Kinetic mixing at strong coupling

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A common feature of many string-motivated particle physics models is additional strongly coupled $U(1)$'s. In such sectors, electric and magnetic states have comparable mass, and integrating out modes also charged under $U(1)$ hypercharge generically yields CP preserving electric kinetic mixing and CP violating magnetic kinetic mixing terms. Even though these extra sectors are strongly coupled, we show that in the limit where the extra sector has approximate $\mathcal{N} = 2$ supersymmetry, we can use formal methods from Seiberg-Witten theory to compute these couplings. We also calculate various quantities of phenomenological interest such as the cross section for scattering between visible sector states and heavy extra sector states as well as the effects of supersymmetry breaking induced from coupling to the minimal supersymmetric Standard Model.

I. INTRODUCTION

As the only known viable theory of quantum gravity, it is clearly important to determine possible low energy manifestations of string theory. One promising route to forging such connections is to examine generic string-motivated scenarios for physics beyond the Standard Model of particle physics.

A generic feature of many string constructions is the presence of additional $U(1)$ gauge fields. These can arise from dimensional reduction of higher p-form potentials, that is, from the "closed string sector" of a model. Another common way such gauge fields arise is from degrees of freedom localized on lower-dimensional branes, that is, from the "open string sector." In many cases, there can be degrees of freedom charged under both the $U(1)$ hypercharge factor of the Standard Model gauge group and one of these extra $U(1)$'s. This motivates the study of kinetic mixing in the context of string phenomenology. For a partial list of references, see, e.g., Refs. [1–17] as well as Refs. [18–28].

But another generic feature of many string constructions is the presence of sectors which are strongly coupled [29]. Indeed, while it is certainly possible to arrange for *some* parameters to remain weakly coupled (as is necessary for realizing the perturbative couplings of our world), it is typically more problematic to arrange for *all* couplings to be small. In the context of closed string parameters, this is the statement that it is easier—albeit less calculable—to produce models with some geometric moduli set at string scale values. In the case of open string sectors, this is the statement that there are extra sectors at strong coupling.

Having such strongly coupled extra sectors is also expected to generate novel phenomenological scenarios. For a review of some recent work on composite dark matter with strong coupling dynamics, see, for example, Ref. [30]. Unparticles with a mass gap [31,32] provide another class of strongly coupled extra sectors with novel signatures.

In this paper, we combine these considerations; that is, we study string-motivated scenarios with an extra $U(1)$ which is strongly coupled. From this perspective, the gauge group of the Standard Model can be approximated as a weakly gauged flavor symmetry. It is natural to expect there to be states (which may be quite heavy) that are charged under both the Standard Model and such extra $U(1)$'s. As far as we are aware, there have been only limited analyses of such systems, with very specialized structure for magnetic objects [11,12]. See Fig. 1 for a depiction of kinetic mixing at strong coupling.
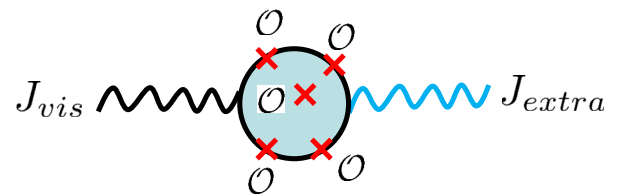

FIG. 1. Depiction of kinetic mixing with a strongly coupled extra sector. In this limit, the standard one loop calculation of kinetic mixing does not apply, and we must instead resort to nonperturbative methods. Integrating out messenger states between the two sectors leads to electric and magnetic kinetic mixing with the visible sector $U(1)$.

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Kinetic mixing between a visible sector $U(1)$ and an extra sector $U(1)$ is captured by the effective Lagrangian,

$$L_{U(1)} = L_{\text{diag}} + L_{\text{mix}} \qquad (1.1)$$

$$L_{\text{diag}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{g^2\theta}{32\pi^2} F'_{\mu\nu} \tilde{F}'^{\mu\nu} \qquad (1.2)$$

$$L_{\text{mix}} = -\frac{\chi_{\text{elec}}}{2} F_{\mu\nu} F'^{\mu\nu} - \frac{\chi_{\text{mag}}}{2} F_{\mu\nu} \tilde{F}'^{\mu\nu}, \qquad (1.3)$$

where $F'_{\mu\nu}$ is the field strength of an extra $U(1)$ with magnetic dual field strength $\tilde{F}'_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} F'^{\rho\sigma}$. Here, we have omitted the theta angle of the visible sector since all its magnetic objects are assumed to be quite heavy. The analog of the fine structure constant in the extra sector is $\alpha_{\text{extra}} = g^2/4\pi$, so that strong coupling corresponds to taking $\alpha_{\text{extra}} \sim O(1)$.

A priori, then, kinetic mixing can occur via both a *CP* preserving and a *CP* violating term:

$$\text{Electric Mixing:} F_{\mu\nu} F'^{\mu\nu} \qquad (1.4)$$

$$\text{Magnetic Mixing:} F_{\mu\nu} \tilde{F}'^{\mu\nu}. \qquad (1.5)$$

Electric kinetic mixing has been heavily studied, starting with Refs. [1,2], and has led to a slew of novel dark matter scenarios. For some examples, see Refs. [33–39].

Magnetic kinetic mixing is far more challenging to study. If we have both electrically and magnetically charged states of comparable mass, we are inherently at strong coupling, and there is no duality transformation available to eliminate terms such as $F_{\mu\nu} \tilde{F}'^{\mu\nu}$. Indeed, another symptom of this fact is that when magnetic monopoles are present, $F_{\mu\nu} \tilde{F}'^{\mu\nu}$ can no longer be expressed as a total derivative because there is no Lorentz invariant formulation of the theory with a vector potential.[1] Indeed, it has been known for some time that the analog of the QCD theta angle plays an important role in the dynamics of Abelian gauge theories with dyons (i.e., states with electric and magnetic charge) [41].

Precisely because the extra $U(1)$ is at strong coupling, standard methods from perturbative quantum field theory do not apply. It is therefore important to see whether we can extract *any* quantitative information about kinetic mixing at strong coupling.

In this paper, we develop a general set of methods to extract these mixing effects. In the limit where the extra sector enjoys approximate $\mathcal{N} = 2$ supersymmetry, we show how to adapt formal methods from Seiberg-Witten theory [42,43] to extract the exact form of electric and magnetic mixing. We also use these methods to extract the spectrum of stable objects and to calculate the leading order

effects of supersymmetry breaking induced from coupling to the minimal supersymmetric Standard Model (MSSM). Additionally, we calculate the leading order contributions to scattering between visible sector states and heavy extra sector states. For some previous uses of extended supersymmetry in the model building literature, see, for example, Ref. [44], and for some discussion on other uses of magnetically charged states of an extra sector, see, e.g., Ref. [45].

In F-theory realizations of the Standard Model (see, e.g., Ref. [46–48] for reviews), the canonical example of such an extra sector is a D3-brane probing a stack of 7-branes with E_8 gauge symmetry [14,15,49–54]. That is, this realizes an $\mathcal{N} = 2$ superconformal field theory with E_8 flavor symmetry [55,56]. Tilting the 7-branes and activating background fluxes then breaks this flavor symmetry down to the Standard Model gauge group, which in particular contains a $U(1) \subset E_8$ which we identify with the hypercharge of the Standard Model.

Approximate conformal symmetry of the extra sector means that the overall mass scales of the extra sector are dictated by coupling it to additional sectors. This can include both mass scales associated with the visible sector Standard Model and its embedding in the MSSM and a stringy grand unified theory (GUT), but can also include other decoupled sectors (for example, in gravity mediated supersymmetry breaking scenarios). For this reason, motivated values for approximate $\mathcal{N} = 2$ supersymmetric extra sector states can range from the TeV scale up to the GUT scale. As noted in Ref. [15], partial breaking to $\mathcal{N} = 1$ supersymmetry via T-brane deformations [49] can induce a seesawlike mechanism for dark extra sector states, which in turn can generate sub-TeV mass scales.

We also put some of these considerations together to provide a preliminary analysis of how such extra sectors can serve as toy models for more realistic phenomenology. In particular, we explain how such extra sectors arise in specific string constructions and how to incorporate the leading order effects of supersymmetry breaking. Since the resulting cosmological history greatly depends on the associated mass scales, we mainly illustrate the general contours of how such models work.

The rest of this paper is organized as follows. First, in Sec. II, we discuss in greater detail some additional features of electric and magnetic mixing, as well as the effect such terms make on scattering cross sections. Next, in Sec. III, we show how to apply formal methods from the study of theories with $\mathcal{N} = 2$ supersymmetry to calculate such mixing effects and how to incorporate the leading effects of supersymmetry breaking. Section IV sets up the ingredients needed for theories with a single extra $U(1)$, which we follow with an analysis of kinetic mixing when the extra sector is the rank 1 H_1 Argyres-Douglas theory. In Sec. V, we discuss some aspects of the resulting phenomenology. We present our conclusions in Sec. VI. Some of the results

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[1]See, however, Ref. [40].

presented in this paper also appear in the Ph.D. thesis of A. Malekian [57].

II. ELECTRIC AND MAGNETIC MIXING

Our plan in this section will be to discuss some basic aspects of electric and magnetic kinetic mixing. We also show how to extract some information about how visible states can scatter off of dark dyons of an extra sector. For a complementary account of some aspects of magnetic mixing, see, for example, Refs. [11,12].

Our starting point is a system with r total $U(1)$'s, with effective Lagrangian:

$$L_{U(1)'s} = \sum_i \left(-\frac{1}{4} F^i \cdot F^i + \frac{g_{ii}^2 \theta_{ii}}{32\pi^2} F^i \cdot \tilde{F}^i \right) \quad (2.1)$$

$$+ \sum_{i \neq j} \left(-\frac{\chi_{ij}^{\text{elec}}}{4} F^i \cdot F^j - \frac{\chi_{ij}^{\text{mag}}}{4} F^i \cdot \tilde{F}^j \right). \quad (2.2)$$

Working in terms of the electric and magnetic field strengths, we see two types of interaction terms: those which preserve CP and those which do not,

$$CP \text{ Preserving}: F^i \cdot F^j \quad (2.3)$$

$$CP \text{ Violating}: F^i \cdot \tilde{F}^j, \quad (2.4)$$

which are respectively associated with electric kinetic mixing and magnetic kinetic mixing.

Now, since our extra $U(1)$'s will typically be at strong coupling, it is actually more convenient to make use of a basis of fields in which charge quantization is manifest. By abuse of notation, we shall use the same expression for the field strengths,

$$L_{U(1)'s} = -\frac{1}{4g_{ij}^2} F^i \cdot F^j + \frac{\theta_{ij}}{32\pi^2} F^i \cdot \tilde{F}^j \quad (2.5)$$

$$= -\frac{1}{16\pi} (\text{Im}\tau_{ij} F^i \cdot F^j - \text{Re}\tau_{ij} F^i \cdot \tilde{F}^j), \quad (2.6)$$

where we sum repeated indices and we have introduced the complexified parameter

$$\tau_{ij} = \frac{4\pi i}{g_{ij}^2} + \frac{\theta_{ij}}{2\pi}. \quad (2.7)$$

The original mixing parameters are then given by

$$\chi_{ij}^{\text{elec}} = \frac{\text{Im}\tau_{ij}}{\sqrt{\text{Im}\tau_{ii}}\sqrt{\text{Im}\tau_{jj}}} \quad \text{and}$$

$$\chi_{ij}^{\text{mag}} = -\frac{\text{Re}\tau_{ij}}{\sqrt{\text{Im}\tau_{ii}}\sqrt{\text{Im}\tau_{jj}}}. \quad (2.8)$$

We are interested in extra sectors which contain both monopoles and dyons. Some care must be taken in properly defining a basis of electric and magnetic charges which is also consistent with Dirac quantization. It is convenient to adopt a basis in which all magnetic charges are integral and in which the physically measured electric charges may contain shifts by the various theta angles [41]. So, we introduce $2r$ integers n_i^{elec} and n_{mag}^i and corresponding electric and magnetic charges:

$$Q_i^{\text{elec}} = \left(n_i^{\text{elec}} - \frac{\theta_{ij}}{2\pi} n_{\text{mag}}^j \right) \quad \text{and} \quad Q_{\text{mag}}^i = n_{\text{mag}}^i. \quad (2.9)$$

In our conventions, the electric fields $\vec{E}^i$ and magnetic fields $\vec{B}^i$ for a point particle with these integral values satisfy

$$\vec{\nabla} \cdot \vec{E}^i = 4\pi \delta^3(\vec{x}) \times \left(\frac{1}{\text{Im}\tau} \right)^{ij} n_j^{\text{elec}} \quad \text{and}$$

$$\vec{\nabla} \cdot \vec{B}^i = 4\pi \delta^3(\vec{x}) \times n_{\text{mag}}^i. \quad (2.10)$$

Electric-magnetic duality in this setting amounts to the collection of transformations which preserve the form of the Dirac pairing. We can, without loss of generality, adopt a basis in which the pairing Ω has the block-diagonal form:

$$\Omega = \begin{bmatrix} & \mathbf{1}_{r \times r} \\ -\mathbf{1}_{r \times r} & \end{bmatrix}. \quad (2.11)$$

We shall sometimes write Ω_{IJ} with indices $I, J = 1, \ldots, 2r$; i.e., the index runs over both the electric and magnetic charges.

Nontrivial duality transformations are then captured by $2r \times 2r$ matrices M with integer values subject to the condition

$$M^T \Omega M = \Omega; \quad (2.12)$$

that is, the dualities are captured by $Sp(2r, \mathbb{Z})$ transformations. It acts on the complexified parameter matrix τ_{ij} as

$$\tau \mapsto (A\tau + B)(C\tau + D)^{-1}, \quad (2.13)$$

where we have decomposed M according to the block structure:

$$M_{2r \times 2r} = \begin{bmatrix} A_{r \times r} & B_{r \times r} \\ C_{r \times r} & D_{r \times r} \end{bmatrix} \in Sp(2r, \mathbb{Z}). \quad (2.14)$$

An important aspect of such duality transformations is that we must ensure that our answers are compatible with this $Sp(2r, \mathbb{Z})$ redundancy.[2] It is common to work in a

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[2]More precisely, it may happen that duality transformations may only involve a congruent subgroup of $Sp(2r, \mathbb{Z})$. This is in turn dictated by the precise spectrum of Bogomol'nyi-Prasad-Sommerfield (BPS) objects which transform into one another under various duality transformations. We shall not dwell on this point in what follows.

"fundamental domain" for τ and label all charges with respect to this basis choice. For the purposes of mapping out possible values of parameters, however, it is sometimes convenient to work on the enlarged covering space. Unitarity imposes the condition that

$$\text{Im}\tau > 0, \qquad (2.15)$$

that is, that we have a positive definite matrix of kinetic terms. As we have already remarked, this choice of parametrization contains some redundancies because of the duality group action.

A. Dark Rutherford scattering

Let us now suppose we have fixed a choice of fundamental domain, as well as a basis of electric and magnetic charges. We would like to know how visible sector states interact with hidden sector dyons.

The main idea will be to introduce a fixed background for our various fields. We then consider small fluctuations around this background, which we identify with the visible sector gauge potential. For this approximation to be valid, we really need the extra sector states to be heavy so that we can simply substitute in the background values of the various fields. This can be viewed as a mild generalization of the calculation given in Ref. [58] (see also Ref. [12]).

With this in mind, we shall aim to expand the various field strengths around background values, with fluctuations captured by a vector potential:

$$F_{\mu\nu}^i = F_{\mu\nu}^{i,\text{bkgnd}} + \partial_\mu A_\nu^i - \partial_\nu A_\mu^i. \qquad (2.16)$$

Our goal will be to determine how the vector potentials A_μ^i couple to the background sourced by a dyon. To proceed further, it is helpful to work directly with the electric and magnetic field strengths. The mixing Lagrangian is then given by

$$L_{U(1)'s} = \frac{1}{2g_{ij}^2}(\vec{E}^i \cdot \vec{E}^j - \vec{B}^i \cdot \vec{B}^j) - \frac{\theta_{ij}}{8\pi^2}\vec{E}^i \cdot \vec{B}^j. \qquad (2.17)$$

Since we are working with static pointlike sources, it suffices to consider the coupling of the scalar potential to this background:

$$\vec{E}^i = \vec{E}_{\text{bkgnd}}^i - \vec{\nabla}\varphi^i. \qquad (2.18)$$

Plugging in to our effective Lagrangian, the scalar potential couples to a source term:

$$J_i^{\text{eff}} = \delta^3(\vec{x}) \times (n_i^{\text{elec}} - \text{Re}\tau_{ij}n_{\text{mag}}^j). \qquad (2.19)$$

Consequently, we see that in matrix elements between visible sector currents and a heavy dark dyon, all our amplitudes will be proportional to the quantity

$$\Pi(M_{\text{vis}}, N_{\text{hid}}) = q_{\text{vis}}\left(\frac{1}{\text{Im}\tau}\right)^{\text{vis},j}(n_j^{\text{elec}} - \text{Re}\tau_{jk}n_{\text{mag}}^k), \qquad (2.20)$$

in the obvious notation.

It is tempting to organize this into a single duality invariant expression. Indeed, the scattering amplitude we compute cannot depend on the particular basis of fields we choose to use in performing our calculation. The caveat is that if we perform a duality transformation on the gauge fields and couplings, we must also transform the charges of the external states entering into the scattering amplitude.

So, following the discussion in Ref. [12], we note that the $Sp(2r, \mathbb{Z})$ invariant bilinear between dyonic charges is

$$\Pi(M, N) = M_I \Pi^{IJ} N_J, \qquad (2.21)$$

where

$$M_I = \begin{bmatrix} n_i^{\text{elec}} \\ n_{\text{mag}}^i \end{bmatrix}, \qquad N_J = \begin{bmatrix} n_j^{\text{elec}} \\ n_{\text{mag}}^j \end{bmatrix}, \qquad (2.22)$$

and

$$\Pi^{IJ} = \begin{bmatrix} (\frac{1}{\text{Im}\tau})^{il} & -(\frac{1}{\text{Im}\tau})^{ij}\text{Re}\tau_{jl} \\ -\text{Re}\tau_{ij}(\frac{1}{\text{Im}\tau})^{jl} & \text{Re}\tau_{ij}(\frac{1}{\text{Im}\tau})^{jk}\text{Re}\tau_{kl} + \text{Im}\tau_{il} \end{bmatrix}, \qquad (2.23)$$

in the obvious notation. We view $M_I \Pi^{IJ} N_J$ as calculating the matrix element between a visible sector current associated with M_I and a hidden sector current associated with N_J.

Consider, then, the special case where we have a state with charge M_I which couples to a weakly coupled gauge boson; i.e., this is our "visible sector." Assuming the extra sector state is quite heavy and that the visible sector state has mass m_{vis} and charge q_{vis} and moves with velocity $\vec{v}$, we then get a mild generalization of the standard result for Rutherford scattering (see, e.g., Ref. [59]):

$$\frac{d\sigma}{d\Omega} = \frac{|\Pi(M, N)|^2}{4m_{\text{vis}}^2 v^4 \sin^4\frac{\theta}{2}}. \qquad (2.24)$$

An interesting feature of this formula is the dependence of the cross section on the electric and magnetic charge of the extra sector. In particular, we see that the strength of the magnetic mixing term can have a nontrivial impact on the scattering of dark magnetic states.

We caution that to really apply this formula, we need to have at least one scattering state to be near the free field limit, i.e., we need it to be charged with respect to only a weakly coupled gauge boson, and for the states of the extra sector to be heavy. Thankfully, this is the case of maximal interest for phenomenology, where we consider a visible

sector electron/charged nucleon scattering off of a heavy hidden sector dyon.

It is also convenient to package the contribution to the scattering amplitude in terms of an effective electric charge from the extra sector. We define an effective electric charge for a dark sector state which scatters off a visible sector state:

$$q_{\text{eff}} \equiv \frac{|\Pi(M_{\text{vis}}, N_{\text{hid}})|}{|M_{\text{vis}}|\alpha_{\text{vis}}}. \tag{2.25}$$

Note that since Π is linear in M_{vis}, the overall value of the visible sector charge drops out of this expression.

One might also ask whether we can extend this calculation to a regime in which we do not treat the extra sector as a fixed classical source. This is of particular relevance for strongly coupled sectors where we can typically expect a rich spectrum of composite bound states. When we do this, we need to have much more detailed information about the spectrum of asymptotic scattering states. It is analogous to the problem in QCD of determining the precise form of the parton distribution functions. Nevertheless, we can already see that several novel features will present themselves in the general case. Precisely because we expect a general theory of dyons to include nontrivial bound states with a finite radius, these configurations can have nontrivial angular momentum (as dictated by the Dirac pairing). This already tells us that if we consider a scattering event in which the internal state of the composite object undergoes a transition, conservation of angular momentum will lead to nontrivial selection rules on possible interaction terms. One can view this as a generalization of the Callan-Rubakov effect [60–62].

III. SUPERSYMMETRIC APPROXIMATION

In the previous section, we presented some general considerations on electric and magnetic mixing and explained how in the regime where the dark charged objects are quite heavy we can determine the net effect of magnetic mixing on the visible sector. In particular, many of the same considerations used to study electric kinetic mixing also carry over to this case as well.

This prompts the following question: Can we realize specific examples in which magnetic mixing is generated, and, moreover, can we actually *calculate* the overall strength of such mixing terms? To frame the discussion to follow, let us recall that in a weakly coupled theory, the leading order contribution to kinetic mixing between two $U(1)$'s is

$$\frac{1}{g_{ij}^2} = \sum_{\psi} c^{(\psi)} \frac{q_i^{(\psi)} q_j^{(\psi)}}{16\pi^2} \log\left(\frac{M_{(\psi)}^2}{\mu^2}\right), \tag{3.1}$$

where the sum is over states of mass $M_{(\psi)}$ and the q's are the electric charges under the respective gauge groups.

Additionally, $c^{(\psi)}$ is a numerical prefactor which depends on the spin of the state.

We would like to generalize this calculation to the case where our extra sector states interact with a strongly coupled $U(1)$. The issue we face is that perturbative methods via Feynman diagrams will no longer apply.

To give specific examples of how to integrate out massive dyonic states to calculate possible mixing terms, we shall use the general formalism of supersymmetric gauge theories. Our conventions follow Ref. [63]. Recall that in a supersymmetric gauge theory, we can package the $\mathcal{N} = 1$ vector multiplet (with a gauge field and its superpartner the gaugino as dynamical degrees of freedom) in terms of the superfield $W_\alpha = -i\lambda_\alpha(y) + \ldots$. In this context, the electric and magnetic mixing terms both descend from a single complexified parameter,

$$\tau_{ij} = \frac{4\pi i}{g_{ij}^2} + \frac{\theta_{ij}}{2\pi}, \tag{3.2}$$

and the kinetic term is

$$L_{U(1)'s} = \sum_{i,j} \text{Im} \int d^2\theta \frac{\tau_{ij}}{8\pi} W^{(i)} \cdot W^{(j)} \tag{3.3}$$

$$= \sum_{i,j} -\frac{1}{4g_{ij}^2} F^{(i)} \cdot F^{(j)} + \frac{\theta_{ij}}{32\pi^2} F^{(i)} \cdot \tilde{F}^{(j)}. \tag{3.4}$$

In spite of this canonical holomorphic structure, it is still challenging to extract the parameters τ_{ij} for a theory with both electric and magnetically charged states, even with $\mathcal{N} = 1$ supersymmetry. To proceed further, we now assume that we have $\mathcal{N} = 2$ supersymmetry. Let us hasten to add that this will not require us to extend the $U(1)_{\text{vis}}$ gauge theory to actually have $\mathcal{N} = 2$ supersymmetry. All that is really required is that all extra sector states organize into $\mathcal{N} = 2$ supersymmetry multiplets. Indeed, we shall view the visible sector as a weakly gauged flavor symmetry.

Let us review some basic aspects of $N = 2$ supersymmetric gauge theory. For further details, see, for example, Refs. [64,65]. Now, an $\mathcal{N} = 2$ vector multiplet consists of an $\mathcal{N} = 1$ vector multiplet and an $\mathcal{N} = 1$ chiral multiplet. In our conventions, the scalar component of each $\mathcal{N} = 2$ vector multiplet is a^i. When the a^i have generic values, all states charged under the $U(1)$'s will have picked up a mass, and we can integrate them out.[3] In this limit, then, we get a low energy effective action involving $\mathcal{N} = 2$ Abelian vector multiplets. The key point for us is that the parameters τ_{ij} are given by

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[3] For example, for a weakly coupled $U(1)$ gauge theory in which we have an $\mathcal{N} = 2$ hypermultiplet with electric charge q_{elec}, we have a superpotential coupling $W = \sqrt{2}H^c(q_{\text{elec}}a)H$. So, giving a background value to the scalar a gives a mass to the corresponding hypermultiplet.

$$\frac{\partial a_i^D}{\partial a^j} = \tau_{ij}, \qquad (3.5)$$

where we have introduced the scalar of the magnetic dual theory a_i^D given by the derivative of the $\mathcal{N}=2$ prepotential[4]:

$$\frac{\partial \mathcal{F}}{\partial a^i} = a_i^D. \qquad (3.7)$$

An additional benefit of knowing the specific values of these parameters is that we can also extract the mass M of BPS states with prescribed electric and magnetic charges from the central charge Z. For a state of charge $Q^I = (q_{\text{elec}}^1, ..., a_{\text{elec}}^r; q_1^{\text{mag}}, ..., q_r^{\text{mag}})$ which also transforms in a representation $\mathcal{R}$ of a flavor symmetry G_{flav}, we have

$$Z = \sum_{I,J} \Omega_{IJ} Q^I A^J + \frac{1}{\sqrt{2}} \sum_{b=1}^{\dim \mathcal{R}} q_b m^b, \quad \text{with} \quad M^2 = 2|Z|. \qquad (3.8)$$

Here, we have introduced $A^J = (a_1^D, ..., a_r^D; a^1, ..., a^r)$, which pair with the charges via the Dirac pairing Ω_{IJ} of Eq. (2.11). We have also introduced background mass parameters m_b which transform in the representation $\mathcal{R}$ along with corresponding half integrally quantized charges:

$$q_b \in \frac{1}{2}\mathbb{Z}. \qquad (3.9)$$

Physically, we should view the mass parameters as being specified by weakly gauging a flavor symmetry and moving onto the Coulomb branch. From this perspective, we introduce a complex scalar ϕ in the adjoint representation of G_{flav}. Activating a vacuum expectation value (vev) for this field yields a mass for the hypermultiplet,[5]

$$\frac{m^b}{\sqrt{2}} = \vec{w}^b \cdot \vec{\phi}, \qquad (3.10)$$

where $\vec{w}^b$ is a weight vector for a representation $\mathcal{R}$ of G_{flav}, and

$$\vec{\phi} = \sum_{s=1}^{\text{rk}G} \vec{\alpha}_s \phi^s, \qquad (3.11)$$

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[4]Recall that in terms of $\mathcal{N}=2$ superfields (which by abuse of notation we also denote by a^i and a_i^D), the low energy effective Lagrangian specifies the prepotential $\mathcal{F}(a^i)$ via

$$L_{\text{eff}} = \frac{1}{8\pi} \text{Im} \int d^2\theta d^2\tilde{\theta} (\mathcal{F}(a^i) - a^i a_i^D). \qquad (3.6)$$

[5]For example, in a weakly coupled model, with a hypermultiplet in a representation $\mathcal{R}$, we have the superpotential coupling $\sqrt{2}H^c T_A^{\mathcal{R}} \phi^A H$, where $T_A^{\mathcal{R}}$ are generators of G_{flav} in the representation $\mathcal{R}$.

where the $\vec{\alpha}_s$'s are a basis of positive roots of the flavor symmetry algebra.

To extract the kinetic mixing with a visible sector, as well as the mass of various electric and magnetic states, our task therefore reduces to computing a_i^D as a function of the values a^j and the ϕ^s. In particular, if we identify one of the flavor $U(1)$'s with the visible sector $U(1)$ so that $\phi^{\text{vis}} = a^{\text{vis}}$, we can extract the kinetic mixing term:

$$\tau_{\text{vis},i} = \frac{\partial a_j^D}{\partial \phi^{\text{vis}}} = \frac{\partial a_j^D}{\partial a^{\text{vis}}}. \qquad (3.12)$$

Thankfully, this is precisely what the general method outlined by Seiberg and Witten in Refs. [42,43] provides. The key point for us is that there is an auxiliary Riemann surface and a meromorphic 1-form λ (i.e., a 1-form with simple poles) such that the parameters a^i, a_i^D, and m^b are encoded as contour integrals [42,43]. The presence of marked points can be visualized as the effect of weakly gauging a $U(1)$, i.e., adding a long narrow tube to the Seiberg-Witten curve.

A. Supersymmetry breaking effects

A priori, it could happen that, even if supersymmetry is badly broken in the visible sector, it may be preserved in some approximate form in the extra sector. Indeed, the primary assumption we make throughout this work is the presence of (possibly mildly broken) $\mathcal{N}=2$ supersymmetry in the extra sector. The nature of supersymmetry breaking will of course impact some details of the mass spectrum, as well as the amount of mixing between the visible and hidden sectors. Our aim here will therefore be to focus on aspects which are more generic. In particular, we focus on those contributions which come from coupling to the visible sector.

Since we are working in the limit where we treat the visible sector as a weakly gauged symmetry, we can parametrize possible contributions in terms of nonzero background values to the corresponding $\mathcal{N}=2$ vector multiplet. Assuming these effects are small, we can expand in their auxiliary fields. In terms of $\mathcal{N}=1$ superfields a^{vis} and W^{vis}, we can therefore make the substitutions

$$a^{\text{vis}} \mapsto a^{\text{vis}} + \theta^2 F^{\text{vis}} \quad \text{and} \quad W_\alpha^{\text{vis}} \mapsto W_\alpha^{\text{vis}} + \theta_\alpha D^{\text{vis}}. \qquad (3.13)$$

For example, F-term breaking could arise from a symmetry breaking pattern which also breaks a GUT group to the Standard Model gauge group. D-term breaking will inevitably arise in the MSSM and its extensions due to the D-term potential of the MSSM. Expanding as in line (3.13) is valid provided these mass scales are subdominant compared with supersymmetric mass terms:

$$\frac{F^{\text{vis}}}{M_{\text{SUSY}}^2} \ll 1 \quad \text{and} \quad \frac{D^{\text{vis}}}{M_{\text{SUSY}}^2} \ll 1. \qquad (3.14)$$

Let us begin by tracking the impact of F-term supersymmetry breaking on the hypermultiplets. First of all, we can see that the BPS mass formula will now receive corrections. To see why, note that the mass of a hypermultiplet with electric-magnetic charge vector Q^I and flavor charges q_b has central charge

$$Z_{Q,q}(a, a^D, m) = \sum_{I,J} \Omega_{IJ} Q^I A^J + \frac{1}{\sqrt{2}} \sum_b q_b m^b. \quad (3.15)$$

In particular, a hypermultiplet $H^c \oplus H$ has a superpotential coupling:

$$L_{\text{eff}} \supset \int d^2\theta \sqrt{2} H^c Z_{Q,q} H + \text{H.c.} \qquad (3.16)$$

Expanding around the background of line (3.13), we get

$$Z_{Q,q} \mapsto Z_{Q,q} + \theta^2 F^{\text{vis}} \frac{\partial Z_{Q,q}}{\partial a^{\text{vis}}}. \qquad (3.17)$$

If this is the only effect of supersymmetry breaking, we can calculate the correction to the masses of states in the hypermultiplets:

$$|M_\pm^{\text{Bosons}}|^2 = 2|Z_{Q,q}|^2 \pm \sqrt{2} \left| F^{\text{vis}} \frac{\partial Z_{Q,q}}{\partial a^{\text{vis}}} \right| \quad \text{and}$$

$$|M^{\text{Fermions}}|^2 = 2|Z_{Q,q}|^2. \qquad (3.18)$$

This approximation requires $F^{\text{vis}}/M_{\text{SUSY}}^2 \ll 1$. Observe also that the lightest state in the hypermultiplet is a boson and that the supertrace relation on the mass spectrum is obeyed.

An interesting feature of this answer is that there are actually two distinct contributions to the mass splitting formula. First, we have the expected electric contribution from the mass parameters proportional to $q_b m^b$. For a magnetically charged state, there is another contribution proportional to $\partial a^D / \partial a^{\text{vis}} = \tau_{\text{mix}}$.

Consider next the effects of D-term supersymmetry breaking on the vector multiplets. To track these contributions, we return to our kinetic mixing interactions and make the substitution of line (3.13):

$$L_{\text{eff}} \supset \frac{1}{8\pi} \left(\text{Im}\left(\int d^2\theta \tau_{ij} W^{(i)} \cdot W^{(j)} \right) \right.$$
$$\left. + \text{Im}\left(F^{\text{vis}} \frac{\partial \tau_{ij}}{\partial a^{\text{vis}}} \lambda^{(i)} \cdot \lambda^{(j)} \right) + \text{Im}(\tau_{\text{vis},j} D^{(\text{vis})} \cdot D^{(j)}) \right).$$
$$(3.19)$$

The middle term induces a gaugino mass matrix, which in particular can mix a visible sector gaugino with the extra sector gauginos. The last term specifies an effective Fayet-Iliopoulos (FI) parameter for the extra sector [37,38].

The net combination of contributions, in particular the presence of FI parameters and mass terms for the hypermultiplets, provides multiple ways in which supersymmetry may be partially or fully broken. First of all, in the $\mathcal{N} = 2$ supersymmetric limit, we see that in various weakly coupled models, having a large mass but with an FI parameter switched on will lead to a partial breaking of $\mathcal{N} = 2$ to $\mathcal{N} = 1$ supersymmetry [66]. Additionally, in this case, the vacuum generically sits at the origin of the Coulomb branch, and one of the scalars of the hypermultiplet develops a vev, breaking the $U(1)$, thus screening some charges (the ones which are local with respect to the hypermultiplet charge) and confining others (the ones which are nonlocal with respect to the hypermultiplet charge). In such cases, we do not expect to retain as much analytic control, because $\mathcal{N} = 2$ supersymmetry has been badly broken.

An alternative way to retain more analytic control is to also introduce a superpotential mass term for the Coulomb branch scalar of the extra sector. In the context of string constructions where the extra sector originates from a D3-brane probing a visible sector, this will generically happen in the presence of appropriate fluxes/instanton effects [14,15,49,67–69]. Provided the mass of the Coulomb branch scalar is lower than that of the hypermultiplets, but still higher than the supersymmetry breaking terms from mixing with the visible sector, we can continue to use the $\mathcal{N} = 2$ supersymmetric approximation developed here.

IV. RANK 1 THEORIES

In this section, we turn to some concrete examples of kinetic mixing at strong coupling. For simplicity, we consider rank 1 theories, i.e., those with a single $U(1)$ in the extra sector Coulomb branch.

We further specialize to extra sectors which are obtained from a deformation of a strongly coupled $\mathcal{N} = 2$ superconformal field theory with a flavor symmetry group G_{flav}. This case is particularly well motivated from string constructions, as it arises from a probe D3-brane next to a stack of intersecting 7-branes with exceptional gauge symmetry. In such examples, the Standard Model is realized via the stack of 7-branes, and the D3-brane realizes an extra sector [14,15,49–54]. We can describe these theories as $\mathcal{N} = 1$ deformations of $\mathcal{N} = 2$ superconformal field theories with exceptional flavor symmetry [55,56]. Our discussion of the associated $\mathcal{N} = 2$ Seiberg-Witten geometry follows the presentation and analysis of Ref. [70].

We assume that $U(1)_{\text{vis}}$ corresponds to a weakly gauged subgroup of G_{flav}. There can potentially be additional weakly gauged $U(1)$'s contained in G_{flav}. We therefore denote the local electric and magnetic coordinates as a and

a^D and the various mass parameters as m^b. The central charge of a state with electric charge n_{elec} and magnetic charge n_{mag} transforming in a representation $\mathcal{R}$ of G_{flav} is

$$Z = n_{\text{elec}}a - n_{\text{mag}}a^D + \frac{1}{\sqrt{2}}\sum_{b=1}^{\dim \mathcal{R}} q_b m^b, \quad \text{with}$$

$$M^2 = 2|Z|^2. \tag{4.1}$$

The vacua are parametrized by u, the coordinate on the moduli space of vacua. In physical terms, u is given by the vev of an operator of the strongly coupled field theory. A nonzero value for this operator breaks conformal symmetry and gives masses to the hypermultiplets of the theory.[6] The corresponding Seiberg-Witten curve is given by

$$y^2 = x^3 + f(u,m)x + g(u,m). \tag{4.2}$$

The coefficients f and g are determined by our choice of a strongly coupled theory.

Let us now turn to the Seiberg-Witten differential. In general, we need to introduce a meromorphic 1-form with appropriate periods which captures the spectrum of dyonic states in our theory. In fact, there can be more than one choice, and this is dictated by picking a representation $\mathcal{R}$ for the flavor symmetry group, so we denote the Seiberg-Witten differential by $\lambda_{\mathcal{R}}$. Physically, however, the coupling constants will not depend on this choice. In more formal terms, we are specifying a section of the elliptic fibration over the u-plane. The general form of $\lambda_{\mathcal{R}}$ is

$$\lambda_{\mathcal{R}} = \alpha\frac{x\,dx}{y} + \beta\frac{dx}{y} + \sum_b \gamma_b y_b \frac{dx}{y(x-x_b)}, \tag{4.3}$$

where the coefficients α, β, and γ_b depend on the parameters u and m. Here, y_b is the value of y in Eq. (4.2) evaluated at the point $x = x_b$. The parameters of the effective action are in turn obtained by evaluating the contour integrals,

$$a = \oint_{\gamma_A} \lambda_{\mathcal{R}}, \quad a^D = \oint_{\gamma_B} \lambda_{\mathcal{R}}, \quad \frac{1}{k_{\mathcal{R}}}\frac{m^b}{2\sqrt{2}} = \oint_{x_b} \lambda_{\mathcal{R}}, \tag{4.4}$$

---

[6]In a weakly coupled $SU(2)$ gauge theory, it would be given by Tr ϕ^2, where ϕ is the adjoint valued scalar of the $\mathcal{N} = 2$ vector multiplet. In the case of a strongly coupled theory, this characterization is not available. One symptom of this is that for the H_1 Argyres-Douglas theory [71], for example, the scaling dimension of u is $4/3$, and for the E_8 Minahan-Nemeschansky theory [55,56], it has scaling dimension 6. These scaling dimensions are calculated using the method given in Ref. [72] (see also Ref. [73]).

where we have introduced mass parameters m^b of the weakly gauged flavor symmetry. These m^b transform in the representation $\mathcal{R}$. Here, $k_{\mathcal{R}} = \text{Ind}(\mathcal{R})/n$ with $\text{Ind}(\mathcal{R})$ the index of the representation, which in our conventions is set to 1 for the fundamental representation. Additionally, the parameter $n = 1$ if all mass parameters are associated with a unique pole x_b, and $n = 2$ if each pole x_b is associated with two mass parameters. The additional factor of $1/2$ in the last contour integral is due to the fact that we have a two sheeted Riemann surface but are only encircling the pole on one of the sheets.

Physically, the x_b are marked points associated to long narrow cylinders (i.e., weakly gauged flavor symmetries) and where γ_A and γ_B are a basis of 1-cycles on the Riemann surface such that

$$\gamma_A \cap \gamma_B = 1, \qquad \gamma_A \cap \gamma_A = 0, \qquad \gamma_B \cap \gamma_B = 0. \tag{4.5}$$

Now, our aim is to calculate the kinetic mixing couplings of our model. To this end, we will need to evaluate the derivatives,

$$\tau_{\text{extra}} \equiv \frac{\partial a^D}{\partial a} \quad \text{and} \quad \tau_{\text{mix}} \equiv \frac{\partial a^D}{\partial a^{\text{vis}}}, \tag{4.6}$$

where a^{vis} is the local coordinate of the visible sector Coulomb branch, associated with the weakly gauge visible sector $U(1)$. In this approximation, we also have $\tau_{\text{vis}} \simeq i\infty$. Electric/magnetic duality in the strongly coupled $U(1)$ is the geometric statement that there is in general an ambiguity in defining which 1-cycle of our curve is γ_A and which is γ_B. A duality invariant way to parametrize the strength of the extra sector coupling is in terms of the Klein invariant J-function,

$$J(\tau_{\text{extra}}) = \frac{4f^3}{4f^3 + 27g^2}, \tag{4.7}$$

which satisfies $J(i) = 1$ and $J(e^{2\pi i/6}) = 0$.

Our plan in the remainder of this section will be to illustrate how to calculate the explicit form of these mixing terms. We first present the expressions for the period integrals. We will need these in order to extract numerical quantities of interest. After this, we turn to a concrete model which exhibits strong coupling. We calculate the electric and magnetic kinetic mixing parameters in this model and also determine the spectrum of lightest stable charged objects. One can view this as defining an interesting phenomenological scenario in its own right, though from the perspective of a complete string theory construction, it is better viewed as a toy model.

A. Elliptic integrals

Since our eventual aim is to extract numerical values of the magnetic mixing, we will need explicit expressions for the contour integrals of line (4.4). Following Refs. [74–76],

we introduce a basis of three elliptic integrals which we use to express the contour integrals of the Seiberg-Witten differential around the 1-cycles of the Seiberg-Witten curve. In addition to the contours encircling the poles, we have 1-cycles which encircle the roots of the cubic in x appearing in Eq. (4.2),

$$y^2 = x^3 + fx + g = (x - e_1)(x - e_2)(x - e_3), \quad (4.8)$$

where the roots of the cubic are

$$e_i = -\frac{1}{\xi^{i-1}} \left(\frac{2}{3\Lambda}\right)^{1/3} f + \frac{\xi^{i-1}}{3} \left(\frac{3\Lambda}{2}\right)^{1/3} \quad \text{with}$$

$$\Lambda = -9g + \sqrt{3}\sqrt{4f^3 + 27g^2} \quad \text{and} \quad \xi = e^{2\pi i/3}. \quad (4.9)$$

We take a basis in which for u and m real the cycle γ_A is given by encircling e_2 and e_3, and the cycle γ_B encircles e_1 and e_2. Using the presentation in Ref. [76], we have the explicit form of the contour integrals in terms of elliptic integrals:

$$I_A^{(1)} = \oint_{\gamma_A} \frac{dx}{y} = \frac{4}{(e_1 - e_3)^{1/2}} K(k) \quad (4.10)$$

$$I_A^{(2)} = \oint_{\gamma_A} \frac{xdx}{y} = \frac{4}{(e_1 - e_3)^{1/2}} [e_1 K(k) + (e_3 - e_1)E(k)]$$

$$(4.11)$$

$$I_A^{(3)}(c) = \oint_{\gamma_A} \frac{dx}{y(x - c)}$$

$$= \frac{4}{(e_1 - e_3)^{3/2}} \left[\frac{1}{1 - \tilde{c} + p} K(k) \right.$$

$$\left. + \frac{4p}{1 + p} \frac{1}{(1 - \tilde{c})^2 - p^2} \Pi_1 \left(\nu(c), \frac{1 - p}{1 + p}\right) \right]$$

$$(4.12)$$

with

$$k^2 = \frac{e_2 - e_3}{e_1 - e_3}, \qquad p^2 = \frac{e_2 - e_1}{e_3 - e_1},$$

$$\tilde{c} = \frac{c - e_3}{e_1 - e_3}, \qquad \nu(c) = -\left(\frac{1 - \tilde{c} + p}{1 - \tilde{c} - p}\right)^2 \left(\frac{1 - p}{1 + p}\right)^2.$$

$$(4.13)$$

Similar considerations hold for the integrals around γ_B by interchanging e_1 and e_3.

In the above, we have introduced the elliptic integrals (see, e.g., Refs. [77]),

$$K(k) = \int_0^1 \frac{dx}{[(1 - x^2)(1 - k^2 x^2)]^{1/2}} \quad (4.14)$$

$$E(k) = \int_0^1 dx \left(\frac{1 - k^2 x^2}{1 - x^2}\right)^{1/2} \quad (4.15)$$

$$\Pi_1(\nu, k) = \int_0^1 \frac{dx}{[(1 - x)^2(1 - k^2 x^2)]^{1/2}(1 + \nu x^2)}, \quad (4.16)$$

which in Mathematica are respectively given by $K(k) = \texttt{EllipticK}[k^2]$, $E(k) = \texttt{EllipticE}[k^2]$, $\Pi_1(\nu, k) = \texttt{EllipticPi}[-\nu, k^2]$.

In obtaining numeric results, we must be mindful of a few subtleties. First of all, the actual period integral expressions will depend on a basis of electric and magnetic charges for the visible and extra sector. This can lead to shifts in the evaluation of period integrals by contributions proportional to $m/\sqrt{2}$. Our guiding principle is that we recover the correct asymptotics for all periods and masses in suitable decoupling limits.

An additional subtlety has to do with the specific implementation in Mathematica. In the numerical evaluation of these expressions, we will encounter branch cuts in the roots of the cubic in x. To account for this, we fix one patch of values of the parameters for m real and for small phases of u and then continue to other values by permuting the roots of the cubic to retain smooth behavior for all numerically evaluated quantities.

B. H_1 Argyres-Douglas Theory

We now turn to a detailed analysis in the case where the extra sector is a deformation of the H_1 Argyres-Douglas theory [71]. This is also sometimes referred to as the "A_3 Argyres-Douglas theory" because of the way it is engineered by taking type IIB string theory on the background $\mathbb{R}^{3,1} \times X$, where X is a noncompact Calabi-Yau threefold with a local A_3 singularity [78–80].

This is a four-dimensional $\mathcal{N} = 2$ superconformal field theory which enjoys an $SU(2)$ flavor symmetry.[7] Now, in this theory, there is a single $U(1)$ subalgebra of $SU(2)$, so we have our Coulomb branch parameter u and a single complex scalar parametrizing breaking patterns of the flavor symmetry. It therefore suffices to introduce mass parameters m^1 and m^2 transforming in the doublet representation. Returning to Eq. (3.10), we have

$$\frac{m^1}{\sqrt{2}} = \phi \quad \text{and} \quad \frac{m^2}{\sqrt{2}} = -\phi, \quad (4.17)$$

---

[7]The name H_1 simply comes from the fact that in an F-theory construction of this model, we have a D3-brane probing a nonperturbative bound state of (p, q) 7-branes with $SU(2)$ flavor symmetry. Indeed, in F-theory, there are two distinct ways to realize an $SU(2)$ gauge symmetry on a 7-brane, one which is perturbative and is called A_1 (realized by a type I_2 fiber) and one which is nonperturbative and is called H_1 (realized by a type III fiber). For additional discussion on this point, see, e.g., Refs. [81] and [82].

so we can work in terms of a single mass parameter $m = \sqrt{2}\phi$.

The Seiberg-Witten curve and Seiberg-Witten differential in the fundamental representation are (see Ref. [70])

$$y^2 = x^3 + ux + w_2 \qquad (4.18)$$

$$\lambda = \frac{\sqrt{2}}{4\pi i} \left(\frac{u}{3} + \frac{m_1 y_1}{x} \right) \frac{dx}{y}, \qquad (4.19)$$

where

$$w_2 = -4m_1 m_2 = 4m^2 \qquad (4.20)$$

is the mass dependent quadratic Casimir and $y_1 = 2m_1$. Physically, one can view w_2 as the gauge invariant operator proportional to $\mathrm{Tr}\phi^2$ we would get from weakly gauging the $SU(2)$ flavor symmetry. In the above, we used Eq. (4.4) with $k_{\mathcal{R}} = 1/2$ (since we have the Seiberg-Witten (SW) differential in the fundamental representation, but there is a single pole).

As a first step toward understanding the parameter space of our model, we compute the Klein-Invariant J-function:

$$J(\tau_{\mathrm{extra}}) = \frac{4u^3}{4u^3 + 27(4m^2)^2}. \qquad (4.21)$$

So, depending on the parameters, we can either be at strong coupling or weak coupling. For example, three canonical values of interest are

$$\tau_{\mathrm{extra}} = i \quad \text{for } m = 0 \qquad (4.22)$$

$$\tau_{\mathrm{extra}} = e^{2\pi i/6} \quad \text{for } u = 0 \qquad (4.23)$$

$$\tau_{\mathrm{extra}} \simeq i\infty \quad \text{for } \left(\frac{u}{3}\right)^3 + 4m^4 = 0. \qquad (4.24)$$

The parameters u and m each implicitly specify mass scales. More precisely, because the H_1 Argyres-Douglas theory is (at the origin of moduli space) actually a superconformal field theory, homogeneity allows us to fix the scaling of u and m as a function of energy scales. We have

$$u \sim \mathrm{Mass}^{4/3} \quad \text{and} \quad m \sim \mathrm{Mass}. \qquad (4.25)$$

The fractional power in the scaling of u is one of the hallmarks of a strongly coupled superconformal field theory. This leaves us with one unfixed dimensionless ratio, m^4/u^3.

Depending on the phenomenological scenario, the actual mass scales involved could be anywhere from the GUT scale down to the TeV or sub-TeV scale. For example, in

many string-motivated scenarios, it is natural to take $m \sim 10^{16}$ GeV since this is the implicit scale set by separating the various 7-branes from each other. On the other hand, if we assume that the dominant contribution to conformal symmetry breaking is set by supersymmetry breaking effects, a far lower reference scale is also possible.

We now turn to the calculation of the periods a and a^D and their derivatives. We have

$$a = \frac{\sqrt{2}}{4\pi i} \left(\frac{2u}{3} I_A^{(1)} + w_2 I_A^{(3)}(0) \right) - \frac{2}{3}\frac{m}{\sqrt{2}} \qquad (4.26)$$

$$a^D = \frac{\sqrt{2}}{4\pi i} \left(\frac{2u}{3} I_B^{(1)} + w_2 I_B^{(3)}(0) \right) + \frac{2}{3}\frac{m}{\sqrt{2}}, \qquad (4.27)$$

with $w_2 = 4m^2$, as per Eq. (4.20). Let us make a few comments on the presence of the terms proportional to m in our period integrals. Strictly speaking, this last piece is just an artifact of how we pick a basis of contour integrals, i.e., how we choose to define our basis of electric charges with respect to the visible sector. The choice in the above equation comes from imposing the condition that as we take the $m \to \infty$ decoupling limit, a and a^D should be independent of m. Additionally, we pass to a theory with no continuous flavor symmetry and in which the asymptotic value of $a^D/a \to \exp(2\pi i/6)$; i.e., the value of τ_{extra} in this limit is frozen. This induces a flow from the H_1 Argyres-Douglas theory to what is known as the H_0 Argyres-Douglas theory.

The first derivatives of the periods provide us with the complexified gauge coupling and the mixing parameter:

$$\tau_{\mathrm{extra}} \equiv \frac{\partial a^D}{\partial a} = \frac{\partial a^D/\partial u}{\partial a/\partial u} \quad \text{and} \quad \tau_{\mathrm{mix}} \equiv \frac{\partial a^D}{\partial \phi} = \sqrt{2}\frac{\partial a^D}{\partial m}. \qquad (4.28)$$

Taking one more derivative provides us with the terms which appear in gaugino mixing (after supersymmetry breaking) as well as the coupling between the Coulomb branch scalar and the $U(1)$ gauge fields. In evaluating these derivatives, we must treat a and ϕ as independent variables.

To get a sense of the overall values of these coupling constants, and to emphasize the point that these really are calculable quantities, we present a few of the numerically evaluated derivatives obtained via our method. In general, it is challenging to obtain a set of parameters which remains in a single fundamental domain (i.e., a single basis of electric and magnetic charges). To bypass these issues and get a sensible class of examples, we hold fixed $u = 0.1$, with m in powers of 5. For the first derivatives of a^D, we have

	$\tau_{\text{extra}} = \partial a^D/\partial a$	$\tau_{\text{mix}} = \partial a^D/\partial \phi$
$m = 0.04$	$0.13 + 0.99i$	$(-2.0 + 1.0i) \times 10^{-1}$
$m = 0.2$	$0.45 + 0.90i$	$(-2.4 + 3.5i) \times 10^{-2}$
$m = 1.0$	$0.50 + 0.87i$	$(-2.6 + 4.4i) \times 10^{-3}$
$m = 5.0$	$0.50 + 0.87i$	$(-3.0 + 5.2i) \times 10^{-4}$

$$(4.29)$$

and for the second derivatives of a^D, we have

	$\partial^2 a^D/\partial a \partial a$	$\partial^2 a^D/\partial a \partial \phi$	$\partial^2 a^D/\partial \phi \partial \phi$
$m = 0.04$	$3.6 \times 10^0 - 5.4i$	$3.7 - 2.4i$	$(5.1 - 1.4i) \times 10^0$
$m = 0.2$	$3.1 \times 10^{-1} - 3.4i$	$-0.7 - 2.1i$	$(2.4 - 2.9i) \times 10^{-1}$
$m = 1.0$	$6.9 \times 10^{-3} - 0.65i$	$-0.2 - 0.4i$	$(4.9 - 8.0i) \times 10^{-3}$
$m = 5.0$	$1.6 \times 10^{-4} - 0.13i$	$-0.05 - 0.08i$	$(1.1 - 1.9i) \times 10^{-4}$

$$(4.30)$$

Let us stress that the physically more meaningful quantity is given by a duality invariant expression such as a scattering amplitude, as in our discussion in Sec. II. The reason is that to get a proper notion of the overall strength of kinetic mixing, we also need to know the spectrum of charges in the extra sector which can couple to the visible sector.

1. BPS spectrum

For various model building considerations, it is important to know the spectrum of stable objects in our system and their charges in some duality frame under both the extra sector $U(1)$ and the visible sector $U(1)$. In more realistic models where supersymmetry is broken, the spectrum will be deformed with a mass splitting specified as in our discussion around Eq. (3.18). A nonzero mass splitting within a multiplet also means that there can now be nontrivial decays to the lowest mass state. With an unbroken $U(1)$, however, this bottom component will be stable. We therefore view the $\mathcal{N} = 2$ supersymmetric approximation as telling us the leading order structure of stable objects in our theory.

Let us now turn to the BPS spectrum of the H_1 Argyres-Douglas theory. With the explicit form of the period integrals in hand, we can also determine the lightest BPS particles at any point on the Coulomb branch. For early work on the BPS spectrum of Argyres-Douglas theories, see Ref. [80]. In general terms, the spectrum of stable BPS states in the system will depend on the value of the Coulomb branch parameter and mass parameters of the model. An additional feature is that we should expect "wall-crossing phenomena" in which the spectrum of stable

objects actually changes as we cross real codimension-1 loci in the moduli space of vacua [42,83,84].

Returning to the BPS formula for the mass of our states given in Eq. (4.1), we have for a state of the rank 1 H_1 Argyres-Douglas theory

$$Z = n_{\text{elec}} a - n_{\text{mag}} a^D + q_{\text{flav}} \frac{m}{\sqrt{2}}, \quad \text{with} \quad M^2 = 2|Z|^2,$$

$$(4.31)$$

so we see that if we take $m \to \infty$, a state with nonzero charge with respect to the flavor symmetry will develop a large mass.

It is also possible to arrange for the flavor neutral state to be lightest by appropriately tuning the parameters and moduli of the theory. For example, we can ensure that we have an approximately massless state by working in the special limit where the discriminant is nearly zero,

$$4u^3 + 27(w_2)^2 \simeq 0,$$

$$(4.32)$$

with $w_2 = 4m^2$ given by Eq. (4.20). Indeed, in this case, the length of the cycle used to generate the period a collapses to zero size, and the corresponding BPS mass of a $U(1)_{\text{extra}}$ electrically charged state will be zero. In the special case of the H_1 Argyres-Douglas theory, we can also see that when u and m are both nonzero, the coupling constant τ_{extra} will be near $i\infty$, i.e., the point of weak coupling.

Let us now turn to the calculation of the BPS spectrum of the theory in the Coulomb phase. There are by now various methods for performing such a calculation. These include the method of "BPS quivers," e.g., Refs. [85–89], as well as the method of spectral networks, e.g., Refs. [90,91].

Since we have an explicit presentation for all of the period integrals and we can track the dependence on moduli, we shall use the method of BPS quivers.

The main idea in the BPS quiver method is to recognize that all of the BPS particles are obtained as bound states of smaller elementary constituent particles. The number of independent charges for these particles is completely fixed by the number of $U(1)$ factors of the model. For each gauged $U(1)$, we get two charges (one electric and one magnetic), while for each $U(1)$ flavor symmetry, we get one charge (just electric). The dynamics governing the stability of a configuration is encoded by a supersymmetric quiver quantum mechanics (SQM) with four conserved supercharges [92]. The quiver is determined by the elementary constituents as follows: it has nodes in one-to-one correspondence with the charges of the elementary constituents and directed arrows between two such quiver nodes specified by the Dirac pairing for these charges. In string theory terms, we view the nodes as candidate BPS objects and the directed arrows as open strings which stretch from one BPS object to the next. The existence of a bound state of given charge corresponds to the existence of a ground state for the corresponding SQM [92,93].

For the Argyres-Douglas theory, the total number of generators of the Coulomb branch charge lattice is $2 + 1 = 3$. Indeed, we can express the charge of a candidate state as a three component vector which we write as a linear combination of the form

$$(n_{\text{elec}}, n_{\text{mag}}, q_{\text{flav}}) = \gamma = \sum_{i=1}^{2r+f} n_i \gamma_i \quad \text{for } n_i \geq 0. \quad (4.33)$$

Here, the γ_i are the constituent charges out of which all other stable bound states are constructed.

Now, as we vary the value of the complex phase in u, we can expect some new bound states to enter or exit the spectrum. For the H_1 Argyres-Douglas theory, the full list of candidate states is dictated by the root space of the corresponding A_3 lattice [80]:

$$\text{Candidate Charges} = \{\pm\gamma_1, \pm\gamma_2, \pm\gamma_3, \pm(\gamma_1 + \gamma_2),$$
$$\pm(\gamma_2 + \gamma_3), \pm(\gamma_1 + \gamma_2 + \gamma_3)\}.$$
$$(4.34)$$

For this model, all the stable BPS states are hypermultiplets.[8]

The task of finding the spectrum of stable states therefore decomposes into two pieces. First, we need to determine a good quiver basis $\{\gamma_i\}_i$ in the sense of Refs. [87,93], and, second, we determine which values of

n_i in Eq. (4.33) lead to stable particles.[9] The actual presentation of the quiver as well as the spectrum of stable particles will depend on the particular region of moduli space where we are located. The basis of charges we use to construct our bound states will change, i.e., we have a transformation of the form,

$$\gamma_i \mapsto \gamma_i' = \sum_j M_{ij} \gamma_j, \quad (4.35)$$

for M_{ij} an integer valued matrix.[10] This leads to a "mutation" or Seiberg duality on the quiver SQM. The candidate physical charges of line (4.34) can also change; i.e., we build our spectrum of candidates using γ_i' rather than γ_i. A mutation simply reflects the fact that the structure of composite objects may change as we change the moduli/parameters of the model: an object which looks elementary in one frame, can look like a bound state in another. In string theory terms, this means that we must alter the BPS states used to construct bound states, and correspondingly the spectrum of open strings will also change. On top of that, the actual spectrum of stable BPS states can change as we move in moduli space (wall-crossing phase transitions).

Since we have an explicit presentation of the various period integrals, it is straightforward for us to sweep over possible choices of charge assignments. The main complication is to ensure that we have indeed found all of the stable particles at a given point in the moduli space, i.e., wall crossing.

At a qualitative level, there are three general regimes of possible interest:

$$\text{Large Mass:} \, |m| \gg |u^{3/4}| \quad (4.36)$$

$$\text{Tuned Mass:} \, \left(\frac{u}{3}\right)^3 + 4m^4 \simeq 0 \quad (4.37)$$

$$\text{Small Mass:} \, |m| \ll |u^{3/4}|. \quad (4.38)$$

For illustrative purposes, we study in detail the large mass regime. We shall also explain how a similar analysis applies at small mass parameters.

Consider, then, the large mass regime. Here, we have $\tau_{\text{extra}} \simeq e^{2\pi i/6} \simeq e^{2\pi i/3}$, so we are at strong coupling. An additional simplification is that we always expect the lightest object to be neutral under the flavor symmetry. To determine the spectrum near this point, it is helpful to

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[8]Borrowing from standard techniques in soliton theory, the spin of a BPS multiplet is determined "quantizing" the moduli space of vacua for the SQM [92,94] (see also Ref. [95]).

[9]We find, however, that the technical definition specified in Refs. [87,93] for a good quiver basis is not enough to determine it uniquely; there is an extra condition (compatibility among mutations and wall crossings) which needs to be imposed. The details of this point are discussed in the Appendix.

[10]The precise form of the allowed matrices M_{ij} is subject to the same caveat discussed in footnote 2.

rely on the existing analysis of BPS quivers presented, for example, in Refs. [87,89]. For the H_1 theory, there are always at least three stable BPS states corresponding to the three nodes of the BPS quiver. These are always $\mathcal{N} = 2$ hypermultiplets. In addition to these three states, there can in principle be others which are also stable.

We find that when $|u| = 0.1$ and $m = 1$, we are effectively in the large mass regime. So, let us turn to an analysis of the BPS quiver in this regime. To illustrate, suppose that we hold fixed the parameters:

$$u = 0.1 \exp(i\theta), \quad m = 1. \tag{4.39}$$

When $5\pi/3 \leq \theta \leq 2\pi$, the quiver SQM governing the dynamics of the BPS solitons is the quiver with nodes,

$$\text{BPS Quiver}: \gamma_1 \rightarrow \gamma_2 \leftarrow \gamma_3, \tag{4.40}$$

with

Node\charge	n_{elec}	n_{mag}	q_{flav}
γ_1	+1	+1	+1/2
γ_2	−1	0	0
γ_3	+1	+1	−1/2

$$(4.41)$$

so the three hypermultiplets with charges γ_1, γ_2, and γ_3 are the elementary BPS states in this region of moduli space. Notice that we have a stable "dark electron" with charge γ_2. This is a stable BPS particle which is neutral under the flavor symmetry. For these values of θ, there is an additional bound state with charge $\gamma_2 + \gamma_3$ in the spectrum. Now, as we vary the phase θ, we can expect that some of these objects ceases to be elementary and decay to other stable constituents. To figure out the possible changes as we move around, we need to explore the various mutants quiver SQMs occurring as we vary θ in line (4.39). The actual pattern of wall crossings is analyzed in details in the Appendix. The precise structure of the BPS spectrum as a function of θ is plotted in Fig. 2. We find that in sweeping over all values of the phases for $|u| \sim 0.1$, the dark electron with charge γ_2 remains a stable object of the spectrum.

Let us also note that, although the "dark dyon" with charge $\gamma_1 + \gamma_2 + \gamma_3$ has lower mass than its flavor charged counterparts, it is nevertheless not a stable object in the large mass regime for $|u| \sim 0.1$. Rather, it can enter the spectrum as we decrease the value of m (see Fig. 3 as well as the Appendix).

Similar analyses can be carried out for all of the regions of moduli space and mass parameter space. An important point is that near the region $m = 0$, we also have a restored $SU(2)$ flavor symmetry, so as a consequence, the states have a mass degeneracy compatible with this fact. Another interesting feature close to this region is that the state of charge γ_2 is not always the lightest in the spectrum.

2. Dark electron and dark dyon

From our analysis of the mass spectrum of the H_1 Argyres-Douglas theory, we can also draw some conclusions about the spectrum of stable particles which are neutral under the visible sector gauge coupling. For both the large and small mass regimes, the states of charge $\pm\gamma_2$ are stable. We refer to this as a dark electron since it only has electric charge under the extra sector $U(1)$. Additionally, in some regions of parameter space, there is another flavor neutral state which in a suitable basis of electric and magnetic charges has charge vector $\pm(\gamma_1 + \gamma_2 + \gamma_3)$ which we refer to as the dark dyon since it has both electric and magnetic charge under the extra sector $U(1)$.

Now, even when these states are unstable, they can still play an important role in scattering events between the visible and hidden sector. The reason is that with a sufficiently energetic process in the extra sector, we may still be able to generate such charged states. Since we can also calculate the effects of kinetic mixing, we now ask
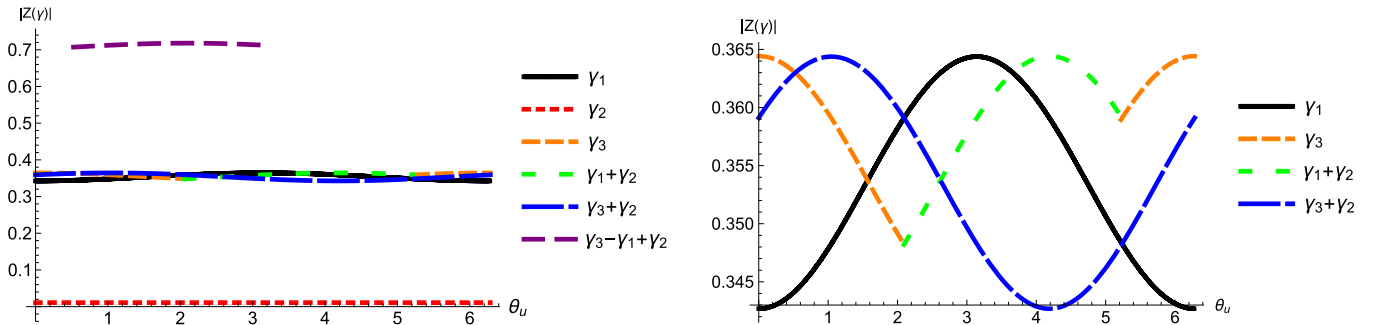

FIG. 2.   LEFT: Plot of the spectrum of masses for the stable states as a function of θ in the large mass regime. For numerical purposes, we take $u = 0.1 \exp(i\theta)$ and $m = 1$. Notice that a BPS state with charge $\gamma_3 - \gamma_1 + \gamma_2$ enters the spectrum in the region $0.525 < \theta < 3.65$. This is possible precisely because the BPS quiver relevant in that region is a mutant of the one in line (4.40) (see the Appendix for the details). RIGHT: Magnified region of the plot of $|Z|$ with $u = 0.1 \exp(i\theta)$ and $m = 1$, which shows γ_3 destabilizing and $\gamma_1 + \gamma_2$ stabilizing in complementary regions.
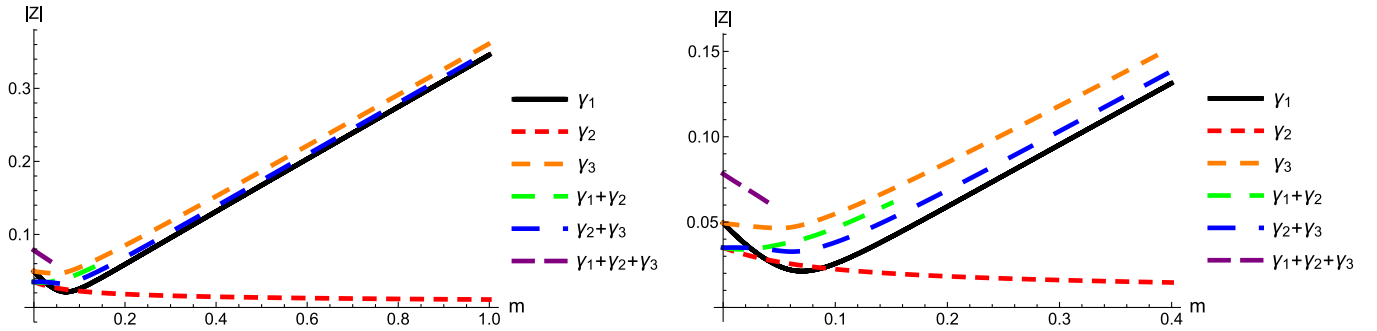
FIG. 3.   LEFT: Plot of the spectrum of masses for the stable states as a function of m interpolating from the large mass regime $m = 1$ to $m = 0$. For numerical purposes, we take $u = 0.1 \exp(i5.5)$. The whole deformation is covered by the BPS quiver in line (4.40); no mutation occurs. RIGHT: Magnified region of the plot which shows 1) the region where the dark dyon stabilizes and 2) the small region where the dark electron ceases to be the lightest massive excitation.

what the effective electric charge under the visible sector $U(1)$ is for each of these states. The effective electric charge follows from our formula for dark Rutherford scattering presented in Eq. (2.25). To keep the analysis simple yet tractable, we shall primarily focus on the single slice of parameters $u = 0.1$, with m varying by powers of 5.

As in our earlier analyses, we work in dimensionless units; i.e., depending on the scale of conformal symmetry breaking (dictated by its coupling to other sectors), the actual mass of the state could be anywhere from the TeV scale to the GUT scale. Here, then, is the list of effective electric charges as we vary the value of m:

Dark electron	$m = 0.04$	$m = 0.2$	$m = 1.0$	$m = 5.0$
$\|q_{\text{eff}}(\gamma_2)\|$	2.0×10^{-2}	1.0×10^{-3}	1.3×10^{-5}	1.8×10^{-7}
$\|Z(\gamma_2)\|$	2.9×10^{-2}	1.8×10^{-2}	1.1×10^{-2}	6.3×10^{-3}

$$(4.42)$$

Dark dyon	$m = 0.04$	$m = 0.2$	$m = 1.0$	$m = 5.0$
$\|q_{\text{eff}}(\gamma_1 + \gamma_2 + \gamma_3)\|$	4.0×10^{-1}	8.1×10^{-2}	1.0×10^{-2}	1.2×10^{-3}
$\|Z(\gamma_1 + \gamma_2 + \gamma_3)\|$	5.6×10^{-2}	3.2×10^{-2}	1.9×10^{-2}	1.1×10^{-2}

$$(4.43)$$

where for reference we have also included the corresponding values of the central charge. Again, we emphasize that the dark dyon is not stable in some regions of parameter space, e.g., in the large mass regime $|m| \gtrsim 1$.

V. PHENOMENOLOGICAL TOY MODELS

Having spelled out the main technical elements of how to compute kinetic mixing at strong coupling, we now turn to some aspects of how these models embed in more realistic phenomenological scenarios. Even so, we will keep our discussion at the level of toy models, using the H_1 Argyres-Douglas theory as our primary example.

Indeed, in the context of string constructions, the Argyres-Douglas theory should be viewed as a subsector of a more complete model. From a bottom up perspective, however, we can view deformations of the H_1 Argyres-Douglas theory as a candidate extra sector in its own right. Even in this case, however, there are several moving parts which can impact the resulting phenomenology.

The rest of this section is organized as follows. First, we place the H_1 Argyres-Douglas theory in the context of more general stringy constructions which incorporate the Standard Model. After this, we explain how different scales of conformal symmetry breaking lead to different types of phenomenological scenarios.

A. String-motivated examples

One of the motivations for this work is the fact that string constructions typically contain extra $U(1)$'s which can mix with the visible sector $U(1)$. To illustrate the general suite of ideas, we focus on the class of extra sectors introduced in Refs. [14,15,49,50,96]. In these models, the Standard Model is realized from a stack of intersecting 7-branes, and the extra sector is realized by a probe D3-brane. This D3-brane is energetically attracted to the visible sector by the same mechanism which generates quark and lepton masses and mixing angles [67,97] (see also Refs. [68,69]). As a passing remark, we note that in constructions of the Standard Model via heterotic M-theory, a similar class

of extra sectors is realized by M5-branes wrapped on a curve of the compactification manifold.

A priori, there may be other local minima for the D3-brane, so fluxes may localize it at other points of the compactification manifold. Indeed, we can expect there to typically be many such D3-branes. The total number in a general type IIB background is given by the formula [98]

$$N_{D3} = \frac{\chi(CY_4)}{24} + \int_B H_{NS} \wedge H_{RR}, \qquad (5.1)$$

where $\chi(CY_4)$ is the Euler characteristic of the elliptically fibered Calabi-Yau fourfold used to define an F-theory background and H_{NS} and H_{RR} are 3-form fluxes which are integrated over the six-dimensional internal spacetime B. Values of $\chi(CY_4)/24$ can range from $O(10^2)$ to $O(10^4)$ (see, e.g., Refs. [99,100]), so depending on the choice of background fluxes, one can contemplate scenarios with either many D3-branes or only a small number.

One of the interesting features of kinetic mixing is that, because it comes from integrating out heavy states to generate marginal couplings, we can expect there to be possible contributions to electric and magnetic kinetic mixing even for those D3-branes which are far removed from our visible sector stack. So, even for extra sector models where other direct couplings to the Standard Model are suppressed (as they typically will be), kinetic mixing at strong coupling can still survive.

Let us now turn to more details of the resulting effective field theory on a D3-brane. In the limit where the D3-brane is close to the Standard Model stack of intersecting 7-branes, we can visualize this extra sector as an $\mathcal{N} = 2$ superconformal field theory with E_8 flavor symmetry [55,56] which is subject to $\mathcal{N} = 1$ relevant and marginal deformations which induce a flow to an $\mathcal{N} = 1$ superconformal field theory in which the flavor symmetry of this IR theory includes the gauge group of the Standard Model [14,15,49,50,96].

We organize our discussion according to the decomposition of $SU(5)_{\text{GUT}} \times SU(5)_{\perp} \subset E_8$, with corresponding mass deformations valued in the adjoint representations; i.e., we schematically write ϕ_{GUT} and $\phi_{\perp}$ for these Coulomb branch parameters. Geometrically, the main idea is that the Coulomb branch parameter u describes the position of a D3-brane normal to the $SU(5)_{\text{GUT}}$ 7-brane. There are also two complex directions u_1 and u_2 parallel to the 7-brane. In the associated field theory, $u_1 \oplus u_2$ parametrize a decoupled hypermultiplet. To get an $\mathcal{N} = 1$ deformation, we therefore allow the mass parameters of the theory to depend on u_1 and u_2, so we make the substitution $\phi_{\perp} \mapsto \phi_{\perp}(u_1, u_2)$. Additionally, we need not require that $\phi_{\perp}$ is even diagonal. We can also consider mass deformations which break $SU(5)_{\text{GUT}}$ to $SU(3) \times SU(2) \times U(1)$, i.e., by taking a mass deformation in the same direction as $U(1)_Y$.

To apply the methods of the present paper, we must also assume that the deformation to an $\mathcal{N} = 1$ vacuum is sufficiently mild; i.e., we have a "short flow" from a neighboring $\mathcal{N} = 2$ theory. Now, even though we only have $\mathcal{N} = 1$ supersymmetry, there is still a notion of a Seiberg-Witten curve, with the Seiberg-Witten differential now replaced by a meromorphic 4-form of a noncompact Calabi-Yau fourfold. The main caveat to extracting numerical estimates, however, is that the physical couplings may now receive nontrivial contributions from wave function renormalization. This shows up quite directly in other contexts as corrections to the scaling dimensions of operators in the deformed theory; see, e.g., Refs. [15,50].

While we leave a complete analysis of this more involved case to future work, it is interesting to already explore some of the general features of these models. First of all, we see that if we take most of the mass parameters to be of the GUT scale or higher, then the lightest states which can meaningfully participate at low energies will be those which are neutral under the flavor symmetries. As we have already seen in the H_1 Argyres-Douglas theory, there is a lightest state which is neutral under all such flavor symmetries, with mass controlled primarily by the Coulomb branch parameter. Additionally, we can reincorporate some of the effects of heavier states of the model. These will show up as line operators (that is, heavy quarks) of the theory, and we can also contemplate bound states of comparatively light objects to these line operators. The excitation scale for these heavy objects can naturally be at the GUT scale or higher, so in this sense, their direct relevance for phenomenology may be more limited. It is interesting to note, however, that in some cases, we can tune parameters of the string-based model to realize excitations of these objects at lower energy scales. Indeed, an intriguing novelty of rank 1 theories with larger flavor symmetry groups such as E_6, E_7, and E_8 is the presence of whole Regge trajectories of stable objects in certain ranges of moduli space [89,101–103]. This clearly leads to a rich class of possibilities, which would be quite interesting to study in future work.

B. Mass scales

To make more contact with model building considerations, we clearly need to specify possible mass scales for our model. Since we have an extra sector with approximate conformal symmetry, we expect that the masses of the extra sector states will be dictated by the scale of conformal symmetry breaking. Even in this case, however, we can get different mass hierarchies, since, as we saw in the case of the H_1 Argyres-Douglas theory, taking the mass parameter m very large still leaves us with a light state which we referred to as the dark electron. In other regimes of parameter space, this can also be accompanied by a dark dyon. Let us step through the different kinds of scenarios associated with each sort of mass scale.

1. GUT scale masses

Suppose we take the simplest scenario in which all hypermultiplets have GUT scale masses. This possibility is also well motivated in the context of string constructions. In this case, we expect to be left at low energies with a collection of $U(1)$ gauge bosons and their $\mathcal{N} = 2$ superpartners. Transmission of supersymmetry breaking to the extra sector will then lead to further mass splittings amongst the states.

The phenomenological bounds on extra decoupled $U(1)$'s are quite weak, since without any charged states from the extra sector, there is no way to directly detect these vector bosons.

The caveat to this statement is that we also have the $\mathcal{N} = 2$ superpartners, which include a gaugino and a decoupled $\mathcal{N} = 1$ chiral multiplet. As we have already remarked, the extra sector gauginos can mix with visible sector gauginos. These mixing terms depend on the details of supersymmetry breaking, but we have shown in Sec. III how to calculate these contributions in certain supersymmetry breaking scenarios by computing the second derivatives of a^D with respect to a and m. Some aspects of the phenomenology of these photini mixing have been studied, for example, in Ref. [104].

Consider next the $\mathcal{N} = 1$ chiral multiplet. In the limit of exact $\mathcal{N} = 2$ supersymmetry, the presence of a complex scalar with no potential suggests the presence of a modulus, which, if left unstabilized, can lead to a cosmological history in which the energy density is dominated by such a rolling scalar.

There is a simple way to ameliorate this issue by introducing an overall superpotential deformation of the system, i.e., $W(u)$, for the Coulomb branch parameter. For us to continue to use our $\mathcal{N} = 2$ supersymmetric approximation, we simply need to require that the mass scale for the scalar is small compared to those of the charged states, i.e., that $m_a \ll \langle a \rangle$, with a the local expression for the Coulomb branch scalar. This is technically natural since such mass terms are conformally suppressed in this class of models [15,49].

2. TeV and sub-TeV scale masses

It is also natural to consider scenarios in which some of the extra sector states have masses far below the GUT scale. For example, if the D3-brane remains close to the Standard Model stack, we can still expect some flavor neutral hypermultiplets to survive to much lower energies. Again, this is technically natural since a superpotential deformation for the Coulomb branch parameter can be conformally suppressed [15,49]. In such cases, transmission of supersymmetry breaking to the extra sector will also contribute to the masses of these states. We can also see from our analysis near lines (4.42) and (4.43) that the effective electric charge for these flavor neutral states can be quite small. For some discussion on cosmological constraints on millicharged particles as well as scenarios with an exactly

massless $U(1)$ decoupled from the Standard Model, see, respectively, Refs. [105] and [106] (see also Ref. [107]).

In the TeV scale mass range, much of the phenomenology is dictated by whether the extra sector $U(1)$ is electrically screened/magnetically confined or remains as a long range force carrier. Some aspects of the former case were studied in detail in Ref. [15], to which we refer the interested reader for further details. In this case, we get string-motivated examples of asymmetric dark matter models with order 10 GeV masses for dark matter. The sub-TeV mass scale originates from a seesawlike mechanism for dark states connected with partial breaking to $\mathcal{N} = 1$ supersymmetry [15]. Even lower mass scales are potentially possible, though the presence of heavier extra sector states charged under the visible sector means that we must exercise some care in building such models.

If, on the other hand, we assume that the extra sector $U(1)$ remains as a long range force carrier, then we have a conserved electric and magnetic charge, and so we can also expect there to be stable dark states. We have also seen that visible sector charged states can be decoupled.

Assuming we have a TeV scale dark state, we can estimate its cosmological relic abundance. The fact that we have kinetic mixing with the visible sector as well as a strongly coupled extra sector means that the overall thermally produced relic abundance will be lower than that of the standard weakly interacting massive particle (WIMP) example. For example, letting Ω_{extra} denote the relic abundance of such an extra sector state and Ω_{DM} that of WIMP dark matter, we have

$$\frac{\Omega_{\text{extra}}}{\Omega_{\text{DM}}} \sim \frac{\alpha_{\text{WIMP}}^2}{\alpha_{\text{extra}}^2} \frac{M_{\text{extra}}^2}{M_{\text{WIMP}}^2} \sim \left(\frac{10^{-3}}{\alpha_{\text{extra}}^2}\right)\left(\frac{M_{\text{extra}}}{1 \text{ TeV}}\right)^2, \quad (5.2)$$

so we see that if our extra sector states are around the TeV scale, an order 1 value for α_{extra} suppresses the overall contribution of these states. For this reason, we see that any individual extra sector will make only a small contribution to the net relic abundance; i.e., we can easily satisfy various cosmological bounds. Observe also that if we have a nonthermal epoch in the evolution of the Universe, i.e., one with a late decaying scalar, it can also be beneficial to overproduce this relic abundance, as is common in some string based constructions [108,109].

Aside from their potential role in cosmology (if we have multiple decoupled extra sectors to obtain a suitable relic abundance), we now have the strongly coupled analog of extra charged states which could be generated in collider experiments. Indeed, we have also explained how these extra sector states can produce an effective electric charge [cf. Eq. (2.25)]. This leads to generalizations of the standard Z' scenario, which would be interesting to study further. It is important to emphasize, however, that the strongly coupled nature of the extra sector means that some of the implicit assumptions usually made in the analysis of

Z' models should be revisited before drawing any definite conclusions on this class of models. We leave a full analysis of this possibility for future work.

VI. CONCLUSIONS

Kinetic mixing at strong coupling is well motivated from both a top down and bottom up perspective. We have shown how to extract the leading order mixing terms for an extra sector with approximate $\mathcal{N} = 2$ supersymmetry and commented on their potential role in phenomenological scenarios. In the remainder of this section, we discuss some avenues of future investigation.

It would be interesting to extend our analysis to larger unbroken flavor symmetry groups for the extra sector. In particular, theories with exceptional flavor symmetry have a rich spectrum of BPS objects which can also figure into model building considerations.

A related question is how to carry over our results to models in which $\mathcal{N} = 2$ supersymmetry is broken to $\mathcal{N} = 1$ or $\mathcal{N} = 0$ supersymmetry. Provided these supersymmetry breaking effects are sufficiently mild, we anticipate that the formal techniques developed here should be more broadly applicable.

Finally, it is tempting to speculate that, because our $\mathcal{N} = 2$ sector contains a scalar modulus with a flat potential, this mode could play the role of an inflaton in slow roll inflation [14], with reheating triggered by reaching the origin of moduli space. This suggests yet another potential role for such extra sectors.

APPENDIX: DETAILS ON THE BPS SPECTRUM OF THE H_1 MODEL

The BPS quiver computation of the BPS spectrum is achieved by means of quiver representation theory [92]. By a standard geometric invariant theory argument, the Higgs branch moduli space of the quiver SQM corresponding to a state of charge γ is equivalent to the moduli space of stable representations of dimension vector $(N_1, \ldots, N_n)$, where $\gamma = \sum_i N_i \gamma_i$ and γ_i is a good quiver basis in the sense of Refs. [87,93] (i.e., one for which 1) the coefficients N_i are either all non-negative integers or nonpositive ones and 2) $\mathrm{Im}Z(\gamma_i) > 0 \ \forall \ i = 1, \ldots, n$). In our case, we find two candidates of good quiver basis for the H_1 model at $u \sim 0.1$ and $m \sim 1$: the one outlined in the main body of
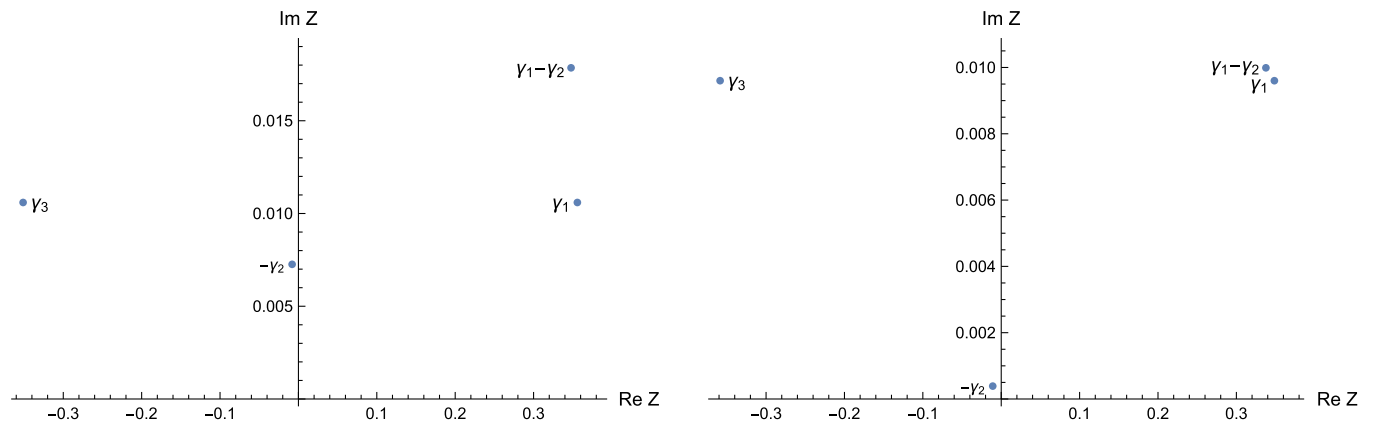

FIG. 4. LEFT: Putative quiver basis $\gamma_1, -\gamma_2, \gamma_3$ for $\pi < \theta < 5\pi/3$. RIGHT: Same basis as θ approaches $5\pi/3$ from the left; one can see that the bound state $\gamma_1 - \gamma_2$ destabilizes by wall crossing, which is in contradiction with the mutation rule for the basis elements as $Z(-\gamma_2)$ exits the upper Z-plane from the negative real axis.
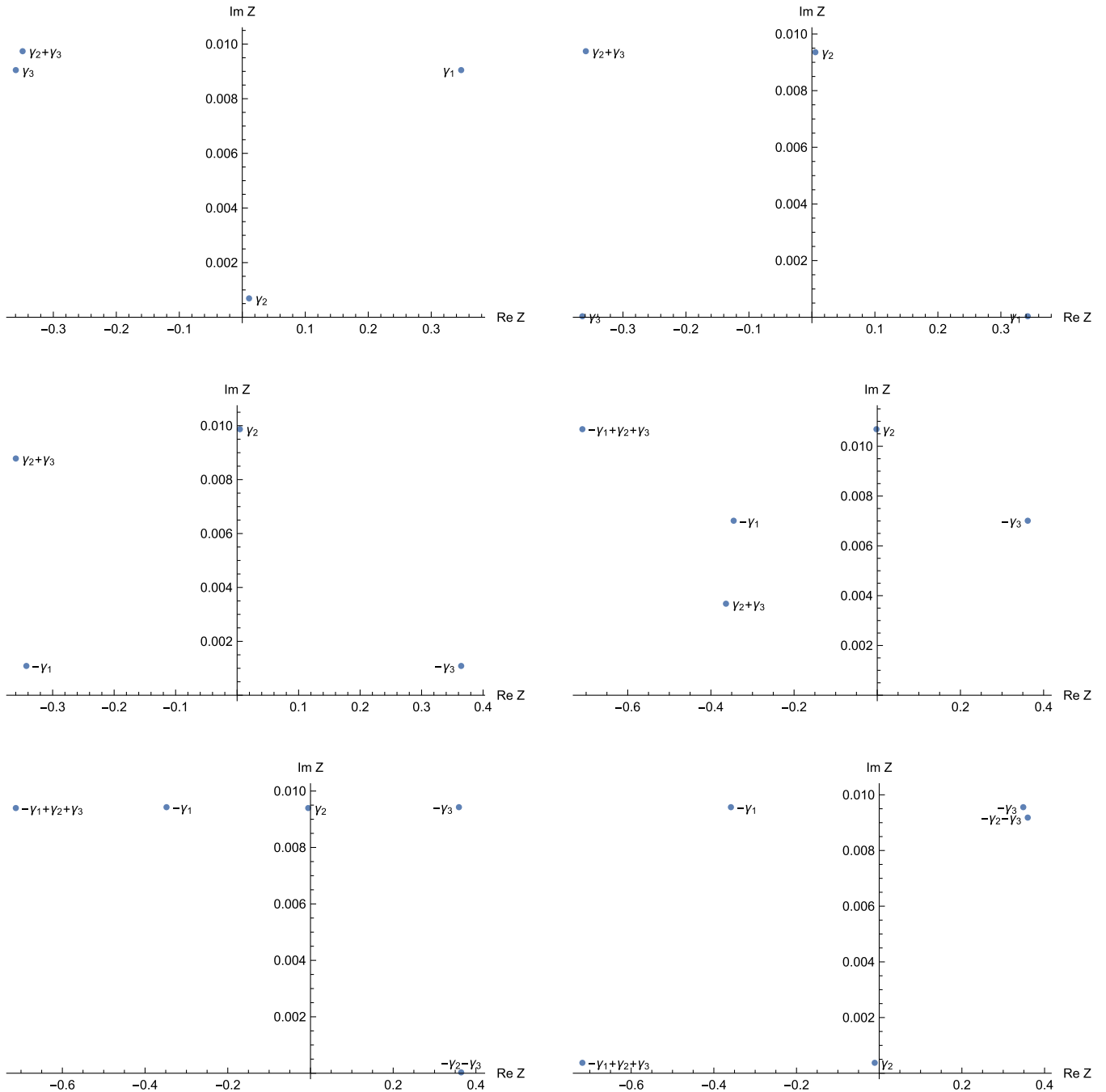
FIG. 5.   UPPER LEFT: Stable states with charges $\gamma_1, \gamma_2, \gamma_3, \gamma_2 + \gamma_3$ for $\theta = 5.3$. UPPER RIGHT: Double mutation at $\theta = 2\pi$. CENTER LEFT: New quiver basis valid for $0 < \theta < 1.05$. CENTER RIGHT: The wall crossing at which the state of charge $\gamma_2 + \gamma_3 - \gamma_1$ enters in the spectrum occurs at $\theta = 0.525$, and here we plot the stable states in the Z-plane at $\theta = 0.7$. DOWN LEFT: Right after the $\theta \approx 1.05$ mutation at $\gamma_2 + \gamma_3$. DOWN RIGHT: $\theta \approx 2.06$ right before the double mutation at $-\gamma_1 + \gamma_2 + \gamma_3$ and γ_2 and the wall-crossing leading to the disappearence of γ_3 from the spectrum and the appearance of $\gamma_1 + \gamma_2$.

the text and the one given by $\gamma_1, (-\gamma_2), \gamma_3$ in the region $\pi < \theta < 5\pi/3$. Consider the former. See the lhs of Fig. 4 to see that indeed it meets requirements 1 and 2 of Refs. [87,93]. The candidate basis $\gamma_1, (-\gamma_2), \gamma_3$ does not mutate to $\gamma_1, \gamma_2, \gamma_3$ at $\theta = 5\pi/3$, but rather it mutates to $\gamma_1 - \gamma_2, \gamma_2, \gamma_3 - \gamma_2$, which leads to an inconsistency. The quiver for this putative basis would be

$$\gamma_1 \leftarrow (-\gamma_2) \rightarrow \gamma_3, \qquad (A1)$$

from which we see that the representation with dimension vector (1,1,0) is indeed stable. As θ approaches $5\pi/3$ from the left, one can see from Fig. 4 that $Z(-\gamma_2)$ exits the upper Z-plane from the negative real axis, which triggers the quiver mutation from the quiver in Eq. (A1) to
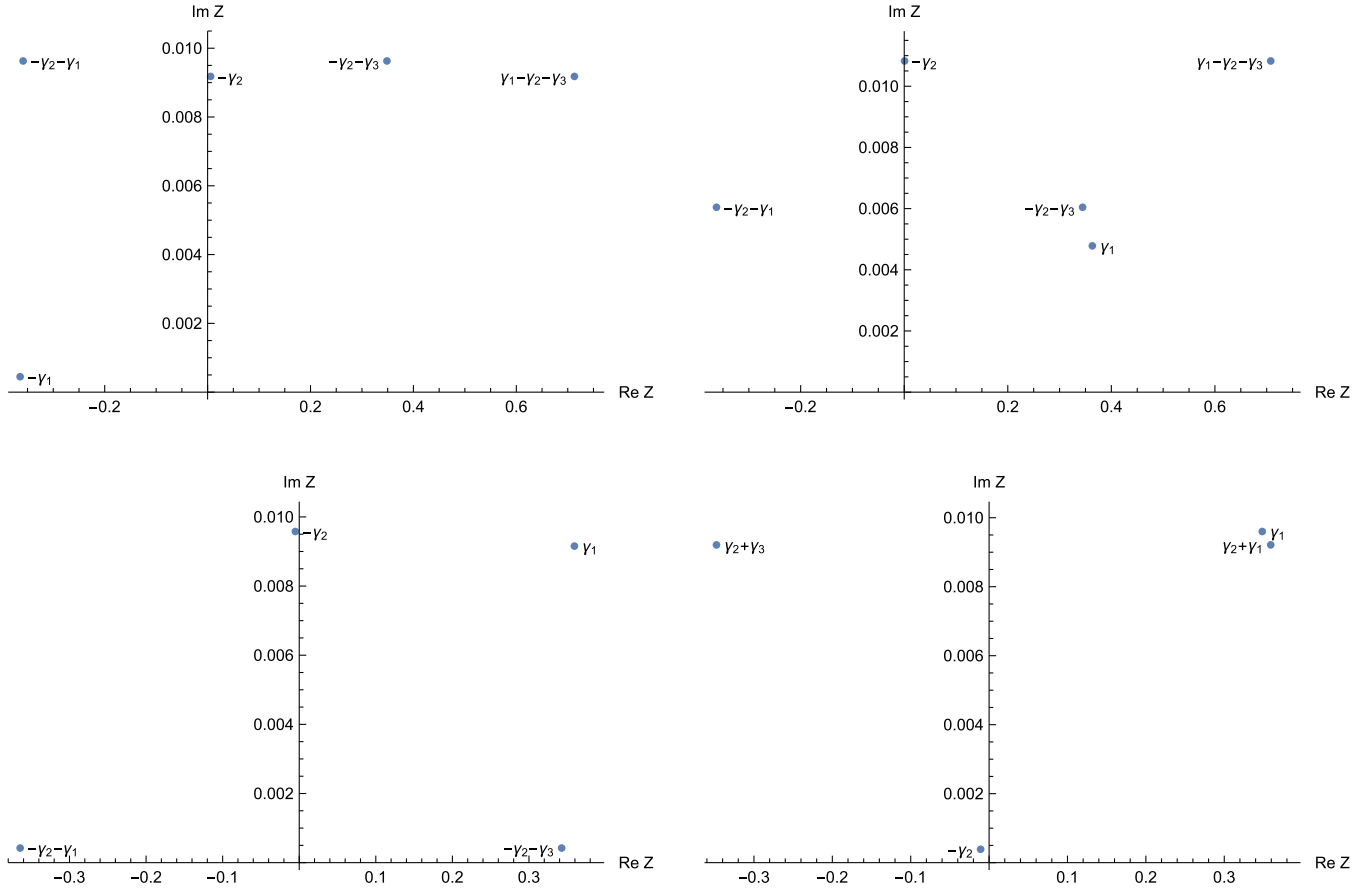
FIG. 6.   UPPER LEFT: $\theta \approx 3.1$ right before the mutation at π. UPPER RIGHT: $\theta \approx 3.6$ right before the wall crossing leading to the decay of the state $\gamma_1 - \gamma_2 - \gamma_3$. DOWN LEFT: Right before the mutation at $\theta \approx 4.15$. DOWN RIGHT: Right before the wall crossings destabilizing $\gamma_1 + \gamma_2$ and stabilizing γ_3 while γ_2 mutates at $\theta \approx 5.2$.

$$\gamma_1 - \gamma_2 \rightarrow \gamma_2 \leftarrow \gamma_3 - \gamma_2. \tag{A2}$$

At the same time, the state $\gamma_1 - \gamma_2$ is wall crossing away (getting unstable and disappearing from the spectrum), which is inconsistent with the charges on the nodes of the mutated quiver, because quiver nodes always correspond to stable particles. This rules out the candidate basis $\gamma_1, (-\gamma_2), \gamma_3$ with respect to the one we use in the main body of the text, which does not lead to such inconsistencies.

Let us proceed by reviewing the computation of the BPS spectra we summarized in Figs. 2 and 3. Let us first consider the large mass regime with $u = 0.1$ and $m = e^{i\theta}$.

For $5\pi/3 < \theta < 2\pi$, the quiver basis we start with gives a BPS quiver,

$$\gamma_1 \rightarrow \gamma_2 \leftarrow \gamma_3.$$

The corresponding central charges and stable states are depicted in Fig. 5, and we have BPS spectrum

$$\gamma_1, \gamma_2, \gamma_3, \gamma_2 + \gamma_3, \quad \text{and} \quad CPT \text{ conjugates.} \tag{A3}$$

At $\theta = 2\pi$, a double mutation occurs (see Fig. 5): $Z(\gamma_1)$ and $Z(\gamma_3)$ exit the upper Z-plane simultaneously. The mutated quiver is

$$-\gamma_1 \leftarrow \gamma_2 + \gamma_3 \rightarrow -\gamma_3. \tag{A4}$$

The BPS spectrum remains the one in (A3), and the stable particles have charges $-\gamma_1, \gamma_2 + \gamma_3, -\gamma_3, \gamma_2$. Now, γ_2 appears as a stable bound state with dimension vector $(0,1,1)$ for the A_3 quiver in line (A4), and adding CPT conjugates, one gets the same charges as in line (A3). At $\theta \sim 0.525$, a wall-crossing phase transition occurs, and the BPS state with charge $\gamma_2 + \gamma_3 - \gamma_1$ stabilizes; the BPS spectrum becomes

$$-\gamma_1, \gamma_2 + \gamma_3, -\gamma_3, \gamma_2, \gamma_2 + \gamma_3 - \gamma_1 \quad \text{and} \quad CPT \text{ conjugates.} \tag{A5}$$

At $\theta \approx 1.05$, another mutation occurs (see Fig. 5), the charge $\gamma_2 + \gamma_3$ exits the upper half Z-plane, and the BPS quiver becomes
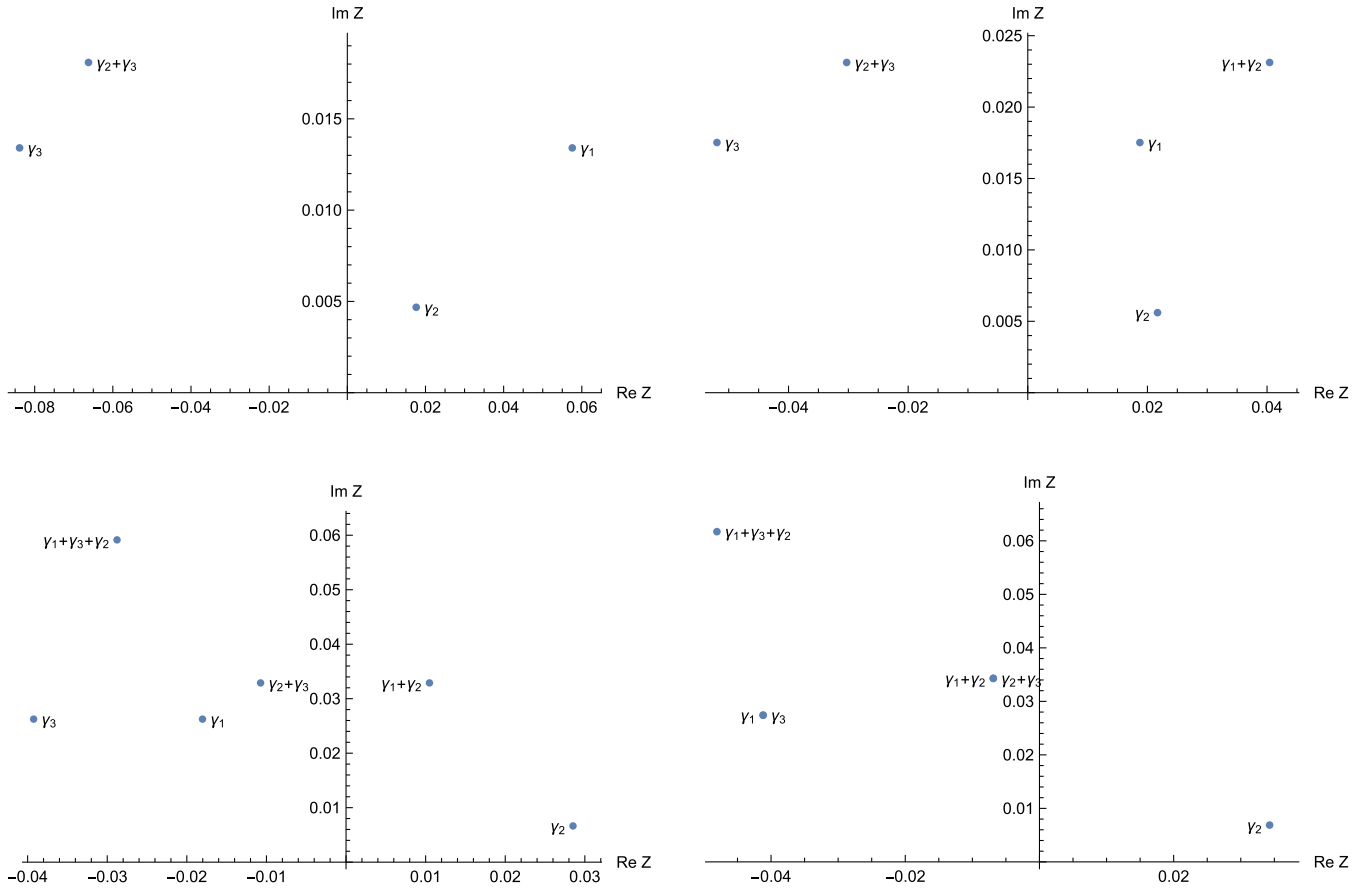
FIG. 7.   UPPER LEFT: $m \approx 0.2$ same spectrum as at large mass for $\theta = 5.5$. UPPER RIGHT: $m \approx 0.1$; the state $\gamma_1 + \gamma_2$ enters the spectrum. DOWN LEFT: $m \approx 0.03$ right after the mutation leading to the stabilization of the dark dyon. DOWN RIGHT: $m \approx 0$; the flavor symmetry gets restored.

$$\gamma_2 + \gamma_3 - \gamma_1 \rightarrow -(\gamma_2 + \gamma_3) \leftarrow -\gamma_2. \qquad \text{(A6)}$$

The spectrum is still as in line (A5), but now γ_3 is a bound state corresponding to the dimension vector $(0,1,1)$. At $\theta \approx 2.06$, both the charge $\gamma_2 + \gamma_3 - \gamma_1$ and the charge γ_2 exit the upper half Z-plane. Moreover, the state with charge γ_3 destabilizes, while the state with charge $-(\gamma_1 + \gamma_2)$ stabilizes (see Fig. 5). The mutated BPS quiver is

$$-\gamma_2 - \gamma_3 + \gamma_1 \leftarrow -\gamma_1 \rightarrow -\gamma_2, \qquad \text{(A7)}$$

The new BPS spectrum is

$$-\gamma_2 - \gamma_3 + \gamma_1, -\gamma_1, -\gamma_2, -\gamma_2 - \gamma_3, -\gamma_1 - \gamma_2$$
$$\text{and} \quad CPT \text{ conjugates.} \qquad \text{(A8)}$$

At $\theta = \pi$ (see Fig. 6), the charge γ_1 mutates, and one has the quiver

$$-\gamma_2 - \gamma_3 \rightarrow \gamma_1 \leftarrow -\gamma_2 - \gamma_1. \qquad \text{(A9)}$$

At $\theta \approx 3.65$, there is a wall crossing, and the spectrum becomes

$$\gamma_1, -\gamma_2, -\gamma_2 - \gamma_3, -\gamma_1 - \gamma_2 \quad \text{and} \quad CPT \text{ conjugates.} \qquad \text{(A10)}$$

At $\theta \approx 4.15$, we have another double mutation (see Fig. 6),

$$\gamma_2 + \gamma_3 \leftarrow -\gamma_2 \rightarrow \gamma_2 + \gamma_1, \qquad \text{(A11)}$$

followed at $\theta \approx 5\pi/3$ by a mutation at γ_2 and simultaneously two wall crossings leading to the destabilization of the state with charge $\gamma_2 + \gamma_1$ and the stabilization of the state with charge γ_3, bringing us back to the original spectrum at $\theta > 5\pi/3$ (see Fig. 6).

The BPS spectrum in Fig. 3 was obtained for $u = 0.1e^{i5.5}$ tuning m from 0 to 1. The whole line is covered by the quiver

$$\gamma_1 \rightarrow \gamma_2 \leftarrow \gamma_3. \qquad \text{(A12)}$$

The relevant pattern of wall crossings in the Z-plane is illustrated in Fig. 7: for $0.2 < m < 1$, the spectrum is constant. In the region $0.03 < m < 0.2$, a series of wall crossing occurs leading to a maximal chamber with stable states

$$\gamma_1, \gamma_2, \gamma_3, \gamma_1 + \gamma_2, \gamma_2 + \gamma_3, \gamma_1 + \gamma_2 + \gamma_3. \quad \text{(A13)}$$

It should be possible to reproduce our results using the spectral networks as in Ref. [110].

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