Erratum: Quantization of unstable linear scalar fields in static spacetimes [Phys. Rev. D 88, 124005 (2013)]

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Unlike what is stated in the last paragraph of Sec. III B, the representation of the "one-particle Hilbert space" $\mathcal{H}$ in terms of $L^2_{\mathbb{C}}$ discussed at the end of Sec. III A is useful to represent the one-particle dynamics when the field is unstable. The rectification of that statement has no consequences for any of the other results presented in the paper.

As mentioned below Eq. (69), the time-translation operator $T(t)$ maps $\mathcal{H}$ into $\tilde{\mathcal{A}}_{\mathbb{C}} = \mathcal{H} \oplus \bar{\mathcal{H}}$, in general, due to the properties of $T_-(t)$, the time translation on the unstable sector. However, from Eq. (68) we see that the restriction of $T_-(t)$ to $\mathcal{H}^-$, the subspace of $\mathcal{H}$ associated with the instability, can also be written as

$$T_-(t)\upharpoonright\mathcal{H}^- = \cosh(t|\Lambda|^{\frac{1}{2}})\mathbb{I} + i\sinh(t|\Lambda|^{\frac{1}{2}})\mathbf{E}, \tag{1}$$

where $\mathbb{I}$ is the identity and $\mathbf{E}$ is the matrix defined in Eq. (111). As an operator on $\tilde{\mathcal{A}}_{\mathbb{C}}$, the matrix $\mathbf{E}$ is an isometry satisfying $\mathbf{E}^2 = \mathbb{I}$ and mapping $\mathcal{H}$ into its complex-conjugate space $\bar{\mathcal{H}}$ and vice versa—for this last property, see comment below Eq. (112). The relation between the inner products of $\mathcal{H}$ and $\bar{\mathcal{H}}$, in particular, allows us to interpret $\mathbf{E}$ as an antilinear operator mapping $\mathcal{H}$ into itself. Therefore, we have established that $\mathbf{E}$ defines a conjugation on $\mathcal{H}$, and $T(t)$ can be seen as an operator mapping $\mathcal{H}$ into itself. We mention in passing that this result is an instance of the more general symplectic transformation approach of Segal [1] and Shale [2] for Bogoliubov transformations—see also the notes for Sec. XI.15 of Ref. [3].

Consider then $\psi = (f, p) \in \mathcal{P}$ and the vector $K\psi \in \mathcal{H}$. Since the time translation now takes $\mathcal{H}$ into itself, we have

$$\|T(t)K\psi\|^2_{\mathcal{H}} = \|e^{-it|\Lambda|^{\frac{1}{2}}}v_+\|^2_{L^2_{\mathbb{C}}} + \|\cosh(t|\Lambda|^{\frac{1}{2}})v_-\|^2_{L^2_{\mathbb{C}}} + \|\sinh(t|\Lambda|^{\frac{1}{2}})v_-\|^2_{L^2_{\mathbb{C}}}, \tag{2}$$

where $v_\pm \equiv Q_{I^\pm}v$, $v \equiv (Xf + iX^{\dagger -1}p)/\sqrt{2}$, $Q_{I^\pm}$ are the spectral projections of Λ appearing in Eq. (56), and $X = |\Lambda|^{\frac{1}{4}}\|\varkappa\|^{-\frac{1}{2}}$. Hence, if we take $\mathcal{H} = L^2_{\mathbb{C}}$ and implement K through the map k defined at the end of Sec. III A, then Eq. (2) allows us to represent $T(t)\upharpoonright\mathcal{H}$ as $e^{-it|\Lambda|^{\frac{1}{2}}}$ on the stable sector and as $\sqrt{2}\cosh\left(t|\Lambda|^{\frac{1}{2}} - \frac{i\pi}{4}\right)$ on the unstable one.

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