Supersymmetric models on magnetized orbifolds with flux-induced Fayet-Iliopoulos terms

Hiroyuki Abe,[1,*] Tatsuo Kobayashi,[2,†] Keigo Sumita,[1,‡] and Yoshiyuki Tatsuta[1,§]

[1]*Department of Physics, Waseda University, Tokyo 169-8555, Japan*
[2]*Department of Physics, Hokkaido University, Sapporo 060-0810, Japan*
(Received 31 October 2016; published 9 January 2017)

We study supersymmetric (SUSY) models derived from the ten-dimensional SUSY Yang-Mills theory compactified on magnetized orbifolds, with nonvanishing Fayet-Iliopoulos (FI) terms induced by magnetic fluxes in extra dimensions. Allowing the presence of FI-terms relaxes a constraint on flux configurations in SUSY model building based on magnetized backgrounds. In this case, charged fields develop their vacuum expectation values to cancel the FI-terms in the D-flat directions of fluxed gauge symmetries, which break the gauge symmetries and lead to a SUSY vacuum. Based on this idea, we propose a new class of SUSY magnetized orbifold models with three generations of quarks and leptons. Especially, we construct a model where the right-handed sneutrinos develop their vacuum expectation values which restore the supersymmetry but yield lepton number violating terms below the compactification scale, and show their phenomenological consequences.

I. INTRODUCTION

There remain several puzzles in the standard model (SM) even though the last missing piece, the Higgs boson, was discovered at the Large Hadron Collider [1]. In particular, an origin of the flavor structure is a remarkable one in particle physics, that is, the reason why our world consists of three generations of the quarks and the leptons, what is more, with hierarchical Yukawa couplings.

The extra dimensions of space are known as great candidates for the new physics beyond the SM, which provides a source of hierarchy [2] for explaining the flavor structure and they have been studied actively so far. This is also preferable from a theoretical point of view because superstring theories, candidates for the unified theory, are defined in ten-dimensional (10D) spacetime. We usually consider ten- or other higher-dimensional supersymmetric Yang-Mills (SYM) theory to study compactifications of the extra dimensional space, because they provide simple frameworks for such a purpose and, even more, are well motivated by superstring theories. We expect that a non-trivial structure in the extra compact space generates the observed flavor structures of the SM and realistic models would be obtained as four-dimensional (4D) effective field theories of the higher-dimensional SYM theories.

It is known that magnetic fluxes in the compact space have a potential to realize the hierarchical flavor structures [3,4]. Furthermore, in higher-dimensional SYM theories compactified on a product of two-dimensional (2D) tori with magnetic fluxes, an analytic form of zero-mode wave functions can be obtained by solving the Dirac equation, which lead to an explicit form of 4D effective Yukawa couplings [4]. In accordance with this result, a concrete model consistent with the minimal supersymmetric SM (MSSM) was constructed with a (semi-)realistic pattern of the masses and the mixing angles of quarks and leptons [5,6].

We can utilize Z_N orbifolding for constructing more realistic models well in the magnetized toroidal compactifications. While the three-generation structure is uniquely given with a $\Delta(27)$ flavor symmetry on magnetized tori without orbifolding, the orbifold projections as well as certain classes of Wilson lines lead to a broad variety of three-generation structures with different types of flavor symmetries [7], and furthermore it can eliminate some phenomenologically disfavored extra massless field contents. The three-generation structures were systematically studied with Z_2 orbifolds [8,9] and $Z_{3,4,6}$ ones [10].

It is not straightforward to find a supersymmetric (SUSY) SM vacuum on magnetized backgrounds, because magnetic flux in extra compact space generically produces the Fayet-Iliopoulos (FI) term [11] for the hypercharge and/or extra $U(1)$ factors. When there are charged fields without their mass terms in the superpotential, the FI-term makes the charged fields develop nonvanishing vacuum expectation values (VEVs), which break the fluxed gauge symmetry and lead to sizable D-term contributions to the charged scalar masses. Thus, in previous works [5,12], it has been required that the FI-terms produced on three 2D tori are canceled out by each other in the SUSY model building based on magnetized SYM theories.

In the present paper, we allow configurations of magnetic fluxes which induce nonvanishing FI-terms for extra $U(1)$ gauge symmetries other than the $U(1)$ hypercharge to

construct a new class of models. An anomalous $U(1)$ symmetry with a nonvanishing FI-term has been intensively studied in generic 4D SUSY models, motivated by string models [13]. In such works, they found that the presence of FI-terms can play a significant role in the phenomenology of particle physics and cosmology. This motivates us to study the magnetized orbifold models with FI-terms classically produced by the magnetic fluxes and we expect that the models with FI-terms lead to phenomenological implications very different from those without the FI-terms.

In the presence of a FI-term in a $U(1)$ sector, charged scalars tend to develop their VEVs for the D-term to vanish. In the magnetized models, the typical scale of the VEVs is almost equal to the compactification scale. Since that is usually as high as the grand unification theory (GUT) scale or Planck scale, the responsible fields must be a SM singlet. In this paper, we especially focus on the right-handed sneutrinos, which are SM singlets but can play phenomenologically relevant roles in the MSSM sector, and consider their nonvanishing VEVs in the D-flat directions of fluxed $U(1)$ symmetries to cancel the FI-term.

This paper is organized as follows. In Sec. II, we give an overview of magnetized orbifold models and show some basic ideas for realizing a new class of models with flux-induced FI-terms. We first explain the D-terms in the toroidal compactification of 10D SYM theories with magnetic fluxes and the orbifolding in Sec. II A, and show how to construct the MSSM-like models by introducing flux-induced FI-terms in some $U(1)$ subgroups of $U(8)$ SYM theory on a magnetized orbifold in Sec. II B.

Subsequently, in Sec. III, we construct a SUSY model with flux-induced FI-terms for certain $U(1)$ subgroups of $U(8)$, which make the right-handed sneutrinos develop their VEVs in the D-flat directions leading to a SUSY minimum. We show the almost unique flux configuration which realizes three generations of quarks and leptons at the SUSY minimum where the sneutrinos have VEVs in Sec. III A. In Sec. III B, we show the superpotential of our model and discuss a relation between the textures of the μ-terms and the lepton number violating mass terms. Their interplay modifies the flavor structure of the leptons. We estimate the mass ratios and mixing angles of the quarks and the leptons in a numerical calculation in Sec. III C, where we show that our model leads to a hopeful spectrum of the SM matter fields. Finally, we summarize our result and discuss its future prospect in Sec. IV.

The analytic forms of Yukawa couplings are shown in the Appendix for the model shown in Sec. III A.

II. FLUX-INDUCED FI-TERMS ON MAGNETIZED ORBIFOLDS

We briefly review the magnetized compactification with/without orbifolding in 10D SYM theories. It will be shown that the magnetic fluxes break the gauge symmetry down to a product of several unbroken gauge

subgroups and bifundamental gaugino fields of the unbroken subgroups have degenerate zero modes with their conjugate ones eliminated; that is, generations of chiral fermions like the SM are obtained.

The magnetic fluxes, in general, produce FI-terms for some of the unbroken gauge subgroups. In previous works [5,12] for SUSY model building, it has been required that the FI-terms do not appear for any of the subgroups; otherwise SUSY is broken or those can lead to color and/or electromagnetism breaking vacua in some cases. The conditions for the FI-terms to vanish is so strong that we have been found a few configurations of magnetic fluxes which lead to three generations of the quarks and leptons preserving SUSY so far [12].

In the present paper, we consider model building such that the flux-induced FI-terms are nonvanishing for some $U(1)$s [other than the hypercharge $U(1)$] and the charged (but SM-singlet) scalar fields develop their VEVs to cancel the FI-terms in the D-term potential, leading to a SUSY minimum of the scalar potential. Then we will see in the following and in the next section that the allowed flux configurations for the SUSY model building and the resultant phenomenologies are quite different from those with vanishing FI-terms.

A. *D*-terms on magnetized tori

First we review the 10D SYM theories compactified on magnetized tori and orbifolds. We exclusively consider a product of three 2D tori as a six-dimensional extra compact space, which are described by complex coordinates $(z_i, \bar{z}_i)$ $(i = 1, 2, 3)$. The 10D SYM theory contains a pair of 10D gauge field A_M ($M = 0, 1, \ldots, 9$) and 10D Majorana-Weyl spinor field λ.

We decompose them into a 4D vector A_μ ($\mu = 0, 1, 2, 3$), three 4D complex scalars φ_i, and four 4D Weyl spinor fields $\lambda_{\pm\pm\pm}$ as

$$A_M = (A_\mu, \varphi_i), \qquad \lambda = (\lambda_{+++}, \lambda_{+--}, \lambda_{-+-}, \lambda_{--+}).$$

Here, the complex scalar field φ_i is defined in accordance with the complex coordinate z_i. Each subscript "$\pm$" accompanied with 4D Weyl spinors $\lambda_{\pm\pm\pm}$ represents the chirality on one of the three 2D tori. 4D Weyl spinors with other combinations of $\pm$'s, e.g., λ_{---}, do not appear because here the 10D spinor field λ is an eigenstate of a 10D chirality operator with a positive eigenvalue as a result of the Majorana-Weyl condition.

These component fields form 4D $\mathcal{N} = 1$ supermultiplets with auxiliary fields F_i and D as

$$V \equiv -\theta\sigma^\mu\bar{\theta}A_\mu + i\bar{\theta}\,\bar{\theta}\,\theta\lambda_0 - i\theta\theta\bar{\theta}\bar{\lambda}_0 + \frac{1}{2}\theta\theta\bar{\theta}\,\bar{\theta}\,D,$$

$$\phi_i \equiv \frac{1}{\sqrt{2}}A_i + \sqrt{2}\theta\lambda_i + \theta\theta F_i,$$

where $(\lambda_0, \lambda_1, \lambda_2, \lambda_3) = (\lambda_{+++}, \lambda_{+--}, \lambda_{-+-}, \lambda_{--+})$. In Refs. [14,15], the 10D SYM action is expressed in the 4D $\mathcal{N} = 1$ superspace by using these superfields. We compactify it on a product of the 4D Minkowski spacetime and three 2D tori, $M^4 \times T^2 \times T^2 \times T^2$, and introduce Abelian constant magnetic fluxes on the tori, which was studied in Ref. [16] and its 4D effective action is derived in a systematic way shown there.

Let us consider 10D $U(N)$ SYM theories compactified on three 2D tori with magnetic fluxes of the (1,1) form, e.g.,

$$F_{z_i \bar{z}_i} = 2\pi M^{(i)} = 2\pi \begin{pmatrix} M_a^{(i)} \mathbf{1}_{N_a} & 0 \\ 0 & M_b^{(i)} \mathbf{1}_{N_b} \end{pmatrix}, \quad M_a^{(i)}, \quad M_b^{(i)} \in \mathbb{Z},$$

in the $U(N)$ gauge space with $N = N_a + N_b$, where integer values of $M_a^{(i)}$ and $M_b^{(i)}$ represent the quantized magnetic fluxes and $\mathbf{1}_{N_a}$ denotes the $(N_a \times N_a)$ unit matrix. When the fluxes, $M_a^{(i)}$ and $M_b^{(i)}$, are different from each other, the gauge symmetry is broken as $U(N) \to U(N_a) \times U(N_b)$. In this paper, we will not consider magnetic fluxes of the other forms, i.e., (2,0) and (0,2) forms. On this magnetized background, $U(N)$-adjoint field is decomposed as

$$V = \begin{pmatrix} V_{aa} & V_{ab} \\ V_{ba} & V_{bb} \end{pmatrix}, \qquad \phi_i = \begin{pmatrix} (\phi_i)_{aa} & (\phi_i)_{ab} \\ (\phi_i)_{ba} & (\phi_i)_{bb} \end{pmatrix}. \quad (1)$$

The VEVs of zero modes of auxiliary fields D_a and $(F_i)_a$ in $U(N_a)$-adjoint vector and chiral superfields V_{aa} and $(\phi_i)_{aa}$ are determined by their equations of motion respectively as

$$D_a = \left(\frac{1}{\mathcal{A}^{(1)}} M_a^{(1)} + \frac{1}{\mathcal{A}^{(2)}} M_a^{(2)} + \frac{1}{\mathcal{A}^{(3)}} M_a^{(3)} \right) \times \mathbf{1}_{N_a},$$

$$(F_i)_a = 0 \times \mathbf{1}_{N_a}, \quad (2)$$

where we denote the ith 2D torus area by $\mathcal{A}^{(i)}$. Then the conditions for preserving 4D $\mathcal{N} = 1$ SUSY are simply described by $D_a = 0$, i.e.,

$$\frac{1}{\mathcal{A}^{(1)}} M_a^{(1)} + \frac{1}{\mathcal{A}^{(2)}} M_a^{(2)} + \frac{1}{\mathcal{A}^{(3)}} M_a^{(3)} = 0, \quad (3)$$

as long as we restrict ourselves to the case with vanishing VEVs of fields in the bifundamental representations $(N_a, \overline{N}_b)$ and/or $(\overline{N}_a, N_b)$ carried by $(\phi_i)_{ab}$ and/or $(\phi_i)_{ba}$. Similar arguments hold for $U(N_b)$-adjoint superfields by replacing a with b in Eqs. (2) and (3). These give constraints on magnetic fluxes $M_a^{(i)}$ and $M_b^{(i)}$, and the supersymmetric model building on magnetized tori and orbifolds suffers from them.

The bifundamental fields in $(\phi_i)_{ab}$ feel the $M_{ab}^{(i)} \equiv M_a^{(i)} - M_b^{(i)}$ unit of magnetic fluxes on the ith 2D torus. For the positive values of $M_{ab}^{(i)}$, the magnetic fluxes give rise to $M_{ab}^{(i)}$

TABLE I. The number of degenerate zero modes on magnetized Z_2 orbifolds where $n \in \mathbb{N}$ [8].

M	0	1	2	3	4	5	6	7	8	9	$2n$	$2n+1$
Even	1	1	2	2	3	3	4	4	5	5	$n+1$	$n+1$
Odd	0	0	0	1	1	2	2	3	3	4	$n-1$	n

degenerate zero modes for $(\phi_i)_{ab}$ and $(\phi_{j \neq i})_{ba}$, and their conjugate ones, $(\phi_i)_{ba}$ and $(\phi_{j \neq i})_{ab}$, are then eliminated in the low-energy spectrum, which yield generations of chiral fermions [3,4]. For the negative values of $M_{ab}^{(i)}$, in contrast, $|M_{ab}^{(i)}|$ degenerate zero modes are produced for $(\phi_i)_{ba}$ and $(\phi_{j \neq i})_{ab}$, while $(\phi_i)_{ab}$ and $(\phi_{j \neq i})_{ba}$ have no zero modes. Note that, with the vanishing value of $M_{ab}^{(i)}$, all of representations have a single zero mode with a flat wave function.

Next, we go on to the compactification on magnetized orbifolds. Systematic studies of Z_N ($N = 3,4,6$) orbifolds with magnetic fluxes have been recently done [10]. In this paper we concentrate on the Z_2 orbifolds [8,9], where all field contents are assigned into either even or odd modes under the Z_2 parity. The numbers of degenerate even or odd zero modes produced by the magnetic fluxes are reduced because of the orbifold projection and these zero-modes numbers are shown in Table I [8].

It is also remarkable in our discussion that these Z_2 parities are assigned by respecting the superfield formulation where the 4D $\mathcal{N} = 1$ SUSY is manifest. This determines the transformation low of the superfields under the Z_2. For example, under a Z_2 operation given by $(z_1, z_2, z_3) \to (-z_1, -z_2, z_3)$, they transform as

$$V \to +PVP^{-1},$$
$$\phi_1 \to -P\phi_1 P^{-1},$$
$$\phi_2 \to -P\phi_2 P^{-1},$$
$$\phi_3 \to +P\phi_3 P^{-1},$$

where P is a projection operator ($P^2 = \mathbf{1}$) and is an $N \times N$ matrix in $U(N)$ cases.

B. $U(8)$ models with FI-terms

In the following, we discuss the FI-term induced by the magnetic fluxes in three-generation magnetized orbifold models. The Pati-Salam-like gauge group $U(4)_C \times U(2)_L \times U(2)_R$ can realize a realistic model based on magnetized backgrounds. This is derived from the $U(8)$ SYM theory with magnetic fluxes of the form

$$M^{(i)} = \begin{pmatrix} M_C^{(i)} \times \mathbf{1}_4 & 0 & 0 \\ 0 & M_L^{(i)} \times \mathbf{1}_2 & 0 \\ 0 & 0 & M_R^{(i)} \times \mathbf{1}_2, \end{pmatrix}, \quad (4)$$

which breaks $U(8)$ down to $U(4)_C \times U(2)_L \times U(2)_R$. The representation $(\mathbf{1}, \mathbf{2}, \bar{\mathbf{2}})$ of the respective unbroken subgroups contains the Higgs multiplets. The left-handed and right-handed matter fields are assigned into the representation $(\mathbf{4}, \bar{\mathbf{2}}, \mathbf{1})$ and $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$, respectively. The right-handed sneutrinos (to get nonvanishing VEVs later) are contained in the representation $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$.

These magnetic fluxes generically produce FI-terms for the Abelian part of each unbroken gauge subgroup. For example, in the $U(4)_C$ sector, the magnetic fluxes on the three 2D tori yield constant contributions in the D-term of $U(4)_C$,

$$D_C = \left(\frac{1}{\mathcal{A}^{(1)}} M_C^{(1)} + \frac{1}{\mathcal{A}^{(2)}} M_C^{(2)} + \frac{1}{\mathcal{A}^{(3)}} M_C^{(3)} \right) \times \mathbf{1}_4, \quad (5)$$

like in Eq. (2), due to the presence of FI-term for the $U(4)_C$ vector superfield induced by the fluxes $M_C^{(i)} \times \mathbf{1}_4$. In this paper, we adopt such flux configurations, which generate nonvanishing FI-terms for the fluxed $U(1)$ vector multiplet, and consider the case that some SM singlets develop their VEVs to cancel the FI-terms in the 4D effective field theories [17] restoring the $\mathcal{N} = 1$ SUSY.

Let us consider the D-term of the $U(N_a)$ subgroup with the VEVs of matter fields in its fundamental representation. The coupling of such a matter chiral superfield Φ and the gauge multiplet is described by

$$\int d^4\theta (\Phi^*)_i (e^V)_{ij} (\Phi)_j,$$

where V is the $U(N_a)$ gauge superfield, and $i, j = 1, 2, \ldots, N_a$ are now $U(N_a)$ indices. Without the flux-induced FI-term, the D-term of $U(N_a)$ is given by $D_{ij} = \langle (\Phi^*)_i \rangle \langle (\Phi)_j \rangle$ which can be always diagonalized as

$$D_{ij} = \text{diag}(x, 0, \ldots, 0), \quad (6)$$

by a certain $U(N_a)$ rotation, where x is a real constant. In the presence of the FI-term, a SUSY vacuum is obtained when the following condition is satisfied:

$$\delta_{ij} \left(\frac{1}{\mathcal{A}^{(1)}} M_a^{(1)} + \frac{1}{\mathcal{A}^{(2)}} M_a^{(2)} + \frac{1}{\mathcal{A}^{(3)}} M_a^{(3)} \right) + \langle (\Phi^*)_i \rangle \langle (\Phi)_j \rangle = 0. \quad (7)$$

For $N_a > 1$, this cannot be satisfied because the first contribution is rank N_a but the second one is rank 1 as we see in Eq. (6). Therefore we find that Eq. (7) can be satisfied only in the case with $N_a = 1$.

From the above argument we expect that, when the unbroken subgroup which has the nonvanishing FI-term is $U(1)$, the FI-term can be canceled by the VEVs of charged fields. In this case, the scale of VEVs is comparable to the

compactification scale, which would be typically set to M_{GUT} or M_{Planck}. In the Pati-Salam-like model obtained by the flux configuration (4), such a large value of VEV is phenomenologically allowed only for the right-handed sneutrinos. We identify them as the responsible field for canceling the FI-term. In the following, we adopt the flux configurations where all the unbroken gauge subgroups related to the right-handed neutrinos are $U(1)$, and consider the case that their flux-induced FI-terms are canceled by the VEVs of right-handed sneutrinos, yielding a new class of SUSY vacua.

In the Pati-Salam-like model, the right-handed neutrinos are carried by the bifundamental representation of $U(4)_C \times U(2)_R$. In accordance with the above discussion, these two gauge groups have to be further broken by the magnetic fluxes down to $U(3)_C \times U(1)_\ell$ and $U(1)_r \times U(1)_{r'}$ from the beginning. This gauge symmetry breaking is realized by the magnetic fluxes of the form,

$$M^{(i)} = \begin{pmatrix} M_C^{(i)} \times \mathbf{1}_3 & 0 & 0 & 0 & 0 \\ 0 & M_\ell^{(i)} & 0 & 0 & 0 \\ 0 & 0 & M_L^{(i)} \times \mathbf{1}_2 & 0 & 0 \\ 0 & 0 & 0 & M_r^{(i)} & 0 \\ 0 & 0 & 0 & 0 & M_{r'}^{(i)} \end{pmatrix},$$

$$(8)$$

where each flux number of $M_C^{(i)}, M_\ell^{(i)}, M_L^{(i)}, M_r^{(i)}$, and $M_{r'}^{(i)}$ takes a different value from the others on at least one of three 2D tori $i = 1, 2, 3$; otherwise the unbroken gauge symmetry is enhanced. Note that this form of the magnetic fluxes can be shifted as $M^{(i)} \to M^{(i)} + \alpha \times \mathbf{1}_8$ without changing the spectrum in the low-energy effective field theory, and in the following we set the value of $M_C^{(i)}$ to vanish by using this degree of freedom.

On this magnetized background, the gauge symmetry is broken as $U(8) \to U(3)_C \times U(1)_\ell \times U(2)_L \times U(1)_r \times U(1)_{r'}$. We can then assign the MSSM fields into the decomposed adjoint fields as

$$\Phi_{\text{adj}} = \begin{pmatrix} * & * & Q & * & * \\ * & * & L & * & * \\ * & * & * & H_u & H_d \\ U & N & * & * & * \\ D & E & * & * & * \end{pmatrix}. \quad (9)$$

The substructure of this 8×8 matrix Φ_{adj} is exactly the same as Eq. (8). The fields denoted by Q, L, U, D, N, E, H_u, and H_d correspond to the left-handed quarks, the left-handed leptons, the right-handed up-type quarks, the

right-handed down-type quarks, the right-handed neutrinos, the right-handed charged leptons, the up-type Higgs fields, and the down-type Higgs fields, respectively. The other elements symbolically expressed by $*$ represent extra fields which can be eliminated by the interplay between the magnetic fluxes and orbifold projections. The VEVs of the right-handed sneutrinos can give rise to new contributions in $U(1)_\ell$ and $U(1)_r$ D-terms. The sneutrino VEVs break one linear combination of these two $U(1)$'s while the other (orthogonal) combination is preserved, and the latter one is a part of the $U(1)$ hypercharge.

Because it is required for our purpose that each flux number of $M_C^{(i)}$, $M_\ell^{(i)}$, $M_L^{(i)}$, $M_r^{(i)}$, and $M_{r'}^{(i)}$ in Eq. (8) takes a different value from the others on at least one of three 2D tori, in order to obtain three generations of quarks Q, U, D and leptons L, N E, certain orbifold projections are necessary. Without orbifolding, the three-generation structure is generated by $M = 3$ units of magnetic fluxes exclusively, and any flux configurations which induce three generations of the quarks and leptons cannot realize the required pattern of gauge symmetry breaking. In contrast, as we find in Table I, three generations appear with $M = 4$, 5, 7, or 8 units of magnetic fluxes [18], which allow us to construct three-generation models with the desired patterns of gauge symmetry breaking for our purpose.

If we do not allow any nonvanishing VEVs of off-diagonal (bifundamental) fields in Eq. (9) in our model building, the SUSY preserving conditions (3) for all the unbroken gauge subgroups $a = C, \ell, L, r$, and r' severely restrict the patterns of original flux configurations. Indeed, SUSY configurations of the magnetic fluxes which lead to a product gauge group with more than three subgroups have never been found [19]. On the other hand, if we consider a situation that any off-diagonal fields in Eq. (9), especially the right-handed sneutrinos denoted by N from the phenomenological viewpoint in the Pati-Salam-like model, develop their nonvanishing VEVs, there appear to be additional contributions in the D-flat conditions and the SUSY preserving condition (3) is modified as

$$\frac{1}{\mathcal{A}^{(1)}} M^{(1)} + \frac{1}{\mathcal{A}^{(2)}} M^{(2)} + \frac{1}{\mathcal{A}^{(3)}} M^{(3)} + X = 0, \quad (10)$$

where $M^{(i)}$ is shown in Eq. (8), and the (8×8) matrix X represents the contributions due to the VEVs $\langle \tilde{\nu}_i \rangle$ of right-handed sneutrinos. Because the right-handed neutrinos are charged under $U(1)_\ell$ and $U(1)_r$, the matrix X is described as

$$X = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & qx & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -qx & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (11)$$

which is parametrized by $x = \sum_i \langle \tilde{\nu}_i \rangle^2$ and $q = \pm 1$. The modified SUSY condition (10) allows a new class of SUSY vacua on magnetized orbifolds, which we demonstrate in the next section.

III. SUPERSYMMETRIC MODELS WITH FI-TERMS

Magnetized orbifolds provide a wide variety of three-generation structures. One of the key points to construct phenomenological models is that three-generation structures for quarks and leptons must be produced on a single 2D torus, otherwise the rank of Yukawa matrices is reduced to one. For this reason, we concentrate on a 2D torus for a while. The model building based on magnetized Z_2 orbifolds was studied systematically in Refs. [9,19]. On (untwisted) magnetized Z_2 orbifolds, three generations of chiral fermions are produced by the $|M| = 4$, 5 units of fluxes for Z_2 even modes, and $|M| = 7$, 8 for odd modes as shown in Table I. There is a severe constraint, as well as the SUSY preserving condition, on the flux configurations due to the requirement that the numbers of H_u and H_d have to be equal to each other in order to avoid the anomaly of $U(1)$ hypercharges.

A. The three-generation model

A systematic search performed in Ref. [19] shows that there are only four patterns of magnetic fluxes and Z_2 parity assignments on one of three 2D tori, which would be available for our purpose, namely, the gauge symmetry is suitably broken as $U(8) \to U(3)_C \times U(1)_\ell \times U(2)_L \times U(1)_r \times U(1)_{r'}$, and three generations of quarks and leptons as well as pairs of H_u and H_d are obtained. One of them adopted in the following analysis is shown in Table II, where five pairs of the Higgs fields appear. Note that the assignment of Z_2 parity in this model is consistent with the nonvanishing Yukawa couplings required in (MS)

TABLE II. An example of magnetic fluxes felt by MSSM fields and the parity assignments for them under the Z_2 projection $z_i \to -z_i$ is shown.

	Number of Fluxes	Z_2 Parity	Number of Zero Modes
Q	$M_C^{(i)} - M_L^{(i)} = -4$	Even	3
L	$M_\ell^{(i)} - M_L^{(i)} = -5$	Even	3
U	$M_r^{(i)} - M_C^{(i)} = -5$	Even	3
D	$M_{r'}^{(i)} - M_C^{(i)} = -8$	Odd	3
N	$M_r^{(i)} - M_\ell^{(i)} = -4$	Even	3
E	$M_{r'}^{(i)} - M_\ell^{(i)} = -7$	Odd	3
H_u	$M_L^{(i)} - M_r^{(i)} = 9$	Even	5
H_d	$M_L^{(i)} - M_{r'}^{(i)} = 12$	Odd	5

SM. Other three patterns are obtained by exchanging the assignment of flux and parity between the quark and lepton sectors, and/or, the up and down sectors in the above example.

The flux configurations and the Z_2 parity assignment on the other two 2D tori are mainly determined in order to satisfy the modified version of the SUSY preserving condition (10) and induce no extra generations of quarks and leptons. These two conditions are indeed so severe that there exists one and only possible configuration which we find in a systematic search. That is given by

$$M^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & -8 & 0 \\ 0 & 0 & 0 & 0 & -5 \end{pmatrix},$$

$$M^{(2)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix},$$

$$M^{(3)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \qquad (12)$$

and

$$X = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

This matrix X corresponds to Eq. (11) with $q = x = +1$. These satisfy the SUSY preserving conditions (10) with $\mathcal{A}^{(1)}/\mathcal{A}^{(2)} = 4$ and $\mathcal{A}^{(1)}/\mathcal{A}^{(3)} = 9$. In this case, the sneutrino VEVs are given in the unit of $1/\sqrt{\mathcal{A}^{(1)}}$, which we identify with the compactification scale because this is just the mass scale of the first excited Kaluza-Klein mode. Note that, exchanging flux configurations on the three 2D tori leads to different models on first glance, but they are physically equivalent to each other. It is just a matter of labeling the complex coordinates of three 2D tori.

TABLE III. We summarize the effective magnetic fluxes felt by each of the MSSM fields and the suitable Z_2 parities.

	$(z_1, \bar{z}_1)$	$(z_2, \bar{z}_2)$	$(z_3, \bar{z}_3)$
Q	−4, even	+1, even	0, even
L	−5, even	+1, even	0, even
U	−8, odd	0, even	+1, even
D	−5, even	−1, even	+1, even
N	−7, odd	0, even	+1, even
E	−4, even	−1, even	+1, even
H_u	+12, odd	−1, even	−1, even
H_d	+9, even	0, even	−1, even

On this magnetized background, we can obtain the three generations of quarks and leptons and the five pairs of Higgs fields when the Z_2 parities on each 2D torus $(z_i, \bar{z}_i)$ are assigned as shown in Table III. The orbifolding in $(z_2, \bar{z}_2)$ and $(z_3, \bar{z}_3)$ directions are not necessary to realize the three generations, but we impose them on the two 2D tori because they are useful to eliminate extra field contents, such as phenomenologically disfavored chiral exotics and massless adjoint fields. Note that, all the MSSM fields are assigned to the Z_2 even mode on these two 2D tori otherwise they are eliminated in the 4D low-energy spectrum.

We consider a $T^6/Z_2 \times Z_2'$ orbifold to realize the desired parity assignment for our purpose. The orbifold projection operators are assigned as mentioned at the end of Sec. II A; $\phi_i \to \pm P\phi_i P^{-1}$. We find that the following ones lead to the desirable Z_2 parities:

$$Z_2 : (z_1, z_2, z_3) \to (-z_1, -z_2, z_3)$$

$$\text{with} \quad P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$Z_2' : (z_1, z_2, z_3) \to (z_1, -z_2, -z_3)$$

$$\text{with} \quad P' = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}. \qquad (13)$$

On this magnetized orbifold, chiral superfields ϕ_i produce the following zero modes:

$$\phi_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & H_u & H_d \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & S & 0 \end{pmatrix},$$

$$\phi_2 = \begin{pmatrix} 0 & 0 & Q & 0 & 0 \\ 0 & 0 & L & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\phi_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ U & N & 0 & 0 & 0 \\ D & E & 0 & 0 & 0 \end{pmatrix}.$$

Thanks to the structural interplay between chirality projections caused by the magnetic fluxes (12) and the above orbifold projections (13), almost all the phenomenologically unwanted extra massless modes are eliminated in the 4D spectrum, except for two generations of chiral exotic modes S which have the same hypercharge as the right-handed electrons. We also find that, in another similar pattern of suitable $T^6/Z_2 \times Z_2'$ orbifold, these exotics remain after all. In the following, we propose a way to eliminate them for getting realistic models.

It seems that the simplest way to eliminate the exotics is to twist the orbifold boundary conditions. The zero-mode structure on twisted orbifolds is different from that on untwisted ones [20], and it is also known that this twisting is equivalent to turning on a gauge field background with a vortex configuration [21]. Although we will skip the details here, we explain only the essence we need for our purpose, that is, how to eliminate the exotics without spoiling the nonvanishing Yukawa couplings among quarks, leptons, and Higgs bosons obtained above.

Let us consider a twisted boundary condition of orbifolding in the $\mathrm{Re}z_3$ or $\mathrm{Im}z_3$ direction, only in the $U(1)_{r'}$ sector. The twisting phase is uniquely specified on Z_2 orbifolds. We notice that only the exotics S feel a vanishing magnetic flux on the third 2D torus in the $U(1)_{r'}$ sector. As a consequence, their zero modes are eliminated by the twist, because the vanishing flux gives rise to a flat zero-mode wave function which cannot satisfy the twisted boundary condition. As for the other $U(1)_{r'}$ charged fields, D, E, and H_d, the number of their zero modes are not changed since they feel $|M| = 1$ units of magnetic fluxes. All of the other $U(1)_{r'}$ singlet fields are obviously unaffected by this twisting. Thus, we can eliminate the exotic field S in the low-energy spectrum and derive a

TABLE IV. We show the transformation low of the MSSM fields under the Z_3 symmetry with $\alpha = \mathrm{Exp}(2\pi i/3)$.

Q	U	D	L	N	E	H_u	H_d
1	α^2	α	α^2	1	α^2	α	α^2

MSSM-like model without any of the massless extra fields. This is one of the great features of our model.

The VEVs of the right-handed sneutrinos lead to the lepton number violating term,

$$y^\nu \langle \tilde{\nu}_R \rangle L H_u, \tag{14}$$

in the superpotential. This term clearly breaks the usual R parity but we can find that our model has another discrete symmetry to prohibit the rapid proton decay. That is a Z_3 symmetry, so-called baryon triality [22], under which the MSSM fields transform in accordance with the charge assignment shown in Table IV. This symmetry allows the presence of the μ-term and lepton number violating terms but not baryon number violations, suppressing the proton decay. Within the MSSM matter contents, this can be an anomaly-free discrete gauge symmetry. Discrete symmetries without an anomaly means that such symmetries cannot be violated even by nonperturbative effects. Although our model contains extra heavy Higgs fields other than the MSSM Higgs fields, this Z_3 symmetry can be anomaly-free because the copies of (H_u, H_d) cannot contribute to the anomaly with the charge assignment shown in Table IV [23]. We stop discussing the whole anomalies here because it is necessary to construct a full system that contains hidden sectors as well as the MSSM sector to study them completely.

B. Mass eigenstates with sneutrino VEVs

We study the phenomenological impact of the lepton number violating mass term (14) in the superpotential. The total superpotential of our model is given by

$$W = y^u_{ijm} Q_i U_j H_{um} + y^d_{ijm} Q_i D_j H_{dm} + y^\nu_{ijm} L_i N_j H_{um}$$
$$+ y^e_{ijm} L_i E_j H_{dm} + \mu_{mn} H_{um} H_{dn} + \tilde{M}_{im} L_i H_{um},$$

where i, $j = 1$, 2, 3 and m, $n = 1$, 2, 3, 4, 5. The right-handed neutrino superfield N_j represents the fluctuation around the vacuum with nonvanishing sneutrino VEVs in the D-flat direction satisfying Eq. (10). The last term violating the lepton number is generated by the VEVs of sneutrinos $\tilde{\nu}_j$ as

$$\tilde{M}_{im} L_i H_{um} = (y^\nu_{i1m} \langle \tilde{\nu}_1 \rangle + y^\nu_{i2m} \langle \tilde{\nu}_2 \rangle + y^\nu_{i3m} \langle \tilde{\nu}_3 \rangle) L_i H_{um}. \tag{15}$$

On the other hand, the second-to-last term, the so-called μ-term, does not appear perturbatively in the 10D SYM

theory compactified on tori, but is necessary for realizing electroweak symmetry breaking and some other phenomenological reasons. Because our magnetized SYM model is expected to be embedded into some D-brane configurations or other stringy setup, we assume that certain nonperturbative effects [26], higher-dimensional operators, or some other extrinsic effects generate this term in our model and here treat the components of this (5×5) matrix $\tilde{M}_{im}$ as parameters.

Let us consider the following rotation of the basis to diagonalize the mass terms of H_u, H_d and L:

$$H'_u = \mathcal{U}H_u, \qquad \begin{pmatrix} L' \\ H'_d \end{pmatrix} = \mathcal{V}\begin{pmatrix} L \\ H_d \end{pmatrix}, \qquad (16)$$

where $\mathcal{U}$ and $\mathcal{V}$ are (5×5)- and (8×8)-unitary matrices, respectively. The superpotential (15) is rewritten as

$$W = y'^u_{ijm}Q_iU_jH'_{um} + y'^d_{ijm}Q_iD_jH'_{dm} + y'^\nu_{ijm}L'_iU_jH'_{um}$$
$$+ y'^e_{ijm}L'_iE_jH'_{dm} + \tilde{\mu}_{mn}H'_{um}H'_{dn} + W_{\not L}, \qquad (17)$$

where

$$\tilde{\mu}_{mn} = \mathrm{diag}(m_1, m_2, ..., m_5). \qquad (18)$$

We identify H'_{u1} and H'_{d1} with the MSSM Higgs fields, and the first entry m_1 in Eq. (18) corresponds to the μ parameter of the MSSM. The other $H'_{um\neq1}$ and $H'_{dm\neq1}$ must be heavy enough to suppress the flavor changing neutral currents (FCNCs) and we assume $m_{m\neq1} \gtrsim \mathcal{O}(10\ \mathrm{TeV})$.

The last term $W_{\not L}$ in the superpotential (17) represents the lepton number violating terms in the present diagonal basis of H'_u, H'_d and L',

$$W_{\not L} = \lambda_1QDL' + \lambda_2L'L'E + \lambda_3NH'_uH'_d + \lambda_4EH'_dH'_d, \quad (19)$$

which are generated from the Yukawa couplings in the original basis. Thanks to the baryon triality, we need not be concerned about the proton decay process caused by $W_{\not L}$. However, studying the effect of these terms on the collider physics is interesting, because we can explicitly calculate all of the coupling constants except for the μ parameter in our model. This is one of the attractive features for building models based on magnetized toroidal compactifications.

The Yukawa couplings y^u_{ijm}, y^d_{ijm}, y^ν_{ijm}, and y^e_{ijm} in the original basis are determined by the magnetic fluxes (12) and the projection operators (13), and their analytic forms can be derived. With these Yukawa couplings, the VEVs of multiple Higgs fields H_{um} and H_{dm} generate the mass matrices of quarks and leptons, e.g.,

$$(M_u)_{ij} = y^u_{ij1}\langle H_{u1}\rangle + y^u_{ij2}\langle H_{u2}\rangle + y^u_{ij3}\langle H_{u3}\rangle$$
$$+ y^u_{ij4}\langle H_{u4}\rangle + y^u_{ij5}\langle H_{u5}\rangle,$$

in the original basis. Here we remark that the sneutrino VEVs deform the lepton mass matrices through the diagonalization of H_u, H_d and L, because the part of the diagonalizing matrix $\mathcal{V}$ which rotates L_i in Eq. (16) can be nonunitary [even though the whole (8×8) matrix $\mathcal{V}$ is unitary]. We show this explicitly in the following based on a simplified situation. We here note that such an effect will be taken into account when we evaluate the mass ratios and mixing angles of quarks and leptons in Sec. III C.

We can extract information about the desired structure of the matrix μ_{mn} which we expect to be generated by extrinsic effects. For such a purpose, let us consider a simplified situation. We denote the relevant part of the superpotential by

$$W_{LH} = \mu_{mn}H_{um}H_{dn} + \tilde{M}_{im}L_iH_{um}.$$

Instead of the rotation (16), let us suppose that the rotation of

$$H_{um} \to H'_{um} = \mathcal{U}^{(u)}_{mn}H_{un},$$
$$H_{dm} \to H'_{dm} = \mathcal{U}^{(d)}_{mn}H_{dn},$$
$$L_i \to L'_i = \mathcal{V}_{ij}L_j, \qquad (20)$$

can diagonalize the matrices μ_{mn} and $\tilde{M}_{im}$ simultaneously as

$$W_{LH} = \sum_{i=1}^{3}(\mu'_iH'_{ui}H'_{di} + \tilde{M}'_iL'_iH'_{ui}) + \sum_{q=4}^{5}\mu'_qH'_{uq}H'_{dq},$$

where μ'_i and μ'_q are the eigenvalues of matrix μ_{mn}. The three singular values of the (3×5) matrix $\tilde{M}_{im}$ are represented by $\tilde{M}'_i$, which we can calculate explicitly on concrete magnetized backgrounds.

After the subsequent rotation,

$$H'_{di} \to H''_{di} \equiv \frac{1}{\sqrt{(\mu'_i)^2 + (\tilde{M}'_i)^2}}(\mu'_iH'_{di} + \tilde{M}'_iL'_i),$$
$$L'_i \to L''_i \equiv \frac{1}{\sqrt{(\mu'_i)^2 + (\tilde{M}'_i)^2}}(-\tilde{M}'_iH'_{di} + \mu'_iL'_i),$$

we find the final form of the superpotential as

$$W_{LH} = \sum_{i=1}^{3}\sqrt{(\mu'_i)^2 + (\tilde{M}'_i)^2}\, H'_{ui}H''_{di} + \sum_{q=4}^{5}\mu'_qH'_{uq}H'_{dq}.$$

As we mentioned, in this diagonal basis of L'', H'_u and H''_d, the mass matrices of charged leptons and neutrinos are deformed, e.g.,

$$(M_e)_{ij} = \mathcal{M}_{ik}\mathcal{V}^*_{kl}(y^e_{lj1}\langle H_{d1}\rangle + y^e_{lj2}\langle H_{d2}\rangle + y^e_{lj3}\langle H_{d3}\rangle$$
$$+ y^e_{lj4}\langle H_{d4}\rangle + y^e_{lj5}\langle H_{d5}\rangle),$$

where the unitary matrix $\mathcal{V}_{ij}$ is given in Eq. (20), and

$$\mathcal{M} = \begin{pmatrix} \frac{\mu_1'}{\sqrt{(\mu_1')^2 + (\tilde{M}_1')^2}} & 0 & 0 \\ 0 & \frac{\mu_2'}{\sqrt{(\mu_2')^2 + (\tilde{M}_2')^2}} & 0 \\ 0 & 0 & \frac{\mu_3'}{\sqrt{(\mu_3')^2 + (\tilde{M}_3')^2}} \end{pmatrix}. \quad (21)$$

Matrices $\mathcal{M}$ and $\mathcal{V}$ can change the mass eigenvalues and mixing angles of the leptons because their product $\mathcal{MV}$ is not unitary.

In general, the largest one among the three values of $\tilde{M}_i'$ is of $\mathcal{O}(1) \times \langle \tilde{\nu} \rangle$, where $\langle \tilde{\nu} \rangle$ represents the typical scale of sneutrino VEVs, and a numerical analysis tells us that the other two values cannot be smaller than $\mathcal{O}(10^{-3}) \times \langle \tilde{\nu} \rangle$ in a wide parameter space. The scale of $\langle \tilde{\nu} \rangle$ is comparable to the compactification scale because the flux-induced FI-terms are canceled by the sneutrino VEVs in our model. If $\langle \tilde{\nu} \rangle \sim M_{\text{GUT}} \sim 10^{16}$ GeV, the values of $\tilde{M}_i'$ are realized to be inside $10^{13} \sim 10^{16}$ GeV. In this case, we want also μ_i' ($i = 1, 2, 3$) to be so heavy because the effective Yukawa couplings of left-handed leptons L'' have the following factor:

$$\mu_i' \Big/ \sqrt{(\mu_i')^2 + (\tilde{M}_i')^2}. \quad (22)$$

When $\mu_i' \ll \tilde{M}_i'$, this would lead to an exceeding suppression, which causes some problems, clearly, in the charged-lepton sector.

For this reason, we expect that the matrix μ_{mn} has the typical scale of $\mathcal{O}(M_{\text{GUT}})$, and the rank of μ_{mn} is required to be at least three (the full rank is five). In order to suppress the FCNC due to the extra Higgs multiplets and realize a "natural" SUSY scenarios, one finds that the desirable rank of this matrix is four. We can then identify either $\{H_{u4}', H_{d4}'\}$ or $\{H_{u5}', H_{d5}'\}$ with the MSSM Higgs doublets, because the others must be heavy owing to $\tilde{M}_i'$ discussed above. When the matrix μ_{mn} is rank deficient, we can further infer its structure, because massive linear combinations of H_{um} indicated by the matrix $\tilde{M}_{im}$ must also be mass eigenstates of μ_{mn} with nonvanishing mass eigenvalues. Otherwise, some or all of the left-handed leptons are decoupled from the other MSSM matter fields. This clearly restricts the texture of the matrix μ_{mn}. Although we are studying the simplified situation given in Eq. (20), a similar discussion could be available also in more general cases.

We can adopt an alternative scenario with tiny values of the neutrino Yukawa couplings. It is known that a global suppression factor of Yukawa couplings can be induced in some special SYM systems compactified on magnetized tori, and the suppression can be strong enough to explain the tiny neutrino masses [31]. In the case with the tiny neutrino Yukawa couplings, the mass of $\tilde{M}_{im}L_iH_{um}$, is very light even when $\langle \tilde{\nu} \rangle \sim M_{\text{GUT}}$, because the mass is given by a product of neutrino Yukawa couplings and sneutrino VEVs as shown in Eq. (15). In this case, the mass of $\mu_{mn}H_{um}H_{un}$ dominates $\tilde{M}_i'$, and there is no constraint on μ_{mn} to avoid exceeding suppressions of lepton Yukawa matrices as discussed below Eq. (22). As a result, this alternative scenario permits that the typical scale of μ_{mn} can be much lower than M_{GUT} [but that should be at least $\mathcal{O}(10)$ TeV in order to avoid the dangerous FCNC processes]. This is very different from the previous scenario.

For example, let us consider the case that a suppression factor of $\mathcal{O}(10^{-12})$ is realized for the neutrino Yukawa couplings. The heaviest neutrino (Dirac) mass is then estimated as $m_\nu \sim v_u \times \mathcal{O}(10^{-12}) \sim \mathcal{O}(0.1)$ eV (v_u is the VEV of the up-type Higgs field of the MSSM). We find the three singular values $\tilde{M}_i'$ are roughly of $\mathcal{O}(10 \sim 10^4)$ GeV, which cannot induce the destructive suppression in the factor (22) even when μ_i' is comparable to the electroweak scale. For the case with $\mu_{mn} \gg \mathcal{O}(10)$ TeV, the natural SUSY scenarios require that the rank of μ_{mn} should be four. However, in the case with $\mu_{mn} \sim \mathcal{O}(10)$ TeV, the full rank of matrix μ_{mn} might be consistent with low-scale SUSY breaking scenarios.

C. Mass ratios and mixing angles of quarks and leptons

We have constructed a MSSM-like model with the concrete configuration of magnetic fluxes (12) on the $Z_2 \times Z_2'$ orbifold characterized by the projection operators (13). Finally, we study the masses and mixing angles of quarks and leptons in our model.

The 4D effective Yukawa couplings in magnetized orbifold models can be expressed as linear combinations of η_N defined by

$$\eta_N \equiv \vartheta \begin{bmatrix} N/M \\ 0 \end{bmatrix} (0, \tau M), \quad (23)$$

where M and N are determined by the magnetic fluxes, and the Jacobi-theta function is given by

$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (p, q) = \sum_{\ell \in \mathbb{Z}} e^{\pi i (a+\ell)^2 q} e^{2\pi i (a+\ell)(b+p)}.$$

In our model, the parameter τ in Eq. (23) is identified with the complex structure of the first 2D torus in $(z_1, \bar{z}_1)$ directions where the flavor structure of SM is produced. The parameter M is given by a product of the effective magnetic fluxes felt by the left-handed matter, the right-handed matters, and the Higgs fields on this 2D torus. Specifically, for the up-type quarks, the value of M is $4 \times 8 \times 12 = 384$. It is similarly given as $M = 180, 420$, and 180 for the down-type quarks, neutrinos, and charged leptons, respectively. With these numerical values, certain

TABLE V. The sample theoretical values of the mass ratios of quarks and charged leptons as well as CKM and PMNS mixing matrices are shown. We quote the observed values from Ref. [34].

	Sample Values	Observed
$(m_u/m_t, m_c/m_t)$	$(6.6 \times 10^{-5}, 6.8 \times 10^{-2})$	$(1.3 \times 10^{-5}, 7.4 \times 10^{-3})$
$(m_d/m_b, m_s/m_b)$	$(2.0 \times 10^{-4}, 3.5 \times 10^{-2})$	$(1.1 \times 10^{-3}, 2.3 \times 10^{-2})$
$(m_e/m_\tau, m_\mu/m_\tau)$	$(1.7 \times 10^{-3}, 1.6 \times 10^{-2})$	$(2.9 \times 10^{-4}, 6.0 \times 10^{-2})$
$|V_{\text{CKM}}|$	$\begin{pmatrix} 0.97 & 0.24 & 0.030 \\ 0.23 & 0.95 & 0.22 \\ 0.081 & 0.20 & 0.98 \end{pmatrix}$	$\begin{pmatrix} 0.97 & 0.23 & 0.0035 \\ 0.23 & 0.97 & 0.041 \\ 0.0087 & 0.040 & 1.0 \end{pmatrix}$
$|V_{\text{PMNS}}|$	$\begin{pmatrix} 0.91 & 0.37 & 0.13 \\ 0.32 & 0.90 & 0.29 \\ 0.23 & 0.23 & 0.95 \end{pmatrix}$	$\begin{pmatrix} 0.82 & 0.55 & 0.16 \\ 0.51 & 0.58 & 0.64 \\ 0.26 & 0.61 & 0.75 \end{pmatrix}$

suitable hierarchies for the masses and the mixing angles can be reasonably obtained thanks to the Gaussian profile in η_N [9,19].

The analytic forms of the Yukawa couplings (before the right-handed sneutrinos develop their VEVs) are explicitly shown in the Appendix. These also determine the texture of the lepton number violating mass $\tilde{M}$ (15) as discussed in the previous section. Based on them, we analyze the mass ratios of quarks and charged leptons as well as Cabibbo-Kobayashi-Maskawa (CKM) [32] and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [33] mixing angles but not neutrino mass squared differences here, because the neutrino mass spectrum depends on whether they are Dirac or Majorana.

The numerical analyses are simply performed for some sample values of parameters τ, $\mathcal{M}_{ii}$ and VEVs $\langle \tilde{\nu}_i \rangle$, $v_{um} = \langle H_{um} \rangle$, and $v_{dm} = \langle H_{dm} \rangle$. Note that $\mathcal{M}_{ii}$ is a diagonal entry of matrix $\mathcal{M}$ given in Eq. (21), and can be controlled by $\tilde{\mu}_i$ although the value of $\tilde{M}'_i$ is determined by the other parameters. The VEVs must satisfy the following conditions:

$$\sum_{i=1}^{3} \langle \tilde{\nu}_i \rangle^2 = 1, \qquad \sum_{m=1}^{5} v_{um}^2 = v_u^2, \qquad \sum_{m=1}^{5} v_{dm}^2 = v_d^2,$$

where v_u and v_d are the VEVs of the MSSM Higgs doublets. The first one is required to satisfy the modified SUSY condition (10) on the magnetized background (12). Since our interest here is the mass ratios and mixing angles, the ratios of these VEVs are important. Note also that the effects of renormalization group equations are not included in the analysis.

It is found that the following set

$$\tau = 5i, \quad \langle \tilde{\nu}_3 \rangle = 1, \quad \mathcal{M}_{22}/\mathcal{M}_{11} = 3, \quad \mathcal{M}_{33}/\mathcal{M}_{11} = 25,$$
$$v_{u3}/v_{u5} = 2, \quad v_{u4}/v_{u5} = 4, \quad v_{d3}/v_{d2} = 12, \quad v_{d4}/v_{d2} = 14,$$

and $\langle \tilde{\nu}_1 \rangle = \langle \tilde{\nu}_2 \rangle = v_{u0} = v_{u1} = v_{d0} = v_{d4} = 0$ leads to a hopeful pattern of the mass ratios and the mixing angles as

shown in Table V. Although there are some unacceptable deviations from the observed values especially in the quark sector, we remark that these theoretical values are derived from very limited sample choices of parameters. We expect that a more realistic pattern would be obtained by thorough analyses which remain as future works. It is interesting that the observed Cabibbo angle is obtained even in this simple analysis, which is almost unchanged by the renormalization group equation effects [35].

IV. SUMMARY

We have studied a new class of supersymmetric models on a magnetized orbifold, where the nonvanishing FI-terms are induced by magnetic fluxes in the extra compact space. Scalar fields charged under the fluxed gauge symmetries tend to develop nonvanishing VEVs in the D-flat directions to cancel the FI-terms and SUSY is recovered on such vacua. This idea has broadened the variety of magnetized models. Especially, as a concrete phenomenological example, we have analyzed the case that the fluxed gauge symmetries possessing nonvanishing FI-terms are two $U(1)$ symmetries under which the right-handed neutrinos are charged. In this case, the sneutrino VEVs along the D-flat directions cancel the FI-terms out leading to a new class of SUSY vacua, where all the unwanted chiral exotics and massless adjoint fields, which generically appear in string or string-inspired models, are eliminated completely.

We have also studied the phenomenology of this model focusing on the effects of right-handed sneutrino VEVs which induce a mass term LH_u in the superpotential. It violates the lepton number, and our model does not have the R parity that is usually assumed to suppress the proton decay. Instead, we have found that our model has the Z_3 symmetry, called the baryon triality, which forbids baryon number violating processes and ensures the long lifetime of a proton to be consistent with the nonobservations of its decay. After diagonalizing the whole mass matrices of L, H_u, and H_d, the lepton number violating masses are

eliminated and consequently the lepton flavor structure is modified in this new basis. Such a correction for leptonic Yukawa couplings is determined by the interplay between the lepton number violating mass and the SUSY Higgs mass (so called μ parameters) in the superpotential.

By introducing Higgs VEVs, we have finally performed a rough analysis for parameters which yield semirealistic flavor structures, and shown sample theoretical values of mass ratios and mixing angles of quarks and leptons. Yukawa couplings in certain magnetized orbifold models have a texture which induces suitable hierarchies reasonably and yields a semirealistic pattern of the hierarchies without hierarchical input parameters [19]. The texture is modified in our model due to the lepton number violating mass. Although the rough estimation in Sec. III C shows some deviations from the observed values, we expect that they would be improved by the thorough analyses in future works, where CP-violating phases should also be studied (CP-violating phases of the quark sector in the magnetized orbifold models were recently studied in Ref. [36]).

Accepting flux-induced FI-terms provides a new class of SUSY models in SYM theories compactified on magnetized tori/orbifolds. We expect that the scenario of cancellation between the FI-terms and the VEVs of bifundamental fields can be applied to the other model building for visible (e.g., from other gauge groups [37]), hidden (e.g., SUSY breaking [38]), and moduli stabilization sectors. Furthermore, if the localized fluxes like vortex configurations [21] also contribute to the FI-terms, it might be possible to generate nontrivial wave function profiles of charged fields [39] as in the five-dimensional SUSY [40] and supergravity [41] models. It is also interesting to consider the fluxed $U(1)$ symmetry to be anomalous, and study the combination of flux-induced FI-terms and loop-corrected ones [13] caused by the anomaly.

APPENDIX: YUKAWA COUPLINGS

We show the analytic forms of Yukawa couplings for the model given in Sec. III A, where all the (super)fields Q_i,

$U_j, D_i, L_i, N_j, E_j, H_{um}$, and H_{dm} are in their original basis before the sneutrinos develop VEVs.

For the quark sector, the Yukawa couplings involving Q_i, U_j, and H_{um} are given by

$$
y_{ij1}^u = \begin{pmatrix} y_b & 0 & -y_l \\ 0 & \frac{1}{\sqrt{2}}(y_e - y_i) & 0 \\ -y_f & 0 & y_h \end{pmatrix},
$$

$$
y_{ij2}^u = \begin{pmatrix} 0 & y_c - y_k & 0 \\ \frac{1}{\sqrt{2}}(y_b - y_h) & 0 & \frac{1}{\sqrt{2}}(y_f - y_l) \\ 0 & 0 & 0 \end{pmatrix},
$$

$$
y_{ij3}^u = \begin{pmatrix} -y_j & 0 & y_d \\ 0 & \frac{1}{\sqrt{2}}(y_a - y_m) & 0 \\ y_d & 0 & -y_j \end{pmatrix},
$$

$$
y_{ij4}^u = \begin{pmatrix} 0 & 0 & 0 \\ \frac{1}{\sqrt{2}}(y_f - y_l) & 0 & \frac{1}{\sqrt{2}}(y_b - y_h) \\ 0 & y_c - y_k & 0 \end{pmatrix},
$$

$$
y_{ij5}^u = \begin{pmatrix} y_h & 0 & -y_f \\ 0 & \frac{1}{\sqrt{2}}(y_e - y_i) & 0 \\ -y_l & 0 & y_b \end{pmatrix}, \tag{A1}
$$

where

$$
\begin{aligned}
y_a &= \eta_0 + \eta_{96} + \eta_{192} + \eta_{96}, \\
y_b &= \eta_4 + \eta_{100} + \eta_{188} + \eta_{92}, \\
y_c &= \eta_8 + \eta_{104} + \eta_{184} + \eta_{88}, \\
y_d &= \eta_{12} + \eta_{108} + \eta_{180} + \eta_{84}, \\
y_e &= \eta_{16} + \eta_{112} + \eta_{176} + \eta_{80}, \\
y_f &= \eta_{20} + \eta_{116} + \eta_{172} + \eta_{76}, \\
y_g &= \eta_{24} + \eta_{120} + \eta_{168} + \eta_{72}, \\
y_h &= \eta_{28} + \eta_{124} + \eta_{164} + \eta_{68}, \\
y_i &= \eta_{32} + \eta_{128} + \eta_{160} + \eta_{64}, \\
y_j &= \eta_{36} + \eta_{132} + \eta_{156} + \eta_{60}, \\
y_k &= \eta_{40} + \eta_{136} + \eta_{152} + \eta_{56}, \\
y_l &= \eta_{44} + \eta_{140} + \eta_{148} + \eta_{52}, \\
y_m &= \eta_{48} + \eta_{144} + \eta_{144} + \eta_{48},
\end{aligned}
$$

while those among Q_i, D_j, and H_{dm} are given by

$$y_{ij1}^d = \begin{pmatrix} \eta_0 & \sqrt{2}\eta_{36} & \sqrt{2}\eta_{72} \\ \sqrt{2}\eta_{45} & \eta_9 + \eta_{81} & \eta_{27} + \eta_{63} \\ \eta_{90} & \sqrt{2}\eta_{54} & \sqrt{2}\eta_{18} \end{pmatrix},$$

$$y_{ij2}^d = \begin{pmatrix} \frac{1}{\sqrt{2}}(\eta_{20} + \eta_{40}) & \eta_4 + \eta_{76} & \eta_{32} + \eta_{68} \\ \eta_5 + \eta_{85} & \frac{1}{\sqrt{2}}(\eta_{31} + \eta_{41} + \eta_{49} + \eta_{59}) & \frac{1}{\sqrt{2}}(\eta_{13} + \eta_{23} + \eta_{67} + \eta_{77}) \\ \sqrt{2}\eta_{50} & \eta_{44} + \eta_{64} & \eta_{22} + \eta_{58} \end{pmatrix},$$

$$y_{ij3}^d = \begin{pmatrix} \frac{1}{\sqrt{2}}(\eta_{20} + \eta_{40}) & \eta_{44} + \eta_{64} & \eta_8 + \eta_{28} \\ \eta_{35} + \eta_{55} & \frac{1}{\sqrt{2}}(\eta_1 + \eta_{19} + \eta_{71} + \eta_{89}) & \frac{1}{\sqrt{2}}(\eta_{17} + \eta_{37} + \eta_{53} + \eta_{73}) \\ \sqrt{2}\eta_{10} & \eta_{26} + \eta_{46} & \eta_{62} + \eta_{82} \end{pmatrix},$$

$$y_{ij4}^d = \begin{pmatrix} \frac{1}{\sqrt{2}}(\eta_{60} + \eta_{80}) & \eta_{24} + \eta_{84} & \eta_{12} + \eta_{48} \\ \eta_{15} + \eta_{75} & \frac{1}{\sqrt{2}}(\eta_{21} + \eta_{39} + \eta_{51} + \eta_{69}) & \frac{1}{\sqrt{2}}(\eta_3 + \eta_{33} + \eta_{57} + \eta_{87}) \\ \sqrt{2}\eta_{30} & \eta_6 + \eta_{26} & \eta_{42} + \eta_{78} \end{pmatrix},$$

$$y_{ij5}^d = \begin{pmatrix} \frac{1}{\sqrt{2}}(\eta_{60} + \eta_{80}) & \eta_{16} + \eta_{56} & \eta_{52} + \eta_{88} \\ \eta_{25} + \eta_{65} & \frac{1}{\sqrt{2}}(\eta_{11} + \eta_{29} + \eta_{61} + \eta_{79}) & \frac{1}{\sqrt{2}}(\eta_7 + \eta_{43} + \eta_{47} + \eta_{83}) \\ \sqrt{2}\eta_{70} & \eta_{34} + \eta_{74} & \eta_2 + \eta_{38} \end{pmatrix}. \tag{A2}$$

For the lepton sector, the Yukawa couplings between L_i, N_j, and H_{um} are given by

$$y_{ij1}^\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}(\eta_5 - \eta_{65}) & \sqrt{2}(\eta_{185} - \eta_{115}) & \sqrt{2}(\eta_{55} + \eta_{125}) \\ \eta_{173} - \eta_{103} - \eta_{187} + \eta_{163} & \eta_{67} - \eta_{137} - \eta_{53} + \eta_{17} & \eta_{113} - \eta_{43} - \eta_{127} + \eta_{197} \\ \eta_{79} - \eta_{149} - \eta_{19} + \eta_{89} & \eta_{101} - \eta_{31} - \eta_{199} + \eta_{151} & \eta_{139} - \eta_{209} - \eta_{41} + \eta_{29} \end{pmatrix},$$

$$y_{ij2}^\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}(\eta_{170} - \eta_{110}) & \sqrt{2}(\eta_{10} - \eta_{130}) & \sqrt{2}(\eta_{190} + \eta_{50}) \\ \eta_2 - \eta_{142} - \eta_{58} + \eta_{82} & \eta_{178} - \eta_{38} - \eta_{122} + \eta_{158} & \eta_{62} - \eta_{202} - \eta_{118} + \eta_{22} \\ \eta_{166} - \eta_{26} - \eta_{194} + \eta_{94} & \eta_{74} - \eta_{206} - \eta_{46} + \eta_{94} & \eta_{106} - \eta_{34} - \eta_{134} + \eta_{146} \end{pmatrix},$$

$$y_{ij3}^\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}(\eta_{75} - \eta_{135}) & \sqrt{2}(\eta_{165} - \eta_{45}) & \sqrt{2}(\eta_{15} - \eta_{195}) \\ \eta_{177} - \eta_{33} - \eta_{117} + \eta_{93} & \eta_3 - \eta_{207} - \eta_{123} + \eta_{87} & \eta_{183} - \eta_{27} - \eta_{57} + \eta_{153} \\ \eta_9 - \eta_{201} - \eta_{51} + \eta_{81} & \eta_{171} - \eta_{39} - \eta_{129} + \eta_{81} & \eta_{69} - \eta_{141} - \eta_{111} + \eta_{99} \end{pmatrix},$$

$$y_{ij4}^\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}(\eta_{100} - \eta_{140}) & \sqrt{2}(\eta_{80} - \eta_{200}) & \sqrt{2}(\eta_{160} - \eta_{20}) \\ \eta_{68} - \eta_{208} - \eta_{128} + \eta_{152} & \eta_{172} - \eta_{32} - \eta_{52} + \eta_{88} & \eta_8 - \eta_{148} - \eta_{188} + \eta_{92} \\ \eta_{184} - \eta_{44} - \eta_{124} + \eta_{164} & \eta_4 - \eta_{136} - \eta_{116} + \eta_{164} & \eta_{176} - \eta_{104} - \eta_{64} + \eta_{76} \end{pmatrix},$$

$$y_{ij5}^\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}(\eta_{145} - \eta_{205}) & \sqrt{2}(\eta_{95} - \eta_{25}) & \sqrt{2}(\eta_{85} - \eta_{155}) \\ \eta_{107} - \eta_{37} - \eta_{47} + \eta_{23} & \eta_{73} - \eta_{143} - \eta_{193} + \eta_{157} & \eta_{167} - \eta_{97} - \eta_{13} + \eta_{83} \\ \eta_{61} - \eta_{131} - \eta_{121} + \eta_{11} & \eta_{179} - \eta_{109} - \eta_{59} + \eta_{11} & \eta_1 - \eta_{71} - \eta_{181} + \eta_{169} \end{pmatrix}. \tag{A3}$$

As for the Yukawa couplings y_{ijm}^e involving L_i, E_j, and H_{dm}, the 3×3 matrices y_{ijm}^e for each m are equivalent to the corresponding transposed matrices for the down-type quarks, that is, $y_{ijm}^e = y_{jim}^d$.

[1] G. Aad *et al.* (ATLAS Collaboration), Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys. Lett. B 716, 1 (2012); S. Chatrchyan *et al.* (CMS Collaboration), Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, Phys. Lett. B 716, 30 (2012).

[2] N. Arkani-Hamed and M. Schmaltz, Hierarchies without symmetries from extra dimensions, Phys. Rev. D 61, 033005 (2000).

[3] C. Bachas, A way to break supersymmetry, arXiv:hep-th/9503030.

[4] D. Cremades, L. E. Ibanez, and F. Marchesano, Computing Yukawa couplings from magnetized extra dimensions, J. High Energy Phys. 05 (2004) 079.

[5] H. Abe, T. Kobayashi, H. Ohki, A. Oikawa, and K. Sumita, Phenomenological aspects of 10D SYM theory with magnetized extra dimensions, Nucl. Phys. B870, 30 (2013).

[6] H. Abe, J. Kawamura, and K. Sumita, The Higgs boson mass and SUSY spectra in 10D SYM theory with magnetized extra dimensions, Nucl. Phys. B888, 194 (2014).

[7] H. Abe, K.-S. Choi, T. Kobayashi, and H. Ohki, Non-Abelian discrete flavor symmetries from magnetized/intersecting brane models, Nucl. Phys. B820, 317 (2009); H. Abe, K.-S. Choi, and T. Kobayashi, H. Ohki, Magnetic flux, Wilson line, and orbifold, Phys. Rev. D 80, 126006 (2009); Flavor structure from magnetic fluxes and non-Abelian Wilson lines, Phys. Rev. D 81, 126003 (2010); H. Abe, T. Kobayashi, H. Ohki, K. Sumita, and Y. Tatsuta, Non-Abelian discrete flavor symmetries of 10D SYM theory with magnetized extra dimensions, J. High Energy Phys. 06 (2014) 017.

[8] H. Abe, T. Kobayashi, and H. Ohki, Magnetized orbifold models, J. High Energy Phys. 09 (2008) 043.

[9] H. Abe, K. S. Choi, T. Kobayashi, and H. Ohki, Three generation magnetized orbifold models, Nucl. Phys. B814, 265 (2009).

[10] T. h. Abe, Y. Fujimoto, T. Kobayashi, T. Miura, K. Nishiwaki, and M. Sakamoto, Operator analysis of physical states on magnetized T^2/Z_N orbifolds, Nucl. Phys. B890, 442 (2014); T. h. Abe, Y. Fujimoto, T. Kobayashi, T. Miura, K. Nishiwaki, M. Sakamoto, and Y. Tatsuta, Classification of three-generation models on magnetized orbifolds, Nucl. Phys. B894, 374 (2015); Y. Matsumoto and Y. Sakamura, Yukawa couplings in 6D gauge Higgs unification on T^2/Z_N with magnetic fluxes, Prog. Theor. Exp. Phys. (2016) 053B06; Y. Fujimoto, T. Kobayashi, K. Nishiwaki, M. Sakamoto, and Y. Tatsuta, Comprehensive analysis of Yukawa hierarchies on T^2/Z_N with magnetic fluxes, Phys. Rev. D 94, 035031 (2016).

[11] P. Fayet and J. Iliopoulos, Spontaneously broken super-gauge symmetries and Goldstone spinors, Phys. Lett. 51B, 461 (1974).

[12] H. Abe, T. Kobayashi, H. Ohki, K. Sumita, and Y. Tatsuta, Flavor landscape of 10D SYM theory with magnetized extra dimensions, J. High Energy Phys. 04 (2014) 007.

[13] M. B. Green and J. H. Schwarz, Anomaly cancellation in supersymmetric $D = 10$ gauge theory and superstring theory, Phys. Lett. 149B, 117 (1984).

[14] N. Marcus, A. Sagnotti, and W. Siegel, Ten-dimensional supersymmetric Yang-Mills theory in terms of four-dimensional superfields, Nucl. Phys. B224, 159 (1983).

[15] N. Arkani-Hamed, T. Gregoire, and J. G. Wacker, Higher dimensional supersymmetry in 4D superspace, J. High Energy Phys. 03 (2002) 055.

[16] H. Abe, T. Kobayashi, H. Ohki, and K. Sumita, Superfield description of 10D SYM theory with magnetized extra dimensions, Nucl. Phys. B863, 1 (2012).

[17] This situation may be realized by some non-Abelian gauge backgrounds in the original higher-dimensional field theory, where certain off-diagonal elements of higher-dimensional gauge field have constant backgrounds. In this case, however, such off-diagonal constants could affect the wave function profiles of zero modes. We will not discuss such a case here but consider VEVs in the D-flat directions of 4D effective field theory, minimizing the 4D scalar potential.

[18] An $M = 6$ unit of magnetic flux on twisted orbifolds [20] can also induce three-generation structure.

[19] H. Abe, T. Kobayashi, K. Sumita, and Y. Tatsuta, Gaussian Froggatt-Nielsen mechanism on magnetized orbifolds, Phys. Rev. D 90, 105006 (2014).

[20] T. H. Abe, Y. Fujimoto, T. Kobayashi, T. Miura, K. Nishiwaki, and M. Sakamoto, Z_N twisted orbifold models with magnetic flux, J. High Energy Phys. 01 (2014) 065.

[21] W. Buchmuller, M. Dierigl, F. Ruehle, and J. Schweizer, Chiral fermions and anomaly cancellation on orbifolds with Wilson lines and flux, Phys. Rev. D 92, 105031 (2015).

[22] L. E. Ibanez and G. G. Ross, Discrete gauge symmetries and the origin of baryon and lepton number conservation in supersymmetric versions of the standard model, Nucl. Phys. B368, 3 (1992).

[23] For anomalies of discrete symmetries, see [24,25] and references therein.

[24] T. Araki, T. Kobayashi, J. Kubo, S. Ramos-Sanchez, M. Ratz, and P. K. S. Vaudrevange, (Non-)Abelian discrete anomalies, Nucl. Phys. B805, 124 (2008).

[25] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, and M. Tanimoto, Non-Abelian discrete symmetries in particle physics, Prog. Theor. Phys. Suppl. 183, 1 (2010).

[26] See, e.g., [27–30].

[27] R. Blumenhagen, M. Cvetic, and T. Weigand, Spacetime instanton corrections in 4D string vacua: The Seesaw mechanism for D-Brane models, Nucl. Phys. B771, 113 (2007).

[28] L. E. Ibanez and A. M. Uranga, Neutrino Majorana masses from string theory instanton effects, J. High Energy Phys. 03 (2007) 052.

[29] M. Cvetic, R. Richter, and T. Weigand, Computation of D-brane instanton induced superpotential couplings: Majorana masses from string theory, Phys. Rev. D 76, 086002 (2007).

[30] T. Kobayashi, Y. Tatsuta, and S. Uemura, Majorana neutrino mass structure induced by rigid instantons on toroidal orbifold, Phys. Rev. D 93, 065029 (2016); T. Kobayashi, S. Nagamoto, and S. Uemura, Modular symmetry in magnetized/intersecting D-brane models, arXiv:1608.06129.

[31] K. Sumita, Quasi-localized wave functions on magnetized tori and tiny neutrino Yukawa couplings, J. High Energy Phys. 01 (2016) 067.

[32] M. Kobayashi and T. Maskawa, *CP* violation in the renormalizable theory of weak interaction, Prog. Theor. Phys. **49**, 652 (1973).

[33] B. Pontecorvo, Neutrino experiments and the problem of conservation of leptonic charge, Zh. Eksp. Teor. Fiz. **53**, 1717 (1967) [Sov. Phys. JETP **26**, 984 (1968)]; Z. Maki, M. Nakagawa, and S. Sakata, Remarks on the unified model of elementary particles, Prog. Theor. Phys. **28**, 870 (1962).

[34] J. Beringer *et al.* (Particle Data Group Collaboration), Review of Particle Physics (RPP), Phys. Rev. D **86**, 010001 (2012).

[35] K. Sasaki, Renormalization group equations for the Kobayashi-Maskawa matrix, Z. Phys. C **32**, 149 (1986).

[36] T. Kobayashi, K. Nishiwaki, and Y. Tatsuta, *CP*-violating phase on magnetized toroidal orbifolds, arXiv:1609.08608.

[37] K. S. Choi, T. Kobayashi, R. Maruyama, M. Murata, Y. Nakai, H. Ohki, and M. Sakai, E(6,7,8) magnetized extra dimensional models, Eur. Phys. J. C **67**, 273 (2010); T. Kobayashi, R. Maruyama, M. Murata, H. Ohki, and M. Sakai, Three-generation models from E$_8$ magnetized extra dimensional theory, J. High Energy Phys. 05 (2010) 050;

H. Abe, T. Kobayashi, H. Otsuka, and Y. Takano, Realistic three-generation models from SO(32) heterotic string theory, J. High Energy Phys. 09 (2015) 056.

[38] H. Abe, T. Kobayashi, and K. Sumita, Dynamical supersymmetry breaking on magnetized tori and orbifolds, Nucl. Phys. **B911**, 606 (2016).

[39] H. M. Lee, H. P. Nilles, and M. Zucker, Spontaneous localization of bulk fields: The Six-dimensional case, Nucl. Phys. **B680**, 177 (2004).

[40] S G. Nibbelink, H. P. Nilles, and M. Olechowski, Spontaneous localization of bulk matter fields, Phys. Lett. B **536**, 270 (2002); Instabilities of bulk fields and anomalies on orbifolds, Nucl. Phys. **B640**, 171 (2002); H. Abe, T. Higaki, and T. Kobayashi, Wave function profile and SUSY breaking in 5-D model with Fayet-Iliopoulos term, Prog. Theor. Phys. **109**, 809 (2003).

[41] H. Abe, K. Choi, and I. W. Kim, Fayet-Iliopoulos terms in 5-D orbifold supergravity, J. High Energy Phys. 09 (2004) 001; F. P. Correia, M. G. Schmidt, and Z. Tavartkiladze, (BPS) Fayet-Iliopoulos terms in 5-D orbifold SUGRA, Phys. Lett. B **613**, 83 (2005); H. Abe and K. Choi, Gauged U(1)(R) symmetries and Fayet-Iliopoulos terms in 5-D orbifold supergravity, J. High Energy Phys. 12 (2004) 069.