PHYSICAL REVIEW D **95,** 014031 (2017)

Heavy pentaquark states and a novel color structure

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(Received 12 August 2016; published 30 January 2017)

Encouraged by the observation of the pentaquark states $P_c^+(4380)$ and $P_c^+(4450)$, we propose a novel color flux-tube structure, a pentagonal state, for pentaquark states within the framework of a color flux-tube mode involving a five-body confinement potential. Numerical results on the heavy pentaquark states indicate that the states with three color flux-tube structures, diquark, octet, and pentagonal structures, have the closest masses, which can therefore be called QCD isomers, analogous to isomers in chemistry. The pentagonal structure has the lowest energy. The state $P_c^+(4380)$ can be described as the compact pentaquark state $uudc\bar{c}$ with the pentagonal structure and $J^P = \frac{3}{2}^-$ in the color flux-tube model. The state $P_c^+(4450)$ can not be accommodated into the color flux-tube model. The heavy pentaquark states $uudc\bar{b}$, $uudb\bar{c}$, and $uudb\bar{b}$ are predicted in the color flux-tube model. The five-body confinement potential, based on the color flux-tube picture as a collective degree of freedom, is a dynamical mechanism in the formation of the compact heavy pentaquark states.

DOI: 10.1103/PhysRevD.95.014031

I. INTRODUCTION

In the constituent quark models, baryons and mesons (conventional hadrons) are assumed to be composed of three valence quarks qqq and a valence quark q and a valence antiquark $\bar{q}$, respectively. Quantum chromodynamics (QCD) does not deny the existence of exotic hadrons besides the $q\bar{q}$-meson and qqq-baryon paradigm. Searching for exotic hadrons has been one of the most significant research topics of hadronic physics since the pioneering work by Gell-Mann [1], in which mesons and baryons can also be, respectively, tetraquark and pentaquark states if the excitation of a sea quark pair $q\bar{q}$ is taken into account. Exotic hadrons, if they really exist, may contain more information about the low-energy QCD than that of conventional hadrons. In recent years, a number of experiments have witnessed the proliferation of members of the exotic hadron family. The charged tetraquark states Z_b [2] and Z_c [3], dibaryon resonance state d^* [4], tetraquark state $X(5568)$ [5], and charmonium-pentaquark states $P_c^+(4380)$ and $P_c^+(4450)$ [6] have been giving us a stimulating glance into the abundant multiquark hadronic world and providing an excellent opportunity to explore the fundamental freedom playing an essential role in the multiquark hadron states and hadron-hadron interaction.

The hidden charmed states $P_c^+(4380)$ and $P_c^+(4450)$ were recently reported by LHCb Collaboration in the $J/\psi p$ invariant mass spectrum in the $\Lambda_b^0 \to J/\psi K^- p$ process [6]. Their masses and decay widths from a fit using Breit-Wigner amplitudes are

$$M_{4380} = 4380^{+8+29}_{-8-29}\ \text{MeV}, \qquad \Gamma_{4380} = 205^{+18+86}_{-18-86}\ \text{MeV},$$
$$M_{4450} = 4449.8^{+1.7+2.5}_{-1.7-2.5}\ \text{MeV}, \qquad \Gamma_{4450} = 39^{+5+19}_{-5-19}\text{MeV}.$$

The $J/\psi p$ decay modes of the two P_c^+ states suggest that, regardless of their internal dynamics, they must have a minimum intrinsic quark content $uudc\bar{c}$ with an isospin $I = \frac{1}{2}$. However, their total angular momentum and parity J^P cannot be completely determined up till now, which may be $(\frac{3}{2}^-, \frac{5}{2}^+)$, $(\frac{3}{2}^+, \frac{5}{2}^-)$, or $(\frac{5}{2}^+, \frac{3}{2}^-)$. A large amount of interpretations in different theoretical frameworks have therefore been proposed to reveal the underlying structures of these two pentaquark states so far, such as meson-baryon molecule states [7], diquark-diquark-antiquark states [8], compact and loose diquark-triquark states [9], kinematic effects [10], nucleon-$\psi(2S)$ bound state [11], proton-χ_{c1} state [12], etc. What is eventually the true physical picture of these two pentaquark states? Further experimental and theoretical work are therefore needed to clear the current complicated situation. In addition, the large mass of the pentaquark states $P_c^+(4380)$ and $P_c^+(4450)$ mainly comes from the large masses of the heavy charm quark and antiquark $c\bar{c}$. Consequently, a natural question is that what could be the analogous heavy pentaquark states, such as $uudc\bar{b}$, $uudb\bar{c}$, and $uudb\bar{b}$.

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QCD has been widely accepted as the fundamental theory to describe the interactions among quarks and gluons and the structure of hadrons, in which color confinement is a long distance behavior whose understanding continues to be a challenge in theoretical physics. It is well-known that color flux-tube-like structures emerge by analyzing the chromoelectric fields between static quarks in lattice numerical simulations [13]. Such color flux-tube structures naturally lead to a linear confinement potential between static color charges and to a direct numerical evidence of color confinement [14]. A color flux-tube starts from each quark and ends at an antiquark or a Y-shaped junction, where three flux tubes are either annihilated or created [15]. The color flux-tube structures for mesons and baryons seem to be unique and simple. A quark and an antiquark in mesons are connected through a color flux tube. Three quarks in baryons are connected by a Y-shaped color flux tube into a color singlet. In general, a multiquark state with $N+1$ particles can be generated by replacing a quark or an antiquark in an N-particle state by a Y-shaped junction and two antiquarks or two quarks. In this way, any multiquark state must possess a large number of different topological structures of internal color flux-tube configurations.

It is a well-known fact that the nuclear force in the QCD world and the molecule force in the quantum electrodynamics (QED) world are very similar except for the length and energy scale difference. Furthermore, the color flux tubes in a hadron should also be very analogous to the chemical bond in a molecule. Like the organic world full of variety because of the chemical bonds, i.e., isomers, the multiquark hadron world may be equally or even more diverse due to the color flux-tube structure, which here can be similarly called QCD isomeric compounds. Theoretically, QCD is more complicated than QED so that it is natural to expect that the structures of QCD matters are abundant, even more various than that of QED matters.

In the previous work, we advanced possible color flux-tube structures, so-called QCD quark cyclobutadiene and QCD benzene, for tetra-quark and six-quark states, respectively, within the framework of a color flux-tube model based on the lattice QCD (LQCD) picture and traditional quark models [16,17]. In the paper, we propose and study a novel color flux-tube structure, called a pentagonal state, for the heavy pentaquark states to attempt to enrich the knowledge of the inner structures of multiquark states. In addition, the heavy pentaquark states are also systematically investigated in the color flux-tube model, which may be useful for exploring exotic baryons in future experiments.

This paper is organized as follows: four possible color flux-tube structures for the heavy pentaquark states and the Hamiltonian in the color flux-tube model are given in Sec. II. The numerical calculations and discussions on the heavy petaquark states are presented in Sec. III. A brief summary is given in the last section.

II. COLOR FLUX-TUBE STRUCTURES AND MODEL HAMILTONIAN

Four possible color flux-tube structures of the pentaquark state $uudc\bar{c}$ are presented in Fig. 1, in which q_i stands for a light quark u or d, and the codes of the quarks (antiquarks) q, q, c, q, and $\bar{c}$ are assumed to be 1, 2, 3, 4, and 5, respectively. Their positions are denoted as $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$, $\mathbf{r}_4$, and $\mathbf{r}_5$; $\mathbf{y}_i$ represents the ith Y-shaped junction where three color flux-tubes meet. The color flux-tube structure (**1**) is, a color singlet, a loose baryon-meson molecule state $[qqc]_{\mathbf{1}}[q\bar{c}]_{\mathbf{1}}$; the subscripts represent color dimensions. The pentaquark states $P_c^+(4380)$ and $P_c^+(4450)$ were discussed in this picture due to their proximity to baryon-meson thresholds within different theoretical framework [7]. The color flux-tube structures (**2**), (**3**), and (**4**) are hidden color states. The pentaquark state with the flux-tubes structure (**2**) is a color octet state $[[qqc]_{\mathbf{8}}[q\bar{c}]_{\mathbf{8}}]_{\mathbf{1}}$, which generally has high energies due to a repulsive interaction between the colored subclusters $[qqc]_{\mathbf{8}}$ and $[q\bar{c}]_{\mathbf{8}}$. The color flux-tube structure (**3**) is a diquark-diquark-antiquark state $[[qq]_{\bar{\mathbf{3}}}[cq]_{\bar{\mathbf{3}}}\bar{c}]_{\mathbf{1}}$, which interacts through the color force due to gluon exchange or flavor-dependent force due to meson exchange. The pioneer application of the diquark model applied to explain the structure of the pentaquark Θ^+ was done by Jaffe and Wilczek [18]. The last structure is the so-called pentagonal state, which can be generated by means of exciting two Y-shape junctions and a color flux-tube between c and q_1 or $\bar{c}$ and q_1 from the vacuum based on the second or third structure, respectively. One can suppose that the recombination of color flux-tubes is faster than the motion of the quarks because the quarks in the constituent quark model are massive. Subsequently, the

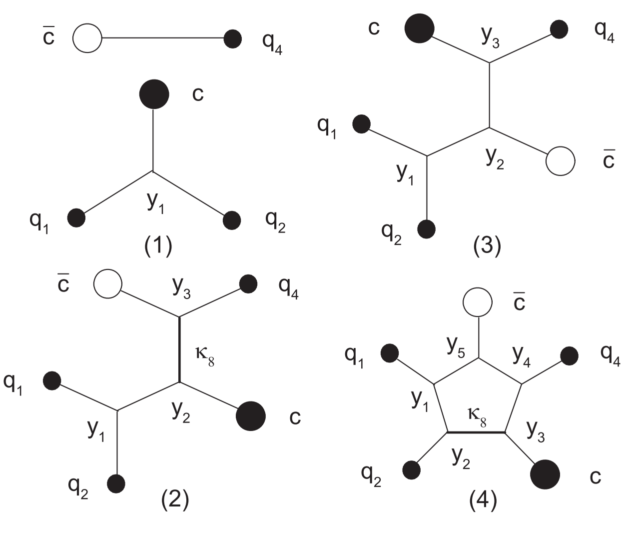

FIG. 1. Four possible color flux-tube structures for the pentaquark state $uudc\bar{c}$.

ends of five compound flux-tubes can meet each other in turn to establish a closed color flux-tube structure, a pentagon-$\mathbf{y}_1\mathbf{y}_3\mathbf{y}_2\mathbf{y}_4\mathbf{y}_5$. According to the overall color singlet and $SU(3)$ color coupling rule, the color flux-tube $\mathbf{y}_2\mathbf{y}_3$ is **8** dimension and the others are **3** or $\bar{\mathbf{3}}$ dimension. It is worth mentioning that the counterpart of the pentagonal state in the QED world, the hydrocarbon C_5H_5 (or generally speaking $C_{2n+1}H_{2n+1}$, $n \in N$) does not seem to exist.

The interactions among quarks is one of the significant quantities for the study of the multiquark system in quark models. LQCD investigations on mesons, baryons, and tetraquark and pentaquark states reveal Y-shaped flux-tube structures [19], which work as a collective degree of freedom connecting all particles to form an overall color singlet hadron. The interactions obey the Coulomb potential plus Y-type linear confinement potential proportional to the minimum of the sum of the lengthen of all color flux tubes [19]. A multiquark color flux-tube model has been developed based on the LQCD picture involving a multibody confinement potential with a harmonic interaction approximation, i.e., a sum of the square of the length of flux tubes rather than a linear one is assumed to simplify the calculation [17,20]. The approximation is justified with the following two reasons: one is that the spatial variations in separation of the quarks (lengths of the flux tube) in different hadrons do not differ significantly, so the difference between the two functional forms is small and can be absorbed in the adjustable parameter, the stiffness of color flux tubes. The other is that we are using a nonrelativistic dynamics in the study. As was shown long ago [21], an interaction energy that varies linearly with separation between fermions in a relativistic first order differential dynamics has a wide region in which a harmonic approximation is valid for the second order (Feynman–Gell-Mann) reduction of the equations of motion. The comparative studies also indicated that the difference between the quadratic confinement potential and the linear one is very small [17,20].

Within the picture of color flux tubes, the quadratic confinement potential is believed to be flavor independent [22–24]. According to the color flux-tube structures of mesons and baryons in Fig. 1 (1), the confinement potential of mesons and baryons in the color flux-tube model can be written as

$$\begin{aligned} V^C_{\min}(2) &= K(\mathbf{r}_4-\mathbf{r}_5)^2, \\ V^C(3) &= K((\mathbf{r}_1-\mathbf{y}_1)^2+(\mathbf{r}_2-\mathbf{y}_1)^2+(\mathbf{r}_3-\mathbf{y}_1)^2). \end{aligned} \tag{1}$$

K is the stiffness of the three-dimension color flux tube. The minimum of the confinement potential of baryons can be obtained by taking the variation of the confinement potential with respect to $\mathbf{y}_1$ and has therefore the following form:

$$V^C_{\min}(3) = K\left(\left(\frac{\mathbf{r}_1-\mathbf{r}_2}{\sqrt{2}}\right)^2+\left(\frac{2\mathbf{r}_3-\mathbf{r}_1-\mathbf{r}_2}{\sqrt{6}}\right)^2\right). \tag{2}$$

The confinement potential $V^C_i(5)$ ($i = 1, 2, 3, 4$) for the pentaquark states $uudc\bar{c}$ with the ith color flux-tube structure listed in Fig. 1 can be expressed as

$$\begin{aligned} V^C_1(5) = K(&(\mathbf{r}_1-\mathbf{y}_1)^2+(\mathbf{r}_2-\mathbf{y}_1)^2+(\mathbf{r}_3-\mathbf{y}_1)^2 \\ &+(\mathbf{r}_4-\mathbf{r}_5)^2), \end{aligned} \tag{3}$$

$$\begin{aligned} V^C_2(5) = K(&(\mathbf{r}_1-\mathbf{y}_1)^2+(\mathbf{r}_2-\mathbf{y}_1)^2+(\mathbf{r}_3-\mathbf{y}_2)^2 \\ &+(\mathbf{r}_4-\mathbf{y}_3)^2+(\mathbf{r}_5-\mathbf{y}_3)^2+(\mathbf{y}_1-\mathbf{y}_2)^2 \\ &+\kappa_8(\mathbf{y}_2-\mathbf{y}_3)^2), \end{aligned} \tag{4}$$

$$\begin{aligned} V^C_3(5) = K(&(\mathbf{r}_1-\mathbf{y}_1)^2+(\mathbf{r}_2-\mathbf{y}_1)^2+(\mathbf{r}_3-\mathbf{y}_2)^2 \\ &+(\mathbf{r}_4-\mathbf{y}_3)^2+(\mathbf{r}_5-\mathbf{y}_3)^2+(\mathbf{y}_1-\mathbf{y}_2)^2 \\ &+(\mathbf{y}_2-\mathbf{y}_3)^2), \\ V^C_4(5) = K&\sum_{i=1}^{5}(\mathbf{r}_i-\mathbf{y}_i)^2+K((\mathbf{y}_1-\mathbf{y}_2)^2+(\mathbf{y}_2-\mathbf{y}_3)^2 \\ &+(\mathbf{r}_3-\mathbf{y}_4)^2+\kappa_8(\mathbf{y}_4-\mathbf{y}_5)^2+(\mathbf{y}_5-\mathbf{y}_1)^2). \end{aligned} \tag{5}$$

The relative stiffness parameter κ_8 of the color **8** dimension flux tube is $\kappa_8 = \frac{C_8}{C_3}$ [25], where C_8 and C_3 are the eigenvalues of the Casimir operator associated with the $SU(3)$ color representation on either end of the color flux tube, namely $C_3 = \frac{4}{3}$ and $C_8 = 3$.

The confinement potential $V^C_i(5)$ can be simplified into the sum of five independent harmonic oscillators by taking the variation with respect to $\mathbf{y}_i$ and then diagonalizing the matrix of the confinement potential. Finally, the confinement potential can be expressed as

$$V^C_i(5) = K\sum_{j=1}^{5} k_{ij}\mathbf{R}^2_{ij}. \tag{6}$$

For the sake of simplicity, the eigenvalue k_{ij} can be written in the form

$$k = \begin{pmatrix} 1 & 1 & 2 & 0 & 0 \\ 0.406 & 1 & 0.820 & 1 & 0 \\ 1 & 0.333 & 0.714 & 1 & 0 \\ 0.580 & 0.783 & 0.638 & 0.862 & 0 \end{pmatrix}. \tag{7}$$

$\mathbf{R}_{ij}$ is an eigenvector corresponding to the eigenvalue k_{ij}. A vector $\mathbf{R}_i$ for the ith color flux-tube structure in Fig. 1 can be constructed as $\mathbf{R}_i = (\mathbf{R}_{i1}\mathbf{R}_{i2}\mathbf{R}_{i3}\mathbf{R}_{i4}\mathbf{R}_{i5})^T$ and $\mathbf{R}_i = \mathbf{M}_i\mathbf{r}$, the vector $\mathbf{r} = (\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3\mathbf{r}_4\mathbf{r}_5)^T$. The ith transformation matrix $\mathbf{M}_i$ has the following forms:

$$\mathbf{M}_1=\begin{pmatrix} 0.707 & -0.707 & 0 & 0 & 0 \\ 0.408 & 0.408 & -0.816 & 0 & 0 \\ 0 & 0 & 0 & 0.707 & -0.707 \\ 0.365 & 0.365 & 0.365 & -0.548 & -0.548 \\ 0.447 & 0.447 & 0.447 & 0.447 & 0.447 \end{pmatrix},$$

$$\mathbf{M}_2=\begin{pmatrix} 0.537 & 0.537 & -0.198 & -0.438 & -0.438 \\ 0.707 & -0.707 & 0 & 0 & 0 \\ 0.107 & 0.107 & -0.872 & 0.329 & 0.329 \\ 0 & 0 & 0 & -0.707 & 0.707 \\ 0.447 & 0.447 & 0.447 & 0.447 & 0.447 \end{pmatrix},$$

$$\mathbf{M}_3=\begin{pmatrix} 0.707 & -0.707 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & -0.5 & -0.5 \\ 0.224 & 0.224 & -0.894 & 0.224 & 0.224 \\ 0 & 0 & 0 & -0.707 & 0.707 \\ 0.447 & 0.447 & 0.447 & 0.447 & 0.447 \end{pmatrix},$$

$$\mathbf{M}_4=\begin{pmatrix} 0.632 & 0.195 & -0.512 & -0.512 & 0.195 \\ 0.632 & -0.512 & 0.195 & 0.195 & -0.512 \\ 0 & 0.688 & 0.162 & -0.162 & -0.688 \\ 0 & 0.162 & -0.688 & 0.688 & -0.162 \\ 0.447 & 0.447 & 0.447 & 0.447 & 0.447 \end{pmatrix}.$$

One-gluon-exchange interaction (coulomb interaction plus color-magnetic interaction) is very important because of the responsibility for the mass splitting in the hadron spectra, it takes the standard form and can be read as [26]

$$V_{ij}^G=\frac{\alpha_s}{4}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\left(\frac{1}{r_{ij}}-\frac{2\pi\delta(\mathbf{r}_{ij})\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j}{3m_im_j}\right), \tag{8}$$

where m_i is the mass of the ith quark (antiquark), the symbols $\boldsymbol{\lambda}$ and $\boldsymbol{\sigma}$ are the color SU(3) Gell-Mann and spin $SU(2)$ Pauli matrices, respectively. The running strong coupling constant α_s takes the following form:

$$\alpha_s(\mu_{ij})=\frac{\alpha_0}{\ln\frac{\mu_{ij}^2}{\Lambda_0^2}}. \tag{9}$$

The function $\delta(\mathbf{r}_{ij})$ should be regularized; the regularization is justified based on the finite size of the constituent quark and should, therefore, be flavor dependent [27],

$$\delta(\mathbf{r}_{ij})=\frac{1}{4\pi r_{ij}r_0^2(\mu_{ij})}e^{-r_{ij}/r_0(\mu_{ij})}, \tag{10}$$

where μ_{ij} is the reduced mass of two interacting particles q_i (or $\bar{q}_i$) and q_j (or $\bar{q}_j$), $r_0(\mu_{ij})=r_0/\mu_{ij}$.

To sum up, the color flux-tube model Hamiltonian H_n for mesons, baryons, and pentaquark states can be universally expressed as,

$$H_n=\sum_{i=1}^n\left(m_i+\frac{\mathbf{p}_i^2}{2m_i}\right)-T_c+\sum_{i>j}^n V_{ij}^G+V_{\min}^C(n). \tag{11}$$

T_c is the center-of-mass kinetic energy of the state, $\mathbf{p}_i$ is the momentum of the ith quark (antiquark), respectively. The tensor and spin-orbit forces between quarks are omitted in the present calculation because, for the lowest energy states which we are interested in here, their contributions are small or zero.

III. NUMERICAL CALCULATIONS AND DISCUSSIONS

The stiffness K of the three-dimension color flux tube is considered as a fixed parameter and taken to be 700 MeV fm^{-2}. The seven adjustable model parameters, m_u, m_s, m_c, m_b, Λ_0, r_0, α_0, and their errors can be fixed by fitting the mass spectra of the ground states of heavy mesons and baryons using the MINUIT program, which are presented in Tables I and II, respectively. The mass spectra can be obtained by solving the two-body and three-body Schrödinger equation

$$(H_n-E_n)\Phi_{IJ}^n=0, \tag{12}$$

with the Rayleigh-Ritz variational principle, where $n=2$ and 3; the details of the construction of the wave functions of baryons and mesons can be found in the papers [22,23]. The mass errors of heavy mesons and baryons ΔE_n introduced by the parameter uncertainty Δx_i can be calculated by the formula of error propagation,

$$\Delta H_n=\sum_{i=1}^7\left|\frac{\partial H_n}{\partial x_i}\right|\Delta x_i, \tag{13}$$

$$\Delta E_n\approx\langle\Phi_{IJ}^n|\Delta H_n|\Phi_{IJ}^n\rangle, \tag{14}$$

where x_i and Δx_i represent the ith adjustable parameter and its error, respectively, which are listed in Table II.

Next, let us discuss the pentaquark states $uudc\bar{c}$ within the framework of diquark-diquark-antiquark $[ud][cd]\bar{c}$. The diquarks $[ud]$ and $[cu]$ are considered as without internal orbital excitation, and the angular excitation L is assumed to occur only between two subclusters $[ud\bar{c}]$ and $[cd]$ if orbital excitation is permitted, which induces the lower

TABLE I. Adjustable parameters in the color flux-tube model. (units: m_u, m_s, m_c, m_b, Λ_0, MeV; r_0, MeV · fm; α_0, dimensionless).

Parameters	x_i	Δx_i	Parameters	x_i	Δx_i
m_u	230.06	0.28530	α_0	4.6945	0.00499
m_s	473.29	0.23195	Λ_0	30.241	0.03927
m_c	1701.3	0.30672	r_0	81.481	0.05267
m_b	5047.0	0.44204			

TABLE II. Ground state heavy-meson and baryon spectra, unit in MeV.

States	$E_2 \pm \Delta E_2$	PDG	States	$E_2 \pm \Delta E_2$	PDG
$D^\pm$	1879 ± 2	1869	D^*	2039 ± 2	2007
$D_s^\pm$	1952 ± 2	1968	D_s^*	2144 ± 2	2112
η_c	2949 ± 3	2980	J/Ψ	3168 ± 2	3097
B^0	5285 ± 2	5280	B^*	5343 ± 2	5325
B_s^0	5352 ± 2	5366	B_s^*	5429 ± 2	5416
B_c	6254 ± 2	6277	B_c^*	6396 ± 2	...
η_b	9374 ± 3	9391	$\Upsilon(1S)$	9536 ± 3	9460

States	$E_3 \pm \Delta E_3$	PDG	States	$E_3 \pm \Delta E_3$	PDG
N	945 ± 4	939	Λ	1128 ± 4	1115
Σ	1204 ± 3	1195	Ξ	1345 ± 3	1315
Δ	1230 ± 3	1232	Σ^*	1391 ± 3	1385
Ξ^*	1537 ± 2	1530	Ω	1677 ± 2	1672
Λ_c^+	2278 ± 4	2285	Σ_c	2437 ± 3	2445
Σ_c^*	2508 ± 3	2520	Ξ_c	2460 ± 3	2466
Ξ_c^*	2626 ± 2	2645	Ω_c^0	2703 ± 2	2695
Ω_c^{0*}	2774 ± 2	2766	Λ_b^0	5596 ± 4	5620
Σ_b	5786 ± 3	5808	Σ_b^*	5812 ± 3	5830
Ξ_b	5765 ± 3	5790	Ξ_b^*	5917 ± 3	...
Ω_b^-	6034 ± 2	6071			

relative kinetic energy between the two subclusters because of the bigger reduced mass. Therefore, the parity of the pentaquark states $uudc\bar{c}$ is $(-1)^{L+1}$. In this way, the wave function of the pentaquark states $uudc\bar{c}$ with quantum numbers IJ^P can be expressed as

$$\Phi_{IJ}^5 = \sum_z c_z [\mathcal{A}_{uud} [\psi_{c_1 s_1 f_1}^{[ud]} \psi_{c_2 s_2 f_2}^{[cu]} \psi_{c_3 s_3 f_3}^{\bar{c}}]_{IS} \psi_{LM}^G]_{IJ}. \quad (15)$$

The intermediate quantum numbers c_i, s_i, and i_i stand for the color, spin, and isospin, respectively, the subscript $i = 1$, 2, and 3. The details of the wave functions ψ_{c_i, s_i, i_i} are omitted here. All [ ]s represents all possible Clebsch-Gordan (C-G) couplings. The c_z is a C-G coefficient, $z = \{c_1, s_1, i_1, c_2, s_2, i_2, c_3, s_3, i_3, I, S, J\}$.

The ψ_{LM}^G is the total spatial wave function of the pentaquark states, in which the part of the identical particles uud are assumed to be symmetrical because we are interested in the low energy states here. In this way, the color-spin-isospin wave functions of the three identical quarks uud should be antisymmetrical due to the Pauli principle, the antisymmetrized operator $\mathcal{A}_{uud} = 1 - P_{14} - P_{24}$, which only operates on color, spin, and isospin parts of the wave function because the orbital part is symmetrical.

In order to obtain the symmetrical spatial wave functions of three identical quarks uud, we can define a set of cyclic Jacobi coordinates $\mathbf{r}_{ij}$, $\mathbf{R}_k$, $\mathbf{T}_{ij}$, and $\mathbf{Q}_{ijk}$ for the cyclic permutations of $(i, j, k) = (1, 2, 4)$,

$$\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j, \qquad \mathbf{R}_k = \mathbf{r}_3 - \mathbf{r}_k, \qquad \mathbf{T}_{ij} = \frac{\mathbf{r}_i + \mathbf{r}_j}{2} - \mathbf{r}_5,$$

$$\mathbf{Q}_{ijk} = \frac{m_u \mathbf{r}_i + m_u \mathbf{r}_j + m_c \mathbf{r}_5}{2m_u + m_c} - \frac{m_c \mathbf{r}_3 + m_u \mathbf{r}_k}{m_u + m_c}. \quad (16)$$

In this way, the total orbital wave function ψ_{LM}^G can be expressed as

$$\psi_{LM}^G = \sum_{i,j,k} \phi_{00}^G(\mathbf{r}_{ij}) \phi_{00}^G(\mathbf{R}_k) \phi_{00}^G(\mathbf{T}_{ij}) \phi_{LM}^G(\mathbf{Q}_{ijk}). \quad (17)$$

The Gaussian expansion method (GEM) has been proven to be rather powerful in solving a few-body problem [28], in which the relative motion wave function $\phi_{lm}^G(\mathbf{x})$ can be expanded as the superposition of many single Gaussian functions with a different size ν_k

$$\phi_{lm}^G(\mathbf{x}) = \sum_{k=1}^{k_{\max}} c_k N_{kl} x^l e^{-\nu_k x^2} Y_{lm}(\hat{\mathbf{x}}). \quad (18)$$

The expansion coefficient c_k can be determined by the dynamics of the pentaquark system. The other details of the wave function $\phi_{lm}^G(\mathbf{x})$ can be found in the paper [28].

The color flux-tube structure specifies how the colors of quarks and antiquarks are coupled to form an overall color singlet. Similarly, the color wave functions of the baryon-meson molecules and color octet states can be constructed in the model study. It is, however, difficult to construct the color wave function of the novel color flux-tube structure, pentagonal state, only using quark degrees of freedom if no explicit gluon is introduced in the quark models. In fact, it is difficult to introduce an explicit gluon degree of freedom in the nonrelativistic quark models because of the zero mass of gluons. Furthermore, the predictive power of quark models will be reduced due to the increase of model parameters even if the constituent gluons can be introduced. The wave function of the pentagonal structure is therefore assumed to be the same as that of the diquark-diquark-antiquark structure to estimate the energy of the pentaquark states with pentagonal structure in the present work.

Subsequently, the color flux-tube model with the model parameters listed in the Table II is extended to study the properties of the heavy pentaquark states. The converged numerical results E_5's can be obtained by solving a five-body Schrödinger equation

$$(H_5 - E_5)\Phi_{IJ}^5 = 0. \quad (19)$$

with the Rayleigh-Ritz variational principle under the conditions of $k_{\max} = 5$, $r_1 = 0.3$ fm, and $r_{k_{\max}} = 2.0$ fm. The error ΔE_5 can be calculated as ΔE_2 and ΔE_3.

The energies $E_5 \pm \Delta E_5$ of the ground states of the heavy pentaquark states $uudc\bar{c}$, $uudc\bar{b}$, $uudb\bar{c}$, and $uudb\bar{b}$ with three different color flux-tube structures, diquark-diquark-antiquark (Diquark), color octet state (Octet), and

TABLE III. The energies $E_5 \pm \Delta E_5$ of the ground states of the heavy pentaquark states $uudc\bar{c}$, $uudb\bar{c}$, $uudc\bar{b}$, and $uudb\bar{b}$ with J^P and three color structures in the color flux-tube model, unit in MeV.

Flavors	J^P	Octet	Diquark	Pentagon	Candidate
	$\frac{1}{2}^-$	4402 ± 5	4344 ± 5	4303 ± 5	...
$uudc\bar{c}$	$\frac{3}{2}^-$	4473 ± 5	4405 ± 5	4369 ± 5	$P_c^+(4380)$
	$\frac{5}{2}^-$	4616 ± 4	4567 ± 4	4516 ± 4	$P_c^+(4450)$?
	$\frac{1}{2}^-$	7612 ± 5	7609 ± 5	7564 ± 5	...
$uudb\bar{c}$	$\frac{3}{2}^-$	7634 ± 5	7631 ± 5	7587 ± 5	...
	$\frac{5}{2}^-$	7812 ± 4	7788 ± 4	7738 ± 4	...
	$\frac{1}{2}^-$	7650 ± 5	7618 ± 5	7573 ± 5	...
$uudc\bar{b}$	$\frac{3}{2}^-$	7702 ± 5	7658 ± 5	7613 ± 5	...
	$\frac{5}{2}^-$	7817 ± 4	7790 ± 4	7740 ± 4	...
	$\frac{1}{2}^-$	10747 ± 5	10616 ± 6	10587 ± 6	...
$uudb\bar{b}$	$\frac{3}{2}^-$	10767 ± 5	10622 ± 5	10592 ± 5	...
	$\frac{5}{2}^-$	10947 ± 5	10935 ± 5	10892 ± 5	...

pentagonal state (Pentagon), under the assumptions of total spin $S = \frac{1}{2}$, $S = \frac{3}{2}$, and $S = \frac{5}{2}$ are systematically calculated and presented in Table III. Their corresponding J^P are therefore $\frac{1}{2}^-$, $\frac{3}{2}^-$, and $\frac{5}{2}^-$ because of $L = 0$. It can be seen from Table III that the energy errors ΔE_5 are very small, just several MeVs. The bigger the angular momentum J, the higher the energy E_5 of the pentaquark states with the same quark content. The energies of the pentaquark states with the same flavor and quantum numbers but three different color flux-tube structure are close, the difference among them mainly comes from the contribution of one-gluon-exchange interaction. The previous investigation on the six-quark state indicated that the energy difference among different color flux-tube structures is very small if one-gluon-exchange interaction is not involved [17]. These different color flux-tube structures with the same flavor can therefore be called QCD isomers analogous to the isomers in the QED world, which have a different chemical bond structure but the same atom constituent. The energy of the pentaquark states with a ringlike color flux-tube structure is lower than that of the state with chainlike structures in the color flux-tube model with quadratic confinement potential because the ringlike structure is easier to shrink into a compact multiquark state than chainlike structures. In this way, the energy of the pentaquark with the pentagonal structure is lower than that of the diquark structure. However, the energy of the color octet state is higher than the diquark structure mainly because of a repulsive one-gluon-exchange interaction in the two colored octet subclusters. In addition, it is worth mentioning that the baryon-meson molecule configuration can not be formed in the color flux-tube model because there does not exist a binding mechanism except a one-gluon-exchange interaction, which is not enough to bind a baryon and a meson into a loose hadron molecule state.

The energy of the state $uudc\bar{c}$ with the pentagonal structure and $J^P = \frac{3}{2}^-$ is 4369 ± 5 MeV in the color flux-tube model, see Table III, which is highly consistent with experimental data of the state $P_c^+(4380)$. It is therefore possible to explain the state $P_c^+(4380)$ as the state $uudc\bar{c}$ with $J^P = \frac{3}{2}^-$, which is supported by a large number of theoretical studies [7–9]. The energy of the state $uudc\bar{c}$ with the pentagonal structure and $J^P = \frac{5}{2}^-$ is 4516 ± 4 MeV in the color flux-tube model, which is a little higher than that of the state $P_c^+(4450)$ and, however, agrees with the conclusions in several researches [29–31]. The energies of the states $uudc\bar{c}$ with positive parity ($L = 1$) and total spin $S = \frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ are, respectively, 4602 ± 5 MeV, 4632 ± 5 MeV, and 4781 ± 4 MeV in the color flux-tube model (the spin-orbit interaction is very weak and therefore not taken into account here [3]), which are much higher than the energies of the two P_c states and close to the prediction on the states in the work [30]. Therefore, these positive parity states should not be the main component of the two P_c^+ states in the color flux-tube model. In this way, the optimum assignment of the main component of the state $P_c^+(4450)$ from the mass seems to be the state $uudc\bar{c}$ with $J^P = \frac{5}{2}^-$ in the color flux-tube model. However, the negative parity is contradictive with the assignment of the opposite parity of the two P_c^+ states reported by the LHCb Collaboration. The state $P_c^+(4450)$ is therefore difficult to be accommodated into the color flux-tube model and worth further research in the future.

The expected lowest energy of the state $uudc\bar{c}$ with the pentagonal structure and $J^P = \frac{1}{2}^-$ is 4303 ± 5 MeV in the color flux-tube model, which is close to the prediction on the state in the work [30]. The energies of the hidden beauty pentaquark states $uudb\bar{b}$ with different quantum numbers and structures are similarly estimated in the color flux-tube model, which are lower than those of the states in the researches [31,32]. The energies of the states $uudb\bar{b}$ in the color flux-tube model should be underestimated mainly because of the strong Coulomb attractive interaction due to the small distance among heavy quarks, the details can be found in our previous work [22]. In addition, the pentaquark states $uudc\bar{b}$ and $uudb\bar{c}$ are also predicted in the color flux-tube model. The pentaquark states $uudc\bar{b}$ and $uudb\bar{c}$ with $J^P = \frac{5}{2}^-$ almost share the same energies. For $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$, the energies of the states $uudc\bar{b}$ is a little higher than those of the states $uudb\bar{c}$.

The main component analysis of the two P_c^+ states is only based on the mass calculation. The crucial test of the main components should be determined by the systematic study of their decays, which involves a channel coupling calculation containing all possible color flux-tube structures and is left for further research in the future. The five-body color flux-tube is a collective degree of freedom, which acts as a dynamical mechanism and plays an important role in the formation and decay of those compact pentaquark states. Different topological structures of color flux tubes induce the diversity of inner color configurations in the pentaquark states. In general, the pentaquark states should be the mixtures of all possible color flux-tube structures, especially within the range of confinement (about 1 fm). These different structures can transform one another, which can be understood here that the gluon field readjusts immediately to its minimal configuration. In this way, the flip flop of color flux-tube structures may induce a color structure resonance, which can be called a color confined, multiquark resonance state [33].

IV. SUMMARY

Within the framework of the color flux-tube model including a five-body confinement potential, a novel color flux-tube structure, pentagonal structure, for pentaquark states, is presented because of the observation of the two P_c^+ states. The pentagonal structure provide a new insight into the inner structure of the pentaquark states. Numerical calculations on the heavy pentaquark states indicate that three color flux-tube structures, diquark, octet, and pentagonal states, have the closest masses. The pentagonal structure has the lowest energy because the ringlike structure is easier to shrink into a compact multiquark state than chainlike structure. These different color flux-tube structures with the same flavor can be called QCD isomers analogous to QED isomers. The five-body confinement potential based on the color flux tube as a collective degree of freedom plays an important role in the formation of those compact heavy pentaquark states.

The main component of the state $P_c^+(4380)$ can be described as a compact pentaquark state $uudc\bar{c}$ with the pentagonal structure and $J^P = \frac{3}{2}^-$ in the color flux-tube model. Although the lowest mass of the state $uudc\bar{c}$ with $J^P = \frac{5}{2}^-$ is not far from the experimental data of the state $P_c^+(4450)$, it should not be a good candidate of the main component of the state $P_c^+(4450)$ because of the same parity with the state $P_c^+(4380)$ in the color flux-tube model. The states $uudc\bar{c}$ with positive parity have masses much higher than those of the states $P_c^+(4380)$ and $P_c^+(4450)$ in the color flux-tube model. It is therefore hard to describe the state $P_c^+(4450)$ in the color flux-tube model. The heavy pentaquark states $uudc\bar{b}$, $uudb\bar{c}$, and $uudb\bar{b}$ are predicted in the color flux-tube model. These mass calculations on the heavy pentaquark states may be useful for planning future experiments and studying manifestly exotic baryon states to complete the picture of exotic baryons.

ACKNOWLEDGMENTS

This research is partly supported by the National Science Foundation of China under Contracts No. 11305274, No. 11175088, No. 11035006, and No. 11205091, and the Chongqing Natural Science Foundation under Project No. cstc2013jcyjA00014.

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