# Flipped versions of the universal 3-3-1 and the left-right symmetric models in $[SU(3)]^3$: A comprehensive approach

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By considering the 3-3-1 and the left-right symmetric models as low-energy effective theories of the $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$ (for short $[SU(3)]^3$) gauge group, alternative versions of these models are found. The new neutral gauge bosons of the universal 3-3-1 model and its flipped versions are presented; also, the left-right symmetric model and its flipped variants are studied. Our analysis shows that there are two flipped versions of the universal 3-3-1 model, with the particularity that both of them have the same weak charges. For the left-right symmetric model, we also found two flipped versions; one of them is new in the literature and, unlike those of the 3-3-1, requires a dedicated study of its electroweak properties. For all the models analyzed, the couplings of the $Z'$ bosons to the standard model fermions are reported. The explicit form of the null space of the vector boson mass matrix for an arbitrary Higgs tensor and gauge group is also presented. In the general framework of the $[SU(3)]^3$ gauge group, and by using the LHC experimental results and EW precision data, limits on the $Z'$ mass and the mixing angle between $Z$ and the new gauge bosons $Z'$ are obtained. The general results call for very small mixing angles in the range $10^{-3}$ radians and $M_{Z'} > 2.5$ TeV.

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## I. INTRODUCTION

The quantization of the electric charge is an indication that the Standard Model (SM) of the strong, weak, and electromagnetic interactions based on the local gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ might be embedded into a larger gauge structure [1,2]. This feature can be explained by grand unified theories (GUT) which, in general, have a unified coupling constant for all the interactions at an energy given by the GUT scale which is around $10^{16}$ GeV for supersymmetric models. One of the most important results of the GUT is the prediction of the neutrino masses in the $(10^{-5}$–$10^{2})$ eV range [3,4], which is compatible with the present constraints on the neutrino masses [5].

In the late 1970s, the unification theories were under suspicion owing to the prediction of topological defects which are typical GUT predictions; from these considerations, the cosmological inflation scenario was born [6], which proved to be quite useful for solving other cosmological problems, showing in this way that the insight provided by GUT is in the right direction. In general, the unification models based on a simple group, in particular the nonsupersymmetric models, lead to a detectable proton decay [3]. However, when the group is the product of two or more simple groups, the structure not necessarily contains gauge bosons that mediate proton decay [1,7,8]. In this context, the trinification group based on the semisimple group $SU(3)_C \times SU(3)_L \times SU(3)_R$[1] [9–12], results quite convenient from a phenomenological point of view owing to the fact that the baryon number is conserved by the gauge interactions [7]. The original $SU(3)_L \times SU(3)_R$ models with a lepton nonet were first considered by *Y.* Achiman [10,11]; however, earlier work on the $[SU(3)]^3$ group can be traced back up to the seminal works in Refs. [13,14]. Besides, this model has been flexible enough to adjust recent LHC anomalies, for example, the di-photon excess at 750 GeV [15] and the di-boson excess at 1.9 TeV [16,17].

The different $[SU(3)]^3$ models have a rich phenomenology in the Higgs and neutrino sectors [9,17,18]; its rank is 6 (equal to $E_6$), hence the model predicts, in addition to those already present in the SM, two additional heavy vector neutral gauge bosons which constitute one of the most important sources of constraints for the model. In this paper, we undertake a detailed study of the couplings of these new gauge bosons to the SM fermions, in order to put electroweak (EW) and collider constraints on $[SU(3)]^3$.

In general, intricate models are not appealing. A way to look for new models with a moderate content of fermions is

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[1]In trinification, the equality of the coupling constants at the unification scale is assumed, which is equivalent to impose an additional discrete $Z_3$ symmetry (see [9] and references therein). In the present work, such assumption has not been made.

 

to consider flipped versions of the already known models in the literature [19–26]. An exhaustive account of the phenomenology of these models has not been done so far. Our work represents a first step in that direction. The first alternative model was "Flipped $SU(5)$" [19,27], which produces a symmetry breaking for $SO(10)$ GUT down to $SU(5) \otimes U(1)$, where the $U(1)$ factor contributes to the electric charge, and as such, its basic predictions for $\sin^2\theta_W$ and the proton decay are known to be different from those of $SU(5)$. In the present work, we study the flipped versions of the universal 3-3-1 and the left-right symmetric models in $[SU(3)]^3$. That is equivalent to the study of the different embeddings of the SM fermions in the multiplets when the $[SU(3)]^3$ gauge group breaks down to the SM. As a consequence of the reduction in the effective group symmetry, these models predict new $Z'$ bosons at low energies. For a given $Z'$ mass, these vector boson resonances have well determined predictions in low-energy experiments and colliders. For universal models in the $E_6$ context, a systematic study of its alternative models and further references can be found in [25].

The heavy vector bosons $Z'$ are a generic prediction of the physics Beyond the SM (BSM) with an extended EW sector [28]. The detection of one of these resonances at the LHC will shed light on the underlying symmetries of the BSM physics. For the high-luminosity regime, the LHC will have sensitivity for $Z'$ masses under 5 TeV [29,30]; thus, a systematic and exhaustive study of the EW extensions of the SM with a minimal content of exotic fields is mandatory. By imposing universality on the EW extensions of the SM (as it happens in the SM), the possible EW extensions are basically the $E_6$ subgroups [31–33]. $[SU(3)]^3$ is one of the four maximal $E_6$ subgroups; so, an exhaustive study of its neutral current structure is convenient, something done in the present work. As we will show, the couplings of additional gauge bosons to the SM fermions are independent of the Higgs sector and just depend on the $[SU(3)]^3$ symmetries. We also present LHC and EW constraints for these models.

Finally, let us mention that unification is not implicit in our assumptions; so, nonuniversal gauge coupling strengths are used in this study.

The paper is organized as follows: in Sec. II, we review the $[SU(3)]^3$ model and its subgroups. In Sec. III, we calculate the EW couplings for $Z'$ bosons in the $[SU(3)]^3$ subgroup $SU(3)_C \otimes SU(3)_L \times U(1) \otimes U(1)'$. In Sec. IV, we calculate the eigenstates of the most general $[SU(3)]^3$ Higgs potential and, for considering different cases, it is shown that these eigenstates are independent of the Higgs sector. It is also shown that the null space of the $[SU(3)]^3$ Higgs potential corresponds to the photon. In Sec. V, we calculate the EW couplings for the left-right model and its alternative models. In Sec. VI, we impose EW and collider constraints on the $Z$-$Z'$ mixing angle and on the mass of the new neutral $Z'$ gauge bosons. Section VII summarizes our conclusions. Four technical appendixes are presented at the end of the manuscript, in particular, in Appendix C the null vector of the EW vector boson mass matrix is built for an arbitrary Higgs tensor and gauge theory.

## II. THE $[SU(3)]^3$ GROUP

The $[SU(3)]^3$ group [9,31,34] $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R \equiv [SU(3)]^3$ is a maximal subgroup of $E_6$ [35] with the same rank and fundamental representation. The three factor groups are identified in the following way: the first one corresponds to the vectorlike QCD color group $SU(3)_C$, the same as in the SM, and the other two can be identified with the left-right symmetric flavor group $SU(3)_L \otimes SU(3)_R$ extension of the $SU(2)_L \otimes SU(2)_R$, where $SU(2)_L$ in the SM is such that $SU(2)_L \subset SU(3)_L$. Using $\lambda_i$, $i = 1, 2, \ldots, 8$ as the eight Gell-Mann matrices for $SU(3)$ normalized as $\mathrm{Tr}(\lambda_i\lambda_j) = 2\delta_{ij}$, the charge operator for the $[SU(3)]^3$ group may be written as

$$Q = \frac{\lambda_{3L}}{2} \oplus \frac{\lambda_{8L}}{2\sqrt{3}} \oplus \frac{\lambda_{3R}}{2} \oplus \frac{\lambda_{8R}}{2\sqrt{3}}. \tag{1}$$

In this way, each family of fermions is assigned to a 27 as[2]

$$27 = (3, 3, 1) \oplus (1, \bar{3}, 3) \oplus (\bar{3}, 1, \bar{3}),$$

where according to (1), the particle content of each term is

$$(3, 3, 1) = (u, d, D)^T_L,$$
$$(\bar{3}, 1, \bar{3}) = (u^c, d^c, D^c)^T_L,$$
$$(1, \bar{3}, 3) = \begin{pmatrix} N^0 & E^- & e^- \\ E^+ & N^{0c} & \nu_e \\ e^+ & \nu^c_e & M^0 \end{pmatrix}_L,$$

which corresponds to the 27 states in the fundamental representation of $E_6$.

### A. 3-3-1 models from $[SU(3)]^3$

Let us now consider the decomposition of the $[SU(3)]^3$ gauge group into a subgroup $G$ which survives at an intermediate energy scale between the EW scale (245 GeV) and the unification scale; that is $[SU(3)]^3 \supset G$.

Suppose first that $G$ corresponds to the universal 3-3-1 model [37]:

[2]Another convention assigns leptons $\sim(1, \bar{3}, 3)$, quarks $\sim(\bar{3}, 3, 1)$ and antiquarks $\sim(3, 1, \bar{3})$, in this case the assignments of the $SU(3)_C$ representation of the quarks are interchanged with respect to the SM. In the present work, we follow the Robinett and Rosner convention [20,36].

$$G = SU(3)_C \otimes SU(3)_L \otimes U(1)_X \subset SU(3)_C \otimes SU(3)_L \otimes U(1)_a \otimes U(1)_b. \quad (2)$$

By using that $SU(3) \to SU(2)_a \otimes U(1)_b$ the triplet in each nonet goes to a doublet with charge $b$ and a singlet with charge $-2b$, i.e., $3 \to 2_b + 1_{-2b}$. Next by breaking the remaining spin symmetry, i.e., $SU(2)_a \to U(1)_a$, the doublet goes to a couple of singlets,i.e., $2_b + 1_{-2b} \to 1_{a,b} + 1_{-a,b} + 1_{0,-2b}$. Thus, when $SU(3)_R$ breaks into $U(1)_a \otimes U(1)_b$, the following branching rule applies,

$$3_R \longrightarrow (a)(b) + (-a)(b) + (0)(-2b), \quad (3)$$

which implies

$$(3,3,1) \longrightarrow (3,3,0,0),$$
$$(\bar{3},1,\bar{3}) \longrightarrow (\bar{3},1,-a,-b) \oplus (\bar{3},1,a,-b) \oplus (\bar{3},1,0,2b),$$
$$(1,\bar{3},3) \longrightarrow (1,\bar{3},a,b) \oplus (1,\bar{3},-a,b) \oplus (1,\bar{3},0,-2b).$$

Since the nonet (3, 3, 1) is simultaneously a color and a $SU(3)_L$ triplet, the unique possibility for the fermion assignment is

$$(3,3,1) \longrightarrow (3,3,0,0) = (u_L, d_L, D_L)^T_0.$$

For the nonet $(\bar{3},1,\bar{3})$, there are three different fermion assignments in consistency with the three different $SU(2)_X$ spin symmetries[3] [36], $X = I$, $U$ and $V$, i.e.,

$$(\bar{3},1,\bar{3}) \longrightarrow (\bar{3},1,-a,-b) \oplus (\bar{3},1,a,-b) \oplus (\bar{3},1,0,2b) = \begin{cases} (d^c_L)_{-a,-b} \oplus (u^c_L)_{a,-b} \oplus (D^c_L)_{0,2b}, & X = I, \\ (D^c_L)_{-a,-b} \oplus (d^c_L)_{a,-b} \oplus (u^c_L)_{0,2b}, & X = U, \\ (u^c_L)_{-a,-b} \oplus (D^c_L)_{a,-b} \oplus (d^c_L)_{0,2b}, & X = V. \end{cases}$$

We label the three possible fermion assignments with $X = I$, $U$, $V$, which denote weak-$I$-spin, weak-$U$-spin and weak-$V$-spin, respectively. As can be seen, the $[SU(3)]^3$ gauge group produces three different low-energy 3-3-1 fermion structures: the ordinary one presented in Ref. [37] and two more new in the literature as far as we know.

In a corresponding way, there are three different fermion assignments for the nonet $(1,\bar{3},3)$, i.e.,

$$(1,\bar{3},3) \longrightarrow (1,\bar{3},a,b) \oplus (1,\bar{3},-a,b) \oplus (1,\bar{3},0,-2b) = \begin{cases} (E^-_L, N^{0c}_L, \nu^c_{eL})^T_{a,b} \oplus (N^0_L, E^+_L, e^+_L)^T_{-a,b} \\ \quad \oplus (e^-_L, \nu_{eL}, M^0_L)^T_{0,-2b}, \quad X = I, \\ (e^-_L, \nu_{eL}, M^0_L)^T_{a,b} \oplus (E^-_L, N^{0c}_L, \nu^c_{eL})^T_{-a,b} \\ \quad \oplus (N^0_L, E^+_L, e^+_L)^T_{0,-2b}, \quad X = U, \\ (N^0_L, E^+_L, e^+_L)^T_{a,b} \oplus (e^-_L, \nu_{eL}, M^0_L)^T_{-a,b} \\ \quad \oplus (E^-_L, N^{0c}_L, \nu^c_{eL})^T_{0,-2b}, \quad X = V. \end{cases}$$

In correspondence with Eq. (1), the electric charge is now given by

$$Q = I_{L3} + \frac{1}{\sqrt{3}} I_{L8} + c_X X_{R3} + \frac{2 d_X}{\sqrt{3}} X_{R8}, \quad (4)$$

where $X_{R3}$ and $X_{R8}$ are the fermion charges under $U(1)_a$ and $U(1)_b$, respectively, as is shown in Table I, and $c_X$ and $d_X$ are

$$c_I = 1, \quad d_I = 1/2,$$
$$c_U = 0, \quad d_U = -1,$$
$$c_V = -1, \quad d_V = 1/2,$$

where we have taken $b = 1/(2\sqrt{3})$ and $a = 1/2$ in order to have the charges properly normalized as in $E_6$. In Eq. (4), $I_{L3}$ and $I_{L8}$ represent the charges of the fermions in the 27, when these operators act on the triplets; in the nonets the corresponding tridimensional representation are $\lambda_{3L}/2$ and $\lambda_{8L}/2$, respectively [see Eq. (1)]. In the same vein in Eq. (4) with $X = I$, the charges $I_{R3}$ and $I_{R8}$ correspond to $\lambda_{3R}/2$ and $\lambda_{8R}/2$, respectively. The difference between the weak-$U$-spin and the alternative 3-3-1 models (the normal and the flipped one) is the interchange of fermions between the multiplets, something which does not affect the low-energy phenomenology for the neutral sector as we will see in the next section.

### B. Left-right symmetric models from $[SU(3)]^3$

A further step is to take $G = SU(3)_C \otimes SU(2)_L \otimes SU(2)_X \otimes U(1)_f \otimes U(1)_g$, which is obtained by using the branching rule for $SU(3)_{L,R} \longrightarrow SU(2)_{L,X} \otimes U(1)_{f,g}$ as

$$3_L \longrightarrow (2,f) + (1,-2f), \qquad 3_R \longrightarrow (2,g) + (1,-2g),$$

which produces three different ways to reach the $U(1)_Y$ in the SM:

[3] In Appendix A, we briefly review the $SU(2)$ weak-I-spin (or Isospin), weak-$U$-spin and weak-$V$-spin symmetries in $SU(3)$.

TABLE I. Charge assignments for the fundamental representation of the $[SU(3)]^3$ group, the same 27 of the $E_6$ group, under different $U(1)$ symmetries. For the first family, $l$ is the SM lepton doublet, $l^T = (\nu, e^-)$ and $q = (u, d)^T$ is the SM quark doublet. The charge conjugated of the corresponding right-handed weak-isospin singlets are $e^c$, $\nu^c$, $u^c$ and $d^c$. The heavy exotic particles are vector under the SM group, the heavy down quark, $D$ ($D^c$), is an weak-isospin singlet (charge conjugated of the corresponding right-handed chiral projection) of charge $-1/3$ ($+1/3$), $L = (N^0, E^-)^T$ and $L = (E^c, N^{0c})^T$, are additional weak-isospin doublets where $L$ have the same quantum numbers of the SM lepton doublet, and $M^0$ is a singlet under the SM.

| $U(1)'$ [25] | $Q'$ | Charges |
|---|---|---|
| $U_R$ | $2I_{R3}$ | $(e^+, d^c, \bar{L})_{+1} + (l, q, D, D^c, M^0)_0 + (\nu_e^c, u^c, L)_{-1}$ |
| $U_I$ | $2U_{R3}$ | $(\nu_e^c, D^c, L)_{+1} + (\bar{L}, q, u^c, D, e^+)_0 + (M^0, d^c, l)_{-1}$ |
| $U_A$ | $2V_{R3}$ | $(M^0, u^c, l)_{+1} + (L, q, d^c, D, \nu_e^c)_0 + (e^+, D^c, \bar{L})_{-1}$ |
| $U_{33}$ | $2\sqrt{3}I_{L8}$ | $(l, \bar{L}, L)_{-1} + (u^c, d^c, D^c)_0 + (e^+, \nu_e^c, M^0)_{+2} + q_{+1} + D_{-2}$ |
| $U_{21\bar{R}}$ | $-2\sqrt{3}I_{R8}$ | $(e^+, \nu_e^c, \bar{L}, L)_{-1} + (q, D)_0 + (l, M^0)_{+2} + (u^c, d^c)_{+1} + D_{-2}^c$ |
| $U_{21\bar{I}}$ | $-2\sqrt{3}U_{R8}$ | $(M^0, \nu_e^c, l, L)_{-1} + (q, D)_0 + (\bar{L}, e^+)_{+2} + (D^c, d^c)_{+1} + u_{-2}^c$ |
| $U_{21\bar{A}}$ | $-2\sqrt{3}V_{R8}$ | $(M^0, e^+, l, \bar{L})_{-1} + (q, D)_0 + (L, \nu_e^c)_{+2} + (D^c, u^c)_{+1} + d_{-2}^c$ |
| $U(1)_{31R}$ | $I_{BL}$ | $(\bar{L}, L, M^0)_0 + q_{+1/6} + (u^c, d^c)_{-1/6} + (e^+, \nu_e^c)_{+1/2} + l_{-1/2} + D_{+1/3}^c + D_{-1/3}$ |
| $U(1)_{31I}$ | $U_{BL}$ | $(l, L, e^+)_0 + q_{+1/6} + (D^c, d^c)_{-1/6} + (M^0, \nu_e^c)_{+1/2} + \bar{L}_{-1/2} + u_{+1/3}^c + D_{-1/3}$ |
| $U(1)_{31A}$ | $V_{BL}$ | $(l, \bar{L}, \nu_e^c)_0 + q_{+1/6} + (D^c, u^c)_{-1/6} + (M^0, e^+)_{+1/2} + L_{-1/2} + d_{+1/3}^c + D_{-1/3}$ |

$$
\begin{aligned}
(3,3,1) &\longrightarrow (3,2,1,f,0) \oplus (3,1,1,-2f,0),\\
(\bar{3},1,\bar{3}) &\longrightarrow (\bar{3},1,\bar{2},0,-g)) \oplus (\bar{3},1,1,0,2g),\\
(1,\bar{3},3) &\longrightarrow (1,\bar{2},2,-f,g) \oplus (1,\bar{2},1,-f,-2g)\\
&\quad \oplus (1,1,2,2f,g) \oplus (1,1,1,2f,-2g).
\end{aligned}
$$

The underlying breaking behind these branching rules are

$$
\begin{aligned}
(3,3,1) &\longrightarrow (3,2_f,1_0) \oplus (3,1_{-2f},1_0),\\
(\bar{3},1,\bar{3}) &\longrightarrow (\bar{3},1_0,\bar{2}_{-g}) \oplus (\bar{3},1_0,1_{2g}),\\
(1,\bar{3},3) &\longrightarrow (1,\bar{2}_{-f},2_g) \oplus (1,\bar{2}_{-f},1_{-2g})\\
&\quad \oplus (1,1_{2f},2_g) \oplus (1,1_{-2f},1_{2g}).
\end{aligned}
$$

Now the definition of $U(1)_{\mathrm{BLX}} \equiv U(1)_f + U(1)_g$ for $f = g = 1/6$ conducts to the alternative left-right symmetric models,

$$SU(3)_C \otimes SU(2)_L \otimes SU(2)_X \otimes U(1)_{\mathrm{BLX}},$$

with the following particle content for the quark sector:

$$
\begin{aligned}
(3,3,1) &= (u,d,D)_L \longrightarrow (3,2,1,1/6) \oplus (3,1,1,-1/3)\\
&= (u,d)_L \oplus D_L,\\
(\bar{3},1,\bar{3}) &= (u^c,d^c,D^c)_L \longrightarrow (\bar{3},1,\bar{2},-1/6) \oplus (\bar{3},1,1,1/3)\\
&= \begin{cases} (u^c,d^c)_L \oplus D_L^c, & X = I,\\ (D^c,d^c)_L \oplus u_L^c, & X = U,\\ (u^c,D^c)_L \oplus d_L^c, & X = V. \end{cases}
\end{aligned}
$$

For the lepton sector, we have

$$
\begin{aligned}
(1,\bar{3},3) &= \begin{pmatrix} N^0 & E^- & e^- \\ E^+ & N^{0c} & \nu_e \\ e^+ & \nu_e^c & M^0 \end{pmatrix}_L\\
&\longrightarrow (1,\bar{2},2,0) \oplus (1,\bar{2},1,-1/2) \oplus (1,1,2,1/2)\\
&\quad \oplus (1,1,1,0)\\
&= \begin{cases} \begin{pmatrix} E^+ & N^{0c} \\ N^0 & E^- \end{pmatrix}_L \oplus \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \oplus (e^+,\nu_e^c)_L\\ \qquad \oplus M_L^0, \quad X = I,\\ \begin{pmatrix} \nu_e & N^{0c} \\ e^- & E^- \end{pmatrix}_L \oplus \begin{pmatrix} E^+ \\ N^0 \end{pmatrix}_L \oplus (M_L^0,\nu_e^c)_L\\ \qquad \oplus e^+, \quad X = U,\\ \begin{pmatrix} E^+ & \nu_e \\ N^0 & e^- \end{pmatrix}_L \oplus \begin{pmatrix} N^{0c} \\ E^- \end{pmatrix}_L \oplus (M_L^0,e^+)_L\\ \qquad \oplus \nu_e^c, \quad X = V. \end{cases}
\end{aligned}
$$

In the left-right model $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \subset SU(3)_L \otimes SU(3)_R$ (in our notation $SU(2)_L \otimes SU(2)_I \otimes U(1)_{BLI}$), the weak-isospin subgroup ($X = I$) has been used. That is the correct choice for the left-handed sector, but not the only choice for the right-handed one as we have shown already. The weak-$V$-spin symmetric case is a very well known example where $SU(2)_V$ is used instead of $SU(2)_R$; this model is known as the alternative

left-right[4](ALR), which was found in a different way in Ref. [22]. The case $X = U$ is a new alternative model.

## III. 3-3-1 NEUTRAL CURRENTS

For the $[SU(3)]^3$ group, the interaction Lagrangian $-\mathcal{L}_I$ is

$$g_L J^I_{L3\mu} A^{I\mu}_{L3} + g_L J^I_{L8\mu} A^{I\mu}_{L8} + g_R J^X_{R3\mu} A^{X\mu}_{R3} + g_R J^X_{R8\mu} A^{X\mu}_{R8} = g_L J^I_{L3\mu} A^{I\mu}_{L3} + g' J_{Y\mu} B^{\mu} + g_2 J_{2\mu} Z'^{\mu} + g_3 J_{3\mu} Z''^{\mu}, \quad (5)$$

where $A^I_{L3\mu}$, $A^I_{L8\mu}$, $A^X_{R3\mu}$ and $A^X_{R8\mu}$ are the corresponding vector gauge bosons associated with $\lambda^I_{L3}$, $\lambda^I_{L8}$, $\lambda^X_{R3}$ and $\lambda^X_{R8}$, respectively (for a precise definition, see Appendix B). The neutral currents in (5) are given by

$$J^X_{R8\mu} = \sum_i \bar{f}_i \gamma_\mu [\epsilon^{X_{R8}}_{\mathbf{L}}(i) P_{\mathbf{L}} + \epsilon^{X_{R8}}_{\mathbf{R}}(i) P_{\mathbf{R}}] f_i, \quad (6)$$

$$J^X_{R3\mu} = \sum_i \bar{f}_i \gamma_\mu [\epsilon^{X_{R3}}_{\mathbf{L}}(i) P_{\mathbf{L}} + \epsilon^{X_{R3}}_{\mathbf{R}}(i) P_{\mathbf{R}}] f_i, \quad (7)$$

where the chiral charges, $\epsilon_{\mathbf{L},\mathbf{R}}$, are shown in Table AII in Appendix D 1. Notice in our notation that the bold labels $\mathbf{L}$, $\mathbf{R}$ refer to the left and right chiral projections and $L$ and $R$ refer to different $SU(n)$ group structures. By means of an orthogonal matrix we can rotate from the $[SU(3)]^3$ basis of the neutral vector bosons, to a basis where one boson corresponds to the hypercharge, i.e.,

$$\begin{pmatrix} A^I_{L3\mu} \\ B_\mu \\ Z'_\mu \\ Z''_\mu \end{pmatrix} = \mathcal{O}^T \begin{pmatrix} A^I_{L3\mu} \\ A^I_{L8\mu} \\ A^X_{R8\mu} \\ A^X_{R3\mu} \end{pmatrix}, \quad (8)$$

where the orthogonal matrix is

$$\mathcal{O} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\beta & -\sin\beta \\ 0 & 0 & \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (9)$$

It is important to realize that in order to recover the particular case $X = U$, corresponding to the 3-3-1 models, it is necessary to take $\cos\beta = -1$. By replacing this expression in Eq. (5), we obtain

$$g' B_\mu J^\mu_Y = B^\mu (g_L J^I_{L8\mu} \cos\alpha + g_R J^X_{R8\mu} \sin\alpha\cos\beta + g_R J^X_{R3\mu} \sin\alpha\sin\beta); \quad (10)$$

by equating with

$$J_{Y\mu} = \frac{1}{\sqrt{3}} J^I_{L8\mu} + c_X J^X_{R3\mu} + \frac{2d_X}{\sqrt{3}} J^X_{R8\mu}, \quad (11)$$

we get the following three equations:

$$\frac{1}{\sqrt{3}} g' = g_L \cos\alpha, \qquad \frac{2d_X}{\sqrt{3}} g' = g_R \sin\alpha\cos\beta, \quad (12)$$

$$c_X g' = g_R \sin\alpha\sin\beta. \quad (13)$$

From these equations, we have

$$g_R = \sqrt{\frac{N}{F}} g', \qquad \cos\alpha = \frac{g'}{\sqrt{3} g_L}, \qquad \cos\beta = \frac{2d_X}{\sqrt{N}} = d_X, \quad (14)$$

where $N = (3c_X^2 + 4d_X^2) = 4$ and $F = 3 - (g'/g_L)^2$. It is worth noticing that in the three cases considered, i.e., for any value of $X$,

$$g_R = \frac{2 g_L g'}{\sqrt{3 g_L^2 - g'^2}} = \frac{2 g_L \sin\theta_W}{\sqrt{4\cos^2\theta_W - 1}}. \quad (15)$$

From Eqs. (5) and (8), it is possible to get expressions for the neutral currents associated with the $Z'$ and $Z''$ bosons, respectively,

$$g_2 J_{2\mu} = -g_L J^I_{L8\mu} \sin\alpha + g_R J^X_{R8\mu} \cos\alpha\cos\beta + g_R J^X_{R3\mu} \cos\alpha\sin\beta,$$
$$g_3 J_{3\mu} = -g_R J^X_{R8\mu} \sin\beta + g_R J^X_{R3\mu} \cos\beta. \quad (16)$$

From these relations and from Table AII, we can obtain the explicit expressions of the vector and axial charges for the $Z'$ and $Z''$ gauge bosons; these charges are shown in Tables AIV and AV, respectively. The collider and EW constraints are shown in Table II and Fig. 1. A detailed analysis of these constraints is presented in Sec. VI. Finally, we can make use of the defining condition of the orthogonal matrices, $\mathcal{O}^{-1} = \mathcal{O}^T$, and use the matrix (8) to rotate from the $[SU(3)]^3$ basis for the neutral vector bosons to the SM basis, i.e.,

[4]Or alternate left-right model.

TABLE II. 95% C.L. lower mass limits on extra $Z'$ bosons for various models from EW precision data and constraints on $\sin\theta_{ZZ'}$. For comparison, we show in the second column the 95% LHC constraints [39] which have been calculated according to Ref. [29]. In the following columns, we give, respectively, the central value and the 95% C.L. lower and upper limits for $\sin\theta_{ZZ'}$.

| | $M_{Z'}$ [GeV] | | $\sin\theta_{ZZ'}$ | | |
|---|---|---|---|---|---|
| $Z'$ | LHC | EW | $\sin\theta_{ZZ'}$ | $\sin\theta^{\min}_{ZZ'}$ | $\sin\theta^{\max}_{ZZ'}$ |
| $Z_{331G}$ | 2925 | 958 | −0.00007 | −0.0012 | 0.0009 |
| $Z_I^{\mathrm{Tri}}$ | 2492 | 1134 | 0.0003 | −0.0006 | 0.0013 |
| $Z_I$ | 2525 | 1,204 | 0.0003 | −0.0005 | 0.0012 |
| $Z_{LR}^{\mathrm{Tri}}$ | 2693 | 1182 | −0.0004 | −0.0015 | 0.0006 |
| $Z_{LR}$ | 2682 | 998 | −0.0004 | −0.0013 | 0.0006 |
| $Z_{\mathrm{LRU}}$ | 2.588 | 935 | −0.00001 | −0.0011 | 0.0008 |
| $Z_{\mathrm{ALR}}^{\mathrm{Tri}}$ | 2532 | 447 | −0.0004 | −0.0014 | 0.0007 |

$$\begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \\ Z''_\mu \end{pmatrix} = \mathcal{W}\cdot\mathcal{O}^T \begin{pmatrix} A^I_{L3\mu} \\ A^I_{L8\mu} \\ A^X_{R8\mu} \\ A^X_{R3\mu} \end{pmatrix} = \begin{pmatrix} \sin\theta_W & \cos\theta_W & 0 & 0 \\ \cos\theta_W & -\sin\theta_W & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \mathcal{O}^T \begin{pmatrix} A^I_{L3\mu} \\ A^I_{L8\mu} \\ A^X_{R8\mu} \\ A^X_{R3\mu} \end{pmatrix}, \tag{17}$$

where $\mathcal{W}$ and $\theta_W$ are the Weinberg matrix and the Weinberg angle, respectively.

## IV. EIGENSTATES OF THE VECTOR BOSON MASS MATRIX IN $[SU(3)]^3$

In the last section, we saw that it is possible to obtain the SM fields $A_\mu$ and $Z_\mu$ and the extra neutral vector bosons $Z'_\mu$ and $Z''_\mu$ by rotating the $[SU(3)]^3$ basis for the vector fields. By making use of some viable cases for the Higgs potential in the present section, we will show that, independent of the Higgs sector, the null space of the vector boson mass matrix corresponds to the photon, i.e., by rotating the photon component $(A_\mu, 0, 0, 0)^T$ in the SM basis to the $[SU(3)]^3$ basis, we obtain the null space of the vector boson mass matrix. This is a particular example of a more general theorem which is shown in Appendix C. In that sense, the present section is useful to provide a context for this demonstration. The same is not true for the eigenvalues of the vector mass matrix which strongly depend on the Higgs sector. In the fundamental representation of the $[SU(3)]^3$ group, the neutral components are in the leptonic sector $(1, \bar{3}, 3)$; if we put the Higgs field $\Phi$ in the same representation the corresponding transformation properties are

$$\Phi' = U_L \Phi U_R^\dagger, \qquad U_{L,R} = \exp(-i\theta^a(x)\lambda^a_{L,R}/2). \tag{18}$$

Requiring gauge invariance, the covariant derivative is

$$D_\mu\Phi = \partial_\mu\Phi - \frac{i}{2}(g_L\lambda^a A^a_{\mu L}\Phi - g_R\Phi\lambda^a A^a_{\mu R}), \tag{19}$$

which transforms in the same way as the Higgs fields, i.e.,

$$(D_\mu\Phi)' = U_L D_\mu \Phi U_R^\dagger, \tag{20}$$

as it is required to build the gauge invariant kinetic term. The Higgs sector of the $[SU(3)]^3$ model contains two complex scalar field nonets, $\Phi_1$ and $\Phi_2$. The most general vacuum expectation value (VEV) for these fields are [18]

$$\langle\Phi_1\rangle = \begin{pmatrix} v_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & M_I \end{pmatrix}, \quad \langle\Phi_2\rangle = \begin{pmatrix} v_2 & 0 & 0 \\ 0 & b_2 & b_3 \\ 0 & M_R & M_3 \end{pmatrix}, \tag{21}$$

where $\Phi_1$ is diagonal in view of the fact that it is always possible to bring one Higgs VEV into its diagonal form by using the $SU(3)_L \times SU(3)_R$ symmetry. The vector boson masses came from

$$\mathcal{L}_K = +\sum_{i=1,2} \mathrm{Tr}[D_\mu\Phi_i(D^\mu\Phi_i)^\dagger]|_{\Phi_i=\langle\Phi_i\rangle}, \tag{22}$$

which is invariant under the already mentioned gauge transformations [Eqs. (18) and (20)]. We can get rid of the kinetic mixing term $\mathrm{Tr}[D_\mu\Phi_1(D^\mu\Phi_2)^\dagger]$ by redefining the scalar fields in order to cast the Lagrangian into the canonical form. By a rotation in the adjoint representation, we obtain the simplified expression

$$\begin{aligned}\mathcal{L}_K(\langle\Phi_1\rangle,\langle\Phi_2\rangle) = \frac{1}{3}\big(&(g_L A^V_{L8\mu} + g_R A^I_{R8\mu})^2 b_3^2 \\ &+ (g_R A^V_{R8\mu} + g_L A^I_{L8\mu})^2 M_R^2 \\ &+ (g_L A^V_{L8\mu} - g_R A^V_{R8\mu})^2 (b_1^2 + b_2^2) \\ &+ (g_L A^I_{L8\mu} - g_R A^I_{R8\mu})^2 (M_3^2 + M_I^2) \\ &+ (g_L A^U_{L8\mu} - g_R A^U_{R8\mu})^2 (v_1^2 + v_2^2)\big),\end{aligned} \tag{23}$$

where $A^V_{(L,R)8\mu} = -(A^I_{(L,R)8\mu} - \sqrt{3}A^I_{(L,R)3\mu})/2$ and $A^U_{(L,R)8\mu} = -(A^I_{(L,R)8\mu} + \sqrt{3}A^I_{(L,R)3\mu})/2$. By writing the kinetic part in terms of $A^I_{R8\mu}$ and $A^I_{R3\mu}$, the Higgs covariant derivative can be written as

$$\mathcal{L}_K = \frac{1}{2}\mathcal{A}^T\cdot\mathcal{M}\cdot\mathcal{A}, \tag{24}$$

where $\mathcal{A}=(A^I_{L3\mu}A^I_{L8\mu}A^I_{R8\mu}A^I_{R3\mu})^T$, and $\mathcal{M}$ is the gauge boson mass matrix whose elements are given by

$$
\begin{aligned}
M_{11} &= \frac{2}{4}g_L^2(b_1^2+b_2^2+b_3^2+v_1^2+v_2^2),\\
M_{12} &= \frac{-2}{4\sqrt{3}}g_L^2(b_1^2+b_2^2+b_3^2-v_1^2-v_2^2),\\
M_{13} &= \frac{2}{4\sqrt{3}}g_Lg_R(b_1^2+b_2^2-2b_3^2-v_1^2-v_2^2),\\
M_{14} &= \frac{-2}{4}g_Lg_R(b_1^2+b_2^2+v_1^2+v_2^2),\\
M_{24} &= \frac{2}{4\sqrt{3}}g_Lg_R(b_1^2+b_2^2-2M_R^2-v_1^2-v_2^2),\\
M_{34} &= \frac{-2}{4\sqrt{3}}g_R^2(b_1^2+b_2^2+M_R^2-v_1^2-v_2^2),\\
M_{22} &= \frac{2}{12}g_L^2(b_1^2+b_2^2+b_3^2+4M_3^2+4M_I^2\\
&\quad +4M_R^2+v_1^2+v_2^2),\\
M_{33} &= \frac{2}{12}g_R^2(b_1^2+b_2^2+4b_3^2+4M_3^2+4M_I^2\\
&\quad +M_R^2+v_1^2+v_2^2),\\
M_{23} &= \frac{-2}{12}g_Lg_R(b_1^2+b_2^2-2b_3^2+4M_3^2+4M_I^2\\
&\quad -2M_R^2+v_1^2+v_2^2),\\
M_{44} &= \frac{2}{4}g_R^2(b_1^2+b_2^2+M_R^2+v_1^2+v_2^2).
\end{aligned}
$$

The null space of the mass matrix $\mathcal{M}$ is

$$
\mathcal{A}^\mu_{\text{null}}=\mathcal{N}\left(\tfrac{1}{g_L},\ \tfrac{1}{\sqrt{3}g_L},\ \tfrac{1}{\sqrt{3}g_R},\ \tfrac{1}{g_R}\right)A^\mu(x), \quad (25)
$$

where $A^\mu(x)$ is an arbitrary vector field which, as we will see later, corresponds to the photon, and $\mathcal{N}$ is an arbitrary normalization. We can obtain a similar expression for the null eigenvector by inverting Eq. (17),

$$
\begin{aligned}
\mathcal{R}\begin{pmatrix}A_\mu\\0\\0\\0\end{pmatrix} &= g_L\sin\theta_W\begin{pmatrix}\frac{1}{g_L}\\ \frac{1}{\sqrt{3}g_L}\\ \frac{1}{\sqrt{3}g_R}\\ \frac{1}{g_R}\end{pmatrix}A_\mu,\\
\mathcal{R}\begin{pmatrix}0\\Z_\mu\\0\\0\end{pmatrix} &= g_L\frac{\sin^2\theta_W}{\cos\theta_W}\begin{pmatrix}\frac{\cot^2\theta_W}{g_L}\\ -\frac{1}{\sqrt{3}g_L}\\ -\frac{1}{\sqrt{3}g_R}\\ -\frac{1}{g_R}\end{pmatrix}Z_\mu, \qquad (26)
\end{aligned}
$$

where $\mathcal{R}=(\mathcal{W}\cdot\mathcal{O}^T)^T$. In order to get this result, it was necessary to impose $g'=g_L\tan\theta_W$, which is also satisfied in the SM. This shows that the null vector corresponds to the photon as we previously said. By proceeding in a similar way for $Z'_\mu$ and $Z''_\mu$, we find

$$
\begin{aligned}
\mathcal{R}\begin{pmatrix}0\\0\\Z'_\mu\\0\end{pmatrix} &= \frac{g_L}{2}\tan\theta_W\begin{pmatrix}0\\ -\frac{4}{\sqrt{3}g_R}\\ \frac{1}{\sqrt{3}g_L}\\ \frac{1}{g_L}\end{pmatrix}Z'_\mu,\\
\mathcal{R}\begin{pmatrix}0\\0\\0\\Z''_\mu\end{pmatrix} &= \begin{pmatrix}0\\0\\-\sqrt{3}/2\\1/2\end{pmatrix}Z''_\mu. \qquad (27)
\end{aligned}
$$

These eigenvectors are the same for any Higgs sector; we verify this for some particular cases which are easy to tackle analytically. By taking $v_1=v_2$, $b_1=b_3$, $b_2=0$, $M_I=M_R$ and $M_3=0$, there are two limits $M_I\to\infty$ or $b_3\to\infty$, which do not correspond to a realistic potential; however, they serve us to check that in both cases we obtain the eigenvectors in Eq. (26) and Eq. (27). It is possible to build Higgs tensors in the $\bar{3}_L\times 3_L$ and $\bar{3}_R\times 3_R$ representations, these terms give masses to $A^U_{L3}=-(A^I_{L3}-\sqrt{3}A^I_{L8})/2$ and $A^U_{R3}=-(A^I_{R3}-\sqrt{3}A^I_{R8})/2$, respectively; however, their contribution to the vector boson mass matrix do not change the present results.

## V. ALTERNATIVE LEFT-RIGHT MODELS

As we already saw in Sec. II, by choosing other $SU(2)$ spin symmetries, it is possible to find alternative models to the left-right symmetric model which have been studied extensively [1,40–43]. The gauge group of the low-energy effective theory is $G=SU(3)_C\otimes SU(2)_L\otimes SU(2)_X\otimes U(1)_{\text{BLX}}$. In the literature, the spin symmetry $SU(2)_I$ corresponds to $SU(2)_R$[5] and $U(1)_{\text{BLI}}$ corresponds to $U(1)_{B-L}$. The resulting model by choosing $X=V$ is known as the alternative left-right model [22] as we already mentioned in Sec. II. The case $X=U$ is a new model where $U(1)_{\text{BLU}}$ corresponds to the $E_6$ lephophobic model (modulo a normalization which is important for the phenomenology). In this section, we study these models as low-energy effective field theories for $[SU(3)]^3$ [18,44]. The neutral current Lagrangians for these models are

$$
\begin{aligned}
-\mathcal{L}_{\text{NC}} &= g_LJ^I_{L3\mu}A^{I\mu}_{L3}+g_RJ^X_{R3\mu}A^{X\mu}_{R3}+g^X_{BL}J^X_{BL\mu}A^{X\mu}_{BL}\\
&= g_LJ^I_{L3\mu}A^{I\mu}_{L3}+g'J_{Y\mu}B^\mu+g_2J_{2\mu}Z'^\mu, X=I,V, \qquad (28)
\end{aligned}
$$

[5]We do not use the label $R$ for this symmetry because the alternative spin symmetries also are subgroups of $SU(3)_R$.

$$-\mathcal{L}_{\mathrm{NC}} = g_L J^I_{L3\mu} A^{I\mu}_{L3} + g_R J^U_{R8\mu} A^{U\mu}_{R8} + g^U_{BL} J^U_{BL\mu} A^{U\mu}_{BL}$$
$$= g_L J^I_{L3\mu} A^{I\mu}_{L3} + g' J_{Y\mu} B^{\mu} + g_2 J_{2\mu} Z'^{\mu}, X = U. \quad (29)$$

For $X = U$, the weak-$U$-spin operator $U_{R3}$ does not contribute to the charge operator $Q$; so, it is not mandatory to take into account the corresponding current in the Lagrangian; however, $U_{R8}$ is necessary in order to reproduce the electromagnetic charges of the 27. For $X = I$ and $X = V$, from the $[SU(3)]^3$ charges we obtain

$$Q = I_{L3} + c_X X_{R3} + X_{BL}, \qquad X = I, V, \quad (30)$$

$$Q = I_{L3} + \frac{2}{\sqrt{3}}\left(d_U - \frac{1}{2}\right) X_{R8} + X_{BL}, \qquad X = U, \quad (31)$$

where

$$X_{BL} = \frac{1}{\sqrt{3}} I_{L8} + \frac{1}{\sqrt{3}} X_{R8}, \qquad X = I, U, V. \quad (32)$$

The $X_{BLX}$ charges are not $E_6$ normalized and, as can be verified in Table I, for $X = I$ these charges correspond to the $(B-L)/2$ ones, i.e., $I_{BL} = (B-L)/2$. By means of an orthogonal matrix we can rotate from the left-right basis of the NC vector bosons to the $(B, Z')$ basis, i.e.,

$$\begin{pmatrix} B_\mu \\ Z'_\mu \end{pmatrix} = (\mathcal{O}^{I,V}_{BL})^T \begin{pmatrix} A^{I,V}_{R3\mu} \\ A^{I,V}_{LB\mu} \end{pmatrix},$$
$$\begin{pmatrix} B_\mu \\ Z'_\mu \end{pmatrix} = (\mathcal{O}^{U}_{BL})^T \begin{pmatrix} A^{U}_{R8\mu} \\ A^{U}_{LB\mu} \end{pmatrix}, \quad (33)$$

where the orthogonal matrices are

$$\mathcal{O}^{I,V}_{BL} = \begin{pmatrix} \cos\gamma & \sin\gamma \\ \sin\gamma & -\cos\gamma \end{pmatrix}, \quad \mathcal{O}^{U}_{BL} = \begin{pmatrix} \cos\delta & \sin\delta \\ \sin\delta & -\cos\delta \end{pmatrix}.$$

By replacing this expression in Eq. (28), we obtain

$$g' B_\mu J^\mu_Y = B^\mu (g_R J^X_{R3\mu} \cos\gamma + g^X_{BL} J^X_{BL\mu} \sin\gamma), \quad (34)$$

$$g' B_\mu J^\mu_Y = B^\mu (g_R J^X_{R8\mu} \cos\delta + g^X_{BL} J^X_{BL\mu} \sin\delta), \quad (35)$$

for $X = I$, $V$ and $X = U$, respectively. By equating the hypercharge current with

$$J_{Y\mu} = c_X J^X_{R3\mu} + J^X_{BL\mu}, \qquad X = I, V, \quad (36)$$

$$J_{Y\mu} = \frac{2}{\sqrt{3}}\left(d_U - \frac{1}{2}\right) J^X_{R8\mu} + J^X_{BL\mu}, \qquad X = U, \quad (37)$$

we get the equations

$$g_R \cos\gamma = g' c_X, \quad g^X_{BL} \sin\gamma = g', \quad X = I, V,$$
$$g_R \cos\delta = g' \frac{2}{\sqrt{3}}\left(d_U - \frac{1}{2}\right), \quad g^X_{BL} \sin\delta = g', \quad X = U.$$

From these equations, we get

$$\cos\gamma = c_X \frac{g'}{g_R}, \qquad \frac{1}{g'^2} = \frac{1}{(g^X_{BL})^2} + \frac{c_X^2}{g_R^2}, \qquad X = I, V.$$
$$\cos\delta = -\sqrt{3}\frac{g'}{g_R}, \qquad \frac{1}{g'^2} = \frac{1}{(g^X_{BL})^2} + \frac{3}{g_R^2}, \qquad X = U.$$

Because $c_X = 1$ for $X = I, V$ the right gauge coupling must satisfy the inequality $g_R > g' = 0.357$, which is met in $[SU(3)]^3$. For $X = U$, the last equation implies $g_R > \sqrt{3} g' = 0.619$, which automatically excludes the $[SU(3)]^3$ value of the right gauge coupling, i.e., $g_R = 0.435$; however, the typical left-right gauge coupling $g_L = g_R = 0.652$ is still possible. From Eqs. (28) and (33), it is possible to get expressions for the neutral current associated with the $Z'$

$$g_2 J_{2\mu} = -g^X_{BL} J^X_{BL\mu} \cos\gamma + g_R J^X_{R3\mu} \sin\gamma$$
$$= g_L \tan_W \left(\alpha_X J^X_{R3\mu} - \frac{c_X J^X_{BL\mu}}{\alpha_X}\right), \qquad X = I, V,$$
$$g_2 J_{2\mu} = -g^X_{BL} J^X_{BL\mu} \cos\delta + g_R J^X_{R8\mu} \sin\delta$$
$$= g_L \tan_W \left(\alpha_U J^X_{R8\mu} + \frac{\sqrt{3} J^X_{BL\mu}}{\alpha_U}\right), \qquad X = U, \quad (38)$$

where

$$\alpha_X = \sqrt{\left(\frac{g_R}{g_L}\right)^2 \cot^2\theta_W - c_X^2} \xrightarrow{[SU(3)]^3}$$
$$\times \frac{1}{\sqrt{4\cos^2\theta_W - 1}}, \qquad X = I, V,$$
$$\alpha_U = \sqrt{\left(\frac{g_R}{g_L}\right)^2 \cot^2\theta_W - 3}, \qquad X = U. \quad (39)$$

From these expressions, we can obtain the explicit expressions for the vector and axial charges. For $X = I$ and $g_R = g_L$, these charges correspond to those of the left-right model reported in Ref. [28]. In the present work, $g_R$ is determined by the $[SU(3)]^3$ symmetry; thus, by replacing $g_R$ from Eq. (15) in the expression above, we obtain (for $X = I$, $V$) the rhs expression of Eq. (39). For $\sin^2\theta_W = 3/8$, we recover the unification matching condition $g_L = g_R$ for any $X$. However, in the present work, we make use of the $\overline{\mathrm{MS}}$ value for the weak mixing angle, $\sin\theta_W = 0.231$, as we will explain below. From these relations and from Table AII, we can obtain the explicit expressions for the vector and axial charges for the $Z'$ gauge boson, corresponding to the $g_2 J_{2\mu}$ current. For $X = I$,

$V, U$, these charges are shown in Tables AVI, AVII, and AVIII, respectively. The collider and EW constraints are shown in Table II and Figs. 2 and 3. A detailed analysis of these constraints will be presented in the next section.

## VI. ELECTROWEAK AND COLLIDER CONSTRAINTS

We analyze the previously considered neutral gauge bosons and impose limits on the $Z$-$Z'$ mixing angle, $\theta_{Z-Z'}$, and on the masses of the neutral $Z'$ bosons, $M_{Z'}$. In order to obtain the EW precision data (EWPD) constraints, we make use of the special purpose FORTRAN package GAPP (Global Analysis of Particle Properties) [45]. Details of the analysis can be found in Ref. [38,46–48].[6]

In the third column of Table II, the EW constraints are shown. The quantum numbers of the model $Z_{331G}$ correspond to those of $U(1)_{21\bar{I}}$ in Table I. We do not put the superscript Tri on the 3-3-1 model because the charges and the coupling strength of this model are the same as the very well-known universal 3-3-1 model [33,37,49] or the so-called $G$ model in Refs. [29,50]. The vector and axial charges for this model are shown in Table AIV.

The quantum numbers of $Z_I^{\mathrm{Tri}}$ correspond to those of $U_I$ in Table I. This model is known as the inert model which does not couple to up-type quarks [20], and corresponds to the second neutral vector boson or $Z''$ in the $[SU(3)]^3$ group. From Eq. (16) for $X = U$ we can see that the coupling strength of $Z_I^{\mathrm{Tri}}$ is $g_2 = g_R = 0.435$. To get this number in Eq. (15) we use for the weak mixing angle the value $\sin\theta_W = 0.231$, which corresponds to the $\overline{\mathrm{MS}}$ renormalization scheme at the $Z$-pole scale. This value is different of the traditional $E_6$ coupling strength, $g_2 = \sqrt{\frac{5}{3}} g_L \tan\theta_W = 0.4615$. The constraints by using the $E_6$ coupling strength correspond to those of $Z_I$ in Table II. The inequality of the couplings is reflected in the EW and LHC constraints. The axial and vector couplings of this model are shown in Table AV.

In Table II, we also distinguish between $Z_{LR}$, which assume the equality between the left and right gauge couplings, i.e., $g_R = g_L = 0.652$, and $Z_{LR}^{\mathrm{Tri}}$ for which the right coupling strength is dictated by $[SU(3)]^3$, i.e., $g_R = 0.435$. This inequality between the left and right couplings makes the chiral charges different which is the reason of the disparity in the constraints in Table II.

We observe that for the alternative left-right model (ALR) the EW constraints are weak compared to other typical $E_6$ models in the literature (except the $Z_\psi$ which only has axial couplings to the SM particles and the leptophobic model $Z_L$), which is a well known feature of this model [51].

[6]An update of Ref. [38] will be presented soon.

As we already saw in Sec. V, there is another alternative model for $I_{BL} = (B - L)/2$, the $U_{BL}$, which, to the best of our knowledge, has not been studied before. This model is $U$-spin symmetric [i.e., $SU(2)_U$], and it has as the main feature that it is leptophobic in the limit $g_R \to 0.619^+$. In the aforementioned limit, $\alpha_U \to 0$ and the lepton couplings are proportional to $\alpha_U$; however, because in this limit the quarks couplings go as $\sim 1/\alpha_U$, in many observables these effects compensate each other in such a way that the EW constraints are not trivial for this model.

In Figs. 1 and 2, the 90% exclusion contours for the universal 3-3-1 model $Z_{331G}$, its corresponding $Z'' = Z_I^{\mathrm{Tri}}$ in the $[SU(3)]^3$ model, in the left-right symmetric model $Z_{LR}^{\mathrm{Tri}}$, and in its alternative version the $Z_{\mathrm{ALR}}^{\mathrm{Tri}}$ are shown. The plots for $Z_{LR}^{\mathrm{Tri}}$ and the inert model $Z_I^{\mathrm{Tri}}$ are comparable with $Z_{LR}$ and $Z_I$ in Ref. [38]. Because $g_R > 0.619$ as we already saw in Sec. V, it is not possible to have the $Z_{\mathrm{LRU}}$ coming from a low-energy $[SU(3)]^3$ effective model; however, by choosing $g_R = g_L = 0.652$ this model is feasible. The corresponding EW and LHC constraints are shown in Table II and Fig. 3.

In Ref [39], the ATLAS detector data on dilepton production was used to search for high-mass resonances decaying to dielectron or dimuon final states. The experiment analyze proton-proton collisions at a center of mass energy of 8 TeV and an integrated luminosity of 20.3 $\mathrm{fb}^{-1}$ in the dielectron channel, and 20.5 $\mathrm{fb}^{-1}$ in the dimuon channel. From this data they report 95% CL upper limits on the total cross section of $Z^0$ decaying to dilepton final states. From these results, and following our earlier analysis [29], we obtain the 95% C.L. lower mass limits for all the models mentioned above. These limits are shown in the second column in Table II and they correspond to the red dashed line in Figs. 1 and 2.

## VII. CONCLUSIONS

In this work, we analyzed all the possible embeddings of the 3-3-1 and 3-2-2-1 models present in the $[SU(3)]^3$ gauge group. By considering the weak-$U$-spin and weak-$V$-spin symmetries in $SU(3)_R$ besides the usual weak-$I$-spin symmetry [best known as $SU(2)_R$] we found two flipped versions of the 3-3-1 model, with the particularity that the $Z'$ axial and vector charges are identical for the three spin symmetries; hence, they are not a new source of phenomenological results. In Appendix B, we showed that the reason behind these results is that, just for these models, the corresponding neutral current Lagrangians are related each other by unitary transformations. For the left-right symmetric model, we also found two flipped versions one of them not reported in the literature as far as we know. This new model is denoted as $Z_{\mathrm{RLU}}$ and it corresponds to a second alternative model of the left-right model $Z_{LR}$ (the first alternative model is $Z_{\mathrm{ALR}}$ which is well known in the literature [22]). In several respects, the $Z_{\mathrm{LRU}}$ model is

different of $Z_{LR}$ and $Z_{\mathrm{ALR}}$; for example, it is not viable as a low-energy effective theory, unless we make it left-right symmetric, which is a typical assumption of the $Z_{LR}$ and $Z_{\mathrm{ALR}}$ models. This model has as the main feature that it is leptophobic in the limit $g_R \to 0.619^{+}$. In the aforementioned limit, $\alpha_U \to 0$ and the lepton couplings are proportional to $\alpha_U$; however, because in this limit the quarks couplings go as $\sim 1/\alpha_U$, in many observables these effects compensate each other in such a way that the EW constraints are not trivial.

We also calculated the eigenstates of the $[SU(3)]^3$ Higgs potential and, by considering different cases, it was shown that these eigenstates are independent of the Higgs sector. It was also shown that the null space of the $[SU(3)]^3$ vector boson mass matrix corresponds to the photon. As a generalization of these results, we gave the explicit form of the null vector of the EW vector boson mass matrix for an arbitrary Higgs tensor and an arbitrary gauge group.

By using the LHC experimental results and EW precision data, new limits on the $Z'$ mass $M_{Z'}$ and the mixing angle $\theta_{Z-Z'}$ are imposed. From this analysis we found lower limits on $M_{Z'}$ of the order of 2.5 TeV, while the mixing angle was found to be constrained to values of the order of $10^{-3}$ radians.

The scope of the present work is not limited to the $[SU(3)]^3$ group. In Ref. [25], the full set of alternatives breakings in $E_6$ was shown, the next step is to extend our analysis to $E_6$, which has as subgroups the most promising and best-known electroweak extensions of the standard model.

## ACKNOWLEDGMENTS

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## APPENDIX A: THE WEAK-*I*, WEAK-*U* AND WEAK-*V* SPIN SYMMETRIES

The $SU(3)$ algebra is invariant under any unitary transformation, i.e.,

$$[\lambda_a/2, \lambda_b/2] = i f_{abc} \lambda_c/2 \to [\lambda'_a/2, \lambda'_b/2] = i f_{abc} \lambda'_c/2,$$

where $\lambda'_a = U \lambda_a U^{\dagger}$. By requiring that $\lambda_3$ and $\lambda_8$ be mapped to diagonal matrices a form of the unitary matrices is (there are several ways to choose $U$ and $V$)

$$U = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \qquad V = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

Additionally, these matrices satisfy $U^2 = U^{\dagger}$, and $V^2 = V^{\dagger}$; from these relations and unitarity we obtain $U^3 = V^3 = 1$. The operators corresponding to $\lambda_3$ and $\lambda_8$ Gell-Mann generators for the weak-*I*-spin (or Isospin), weak-*U*-spin and weak-*V*-spin are

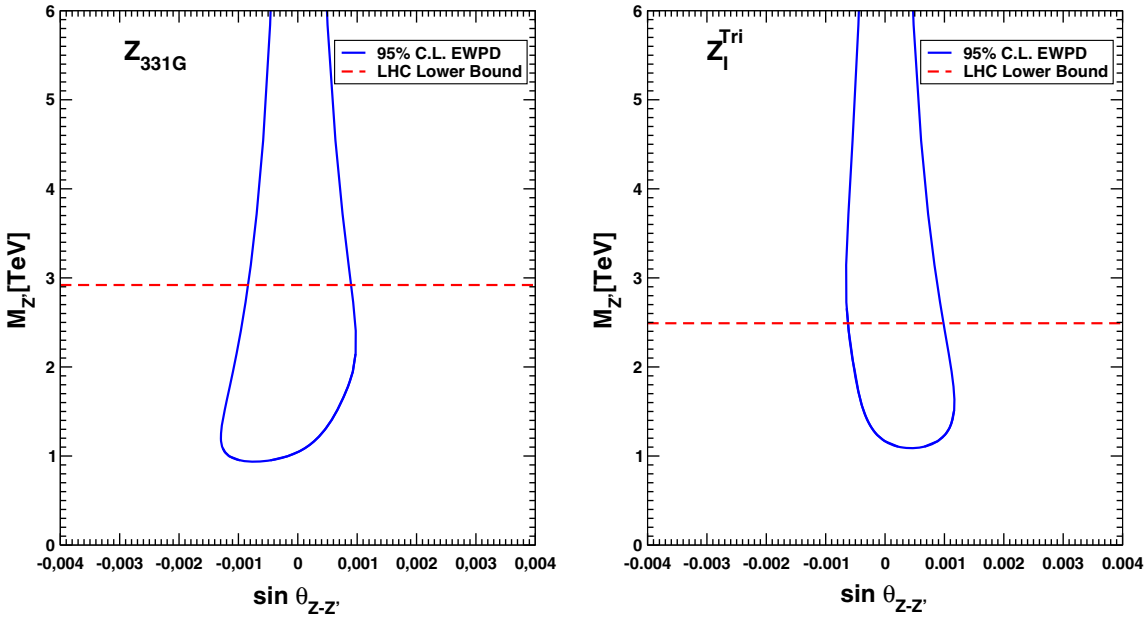


FIG. 1. The continuous blue line represents the 90% C.L. exclusion contour in $M_{Z'}$ vs $\sin\theta_{ZZ'}$ for the universal 3-3-1 model which has charges and coupling strength according to Eq. (16) with $X = I$. The axial and vector charges for this model are shown in Table AIV. The inert model $Z_I^{\mathrm{Tri}}$ has the same charges as the $E_6$ motivated $Z_I$, but with the coupling strength dictated by $[SU(3)]^3$ according to Eq. (16) for $X = U$, i.e., $g_3 = g_R = 0.435$.The axial and vector charges for this model are shown in Table AV. The corresponding plot for the $E_6$ motivated $Z_I$ is shown in Ref. [38]. The red dashed line is the 95% C.L. lower mass limit obtained from ATLAS data [39].

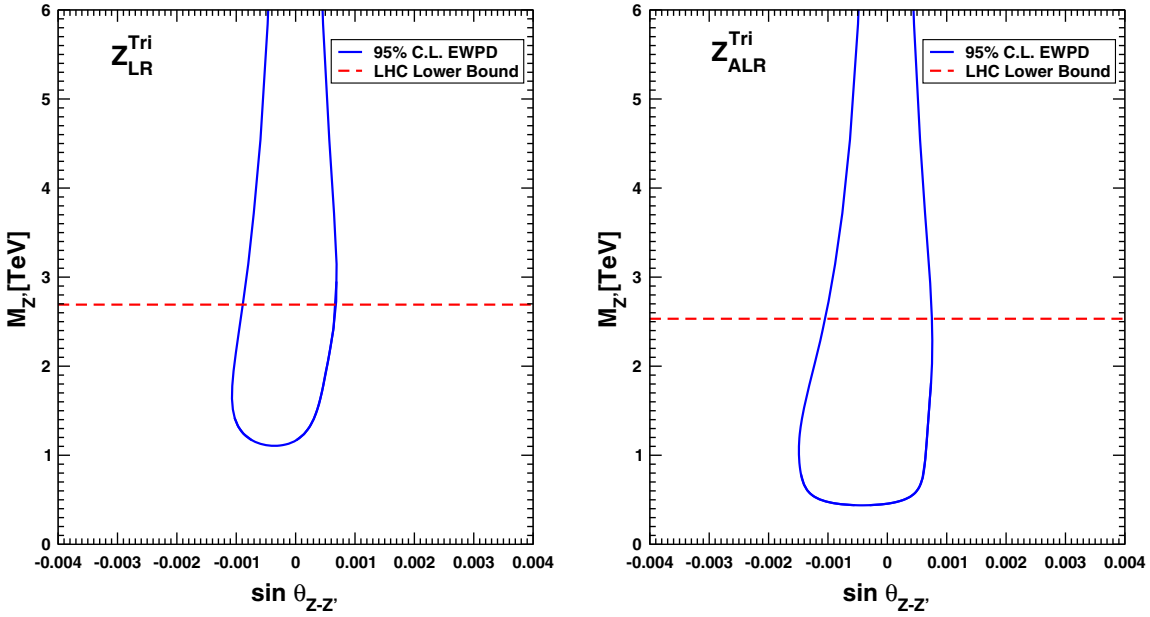


FIG. 2. The continuous blue line represents the 90% C.L. exclusion contour in $M_{Z'}$ vs $\sin\theta_{ZZ'}$ for the left-right symmetric model $Z^{\mathrm{Tr}}_{LR}$ and the alternative left-right Model $Z^{\mathrm{Tr}}_{\mathrm{ALR}}$ with the right coupling strength dictated by $[SU(3)]^3$, i.e., $g_R = 0.435$ for $\sin\theta_W = 0.231$ (see Eq. (38) for $X = I$ and $X = V$, respectively). The axial and vector charges for the left-right and the ALR model are shown in Table AVI and Table AVII. The corresponding plot for the left-right symmetric model with $g_R = g_L = 0.652$ is shown in Ref. [38]. The red dashed line is the 95% C.L. lower mass limit obtained from ATLAS data [39].

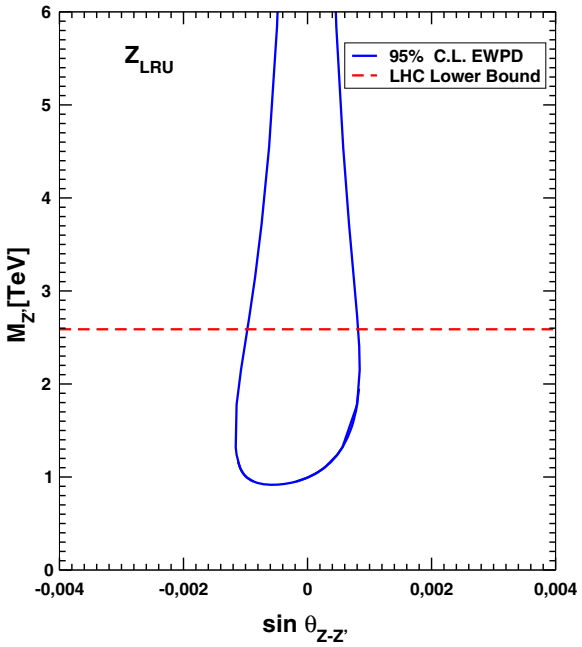


FIG. 3. The continuous blue line represents the 90% C.L. exclusion contour in $M_{Z'}$ vs $\sin\theta_{ZZ'}$ for the LRU model $Z^{\mathrm{Tr}}_{\mathrm{LRU}}$ with $g_R = g_L = 0.652$ for $\sin\theta_W = 0.231$ (See Eq. (38) for $X = U$). The axial and vector charges for the left-right and the LRU model are shown in Table AVI and Table AVII. The red dashed line is the 95% C.L. lower mass limit obtained from ATLAS data [39].

$$\lambda_3^I = \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_3^U = U\lambda_3 U^\dagger = -\frac{1}{2}(\lambda_3 - \sqrt{3}\lambda_8) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

$$\lambda_3^V = V\lambda_3 V^\dagger = -\frac{1}{2}(\lambda_3 + \sqrt{3}\lambda_8) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

and

TABLE AI. The $SU(3)$ algebra is invariant under a unitary transformation. By requiring that $\lambda_3$ and $\lambda_8$ be mapped to diagonal matrices there are two possible choices, U and $V = U^\dagger$. Additionally, these matrices satisfy $U^2 = U^\dagger$, and $V^2 = V^\dagger$; from these relations and unitarity we obtain $U^3 = V^3 = 1$. The latter identity allows us to verify the Table entries.

| $U\lambda_i U^\dagger = \lambda_i^U$ | | | | | | $V\lambda_i V^\dagger = \lambda_i^V$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda_1^U$ | $\lambda_2^U$ | $\lambda_4^U$ | $\lambda_5^U$ | $\lambda_6^U$ | $\lambda_7^U$ | $\lambda_1^V$ | $\lambda_2^V$ | $\lambda_4^V$ | $\lambda_5^V$ | $\lambda_6^V$ | $\lambda_7^V$ |
| $\lambda_6$ | $\lambda_7$ | $\lambda_1$ | $-\lambda_2$ | $\lambda_4$ | $-\lambda_5$ | $\lambda_4$ | $-\lambda_5$ | $\lambda_6$ | $-\lambda_7$ | $\lambda_1$ | $\lambda_2$ |

$$\lambda_8^I = \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},$$

$$\lambda_8^U = U\lambda_8 U^\dagger = -\frac{1}{2}(\lambda_8 + \sqrt{3}\lambda_3) = \frac{1}{\sqrt{3}}\begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\lambda_8^V = V\lambda_8 V^\dagger = -\frac{1}{2}(\lambda_8 - \sqrt{3}\lambda_3) = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Upon the unitary transformations $U$ and $V$, the $SU(3)$ Gell-Mann matrices $\lambda_i$ are mapped to $\lambda_i^U$ and $\lambda_i^V$ as it is shown in Table AI. These alternative representations for the $SU(3)$ algebra are relevant only for $SU(3)_R$ in the $[SU(3)]^3$ group. The representation of the Gell-Mann matrices in $SU(3)_L$ is fixed by the phenomenology of the SM.

## APPENDIX B: $U$ AND $V$ IN THE ADJOINT REPRESENTATION

The eight gauge bosons associated with the $SU(3)_R$ are written by convenience as

$$\frac{1}{2}A^I_{a\mu}\lambda^I_a = \frac{1}{2}A^I_{b\mu}\bar{U}^{Tb}{}_a U_a{}^c\lambda^I_c = \frac{1}{2}(U_a{}^b A^I_{b\mu})(U_a{}^c\lambda^I_c) \equiv \frac{1}{2}A^U_{a\mu}\lambda^U_a,$$

where the bar in $\bar{U}$ stands for complex conjugation. Because $U$ is real $U_a{}^b = \bar{U}_a{}^b$. In the adjoint representation, $U$ is a $8\times 8$ matrix; however, it is reducible to a couple of $3\times 3$ matrices and one $2\times 2$ matrix. The three-dimensional matrices mix the generators associated with the charged bosons while the two-dimensional one mix the diagonal generators associated with the neutral ones

$$\begin{pmatrix} \lambda_1 \\ \lambda_4 \\ \lambda_6 \end{pmatrix} \xrightarrow{U} \begin{pmatrix} \lambda_1^U \\ \lambda_4^U \\ \lambda_6^U \end{pmatrix} = \begin{pmatrix} \lambda_6 \\ \lambda_1 \\ \lambda_4 \end{pmatrix},$$

$$\begin{pmatrix} \lambda_2 \\ \lambda_5 \\ \lambda_7 \end{pmatrix} \xrightarrow{U} \begin{pmatrix} \lambda_2^U \\ \lambda_5^U \\ \lambda_7^U \end{pmatrix} = \begin{pmatrix} \lambda_7 \\ -\lambda_2 \\ -\lambda_5 \end{pmatrix}.$$

The diagonal generators are mapped to

$$\begin{pmatrix} \lambda_3 \\ \lambda_8 \end{pmatrix} \xrightarrow{U} \begin{pmatrix} \lambda_3^U \\ \lambda_8^U \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} \lambda_3 - \sqrt{3}\lambda_8 \\ \lambda_8 + \sqrt{3}\lambda_3 \end{pmatrix}.$$

We want to make use of this symmetry to rewrite the neutral current

$$J^I_{a\mu}A^{I\mu}_a = J^I_{b\mu}\bar{U}^{Tb}{}_a U_a{}^c A^{I\mu}_c = J^I_{b\mu}\bar{V}^{Tb}{}_a V_a{}^c A^I_{c\mu}, \quad \text{(B1)}$$

by defining

$$A^{U\mu}_a \equiv U_a{}^c A^{I\mu}_c, \qquad A^{V\mu}_a \equiv V_a{}^c A^{I\mu}_c,$$
$$J^U_{a\mu} \equiv J^I_{b\mu}\bar{U}^{Tb}{}_a = U_a{}^b J^I_{b\mu}, \qquad J^A_{a\mu} \equiv J^I_{b\mu}\bar{V}^{Tb}{}_a = V_a{}^b J^I_{b\mu},$$

where we take into account that $U$ is a real matrix. By replacing these results in (B1) we obtain

$$J^I_{a\mu}A^{I\mu}_a = J^U_{a\mu}A^{U\mu}_a = J^V_{a\mu}A^{V\mu}_a. \quad \text{(B2)}$$

With these expressions it is possible to build the Lagrangian term $-\mathcal{L}_I = g_R J^X_{R8\mu}A^{X\mu}_{R8}$. It is important to stress that for the 3-3-1-1 models in $[SU(3)]^3$ the neutral current Lagrangians of the alternative models are related each other by a unitary transformation; however, in general, that is not true for alternative models.

## APPENDIX C: THE NULL SPACE OF THE VECTOR BOSON MASS MATRIX FOR AN ARBITRARY HIGGS REPRESENTATION AND GAUGE GROUP

In this section, we will show that for any Higgs potential there is a null vector for the mass matrix $M^{ab}$ of the neutral gauge vector bosons. The explicit form of the vector is[7] $A^a_\mu = \frac{c^a}{g^a}A(x)_\mu$, where the $c^a$ are the coefficients of the group generators in the charge operator, i.e., $Q = c^a T^a$, the $g^a$ is the coupling strength associated with the $A^a_\mu$ vector field and $A(x)_\mu$ must be identified with the photon field. For a simple group, all the $g^a$ are identical; however, they may be different for semisimple groups.

### 1. Rank 1 tensors

For a rank 1 tensor, we can obtain the vector mass matrix from the Higgs covariant derivative

$$\mathcal{L}_{\rm K} = \mathrm{Tr}((D_\mu\phi^i)^\dagger D_\mu\phi^i)|_{\phi^i=v^i} = \frac{1}{2}A^a_\mu M^{ab}A^{b\mu}, \quad \text{(C1)}$$

where $v^i$ are the components of the vacuum expectation value vector. This vector satisfies $Q.v = 0$ since the charge operator must annihilate the vacuum. By taking the components of the vector boson as $A^a_\mu = \frac{c^a}{g^a}A(x)_\mu$, the covariant derivative becomes zero

$$D_\mu\phi^i|_{\phi^i=v^i} = -ig^a A^a_\mu T^a\phi^i|_{\phi^i=v^i} = -i\left(g^a\frac{c^a}{g^a}T^a A(x)_\mu\right)_{ji} v^i = -A(x)_\mu Q_{ji}v^i = 0,$$

where $A(x)_\mu$ is an arbitrary vector function of $x$, which can be identified with the photon field. From Eq. (C1) we get

[7]Modulo a normalization.

$$A^a_\mu M^{ab} A^{b\mu} = 0, \tag{C2}$$

showing that $A^a_\mu = c^a A(x)_\mu$ is a null space vector of $M^{ab}$.

### 2. Rank 2 tensors

For a rank 2 tensor, the analysis is quite similar. The gauge transformation of a rank two tensor under the gauge group is

$$\Phi^{i'j'} = U^{i'}_i U^{j'}_j \Phi^{ij},$$

where the gauge group transformation $U^{i'}_i(\theta(x))$ is a function of the local coordinate $x$. This allows us to define the covariant derivative as

$$D_\mu \Phi^{ij} = \partial_\mu \Phi^{ij} - ig^a (T^a A^a_\mu)^i_\alpha \Phi^{\alpha j} - ig^a (T^a A^a_\mu)^j_\alpha \Phi^{i\alpha}.$$

For the $SU(3)$ gauge group, $T^a = \frac{\lambda^a}{2}$, $g^a = g$, $U(\theta) = \exp(-i\theta^a T^a)$ and the gauge transformation of the vector field is

$$A'_\mu = T^a A'^a_\mu = U(\theta) T^a A^a_\mu U(\theta)^\dagger + \frac{i}{g} U(\theta) \partial_\mu U^\dagger(\theta).$$

We do not lose generality by assuming that the VEV of the Higgs rank 2 tensor is the product of two Higgs scalars in the fundamental representation,[8] i.e., $\Phi^{ij} = \chi^i \xi^j$. In a similar way as we did for the rank 1 tensors, we also build the null vector as $A^a_\mu = \frac{c^a}{g^a} A(x)_\mu$; thus, the covariant derivative is

$$\begin{aligned} D_\mu \Phi^{ij}|_{\Phi=\langle\Phi\rangle} &= -ig^a (T^a A^a_\mu)^i_\alpha \chi^\alpha \xi^j - ig^a (T^a A^a_\mu)^j_\alpha \chi^i \xi^\alpha, \\ &= -iA(x)_\mu (Q^i_\alpha \chi^\alpha \xi^j + Q^j_\alpha \chi^i \xi^\alpha), \\ &= -iA(x)_\mu (q^i \chi^i \xi^j + q^j \chi^i \xi^j) \\ &= -iA(x)_\mu (q^i + q^j) \chi^i \xi^j, \end{aligned}$$

where in the last step we take into account that the charge operator is diagonal, i.e., $Q^i_\alpha \chi^\alpha = q^i \chi^i$ and $Q^j_\alpha \xi^\alpha = q^j \xi^j$. In these expressions, the $q_i$ are the charges of the components of a vector in the fundamental representation. If the component $\langle \Phi^{ij} \rangle$ correspond to the VEV of a Higgs field then $q^i + q^j = 0$ and the kinetic Lagrangian becomes zero,

$$\mathcal{L} = \mathrm{Tr}((D_\mu \Phi^{ij})^\dagger D_\mu \Phi^{ij})|_{\Phi=\langle\Phi\rangle} = \frac{1}{2} A^a_\mu M^{ab} A^{b\mu} = 0.$$

This shows that, as we already demonstrated for the rank 1 tensor, $A^a_\mu = \frac{c^a}{g^a} A(x)_\mu$ is a null vector of the mass matrix $M^{ab}$. The procedure is similar for an arbitrary tensor.

[8] Any component of a matrix can always be written as the tensorial product of two vectors.

## APPENDIX D: $Z'$ COUPLINGS

For the SM extended by a $U(1)'$ extra factor, the neutral current interactions of the fermions are described by the Hamiltonian,

$$H_{\mathrm{NC}} = \sum_{i=1}^{2} g_i Z^0_{i\mu} \sum_f \bar{f} \gamma^\mu (\epsilon^{(i)}_{\mathbf{L}}(f) P_{\mathbf{L}} + \epsilon^{(i)}_{\mathbf{R}}(f) P_{\mathbf{R}}) f, \tag{D1}$$

where $Z^0_{1\mu}$ and $Z^0_{2\mu}$ are the weak basis states such that $Z^0_{1\mu}$ is identified with the neutral gauge boson of the SM, $Z$, and $Z^0_{2\mu}$ with the $Z'$; the index $f$ runs over all the SM fermions in the low-energy neutral current (NC) effective Hamiltonian $H_{\mathrm{NC}}$, and $P_{\mathbf{L}} = (1-\gamma_5)/2$ and $P_{\mathbf{R}} = (1+\gamma_5)/2$. It is convenient to write Eq. (D1) in terms of the vector and axial charges,

$$H_{\mathrm{NC}} = \frac{1}{2} \sum_{i=1}^{2} g_i Z^0_{i\mu} \sum_f \bar{f} \gamma^\mu (G^{(i)}_V(f) - G^{(i)}_A(f)\gamma_5) f, \tag{D2}$$

where the chiral couplings $\epsilon^{(i)}_{\mathbf{L}}(f)$ and $\epsilon^{(i)}_{\mathbf{R}}(f)$ are linear combinations of the vector $G^{(i)}_V(f)$ and axial $G^{(i)}_A(f)$ charges given by $\epsilon^{(i)}_{\mathbf{L}}(f) = [G^{(i)}_V(f) + G^{(i)}_A(f)]/2$ and $\epsilon^{(i)}_{\mathbf{R}}(f) = [G^{(i)}_V(f) - G^{(i)}_A(f)]/2$. The mass eigenstates $Z_{1\mu}$ and $Z_{1\mu}$ are given by

$$\begin{aligned} Z_{1\mu} &= Z^0_{1\mu} \cos\theta + Z^0_{2\mu} \sin\theta, \\ Z_{2\mu} &= -Z^0_{1\mu} \sin\theta + Z^0_{2\mu} \cos\theta. \end{aligned}$$

For the numerical calculations, we use the expressions for the vector and axial charges shown in the Appendices D 1 and D 2, where most of the values in the Tables are being presented for the first time in the literature. We have also used $\sin^2\theta_W = 0.231$ and $g_1 \equiv g/\cos\theta_W = 0.743$.

### 1. The 3-3-1 charges and coupling strength

For $X = U$, $\cos\beta = d_U = -1$ and Eq. (16) reduces to

$$g_2 J_{\mu 2} = -g_L J^I_{L8\mu} \sin\alpha - g_R J^U_{R8\mu} \cos\alpha, \tag{D3}$$

where

$$\begin{aligned} J^I_{L8\mu} &= \sum_i \bar{f}_i \gamma_\mu [\epsilon^{I_{L8}}_{\mathbf{L}}(i) P_L + \epsilon^{I_{L8}}_{\mathbf{R}}(i) P_R] f_i, \\ J^U_{R8\mu} &= \sum_i \bar{f}_i \gamma_\mu [\epsilon^{U_{R8}}_{\mathbf{L}}(i) P_L + \epsilon^{U_{R8}}_{\mathbf{R}}(i) P_R] f_i. \end{aligned}$$

In this way,

$$\begin{aligned} g_2 J_{\mu 2} = \frac{1}{2} \sum_i \bar{f}_i \gamma_\mu (&-g_L \sin\alpha [g^{I_{L8}}_{\mathrm{V}}(i) - g^{I_{L8}}_{\mathrm{A}}(i)\gamma^5] \\ &- g_R \cos\alpha [g^{U_{R8}}_{\mathrm{V}}(i) - g^{U_{R8}}_{\mathrm{A}}(i)\gamma^5]) f_i, \end{aligned}$$

TABLE AII. The chiral charges for the SM particles under the additional $U(1)$ symmetries embedded in the $[SU(3)]^3$ group. $l$ stands for the left-handed doublet $(\nu_L, e_L^-)^T$ and $q$ for the quark left-handed doublet $(u_L, d_L)^T$. For low-energy constraints, only the $Z'$ charges of the SM fermions are involved in the calculation.

| | Chiral charges | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $l$ | $e_R$ | $q$ | $u_R$ | $d_R$ | | $l$ | $e_R$ | $q$ | $u_R$ | $d_R$ |
| $\epsilon^{I_{R3}}$ | 0 | $-\frac{1}{2}$ | 0 | $+\frac{1}{2}$ | $-\frac{1}{2}$ | $\epsilon^{I_{R8}}$ | $\frac{-2}{2\sqrt{3}}$ | $\frac{-1}{2\sqrt{3}}$ | 0 | $\frac{+1}{2\sqrt{3}}$ | $\frac{+1}{2\sqrt{3}}$ |
| $\epsilon^{U_{R3}}$ | $-\frac{1}{2}$ | 0 | 0 | 0 | $+\frac{1}{2}$ | $\epsilon^{U_{R8}}$ | $\frac{1}{2\sqrt{3}}$ | $\frac{2}{2\sqrt{3}}$ | 0 | $\frac{-2}{2\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ |
| $\epsilon^{V_{R3}}$ | $+\frac{1}{2}$ | $+\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | 0 | $\epsilon^{V_{R8}}$ | $\frac{1}{2\sqrt{3}}$ | $\frac{-1}{2\sqrt{3}}$ | 0 | $\frac{1}{2\sqrt{3}}$ | $\frac{-2}{2\sqrt{3}}$ |
| $\epsilon^{I_{BL}}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $+\frac{1}{6}$ | $+\frac{1}{6}$ | $+\frac{1}{6}$ | $\epsilon^{I_{L8}}$ | $-\frac{1}{2\sqrt{3}}$ | $-\frac{1}{\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ | 0 | 0 |
| $\epsilon^{U_{BL}}$ | 0 | 0 | $+\frac{1}{6}$ | $-\frac{1}{3}$ | $+\frac{1}{6}$ | $\epsilon^{V_{BL}}$ | 0 | $-\frac{1}{2}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $-\frac{1}{3}$ |

where

$$g_{\mathrm{V,A}}^{X_{(L,R)8}}(i) = \epsilon_{\mathbf{L}}^{X_{(L,R)8}}(i) \pm \epsilon_{\mathbf{R}}^{X_{(L,R)8}}(i). \tag{D4}$$

Reordering, we have

$$g_2 J_{\mu 2} = \frac{g_{331G}}{2} \sum_i \bar{f}_i \gamma_\mu (G_V^{331G}(i) - G_A^{331G}(i)\gamma^5) f_i,$$

where the vector and axial charges are

$$g_{331G} G_{V,A}^{331G}(i) = -g_L \sin\alpha g_{\mathrm{V,A}}^{I_{L8}}(i) - g_R \cos\alpha g_{\mathrm{V,A}}^{U_{R8}}(i).$$

In the differential cross section, the product $g_{331G} G_{V,A}^{331G}$ always appears, where the $G_{V,A}^{331G}$ are the vector and axial charges in Eq. (D2) and $g_{331G}$ is the corresponding coupling strength. For this reason, it is not necessary to know them separately. Now, given that

$$g' = g_L \tan\theta_W, \qquad g_R = \frac{2 g_L \sin\theta_W}{\sqrt{4\cos^2\theta_W - 1}},$$

$$\cos\alpha = \frac{g'}{\sqrt{3} g_L} = \frac{1}{\sqrt{3}} \tan\theta_W,$$

we take the positive sign of $\sin\alpha$ in agreement with Eq. (12). The expressions for the vector and axial couplings can be cast as

$$\begin{aligned} g_{331G} G_{V,A}^{331G}(i) = {} & \frac{-g_L}{\sqrt{3}\cos\theta_W \sqrt{4\cos^2\theta_W - 1}} \\ & \times ((4\sin^2\theta_W - 3) g_{\mathrm{V,A}}^{I_{L8}}(i) \\ & + 2\sin^2\theta_W g_{\mathrm{V,A}}^{U_{R8}}(i)). \end{aligned} \tag{D5}$$

From Table I, we obtain the chiral charges in Table AII and their corresponding axial and vector expressions in Table AIV. By replacing these expressions in Eq. (D5), we obtain the axial and vector charges as they are shown in

TABLE AIII. The vector and axial charges for the SM particles under the additional $U(1)$ symmetries embedded in the $[SU(3)]^3$ group. For low-energy constraints, only the $Z'$ charges of the SM fermions are involved in the calculation.

| | Vector and axial charges | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $u$ | $d$ | $\nu$ | $e$ | | $u$ | $d$ | $\nu$ | $e$ |
| $g_V^{I_{R3}}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $g_V^{I_{R8}}$ | $\frac{1}{2\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ | $\frac{-2}{2\sqrt{3}}$ | $\frac{-3}{2\sqrt{3}}$ |
| $g_A^{I_{R3}}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | $g_A^{I_{R8}}$ | $\frac{-1}{2\sqrt{3}}$ | $\frac{-1}{2\sqrt{3}}$ | $\frac{-2}{2\sqrt{3}}$ | $\frac{-1}{2\sqrt{3}}$ |
| $g_V^{U_{R3}}$ | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $g_V^{U_{R8}}$ | $\frac{-2}{2\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ | $\frac{3}{2\sqrt{3}}$ |
| $g_A^{U_{R3}}$ | 0 | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $g_A^{U_{R8}}$ | $\frac{2}{2\sqrt{3}}$ | $\frac{-1}{2\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ | $\frac{-1}{2\sqrt{3}}$ |
| $g_V^{V_{R3}}$ | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 1 | $g_V^{V_{R8}}$ | $\frac{1}{2\sqrt{3}}$ | $\frac{-2}{2\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ | 0 |
| $g_A^{V_{R3}}$ | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 0 | $g_A^{V_{R8}}$ | $\frac{-1}{2\sqrt{3}}$ | $\frac{2}{2\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ | $\frac{2}{2\sqrt{3}}$ |
| $g_V^{I_{BL}}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $-\frac{1}{2}$ | $-1$ | $g_V^{I_{L8}}$ | $\frac{1}{2\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ | $\frac{-1}{2\sqrt{3}}$ | $\frac{-3}{2\sqrt{3}}$ |
| $g_A^{I_{BL}}$ | 0 | 0 | $-\frac{1}{2}$ | 0 | $g_A^{I_{L8}}$ | $\frac{1}{2\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ | $\frac{-1}{2\sqrt{3}}$ | $\frac{1}{2\sqrt{3}}$ |
| $g_V^{U_{BL}}$ | $-\frac{1}{6}$ | $+\frac{1}{3}$ | 0 | 0 | $g_V^{V_{BL}}$ | $\frac{1}{3}$ | $-\frac{1}{6}$ | 0 | $-\frac{1}{2}$ |
| $g_A^{U_{BL}}$ | $+\frac{1}{2}$ | 0 | 0 | 0 | $g_A^{V_{BL}}$ | 0 | $\frac{1}{2}$ | 0 | $\frac{1}{2}$ |

TABLE AIV. Couplings for $Z_{331G} \to \bar{f}f$. Here $\eta_{331} = g_{331G}/\sqrt{4\cos^2\theta_W - 1}$ and $g_{331G} = g_1 = g_L/\cos\theta_W$

| $f$ | $g_{331G} G_V^{331G}(f)$ | $g_{331G} G_A^{331G}(f)$ |
|---|---|---|
| $\nu$ | $(\frac{1}{2} - \sin^2\theta_W)\eta_{331}$ | $(\frac{1}{2} - \sin^2\theta_W)\eta_{331}$ |
| $e$ | $3(\frac{1}{2} - \sin^2\theta_W)\eta_{331}$ | $(\sin^2\theta_W - \frac{1}{2})\eta_{331}$ |
| $u$ | $(\frac{4}{3}\sin^2\theta_W - \frac{1}{2})\eta_{331}$ | $-\frac{1}{2}\eta_{331}$ |
| $d$ | $(\frac{1}{3}\sin^2\theta_W - \frac{1}{2})\eta_{331}$ | $(\sin^2\theta_W - \frac{1}{2})\eta_{331}$ |

TABLE AV. Vector and axial couplings $Z_I^{\mathrm{Tri}} \to \bar{f}f$ ($X = U$ case). Here $\eta_{IN} = g_R$.

| $f$ | $g_{IN} G_V^{IN}(f)$ | $g_{IN} G_A^{IN}(f)\eta_{IN}$ |
|---|---|---|
| $\nu$ | $-\frac{1}{2}\eta_{IN}$ | $-\frac{1}{2}\eta_{IN}$ |
| $e$ | $-\frac{1}{2}\eta_{IN}$ | $-\frac{1}{2}\eta_{IN}$ |
| $u$ | 0 | 0 |
| $d$ | $\frac{1}{2}\eta_{IN}$ | $-\frac{1}{2}\eta_{IN}$ |

TABLE AVI. Vector and axial couplings for $Z_{LR}^{\mathrm{Tri}} \to \bar{f}f$ (The $X = I$ case). Here $\eta_{LR} = g_L \tan\theta_W/\sqrt{4\cos^2\theta_W - 1}$.

| $f$ | $g_{LR} G_V^{LR}(f)$ | $g_{LR} G_A^{LR}(f)$ |
|---|---|---|
| $\nu$ | $-\frac{1}{2}(1\text{–}4\cos^2\theta_W)\eta_{LR}$ | $-\frac{1}{2}(1\text{–}4\cos^2\theta_W)\eta_{LR}$ |
| $e$ | $(4\cos^2\theta_W - \frac{3}{2})\eta_{LR}$ | $\frac{1}{2}\eta_{LR}$ |
| $u$ | $\frac{1}{3}(\frac{5}{2} - 4\cos^2\theta_W)\eta_{LR}$ | $-\frac{1}{2}\eta_{LR}$ |
| $d$ | $-\frac{1}{3}(\frac{1}{2} + 4\cos^2\theta_W)\eta_{LR}$ | $\frac{1}{2}\eta_{LR}$ |

TABLE AVII. Vector and axial couplings for $Z^{\rm Tri}_{\rm ALR} \to \bar{f}f$ ($X = V$ case). Here $\eta_{LR} = g_L \tan\theta_W/\sqrt{4\cos^2\theta_W - 1}$.

| $f$ | $g_{\rm ALR} G_V^{\rm ALR}(f)$ | $g_{\rm ALR} G_A^{\rm ALR}(f)$ |
|---|---|---|
| $\nu$ | $\frac{1}{2}\eta_{LR}$ | $\frac{1}{2}\eta_{LR}$ |
| $e$ | $(\frac{3}{2} - 2\cos^2\theta_W)\eta_{LR}$ | $\frac{1}{2}(4\cos^2\theta_W - 1)\eta_{LR}$ |
| $u$ | $\frac{1}{3}(4\cos^2\theta_W - \frac{5}{2})\eta_{LR}$ | $\frac{1}{2}\eta_{LR}$ |
| $d$ | $-\frac{1}{6}(4\cos^2\theta_W - 1)\eta_{LR}$ | $\frac{1}{2}(4\cos^2\theta_W - 1)\eta_{LR}$ |

Table AIV. By defining $g_{331G} = g_L/\cos\theta_W$, as usual for 3-3-1 models, we recover the vector and axial couplings to the $Z'$ boson in the G model [29]. From Eq. (16), for $X = U$ and $X = V$, we obtain exactly the same expression for the axial and vector couplings as the one for the $I$ case in Table AIV. The reason behind this coincidence is that the EW Langrangians $-\mathcal{L}^X = g_R J^X_{R3\mu} A^{X\mu}_{R3} + g_R J^X_{R8\mu} A^{X\mu}_{R8}$ [see Eq. (5)] are related to each other by unitary transformations for the different values of $X = I$, $U$, $V$, as it is shown in Appendix B. The same is not true for the left-right symmetric model and its alternative models as we will see in the next section. The vector and axial charges of the $Z''$ current, $g_2 J_3$, are obtained directly from Eq. (16),

$$g_2 J_2 = \frac{g_{IN}}{2}\sum_i \bar{f}_i \gamma_\mu (G_V^{IN}(i) - G_A^{IN}(i)\gamma^5) f_i. \quad \text{(D6)}$$

Here we use $IN$ instead of $I$ to denote the inert model $Z^{\rm Tri}_I$, in spite of the latter is a more frequent label for this model.[9]

### 2. Couplings for the left-right symmetric model and its alternative versions

From Eq. (38) the neutral current coupled to the $Z'$ boson is given by

[9]That is in order to avoid confusion with the label $I$ for the weak-$I$-spin symmetry.

TABLE AVIII. Vector and axial couplings for $Z_{\rm LRU} \to \bar{f}f$ ($X = U$ case) Couplings $Z' \to \bar{f}f$ for $X = U$. Here $\eta = g_L \tan\theta_W/\sqrt{3}$ and $\alpha_U = \sqrt{(g_R/g_L)^2\cot^2\theta_W - 3}$.

| $f$ | $g_{\rm LRU} G_V^{\rm LRU}(f)$ | $g_{\rm LRU} G_A^{\rm LRU}(f)$ |
|---|---|---|
| $\nu_\alpha$ | $\frac{1}{2}\eta\alpha_U$ | $\frac{1}{2}\eta\alpha_U$ |
| $e_\alpha$ | $\frac{3}{2}\eta\alpha_U$ | $-\frac{1}{2}\eta\alpha_U$ |
| $u_\alpha$ | $-\eta(\alpha_U + \frac{1}{2\alpha_U})$ | $\eta(\alpha_U + \frac{3}{2\alpha_U})$ |
| $d_\alpha$ | $-\frac{1}{2}\eta(\alpha_U + \frac{2}{\alpha_U})$ | $-\frac{1}{2}\eta\alpha_U$ |

$$g_2 J_{2\mu} = g_L \tan\theta_W \left(\alpha_X J^X_{R3\mu} - \frac{c_X J^X_{BL\mu}}{\alpha_X}\right), \qquad X = I, V,$$
$$g_2 J_{2\mu} = g_L \tan\theta_W \left(\alpha_U J^U_{R8\mu} + \frac{\sqrt{3} J^U_{BL\mu}}{\alpha_U}\right), \qquad X = U \quad \text{(D7)}$$

which encompasses the three different $X$ values. From Eq. (D7) we get for $X = I$, $V$, $U$ the vector and axial charges for the left-right, ALR and inert models, respectively,

$$g_2 J_2 = \frac{g_{(A)LR(U)}}{2}\sum_i \bar{f}_i \gamma_\mu (G_V^{(A)LR(U)}(i) - G_A^{(A)LR(U)}(i)\gamma^5) f_i,$$

where the index $(A)LR(U)$ stands for the three models, i.e., $LR$, $ALR$ and $LRU$.

$$g_{(A)LR} G_{V,A}^{(A)LR}(i) = g'\left(\frac{g_{V,A}^{X_{R3}}(i)}{\alpha_X} - c_X \alpha_X g_{V,A}^{X_{BL}}(i)\right),$$
$$g_{\rm LRU} G_{V,A}^{\rm LRU}(i) = g'\left(\frac{g_{V,A}^{U_{R8}}(i)}{\alpha_U} + \sqrt{3}\alpha_U g_{V,A}^{U_{BL}}(i)\right), \quad \text{(D8)}$$

where, $g' = g_L \tan\theta_W$ and $\alpha_I = \alpha_V = 1/\sqrt{(4\cos^2\theta_W - 1)}$. From Table AIII and Eqs. (D8), we get the vector and axial-vector couplings to the $Z'$ boson, which are shown in Tables AVII and AVIII.


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