PHYSICAL REVIEW D **94,** 126020 (2016)

Flavor structure in $SO(32)$ heterotic string theory

Hiroyuki Abe,[1,*] Tatsuo Kobayashi,[2,†] Hajime Otsuka,[1,‡] Yasufumi Takano,[2,§] and Takuya H. Tatsuishi[2,¶]

[1]*Department of Physics, Waseda University, Tokyo 169-8555, Japan*
[2]*Department of Physics, Hokkaido University, Sapporo 060-0810, Japan*
(Received 27 July 2016; published 30 December 2016)

We study the flavor structure in $SO(32)$ heterotic string theory on six-dimensional tori with magnetic fluxes. Specifically, we focus on models with the flavor symmetries $SU(3)_f$ and $\Delta(27)$. In both models, we can realize the realistic quark masses and mixing angles.

DOI: 10.1103/PhysRevD.94.126020

I. INTRODUCTION

Superstring theory is a promising candidate for unified theory to describe all interactions that include gravity and matter, such as quarks, leptons, and Higgs fields. Superstring theory predicts six-dimensional (6D) compact space in addition to four-dimensional (4D) spacetime—i.e., ten-dimensional (10D) spacetime in total. The massless spectrum is completely determined at the perturbative level when one fixes concretely a compactification, i.e., a geometrical and gauge background. Actually, various interesting models have been constructed, and they include the gauge symmetry of the standard model (SM), $SU(3)_C \times SU(2)_L \times U(1)_Y$, and three chiral generations of quarks and leptons. (See [1] for a review.) In some models, supersymmetry (SUSY) remains in 4D, while SUSY is broken in other models. Thus, there are a lot of (semi)realistic models from the viewpoint of massless spectra. The next issue to examine in these models is whether these models can lead to numerically realistic results on the parameters in the SM, e.g., experimental values of gauge couplings and Yukawa couplings, the Higgs potential, the CP phase, etc.

Recently, $SO(32)$ heterotic string theory on toroidal compactification with magnetic fluxes was studied. Several models with the SM gauge group and three chiral generations have been constructed [2]. In addition, one of the interesting aspects in this type of models is that they lead to nonuniversal gauge couplings among the $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ groups, and such nonuniversal corrections depend on magnetic fluxes and Kähler moduli [3]. Then, it is possible that those models with the SM gauge group and three chiral generations lead to gauge couplings that are consistent with experimental values [4]. Note that the $E_8 \times E_8$ heterotic string theory on toroidal compactification cannot lead to such nonuniversal gauge couplings between $SU(3)_C$ and $SU(2)_L$ only by magnetic fluxes.[1] Hence, this nonuniversality is an interesting aspect of $SO(32)$ heterotic string theory, although one-loop threshold corrections can lead to nonuniversal effects on gauge couplings in $E_8 \times E_8$ heterotic string theory [6–8]. (See [9,10] for numerical studies.)

For the next step, we study quark and lepton masses and mixing angles in $SO(32)$ heterotic string theory on toroidal compactification with magnetic fluxes. Because of magnetic fluxes, zero-mode profiles are nontrivially quasilocalized. When zero modes are localized close to each other, their couplings are strong. On the other hand, when they are localized far away from each other, their couplings are suppressed. Indeed, their couplings are given by the Jacobi ϑ function [11]. Thus, we could lead to phenomenologically interesting results on fermion mass matrices.[2] The flavor structure of $SO(32)$ heterotic string theory on a magnetized torus has already been studied in [2], and it was shown that several flavor symmetries appear: $SU(3)_f$, $\Delta(27)$, etc. The appearance of such discrete flavor symmetries as $\Delta(27)$, $\Delta(64)$, and D_4 has been pointed out in heterotic orbifold models [13,14] and intersecting/magnetized D-brane models [15,16], and certain non-Abelian flavor symmetries of note when realizing fermion masses and mixing angles [17–19]. Thus, we study quark masses and mixing angles which are derived from $SO(32)$ heterotic string theory on toroidal compactification with magnetic fluxes. We focus on models with the flavor symmetries $SU(3)_f$ and $\Delta(27)$. We also discuss the lepton sector. Although similar studies in magnetized D-brane models with the $\Delta(27)$ flavor symmetry were done [12], the $SU(3)_f$ flavor models have never been studied.

This paper is organized as follows. In Sec. II, we review $SO(32)$ heterotic string theory on toroidal compactification with magnetic fluxes, and we explain models with the flavor symmetries $SU(3)_f$ and $\Delta(27)$. In Sec. III, we study quark masses and mixing angles in $SU(3)_f$ and $\Delta(27)$

*abe@waseda.jp
†kobayashi@particle.sci.hokudai.ac.jp
‡h.otsuka@aoni.waseda.jp
§takano@particle.sci.hokudai.ac.jp
¶t-h-tatsuishi@particle.sci.hokudai.ac.jp

[1]See, e.g., [5] for 10D super E_8 Yang-Mills models on tori and orbifolds with magnetic fluxes.

[2]See [12] for a similar study on magnetized brane models.

models. In Sec. IV, we also discuss the lepton sector and neutrino and Higgs masses. Section V consists of the conclusion and a discussion.

II. 10D SO(32) SYM THEORY ON MAGNETIZED TORI

In this section, we give a brief review of $SO(32)$ heterotic string theory on the torus compactification with background magnetic fluxes. We also explain their flavor symmetries and Yukawa couplings.

A. Three generation models from $SO(32)$ heterotic string theory

The low-energy effective field theory of $SO(32)$ heterotic string theory is described by 10D $SO(32)$ super Yang-Mills (SYM) theory coupled with supergravity. We compactify the 6D space to three 2-tori $(T^2)_1 \times (T^2)_2 \times (T^2)_3$, with magnetic fluxes.

We break $SO(32)$ gauge group by inserting $U(1)$ magnetic fluxes,

$$SO(32) \to SU(3)_C \times SU(2)_L \times \Pi_{a=1}^{13} U(1)_a. \tag{1}$$

Since $SO(32)$ has 16 Cartan elements $H_i (i = 1, \ldots, 16)$, we define Cartan elements of $SU(3)$ along $H_1 - H_2$, $H_1 + H_2 - 2H_3$ and $SU(2)$ as $H_5 - H_6$. We set Cartan elements of $U(1)_a$ as

$$\begin{aligned}
U(1)_1&: \frac{1}{\sqrt{2}}(0,0,0,0,1,1;0,0,\ldots,0),\\
U(1)_2&: \frac{1}{2}(1,1,1,1,0,0;0,0,\ldots,0),\\
U(1)_3&: \frac{1}{\sqrt{12}}(1,1,1,-3,0,0;0,0,\ldots,0),\\
U(1)_4&: (0,0,0,0,0,0;1,0,\ldots,0),\\
U(1)_5&: (0,0,0,0,0,0;0,1,\ldots,0),\\
&\vdots\\
U(1)_{13}&: (0,0,0,0,0,0;0,0,\ldots,1),
\end{aligned} \tag{2}$$

in the basis H_i. Then, we use the basis in which the nonzero roots have the charges

$$(\underline{\pm 1, \pm 1, 0, \ldots, 0}), \tag{3}$$

under $H_i (i = 1, \ldots, 16)$, where the underline indicates any possible permutations. The gauge group enhances to a larger one if $U(1)$ fluxes are absent or degenerate. For example, if the magnetic flux along $U(1)_3$ is absent, $SU(3)_C$ and $U(1)_3$ enhance to $SU(4)$, with Cartan elements along $H_1 - H_2$, $H_1 + H_2 - 2H_3$, $H_1 + H_2 + H_3 - 3H_4$, in our model building. Those enhanced symmetries can be broken by Wilson lines.

We define three 2-tori $(T^2)_i \simeq \boldsymbol{C}/\Lambda_i$, with $i = 1, 2, 3$, where Λ_i represents two-dimensional lattices generated by $e_1 = 2\pi R_i$ and $e_2 = 2\pi R_i \tau_i$, $\tau_i \in \boldsymbol{C}$. R_i and τ_i are the radii and the complex structure moduli. Then, the 6D metric is given by

$$\begin{aligned}
ds_6^2 &= g_{mn} dx^m dx^n = 2h_{i\bar{j}} dz^i dz^{\bar{j}},\\
g_{mn} &= \begin{pmatrix} g^{(1)} & 0 & 0 \\ 0 & g^{(2)} & 0 \\ 0 & 0 & g^{(3)} \end{pmatrix}, \quad h_{i\bar{j}} = \begin{pmatrix} h^{(1)} & 0 & 0 \\ 0 & h^{(2)} & 0 \\ 0 & 0 & h^{(3)} \end{pmatrix},
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
g^{(i)} &= (2\pi R_i)^2 \begin{pmatrix} 1 & \mathrm{Re}\tau_i \\ \mathrm{Re}\tau_i & |\tau_i|^2 \end{pmatrix},\\
h^{(i)} &= (2\pi R_i)^2 \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix},
\end{aligned} \tag{5}$$

with the real coordinates x^m for $(m, n = 4, \ldots, 9)$ and the complex coordinates $z^i = x^{2+2i} + \tau^i x^{3+2i}$ $(i = 1, 2, 3)$ of the 6D space. We expand $U(1)_a$ magnetic fluxes in the compact space $\bar{f}_a$ with $a = 1, \ldots, 13$ in the basis of Kähler forms, $w_i = i dz^i \wedge d\bar{z}^i / (2\mathrm{Im}\tau_i)$,

$$\bar{f}_a = 2\pi d_a \sum_{i=1}^{3} m_a^i w_i, \tag{6}$$

where d_a indicates normalization factors and m_a^i represents integers or half integers determined by the Dirac quantization condition.

The 10D gauge fields and gaugino fields are decomposed as

$$\begin{aligned}
\lambda(x^\mu, z^i) &= \sum_{\ell,m,n} \chi_{\ell mn}(x^\mu) \otimes \psi_\ell^1(z^1) \otimes \psi_m^2(z^2) \otimes \psi_n^3(z^3),\\
A_M(x^\mu, z^i) &= \sum_{\ell,m,n} \varphi_{\ell mn,M}(x^\mu) \otimes \phi_{\ell,M}^1(z^1) \otimes \phi_{m,M}^2(z^2)\\
&\quad \otimes \phi_{n,M}^3(z^3),
\end{aligned} \tag{7}$$

where $M = 0, 1, \ldots, 9$, $\mu = 0, 1, 2, 3$, and $\phi_{\ell,M}^i(z^i)$, and $\psi_\ell^i(z^i)$ corresponds to the ℓth mode on the ith T^2. $\psi_\ell^i(z^i)$ is the 2D spinor, and we denote the zero mode $\psi_0^i(z^i)$ as

$$\psi_0^i(z^i) = \begin{pmatrix} \psi_+^i(z^i) \\ \psi_-^i(z^i) \end{pmatrix}. \tag{8}$$

Magnetic fluxes (6) can be obtained from the $U(1)_a$ vector potentials

$$A^i_a(z^i) = \frac{\pi m^i_a}{\mathrm{Im}\tau_i}\,\mathrm{Im}((\bar{z}^i + \bar{\zeta}^i_a)dz^i). \tag{9}$$

Note that we included the degree of freedom of the complex Wilson lines $\zeta^i_a = \zeta^{x^{2+2i}}_a + \tau_i \zeta^{x^{3+2i}}_a$.

We use the following gamma matrices on $(T^2)_i$:

$$\Gamma^1_i = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \Gamma^2_i = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{10}$$

satisfying the Clifford algebra, $\{\Gamma^a_i, \Gamma^b_i\} = 2\delta^{ab}$. In holomorphic coordinates, then, we obtain

$$\Gamma^{z^i} = (2\pi R^i)^{-1}\begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}, \qquad \Gamma^{\bar{z}^i} = (2\pi R^i)^{-1}\begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix} \tag{11}$$

from Eq. (5).

The Dirac equation for the zero modes with the representation A and the $U(1)_a$ charge q^A_a is given by

$$iD_i\psi^i_0(z^i) = i(\Gamma^{z^i}\nabla_{z_i} + \Gamma^{\bar{z}^i}\nabla_{\bar{z}_i})\psi^i_0(z^i) = 0, \tag{12}$$

with the covariant derivatives

$$\nabla_{z^i} = \partial_{z^i} - iq^A_a(A^i_a)_{z^i},$$
$$\nabla_{\bar{z}^i} = \partial_{\bar{z}^i} - iq^A_a(A^i_a)_{\bar{z}^i}. \tag{13}$$

The Dirac equations can be rewritten in terms of the components of $\psi^i(z^i)$ as

$$\left[\partial_{\bar{z}^i} + \frac{\pi q^A_a m^i_a}{2\mathrm{Im}\tau^i}\left(z^i + \frac{q^A_a m^i_a \zeta^i_a}{q^A_a m^i_a}\right)\right]\psi^i_+(z^i,\bar{z}^i) = 0, \tag{14}$$

$$\left[\partial_{z^i} - \frac{\pi q^A_a m^i_a}{2\mathrm{Im}\tau^i}\left(\bar{z}^i + \frac{q^A_a m^i_a \bar{\zeta}^i_a}{q^a_A m^i_a}\right)\right]\psi^i_-(z^i,\bar{z}^i) = 0. \tag{15}$$

Here, ψ^i_+ has degenerate zero modes only if $M^i_A = q^A_a m^i_a > 0$, whereas ψ^i_- has degenerate zero modes only if $M^i_A < 0$. Their degeneracy is given by $|M^i_A|$. In addition, the effective Wilson line $\zeta^i_A = \frac{q^A_a m^i_a \zeta^i_a}{q^A_a m^i_a}$ determines the quasilocalization positions of the wave functions of zero modes. Thus, Wilson lines are very important to Yukawa couplings.

If $M^i_A > 0$, wave functions for ψ^i_+ are given by

$$\psi^{iA,I}_+ = \Theta^{I,M^i_A}(z^i + \zeta^i_A, \tau_i), \tag{16}$$

where

$$\Theta^{I,M}(z,\tau) = \mathcal{N}_I \cdot e^{\pi i M z \mathrm{Im}z/\mathrm{Im}\tau} \cdot \vartheta\begin{bmatrix} I/M \\ 0 \end{bmatrix}(Mz, M\tau),$$
$$\vartheta\begin{bmatrix} a \\ b \end{bmatrix}(\nu,\tau) = \sum_{l\in\mathbf{Z}} e^{\pi i(a+l)^2\tau} e^{2\pi i(a+l)(\nu+b)},$$

and normalization factors $\mathcal{N}_I$ are determined, such that

$$\int_{T^2} d^2z\,\Theta^{I,M}(\Theta^{J,M})^* = \delta_{IJ}. \tag{17}$$

The index $I = 0, \ldots, |M^i_A|$ labels degenerate zero modes. The total degeneracy, i.e., the number of generations, is a product of $|M^i_A|$,

$$M_A = |M^1_A||M^2_A||M^3_A|. \tag{18}$$

One can extract candidates for SM particles from an adjoint representation of an $SO(32)$ gauge group with identification of the hypercharge $U(1)_Y = (U(1)_3 + 3\sum_{a=4}^N U(1)_a)/6$, where N depends on the models. (See [2] for details.) These candidates are summarized as follows:

$$\begin{aligned}
&Q\colon \begin{cases} Q_1 = (3,2)_{1,1,1;0,\ldots,0} \\ Q_2 = (3,2)_{-1,1,1;0,\ldots,0} \end{cases}, && L\colon \begin{cases} L_1 = (1,2)_{1,1,-3;0,\ldots,0} \\ L_2 = (1,2)_{-1,1,-3;0,\ldots,0} \end{cases}, \\
&u_R\colon u^a_{R_2} = (3,1)_{0,1,1;\underline{1,0,\ldots,0}}, && d_R\colon d^a_{R_3} = (3,1)_{0,1,1;\underline{-1,0,\ldots,0}}, \\
&e_R\colon u^a_{R_1} = (1,1)_{0,1,-3;\underline{-1,0,\ldots,0}}, && \nu_R\colon n^a_2 = (1,1)_{0,1,-3;\underline{1,0,\ldots,0}}, \\
&H_u\colon \bar{L}^a_4 = (1,2)_{1,0,0;\underline{1,0,\ldots,0}}, && H_d\colon L^a_3 = (1,2)_{1,0,0;\underline{-1,0,\ldots,0}},
\end{aligned} \tag{19}$$

where the indices imply $U(1)_{1,\ldots,13}$ charge $q_{1,\ldots,13}$ and the underlines are possible permutations. Here, we focus on the supersymmetric standard model, e.g., the minimal supersymmetric standard model (MSSM). Here and hereafter, we use the superfield notation. We can discuss the nonsupersymmetric SM similarly.

We need constraints on magnetic fluxes in order to make $U(1)_Y$ massless [2],

$$m_3^i = 0,$$
$$m_{2+2a}^i = -m_{3+2a}^i \left(a = 1, \ldots, \frac{N-3}{2} \right). \qquad (20)$$

Furthermore, we impose K-theory constraints to construct models without heterotic five-branes,

$$\sum_{a=1}^{2} m_a^i = 0 \pmod 2. \qquad (21)$$

We can achieve these conditions by setting

$$M_{Q_2} = 3, \qquad M_{L_2} = 3,$$
$$M_{Q_1} = 0, \qquad M_{L_1} = 0. \qquad (22)$$

For the right-handed sector, we can obtain three generations of quarks and leptons when $\sum_{a=4}^{13} M_{u_{R_2}^a} = -3$. In general, there are many Higgs pairs, H_u and H_d.

B. Flavor symmetries in three generation models

For the left-handed sector, three generations of quark and lepton doublets are realized by 12 cases,

$$M_{Q_2}^i = \begin{cases} (\underline{3, 1, 1}) \\ (\underline{3, -1, -1}) \\ (\underline{-3, -1, 1}). \end{cases} \qquad (23)$$

Since these cases are related to each other by interchanging two tori $(T^2)_i \leftrightarrow (T^2)_j$, or changing signs of magnetic fluxes on two tori $m_a^i \to -m_a^i$, $m_a^j \to -m_a^j$, we can set

$$M_{Q_2}^i = (-3, -1, 1) \qquad (24)$$

without losing generality.

For the right-handed sector, we have a lot of models to realize three generations of quarks and leptons. The first example is obtained as follows:

$$M_{u_{R_2}^4}^i = M_{u_{R_2}^6}^i = M_{u_{R_2}^8}^i, \qquad M_{u_{R_2}^4} = -1, \qquad \sum_{a=4}^{13} M_{u_{R_2}^a} = -3. \qquad (25)$$

In this model, the gauge symmetries develop into a larger one, $\prod_{a=4}^{9} U(1)_a \to SU(3)_u \times SU(3)_d \times SU(2)_R$. Cartan elements of $SU(3)_u$ are $H_4 - H_6$, $H_4 + H_6 - 2H_8$. $SU(3)_d$ and $SU(2)_R$ are given by $H_5 - H_7$, $H_5 + H_7 - 2H_9$ and $H_4 + H_6 + H_8 - H_5 - H_7 - H_9$, respectively. These $SU(3)_{u,d}$ symmetries are flavor symmetries among the right-handed quarks and leptons, as well as the Higgs fields. That is, the right-handed quarks in the up sector (the down sector) are a triplet under $SU(3)_u$ $[SU(3)_d]$. Similarly, the Higgs fields H_u (H_d) are also triplets under $SU(3)_u$ $[SU(3)_d]$, while the right-handed neutrinos (the charged leptons) are a triplet under $SU(3)_u$ $[SU(3)_d]$. Thus, we refer to this model as the $SU(3)_f$ model. The left-handed quarks and leptons are singlets under the $SU(3)_{u,d}$ symmetries.

The second example is obtained as

$$M_{u_{R_2}^4}^i = -M_{Q_2}^i, \qquad \sum_{a=5}^{13} M_{u_{R_2}^a} = 0. \qquad (26)$$

This model has a gauge symmetry $SU(2)_R$ whose Cartan element is $H_4 - H_5$. In addition, this model has the non-Abelian discrete symmetry $\Delta(27)$ [15]. The three generations of the quarks and leptons are triplets under $\Delta(27)$. The Higgs fields are also $\Delta(27)$ triplets.

There are other models which have different flavor structures. We focus on the above two models, the $SU(3)_f$ flavor model and the $\Delta(27)$ flavor model, since they contain good flavor symmetries, leading to simple mass matrices. Throughout this paper, we assume that the gauge couplings of these flavor symmetries are sufficiently suppressed at the low-energy scale, although this depends on the matter contents of the hidden sector. Furthermore, we also assume the existence of the $\mathcal{N} = 1$ supersymmetry to ensure the stability of our system, although it is irrelevant to the flavor structure of the Yukawa coupling.

C. Computation of Yukawa couplings

As shown in the previous section, the wave function of each degenerate mode on tori is quasilocalized at a different point which is controlled by Wilson lines. Since performing an overlap integral derives Yukawa couplings, these couplings can become hierarchical. Let us now compute Yukawa couplings. Yukawa coupling in 4D is given by the product of three overlap integrals on three 2-tori, i.e.,

$$Y_{\mathcal{I}\mathcal{J}\mathcal{K}} = g\lambda_{I_1 J_1 K_1}^{(1)} \lambda_{I_2 J_2 K_2}^{(2)} \lambda_{I_3 J_3 K_3}^{(3)},$$
$$\lambda_{I_i J_i K_i}^{(i)} = \int_{(T^2)_i} d^2 z^i \Theta^{I_i, M_A^i}(z^i + \zeta_A^i, \tau_i) \Theta^{J_i, M_B^i}(z^i + \zeta_B^i, \tau_i) \times (\Theta^{K_i, -M_C^i}(z^i + \zeta_C^i, \tau_i))^*, \qquad (27)$$

where g is the 4D gauge coupling, $\mathcal{I} = (I_1, I_2, I_3)$, $\mathcal{J} = (J_1, J_2, J_3)$, $\mathcal{K} = (K_1, K_2, K_3)$, and we impose invariance under $U(1)_a$ gauge symmetries, $q_a^A + q_a^B + q_a^C = 0$. Note that the Lorentz symmetry of the 6D compact space also leads to the selection rule of allowed Yukawa couplings. For example, the Yukawa coupling, $Y^{(u)} H_u Q_L u_R$, is allowed only if the fermionic components of H_u, Q_L, and u_R have the chiralities $(+,-,-)$, $(-,+,-)$, and $(-,-,+)$ in the 6D compact space, respectively, and other permutations.

By performing an overlap integral, we obtain

$$\lambda_{I_iJ_iK_i} = \frac{\mathcal{N}_{I_i}\mathcal{N}_{J_i}}{\mathcal{N}_{K_i}} e^{\pi i(M_A^i\zeta_A^i \mathrm{Im}\zeta_A^i + M_B^i\zeta_B^i \mathrm{Im}\zeta_B^i + M_C^i\zeta_C^i \mathrm{Im}\zeta_C^i)/\mathrm{Im}\tau^i} \cdot \sum_{m\in Z_{M_A^i+M_B^i}} \vartheta\begin{bmatrix} \frac{M_B^i I_i - M_A^i J_i + M_A^i M_B^i m}{M_A^i M_B^i(-M_C^i)} \\ 0 \end{bmatrix} \times (M_A^i M_B^i(\zeta_A^i - \zeta_B^i), \tau M_A^i M_B^i(-M_C^i)) \cdot \delta_{I_i+J_i+M_A^i m, K_i}. \tag{28}$$

III. QUARK MASSES AND MIXINGS

In this section, we study the mass matrices and mixing angles of quark sector.

A. $SU(3)_f$ model

We begin with the $SU(3)_f$ model. Although there are several $SU(3)_f$ models, we focus on the case $M^i_{u^{c\,4}_{R2}} = (-1, 1, -1)$ such that the Lorentz symmetry of the 6D compact space allows for Yukawa couplings. The three generations of the up-sector (down-sector) right-handed quarks are a triplet under $SU(3)_u$ [$SU(3)_d$]. This model contains, in total, (4×3) pairs of vectorlike Higgs fields, and these up-sector (down-sector) Higgs fields are four triplets under $SU(3)_u$ [$SU(3)_d$]. The degeneracy factor, 4, comes from four chiral zero modes on the first T^2. For simplicity, we concentrate on a single zero mode among four zero modes in order to study the properties of the $SU(3)_f$ flavor model. Note that the difference among four chiral zero modes on the first T^2 is the peak positions of the wave functions, and the peak position can be shifted by varying the Wilson line. This implies that any choice of a single zero mode among four zero modes can lead to an equivalent configuration by varying the Wilson lines. Thus, we consider three pairs of Higgs fields, which are triplets under $SU(3)_u$ and $SU(3)_d$, and we denote them by H_{uK} and H_{dK}, with $K = 0, 1, 2$.

Yukawa coupling terms of the up-sector quarks and three Higgs fields,

$$Y^{(u)}_{IJK} H_{uK} Q_{L_I} u_{R_J}, \tag{29}$$

can be written as

$$Y^{(u)}_{IJ0} = g\begin{pmatrix} \eta_{8,\zeta_{u1}} & 0 & 0 \\ \eta_{4,\zeta_{u1}} & 0 & 0 \\ \eta_{0,\zeta_{u1}} & 0 & 0 \end{pmatrix}, \qquad Y^{(u)}_{IJ1} = g\begin{pmatrix} 0 & \eta_{8,\zeta_{u2}} & 0 \\ 0 & \eta_{4,\zeta_{u2}} & 0 \\ 0 & \eta_{0,\zeta_{u2}} & 0 \end{pmatrix},$$
$$Y^{(u)}_{IJ2} = g\begin{pmatrix} 0 & 0 & \eta_{8,\zeta_{u3}} \\ 0 & 0 & \eta_{4,\zeta_{u3}} \\ 0 & 0 & \eta_{0,\zeta_{u3}} \end{pmatrix}, \tag{30}$$

up to the normalization factors, where $\eta_{n,\zeta_{ui}}$ represents the contributions on Yukawa couplings from the first T^2, and is obtained by use of Eq. (28). In the following analysis, we restrict complex structure moduli τ_i and Wilson lines ζ^i_a are pure imaginary. Then, $\eta_{n,\zeta_{ui}}$ is written as

$$\eta_{n,\zeta_{ui}} = \sum_l e^{-12\pi \mathrm{Im}\tau(\frac{n}{12}+l+\frac{\mathrm{Im}\zeta_{ui}}{\mathrm{Im}\tau_1})^2}, \tag{31}$$

where

$$\zeta_{ui} = (m^1_2 + m^1_{2i+2})m^1_1\zeta^1_1 - (m^1_1 - m^1_{2i+2})m^1_2\zeta^1_2 - (m^1_1 + m^1_2)m^1_{2i+2}\zeta^1_{2i+2}. \tag{32}$$

We obtain $\eta_{0,\zeta_{ui}} \sim 1$ for $\zeta_{ui} = 0$.

Similarly, the down-sector Yukawa couplings are written in the same form, except for the replacement of $\eta_{n,\zeta_{ui}}$ by $\eta_{n,\zeta_{di}}$. Wilson lines for the down sector are defined by

$$\zeta_{di} = (m^1_2 + m^1_{2i+3})m^1_1\zeta^1_1 - (m^1_1 - m^1_{2i+3})m^1_2\zeta^1_2 - (m^1_1 + m^1_2)m^1_{2i+3}\zeta^1_{2i+3}. \tag{33}$$

Here, we assume that these Higgs fields develop their vacuum expectation values (VEVs). This leads to the following mass matrix for the up sector:

$$M^u = g\langle H_{u2}\rangle \begin{pmatrix} \eta_{8,\zeta_{u_1}}\rho_{u1} & \eta_{8,\zeta_{u_2}}\rho_{u2} & \eta_{8,\zeta_{u_3}} \\ \eta_{4,\zeta_{u_1}}\rho_{u1} & \eta_{4,\zeta_{u_2}}\rho_{u2} & \eta_{4,\zeta_{u_3}} \\ \eta_{0,\zeta_{u_1}}\rho_{u1} & \eta_{0,\zeta_{u_2}}\rho_{u2} & \eta_{0,\zeta_{u_3}} \end{pmatrix}, \tag{34}$$

and the down-sector mass matrix

$$M^d = g\langle H_{d2}\rangle \begin{pmatrix} \eta_{8,\zeta_{d_1}}\rho_{d1} & \eta_{8,\zeta_{d_2}}\rho_{d2} & \eta_{8,\zeta_{d_3}} \\ \eta_{4,\zeta_{d_1}}\rho_{d1} & \eta_{4,\zeta_{d_2}}\rho_{d2} & \eta_{4,\zeta_{d_3}} \\ \eta_{0,\zeta_{d_1}}\rho_{d1} & \eta_{0,\zeta_{d_2}}\rho_{d2} & \eta_{0,\zeta_{d_3}} \end{pmatrix}, \tag{35}$$

where

$$\rho_{u1} = \frac{\langle H_{u0}\rangle}{\langle H_{u2}\rangle}, \qquad \rho_{u2} = \frac{\langle H_{u1}\rangle}{\langle H_{u2}\rangle}, \tag{36}$$

$$\rho_{d1} = \frac{\langle H_{d0}\rangle}{\langle H_{d2}\rangle}, \qquad \rho_{d2} = \frac{\langle H_{d1}\rangle}{\langle H_{d2}\rangle}. \tag{37}$$

The mass ratios and mixing angles are determined by the complex structure τ_1 on the first T^2, the Wilson lines ζ_{ui} and ζ_{di}, and the ratios $\rho_{u1}, \rho_{u2}, \rho_{d1}, \rho_{d2}$. In this paper, we treat them as free parameters to fit the data, although they are determined by the stabilization of the moduli and the Higgs fields.

The above matrices for the up sector have the hierarchy $M^u_{ij} \leq M^u_{i'j'}$ for $i \leq i'$, and $j \leq j'$ when $\zeta_{u1} \sim \zeta_{u2} \sim \zeta_{u3} \sim 0$. Down-sector matrices have the same characteristics.

Let us consider the (2×2) lower right submatrix first. Because of the hierarchical structure, the diagonalizing angles of the up- and down-sector mass matrices are estimated as

$$\theta^{u,d}_{23} \sim M^{u,d}_{23}/M^{u,d}_{33}, \tag{38}$$

and the mass ratios are also estimated as

$$(m_2/m_3)^{u,d} \sim |M^{u,d}_{22}/M^{u,d}_{33} - (M^{u,d}_{23}/M^{u,d}_{33})(M^{u,d}_{32}/M^{u,d}_{33})|. \tag{39}$$

Similarly, we can examine the (2×2) upper left submatrix to estimate diagonalizing angles $\theta^{u,d}_{12}$ and $\theta^{u,d}_{13}$, as well as mass ratios. Then, the Cabibbo-Kobayashi-Maskawa (CKM) matrix,

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \tag{40}$$

is estimated as

$$\begin{aligned} |V_{us}| &\sim |\theta^u_{12} - \theta^d_{12}|, \\ |V_{ub}| &\sim |\theta^u_{13} - \theta^d_{13}|, \\ |V_{cb}| &\sim |\theta^u_{23} - \theta^d_{23}|. \end{aligned} \tag{41}$$

These experimental values are

$$\begin{aligned} |V_{us}| &= 0.23, \\ |V_{ub}| &= 0.0041, \\ |V_{cb}| &= 0.041. \end{aligned} \tag{42}$$

The renormalization group flow in the SM leads

$$\begin{aligned} m_u/m_t &\sim 6.5 \times 10^{-6}, \\ m_c/m_t &\sim 3.2 \times 10^{-3}, \\ m_d/m_b &\sim 1.1 \times 10^{-3}, \\ m_s/m_b &\sim 2.2 \times 10^{-2}, \end{aligned} \tag{43}$$

at $\Lambda_{\rm GUT} = 2 \times 10^{16}$ GeV (see, e.g., [20]). The renormalization group flow of the MSSM leads to similar values.

With hierarchical Yukawa matrices, we can estimate mass ratios and mixing angles for the up sector,

$$\begin{aligned} (m_1/m_3)^u &\sim \rho_{u1}\left|\frac{\eta_{8,\zeta_{u1}}}{\eta_{0,\zeta_{u3}}} - \frac{\rho_{u2}}{(m_2/m_3)^u}\frac{\eta_{4,\zeta_{u1}}}{\eta_{0,\zeta_{u3}}}\frac{\eta_{8,\zeta_{u2}}}{\eta_{0,\zeta_{u3}}}\right|, \\ (m_2/m_3)^u &\sim \rho_{u2}\left|\frac{\eta_{4,\zeta_{u2}}}{\eta_{0,\zeta_{u3}}} - \frac{\eta_{4,\zeta_{u3}}}{\eta_{0,\zeta_{u3}}}\frac{\eta_{0,\zeta_{u2}}}{\eta_{0,\zeta_{u3}}}\right|, \\ m^u_3 &\sim g\langle H_{u2}\rangle \eta_{0,\zeta_{u3}} \\ \theta^u_{12} &\sim \frac{\rho_{u2}}{(m_2/m_3)^u}\frac{\eta_{8,\zeta_{u2}}}{\eta_{0,\zeta_{u3}}}, \\ \theta^u_{13} &\sim \frac{\eta_{8,\zeta_{u3}}}{\eta_{0,\zeta_{u3}}}, \\ \theta^u_{23} &\sim \frac{\eta_{4,\zeta_{u3}}}{\eta_{0,\zeta_{u3}}}. \end{aligned} \tag{44}$$

The down sector gives similar expressions.

When $\rho_{ui} \sim \rho_{di} \sim 1$, the ratios of the above parameters bring insufficient hierarchy to realize the mixing angles; thus, we need *tuning* to realize a hierarchical structure. Here, we show an example of a set of parameters, yielding realistic quark masses and mixings. We set

$$\begin{aligned} \tau_1 &= 1.1i, \\ \zeta_{u_i} &= (-0.065i, -0.068i, -0.072i), \\ \zeta_{d_i} &= (0.002i, -0.063i, 0.017i), \\ \rho_{u_i} &= (1, 1), \\ \rho_{d_i} &= (1, 1). \end{aligned} \tag{45}$$

Note that this model contains tuning. For instance, $(m_2/m_3)^u$ is estimated as $|0.056–0.061| = 0.005$ in Eq. (44), indicating that cancellation derives the hierarchical mass ratio. Similar cancellation is required to derive other mass ratios. Since $\rho_{ui}, \rho_{di} \sim \mathcal{O}(1)$ do not suppress mass ratio, we need tuning to realize the hierarchical masses. These parameters lead to the realistic values shown in Table I.

When ρ_{ui}, ρ_{di} are not of $\mathcal{O}(1)$ but are instead hierarchical, we do not need tuning. Next, we show an example without tuning. We set

$$\begin{aligned} \tau_1 &= 1.1i, \\ \zeta_{u_i} &= (0.010i, -0.035i, -0.020i), \\ \zeta_{d_i} &= (-0.020i, -0.084i, -0.070i), \\ \rho_{u_i} &= (0.0021, 0.44), \\ \rho_{d_1} &= (0.18, 0.97), \end{aligned} \tag{46}$$

leading to the results shown in Table II.

TABLE I. Mass ratios and mixings evaluated with values of the complex structure moduli on the first T^2, the Higgs VEVs, and the Wilson lines in Eq. (45).

$(m_u/m_t, m_c/m_t)$	$(6.3 \times 10^{-6}, 4.0 \times 10^{-3})$
$(m_d/m_b, m_s/m_b)$	$(1.6 \times 10^{-3}, 1.9 \times 10^{-2})$
$\lvert V_{\rm CKM}\rvert$	$\begin{pmatrix} 0.97 & 0.23 & 0.012 \\ 0.23 & 0.97 & 0.039 \\ 0.021 & 0.035 & 1.0 \end{pmatrix}$

TABLE II. Mass ratios and mixings evaluated with values of complex structure moduli on the first T^2, the Higgs VEVs, and the Wilson lines in Eq. (46).

$(m_u/m_t, m_c/m_t)$	$(8.7\times10^{-6}, 2.8\times10^{-3})$
$(m_d/m_b, m_s/m_b)$	$(4.4\times10^{-4}, 1.4\times10^{-2})$
$\lvert V_{\rm CKM}\rvert$	$\begin{pmatrix} 0.98 & 0.20 & 0.018 \\ 0.20 & 0.98 & 0.049 \\ 0.0076 & 0.051 & 1.0 \end{pmatrix}$

We are assuming that one pair of Higgs fields, which corresponds to the VEV direction, is light, and others are sufficiently heavy, in order to avoid the flavor changing neutral currents. It is an interesting but challenging issue—but one which is beyond the scope of this paper—to realize such Higgs mass matrices.

B. $\Delta(27)$ model

Let us move on to the $\Delta(27)$ flavor symmetry model. In this model, all of the quarks and leptons are the same type of triplets of $\Delta(27)$.[3] We focus on the case $M^i_{u^{c\,4}_{R2}} = (-3,1,-1)$ to obtain full-rank mass matrices. This model contains $(6 = 2\times 3)$ pairs of vectorlike Higgs fields, and they are two triplets of $\Delta(27)$, which are also the same type of triplets as the quarks and leptons. The degeneracy factor, 6, comes from six chiral zero modes on first T^2.

We use all pairs of the Higgs fields to realize realistic mass matrices, which are two triplets under $\Delta(27)$. We denote them by H_{uK} and H_{dK}, with $K = 0,\ldots,5$. Among them, H_{uK} and H_{dK}, with $K = 0$, 1, 2, correspond to a triplet, while H_{uK} and H_{dK}, with $K = 3$, 4, 5, correspond to another triplet. They lead to the Yukawa coupling term

$$Y^{(u)}_{IJK} H_{uK} Q_{L_I} u_{R_J}, \tag{47}$$

which can be written as

$$Y^{(u)}_{IJ0} = g\begin{pmatrix} \tilde\eta_{0,\zeta_u} & 0 & 0 \\ 0 & 0 & \tilde\eta_{6,\zeta_u} \\ 0 & \tilde\eta_{12,\zeta_u} & 0 \end{pmatrix}, \quad Y^{(u)}_{IJ1} = g\begin{pmatrix} 0 & \tilde\eta_{15,\zeta_u} & 0 \\ \tilde\eta_{3,\zeta_u} & 0 & 0 \\ 0 & 0 & \tilde\eta_{9,\zeta_u} \end{pmatrix},$$
$$Y^{(u)}_{IJ2} = g\begin{pmatrix} 0 & 0 & \tilde\eta_{12,\zeta_u} \\ 0 & \tilde\eta_{0,\zeta_u} & 0 \\ \tilde\eta_{6,\zeta_u} & 0 & 0 \end{pmatrix}, \quad Y^{(u)}_{IJ3} = g\begin{pmatrix} \tilde\eta_{9,\zeta_u} & 0 & 0 \\ 0 & 0 & \tilde\eta_{15,\zeta_u} \\ 0 & \tilde\eta_{3,\zeta_u} & 0 \end{pmatrix},$$
$$Y^{(u)}_{IJ4} = g\begin{pmatrix} 0 & \tilde\eta_{6,\zeta_u} & 0 \\ \tilde\eta_{12,\zeta_u} & 0 & 0 \\ 0 & 0 & \tilde\eta_{0,\zeta_u} \end{pmatrix}, \quad Y^{(u)}_{IJ5} = g\begin{pmatrix} 0 & 0 & \tilde\eta_{3,\zeta_u} \\ 0 & \tilde\eta_{9,\zeta_u} & 0 \\ \tilde\eta_{15,\zeta_u} & 0 & 0 \end{pmatrix}, \tag{48}$$

up to the normalization factors, where $\tilde\eta_{n,\zeta_u}$ again represents the contributions on Yukawa couplings from the first T^2. As the $SU(3)_f$ model, we restrict that complex structure moduli τ, and the Wilson lines ζ_a are purely imaginary. Then $\tilde\eta_{n,\zeta_u}$ is written as

$$\tilde\eta_{n,\zeta_u} = \sum_l \sum_{m=0}^{2} e^{-54\pi {\rm Im}\tau(\frac{n}{54}+\frac{m}{3}+l+\frac{{\rm Im}\zeta_u}{{\rm Im}\tau_1})^2}. \tag{49}$$

Similarly, the down-sector Yukawa couplings are written in the same form, except for the replacement of $\tilde\eta_{n,\zeta_u}$ by $\tilde\eta_{n,\zeta_d}$. The Wilson lines for the up and down sectors are

$$\zeta_u = (m_2^1+m_4^1)m_1^1\zeta_1^1 - (m_1^1-m_4^1)m_2^1\zeta_2^1 - (m_1^1+m_2^1)m_4^1\zeta_4^1 \tag{50}$$

and

$$\zeta_d = (m_2^1+m_5^1)m_1^1\zeta_1^1 - (m_1^1-m_5^1)m_2^1\zeta_2^1 - (m_1^1+m_2^1)m_5^1\zeta_5^1. \tag{51}$$

Note that Y_{IJm} $(m = 0, 1, 2)$ has an opposite hierarchy to Y_{IJm+3}, which is not preferable for realizing a hierarchical Yukawa matrix. We assume that H_{u2}, H_{u3}, and H_{u4} develop their VEVs. Then, the mass matrix of the up-sector quarks is obtained as

$$M^u \approx g\langle H_{u4}\rangle \begin{pmatrix} \tilde\eta_{9,\zeta_u}\rho_{u3} & \tilde\eta_{6,\zeta_u} & \tilde\eta_{12,\zeta_u}\rho_{u2} \\ \tilde\eta_{12,\zeta_u} & \tilde\eta_{0,\zeta_u}\rho_{u2} & \tilde\eta_{15,\zeta_u}\rho_{u3} \\ \rho_{u2}\tilde\eta_{6,\zeta_u} & \tilde\eta_{3,\zeta_u}\rho_{u3} & \tilde\eta_{0,\zeta_u} \end{pmatrix}, \tag{52}$$

where $\rho_{ui} = \frac{\langle H_{ui}\rangle}{\langle H_{u4}\rangle}$, with $i = 2$, 3. For the down sector, $\rho_{d3}\tilde\eta_{9,\zeta_d}$ is too small to realize a down quark mass. Thus, we

[3] There are several types of triplets in $\Delta(27)$ [18].

assume that H_{d0}, as well as H_{d2}, H_{d3} and H_{d4}, develops its VEV. Then, the mass matrix of the down-sector quarks is given by

$$M^d \approx g\langle H_{d4}\rangle \begin{pmatrix} \tilde{\eta}_{0,\zeta_d}\rho_{d0} & \tilde{\eta}_{6,\zeta_d} & \tilde{\eta}_{12,\zeta_d}\rho_{d2} \\ \tilde{\eta}_{12,\zeta_d} & \tilde{\eta}_{0,\zeta_d}\rho_{d2} & \tilde{\eta}_{15,\zeta_d}\rho_{d3} \\ \tilde{\eta}_{6,\zeta_d}\rho_{d2} & \tilde{\eta}_{3,\zeta_d}\rho_{d3} & \tilde{\eta}_{0,\zeta_d} \end{pmatrix}, \quad (53)$$

where $\rho_{di} = \frac{\langle H_{di}\rangle}{\langle H_{d4}\rangle}$, with $i = 0, 2, 3$.

Since $(m_u/m_t)(m_c/m_t) = \det(M^u)/(m_t)^3 \sim \det(Y_{IJ4}/\tilde{\eta}_{0,\xi_u})$ leads to a constraint on $\mathrm{Im}\tau_1$, $(\tilde{\eta}_{6,\zeta_u})(\tilde{\eta}_{12,\zeta_u}) \sim e^{-\frac{4}{3}\pi\mathrm{Im}\tau_1} \approx 2\times 10^{-8}$, we set $\mathrm{Im}\tau_1 = 4.2$. Next, we concentrate on the 2×2 lower right matrices,

$$v_4^{u,d}\begin{pmatrix} \rho_{u,d2}\tilde{\eta}_{0,\zeta_{u,d}} & \rho_{u,d3}\tilde{\eta}_{15,\zeta_{u,d}} \\ \rho_{u,d3}\tilde{\eta}_{3,\zeta_{u,d}} & \tilde{\eta}_{0,\zeta_{u,d}} \end{pmatrix}, \quad (54)$$

leading to

$$\begin{aligned} V_{cb} &\sim \rho_{u3}\tilde{\eta}_{15,\zeta_u}/\tilde{\eta}_{0,\zeta_u} - \rho_{d3}\tilde{\eta}_{15,\zeta_d}/\tilde{\eta}_{0,\zeta_d}, \\ m_c/m_t &\sim \rho_{u2} - (\rho_{u3})^2\tilde{\eta}_{3,\zeta_u}\tilde{\eta}_{15,\zeta_u}/(\tilde{\eta}_{0,\zeta_u})^2, \\ m_s/m_b &\sim \rho_{d2} - (\rho_{d3})^2\tilde{\eta}_{3,\zeta_d}\tilde{\eta}_{15,\zeta_d}/(\tilde{\eta}_{0,\zeta_d})^2. \end{aligned} \quad (55)$$

Then we can estimate $\rho_{u2} \sim 3.2\times 10^{-3}$, $\rho_{d2} \sim 2.2\times 10^{-2}$, $\rho_{u3} - \rho_{d3} \sim \pm 0.36$, assuming $\zeta_u = \zeta_d = 0$. Finally, we use Y_0 to realize m_d. In a way similar to the up-sector mass matrix, we set $\rho_{d0} \sim 1.1\times 10^{-3}$ from the constraint $\det(M^d) \sim \rho_{d0}\rho_{d2}\tilde{\eta}^3_{0,\zeta_d}$. In the following representative parameters,

$$\begin{aligned} \tau &= 4.2i, \\ \zeta_u &= 0.0045i, \\ \zeta_d &= -0.1i, \\ \rho_{ui} &= (0, 0, 0.0053, 0.415, 1, 0), \\ \rho_{di} &= (0.0012, 0, 0.027, 0.56, 1, 0), \end{aligned} \quad (56)$$

we obtain the realistic quark masses and mixings shown in Table III.

TABLE III. Mass ratios and mixings evaluated with values of complex structure moduli on the first T^2, the Higgs VEVs, and the Wilson lines in Eq. (56).

$(m_u/m_t, m_c/m_t)$	$(7.2\times 10^{-6}, 3.2\times 10^{-3})$
$(m_d/m_b, m_s/m_b)$	$(1.1\times 10^{-3}, 2.1\times 10^{-2})$
$\lvert V_{\mathrm{CKM}}\rvert$	$\begin{pmatrix} 0.97 & 0.23 & 0.0019 \\ 0.23 & 0.97 & 0.033 \\ 0.0095 & 0.031 & 1.0 \end{pmatrix}$

IV. LEPTON SECTOR

Here, we provide comments on the lepton sector.

As mentioned in Sec. II A, when magnetic flux and Wilson lines along the $U(1)_3$ direction are vanishing, the $SU(3)_C$ gauge symmetry is enhanced to $SU(4)$. In such a case, the charged lepton mass matrix is the same as the down-sector quark mass matrix. Let us consider the model where this $SU(4)$ is broken only by Wilson lines. That is, we introduce different Wilson lines between the down-sector quarks and the charged lepton sectors. Then the charged lepton mass matrix corresponding to Sec. III A can be written

$$M^l = g\langle H_{d2}\rangle \begin{pmatrix} \eta_{8,\zeta_{l_1}}\rho_{d1} & \eta_{8,\zeta_{l_2}}\rho_{d2} & \eta_{8,\zeta_{l_3}} \\ \eta_{4,\zeta_{l_1}}\rho_{d1} & \eta_{4,\zeta_{l_2}}\rho_{d2} & \eta_{4,\zeta_{l_3}} \\ \eta_{0,\zeta_{l_1}}\rho_{d1} & \eta_{0,\zeta_{l_2}}\rho_{d2} & \eta_{0,\zeta_{l_3}} \end{pmatrix} \quad (57)$$

for the $SU(3)_f$ model. Here, the new parameters in the lepton sector are the Wilson lines, ζ_{l_i}. The experimental values of mass ratios in the charged lepton sector, m_e/m_τ and m_μ/m_τ, are similar to those in the down-sector quarks, m_d/m_b and m_s/m_b. Thus, we can realize the charged lepton mass ratios by setting $\zeta_{l_i} \sim \zeta_{d_i}$. Similarly, we can discuss the charged lepton sector for the $\Delta(27)$ model. Thus, it is straightforward to realize the charged lepton mass ratios in both the $SU(3)_f$ model and the $\Delta(27)$ model.

We may assign the right-handed neutrinos such that they can couple with the left-handed leptons and the up-sector Higgs scalars. That is the assignment in Sec. II. Then, in order to discuss the neutrino masses, we need to study the origin of right-handed Majorana masses. Our models do not include singlets, whose VEVs become right-handed Majorana mass terms in the three-point couplings, because of gauge invariances of extra $U(1)$ symmetries. Thus, right-handed Majorana mass terms would be generated by higher dimensional terms or nonperturbative terms. Such nonperturbative terms may be constrained by extra anomalous $U(1)$ symmetries because factors in the nonperturbative terms, $e^{-aS-b_iT_i}$, have anomalous $U(1)$ charges.

In the $SU(3)_f$ model, the three generations of neutrinos in the above assignment correspond to an $SU(3)_u$ triplet and they have the same extra $U(1)$ charge. Thus, their Majorana mass terms cannot be generated unless the $SU(3)_u$ symmetry is broken. On the other hand, once the $SU(3)_u$ symmetry is broken, such mass terms would be generated, but the pattern depends on the breaking pattern. For example, it is possible to break $SU(3)_u$ such that breaking does not induce a large mass ratio among the triplets and the Majorana mass terms realize large mixing angles.

In the $\Delta(27)$ model, three generations of right-handed neutrinos are $\Delta(27)$ triplets. Again, unless the $\Delta(27)$ symmetry is broken, their Majorana mass terms are not generated. On the other hand, nonperturbative effects may break the $\Delta(27)$ symmetry.[4] In such a case, all entries may be allowed. Because three generations of right-handed neutrinos have the same extra $U(1)$ charges, those entries in the Majorana mass would be of the same order, and we may have large mixing angles.

Also, we can comment on the Higgs μ-term matrix. Our models have no singlets S, which have perturbative three-point couplings with the Higgs pairs, SH_uH_d, such as the next-to-minimal supersymmetric standard model, because extra $U(1)$ symmetries forbid such couplings. Higher order couplings or nonperturbative effects would generate the μ terms. In the $SU(3)_f$ model, H_u and H_d are triplets under $SU(3)_u$ and $SU(3)_d$, respectively. Thus, unless those symmetries are broken, μ terms cannot be generated. Similar to the above comment on the neutrino masses, the pattern of the μ-term matrix depends on their breaking. It is plausible that the triplets develop VEVs similar to $\langle H_{u0}\rangle \sim \langle H_{u1}\rangle \sim \langle H_{u2}\rangle$ and $\langle H_{d0}\rangle = \langle H_{d1}\rangle = \langle H_{d2}\rangle$. The situation of the μ term in the $\Delta(27)$ is similar.

V. CONCLUSION

We have studied quark mass matrices in $SO(32)$ heterotic string theory on 6D tori with magnetic fluxes. We have examined two models, the $SU(3)_f$ flavor model and the $\Delta(27)$ model. In both models, we have realized realistic quark masses and mixing angles by using our parameters, the complex structure, and Wilson lines, as well as Higgs VEV ratios. We could discuss the charged lepton masses similarly.

We have used the complex structure and the Wilson lines as free parameters. It is important to discuss the dynamics for determining those values. Doing so is beyond the scope of this paper.

Our models do not have Majorana right-handed neutrino mass terms at tree level or singlets, such that they have three-point couplings with right-handed neutrinos at tree level and their VEVs induce neutrino mass terms. Majorana right-handed neutrino mass terms may be generated by higher dimensional operators[5] and/or nonperturbative effects. Indeed, nonperturbative computations for inducing Majorana neutrino mass terms were studied in magnetized D-brane models [22,24]. Thus, it would be quite interesting to apply such discussions to $SO(32)$ heterotic string theory. We will study this scenario elsewhere.

ACKNOWLEDGMENTS

H. A. was supported in part by Grant-in-Aid for Scientific Research No. 25800158 from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) in Japan. T. K. was supported in part by Grant-in-Aid for Scientific Research No. 25400252 from MEXT. H. O. was supported in part by Grant-in-Aid for JSPS Fellows No. 26-7296.

[4]See [21,22] for anomalies of non-Abelian discrete symmetries.

[5]See [23] for higher dimensional operators in magnetized brane models.

[1] L. E. Ibanez and A. M. Uranga, *String Theory and Particle Physics: An Introduction to String Phenomenology* (Cambridge University Press, Cambridge, England, 2012).

[2] H. Abe, T. Kobayashi, H. Otsuka, and Y. Takano, Realistic three-generation models from SO(32) heterotic string theory, J. High Energy Phys. 09 (2015) 056.

[3] R. Blumenhagen, G. Honecker, and T. Weigand, Loop-corrected compactifications of the heterotic string with line bundles, J. High Energy Phys. 06 (2005) 020; Supersymmetric (non-)Abelian bundles in the Type I and SO(32) heterotic stringJ. High Energy Phys.08 (2005) 009.

[4] H. Abe, T. Kobayashi, H. Otsuka, Y. Takano, and T. H. Tatsuishi, Gauge coupling unification in SO(32) heterotic string theory with magnetic fluxes, Prog. Theor. Exp. Phys. **2016**, 053B01 (2016).

[5] K. S. Choi, T. Kobayashi, R. Maruyama, M. Murata, Y. Nakai, H. Ohki, and M. Sakai, $E_{6,7,8}$ magnetized extra dimensional models, Eur. Phys. J. C **67**, 273 (2010); T. Kobayashi, R. Maruyama, M. Murata, H. Ohki, and M. Sakai, Three-generation models from E_8 magnetized extra dimensional theory, J. High Energy Phys. 05 (2010) 050.

[6] V. S. Kaplunovsky, One loop threshold effects in string unification, Nucl. Phys. **B307**, 145 (1988); Erratum, Nucl. Phys. **B382**, 436(E) (1992).

[7] L. J. Dixon, V. Kaplunovsky, and J. Louis, Moduli dependence of string loop corrections to gauge coupling constants, Nucl. Phys. **B355**, 649 (1991).

[8] J. P. Derendinger, S. Ferrara, C. Kounnas, and F. Zwirner, On loop corrections to string effective field theories: Field dependent gauge couplings and sigma model anomalies, Nucl. Phys. **B372**, 145 (1992).

[9] L. E. Ibanez, D. Lust, and G. G. Ross, Gauge coupling running in minimal $SU(3) \times SU(2) \times U(1)$ superstring unification, Phys. Lett. B **272**, 251 (1991); L. E. Ibanez and D. Lust, Duality anomaly cancellation, minimal string unification and the effective low-energy Lagrangian of 4-D strings, Nucl. Phys. **B382**, 305 (1992).

[10] H. Kawabe, T. Kobayashi, and N. Ohtsubo, Minimal string unification and constraint on hidden sector, Nucl. Phys.

B434, 210 (1995); T. Kobayashi, Minimal string unification and Yukawa couplings in orbifold models, Int. J. Mod. Phys. A **10**, 1393 (1995); R. Altendorfer and T. Kobayashi, Implications of nonuniversal soft masses on gauge coupling unification, Int. J. Mod. Phys. A **11**, 903 (1996).

[11] D. Cremades, L. E. Ibanez, and F. Marchesano, Computing Yukawa couplings from magnetized extra dimensions, J. High Energy Phys. 05 (2004) 079.

[12] H. Abe, K. S. Choi, T. Kobayashi, and H. Ohki, Three generation magnetized orbifold models, Nucl. Phys. **B814**, 265 (2009); H. Abe, T. Kobayashi, H. Ohki, A. Oikawa, and K. Sumita, Phenomenological aspects of 10D SYM theory with magnetized extra dimensions, Nucl. Phys. **B870**, 30 (2013); H. Abe, T. Kobayashi, K. Sumita, and Y. Tatsuta, Gaussian Froggatt-Nielsen mechanism on magnetized orbifolds, Phys. Rev. D **90**, 105006 (2014); T. h. Abe, Y. Fujimoto, T. Kobayashi, T. Miura, K. Nishiwaki, M. Sakamoto, and Y. Tatsuta, Classification of three-generation models on magnetized orbifolds, Nucl. Phys. **B894**, 374 (2015).

[13] T. Kobayashi, S. Raby, and R. J. Zhang, Searching for realistic 4d string models with a Pati-Salam symmetry: Orbifold grand unified theories from heterotic string compactification on a Z_6 orbifold, Nucl. Phys. **B704**, 3 (2005).

[14] T. Kobayashi, H. P. Nilles, F. Ploger, S. Raby, and M. Ratz, Stringy origin of non-Abelian discrete flavor symmetries, Nucl. Phys. **B768**, 135 (2007).

[15] H. Abe, K. S. Choi, T. Kobayashi, and H. Ohki, Non-Abelian discrete flavor symmetries from magnetized/intersecting brane models, Nucl. Phys. **B820**, 317 (2009).

[16] M. Berasaluce-Gonzalez, P. G. Camara, F. Marchesano, D. Regalado, and A. M. Uranga, Non-Abelian discrete gauge symmetries in 4d string models, J. High Energy Phys. 09 (2012) 059.

[17] G. Altarelli and F. Feruglio, Discrete flavor symmetries and models of neutrino mixing, Rev. Mod. Phys. **82**, 2701 (2010).

[18] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, and M. Tanimoto, Non-Abelian discrete symmetries in particle physics, Prog. Theor. Phys. Suppl. **183**, 1 (2010); An introduction to non-Abelian discrete symmetries for particle physicists, Lect. Notes Phys. **858**, 87 (2012).

[19] S. F. King and C. Luhn, Neutrino mass and mixing with discrete symmetry, Rep. Prog. Phys. **76**, 056201 (2013).

[20] Z. z. Xing, H. Zhang, and S. Zhou, Updated values of running quark and lepton masses, Phys. Rev. D **77**, 113016 (2008).

[21] T. Araki, T. Kobayashi, J. Kubo, S. Ramos-Sanchez, M. Ratz, and P. K. S. Vaudrevange, (Non-)Abelian discrete anomalies, Nucl. Phys. **B805**, 124 (2008).

[22] Y. Hamada, T. Kobayashi, and S. Uemura, Flavor structure in D-brane models: Majorana neutrino masses, J. High Energy Phys. 05 (2014) 116.

[23] H. Abe, K. S. Choi, T. Kobayashi, and H. Ohki, Higher order couplings in magnetized brane models, J. High Energy Phys. 06 (2009) 080.

[24] R. Blumenhagen, M. Cvetic, and T. Weigand, Spacetime instanton corrections in 4D string vacua: The seesaw mechanism for D-brane models, Nucl. Phys. **B771**, 113 (2007); L. E. Ibanez and A. M. Uranga, Neutrino Majorana masses from string theory instanton effects, J. High Energy Phys. 03 (2007) 052; L. E. Ibanez, A. N. Schellekens, and A. M. Uranga, Instanton induced neutrino Majorana masses in CFT orientifolds with MSSM-like spectra, J. High Energy Phys. 06 (2007) 011; S. Antusch, L. E. Ibanez, and T. Macri, Neutrino masses and mixings from string theory instantons, J. High Energy Phys. 09 (2007) 087; M. Cvetic, R. Richter, and T. Weigand, Computation of D-brane instanton induced superpotential couplings: Majorana masses from string theory, Phys. Rev. D **76**, 086002 (2007); T. Kobayashi, Y. Tatsuta, and S. Uemura, Majorana neutrino mass structure induced by rigid instantons on toroidal orbifold, Phys. Rev. D **93**, 065029 (2016).