phi_\parallel(u)+\left(\varepsilon^*_\mu - p'_\mu\frac{\varepsilon^*.x}{p'.x}\right)g_\perp^{(a)}(u) -\frac{1}{2}x_\mu\frac{\varepsilon^*.x}{(p'.x)^2}m_{a_1}^2\bar{g}_3(u)+\mathcal{O}(x^2)\right\},$$

$$\langle a_1(p',\varepsilon)|\bar{d}(x)\gamma_\mu u(0)|0\rangle = -if_{a_1}m_{a_1}\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p'^\rho x^\sigma\int_0^1 du e^{iup'.x}\left\{\frac{g_\perp^{(v)}(u)}{4}+\mathcal{O}(x^2)\right\}; \tag{12}$$

also, two-particle chiral-odd distribution amplitudes are defined by

$$\langle a_1(p',\varepsilon)|\bar{d}(x)\sigma_{\mu\nu}\gamma_5 u(0)|0\rangle = f_{a_1}^\perp\int_0^1 du e^{iup'.x}\left\{(\varepsilon^*_\mu p'_\nu-\varepsilon^*_\nu p'_\mu)\Phi_\perp(u)+\frac{m_{a_1}^2\varepsilon^*.x}{(p'.x)^2}(p'_\mu x_\nu - p'_\nu x_\mu)\bar{h}_\parallel^{(t)}+\mathcal{O}(x^2)\right\},$$

$$\langle a_1(p',\varepsilon)|\bar{d}(x)\gamma_5 u(0)|0\rangle = f_{a_1}^\perp m_{a_1}^2(\varepsilon^*.x)\int_0^1 du e^{iup'.x}\left\{\frac{h_\parallel^{(p)}(u)}{2}+\mathcal{O}(x^2)\right\}. \tag{13}$$

In these expressions, f_{a_1} and $f_{a_1}^\perp$ are decay constants of the axial vector meson a_1 defined as

$$\langle a_1(p',\varepsilon)|\bar{d}(0)\gamma_\mu\gamma_5 u(0)|0\rangle = if_{a_1}m_{a_1}\epsilon^*_\mu,$$
$$\langle a_1(p',\varepsilon)|\bar{d}(0)\sigma_{\mu\nu}\gamma_5 u(0)|0\rangle = f_{a_1}^\perp a_0^\perp(\epsilon^*_\mu p'_\nu-\epsilon^*_\nu p'_\mu), \tag{14}$$

where $a_0^\perp$ refers to the zeroth Gegenbauer moments of $\Phi_\perp$. It should be noted that f_{a_1} is scale independent and conserves G parity, but $f_{a_1}^\perp$ is scale dependent and violates G parity.

Three-particle distribution amplitudes are defined as

$$\langle a_1(p',\varepsilon)|\bar{d}(x)\gamma_\alpha\gamma_5 g_s G_{\mu\nu}(ux)u(0)|0\rangle = p'_\alpha(p'_\nu\varepsilon^*_\mu - p'_\mu\varepsilon^*_\nu)f_{3,a_1}^A\mathcal{A}+\cdots,$$
$$\langle a_1(p',\varepsilon)|\bar{d}(x)\gamma_\alpha g_s\tilde{G}_{\mu\nu}(ux)u(0)|0\rangle = ip'_\alpha(p'_\mu\varepsilon^*_\nu - p'_\nu\varepsilon^*_\mu)f_{3,a_1}^V\mathcal{V}+\cdots, \tag{15}$$

where $\tilde{G}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\rho\lambda}G^{\rho\lambda}$. The value of coupling constants f_{3,a_1}^V and f_{3,a_1}^A for a_1 meson at $\mu=1$ GeV is $f_{3,a_1}^V=(0.0055\pm 0.0027)\,\mathrm{GeV}^2$ and $f_{3,a_1}^A=(0.0022\pm0.0009)\ \mathrm{GeV}^2$ [13]. The three-parton chiral-even distribution amplitudes $\mathcal{A}$ and $\mathcal{V}$ in Eq. (15) are defined as

$$\mathcal{A}=\int\mathcal{D}\underline{\alpha}e^{ip'.x(\alpha_1+u\alpha_3)}\mathcal{A}(\alpha_i),$$
$$\mathcal{V}=\int\mathcal{D}\underline{\alpha}e^{ip'.x(\alpha_1+u\alpha_3)}\mathcal{V}(\alpha_i), \tag{16}$$

where $\mathcal{A}(\alpha_i)$ and $\mathcal{V}(\alpha_i)$ can be approximately written as [13]

$$\mathcal{A}(\alpha_i)=5040(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3^2 + 360\alpha_1\alpha_2\alpha_3^2\left[\lambda_{a_1}^A+\frac{\sigma_{a_1}^A}{2}(7\alpha_3-3)\right],$$
$$\mathcal{V}(\alpha_i)=360\alpha_1\alpha_2\alpha_3^2\left[1+\frac{\omega_{a_1}^V}{2}(7\alpha_3-3)\right] + 5040(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3^2\sigma_{a_1}^V, \tag{17}$$

where α_1, α_2, and α_3 are the momentum fractions carried by the d, $\bar{u}$ quarks and gluon, respectively, in the axial vector meson a_1. The integration measure is defined as

$$\int\mathcal{D}\underline{\alpha}\equiv\int_0^1 d\alpha_1\int_0^1 d\alpha_2\int_0^1 d\alpha_3\delta\left(1-\sum\alpha_i\right). \tag{18}$$

Diagrammatically, the contributions of two- and three-particle LCDAs to the correlation functions are depicted in Fig. 1.

In this step, inserting the full propagator [Eq. (9)] and two-particle as well as three-particle LCDAs [Eqs. (11)–(15)] in the correlation functions [Eq. (10)] traces and then integrals should be calculated. To estimate these calculations, we have used identities as

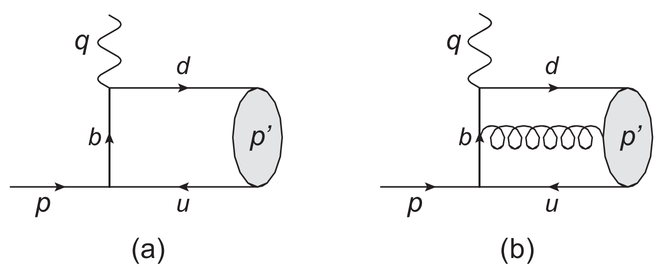


FIG. 1. Leading-order terms in the correlation functions involving the two-particle (a) and three-particle (b).

TABLE I. The Gegenbauer moments of $\Phi_\perp$ and $\Phi_\parallel$ for the a_1 meson and twist-3 LCDA parameters at $\mu = 1$ GeV.

LCDA parameter	$a_1^\perp$	$a_2^\parallel$	$\zeta_{3,a_1}^\perp$	$\omega_{a_1}^\perp$	$\omega_{a_1}^V$
Value	-1.04 ± 0.34	-0.02 ± 0.02	-0.009 ± 0.001	-3.70 ± 0.40	-2.90 ± 0.90

TABLE II. The $B \to a_1$ form factors at zero momentum transfer.

Form factor	$A(0)$	$V_1(0)$	$V_2(0)$	$V_0(0)$	$T_1(0) = T_2(0)$	$T_3(0)$
Value	0.42 ± 0.16	0.68 ± 0.13	0.31 ± 0.16	0.30 ± 0.18	0.44 ± 0.28	0.41 ± 0.18

$$\epsilon_{\alpha\beta\gamma\sigma}\epsilon^{\alpha}_{\mu\nu\lambda} = (\delta_{\nu\beta}\delta_{\lambda\sigma}\delta_{\mu\gamma} - \delta_{\nu\beta}\delta_{\lambda\gamma}\delta_{\mu\sigma} + \delta_{\lambda\beta}\delta_{\nu\gamma}\delta_{\mu\sigma} - \delta_{\lambda\beta}\delta_{\mu\gamma}\delta_{\nu\sigma} + \delta_{\mu\beta}\delta_{\lambda\gamma}\delta_{\nu\sigma} - \delta_{\mu\beta}\delta_{\nu\gamma}\delta_{\lambda\sigma}),$$
$$\epsilon_{\sigma\alpha\beta\mu}\epsilon^{\alpha\beta\rho\lambda} = 2(\delta^{\rho}_{\sigma}\delta^{\lambda}_{\mu} - \delta^{\lambda}_{\sigma}\delta^{\rho}_{\mu}). \tag{19}$$

Now, to get the LCSR for the calculations of the $B \to a_1$ form factors, we equate the coefficients of the corresponding structures from both phenomenological and theoretical sides of the correlation functions and apply Borel transform with respect to the variable p as

$$B_{p^2}(M^2)\frac{1}{(p^2 - m_B^2)^n} = \frac{(-1)^n}{\Gamma(n)}\frac{e^{-\frac{m_B^2}{M^2}}}{(M^2)^n}, \tag{20}$$

in order to suppress the contributions of the higher states and continuum as well as eliminate the subtraction terms. Thus, the form factors are obtained via the LCSR. The explicit expressions for the form factors are presented in Appendix B.

III. NUMERICAL ANALYSIS

In this section, we present our numerical analysis for the form factors and branching ratios of the $B \to a_1\ell^+\ell^-$ decays. In this work, masses are taken in GeV as $m_b = 4.81 \pm 0.03$, $m_\mu = 0.11$, $m_\tau = 1.77$, $m_{a_1} = 1.23 \pm 0.04$, and $m_B = 5.27 \pm 0.01$ [26]. The $f_B = (0.19 \pm 0.02)$ GeV is in agreement with the QCD sum rule result with radiative corrections [12]. The G-parity violating decay constant for the a_1 meson is defined by $f_{a_1}^\perp$ and is equal to $f_{a_1} = (0.23 \pm 0.01)$ GeV at the energy scale $\mu = 1$ GeV [12]. The suitable threshold parameter s_0 is chosen as $s_0 = (33 \pm 1)$ GeV^2, which corresponds to the sum rule calculation [7]. Also, we need to know Gegenbauer moments of $\Phi_\perp$, $\Phi_\parallel$, and G-parity conserving parameters of three-parton LCDAs for the a_1 meson at the scale $\mu = 1$ GeV given in Table I. It should be noted that the value of other parameters such as $\sigma_{a_1}^A$, $\sigma_{a_1}^V$, $\sigma_{a_1}^\perp$, $\lambda_{a_1}^A$, ζ_{3,a_1}^V, $a_0^\parallel$, $a_1^\parallel$, and $a_2^\perp$ is 0 for meson a_1 [13].

TABLE III. Contribution of the b quark free propagator in the form factor values at $q^2 = 0$.

$A(0)$	$V_1(0)$	$V_2(0)$	$V_0(0)$	$T_1(0) = T_2(0)$	$T_3(0)$
0.41	0.67	0.30	0.29	0.43	0.39

We should obtain the region for the Borel mass parameter so that our results for the form factors of the $B \to a_1$ decays are almost insensitive to variation of M^2. We find that the dependence of the form factors on M^2 is small in the interval $M^2 \in [6, 10]$ GeV^2.

Using all these input values and parameters, we can present form factor values at the zero transferred momentum square $q^2 = 0$ in Table II. The errors in Table II are estimated by the variation of the Borel parameter M^2, the variation of the continuum threshold s_0, the variation of b quark mass, and the parameters of the LCDAs. The main uncertainty comes from LCDAs $\Phi_\perp(u)$ and b quark mass m_b, while the other uncertainties are small, constituting a few percent.

The b quark propagator in Eq. (9) consists of the free propagator as well as the one-gluon term. Considering only the free propagator in the QCD calculations, the value of V_1

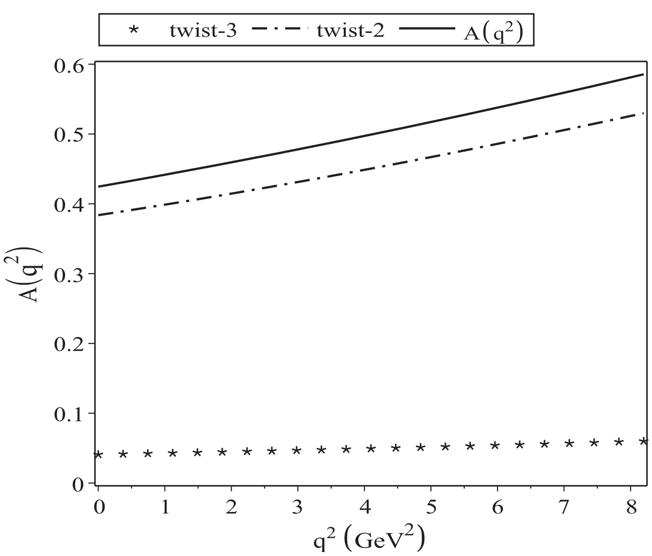


FIG. 2. Form factor A on q^2 as well as the contributions of twist-2 and twist-3 distribution amplitudes in this form factor.

TABLE IV. Transition form factors of the $B \to a_1 \ell^+ \ell^-$ at $q^2 = 0$ in various theoretical approaches. The results of other methods have been rescaled according to the form factor definition in Eq. (2).

Theoretical approach	$A(0)$	$V_1(0)$	$V_2(0)$	$V_0(0)$	$T_1(0) = T_2(0)$	$T_3(0)$
CQM[14]	0.26	0.43	0.14	0.34	0.34	0.19
3PSR [15]	0.31	0.52	0.25	0.76	0.37	0.41
This work	0.42	0.68	0.31	0.30	0.44	0.41

TABLE V. The parameter values for the fitted form factors.

Parameter	A	V_1	V_2	V_0	T_1	T_2	T_3
$F(0)$	0.42	0.68	0.31	0.30	0.44	0.44	0.41
α	1.09	0.73	0.84	0.77	0.57	0.63	0.40
β	0.55	0.35	0.47	0.37	0.38	0.32	3.58

at the zero transferred momentum square $q^2 = 0$ is 0.67, which is about ~99% of the total value, while the contribution of the other part of the propagator is about 1%. Table III shows the contribution of the b quark free propagator in the form factor calculations at $q^2 = 0$. As can be seen in Table III, the main contribution comes from the free propagator. So by taking into account the full propagator instead of the free propagator, correction made in the form factor values at the zero transferred momentum square $q^2 = 0$ is very small.

In this work, the form factors are estimated in the LCSR approach up to twist-4 distribution amplitudes of the axial vector meson a_1. Our calculations show that the most contribution comes from twist-2 functions for all form factors. Also, the LCDAs $\Phi_\perp$ play the most important role in this contribution. Figure 2 depicts the twist-2 and twist-3 contributions in the form factor formula $A(q^2)$. In this form factor, the twist-4 function does not contribute. Several authors have calculated the form factors of the $B \to a_1 \ell^+ \ell^-$ decay via different approaches. To compare the different results, we should rescale them according to the form factor definition in Eq. (2). Table IV shows the values of the rescaled form factors at $q^2 = 0$ according to different approaches. In order to extend our results to the whole physical region $4m_\ell^2 \le q^2 \le (m_B - m_{a_1})^2$, we use the following parametrization of the form factors with respect to q^2 as

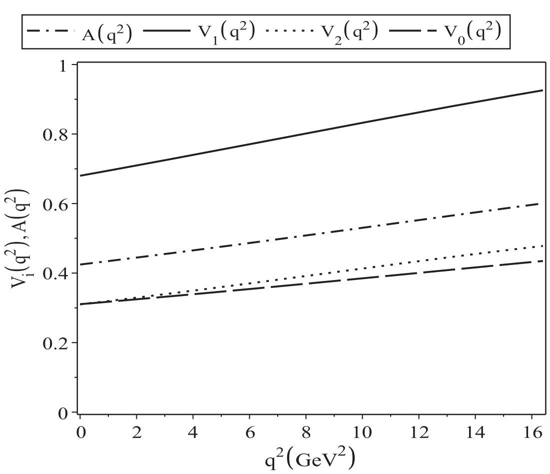
$$F_k(q^2) = \frac{F_k(0)}{1 - \alpha s + \beta s^2}, \tag{21}$$

where $s = q^2/m_B^2$ and $F_k(q^2)$ denote for the form factors, A, $V_i (i = 0, 1, 2)$ and $T_j (j = 1, 2, 3)$. The values of $F_k(0)$, α, and β for the parametrized form factors are given in Table V. The fitted form factors with respect to q^2 are shown in Fig. 3.

We have calculated the form factor values of the $B \to \rho \ell^+ \ell^-$ at $q^2 = 0$ in the LCSR model shown in Table VI. Also, this table contains the results estimated for these form factors in [5,27]. The predicted values by us and Refs. [5,27] are very close to each other in many cases.

In the standard model, the rare semileptonic $B \to a_1 \ell^+ \ell^-$ and $B \to \rho \ell^+ \ell^-$ decays are described via the loop transitions, $b \to d \ell^+ \ell^-$, at quark level. Both mesons a_1 and ρ have the same quark content, but different masses and parities, i.e., ρ is a vector (1^-) and a_1 is an axial vector (1^+). If a_1 behaves as an axial vector partner of the ρ meson, it is expected that $V_1(0)$ for the $B \to a_1$ decays is similar to $A_1(0)$ for the $B \to \rho$ transitions, for example. The values obtained for $V_1(0)$ in Table IV are larger than those for $A_1(0)$ in Table VI. It appears to us that the transition form factors of the $B \to a_1$ decays are quite different from those for $B \to \rho$.

Now, we consider the forward-backward asymmetry A^{FB} for the $B \to a_1 \ell^+ \ell^- (\ell = \mu, \tau)$ decays. The expression of



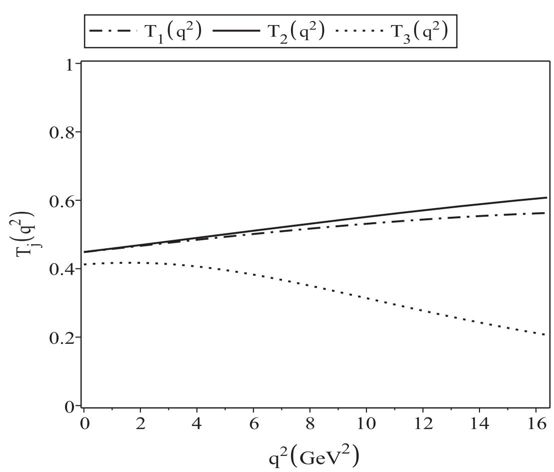


FIG. 3. The form factors A, V_i, and T_j on q^2.

TABLE VI. The form factor values of the $B \to \rho \ell^+ \ell^-$ at $q^2 = 0$.

Model	$V(0)$	$A_1(0)$	$A_2(0)$	$A_0(0)$	$T_1(0) = T_2(0)$	$T_3(0)$
LCSR [27]	0.32 ± 0.03	0.26 ± 0.02	0.22 ± 0.02	0.36 ± 0.04	0.27 ± 0.02	0.14 ± 0.01
LCSR [5]	0.34 ± 0.05	0.26 ± 0.04	0.22 ± 0.03	0.37 ± 0.05	0.29 ± 0.04	0.20 ± 0.03
This work	0.39 ± 0.10	0.30 ± 0.06	0.28 ± 0.05	0.34 ± 0.09	0.32 ± 0.07	0.21 ± 0.04

the A^{FB} is given in [25]. The dependence of A^{FB} for the aforementioned decays on q^2 with and without LD effects is plotted in Fig. 4.

Finally, we can evaluate the branching ratio values for the FCNC $B \to a_1 \ell^+ \ell^-$ decays and the radiative $B \to a_1 \gamma$. For the radiative $B \to a_1 \gamma$ transition, the exclusive decay width is given as [28]

$$\Gamma(B \to a_1 \gamma) = \frac{\alpha_{\rm em} G_F^2 m_b^5}{32\pi^4} |V_{tb} V_{td}^* C_7(m_b) T_1(0)|^2 \times \left(1 - \frac{m_{a_1}^2}{m_B^2}\right)^3 \left(1 + \frac{m_{a_1}^2}{m_B^2}\right). \quad (22)$$

Also, the ratio of the exclusive-to-inclusive radiative decay branching ratio is defined as

$$R \equiv \frac{\mathrm{BR}(B \to a_1 \gamma)}{\mathrm{BR}(B \to X_b \gamma)} = |T_1(0)|^2 \frac{(1 - m_{a_1}^2/m_B^2)^3 (1 + m_{a_1}^2/m_B^2)}{(1 - m_d^2/m_b^2)^3 (1 + m_d^2/m_b^2)}. \quad (23)$$

R is a quantity to test the model dependence of the form factors for the exclusive decay [28]. Using the value of $T_1(0)$, we estimate the branching ratio $\mathrm{Br}(B \to a_1 \gamma) = 5.6 \times 10^{-7}$ and the corresponding ratio $R = 8.81\%$. Our prediction means that about 8.81% of the inclusive $b \to d\gamma$ branching ratio goes into the a_1 channel.

In this paper, we compute the branching ratio values of $B \to a_1 \ell^+ \ell^-$ decays in leading order, completely. For this aim, we have to add the contributions of the weak annihilation amplitude of diagram (c) to the form factor amplitude related to diagrams (a) and (b) in Fig. 5. Diagram (c) is related to the nonfactorizable effects in leading order. They arise from electromagnetic corrections to the matrix elements of purely hadronic operators in the weak effective Hamiltonian. Since the matrix elements of the semileptonic operators $O_{9,10}$ can be expressed through $B \to a_1$ form factors, nonfactorizable corrections contribute to the decay amplitude only through the production of a virtual photon, which then decays into the lepton pair [29].

After calculation of this amplitude, it is straightforward to derive the differential decay rate for $B \to a_1 \ell^+ \ell^-$ decays. The explicit expressions of our calculations for the amplitude and differential decay rate have been presented in Appendix C.

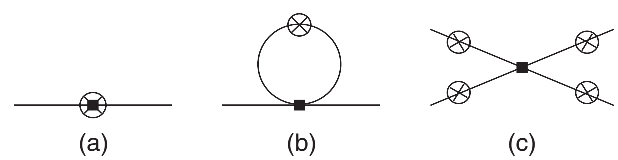


FIG. 5. Factorizable and nonfactorizable contributions in leading order. The circled cross marks the possible insertions of the virtual photon line.

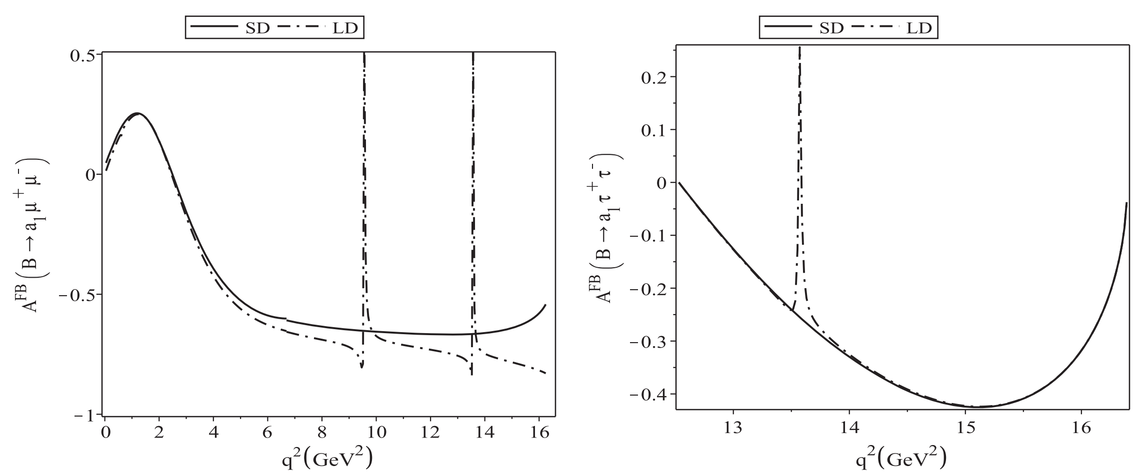


FIG. 4. The dependence of the forward-backward asymmetry on q^2. The solid and dash-dotted lines show the results without and with the LD effects, respectively.

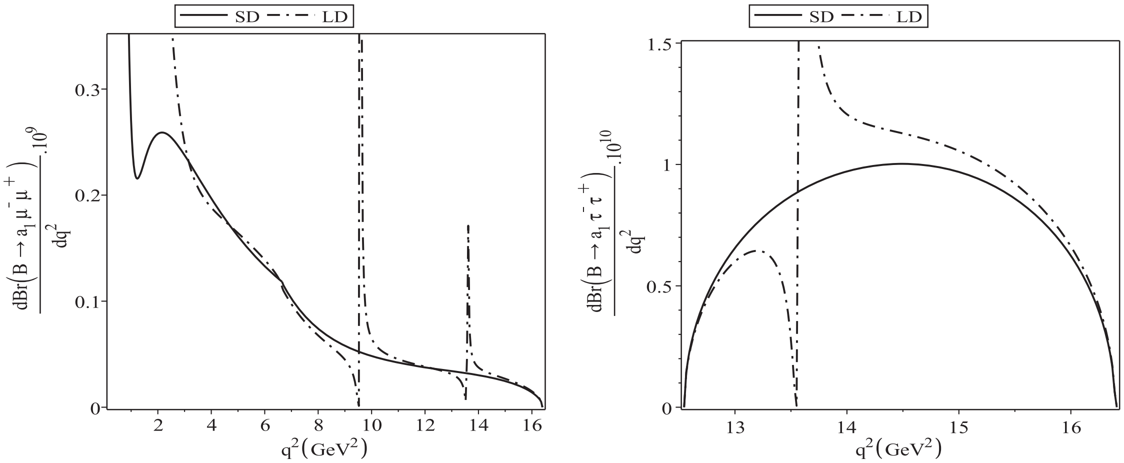


FIG. 6. The differential branching ratios of the semileptonic $B \to a_1$ decays on q^2 with and without LD effects.

We show the dependency of the differential branching ratios of $B \to a_1 \ell^+ \ell^- (\ell = \mu, \tau)$ decays on q^2, with and without LD effects [see Eq. (2)], in Fig. 6. In this figure, the solid and dash-dotted lines show the results without and with the LD effects, respectively. To obtain the branching ratio values of these decays, some cuts around the narrow resonances of J/ψ and ψ' are defined for the muon as

$$\begin{aligned}
&\mathrm{I}:\ 2m_\mu \le \sqrt{q^2} \le M_{J/\psi} - 0.20, \\
&\mathrm{II}:\ M_{J/\psi} + 0.04 \le \sqrt{q^2} \le M_{\psi'} - 0.10, \\
&\mathrm{III}:\ M_{\psi'} + 0.02 \le \sqrt{q^2} \le m_B - m_{a_1},
\end{aligned} \tag{24}$$

and for τ, the following two regions are introduced:

$$\begin{aligned}
&\mathrm{I}:\ 2m_\tau \le \sqrt{q^2} \le M_{\psi'} - 0.02, \\
&\mathrm{II}:\ M_{\psi'} + 0.02 \le \sqrt{q^2} \le m_B - m_{a_1}.
\end{aligned} \tag{25}$$

In Table VII, the branching ratio values for $B \to a_1 \ell^+ \ell^- (\ell = \mu, \tau)$ have been obtained using the regions shown in Eqs. (24) and (25). The results have been neglected for the electron since these are very close to the same as those for the muon.

Our calculation shows that the nonfactorizable correction is less than 1% in branching ratio values in leading order. Therefore, it could be easily ignored in our calculations.

In summary, we investigated the form factors of the FCNC B decays in the a_1 axial vector meson in the LCSR approach up to the twist-4 LCDAs. Considering both the SD and LD effects contributing to the Wilson coefficient C_9^{eff}, the dependence of the forward-backward asymmetry of the decays $B \to a_1 \mu^+ \mu^-$ and $B \to a_1 \tau^+ \tau^-$ was plotted with respect to q^2. Finally, we calculated the branching ratio value for the semileptonic decay $B \to a_1 \gamma$. Also, the branching ratio values of the $B \to a_1 \ell^+ \ell^- (\ell = \mu, \tau)$ decays were completely estimated in leading order by using the nonfactorizable effects. We found that the nonfactorizable correction is very small.

APPENDIX A: TWIST FUNCTION DEFINITIONS

In this appendix, we present the definitions for $\Phi_\parallel$, $\Phi_\perp$, $g_\perp^{(a)}$, $g_\perp^{(v)}$, $h_\parallel^{(t)}$, $h_\parallel^{(p)}$, and g_3.

The functions $\Phi_\parallel$ and $\Phi_\perp$ for the a_1 meson are defined as [13]

$$\begin{aligned}
\Phi_\parallel(u) &= 6u\bar{u}\left[a_0^\parallel + 3a_1^\parallel \xi + a_2^\parallel \frac{3}{2}(5\xi^2 - 1)\right], \\
\Phi_\perp(u) &= 6u\bar{u}\left[1 + 3a_1^\perp \xi + a_2^\perp \frac{3}{2}(5\xi^2 - 1)\right],
\end{aligned} \tag{A1}$$

where $\xi = 2u - 1$. Also u and $\bar{u} = 1 - u$ refer to the momentum fractions carried by the quark and antiquark, respectively, in the axial vector meson a_1. These LCDAs are normalized as the normalization conditions

TABLE VII. The branching ratios of the semileptonic $B \to a_1 \ell^+ \ell^-$ decays including LD effects in three regions.

Mode	I	II	III	I + II + III
$\mathrm{Br}(B \to a_1 \mu^+ \mu^-) \times 10^8$	2.21 ± 0.54	0.25 ± 0.07	0.06 ± 0.01	2.52 ± 0.62
$\mathrm{Br}(B \to a_1 \tau^+ \tau^-) \times 10^9$	Undefined	0.14 ± 0.03	0.17 ± 0.03	0.31 ± 0.06

$$\int_0^1 du\Phi_{\parallel}(u) = 1, \qquad \int_0^1 du\Phi_{\perp}(u) = a_0^{\perp}. \tag{A2}$$

Up to conformal spin 9/2, the approximate expressions for $g_{\perp}^{(a)}$, $g_{\perp}^{(v)}$, $h_{\parallel}^{(t)}$, and $h_{\parallel}^{(p)}$ are taken as

$$\begin{aligned}
g_{\perp}^{(a)}(u) &= \frac{3}{4}(1+\xi^2) + \frac{3}{2}a_1^{\parallel}\xi^3 + \left(\frac{3}{7}a_2^{\parallel} + 5\zeta_{3,a_1}^{V}\right)(3\xi^2-1) + \left(\frac{9}{112}a_2^{\parallel} + \frac{105}{16}\zeta_{3,a_1}^{A} - \frac{15}{64}\zeta_{3,a_1}^{V}\omega_{a_1}^{V}\right) \\
&\quad \times (35\xi^4 - 30\xi^2 + 3) + 5\left[\frac{21}{4}\zeta_{3,a_1}^{V}\sigma_{a_1}^{V} + \zeta_{3,a_1}^{A}\left(\lambda_{a_1}^{A} - \frac{3}{16}\sigma_{a_1}^{A}\right)\right]\xi(5\xi^2-3) - \frac{9}{2}a_1^{\perp}\tilde{\delta}_{+} \\
&\quad \times \left(\frac{3}{2} + \frac{3}{2}\xi^2 + \ln u + \ln\bar{u}\right) - \frac{9}{2}a_1^{\perp}\tilde{\delta}_{-}(3\xi + \ln\bar{u} - \ln u), \\
g_{\perp}^{(v)}(u) &= 6u\bar{u}\left\{1 + \left(a_1^{\parallel} + \frac{20}{3}\zeta_{3,a_1}^{A}\lambda_{a_1}^{A}\right) + \left[\frac{1}{4}a_2^{\parallel} + \frac{5}{3}\zeta_{3,a_1}^{V}\left(1 - \frac{3}{16}\omega_{a_1}^{V}\right) + \frac{35}{4}\zeta_{3,a_1}^{A}\right]\xi(5\xi^2-1)\right. \\
&\quad \left. + \frac{35}{4}\left(\zeta_{3,a_1}^{V}\sigma_{a_1}^{V} - \frac{1}{28}\zeta_{3,a_1}^{A}\sigma_{a_1}^{A}\right)\xi(7\xi^2-3)\right\} - 18a_1^{\perp}\tilde{\delta}_{+}(3u\bar{u} + \bar{u}\ln\bar{u} + u\ln u) - 18a_1^{\perp}\tilde{\delta}_{-}(u\bar{u}\xi + \bar{u}\ln\bar{u} - u\ln u), \\
h_{\parallel}^{(t)}(u) &= 3a_0^{\perp}\xi^2 + \frac{3}{2}a_1^{\perp}\xi(3\xi^2-1) + \frac{3}{2}\left[a_2^{\perp}\xi + \zeta_{3,a_1}^{\perp}\left(5 - \frac{\omega_{a_1}^{\perp}}{2}\right)\right]\xi(5\xi^2-3) + \frac{35}{4}\zeta_{3,a_1}^{\perp}\sigma_{a_1}^{\perp} \\
&\quad \times (35\xi^4 - 30\xi^2 + 3) + 18a_2^{\parallel}\left[\tilde{\delta}_{+}\xi - \frac{5}{8}\tilde{\delta}_{-}(3\xi^2-1)\right] - \frac{3}{2}(1+6a_2^{\parallel})(\tilde{\delta}_{+}\xi[2 + \ln(\bar{u}u)] + \tilde{\delta}_{-}[1 + \xi\ln(\bar{u}/u)]), \\
h_{\parallel}^{(p)}(u) &= 6u\bar{u}\left\{a_0^{\perp} + \left[a_1^{\perp} + 5\zeta_{3,a_1}^{\perp}\left(1 - \frac{1}{40}(7\xi^2-3)\omega_{a_1}^{\perp}\right)\right]\xi + \left(\frac{1}{4}a_2^{\perp} + \frac{35}{6}\zeta_{3,a_1}^{\perp}\sigma_{a_1}^{\perp}\right)\right. \\
&\quad \left. \times (5\xi^2-1) - 5a_2^{\parallel}\left[\tilde{\delta}_{+}\xi + \frac{3}{2}\tilde{\delta}_{-}(1-\bar{u}u)\right]\right\} - 3(1+6a_2^{\parallel})[\tilde{\delta}_{+}(\bar{u}\ln\bar{u} - u\ln u) + \tilde{\delta}_{-}(u\bar{u} + \bar{u}\ln\bar{u} + u\ln u)],
\end{aligned} \tag{A3}$$

where

$$\tilde{\delta}_{\pm} = \frac{f_{a_1}^{\perp}}{f_{a_1}}\frac{m_d \pm m_d}{m_{a_1}}, \qquad \zeta_{3,a_1}^{V(A)} = \frac{f_{3,a_1}^{V(A)}}{f_{a_1}m_{a_1}}.$$

In the $SU(3)$ limit, the normalization conditions for $g_{\perp}^{(a)}$, $g_{\perp}^{(v)}$, $h_{\parallel}^{(t)}$, and $h_{\parallel}^{(p)}$ are defined as

$$\int_0^1 du g_{\perp}^{(a)}(u) = \int_0^1 du g_{\perp}^{(v)}(u) = 1, \qquad \int_0^1 du h_{\parallel}^{(t)}(u) = a_0^{\perp}, \qquad \int_0^1 du h_{\parallel}^{(p)}(u) = a_0^{\perp} + \tilde{\delta}_{-}. \tag{A4}$$

The definition of the function $g_3(u)$ is as follows [30]:

$$g_3(u) = 6u(1-u) + (1-3\xi^2)\left[\frac{1}{7}a_2^{\parallel} - \frac{20}{3}\frac{f_{3,a_1}^{A}}{f_{a_1}m_{a_1}}\right]. \tag{A5}$$

APPENDIX B: FORM FACTOR EXPRESSIONS

In this appendix, the explicit expressions for the form factors of the FCNC $B \to a_1$ decays are presented.

$$A(q^2) = \frac{f_{a_1} m_b}{4m_B^2 f_B}(m_{a_1} - m_B)\left\{\frac{f_{a_1}^{\perp}}{f_{a_1}}\int_{u_0}^1 du \frac{9\Phi_{\perp}(u)}{u} e^{s(u)} + \frac{m_b}{4m_{a_1}}\int_{u_0}^1 du \frac{g_{\perp}^{(v)\prime}(u)}{u} e^{s(u)} \right.$$
$$\left. - \frac{m_b}{4m_{a_1}}\int_{u_0}^1 du \frac{g_{\perp}^{(v)}(u)}{u^2}\left[1 + \frac{\delta_1(u) - 8m_{a_1}^2}{M^2}\right] e^{s(u)} + \frac{f_{a_1}^{\perp} m_{a_1}^2}{f_{a_1}}\int_{u_0}^1 du \frac{32\bar{h}_{\parallel}^{(t)(ii)}(u)}{M^2} e^{s(u)}\right\},$$

$$V_1(q^2) = -\frac{m_b}{8m_B^2 f_B}\frac{f_{a_1}^{\perp}}{(m_B - m_{a_1})}\left\{\frac{1}{2}\int_{u_0}^1 du \frac{7\Phi_{\perp}(u)\delta_1(u)}{u} e^{s(u)} + 2m_{a_1}^2\int_{u_0}^1 du \frac{h_{\parallel}^{(p)}(u)}{u} e^{s(u)} \right.$$
$$- 3\frac{f_{a_1}}{f_{a_1}^{\perp}} m_{a_1} m_b \int_{u_0}^1 du \frac{g_{\perp}^{(a)}(u)}{u} e^{s(u)} - 4m_{a_1}^3 m_b \int_{u_0}^1 du \frac{\bar{g}_3^{(ii)}(u)}{u^2 M^2} e^{s(u)} - 8m_{a_1}^2$$
$$\left. \times \int_{u_0}^1 du \frac{\bar{h}_{\parallel}^{(t)(ii)}(u)}{u^2} e^{s(u)} + 4m_{a_1}^2 \frac{m_b}{f_{a_1}^{\perp}}\int_{u_0}^1 du \int \mathcal{D}\underline{\alpha}\left[\frac{f_{3,a_1}^A \mathcal{A}(\alpha_i) - f_{3,a_1}^V \mathcal{V}(\alpha_i)}{\kappa^2 M^2}\right] e^{s(\kappa)}\right\},$$

$$V_2(q^2) = -\frac{f_{a_1}^{\perp} m_b}{4m_B^2 f_B}(m_B - m_{a_1})\left\{18\int_{u_0}^1 du \frac{\Phi_{\perp}(u)}{u} e^{s(u)} + \frac{4f_{a_1} m_{a_1} m_B}{f_{a_1}^{\perp}}\int_{u_0}^1 du \frac{\phi_a(u)}{u^2 M^2} e^{s(u)} \right.$$
$$+ 4m_{a_1}^2 \int_{u_0}^1 du \frac{h_{\parallel}^{(p)}(u)}{u}(1 + 2u) e^{s(u)} - 8f_{a_1} m_{a_1}^3 m_b \int_{u_0}^1 du \frac{\bar{g}_3^{(ii)}(u)}{u^3 M^4} e^{s(u)}$$
$$\left. + \frac{16 f_{a_1}}{f_{a_1}^{\perp}} m_{a_1} m_b \int_{u_0}^1 du \frac{\Phi_{\parallel}^{(i)}(u)}{u^2 M^2} e^{s(u)} - 16m_{a_1}^2 \int_{u_0}^1 du \frac{\bar{h}_{\parallel}^{(t)(ii)}(u)}{u^2}\left[\frac{2\delta_3(u)}{uM^4} - \frac{3}{2M^2} + \frac{\delta_1(u)}{4uM^4}\right] e^{s(u)}\right\},$$

$$V_0(q^2) = V_3(q^2) + \frac{m_b}{8m_B^2 f_B}\frac{f_{a_1}^{\perp} q^2}{m_{a_1}}\left\{9\int_{u_0}^1 du \frac{\Phi_{\perp}(u)}{u} e^{s(u)} + 2f_{a_1} m_{a_1} m_B \int_{u_0}^1 du \frac{\phi_a(u)}{u^2 M^2} e^{s(u)} \right.$$
$$- 4m_{a_1}^2 \int_{u_0}^1 du \frac{h_{\parallel}^{(p)}(u)}{u}(1 - u) e^{s(u)} - 4\frac{f_{a_1}}{f_{a_1}^{\perp}} m_{a_1}^3 m_b \int_{u_0}^1 du \frac{\bar{g}_3^{(ii)}(u)}{u^3 M^4}(1 - u) e^{s(u)}$$
$$+ \frac{16 f_{a_1}}{f_{a_1}^{\perp}} m_{a_1} m_b \int_{u_0}^1 du \frac{\Phi_{\parallel}^{(i)}(u)}{u^2 M^2} e^{s(u)} + 8m_{a_1}^2 \int_{u_0}^1 du \frac{\bar{h}_{\parallel}^{(t)(ii)}(u)}{u^2}\left[\frac{2\delta_3(u)}{uM^4} - \frac{1}{M^2} + (1 - u)\right.$$
$$\left.\left. \times \left(-\frac{1}{M^2} + \frac{\delta_1(u)}{2uM^4}\right)\right] e^{s(u)}\right\},$$

$$T_1(q^2) = -\frac{f_{a_1} m_b}{8m_B^2 f_B}\left\{m_b\left(\frac{f_{a_1}^{\perp}}{f_{a_1}} + 8\right)\int_{u_0}^1 du \frac{\Phi_{\perp}(u)}{u} e^{s(u)} - 3m_{a_1}\int_{u_0}^1 du g_{\perp}^{(a)}(u) e^{s(u)} \right.$$
$$+ 4m_{a_1}\int_{u_0}^1 du \frac{\phi_a(u)}{u} e^{s(u)} - \frac{f_{a_1}}{m_{a_1}}\int_{u_0}^1 du \frac{g_{\perp}^{(v)\prime}(u)\delta_5(u)}{u} e^{s(u)} - \frac{m_{a_1}}{8}\int_{u_0}^1 du \frac{g_{\perp}^{(v)\prime}(u)}{u}$$
$$\times \left[7 - \frac{\delta_5(u)(8u - 1)}{M^2} + \frac{u\delta_2(u) - \delta_4(u)}{2m_a^2}\right] e^{s(u)} + 4m_{a_1}\int_{u_0}^1 du \frac{\Phi_{\parallel}^{(i)}(u)}{u} e^{s(u)} - 4m_{a_1}^3$$
$$\left. \times \int_{u_0}^1 du \frac{\bar{g}_3^{(ii)}(u)}{uM^2} e^{s(u)} - \frac{16 f_{a_1}^{\perp}}{f_{a_1}} m_{a_1}^2 m_b \int_{u_0}^1 du \frac{\bar{h}_{\parallel}^{(t)(ii)}(u)}{uM^2} e^{s(u)} + 4\frac{f_{3,a_1}^A}{f_{a_1}}\int_{u_0}^1 du \int \mathcal{D}\underline{\alpha}\frac{u\mathcal{A}(\alpha_i)}{\kappa^2}\left[1 + \frac{\delta_1(\kappa)}{M^2}\right] e^{s(\kappa)}\right\},$$

$$
\begin{aligned}
T_2(q^2) = {} & \frac{m_b}{m_B^2 f_B}\frac{f_{a_1}}{m_{a_1}^2 - m_B^2}\bigg\{\frac{m_b f_{a_1}^\perp}{f_{a_1}}\int_{u_0}^1 du \frac{\Phi_\perp(u)\delta_1(u)}{u} e^{s(u)} + \frac{1}{2} m_{a_1}\int_{u_0}^1 du \frac{g_\perp^{(a)}(u)}{u} \\
& \times [\delta_1(u) + 4\delta_5(u)] e^{s(u)} - \frac{1}{16} m_{a_1}\int_{u_0}^1 du \frac{g_\perp^{(v)\prime}(u)\delta_2(u)}{u} e^{s(u)} + \frac{1}{2} m_{a_1}\int_{u_0}^1 du \frac{\phi_a(u)}{u} \\
& \times \delta_1(u) e^{s(u)} + m_{a_1}\int_{u_0}^1 du \frac{g_\perp^{(v)}(u)}{u^2}\bigg[\delta_6(u) + \frac{\delta_1(u)\delta_5(u)}{M^2} + \frac{u\delta_2(u)}{2}\bigg(1 + \frac{\delta_3(u)}{uM^2} + \frac{\delta_7(u)}{u}\bigg) \\
& + u\bigg(m_{a_1}^2 - 2\delta_1(u) + \frac{\delta_4(u)}{2} + \frac{\delta_5(u)\delta_1(u)}{uM^2}\bigg)\bigg] e^{s(u)} + 2m_{a_1}^3\int_{u_0}^1 du \frac{\bar{g}_3^{(ii)}(u)}{u^2}\bigg[5 - \frac{\delta_5(u)}{M^2}\bigg] e^{s(u)} \\
& - 2m_{a_1}\int_{u_0}^1 du \frac{\Phi_\parallel^{(i)}(u)\delta_2(u)}{u} e^{s(u)} - 8\frac{f_{a_1}^\perp}{f_{a_1}} m_{a_1}^2 m_b\int_{u_0}^1 du \frac{\bar{h}_\parallel^{(t)(ii)}(u)}{u^2}\bigg[1 + \frac{\delta_2(u)}{M^2}\bigg] e^{s(u)} \\
& - \frac{1}{2}\frac{f_{3,a_1}^V}{f_{a_1}}\int_{u_0}^1 du \int \mathcal{D}\underline{\alpha}\frac{\mathcal{V}(\alpha_i)}{\kappa^2}\bigg[\delta_4(\kappa) + \frac{\delta_1(\kappa)\delta_2(\kappa)}{M^2} + u m_{a_1}^2\bigg(1 + \frac{\delta_2(\kappa)}{M^2}\bigg)\bigg] e^{s(\kappa)} \\
& - \frac{f_{3,a_1}^A}{f_{a_1}}\int_{u_0}^1 du \int \mathcal{D}\underline{\alpha}\frac{\mathcal{A}(\alpha_i)}{\kappa^2}\bigg[m_{a_1}^2\frac{\delta_2(\kappa)}{M^2} - 4u\bigg(m_{a_1}^2 + \frac{\delta_4(\kappa)}{4} + \frac{\delta_1(\kappa)\delta_2(\kappa)}{M^2}\bigg)\bigg] e^{s(\kappa)}\bigg\},
\end{aligned}
$$

$$
\begin{aligned}
T_3(q^2) = {} & -\frac{f_{a_1} m_b}{4m_B^2 f_B}\bigg\{\frac{8f_{a_1}^\perp}{f_{a_1}} m_b\int_{u_0}^1 du \frac{\Phi_\perp(u)}{u} e^{s(u)} - 4m_{a_1}\int_{u_0}^1 du \frac{g_\perp^{(a)}(u)}{u} e^{s(u)} + 2m_{a_1}^3 \\
& \times \int_{u_0}^1 du \frac{\bar{g}_3^{(ii)}(u)}{u^2}\bigg[\frac{8}{M^2} - \frac{2\delta_5(u)}{M^4}\bigg] e^{s(u)} - \frac{1}{4m_{a_1}}\int_{u_0}^1 du \frac{g_\perp^{(v)\prime}(u)}{u}\bigg[\frac{7}{2}\delta_2(u) + m_{a_1}^2 \\
& - \frac{\delta_1(u)}{4u}\bigg] e^{s(u)} - m_{a_1}\int_{u_0}^1 du \frac{\phi_a(u)}{u^2}\bigg[\frac{1}{u} + \frac{u\delta_1(u) + 2\delta_2(u)}{M^2}\bigg] e^{s(u)} - 4m_{a_1}\int_{u_0}^1 du \frac{\Phi_\parallel^{(i)}(u)}{u^2} \\
& \times \bigg[\frac{u\delta_5(u) - \delta_2(u)}{M^2} - 1\bigg] e^{s(u)} - \frac{1}{4m_{a_1}}\int_{u_0}^1 du \frac{g_\perp^{(v)}(u)}{u}\bigg[\frac{3\delta_1(u)}{uM^2} - \frac{\delta_1(u)}{m_{a_1}^2} + \frac{5\delta_5(u) - 7\delta_3(u)}{M^2} \\
& + \frac{\delta_2(u)}{M^2} - \frac{\delta_1(u)^2}{m_{a_1}^2 M^2}\bigg] e^{s(u)} + \frac{16 f_{a_1}^\perp}{f_{a_1}} m_{a_1}^2 m_b\int_{u_0}^1 du \frac{\bar{h}_\parallel^{(t)(ii)}(u)}{u^2 M^2}\bigg[8 + \frac{2}{u} + \frac{\delta_2(u)}{uM^2}\bigg] e^{s(u)}\bigg\},
\end{aligned}
$$

where

$$
u_0 = \frac{1}{2m_{a_1}^2}\bigg[\sqrt{(s_0 - m_{a_1}^2 - q^2)^2 + 4m_{a_1}^2(m_b^2 - q^2)} - (s_0 - m_{a_1}^2 - q^2)\bigg], \qquad s(u) = -\frac{1}{uM^2}[m_b^2 + u\bar{u}m_{a_1}^2 - \bar{u}q^2] + \frac{m_B^2}{M^2},
$$

$$
\delta_1(u) = m_{a_1}^2(u+2) + \frac{m_b^2}{u} + \frac{q^2}{u}, \qquad \delta_2(u) = u m_{a_1}^2 - \frac{m_b^2}{u} + q^2\frac{u - \bar{u}}{u}, \qquad \delta_3(u) = \frac{m_b^2}{u} - 2q^2\frac{\bar{u}}{u},
$$

$$
\delta_4(u) = 2m_{a_1}^2(u+1) + 2q^2, \qquad \delta_5(u) = u m_{a_1}^2 - \frac{m_b^2}{u} + \frac{q^2(u-2)}{u}, \qquad \delta_6(u) = 2m_{a_1}^2(u+1) + q^2\frac{\bar{u}}{u},
$$

$$
\delta_7(u) = -2\frac{m_b^2}{u} + \frac{q^2}{u}, \qquad f^{(i)}(u) \equiv \int_0^u f(v)dv, \qquad f^{(ii)}(u) \equiv \int_0^u dv\int_0^v d\omega f(\omega),
$$

$$
\phi_a = \int_0^u [\Phi_\parallel - g_\perp^{(a)}(v)]dv, \qquad \kappa = \alpha_1 + u\alpha_3.
$$

APPENDIX C: NONFACTORIZABLE CORRECTION

In this appendix, we present the leading-order results of the nonfactorizable corrections for the amplitude and differential decay rate of the $B \to a_1 \ell^+ \ell^-$ decays. The nonfactorizable corrections for the amplitude are obtained by computing matrix elements of four-quark operators O_{1-6} represented by diagrams (c) in Fig. 5. Using the a_1 and B meson distribution amplitudes and after some calculations, we obtain the expression for amplitude $\mathcal{M}$ as

$$\mathcal{M} = \frac{G_F\alpha_{\rm em}}{2\sqrt{2}\pi}\Bigg(4\rho_1\int_0^1 d\xi\{[i\rho_2\varepsilon_{\mu\nu\alpha\beta}\epsilon_\lambda^{*\nu}p^\alpha p'\beta - \rho_3 p_\mu(\epsilon^*\cdot q) + \rho_3 q_\mu(\epsilon^*\cdot p) - \rho_3(p\cdot q)\epsilon_\mu^*]\varphi_1(\xi)$$
$$+ 2\rho_3(\epsilon^*\cdot p)[Q_b(\bar{\xi}p-q)_\mu A_1(\xi) + Q_u(-\xi p+q)_\mu A_2(\xi)] + im_b\alpha_2\varepsilon_{\mu\nu\alpha\beta}\epsilon_\lambda^{*\nu}n_-^\alpha q^\beta\varphi_2(\xi)$$
$$- m_b\rho_3[(\epsilon^*\cdot q)(n_-)_\mu + (\epsilon^*\cdot n_-)q_\mu]\varphi_3(\xi) + m_b m_B\rho_3\epsilon_\mu^*\varphi_2(\xi) + 2m_b\rho_3(\epsilon^*\cdot n_-)[Q_b(\bar{\xi}p-q)_\mu A_3(\xi)$$
$$+ Q_u(\xi p-q)_\mu A_4(\xi)]\} + 16\rho_4\rho_3(\epsilon^*\cdot n_-)\int_0^1 du\{2[Q_d(up'+q)_\mu A_5(u) + Q_u(\bar{u}p'+q)_\mu A_6(u)] - q_\mu\varphi_4(u)\}$$
$$+ \rho_5[iF_1(q^2)\varepsilon_{\mu\nu\alpha\beta}\epsilon_\lambda^{*\nu}p^\alpha p'^\beta + F_2(q^2)\epsilon_\mu^{*\lambda} - F_3(q^2)(\epsilon^*\cdot q)(p+p')_\mu + F_4(q^2)(\epsilon^*\cdot q)q_\mu]\Bigg)(\ell^-(q_2)\gamma^\mu\ell^+(q_1)),$$

where $\rho_1 = \frac{f_a f_B m_a}{4q^2}$, $\rho_2 = \beta_1 + \beta_2$, $\rho_3 = \beta_1 - \beta_2$, $\rho_4 = \frac{f_a^\perp f_B}{4\hat{s}}$, $\rho_5 = V_{td}V_{tb}^*$, $\beta_1 = V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^* a_4$, $\beta_2 = -V_{tb}V_{td}^* a_6$, $a_1 = C_1 + \frac{C_2}{3}$, $a_4 = C_4 + \frac{C_3}{3}$, and $a_6 = C_6 + \frac{C_5}{3}$. The functions A_i, φ_j, and F_k are defined as

$$A_1(\xi) = \frac{\phi_{B_1}(\xi)m_B^{-2}}{\xi\hat{s} + (1-\xi)(1+\hat{r})}, \qquad A_2(\xi) = \frac{\phi_{B_1}(\xi)m_B^{-2}}{(1-\xi)\hat{s} + \xi(1+\hat{r})}, \qquad A_3(\xi) = \frac{\phi_{B_2}(\xi)m_B^{-2}}{\xi\hat{s} + (1-\xi)(1+\hat{r})},$$
$$A_4(\xi) = \frac{\phi_{B_2}(\xi)m_B^{-2}}{(1-\xi)\hat{s} + \xi(1+\hat{r})}, \qquad A_5(u) = \frac{\phi_\perp(u)m_B^{-2}}{(1-u)\hat{s} + u(1-\hat{r})}, \qquad A_6(u) = \frac{\phi_\perp(u)m_B^{-2}}{u\hat{s} + (1-u)(1-\hat{r})},$$
$$\varphi_1(\xi) = Q_u A_2(\xi) - Q_b A_1(\xi), \qquad \varphi_2(\xi) = Q_b A_3(\xi) - Q_u A_4(\xi),$$
$$\varphi_3(\xi) = Q_b A_3(\xi) + Q_u A_4(\xi), \qquad \varphi_4(u) = Q_d A_5(u) + Q_u A_6(u),$$

and

$$F_1(q^2) = 2\left[\kappa_0 T_1(q^2) - \frac{C_0}{M_0}A(q^2)\right], \qquad F_2(q^2) = \kappa_0 T_2(q^2) - C_0 M_0 V_1(q^2),$$
$$F_3(q^2) = \kappa_0\left[T_2(q^2) + \frac{q^2}{M_0'}T_3(q^2)\right] - \frac{C_0}{M_0}V_2(q^2), \qquad F_4(q^2) = \kappa_0 T_3(q^2) + 2\frac{C_0 m_{a_1}}{q^2}[V_3(q^2) - V_0(q^2)],$$

where $\hat{s} = \frac{q^2}{m_B^2}$, $\hat{r} = \frac{m_a^2}{m_B^2}$, $M_0 = m_B(1-\sqrt{\hat{r}})$, $M_0' = m_B^2(1-\hat{r})$, $\kappa_0 = -\frac{2m_b C_7^{\rm eff}}{q^2}$, and $C_0 = C_9^{\rm eff} + C_{10}$.

Using $\mathcal{M}$, the differential decay rate formula of the semileptonic process of the $B \to a_1\ell^+\ell^-$ is estimated as

$$\frac{d\Gamma(B\to a_1\ell^+\ell^-)}{dq^2} = \frac{G_F^2\alpha_{\rm em}^2 m_B^3}{2^{10}3\pi^5}\lambda^{\frac{1}{2}}(1,\hat{r},\hat{s})\upsilon^{\frac{1}{2}}\Delta,$$

where $\hat{m_\ell} = \frac{m_\ell^2}{m_B^2}$, $\upsilon = 1 - \frac{4\hat{m_\ell}}{\hat{s}}$, and $\lambda = \hat{s}^2 + \hat{r}^2 + 1 - 2\hat{s} - 2\hat{r} - 2\hat{r}\,\hat{s}$. The explicit expression of Δ is given as

$$\Delta = |\rho_1|^2\Bigg\{96\hat{r_b}\,\hat{s}\,|\rho_2|^2 + 32|\rho_3|^2\mathcal{D}(\hat{s},\hat{m_\ell})\left[\frac{\mathcal{C}_1\mathcal{C}_2}{8\hat{r}}\left(\mathcal{C}_1 + \frac{\mathcal{C}_3\mathcal{C}_2}{2\hat{s}}\right) + \left(\hat{s} - \frac{\mathcal{C}_2^2}{2\hat{r}}\right)\left(1 + \frac{\mathcal{C}_2^2}{2\hat{s}}\right)\right.$$
$$\left. - \frac{\mathcal{C}_1^2}{8}\left(1 + \frac{\mathcal{C}_1^2}{2\hat{s}\,\hat{r}}\right)\right]\mathcal{I}_1^2(q^2)\Bigg\} + 8\hat{r_b}|\rho_1|^2|\rho_3|^2\mathcal{D}(\hat{s},\hat{m_\ell})\left[\mathcal{C}_3 - \frac{\mathcal{C}_1\mathcal{C}_2}{2\hat{s}} + \frac{\mathcal{C}_1}{4}\left(1 + \frac{\mathcal{C}_1^2}{2\hat{s}\,\hat{r}}\right)\right]\mathcal{I}_2(q^2)$$
$$+ 16|\rho_1|^2|\rho_2|^2\hat{r_b}(1-\hat{r_b})\mathcal{D}(\hat{s},\hat{m_\ell})\mathcal{I}_2(q^2) - 16\hat{r_b}|\rho_1|^2|\rho_3|^2\mathcal{D}(\hat{s},\hat{m_\ell})\left[\frac{\mathcal{C}_2}{2\hat{s}}\left(\mathcal{C}_1 - \frac{\mathcal{C}_2\mathcal{C}_3}{2\hat{r}}\right) - \frac{\mathcal{C}_2^2\mathcal{C}_1}{2\hat{r}}\right]\mathcal{I}_3(q^2)$$
$$+ 8|\rho_1|^2|\rho_3|^2\mathcal{D}(\hat{s},\hat{m_\ell})[Q_b\mathcal{I}_4(q^2) + Q_u\mathcal{I}_5(q^2)] + 32\hat{r_b}|\rho_1|^2|\rho_3|^2\mathcal{D}(\hat{s},\hat{m_\ell})$$

$$
\begin{aligned}
&\times[Q_b\mathcal{I}_6(q^2)-Q_u\mathcal{I}_7(q^2)]-128\hat{r}_b(\rho_1\rho_4)|\rho_3|^2\mathcal{C}_1\mathcal{D}(\hat{s},\hat{m_\ell})[Q_d\mathcal{I}_8(q^2)+Q_u\mathcal{I}_9(q^2)]\\
&+32|\rho_1|^2|\rho_3|^2\left[\hat{s}(1-\hat{r})^2+\hat{r_b}^2\left(1+\frac{\mathcal{C}_2^2}{8\hat{s}\,\hat{r}}\right)\mathcal{D}(\hat{s},\hat{m_\ell})\right]\mathcal{I}_{10}^2(q^2)-64|\rho_1|^2|\rho_2|^2\hat{r_b}^2(1-\hat{r}_b)^2\\
&\times\mathcal{D}(\hat{s},\hat{m_\ell})\mathcal{I}_{10}^2(q^2)+32\hat{r_b}^2|\rho_1|^2|\rho_3|^2\left(-\hat{s}+\frac{{\mathcal{C}_2}^2}{2\hat{r}}\right)\mathcal{D}(\hat{s},\hat{m_\ell})\mathcal{I}_{11}^2(q^2)+32Q_b\hat{r}_b|\rho_1|^2|\rho_3|^2\\
&\times\mathcal{D}(\hat{s},\hat{m_\ell})\left[\mathcal{I}_{12}(q^2)+\left(\mathcal{C}_1-\frac{\mathcal{C}_2\mathcal{C}_3}{2\hat{r}}\right)\mathcal{I}_{13}(q^2)\right]+16Q_b\hat{r}_b|\rho_1|^2|\rho_3|^2\mathcal{D}(\hat{s},\hat{m_\ell})\left[Q_d\mathcal{I}_{14}(q^2)\right.\\
&\left.+Q_u\left(1-\frac{{\mathcal{C}_3}^2}{4\hat{r}}\right)\mathcal{I}_{15}(q^2)\right]+16Q_b|\rho_1|^2|\rho_3|^2\left(1-\frac{{\mathcal{C}_2}^2}{4\hat{r}}\right)\mathcal{D}(\hat{s},\hat{m_\ell})\mathcal{I}_{16}(q^2)-256Q_b\left(\frac{\rho_1\rho_4}{m_B}\right)|\rho_3|^2\\
&\times\mathcal{D}(\hat{s},\hat{m_\ell})[Q_d\mathcal{I}_{17}(q^2)+Q_u\mathcal{I}_{18}(q^2)]+8Q_u^2|\rho_1|^2|\rho_3|^2\mathcal{D}(\hat{s},\hat{m_\ell})\left(1-\frac{{\mathcal{C}_2}^2}{2\hat{r}}\right)\mathcal{I}_{19}(q^2)\\
&+64\hat{r}_bQ_u|\rho_1|^2|\rho_3|^2\mathcal{D}(\hat{s},\hat{m_\ell})[Q_b\mathcal{I}_{20}(q^2)+Q_u\mathcal{I}_{21}(q^2)]+256Q_b\left(\frac{\rho_1\rho_4}{m_B}\right)|\rho_3|^2\mathcal{D}(\hat{s},\hat{m_\ell})\\
&\times[Q_d\mathcal{I}_{22}(q^2)+Q_u\mathcal{I}_{23}(q^2)]-32\hat{r_b}^2|\rho_1|^2\mathcal{D}(\hat{s},\hat{m_\ell})[Q_b|\rho_4|^2\mathcal{I}_{24}(q^2)-Q_u|\rho_3|^2\mathcal{I}_{25}(q^2)]\\
&+32Q_u\hat{r}_b|\rho_1|^2|\rho_3|^2\mathcal{D}(\hat{s},\hat{m_\ell})\left(\mathcal{C}_1+\frac{\mathcal{C}_2\mathcal{C}_3}{2\hat{r}}\right)\mathcal{I}_{26}(q^2)-128Q_d\hat{r}_b\left(\frac{\rho_1\rho_4}{m_B}\right)|\rho_3|^2\mathcal{D}(\hat{s},\hat{m_\ell})\\
&\times[\mathcal{I}_{27}(q^2)-(\hat{r}_b-1)\mathcal{I}_{28}(q^2)]+\frac{2\rho_1}{m_B^2}\mathrm{Re}(\rho_3\rho_5^*F_2^*(q^2)\mathcal{D}(\hat{s},\hat{m_\ell})\left[\frac{\mathcal{C}_2}{\hat{r}}\left(\mathcal{C}_3-\frac{\mathcal{C}_2\mathcal{C}_1}{2\hat{s}}\right)\right.\\
&\left.+\mathcal{C}_1\left(1+\frac{\mathcal{C}_1^2}{8\hat{r}\,\hat{s}}\right)\right]\mathcal{I}_1(q^2)+2\rho_1\mathrm{Re}[\rho_3\rho_5^*F_3^*(q^2)]\mathcal{D}(\hat{s},\hat{m_\ell})\left[\left(\hat{s}-\frac{{\mathcal{C}_2}^2}{2\hat{r}}\right)\left(\mathcal{C}_1-\frac{\mathcal{C}_2(1-\hat{r})}{\hat{s}}\right)\right.\\
&\left.+\frac{\mathcal{C}_1^2\mathcal{C}_2}{2\hat{r}\,\hat{s}}\right]\mathcal{I}_1(q^2)+\frac{8\rho_1}{m_B^2}\mathrm{Re}[\rho_3\rho_5^*F_2^*(q^2)]\mathcal{D}(\hat{s},\hat{m_\ell})[Q_b\mathcal{I}_{29}(q^2)+Q_u\mathcal{I}_{31}(q^2)]-8\rho_1\\
&\times\mathrm{Re}[\rho_3\rho_5^*F_3^*(q^2)]\mathcal{D}(\hat{s},\hat{m_\ell})\left[Q_b\left(\mathcal{C}_1-\frac{\mathcal{C}_2\mathcal{C}_3}{2\hat{r}}\right)\mathcal{I}_{30}(q^2)-Q_u\left(\mathcal{C}_1+\frac{\mathcal{C}_2\mathcal{C}_3}{2\hat{r}}\right)\mathcal{I}_{32}(q^2)\right]-\frac{8\rho_1}{m_B^2}\frac{\hat{r}_b}{\hat{r}}\\
&\times\mathrm{Re}[\rho_3\rho_5^*F_2^*(q^2)]\mathcal{D}(\hat{s},\hat{m_\ell})\mathcal{C}_2^2\mathcal{I}_{11}(q^2)-4\rho_1\hat{r}_b\mathrm{Re}[\rho_3\rho_5^*F_3^*(q^2)]\mathcal{D}(\hat{s},\hat{m_\ell})\frac{\mathcal{C}_2}{\hat{s}}\left[\left(\hat{s}-\frac{{\mathcal{C}_2}^2}{2\hat{r}}\right)\right.\\
&\left.-\mathcal{C}_2^2\right]\mathcal{I}_{11}(q^2)-\frac{8\rho_1\hat{r}_b}{m_B^2}\mathcal{D}(\hat{s},\hat{m_\ell})\mathrm{Re}[\rho_3\rho_5^*F_2^*(q^2)]\left(1-\frac{{\mathcal{C}_2}^2}{4\hat{r}\,\hat{s}}\right)\mathcal{I}_{10}(q^2)+\frac{8\rho_1\hat{r}_b}{m_B^2}\mathcal{D}(\hat{s},\hat{m_\ell})\\
&\times\mathrm{Re}[\rho_3\rho_5^*F_2^*(q^2)]\left[\frac{Q_b\mathcal{I}_{24}(q^2)+Q_u\mathcal{I}_{25}(q^2)}{\mathcal{I}_{11}(q^2)}\right]-8\rho_1\hat{r}_b\mathrm{Re}[\rho_3\rho_5^*F_3^*(q^2)]\mathcal{D}(\hat{s},\hat{m_\ell})\\
&\times[Q_b\mathcal{I}_{33}(q^2)+Q_u\mathcal{I}_{34}(q^2)]+\frac{64\rho_4}{m_B^3}\mathrm{Re}[\rho_3\rho_5^*F_2^*(q^2)]\mathcal{D}(\hat{s},\hat{m_\ell})\left[\frac{Q_d\mathcal{I}_{27}(q^2)+Q_u\mathcal{I}_{28}(q^2)}{\mathcal{I}_{11}(q^2)}\right]\\
&+\frac{64\rho_4}{m_B}\mathrm{Re}[\rho_3\rho_5^*F_3^*(q^2)]\mathcal{D}(\hat{s},\hat{m_\ell})[Q_d\mathcal{I}_{35}(q^2)+Q_u\mathcal{I}_{36}(q^2)]+8\rho_1\mathrm{Re}[\rho_2\rho_5^*F_1^*(q^2)]\hat{r_b}^2\\
&\times\mathcal{D}(\hat{s},\hat{m_\ell})[\mathcal{I}_1(q^2)+\mathcal{I}_{10}(q^2)]+4|\rho_5|^2\hat{r_b}^2|F_1(q^2)|^2\mathcal{D}(\hat{s},\hat{m_\ell})+2|\rho_5|^2|F_3(q^2)|^2\mathcal{D}(\hat{s},\hat{m_\ell})\\
&\times\left(\hat{s}-\frac{{\mathcal{C}_2}^2}{4\hat{r}\,\hat{s}}\right)\left[\mathcal{C}_3+\hat{r}+1-\frac{(1-\hat{r})^2}{\hat{s}}\right]-\frac{2|\rho_5|^2}{m_B^4}|F_2(q^2)|^2\left[\left(\frac{3\hat{s}}{2}-6\hat{m_\ell}\right)+\mathcal{D}(\hat{s},\hat{m_\ell})\left(1-\frac{{\mathcal{C}_2}^2}{4\hat{r}\,\hat{s}}\right)\right]\\
&+\frac{|\rho_5|^2\mathcal{C}_2}{\hat{r}m_B^2}\mathrm{Re}[F_2(q^2)F_3^*(q^2)]\mathcal{D}(\hat{s},\hat{m_\ell})\left[1-\frac{\mathcal{C}_2(1-\hat{r})}{2\hat{s}}\right],
\end{aligned}
$$

where $\hat{r_b}=\frac{m_b}{m_B}$, $\mathcal{C}_1=1-\hat{r}+\hat{s}$, $\mathcal{C}_2=1-\hat{r}-\hat{s}$, $\mathcal{C}_3=1+\hat{r}-\hat{s}$, and $\mathcal{D}(\hat{s},\hat{m_\ell})=\hat{s}+2\hat{m_\ell}$. The forms of $\mathcal{I}_n$ functions are expressed as

$$\mathcal{I}_1(q^2) = \int_0^1 d\xi\varphi_1(\xi), \qquad \mathcal{I}_2(q^2) = \mathcal{I}_1(q^2)\int_0^1 d\acute{\xi}\varphi_2(\acute{\xi}), \qquad \mathcal{I}_3(q^2) = \mathcal{I}_1(q^2)\int_0^1 d\acute{\xi}\varphi_3(\acute{\xi}),$$

$$\mathcal{I}_4(q^2) = \mathcal{I}_1(q^2)\int_0^1 d\acute{\xi}A_1(\acute{\xi})\left[\left(-\mathcal{C}_1 + \frac{\mathcal{C}_2\mathcal{C}_3}{2\hat{r}}\right)\left(\Delta_1(\acute{\xi}) - \frac{\mathcal{C}_1\Delta_2(\acute{\xi})}{2\hat{s}}\right) + (\mathcal{C}_1\mathcal{C}_3)\left(\Delta_3(\acute{\xi}) - \frac{\mathcal{C}_2\Delta_2(\acute{\xi})}{2\hat{s}}\right)\right],$$

$$\mathcal{I}_5(q^2) = \mathcal{I}_1(q^2)\int_0^1 d\acute{\xi}A_2(\acute{\xi})\left[\left(-\mathcal{C}_1 + \frac{\mathcal{C}_2\mathcal{C}_3}{2\hat{r}}\right)\left(\Delta_4(\acute{\xi}) - \frac{\mathcal{C}_1\Delta_5(\acute{\xi})}{2\hat{s}}\right) + (\mathcal{C}_3\mathcal{C}_1)\left(\Delta_6(\acute{\xi}) - \frac{\mathcal{C}_2\Delta_5(\acute{\xi})}{2\hat{s}}\right)\right],$$

$$\mathcal{I}_6(q^2) = \mathcal{I}_1(q^2)\int_0^1 d\acute{\xi}A_3(\acute{\xi})\left[\Delta_1(\acute{\xi}) - \frac{\mathcal{C}_1\Delta_2(\acute{\xi})}{2\hat{s}} + \mathcal{C}_1\left(\acute{\xi} - \frac{\Delta_2(\acute{\xi})}{2\hat{s}}\right)\right],$$

$$\mathcal{I}_7(q^2) = \mathcal{I}_1(q^2)\int_0^1 d\acute{\xi}A_4(\acute{\xi})\left[\Delta_4(\acute{\xi}) - \frac{\mathcal{C}_1\Delta_5(\acute{\xi})}{2\hat{s}} + \mathcal{C}_1\left(1 - \acute{\xi} - \frac{\Delta_5(\acute{\xi})}{2\hat{s}}\right)\right],$$

$$\mathcal{I}_8(q^2) = \mathcal{I}_1(q^2)\int_0^1 duA_5(u)\left[1 - \frac{\Delta_7(u)}{2\hat{s}}\right], \qquad \mathcal{I}_9(q^2) = \mathcal{I}_1(q^2)\int_0^1 duA_6(u)\left[1 - \frac{\Delta_8(u)}{2\hat{s}}\right],$$

$$\mathcal{I}_{10}(q^2) = \int_0^1 d\xi\varphi_2(\xi), \qquad \mathcal{I}_{11}(q^2) = \int_0^1 d\xi\varphi_3(\xi),$$

$$\mathcal{I}_{12}(q^2) = \mathcal{I}_{10}(q^2)\int_0^1 d\acute{\xi}A_1(\acute{\xi})\left[\Delta_1(\acute{\xi}) + \Delta_4(\acute{\xi}) - \frac{\mathcal{C}_1\Delta_2(\acute{\xi})}{2\hat{s}} - \frac{\mathcal{C}_2\Delta_5(\acute{\xi})}{2\hat{s}} + \mathcal{C}_1\left(\Delta_3(\acute{\xi}) + \Delta_6(\acute{\xi}) - \frac{\mathcal{C}_2\Delta_2(\acute{\xi})}{2\hat{s}}\right)\right],$$

$$\mathcal{I}_{13}(q^2) = \mathcal{I}_{11}(q^2)\int_0^1 d\acute{\xi}A_1(\acute{\xi})\left[\acute{\xi} + \frac{\Delta_2(\acute{\xi})}{2\hat{s}}\right],$$

$$\mathcal{I}_{14}(q^2) = \int_0^1 d\xi A_1(\xi)\int_0^1 d\acute{\xi}A_3(\acute{\xi})\left[\Delta_9(\xi,\acute{\xi}) - \frac{\Delta_2(\xi)\Delta_2(\acute{\xi})}{2\hat{s}}\right],$$

$$\mathcal{I}_{15}(q^2) = \int_0^1 d\xi A_1(\xi)\int_0^1 d\acute{\xi}A_2(\acute{\xi})\left[\Delta_{10}(\xi,\acute{\xi}) - \frac{\Delta_2(\xi)\Delta_5(\acute{\xi})}{2\hat{s}}\right],$$

$$\mathcal{I}_{16}(q^2) = \int_0^1 d\xi A_1(\xi)\int_0^1 d\acute{\xi}A_1(\acute{\xi})\left[Q_b\left(\Delta_9(\xi,\acute{\xi}) - \frac{\Delta_2(\xi)\Delta_2(\acute{\xi})}{2\hat{s}}\right) + Q_u\left(\Delta_{10}(\xi,\acute{\xi}) - \frac{\Delta_2(\xi)\Delta_5(\acute{\xi})}{2\hat{s}}\right)\right],$$

$$\mathcal{I}_{17}(q^2) = \int_0^1 d\xi A_1(\xi)\int_0^1 duA_5(u)\left[\Delta_{11}(\xi,u) - \frac{\Delta_2(\xi)\Delta_7(u)}{2\hat{s}}\right],$$

$$\mathcal{I}_{18}(q^2) = \int_0^1 d\xi A_1(\xi)\int_0^1 duA_6(u)\left[\Delta_{12}(\xi,u) - \frac{\Delta_2(\xi)\Delta_8(u)}{2\hat{s}}\right],$$

$$\mathcal{I}_{19}(q^2) = \int_0^1 d\xi A_2(\xi)\int_0^1 d\acute{\xi}A_2(\acute{\xi})\left[\Delta_{13}(\xi,\acute{\xi}) - \frac{\Delta_5(\xi)\Delta_5(\acute{\xi})}{2\hat{s}}\right],$$

$$\mathcal{I}_{20}(q^2) = \int_0^1 d\xi A_2(\xi)\int_0^1 d\acute{\xi}A_3(\acute{\xi})\left[\Delta_9(\xi,\acute{\xi}) - \frac{\Delta_2(\xi)\Delta_2(\acute{\xi})}{2\hat{s}}\right],$$

$$\mathcal{I}_{21}(q^2) = \int_0^1 d\xi A_2(\xi)\int_0^1 d\acute{\xi}A_4(\acute{\xi})\left[\Delta_{10}(\xi,\acute{\xi}) - \frac{\Delta_2(\xi)\Delta_5(\acute{\xi})}{2\hat{s}}\right],$$

$$\mathcal{I}_{22}(q^2) = \int_0^1 d\xi A_2(\xi)\int_0^1 duA_5(u)\left[\Delta_{14}(\xi,u) - \frac{\Delta_5(\xi)\Delta_7(u)}{2\hat{s}}\right],$$

$$\mathcal{I}_{23}(q^2) = \int_0^1 d\xi A_2(\xi) \int_0^1 du A_6(u) \left[\Delta_{15}(\xi, u) - \frac{\Delta_5(\xi)\Delta_8(u)}{2\hat{s}}\right], \qquad \mathcal{I}_{24}(q^2) = \mathcal{I}_{11}(q^2) \int_0^1 du A_3(\acute{\xi}) \left[\acute{\xi} + \frac{\Delta_2(\acute{\xi})}{2\hat{s}}\right],$$

$$\mathcal{I}_{25}(q^2) = \mathcal{I}_{11}(q^2) \int_0^1 du A_4(\acute{\xi}) \left[1 - \acute{\xi} + \frac{\Delta_5(\acute{\xi})}{2\hat{s}}\right], \qquad \mathcal{I}_{26}(q^2) = \mathcal{I}_{11}(q^2) \int_0^1 du A_2(\acute{\xi}) \left[1 - \acute{\xi} + \frac{\Delta_5(\acute{\xi})}{2\hat{s}}\right],$$

$$\mathcal{I}_{27}(q^2) = \mathcal{I}_{11}(q^2)(\xi) \int_0^1 du A_5(u) \left[1 - \frac{\Delta_7(u)}{2\hat{s}}\right], \qquad \mathcal{I}_{28}(q^2) = \mathcal{I}_{11}(q^2) \int_0^1 du A_6(u) \left[1 + \frac{\Delta_8(u)}{2\hat{s}}\right],$$

$$\mathcal{I}_{29}(q^2) = \int_0^1 d\xi A_1(\xi) \left[\Delta_1(\xi) - \frac{\mathcal{C}_1\Delta_2(\xi)}{2\hat{s}} - \frac{\mathcal{C}_3}{2\hat{r}}\left(\Delta_3(\xi) - \frac{\mathcal{C}_2\Delta_2(\xi)}{2\hat{s}}\right)\right],$$

$$\mathcal{I}_{30}(q^2) = \int_0^1 d\xi A_1(\xi) \left[\Delta_1(\xi) + \Delta_3(\xi) - \frac{\Delta_2(\xi)(1-\hat{r})}{\hat{s}}\right],$$

$$\mathcal{I}_{31}(q^2) = \int_0^1 d\xi A_2(\xi) \left[\Delta_4(\xi) - \frac{\mathcal{C}_2\Delta_5(\xi)}{2\hat{s}} - \frac{\mathcal{C}_3}{2\hat{r}}\left(\Delta_6(\xi) - \frac{\mathcal{C}_2\Delta_2(\xi)}{2\hat{s}}\right)\right],$$

$$\mathcal{I}_{32}(q^2) = \int_0^1 d\xi A_1(\xi) \left[\Delta_4(\xi) + \Delta_6(\xi) - \frac{\Delta_5(\xi)(1-\hat{r})}{\hat{s}}\right],$$

$$\mathcal{I}_{33}(q^2) = \int_0^1 d\xi A_3(\xi) \left[\Delta_1(\xi) + \Delta_3(\xi) - \frac{\Delta_2(\xi)(1-\hat{r})}{\hat{s}}\right],$$

$$\mathcal{I}_{34}(q^2) = \int_0^1 d\xi A_4(\xi) \left[\Delta_4(\xi) + \Delta_6(\xi) - \frac{\Delta_5(\xi)(1-\hat{r})}{\hat{s}}\right],$$

$$\mathcal{I}_{35}(q^2) = \int_0^1 du A_5(u) \left[\Delta_{16}(u) + \Delta_{17}(u) - \frac{\Delta_7(u)(1-\hat{r})}{2\hat{s}}\right],$$

$$\mathcal{I}_{36}(q^2) = \int_0^1 du A_6(u) \left[\Delta_{18}(u) + \Delta_{19}(u) - \frac{\Delta_8(u)(1-\hat{r})}{2\hat{s}}\right],$$

where

$$\Delta_1(\xi) = \frac{1}{2}[(1-2\xi) + \hat{r} - \hat{s}], \qquad \Delta_2(\xi) = \frac{1}{2}[(1-\xi)(1-\hat{r}) - \hat{s}(1+\xi)], \qquad \Delta_3(\xi) = \frac{1}{2}[(2-\xi)(\hat{s}-1) - \xi\hat{r}],$$

$$\Delta_4(\xi) = -\frac{1}{2}[(2\xi-1) + \hat{r} - \hat{s}], \qquad \Delta_5(\xi) = \frac{1}{2}[\xi(1-\hat{r}) + \hat{s}(2-\xi)], \qquad \Delta_6(\xi) = \frac{1}{2}[(1+\xi)(1-\hat{s}) + (1-\xi)\hat{r}],$$

$$\Delta_7(u) = \frac{1}{2}[u(1-\hat{r}) + \hat{s}(2-u)], \qquad \Delta_8(u) = -\frac{1}{2}[(1-u)(1-\hat{r}) + \hat{s}(u+1)],$$

$$\Delta_9(\xi, \acute{\xi}) = \frac{1}{2}[2(1-\xi)(1-\acute{\xi}) - C_1(2-\xi-\acute{\xi}) + 2\hat{s}], \qquad \Delta_{10}(\xi, \acute{\xi}) = \frac{1}{2}[C_1(1-\xi+\acute{\xi}) - 2\hat{s} - 2(1-\xi)\acute{\xi}],$$

$$\Delta_{11}(\xi, u) = \frac{1}{2}[-\mathcal{C}_2 u + \mathcal{C}_1(1-\xi)(1-\hat{r}+\hat{s}) + C_1 u(1-\xi) - 2\hat{s}],$$

$$\Delta_{12}(\xi, u) = -\frac{1}{2}[-\mathcal{C}_2(1-u) + \mathcal{C}_1(1-\xi) + \mathcal{C}_1(1-u)(1-\xi) - 2\hat{s}],$$

$$\Delta_{13}(\xi, \acute{\xi}) = \frac{1}{2}[\xi\acute{\xi} - C_1(\xi+\acute{\xi}) + 2\hat{s}], \qquad \Delta_{14}(\xi, u) = -\frac{1}{2}[\mathcal{C}_1\xi - C_2 u + \mathcal{C}_3 u\xi - 2\hat{s}],$$

$$\Delta_{15}(\xi, u) = \frac{1}{2}[\mathcal{C}_1\xi - \mathcal{C}_2(1-u) + \mathcal{C}_3\xi(1-u) - 2\hat{s}], \qquad \Delta_{16}(u) = \frac{1}{2}[(2u-1)\hat{r} + 1 - \hat{s}],$$

$$\Delta_{17}(u) = \frac{1}{2}[(u+1)(\hat{s}-\hat{r}) + 1 - u], \qquad \Delta_{18}(u) = -\frac{1}{2}[(1-2u)\hat{r} + 1 - \hat{s}], \qquad \Delta_{19}(u) = -\frac{1}{2}[(2-u)(\hat{s}-\hat{r}) + 1 - u].$$

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