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Seven-disk manifold, α-attractors, and B modes

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Cosmological α-attractor models in $\mathcal{N}=1$ supergravity are based on the hyperbolic geometry of a Poincaré disk with the radius square $\mathcal{R}^2=3\alpha$. The predictions for the B modes, $r\approx 3\alpha\frac{4}{N^2}$, depend on moduli space geometry and are robust for a rather general class of potentials. Here we notice that starting with M theory compactified on a 7-manifold with G_2 holonomy, with a special choice of Betti numbers, one can obtain $d=4$, $\mathcal{N}=1$ supergravity with the rank 7 scalar coset $[\frac{SL(2)}{SO(2)}]^7$. In a model where these seven unit size Poincaré disks have identified moduli one finds that $3\alpha=7$. Assuming that the moduli space geometry of the phenomenological models is inherited from this version of M theory, one would predict $r\approx 10^{-2}$ for $N=53$ e-foldings. We also describe the related maximal supergravity and M/string theory models leading to preferred values $3\alpha = 1, 2, 3, 4, 5, 6, 7$.

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I. INTRODUCTION

To compare the predictions of theoretical models with the observational data on inflationary cosmology [1] one has to use some form of the $d=4$ Einstein theory. In particular, one can use $\mathcal{N}=1$ supergravity models making a choice of the Kähler potential and a superpotential to fit the data. Cosmological models called α-attractor models [2–6], based on hyperbolic geometry of a Poincaré disk with the radius square 3α, are in good agreement with the data. The tilt of the spectrum of fluctuations and the level of B models depend on the number of e-foldings N and on the moduli space curvature $\mathcal{R}_{\text{Kähler}}=-\frac{2}{3\alpha}$,

$$n_s\approx 1-\frac{2}{N}, \qquad r\approx 3\alpha\frac{4}{N^2}. \tag{1.1}$$

This prediction is valid for α-attractor models with $\alpha\lesssim O(10)$ for a rather general class of potentials described in [2–6]. The early versions of these models were derived in [2], and the more advanced versions were presented in [3–6]. At the level of phenomenological $\mathcal{N}=1$ supergravity any value of $0<3\alpha<\infty$ is acceptable. In advanced α-attractor models the stabilization of all scalars but the inflaton is possible for all values of α. Therefore one can view the future detection of the B modes, or a new bound on r, as an experimental information about the curvature of the moduli space in these models.

One may try to motivate certain preferred values of the Poincaré disk radius square 3α as originating from a fundamental theory underlying $\mathcal{N}=1$ supergravity. It was already suggested in [3] that the lowest possible value $3\alpha=1$, with one unit size Poincaré disk, is motivated by a maximal superconformal $\mathcal{N}=4$ theory [7] and $\mathcal{N}=4$ pure supergravity without matter [8].

In this note we will study the possible origin of the moduli space geometries in maximal $\mathcal{N}=8$ supergravity and M/string theory. We assume that when the maximally supersymmetric theories are reduced to $\mathcal{N}=1$ phenomenological α-attractor models, some mechanism of generating the required potentials will take place, but the moduli space geometry will be inherited from the more fundamental theories.

In this setting we will find reasonably well motivated models of the Poincaré disk with radius square 3α taking values 1,2,3,4,5,6,7. In particular, the case with the highest value of $3\alpha=7$ suggests that r is only slightly below 10^{-2}.

Joint analysis of the data from the BICEP2/Keck and Planck experiments [1] yields an upper limit on B modes, $r\leq 7\times 10^{-2}$. The new interesting target with preferred values of α originating in M/string theory, for the number of e-foldings $47<N<57$, is now

$$3\alpha=7:\ r\approx 7\frac{4}{N^2}, \qquad 0.86\times 10^{-2}<r<1.3\times 10^{-2}, \tag{1.2}$$

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and the lowest one in the context of maximal $\mathcal{N}=4$ superconformal theory is

$$3\alpha = 1:\ r \approx \frac{4}{N^2}, \qquad 1.2\times 10^{-3} < r < 1.8\times 10^{-3}. \tag{1.3}$$

II. POINCARÉ DISK WITH THE RADIUS SQUARE 3α

Consider the Kähler potential

$$K = -3\alpha \ln(1 - Z\bar{Z}). \tag{2.1}$$

It describes a Poincaré disk with the radius square 3α. The metric of the moduli space is $g_{Z\bar{Z}} = K_{Z\bar{Z}} = \frac{3\alpha}{(1-Z\bar{Z})^2}$. The Kähler manifold curvature computed from this metric depends on α,

$$\mathcal{R}_{\text{Kähler}} = -g^{-1}_{Z\bar{Z}}\partial_Z\partial_{\bar{Z}} \log g_{Z\bar{Z}} = -\frac{2}{3\alpha}. \tag{2.2}$$

The kinetic term for the complex scalar field is

$$ds^2 = \frac{3\alpha}{(1-Z\bar{Z})^2} dZ d\bar{Z} = \frac{dx^2+dy^2}{(1-\frac{x^2+y^2}{3\alpha})^2}, \tag{2.3}$$

where $Z = (x+iy)/\sqrt{3\alpha}$.

Advanced models of inflation which use the Kähler potential above as a part of the total moduli space geometry typically have another chiral superfield,[1] often called a "stabilizer" superfield S in [3–6] so that

$$K = -3\alpha \ln(1 - Z\bar{Z}) + S\bar{S}. \tag{2.4}$$

In these models the light inflaton is in the $Z+\bar{Z}$ direction, whereas the remaining three scalars, the complex stabilizer S, and the sinflaton $Z-\bar{Z}$ are stabilized by the potentials to the values $S = Z-\bar{Z} = 0$. It was shown in [5] that such a stabilization of the sinflaton is possible for all values of α, whereas stabilizing S at zero might require additional terms $(S\bar{S})^2$ in the Kähler potential.[2] An even more interesting possibility is to use models with constrained orthogonal superfields $S^2[x,\theta]=0$ and $S(Z-\bar{Z})[x,\theta,\bar{\theta}]=0$ [6], where the "unwanted scalars" S and $Z-\bar{Z}$ are not fundamental fields; they depend on fermions and do not participate in cosmological evolution. The constraints on the curvature of the moduli space in [11] do not apply to advanced α-attractor models, with inflaton and stabilizer, where all moduli, with exception of the inflaton, are stabilized. In these models the observable combinations of the slow-roll parameters $n_s = 1 - 6\epsilon + 2\eta$ and $r = 16\epsilon$ in Eq. (1.1) are in good agreement with the data, and there is no η problem for all values of α.

For the vanishing sinflaton the kinetic term becomes in terms of the inflaton $Z = \bar{Z} = \tanh\frac{\varphi}{\sqrt{6\alpha}}$

$$3\alpha \frac{\partial_\mu Z \partial^\mu Z}{(1-Z)^2} = \frac{1}{2}(\partial_\mu\varphi)^2. \tag{2.5}$$

In these models the potentials depend on a geometric variable $Z = \bar{Z}$,

$$V = V\left(\tanh\frac{\varphi}{\sqrt{6\alpha}}\right). \tag{2.6}$$

A. HALF-PLANE VARIABLES

One can use an alternative description of the same physical system by making a choice $\frac{1+Z}{1-Z} = -i\tau$,

$$K = -3\alpha \ln(-i(\tau - \bar{\tau})). \tag{2.7}$$

The kinetic term for the complex scalar field is

$$ds^2 = 3\alpha \frac{d\tau d\bar{\tau}}{(2\text{Im}\tau)^2}. \tag{2.8}$$

In this form the kinetic term has an $SL(2,\mathbb{R})$ symmetry

$$\tau' = \frac{a\tau + b}{c\tau + d}, \qquad ad - bc \neq 0, \tag{2.9}$$

where a, b, c, d are real numbers and

$$\frac{d\tau d\bar{\tau}}{(\tau-\bar{\tau})^2} = \frac{d\tau' d\bar{\tau}'}{(\tau'-\bar{\tau}')^2}. \tag{2.10}$$

When the sinflaton $\tau + \bar{\tau}$ vanishes at $\tau = -\bar{\tau} = ie^{\sqrt{\frac{2}{3\alpha}}\varphi}$,

$$3\alpha \frac{d\tau d\bar{\tau}}{(2\text{Im}\tau)^2} = \frac{1}{2}(\partial_\mu\varphi)^2. \tag{2.11}$$

III. SEVEN-DISK GEOMETRY IN MAXIMAL SUPERGRAVITY

Before looking at M theory on a 7-manifold with G_2 holonomy we will explain the origin of the seven-disk geometry starting from $D=4$, $\mathcal{N}=8$ supergravity. M theory/$d=11$ supergravity can be compactified on a

[1] In the case of a single superfield there are consistent inflation models for all α, including $\alpha \le 1$; see for example [3,9]. However, this case requires the modification of the Kähler potential as shown in Eq. (24) in [3], and in Eq. (1.4) in [9]. The relevant modification changes the value of the curvature [3,10].

[2] In earlier models with the S dependence inside the logarithm one may use the bisectional curvature $R_{Z\bar{Z}S\bar{S}}$ to stabilize the inflaton partner for all values of α; see for example [3], Eq. (29).

7-torus, which leads to $d = 4$ maximal $\mathcal{N} = 8$ supergravity [12] upon dualization of the form fields. This model has 70 scalars in the coset space $\frac{E_{7(7)}}{SU(8)}$ and $E_{7(7)}$ symmetry. Following [13], we consider truncation of $\mathcal{N} = 8$ supergravity [12] to $\mathcal{N} = 4$ supergravity interacting with six $\mathcal{N} = 4$ vector multiplets. The $E_{7(7)}$ symmetry is decomposed as follows:

$$E_{7(7)} \supset [SL(2)] \times SO(6,6). \tag{3.1}$$

The 70 scalars of $\mathcal{N} = 8$ supergravity [12] are first truncated to

$$70 \to 2 + 36. \tag{3.2}$$

In the next step one takes into account that

$$SO(6,6) \supset SO(2,2) \times SO(2,2) \times SO(2,2) \tag{3.3}$$

and

$$36 \to 3 \times 4 \tag{3.4}$$

so that

$$70 \to 2(1+6) = 2 \times 7 = 14. \tag{3.5}$$

This truncation has a Kähler structure supporting $\mathcal{N} = 1$ supersymmetry. One can identify seven Poincaré disks due to the decomposition

$$E_{7(7)}(\mathbb{R}) \supset [SL(2,\mathbb{R})]^7. \tag{3.6}$$

The original kinetic term is reduced to a form with the Kähler potential of the form

$$K = -\sum_{i=1}^{7} \ln(-i(\tau_i - \bar{\tau}_i)) \tag{3.7}$$

with seven pairs of independent scalars and the $[SL(2,\mathbb{R})]^7$ symmetry, a seven-disk manifold. The fact that the disk of the $SL(2)$ commuting with $SO(6,6)$ has the same Kähler curvature of the other six $SL(2)/SO(2)$ (each separately corresponding to $\alpha = 1/3$) can be understood by string triality arguments [14] and by the underlying special geometry of the $N = 2$ truncation [15].

IV. M THEORY ON A 7-MANIFOLD WITH G_2 HOLONOMY

Instead of a compactification on a 7-torus, one can compactify M theory on a 7-manifold with G_2 holonomy. The early investigation of G_2 holonomy in physics was performed in [16], with a review of the first 20 years in [17]. One of the most recent applications of this compactification can be found in [18], and, of course, many more studies of M theory on G_2 were performed over the years.

Here we will focus on the model studied in [19,20], which requires the following choice of the Betti numbers:

$$(b_0, b_1, b_2, b_3) = (1, 0, 0, 7). \tag{4.1}$$

This theory is identified with the maximal rank reduction on the 7-torus and leads directly to $d = 4$, $\mathcal{N} = 1$ "curious supergravity" where seven complex scalars are coordinates of the coset space

$$\left[\frac{SL(2,\mathbb{R})}{SO(2)}\right]^7. \tag{4.2}$$

The corresponding Kähler potential describing the scalar sector of this theory is the one in Eq. (3.7) with seven pairs of independent scalars and the $[SL(2,\mathbb{R})]^7$ symmetry. This model is one of the "four curious supergravities" defined in [20]. The other three have $\mathcal{N} = 2$, $\mathcal{N} = 4$, $\mathcal{N} = 8$ supersymmetries, and we are interested only in $\mathcal{N} = 1$ "curious supergravity." It has the field content defined by Betti numbers: the $d = 4$ fields originating from the $d = 11$ metric g_{MN} are

$$\begin{aligned} g_{\mu\nu} &\to b_0 = 1, \\ A_\mu &\to b_1 = 0, \\ \mathcal{A} &\to b_1 + b_3 = 7. \end{aligned} \tag{4.3}$$

The ones from $d = 11$ gravitino ψ_M are

$$\begin{aligned} \psi_\mu &\to b_0 + b_1 = 1, \\ \chi &\to b_2 + b_3 = 7. \end{aligned} \tag{4.4}$$

The ones from the 3-form A_{MNP} are

$$\begin{aligned} A_{\mu\nu\rho} &\to b_0 = 1, \\ A_{\mu\nu} &\to b_1 = 0, \\ A_\mu &\to b_2 = 0, \\ A &\to b_3 = 7. \end{aligned} \tag{4.5}$$

To summarize, the field content of the M theory compactified on a 7-manifold with G_2 holonomy and Betti numbers (4.1) is a metric, a gravitino, and a 3-form (which has no degrees of freedom but affects trace anomaly)

$$g_{\mu\nu}, \psi_\mu, A_{\mu\nu\rho} \tag{4.6}$$

and seven scalars, seven spin 1/2 fields, and seven pseudoscalars

$$\tau_i = \mathcal{A}_i + iA_i, \chi_i. \tag{4.7}$$

The corresponding Kähler geometry is the seven-disk manifold in (3.7).

For generic Betti numbers (b_0, b_1, b_2, b_3) these models are known to have a generalized mirror symmetry, which flips one set of Betti numbers into the other one,

$$(b_0, b_1, b_2, b_3) \to (b_0, b_1, b_2 - \rho/2, b_3 + \rho/2), \quad (4.8)$$

and $\rho \equiv 7b_0 - 5b_1 + 3b_2 - b_3$ changes the sign. One of the reasons the model we describe here was given the name curious supergravity is that it has $\rho = 0$; it is a *self-mirror* in the above sense. It also means that it has a vanishing Weyl anomaly $g_{\mu\nu}\langle T^{\mu\nu}\rangle = -\frac{\rho}{24\times 32\pi^2} \times R^{*\mu\nu\rho\sigma}R^{*}_{\mu\nu\rho\sigma} = 0$, and the presence of the 3-form $A_{\mu\nu\rho}$ is important for this.

To connect this compactified M theory model to α-attractor geometry we can make a choice that all moduli in our seven unit radius disks in (3.7) are identified, namely

$$3\alpha = 7: \ \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6 = \tau_7 \equiv \tau. \quad (4.9)$$

We are left with one Poincaré disk of the radius square 7 times larger than the unit radius square,

$$K = -\sum_{i=1}^{7} \ln(-i(\tau_i - \bar{\tau}_i)) = -7\ln(-i(\tau - \bar{\tau})), \quad (4.10)$$

$$ds^2 = 7\frac{d\tau d\bar{\tau}}{(2\mathrm{Im}\tau)^2}. \quad (4.11)$$

The following interpretation of this identification can be suggested: the diagonal components of the internal space metric g_{ij} are taken to be the same in all seven directions, $g_{ij} \sim \delta_{ij}$, and the 3-form A_{ijk}, which leads to seven pseudoscalars in $d = 4$, since $b_3 = 7$, is also the same in all directions. An analogous identification was performed in [21], where an early dimensional reduction of superstring theories was studied. The resulting $d = 4$, $\mathcal{N} = 1$ supergravity, neglecting the matter fields C in [21], has the following Kähler manifold:

$$K = -\ln(-i(s - \bar{s})) - 3\ln(-i(t - \bar{t})). \quad (4.12)$$

We will show in the next section that using string theory compactification on a product of 3-tori $T_2 \times T_2 \times T_2 \subset T_6$ one can get the seven-disk geometry.

$$\begin{aligned} K = &-\ln(-i(s - \bar{s})) - \ln(-i(t_1 - \bar{t}_1)) \\ &- \ln(-i(t_2 - \bar{t}_2)) - \ln(-i(t_3 - \bar{t}_3)) \\ &- \ln(-i(u_1 - \bar{u}_1)) - \ln(-i(u_2 - \bar{u}_2)) \\ &- \ln(-i(u_3 - \bar{u}_3)). \end{aligned} \quad (4.13)$$

Thus, the model (4.12) in [21] corresponds to the one in (4.13) under the condition that

$$t_1 = t_2 = t_3 = t, \qquad u_1 = u_2 = u_3 = \text{const}. \quad (4.14)$$

This means that some fields of higher-dimensional geometry were discarded, for example, all u_i fields and the difference between t_i fields. If instead we would impose on (4.13) the condition

$$s = t_1 = t_2 = t_3 = u_1 = u_2 = u_3 = \tau, \quad (4.15)$$

we would reproduce the Kähler geometry (4.10) of the single Poincaré disk of the radius square $3\alpha = 7$. In an analogous manner we can get other values

$$3\alpha = \{1, 2, 3, 4, 5, 6, 7\} \quad (4.16)$$

by requiring that

$$\begin{aligned} &3\alpha = 7: \ \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6 = \tau_7 \equiv \tau \\ &3\alpha = 6: \ \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6 \equiv \tau, \quad \tau_7 = \text{const}, \\ &3\alpha = 5: \ \tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 \equiv \tau, \quad \tau_6 = \tau_7 = \text{const}, \\ &3\alpha = 4: \ \tau_1 = \tau_2 = \tau_3 = \tau_4 \equiv \tau, \quad \tau_5 = \tau_6 = \tau_7 = \text{const} \\ &3\alpha = 3: \ \tau_1 = \tau_2 = \tau_3 \equiv \tau, \quad \tau_4 = \tau_5 = \tau_6 = \tau_7 = \text{const}, \\ &3\alpha = 2: \ \tau_1 = \tau_2 \equiv \tau, \quad \tau_3 = \tau_4 = \tau_5 = \tau_6 = \tau_7 = \text{const}, \\ &3\alpha = 1: \ \tau_1 \equiv \tau, \quad \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6 = \tau_7 = \text{const}. \end{aligned} \quad (4.17)$$

We illustrate in Fig. 1 the features of α-attractor models [2,3,6] with the seven-disk geometry using the recent discussion of B modes in the CMB-S4 Science Book [22]. We show in Fig. 1 predictions of α-attractor models with seven-disk geometry in the $n_s - r$ plane for $N \sim 55$, for the minimal value $3\alpha = 1$ and for the maximal value $3\alpha = 7$.

The constraints on the fields presented in Eqs. (4.14) and (4.17) are expected to emerge as a consequence of specifically designed potentials, which align the fields in the desirable direction. We have not yet presented such potentials here; these are still to be constructed. Examples when the structure of the potential was designed to put constraints on the field of the theory are known in cosmological literature. A relevant mechanism was invented in [23] in models known as aligned natural inflation. In the simplest case with two axions the potential enforces one combination of axions to be light and serve as an inflaton, whereas the other one is heavy and drops from the evolution. This type of alignment was also generalized to many axion models. Another example is N-flation in [24] where there are N axions. In polar coordinates only the radial direction is a light inflaton field, and all other angular

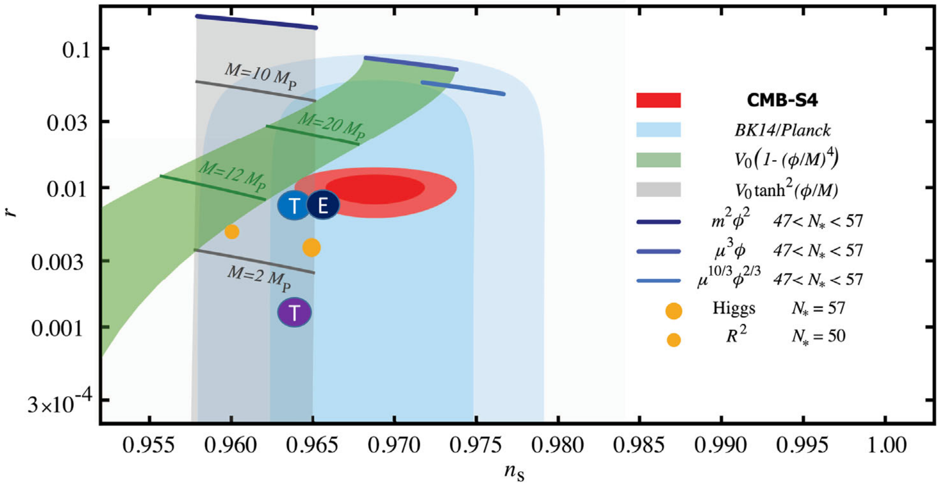

FIG. 1. Taken from [22], this figure represents a forecast of CMB-S4 constraints in the $n_s - r$ plane for a fiducial model with $r = 0.01$. Here the grey band shows predictions of the subclass of α-attractor models [2,3,6]. We have added to this figure a blue circle with the letter T inside it corresponding to the highest preferred value $3\alpha = 7$ and the purple one corresponding to the lowest preferred value $3\alpha = 1$ in a seven-disk geometry. All intermediate cases $3\alpha = \{1,2,3,4,5,6,7\}$ are between these two. They all describe the class of α-attractor models with $V \sim \tanh^2(\varphi/\sqrt{6\alpha})$, so-called quadratic T models. The quadratic E models with $V \sim (1 - e^{\sqrt{2/3\alpha}\varphi})^2$ tend to be slightly to the right of the T models; see [2]. We show them as a navy circle with the letter E inside it.

variables drop from the evolution under certain assumptions specified in these class of models.

V. VALUES OF 3α IN STRING THEORY

Here we will show how to derive the seven-disk geometry (4.13) in string theory. We start with the derivation of noncompact symmetries in string theory following [25,26]. The toroidal compactification to $d = 4$ of the $\mathcal{N} = 1$ supergravity/string theory in $d = 10$ spacetime leads to scalars in $\frac{SO(6,6)}{SO(6)\times SO(6)}$ coset space[3] upon truncation of nongeometric moduli from the $d = 10$ vector multiplets.

As the result of the dimensional reduction one finds a $d = 4$ action for the scalars of the following form:

$$\int d^4x\sqrt{-g}e^{-\phi}(\mathcal{L}_1 + \mathcal{L}_2). \tag{5.1}$$

Here

$$\mathcal{L}_1 = R + g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}, \tag{5.2}$$

and

$$\mathcal{L}_2 = \frac{1}{8}\,\mathrm{tr}(\partial_\mu M^{-1}\partial^\mu M). \tag{5.3}$$

Here M is a symmetric $O(6,6)$ matrix

$$M = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix}, \tag{5.4}$$

where $G_{\alpha\beta}$ and $B_{\alpha\beta}$ are the internal space metric and a 2-form, $\alpha, \beta = 1, \ldots, 6$. Together they represent the 36 coordinates of the coset space $\frac{SO(6,6)}{SO(6)\times SO(6)}$, and we recover the moduli space of the 6-torus T_6 in string theory. We now would like to perform the truncation of the 6-torus to three T_2 so that

$$T_2 \times T_2 \times T_2 \subset T_6. \tag{5.5}$$

This corresponds to the reduction $SO(6,6) \supset [SO(2,2)]^3$ and analogous reduction on the coset representative

$$\frac{SO(6,6)}{SO(6)\times SO(6)} \to \left[\frac{SO(2,2)}{SO(2)\times SO(2)}\right]^3. \tag{5.6}$$

This means that we keep the following nine components of $G_{\alpha\beta}$:

$$G_{(IJ)} = (g_{11}, g_{22}, g_{12}; g_{33}, g_{44}, g_{34}; g_{55}, g_{66}, g_{56}), \tag{5.7}$$

and three components of $B_{\alpha\beta}$

[3]In general, in the case of the heterotic string theory one finds scalars in the $\frac{SO(6,6+n)}{SO(6)\times SO(6+n)}$ coset space. Here the scalars in the $\frac{SO(6,6)}{SO(6)\times SO(6)}$ part of the coset space originate from the geometric moduli, whereas the additional ones with $n \neq 0$ originate from the matter vector multiplets in $d = 10$. If we keep some of the vector multiplets, so that $n > 0$ we do not find models with Poincaré disk geometry.

$$B_{[IJ]} = (b_{12} \equiv b_1, b_{34} \equiv b_2, b_{56} \equiv b_3). \tag{5.8}$$

We also introduce notation

$$g_1 \equiv g_{11}g_{22} - g_{12}^2, \qquad g_2 \equiv g_{33}g_{44} - g_{34}^2,$$
$$g_3 \equiv g_{55}g_{66} - g_{56}^2. \tag{5.9}$$

Now we observe that the coset $\frac{SO(2,2)}{SO(2)\times SO(2)}$ is isomorphic to $\frac{SL(2,\mathbb{R})}{SO(2)} \times \frac{SL(2,\mathbb{R})}{SO(2)}$, and so we can package the $SO(2,2)$ matrix into an $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ matrix. We will do this for all three copies of $\frac{SO(2,2)}{SO(2)\times SO(2)}$ cosets, following an example of one of them in [26]. We have four real scalars from g_{11}, g_{22}, g_{12}, b_{12}. We package them as follows: $t_1 \equiv b_1 + i\sqrt{g_1}$ and $u_1 \equiv \frac{g_{12}}{g_{22}} + i\frac{\sqrt{g_1}}{g_{22}}$. The inverse relation is for the 2×2 matrices

$$G = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} = \frac{\mathrm{Im}t_1}{\mathrm{Im}u_1}\begin{pmatrix} u_1 u_1^* & \mathrm{Re}u_1 \\ \mathrm{Re}u_1 & 1 \end{pmatrix}, \tag{5.10}$$

$$B = \begin{pmatrix} 0 & b_{12} \\ b_{21} & 0 \end{pmatrix} = \mathrm{Re}t_1 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{5.11}$$

In the same way we can organize all six complex scalars; three of them are often called Kähler moduli,

$$t_1 = b_1 + i\sqrt{g_1}, \qquad t_2 = b_2 + i\sqrt{g_2}, \qquad t_3 = b_3 + i\sqrt{g_3}, \tag{5.12}$$

and the other three are called complex structure moduli,

$$u_1 = \frac{g_{12}}{g_{22}} + i\frac{\sqrt{g_1}}{g_{22}}, \qquad u_2 = \frac{g_{34}}{g_{44}} + i\frac{\sqrt{g_2}}{g_{44}},$$
$$u_3 = \frac{g_{56}}{g_{66}} + i\frac{\sqrt{g_3}}{g_{66}}. \tag{5.13}$$

This corresponds to reorganizing $[\frac{SO(2,2)}{SO(2)\times SO(2)}]^3$ into $[\frac{SL(2,\mathbb{R})}{SO(2)}]^6$. The corresponding Kähler potentials are $K(t_i,\bar{t}_i) = -\ln(-i(t_i - \bar{t}_i))$ and $K(u_i,\bar{u}_i) = -\ln(-i(u_i - \bar{u}_i))$.

One more important step here is to switch from the string frame as in (5.1) to the Einstein frame in $d = 4$, which is a well known procedure of rescaling the metric; see for example [21]. As the result, we find an action with $\mathcal{N} = 1$ supersymmetry and seven complex scalars. The axion, dual to $H_{\mu\nu\lambda}$, and dilaton as shown in Eq. (5.2) form a complex scalar

$$s = a + ie^{\phi} \tag{5.14}$$

with the Kähler potential $K = -\ln(-i(s - \bar{s}))$. The complete Kähler potential of the string theory dimensionally reduced on $T_2 \times T_2 \times T_2 \subset T_6$ is now given by the expression in (4.13) in the previous section, as promised there.

Thus here again we reproduced the seven Poincaré disk geometry of the unit radius each. We may now study the same cases as we did in the previous section: the conclusion is as in M theory compactified on the 7-manifold with G_2 holonomy in Eq. (4.16) which gives us seven possible values of r, according to (1.1), for example, for $N = 55$,

$$r \approx \{1.3, 2.6, 3.9, 5.2, 6.5, 7.8, 9.1\} \times 10^{-3}. \tag{5.15}$$

VI. CONCLUSION

In conclusion, we made an assumption that the moduli space geometry of the phenomenological $\mathcal{N} = 1$ α-attractor models in [2,3,6] is inherited from the M theory compactified on the 7-manifold with G_2 holonomy to a "curious $\mathcal{N} = 1$ supergravity" [20], or from truncated $\mathcal{N} = 8$ maximal supergravity, or from toroidally compactified string theory. In such a case we argued that the possible cosmological α-attractor models might come with the values of $3\alpha = 1$, 2, 3, 4, 5, 6, 7 when some of the higher dimensional fields are discarded, following the procedure employed in the past in [21] and presented in Eq. (4.17). To make a step from preferred values for 3α to a realistic prediction we would need to find the origin of the suitable class of potentials in these theories.

The relevant preferred values of the ratio of the tensor to scalar fluctuations during inflation are shown in Eq. (5.15). We illustrated the position of these models in the $n_s - r$ plane in Fig. 1. The highest one, $r \approx 10^{-2}$, will be the first interesting target for the B-mode experiments as well as for the theoretical studies of realistic cosmological models based on the seven-disk geometry.

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