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Flavor from the double tetrahedral group without supersymmetry

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We consider a class of flavor models proposed by Aranda, Carone and Lebed, relaxing the assumption of supersymmetry and allowing the flavor scale to float anywhere between the weak and Planck scales. We perform global fits to the charged fermion masses and Cabibbo-Kobayashi-Maskawa angles, and consider the dependence of the results on the unknown mass scale of the flavor sector. We find that the typical Yukawa textures in these models provide a good description of the data over a wide range of flavor scales, with a preference for those that approach the lower bounds allowed by flavor-changing-neutral-current constraints. Nevertheless, the possibility that the flavor scale and Planck scale are identified remains viable. We present models that demonstrate how the assumed textures can arise most simply in a nonsupersymmetric framework.

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I. INTRODUCTION

There is a vast literature on models that attempt to explain the observed hierarchy of fermion masses by means of horizontal symmetries. In this paper, we revisit one such model, proposed by Aranda, Carone and Lebed, based on the double tetrahedral group T' [1,2]. Prior to this work, it had been shown that supersymmetric grand unified theories with U(2) flavor symmetry predict simple forms for the Yukawa matrices, ones that provide a successful description of charged fermion masses and the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [3,4]. The authors of Refs. [1,2] posed a simple question: What is the smallest discrete flavor group that predicts the same form for the Yukawa textures? The answer to this question was determined by the specific group theoretic properties of U(2) that were utilized in the most successful U(2) models [4]:

(1) U(2) models involved fields in **1**, **2** and **3** dimensional representations (reps). Matter fields of the three generations were embedded into $\mathbf{2} \oplus \mathbf{1}$ dimensional reps; the fact that the third generation fields were treated differently allowed the model to accommodate an order one (i.e., a flavor-group-invariant) top quark Yukawa coupling. The flavor-symmetry-breaking fields, called flavons, appeared in all three of these representations.

(2) In each Yukawa matrix, the two-by-two block associated with the first two generations decomposed into an antisymmetric and symmetric part. These followed from the couplings of the **1** and **3**-dimensional flavon fields, respectively, due to the group multiplication rule

$$\mathbf{2} \otimes \mathbf{2} = \mathbf{3} \oplus \mathbf{1}. \tag{1.1}$$

(3) The U(2) symmetry was broken to a U(1) subgroup that rotated all first generation fields by a phase. This U(1) symmetry was subsequently broken at a lower energy scale than that of the original U(2) symmetry. Since Yukawa couplings emerge as a ratio of a symmetry-breaking scale to a cut off, the sequential breaking of the flavor symmetry explains why the Yukawa couplings associated with first generation were smaller than those of the heavier generations.

The group T' is special in that it is the smallest discrete group that has **1**, **2** and **3**-dimensional representations, as well as the multiplication rule $\mathbf{2} \otimes \mathbf{2} = \mathbf{3} \oplus \mathbf{1}$. We will briefly review the representations and multiplication rules for T' symmetry in Sec. II. Following Refs. [1,2], the appropriate symmetry breaking sequence is achieved if the flavor group includes an Abelian factor, so that $G_F = T' \times Z_3$. Then the breaking pattern of the U(2) model

$$U(2) \xrightarrow{\epsilon} U(1) \xrightarrow{\epsilon'} \text{nothing}, \tag{1.2}$$

is mimicked by

$$T' \times Z_3 \xrightarrow{\epsilon} Z_3^D \xrightarrow{\epsilon'} \text{nothing}. \tag{1.3}$$

Here we have indicated the scale of each symmetry breaking via the dimensionless parameters ϵ and ϵ', which represent the ratio of a symmetry-breaking vacuum expectation value (vev) to the cutoff of the effective theory. We refer to the cutoff as the flavor scale, M_F, henceforth. A useful way to understand the connection between Eq. (1.2) and (1.3) is to consider the $\mathrm{SU}(2) \times \mathrm{U}(1)$ subgroup of U(2); The T' factor is a subgroup of the SU(2) factor while Z_3 is a subgroup of the U(1). The Z_3 factor remaining after the first step in the symmetry-breaking chain in Eq. (1.3) also transforms all first generation fields by a phase and will be specified later.

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The $T' \times Z_3$ model defined in this way reproduces the successful Yukawa textures of the U(2) models, but with a much smaller symmetry group. For other productive applications of T' symmetry in flavor model building, we refer the reader to Ref. [5].

The T' models of Refs. [1,2] were constructed more than 16 years ago, when it was widely assumed that weak-scale supersymmetry was the likely solution to the gauge hierarchy problem. The numerical study of the Yukawa textures in these references assumed supersymmetric renormalization group equations to relate the predictions of the theory at the flavor scale M_F to those at observable energies. Superpartners were taken to have masses just above the electroweak scale, while M_F was identified with the scale of supersymmetric grand unification, $\sim 2 \times 10^{16}$ GeV. The latter choice was motivated by the most elegant T' models, which were formulated in the context of an $SU(5)$ grand unified theory. Some of the essential features of the Yukawa textures followed from the combined restrictions of the flavor and grand unified symmetries.

At the present moment, however, the status of weak-scale supersymmetry as a necessary ingredient in model building is far less certain. The latest data from the LHC has found no evidence for supersymmetry. Of course, this may simply mean that the scale of the superpartner masses is slightly higher than what one might prefer from the perspective of naturalness; this interpretation would have little effect on the results of Refs. [1,2]. On the other hand, the LHC may be hinting that there is no necessary connection between the weak scale and the scale of supersymmetry breaking. In this case, one might entertain the possibility that the supersymmetry breaking scale is associated with the only higher physical mass scale whose existence is well established: the Planck scale. For example, it has been suggested in Ref. [6] that the shallowness of the Higgs potential may be explained by Planck-scale supersymmetry breaking, assuming that supersymmetry is still relevant for a quantum gravitational completion. This latter assumption itself has been challenged in Ref. [7], where it has been noted that there are consistent string theories that are fundamentally nonsupersymmetric and whose low-energy limit could include the standard model. Whether supersymmetry is broken at the Planck scale, or not present at any scale, one might attempt to address the hierarchy between the weak scale and Planck scale, for example, by anthropic selection, or by Higgs field relaxation [8], or by mechanisms not yet known. Alternatively, one might pursue the idea that quantum gravitational physics does not contribute to scalar field quadratic divergences in the way that one expects naively from effective field theory arguments [9]. In this paper, we remain completely agnostic on the issue of naturalness. We instead investigate a question that can be addressed in a more definitive and quantitative way: how well do the T' flavor models in Refs. [1,2] work if there is no supersymmetry below the Planck scale?

We begin our study by assuming a standard form for the Yukawa textures expected in models with $T' \times Z_3$ symmetry and perform a global fit to the charged fermion masses and CKM elements assuming that the predictions at the flavor scale M_F are related to those at the weak scale via nonsupersymmetric renormalization group equations.[1] In the absence of supersymmetry, we no longer have gauge coupling unification and therefore do not consider grand unified embeddings. The flavor scale is taken as a free parameter that may vary anywhere from the TeV scale to the Planck scale. By study of the goodness of these fits, we consider whether there is any preference for a higher or lower flavor scale within the specified range. If one were to find acceptable results for values of M_F near the Planck scale, one might conclude that the model is consistent with a minimal scenario in which there are no other energy scales of physical relevance other than the weak and the Planck scale. On the other hand, if one were to find acceptable results for M_F closer to the lower bounds from flavor-changing-neutral-current processes, then one might obtain interesting predictions for observable indirect effects of heavy particles associated with the flavor sector.

Our paper is organized as follows. In the next section, we briefly review the flavor models of interest and present a parameterization of the Yukawa matrix textures that typically arise in these models at the flavor scale M_F. In Sec. III, we study the predictions that follow from these textures by a nonsupersymmetric renormalization group analysis, including global fits to the current data on charged fermion masses and CKM elements. In Sec. IV, we point out the largest indirect effects of heavy flavor-sector particles on flavor-changing-neutral current processes in the case where M_F is low. In Sec. V, we address model building issues: supersymmetric models have two Higgs doublets (in order to cancel anomalies) and have a superpotential that is constrained by holomorphicity; these requirements are absent in the nonsupersymmetric case. Hence, in this section we show how the textures assumed in Sec. III may arise in nonsupersymmetric T' models. In the final section, we summarize our conclusions.

II. TYPICAL YUKAWA TEXTURES FROM T-PRIME SYMMETRY

The group T' is discussed at length in Ref. [2]. Here we summarize only the most basic properties relevant to the

[1]Note that we do not consider neutrino physics in the present work due to the additional model dependence affecting that sector of the theory. For example, the structure of the theory is different depending on whether neutrino masses are Dirac or Majorana, whether the Majorana masses arise via a seesaw mechanism or via coupling to electroweak triplet Higgs fields, and whether additional neutral fermions are present with which the neutrinos can mix. We reserve such a study for future work.

present discussion: The group has 24 elements. This includes 12 elements that correspond to the 12 proper rotations that take a regular tetrahedron into coincidence with itself, with choices of Euler angles that are less than 2π. The remaining 12 elements are the first set times an element called R that corresponds to a 2π rotation. As we indicated earlier, T' has **1**, **2** and **3**-dimensional representations, that we specify more precisely below. For odd-dimensional representations, R acts trivially and the action of the group T' is not distinguishable from that of the tetrahedral group T. For the even-dimensional representations, however, R acts nontrivially; this reflects the fact that T' is a subgroup of SU(2) and that spinors flip sign under a rotation by 2π.

The complete list of T' representations is as follows: there is a trivial singlet, $\mathbf{1}^0$, two nontrivial singlets, $\mathbf{1}^{\pm}$, three doublets, $\mathbf{2}^0$ and $\mathbf{2}^{\pm}$, and one triplet, **3**. The different singlet and doublet representations are distinguished by how they transform under a Z_3 subgroup, generated by the group element called g_9 in Ref. [2]. This is indicated by the triality superscript; when we multiply representations, trialities add under addition modulo three. Keeping this in mind, the rules for multiplying representations are then specified by

$$\begin{aligned}
&\mathbf{1}\otimes\mathbf{R}=\mathbf{R}\otimes\mathbf{1}\quad \text{for any rep } \mathbf{R},\\
&\mathbf{2}\otimes\mathbf{2}=\mathbf{3}\oplus\mathbf{1},\\
&\mathbf{2}\otimes\mathbf{3}=\mathbf{3}\otimes\mathbf{2}=\mathbf{2}^0\oplus\mathbf{2}^+\oplus\mathbf{2}^-,\\
&\mathbf{3}\otimes\mathbf{3}=\mathbf{3}\oplus\mathbf{3}\oplus\mathbf{1}^0\oplus\mathbf{1}^+\oplus\mathbf{1}^-.
\end{aligned}\tag{2.1}$$

As we indicated in the Introduction, the models of interest are based on the flavor group $G_F = T'\times Z_3$, which includes a Z_3 subgroup that rotates all first-generation matter fields by a phase. We now identify that subgroup. In the models of Ref. [2], the first two generations are assigned to the $\mathbf{2}^0$ representation,[2] in which the element g_9 is given by

$$\mathrm{g}_9(\mathbf{2}^0)=\begin{pmatrix}\eta^2 & 0\\ 0 & \eta\end{pmatrix},\tag{2.2}$$

where $\eta\equiv e^{2\pi i/3}$. However, the matter fields may also transform under the Z_3 factor that commutes with T'. We represent charge assignments under this Z_3 by an additional triality index 0, + and −, corresponding to the phase rotations 1, η and η^2. The diagonal subgroup of the Z_3 subgroup generated by g_9 and the Z_3 factor that commutes with T' is the intermediate symmetry that we desire; we call this subgroup Z_3^D. If we assign the first two generations to the rep $\mathbf{2}^{0-}$, then the action of Z_3^D is through powers of the product

$$\mathrm{g}_9(\mathbf{2}^0)\cdot\eta^2=\begin{pmatrix}\eta & 0\\ 0 & 1\end{pmatrix},\tag{2.3}$$

which provides the desired first generation phase rotation.

Assigning the three generations of matter fields to the $T'\times Z_3$ reps $\mathbf{2}^{0-}\oplus\mathbf{1}^{00}$ yields the following transformation properties of the Yukawa matrices:

$$Y_{U,D,E}\sim\begin{pmatrix}[\mathbf{3}^-\oplus\mathbf{1}^{0-}] & [\mathbf{2}^{0+}]\\ [\mathbf{2}^{0+}] & [\mathbf{1}^{00}]\end{pmatrix}.\tag{2.4}$$

The models of interest include a set of flavon fields, A_{ab}, ϕ_{ab} and S_{ab}, which transform as $\mathbf{1}^{0-}$, $\mathbf{2}^{0+}$ and $\mathbf{3}^-$, respectively. When the $T'\times Z_3$ symmetry is broken to Z_3^D, the doublet and triplet flavons acquire the vacuum expectation values (VEVs)

$$\frac{\langle\phi\rangle}{M_F}\sim\begin{pmatrix}0\\ \epsilon\end{pmatrix},\qquad \frac{\langle S\rangle}{M_F}\sim\begin{pmatrix}0 & 0\\ 0 & \epsilon\end{pmatrix},\tag{2.5}$$

where we use $\sim$ when we omit possible order one factors. This is the most general pattern of nonvanishing entries that is consistent with the unbroken Z_3^D symmetry defined by Eq. (2.3). Yukawa couplings involving first-generation fields are generated only after the Z_3^D symmetry is broken at a lower scale; in analogy to the U(2) models of Refs. [3,4], it is assumed that this is accomplished solely through the vev of the flavon A_{ab},

$$\frac{\langle A\rangle}{M_F}\sim\begin{pmatrix}0 & \epsilon'\\ -\epsilon' & 0\end{pmatrix},\tag{2.6}$$

where $\epsilon'<\epsilon$. This sequential breaking $T'\times Z_3\xrightarrow{\epsilon} Z_3^D\xrightarrow{\epsilon'}$nothing yields a Yukawa texture for the up quarks, down quarks and leptons of the form

$$Y_{U,D,E}\sim\begin{pmatrix}0 & \epsilon' & 0\\ -\epsilon' & \epsilon & \epsilon\\ 0 & \epsilon & 1\end{pmatrix},\tag{2.7}$$

where we have suppressed $\mathcal{O}(1)$ operator coefficients.

The forms of the Yukawa matrices obtained in Eq. (2.7) are inadequate, given the known differences between the up-, down- and charged-lepton masses. The top quark Yukawa coupling is of order one, while the all others are substantially smaller, suggesting an additional overall suppression factor is desirable in Y_D and Y_E. Moreover, the hierarchy of quark masses is more extreme in the up-quark sector than in the down; for example, the quark mass ratios renormalized at the supersymmetric grand unified scale are given approximately by [10]

[2]This choice is motivated by the cancellation of discrete gauge anomalies. See Ref. [2] for details.

$$m_d :: m_s :: m_b = \lambda^4 :: \lambda^2 :: 1 \quad \text{while}$$
$$m_u :: m_c :: m_t = \lambda^8 :: \lambda^4 :: 1, \tag{2.8}$$

where $\lambda \approx 0.22$ is the Cabibbo angle. This suggests that an additional suppression in the 1-2 block of Y_U is also desirable. We call these suppression factors ρ and ξ, which modify the textures of Eq. (2.7) as follows:

$$Y_U \sim \begin{pmatrix} 0 & \epsilon'\rho & 0 \\ -\epsilon'\rho & \epsilon\rho & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}, \qquad Y_D \sim \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}\xi,$$
$$Y_E \sim \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}\xi. \tag{2.9}$$

Clearly, the smallness of ρ and ξ does not follow directly from the assumed flavor symmetry breaking, but requires additional symmetries and/or dynamics. In the U(2) models of Refs. [3,4] and the T' models of Refs. [1,2], ξ is assumed to arise from mixing in the Higgs sector of the theory, while the origin of ρ is understood in terms of a grand unified embedding. Flavon charge assignments under the unified gauge group can cause Yukawa entries to arise at higher order in $1/M_F$ than they would otherwise. In the nonsupersymmetric T' models that we discuss in Sec. V, we will neither have an extended Higgs sector nor a grand unified embedding; we will, however, show how ρ and ξ may arise simply by a small extension of the flavor symmetry.

All other differences between Y_U, Y_D and Y_E can now be accommodated by the choice of the undetermined $\mathcal{O}(1)$ operator coefficients, identified according to naive dimensional analysis. We generally require these to be between $1/3$ and 3 in magnitude; the precise range is a matter of taste, but our choice is consistent with the assumptions of Refs. [1,2]. Variations in the operator coefficients are then sufficient, for example, to account for differences between Y_D and Y_E that are attributed to group theoretic factors of 3 in grand unified theories [11]. We parametrize the Yukawa matrices in terms of coefficients u_i, d_i and ℓ_i as follows:

$$Y_U = \begin{pmatrix} 0 & u_1\epsilon'\rho & 0 \\ -u_1\epsilon'\rho & u_2\epsilon\rho & u_3\epsilon \\ 0 & u_4\epsilon & u_5 \end{pmatrix},$$
$$Y_D = \begin{pmatrix} 0 & d_1\epsilon' & 0 \\ -d_1\epsilon' & d_2\epsilon & d_3\epsilon \\ 0 & d_4\epsilon & d_5 \end{pmatrix}\xi,$$
$$Y_E = \begin{pmatrix} 0 & \ell_1\epsilon' & 0 \\ -\ell_1\epsilon' & \ell_2\epsilon & \ell_3\epsilon \\ 0 & \ell_4\epsilon & \ell_5 \end{pmatrix}\xi. \tag{2.10}$$

These forms will be used to define the Yukawa matrices at the flavor scale M_F in the numerical study presented in the following section.

III. NUMERICAL ANALYSIS

We numerically evolve the Yukawa matrices in Eq. (2.10), using the one-loop, nonsupersymmetric renormalization group equations (RGEs). The flavor scale M_F is taken to be variable, while the scale of observable energies is chosen to be the mass of the Z boson, m_Z. We omit all weak-scale threshold corrections. The RGEs are given by [12]

$$\frac{dg_i}{dt} = \frac{b_i^{\rm SM}}{16\pi^2} g_i^3, \tag{3.1}$$

$$\frac{dY_U}{dt} = \frac{1}{16\pi^2} \times \left(-\sum_i c_i^{\rm SM} g_i^2 + \frac{3}{2} Y_U Y_U^\dagger - \frac{3}{2} Y_D Y_D^\dagger + Y_2(S)\right) Y_U, \tag{3.2}$$

$$\frac{dY_D}{dt} = \frac{1}{16\pi^2} \times \left(-\sum_i c_i'^{\rm SM} g_i^2 + \frac{3}{2} Y_D Y_D^\dagger - \frac{3}{2} Y_U Y_U^\dagger + Y_2(S)\right) Y_D, \tag{3.3}$$

$$\frac{dY_E}{dt} = \frac{1}{16\pi^2} \left(-\sum_i c_i''^{\rm SM} g_i^2 + \frac{3}{2} Y_E Y_E^\dagger + Y_2(S)\right) Y_E, \tag{3.4}$$

where

$$Y_2(S) = \mathrm{Tr}[3Y_U Y_U^\dagger + 3Y_D Y_D^\dagger + Y_E Y_E^\dagger]. \tag{3.5}$$

Here, the g_i are the gauge couplings, Y_U, Y_D and Y_E are the Yukawa matrices, and $t = \ln\mu$ is the log of the renormalization scale. The SU(5) normalization of g_1 is assumed. In the absence of supersymmetry [12],

$$b_i^{\rm SM} = \left(\frac{41}{10}, \quad -\frac{19}{6}, \quad -7\right), \tag{3.6}$$

and

$$c_i^{\rm SM} = \left(\frac{17}{20}, \quad \frac{9}{4}, \quad 8\right), \qquad c_i'^{\rm SM} = \left(\frac{1}{4}, \quad \frac{9}{4}, \quad 8\right),$$
$$c_i''^{\rm SM} = \left(\frac{9}{4}, \quad \frac{9}{4}, \quad 0\right). \tag{3.7}$$

The $\overline{\rm MS}$ gauge couplings are chosen to satisfy the boundary conditions

$$\alpha_1^{-1}(m_Z) = 59.01,$$
$$\alpha_2^{-1}(m_Z) = 29.59,$$
$$\alpha_3^{-1}(m_Z) = 8.44, \tag{3.8}$$

where $\alpha_i = g_i^2/4\pi$. These were computed using the values of $\alpha_{\rm EM} = e^2/4\pi = 127.950$ and $\sin^2\hat{\theta}_W = 0.23129$ renormalized at m_Z [13] as well as

$$e = g_Y \cos\hat{\theta}_W = g_2 \sin\hat{\theta}_W \quad \text{and} \quad g_1 = \sqrt{5/3}\, g_Y, \tag{3.9}$$

where the latter equation converts the standard model hypercharge gauge coupling to SU(5) normalization [14]. The QCD coupling is given directly in Ref. [13].

At the flavor scale M_F, the Yukawa matrices are given by Eq. (2.10). For a given numerical choice of symmetry-breaking parameters and operator coefficients, the Yukawa matrices are run down to the scale m_Z and diagonalized. In addition to the nine fermion mass eigenvalues, three CKM mixing angles can be compared to experimental data. (In this work, we do not consider the CKM phase, which is not constrained by the flavor symmetry.) Equivalently, we take the predictions of the theory to consist of the nine fermion masses and the magnitudes of the three CKM elements, V_{us}, V_{ub} and V_{cb}.

To optimize the choice of parameters and operator coefficients for a given choice of flavor scale M_F, we follow the approach of Ref. [2] and minimize the function

$$\begin{aligned}\tilde{\chi}^2 = &\sum_{i=1}^{9}\left(\frac{m_i^{\rm th} - m_i^{\rm exp}}{\Delta m_i^{\rm exp}}\right)^2 + \left(\frac{|V_{us}^{\rm th}| - |V_{us}^{\rm exp}|}{\Delta V_{us}^{\rm exp}}\right)^2 \\ &+ \left(\frac{|V_{ub}^{\rm th}| - |V_{ub}^{\rm exp}|}{\Delta V_{ub}^{\rm exp}}\right)^2 + \left(\frac{|V_{cb}^{\rm th}| - |V_{cb}^{\rm exp}|}{\Delta V_{cb}^{\rm exp}}\right)^2 . \\ &+ \sum_{i=1}^{5}\left(\frac{\ln|u_i|}{\ln 3}\right)^2 + \sum_{i=1}^{5}\left(\frac{\ln|d_i|}{\ln 3}\right)^2 + \sum_{i=1}^{5}\left(\frac{\ln|\ell_i|}{\ln 3}\right)^2 .\end{aligned} \tag{3.10}$$

Here, the quantities with the superscript th refer to the predictions of the theory, obtained as we have described previously. The quantities with the superscript exp refer to the experimental data, taken from Ref. [13], and written as $X \pm \Delta X$, where the second term is the experimental uncertainty. Since we have omitted two-loop corrections and threshold effects, we take this uncertainty into account in the same way as Ref. [2]: we inflate experimental error bars to 1% of the central value if the experimental error is smaller than this. The terms involving ratios of logarithms in Eq. (3.10) ensure that the operator coefficients remain near unity [2].

We have called the function we minimize $\tilde{\chi}^2$ to make clear that it differs from the conventional χ^2 function one would define in a simple least-squares fit. The latter cannot be sensibly formulated for the purpose of our analysis. A conventional χ^2 function only involves differences between the theoretical predicted values and the experimental measurements. The conventional χ^2 function that would replace our Eq. (3.10) would thus involve the sum of 12 terms that are a function of 19 parameters. This means that the numbers of degrees of freedom is negative and the conventional χ^2 probability distribution is not defined. This reflects the fact that we could choose parameter values to set a conventional χ^2 function identically to zero (i.e., there would be nothing to fit)[3] Doing so, however, is not adequate since this does not prevent a parameter value from exceeding the limits that assure a valid effective field theory. For example, a choice of parameters that gives a very good match to all the experimental central values but includes an operator coefficient that is, for example, 17.3, would be in wild conflict with the assumption that we have a valid effective field theory description. The $\tilde{\chi}^2$ function, on the other hand, includes additional terms that give weight to the theoretical constraint that the effective theory remain valid and consistent with naive dimensional analysis. Any alternative way of imposing such a theoretical constraint, which necessarily involves adding additional terms to the function that is minimized that are independent of the output predictions of the theory, would not be a conventional χ^2 function with the conventional statistical interpretation. Hence, we opt for a form that is both simple and consistent with what has been used in the past literature [2]. The quantity $\tilde{\chi}^2$ is useful in that it allows us to quantify the comparison of one of our fits to another. To interpret the meaning of a given value of $\tilde{\chi}^2$ in absolute terms, one then directly inspects the fit output, as we will discuss later. Since the u_i, d_i and ℓ_i are not treated as free parameters, we might expect qualitatively that a good fit will have a $\tilde{\chi}^2 \approx 8$, corresponding to 12 pieces of experimental data minus 4 unconstrained parameters (ϵ, ϵ', ρ and ξ). We will see that this is consistent with our numerical results.

A plot of $\tilde{\chi}^2$ as a function of the flavor scale M_F is shown in Fig. 1. The two curves in this figure correspond to the cases were the coefficient u_4 is allowed to float, or is fixed to zero. [In the latter case, the sum over the u_i in the second line of Eq. (3.10) omits $i = 4$.] These cases are motivated

[3]Note that there is one way that one could do a conventional χ^2 fit, namely, if one arbitrarily fixes a subset of the model parameters. This approach, however, is not adequate: Imagine if one fixed 14 of the 19 model parameters, and fit the 12 predictions of the theory to the data in terms of the 5 free parameter values. There are over 11,000 different ways of choosing the set of free parameters in this example and no physical basis for choosing one set over another, nor for determining the precise values to which the fixed parameters should be set. We therefore follow an approach where all the parameters are allowed to float. Note that in the one case where we do fix a parameter value, i.e., $u_4 = 0$, there is a specific physics justification that follows from the model building considerations discussed in Sec. V.

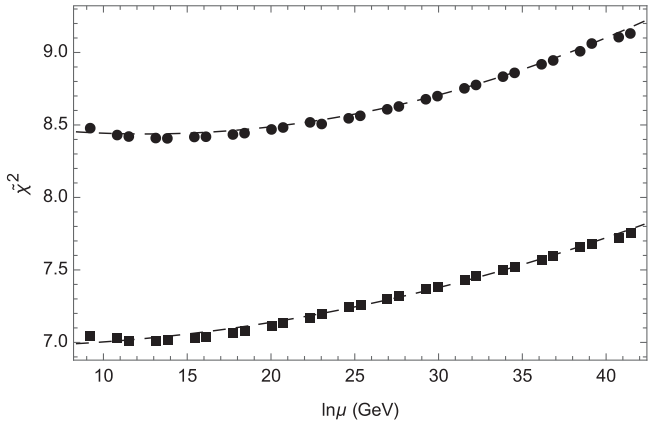

FIG. 1. Minimum $\tilde{\chi}^2$ values as a function of M_F, for two different model assumptions.

by two variants of the Yukawa textures that may arise in explicit models, as we show in Sec. V. Over the entire range of M_F we find good fits with $\tilde{\chi}^2 \approx 8$, but with clear and monotonic improvement in $\tilde{\chi}^2$ toward smaller values of M_F. In addition, the case where the operator corresponding to u_4 is absent from the theory (i.e., where u_4 is fixed to zero), which we will see corresponds to more minimal model-building assumptions, provides a better description of the data than the case where it is present. We present two examples of our results in Tables I and II, for $M_F = 10^6$ GeV and 10^{18} GeV, respectively, both in the case where $u_4 = 0$. The first choice corresponds to a flavor scale of the same order as the lower bounds from flavor-changing neutral current processes, as we discuss further in the next section, while the second is of the same order as the Planck scale. Interestingly, the latter demonstrates that the model is consistent with the possibility that there are only two important physical scales in nature, the weak and the Planck scales (with flavor associated with the latter) so that no additional hierarchies or fine-tuning need to be considered.

TABLE I. Fit parameters and observables for $M_F = 10^6$ GeV with $\chi^2 = 7.021$. In this example, the operator corresponding to u_4 is absent from the theory. All masses are given in GeV. (Note that m_t is the $\overline{\text{MS}}$ mass, not the pole mass.)

Best fit parameters		
$\epsilon = 0.182$, $\epsilon' = 0.005$, $\rho = 0.029$, $\xi = 0.014$		
$u_1 = 1.131$	$d_1 = 1.162$	$\ell_1 = 0.651$
$u_2 = 0.921$	$d_2 = -0.631$	$\ell_2 = -0.710$
$u_3 = -0.575$	$d_3 = 1.024$	$\ell_3 = -1.242$
$u_4 = 0$ (fixed)	$d_4 = 2.375$	$\ell_4 = -1.244$
$u_5 = 0.628$	$d_5 = -0.931$	$\ell_5 = -0.637$
Observable	Expt. Value [13]	Fit Value
m_u	$(2.3 \pm 0.6) \times 10^{-3}$	1.4×10^{-3}
m_c	1.275 ± 0.025	1.277
m_t	160 ± 4.5	160.1
m_d	$(4.8 \pm 0.4) \times 10^{-3}$	4.18×10^{-3}
m_s	$(9.5 \pm 0.5) \times 10^{-2}$	9.84×10^{-2}
m_b	4.18 ± 0.03	4.18
m_e	$(5.11 \pm 1\%) \times 10^{-4}$	5.11×10^{-4}
m_μ	$0.106 \pm 1\%$	0.106
m_τ	$1.78 \pm 1\%$	1.78
$\lvert V_{us}\rvert$	$0.225 \pm 1\%$	0.226
$\lvert V_{ub}\rvert$	$(3.55 \pm 0.15) \times 10^{-3}$	3.58×10^{-3}
$\lvert V_{cb}\rvert$	$(4.14 \pm 0.12) \times 10^{-2}$	4.13×10^{-2}

TABLE II. Fit parameters and observables for $M_F = 10^{18}$ GeV with $\chi^2 = 7.762$. In this example, the operator corresponding to u_4 is absent from the theory. All masses are given in GeV. (Note that m_t is the $\overline{\text{MS}}$ mass, not the pole mass.)

Best fit parameters		
$\epsilon = 0.131$, $\epsilon' = 0.004$, $\rho = 0.02$, $\xi = 0.011$		
$u_1 = 1.005$	$d_1 = 1.005$	$\ell_1 = 0.847$
$u_2 = 1.01$	$d_2 = -0.64$	$\ell_2 = -0.633$
$u_3 = -0.458$	$d_3 = 1.024$	$\ell_3 = -1.193$
$u_4 = 0$ (fixed)	$d_4 = 2.397$	$\ell_4 = -1.199$
$u_5 = 0.369$	$d_5 = -0.676$	$\ell_5 = -0.847$
Observable	Expt. Value [13]	Fit Value
m_u	$(2.3 \pm 0.6) \times 10^{-3}$	1.4×10^{-3}
m_c	1.275 ± 0.025	1.277
m_t	160 ± 4.5	160.4
m_d	$(4.8 \pm 0.4) \times 10^{-3}$	4.2×10^{-3}
m_s	$(9.5 \pm 0.5) \times 10^{-2}$	9.8×10^{-2}
m_b	4.18 ± 0.03	4.18
m_e	$(5.11 \pm 1\%) \times 10^{-4}$	5.11×10^{-4}
m_μ	$0.106 \pm 1\%$	0.106
m_τ	$1.78 \pm 1\%$	1.78
$\lvert V_{us}\rvert$	$0.225 \pm 1\%$	0.226
$\lvert V_{ub}\rvert$	$(3.55 \pm 0.15) \times 10^{-3}$	3.58×10^{-3}
$\lvert V_{cb}\rvert$	$(4.14 \pm 0.12) \times 10^{-2}$	4.13×10^{-2}

Note that Tables I and II correspond to the extreme values of $\tilde{\chi}^2$ on the lower curve of Fig. 1 and show directly that all the predictions of the theory are within one, or occasionally two, standard deviations of the experimental data, with model parameters consistent with naive dimensional analysis. One can then infer that every point on the lower curve of Fig. 1 provides a reasonably good description of the data in comparison to these reference points, over the entire range of flavor scales studied, with a slight preference for lower values. Similar qualitative conclusions can be drawn about the upper curve in the same figure, though, for the sake of brevity, we omit the corresponding fit tables.

IV. DIRECT LOWER BOUNDS ON THE FLAVOR SCALE

Our results in Fig. 1 indicate that typical T' Yukawa textures provide a good description of charged fermion

masses and CKM angles over a wide range of M_F, but with a preference for values closer to the TeV scale than to the Planck scale. The lowest possible values of M_F are separately constrained by flavor-changing-neutral-current (FCNC) processes that receive contributions from heavy flavor-sector fields. In this section, we provide some estimates of the lower bounds on M_F following from $K^0-\overline{K}^0$, $D^0-\overline{D}^0$, $B^0-\overline{B}^0$ and $B_s^0-\overline{B}_s^0$ mixing. In addition, we give the branching fractions predicted for the largest flavor-changing neutral meson decays, which also violate lepton flavor.

The new physics contributions to the FCNC processes of interest come from flavon exchange, or more precisely, the exchange of the physical fluctuations about the flavon vevs. We identify these as follows:

$$\phi=\begin{pmatrix}\varphi_1\\ \epsilon M_F+\varphi_2\end{pmatrix},\qquad S_{ab}=\begin{pmatrix}\tilde{S}_{11} & \tilde{S}_{12}\\ \tilde{S}_{12} & \epsilon M_F+\tilde{S}_{22}\end{pmatrix},$$
$$A_{ab}=\begin{pmatrix}0 & \epsilon' M_F+\tilde{A}\\ -\epsilon' M_F-\tilde{A} & 0\end{pmatrix}, \tag{4.1}$$

where the φ_i, the $\tilde{S}_{ij}$ and $\tilde{A}$ are complex scalar fields. The couplings to standard model fermions originate from the same operators that gave us the Yukawa couplings. As an example, let us consider the origin of $\Delta S=2$ operators, where S here refers to strangeness. We focus on the largest flavor-changing effects, ones that are present even in the absence of a rotation from the gauge to mass eigenstate basis. Let Ψ be a three-component column vector with the elements d, s and b. Then the flavon-quark-antiquark vertex in the down sector follows from

$$\mathcal{L}\supset-\frac{v}{\sqrt{2}}(\overline{\Psi}_L Y_D \Psi_R+\text{H.c.}), \tag{4.2}$$

where we have set the standard model Higgs field to its vev $v/\sqrt{2}$, where $v=246$ GeV, and where

$$Y_D=\left(\begin{array}{c|c}S_{ab}/M_F+A_{ab}/M_F & \phi/M_F\\ \hline \phi/M_F & 1\end{array}\right)\xi, \tag{4.3}$$

with the flavons S, A and ϕ given by Eq. (4.1), and ξ is the dimensionless suppression factor defined earlier. (We provide an origin for ξ and ρ in the next section.) The flavon couplings involving fermions of the first two generations only are given by

$$d_1\frac{v\xi}{\sqrt{2}M_F}(\overline{d}_L\tilde{A}s_R-\overline{s}_L\tilde{A}d_R)$$
$$-d_2\frac{v\xi}{\sqrt{2}M_F}(\overline{d}_L\tilde{S}_{12}s_R+\overline{s}_L\tilde{S}_{12}d_R)+\text{H.c.} \tag{4.4}$$

Four-fermion operators are obtained by integrating out the heavy fields. It follows that the $\Delta S=2$ operator that contributes to the $K^0-\overline{K}^0$ mass splitting is

$$\mathcal{O}_{\Delta S=2}=-\left(\frac{d_1^2}{m_{\tilde{A}}^2}+\frac{d_2^2}{m_{\tilde{S}_{12}}^2}\right)\frac{v^2\xi^2}{2M_F^2}[\overline{d}_L s_R\overline{d}_R s_L], \tag{4.5}$$

where the d_i are the same order one coefficients defined in Eq. (2.10). As the flavon masses are not known exactly, we assume that they are of the same order as the symmetry-breaking scale associated with the given flavon; in the present example,

$$m_{\tilde{S}_{12}}\sim\epsilon M_F\quad\text{and}\quad m_{\tilde{A}}\sim\epsilon' M_F. \tag{4.6}$$

Moreover, we pick numerical values of ϵ, ϵ', ρ and ξ that are characteristic of the values found in our global fits for M_F below ~1000 TeV:

$$\epsilon\sim 0.1,\qquad \xi\sim 0.03,\qquad \rho\sim 0.02. \tag{4.7}$$

We set all order one coefficients equal to one. With these assumptions, the new physics contribution to the neutral pseudoscalar meson mass splittings, Δm, may be expressed as a function of the scale M_F. In general, given a $\Delta F=2$ interaction of the form $c\mathcal{O}$, where c is the operator coefficient and F represents either strange (S), charm (C) or bottom (B), the mass splitting is given by

$$\Delta m=\frac{c}{m_{P^0}}|\langle P^0|\mathcal{O}|\overline{P}^0\rangle|, \tag{4.8}$$

where $P^0(\overline{P}^0)$ is the pseudoscalar meson (anti-meson) in question, and the states are relativistically normalized. For an operator of the form

$$\mathcal{O}=\frac{1}{4}[\overline{h}^\alpha(1-\gamma^5)\ell^\alpha][\overline{h}^\beta(1+\gamma_5)\ell^\beta], \tag{4.9}$$

where h, ℓ represent the heavy (light) quark flavors and α, β are color indices, the matrix element in Eq. (4.8) is given by [15]

$$\langle P^0|\mathcal{O}|P^0\rangle=\frac{1}{2}B_{P^0}\frac{m_{P^0}^4 f_{P^0}^2}{(m_h+m_\ell)^2}, \tag{4.10}$$

in the case where $P^0=K^0$ or D^0. Here, B_{P^0} is the bag parameter, m_{P^0} and f_{P^0} are the mass and decay constants of the meson and m_ℓ, m_h are the masses of the quarks that make up the meson. For $P^0=B^0$ or B_s^0, the matrix element is given by [16]

$$\langle P^0|\mathcal{O}|P^0\rangle=\frac{1}{2}B_{P^0}f_{P^0}^2 m_{P^0}^2\left[\left(\frac{m_{P^0}}{m_h+m_\ell}\right)^2+\frac{1}{6}\right]. \tag{4.11}$$

As computed on the lattice, the bag parameter in Eq. (4.10) is defined by the expression as shown [15], omitting the additional term proportional to $1/6$ that is retained in Eq. (4.11); in the case where $P^0=K^0$ or D^0, the effect of

TABLE III. Lower bounds on the flavor scale. See the text for definitions of our notation.

Mass Splitting	Operator	M_F Lower Bound
$K^0-\bar{K}^0$	$-d_2^2\frac{1}{m^2_{\tilde{s}_{12}}}\frac{v^2\xi^2}{2M_F^2}\bar{d}_L s_R \bar{d}_R s_L$	85 TeV
$B^0-\bar{B}^0$	$-d_3 d_4\frac{1}{m^2_{\tilde{\varphi}_1}}\frac{v^2\xi^2}{2M_F^2}\bar{d}_L b_R \bar{d}_R b_L$	22 TeV
$B_s^0-\bar{B}_s^0$	$-d_3 d_4\frac{1}{m^2_{\tilde{\varphi}_2}}\frac{v^2\xi^2}{2M_F^2}\bar{b}_L s_R \bar{b}_R s_L$	14 TeV
$D^0-\bar{D}^0$	$-u_2^2\frac{1}{m^2_{\tilde{s}_{12}}}\frac{v^2\rho^2}{2M_F^2}\bar{u}_L c_R \bar{u}_R c_L$	14 TeV

this term is negligible. All masses and mass splittings were obtained from the Review of Particle Properties [13], all decay constants were obtained from Ref. [17], the bag parameters for $\Delta S=2$ and $\Delta C=2$ were obtained from Ref. [15], and the bag parameters for $\Delta B=2$ were obtained from Ref. [16]. To estimate the lower bound on M_F, we assume that the experimentally observed mass splittings are consistent with the standard model predictions and require that the new physics contributions not exceed the current 2σ experimental uncertainty. Such an approach is sufficient for an estimate given the theoretical uncertainties involved in determining the new physics contribution itself. Our results are shown in Table III. As one might expect, we obtain the tightest bound from the $K^0-\overline{K^0}$ mass splitting, which requires $M_F \gtrsim 85$ TeV.

Flavon exchange between quarks and leptons can also lead to flavor-changing neutral meson decays. We again focus on operators that are flavor-changing in the absence of a rotation of the fields from the gauge to mass eigenstate basis. The largest effects are shown in Table IV. The relevant operators are of the form $\mathcal{O}^{ijkn}_{qde} \equiv (\overline{\ell}_i e_j)(\overline{d}_k q_n)$, in the notation of Ref. [18]; in the same reference, bounds on the operator coefficients are conveniently summarized. We translate these into bounds on the scale M_F which, as can be seen from Table IV, are much weaker that those coming from the pseudoscalar meson mass splittings. Therefore, we also show the predicted branching fractions with M_F set equal to our lower bound from $K^0-\overline{K^0}$ mixing. It is clear that the predicted branching fractions are far below the experimental bounds and unlikely to have observable consequences. Note that we have only considered CP conserving processes and it is generally known that bounds on CP violation in the neutral kaon system tends to give a better bound on the scale of new physics by about an order of magnitude compared to the CP-conserving FCNC bounds. Given the smallness of these branching fractions, this fact does not change our qualitative conclusions, so we do not pursue that issue further.

V. NONSUPERSYMMETRIC MODELS

In the renormalization group analysis of Sec. III, the Yukawa matrices Y_i are defined by

$$\mathcal{L}_m = \frac{v}{\sqrt{2}}\overline{\psi^i_L} Y_i \psi^i_R + \text{H.c.}, \tag{5.1}$$

where $i = U, D$ or E and generation indices are suppressed. In order to replicate the Yukawa textures of the supersymmetric models of Refs. [1,2], we assign the right-handed fermions of the three generations to the $T' \times Z_3$ representations $\mathbf{2}^{0-} \oplus \mathbf{1}^{00}$. Hence, for example, we would assign the first two generations of the charge-2/3 quarks according to $(u^c_L, c^c_L) \sim (u_R, c_R) \sim \mathbf{2}^{0-}$, where the superscript "c" refers to charge conjugation; since $\overline{\psi} = i\psi^{cT}\gamma^0\gamma^2$, this is equivalent to specifying the transformation properties of the Dirac adjoints $(\overline{u_L}, \overline{c_L})$. We then identify the following transformation properties for the various blocks of the Y_i,

$$Y_{U,D,E} \sim \begin{pmatrix} [\mathbf{3}^- \oplus \mathbf{1}^{0-}] & [\mathbf{2}^{0+}] \\ [\mathbf{2}^{0+}] & [\mathbf{1}^{00}] \end{pmatrix}, \tag{5.2}$$

i.e., Eq. (2.10) (or Eq. (4.1) in Ref. [2]), which omits any additional symmetries that may be needed to explain the suppression factors ρ and ξ. As in the supersymmetric model, the transformation properties given in Eq. (5.2) determine the allowed flavon couplings. However, in the supersymmetric case, Eq. (5.2) dictates the form of terms in the superpotential, which is required to be a holomorphic function of the superfields. The absence of this constraint in the nonsupersymmetric case could lead, in principle, to additional flavon couplings that are not present in the supersymmetric theory. However, we see that as far as the

TABLE IV. Lower bound on M_F for the largest flavor-changing decays. The predicted branching fraction for M_F set equal to the K^0-$\bar{K}^0$ mixing bound is also shown.

Decays	BF (Ref. [13])	Operator	M_F Lower Bound	BF ($M_F = 85$ TeV)
$K^0_L \to \bar{\mu}e$	$<4.7\times10^{-12}$	$-d_2\ell_2\frac{1}{m^2_{\tilde{s}_{12}}}\frac{v^2\xi^2}{2M_F^2}\bar{e}_L\mu_R\bar{s}_R d_L$	9.8 TeV	1.5×10^{-19}
$B^0 \to \bar{\tau}e$	$<2.8\times10^{-5}$	$-d_4\ell_3\frac{1}{m^2_{\tilde{\varphi}_1}}\frac{v^2\xi^2}{2M_F^2}\bar{e}_L\tau_R\bar{d}_R b_L$	0.62 TeV	2.3×10^{-22}
$B^0_s \to \bar{\tau}\mu$	⋯	$-d_3\ell_4\frac{1}{m^2_{\tilde{\varphi}_2}}\frac{v^2\xi^2}{2M_F^2}\bar{s}_L b_R\bar{\mu}_R\tau_L$	⋯	3.2×10^{-22}

ϕ, S and A flavons are concerned, this is not the case: each has a nontrivial Z_3 charge, which prevents new flavon couplings at the same order that involve the complex conjugates of these fields.

In the supersymmetric theories of Refs. [1,2], the additional suppression factors associated with the parameters ρ and ξ required the introduction of additional fields and symmetries. For example, in the simplest unified $T' \times Z_3$ model of Refs. [1,2], SU(5) charge assignments of the flavon fields are responsible for forbidding the coupling of the A and S flavons in Y_U at lowest order in $1/M_F$. However, these couplings emerge via higher-order operators that involve a flavor-singlet, SU(5) adjoint field $\Sigma \sim \mathbf{24}$, just as in earlier models based on U(2) flavor symmetry [4]. The suppression associated with the parameter ξ, on the other hand, was assumed to arise via mixing in the Higgs sector, a reasonable possibility since supersymmetric models require more than one Higgs doublet.

Here we will also achieve the additional suppression factors by means of additional fields and symmetries. However, the additional symmetry will be much smaller than the product of supersymmetry and a grand unified gauge group. (The latter, of course, would not be appropriate for the nonsupersymmetric case where the gauge couplings do not unify.) We will simply assume an additional Z_3 factor, so that the flavor group is $G_F^{\text{new}} = T' \times (Z_3)^2$ Defining one of the elements of the new Z_3 factor as $\omega = \exp(2i\pi/3)$, the only standard model fields that transform nontrivially under this symmetry are

$$H \to \omega H \quad \text{and} \quad t_R \to \omega t_R, \tag{5.3}$$

where H is the standard model Higgs field and t_R is the right-handed top quark. In the standard model, H couples to Y_D and Y_E, while $\sigma^2 H^*$ couples to Y_U. Hence, when the new Z_3 symmetry is unbroken, the assignments in Eq. (5.3) forbid Y_D and Y_E entirely, as well as the first two columns of Y_U. How one proceeds with the model building depends on the desired relative sizes of ϵ, ϵ', ρ and ξ. For example, for some choices of M_F, it is possible to find numerical results that are consistent with the simple possibility $\epsilon \sim \rho \sim \xi$, up to order one factors. In this case, we assume the symmetry-breaking pattern

$$T' \times (Z_3)^2 \xrightarrow{\epsilon} Z_3^D \xrightarrow{\epsilon'} \text{nothing}, \tag{5.4}$$

where the intermediate Z_3^D factor is exactly the same one as in the original theory, that transforms all first generation fields by a phase; in this case, the new Z_3 symmetry is broken at the first step in the symmetry-breaking chain. We introduce two new flavon fields

$$\rho_0 \to \omega^2 \rho_0 \quad \text{and} \quad \tilde{\phi} \to \omega \tilde{\phi}, \tag{5.5}$$

where $\tilde{\phi}$ transforms like $\phi \sim \mathbf{2}^{0+}$ under the original flavor group. With the assumed symmetry breaking pattern, the ρ_0 field and one component of the $\tilde{\phi}$ doublet can develop vevs of order ϵM_F. The Z_3 charges of these fields now allow us to rebuild our otherwise forbidden Yukawa matrices as follows:

(i) For Y_D and Y_E, we may generate matrices proportional to the standard form if we replace H by $H\rho_0$; it follows that $\langle \rho_0 \rangle / M_F$ is identified with the suppression factor ξ, which we now predict to be of order ϵ, up to an order one factor. One might worry that we could obtain a lower-order contribution from operators that do not involve ρ_0, but involve $\tilde{\phi}^*$ instead, which also transforms under the new Z_3 factor as $\tilde{\phi}^* \to \omega^2 \tilde{\phi}^*$. However, this does not occur since $\tilde{\phi}^* \sim \mathbf{2}^{0-}$ under the original flavor symmetry, which is not one of the representations that leads to a lowest order coupling. On the other hand, the product $\rho_0^* \tilde{\phi}$ does couple at the same order as $\rho_0 \phi$; however, this additional contribution does nothing to the form of the resulting Yukawa textures beyond a redefinition of the order one coefficients.

(ii) For Y_U, the two-by-two block associated with the flavons A and S can now be recovered via operators involving $\rho_0^* A$ and $\rho_0^* S$. Hence, the parameter we called ρ previously is now predicted to be of the same order as ξ. In an analogous way, the 3-1 and 3-2 entries of Y_U can couple to the product $\rho_0^* \phi$, but this transforms in the same way as $\tilde{\phi}$, which may couple at lower-order. Hence the canonical Y_U texture with an additional suppression in only the upper-left two-by-two block is obtained. Note that we could simply omit $\tilde{\phi}$ from the theory and ignore the corresponding entries in Y_U; this leads to an alternative texture in which $u_4 = 0$ in Eq. (2.10), neglecting corrections from higher-order operators. This was the alternative possibility considered in Sec. III. It is worth noting that in the case where the $\tilde{\phi}$ is omitted from the theory, there is no longer a necessary connection between the scale of the additional Z_3 breaking and the scale of the T' doublet vev, ϵM_F. In this case, we could vary this additional scale independently so that ρ and ξ are still comparable, but intermediate in size between ϵ and ϵ'. This construction would be compatible with the numerical results in Tables I and II.

In summary, we have provided an existence proof that the textures considered in our numerical analysis may arise in a relatively simple way in a nonsupersymmetric framework.

VI. CONCLUSIONS

In this paper, we have reconsidered models of flavor based on the non-Abelian discrete flavor group T' that were proposed in Ref. [1,2]. We have relaxed two assumptions made in these studies, that the models are supersymmetric and that the scale of the flavor sector is around the scale of supersymmetric grand unification. Our numerical study found that T' models without supersymmetry provide a viable description of charged fermion masses and CKM angles for a range of values of the flavor scale M_F. We find that identification of M_F with the reduced Planck scale is a viable possibility, consistent with a simple picture in which no new physics appears between the weak and gravitational scales. However, we also find that our fits improve monotonically as M_F is lowered toward the lower bound dictated by the constraints from flavor-changing-neutral-current processes. In the case where M_F is as low as possible, we identified the largest flavor-changing neutral current effects that result from the exchange of heavy flavor-sector fields; these could provide indirect probes of the model. We then showed how the form of the Yukawa textures that we studied, which were the same as, or closely related to, those described in Refs. [1,2], can nonetheless arise in a nonsupersymmetric framework, where there is only a single Higgs doublet field and where the interactions do not originate from a superpotential, a holomorphic function of the fields. The models we described are arguably simpler than their supersymmetric counterparts; in the nonsupersymmetric case, we needed only to extend the original flavor-group by a Z_3 factor to obtain the desired Yukawa textures shown in Eq. (2.10), while avoiding the well-known complications that come with a grand unified Higgs sector. Extending the present study to include the neutrino sector is more model dependent, but would be interesting for future work.

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