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Holographic Brownian motion at finite density

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Brownian motion of a heavy charged particle at zero and small (but finite) temperature is studied in the presence of finite density. We are primarily interested in the dynamics at (near) zero temperature which is holographically described by motion of a fundamental string in a (near-)extremal Reissner-Nordström black hole. We analytically compute the functional form of the retarded Green function for small frequencies and extract the dissipative behavior at and near zero temperature.

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I. INTRODUCTION

AdS/CFT or more generally gauge/gravity duality [1–4] has been serving as a great weapon in a theoretician's armory to study strongly coupled systems analytically for almost the last two decades. Although for most of the cases its predictions are qualitative, there are instances (see for example the famous η/s computation in Ref. [5]) when it relates the very formal theoretical framework to real life experiments. Since its discovery this duality has glued many phenomena appearing in apparently different branches of physics together. Studying Brownian motion of a heavy particle using the classical gravity technique is one such example [6,7] where holography relates a statistical system to a gravitational one. The dual gravity description involves a long fundamental string stretching from the boundary of the anti-de Sitter (AdS) space into the black hole horizon (see Fig. 1). Numerous works [8–18] have been done by elaborating on different aspects of this setup.

Integrating out[1] the whole string in that background gives an effective description of the heavy particle at the boundary. Its dynamics is governed by a Langevin equation. For a particle with mass[2] M which is moving with velocity v, the Langevin equation reads

$$M\frac{dv}{dt}+\gamma v=\xi(t) \tag{1.1}$$

$$\text{with}\quad \langle\xi(t)\xi(t')\rangle=\Gamma\delta(t-t'), \tag{1.2}$$

where γ is the viscous drag, ξ is the random force on the particle, and Γ quantifies the strength of the "noise" (i.e, random force). The second equation is one of the many avatars of the celebrated fluctuation-dissipation theorem. One can write down a generalized version[3] of this equation

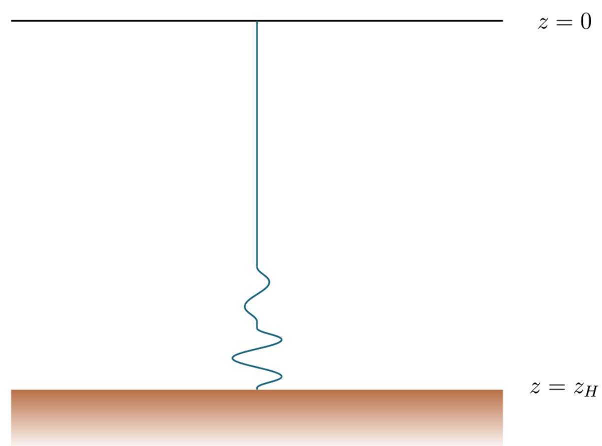

FIG. 1. Gravity set up describing Brownian motion.

$$M_0\frac{d^2x(t)}{dt^2}+\int_{-\infty}^{t}dt'G_R(t,t')x(t')=\xi(t)$$
$$\langle\xi(t)\xi(t')\rangle=iG_{\rm sym}(t,t'). \tag{1.3}$$

$G_R(t,t')$ is thus the same as $\gamma(t-t')$ for the choice of the lower limit of the integral, $t_0=-\infty$, and $iG_{\rm sym}(t,t')$ is the same as $\Gamma(t-t')$.

In frequency space, the generalized Langevin equation takes the following form:

$$[-M_0\omega^2+G_R(\omega)]x(\omega)=\xi(\omega)\quad \langle\xi(-\omega)\xi(\omega)\rangle=iG_{\rm sym}(\omega). \tag{1.4}$$

If the retarded Green function, $G_R(\omega)$, is expanded for small frequencies, the coefficient of $\omega^2(i.e, \frac{d^2x(t)}{dt^2})$ adds

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[1]We mostly follow the Green function language of Ref. [7] to describe Brownian motion.

[2]We will see that this is actually "renormalized" mass. The correction to the *bare mass* (M_0) of the Brownian particle will come from the retarded Green function.

[3]See, for example, Refs. [7,9] for a review of path integral derivation of this generalized Langevin equation. Also notice that this equation is written in terms of the bare mass (M_0) of the Brownian particle.

to the bare mass of the particle, and the coefficient of ω(i.e., $\frac{dx(t)}{dt}$) will show off as the drag term[4]

$$G_R(\omega) = -i\gamma\omega - \Delta M\omega^2 + \cdots \quad (1.5)$$

After defining the renormalized mass as

$$M \equiv M_0 + \Delta M,$$

these generalized Langevin equations (1.4) [up to $\mathcal{O}(\omega^2)$] take the standard form (1.1) and (1.2).

From the above discussion, it is quite clear that if we are interested in studying dissipation for a Brownian particle we just need to compute the retarded Green function, $G_R(\omega)$. We can calculate this quantity using different holographic techniques [19,20] depending on the physical systems.

In Ref. [9], Brownian motion for a heavy quark in $1+1$ dimensions was studied following Ref. [7] which used the prescription of Refs. [19,21] to compute the boundary Green function. The calculation of Ref. [9] was performed in the BTZ black hole background where the system is exactly solvable. The main result was to obtain dissipation for the heavy quark even at zero temperature. The result might look very counterintuitive and unphysical at first sight because at zero temperature the thermal fluctuations go to zero and therefore the Brownian particle should stop dissipating energy. But this zero temperature dissipation has its origin in radiation of an accelerated charged particle. The force term[5] in the Langevin equation at zero temperature was of the form

$$F(\omega) = -i\frac{\sqrt{\lambda}}{2\pi}\omega^3 x(\omega). \quad (1.6)$$

Therefore, the integrated energy loss is given by

$$\Delta E = \frac{\sqrt{\lambda}}{2\pi}\int dt a^2. \quad (1.7)$$

It is known from classical electrodynamics that the energy loss[6] due to radiation is proportional to the square of the acceleration (a). See Refs. [10,26–37] for related works.

Dissipation at zero temperature is a fascinating phenomenon. Its emergence from the retarded Green function signifies that $G_R(\omega)$ actually contains information at the quantum level (by "quantum" here, we mean dynamics at $T=0$). Brownian motion of a particle is usually studied at finite temperature. The system is driven by fluctuations which are thermal in nature, and therefore if the temperature is taken to zero then $G_R(\omega)$ must vanish, too. But the $G_R(\omega)$ we obtain from holography contains information of both thermal and quantum fluctuations for the boundary theory. Although at finite T thermal fluctuations dominate over the quantum ones at very small T, the latter ones are much more important. The main aim of this paper is to understand how a heavy particle's (quark's) motion at finite density (chemical potential $\mu \neq 0$) is described at and near $T=0$. The dual gravity theory should contain a (near-) extremal[7] charged black hole. (See Refs. [38,39] for some results in this setup).

For high temperature regime ($\mu \ll T$), the effect of the charge of the nonextremal black hole can be neglected at the leading order, and $G_R(\omega)$ can be computed in small μ and small ω expansions using the methods followed in Refs. [7,19].

In this paper, we would like to see how the system behaves near $T=0$. Therefore, the other limit $\mu \gg T$, i.e, the low temperature regime, is of more interest to us. We will see that in this regime usual perturbation techniques for small T and small ω will not work because of a double pole for the ω^2 term in the string equation of motion in the extremal black hole background. Due to this double pole, near horizon dynamics is extremely sensitive to ω. To get around this problem, we will adopt the *matching technique*[8] described in Ref. [20] where the authors studied non-Fermi liquids using holography.

The rest of the paper is organized as follows. In Sec. II, we quickly review the Reissner-Nordström (RN) black hole in asymptotically AdS space time. The main purpose is

[4]More terms with higher powers in ω will also be generated in this small frequency expansion. Their interpretations are outside the scope of standard Langevin equation (1.1). But from properties of the Green functions, it is well known that imaginary part of the retarded Green function causes dissipation. Thus, odd powers in ω are responsible for dissipation. Actually, the ω^3 term signifies dissipation at zero temperature [9,18] in the absence of matter density.

[5]This force formula is the same as the "Abraham-Lorentz force" [22] in classical electrodynamics. For electromagnetic theory the coupling constant is $\frac{q^2}{6\pi\epsilon_0}$ where q is the charge of the accelerating particle and ϵ_0 is the vacuum permittivity. In our case the coupling is given by $\frac{\sqrt{\lambda}}{2\pi}$. This similarity is remarkable since in our holographic context the boundary theory is highly nonlinear unlike electrodynamics.

[6]This formula for an accelerated quark was obtained first by Mikhailov in Ref. [23] by a very different approach. The factor $\frac{\sqrt{\lambda}}{2\pi}$ is essentially the Bremsstrahlung function $B(\lambda, N)$ ($2\pi B(\lambda, N) = \frac{\sqrt{\lambda}}{2\pi}$) identified in Ref. [24] as occurring in many other physical quantities (such as the cusp anomalous dimension introduced by Polyakov [25]).

[7]The zero temperature dissipation for a theory dual to pure AdS_{d+1} and black holes in AdS_{d+1} has been calculated in Ref. [18]. Just on dimensional grounds, $G_R(\omega) \sim -i\omega^3$. The coefficient depends on the background. The cause of this dissipation is the radiation due to the accelerated quark.

[8]This matching technique is familiar to string theorists from the brane absorption calculations that led to the discovery of AdS/CFT correspondence. For example, see Refs. [40,41]. Maldacena used a similar technique in his famous decoupling argument [1].

to spell out the notations and conventions that we will be following through out the article. Section III contains the analytic computation of the retarded Green function at zero temperature using the matching technique. We also list some of its properties in detail. The retarded Green function at small but finite temperature is analyzed in Sec. IV. We mainly discuss how $G_R(\omega)$ gets small T corrections. Section V summarizes the main results and their interpretations, assumptions we make, and some future directions. Section VI contains some concluding remarks.

II. ADS-RN BLACK HOLE BACKGROUND

The AdS-RN black hole[9] is a solution to the Einstein-Maxwell equation with a negative cosmological constant,

$$S_{\rm EM} = \frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{-g}\left[R + \frac{d(d-1)}{L_{d+1}^2} + \frac{L_{d+1}^2}{g_F^2}F_{MN}F^{MN}\right]. \tag{2.1}$$

R is the Ricci scalar, and g_F is the dimensionless gauge coupling in the bulk. L_{d+1} is a length scale (known as the AdS radius), and κ^2 is Newton's constant. Notice that we can always redefine the gauge field by absorbing the dimensionless coupling g_F^2 into F_{MN}. Thus, we can fix g_F to 1 without loss of generality. The $(d+1)$-dimensional metric and gauge field that satisfy the corresponding equations of motion are given by

$$ds^2 = \frac{L_{d+1}^2}{z^2}(-f(z)dt^2 + d\vec{x}^2) + \frac{L_{d+1}^2}{z^2}\frac{dz^2}{f(z)}, \tag{2.2}$$

where

$$f(z) = 1 + Q^2 z^{2d-2} - Mz^d$$
$$A_t(z) = \mu\left(1 - \frac{z^{d-2}}{z_0^{d-2}}\right).$$

Q, M, and z_0 are constant parameters which are the black hole charge, black hole mass, and horizon radius, respectively. μ is the chemical potential, $\vec{x} \equiv (x^1, x^2 \ldots x^{d-1})$, and z is the radial coordinate in the bulk such that the boundary of this space is at $z = 0$.

Notice that if we put $f(z) = 1$ we get back pure AdS_{d+1} in the Poincaré patch. This nontrivial function $f(z)$ indicates that the physics changes as we move along the radial direction.

At the horizon, $f(z_0) = 0$. Therefore, we can express M as

$$M = z_0^{-d} + Q^2 z_0^{d-2}. \tag{2.3}$$

Now, Q can be expressed in terms of chemical potential (μ),

$$Q = \sqrt{\frac{2(d-2)}{d-1}}\frac{\mu}{z_0^{d-2}}. \tag{2.4}$$

And the Hawking temperature

$$T = \frac{d}{4\pi z_0}\left(1 - \frac{d-2}{d}Q^2 z_0^{2d-2}\right). \tag{2.5}$$

Actually, Q, M, and z_0 are related to charge density, energy density, and entropy density in the boundary theory, respectively. Q is charge density up to some numbers. Let us introduce a new length scale z_* to express Q as

$$Q \coloneqq \sqrt{\frac{d}{d-2}}\frac{1}{z_*^{d-1}}. \tag{2.6}$$

We also define $\mu_* = \frac{1}{z_*}$. Note that $z_* \geq z_0$ to avoid the naked singular geometry. (This is equivalent to the $M \geq Q$ condition.)

There are two distinct situations possible: *extremal* ($T = 0$) and *nonextremal* ($T \neq 0$).

A. Extremal black hole

When the Hawking temperature is zero, the black hole is called extremal. An extremal black hole contains the maximum possible charge. The "blackening function" becomes

$$f(z) = 1 + \frac{d}{d-2}\frac{z^{2d-2}}{z_*^{2d-2}} - \frac{2(d-1)}{d-2}\frac{z^d}{z_*^d}. \tag{2.7}$$

The near horizon region for the extremal black hole becomes $\mathrm{AdS}_2 \times \mathbb{R}^{d-1}$,

$$ds^2 = \frac{L_2^2}{\zeta^2}(-dt^2 + d\zeta^2) + \mu_*^2 L_{d+1}^2 d\vec{x}^2 \tag{2.8}$$

$$A_t(\zeta) = \frac{1}{\sqrt{2d(d-1)}}\frac{1}{\zeta}, \tag{2.9}$$

where $\zeta \coloneqq \frac{z_*^2}{d(d-1)(z_*-z)}$ and L_2 is the radius[10] of the AdS_2 and is related to L_{d+1} by the following relation:

$$L_2 = \frac{1}{\sqrt{d(d-1)}}L_{d+1}.$$

B. Non-extremal black hole

Generically, charged black holes (BHs) have nonvanishing temperature. We will be interested in studying Brownian motion at finite density and finite temperature

[9] The solution we will be working with has a planar horizon with topology $\mathbb{R}^{d-1}$. Therefore, it is really a black brane rather than a black hole.

[10] Note that $L_2 < L_{d+1}$ for $d \geq 3$.

(T) but with $T \ll \mu$. We want to be in this regime because the near horizon geometry will become AdS_2-BH $\times \mathbb{R}^{d-1}$,

$$ds^2 = \frac{L_2^2}{\zeta^2}\left(-g(\zeta)dt^2 + \frac{d\zeta^2}{g(\zeta)}\right) + \mu_*^2 L_{d+1}^2 d\vec{x}^2 \tag{2.10}$$

$$A_t(\zeta) = \frac{1}{\sqrt{2d(d-1)}}\frac{1}{\zeta}\left(1 - \frac{\zeta}{\zeta_0}\right), \tag{2.11}$$

where $g(\zeta) \coloneqq (1 - \frac{\zeta^2}{\zeta_0^2})$, $\zeta_0 \coloneqq \frac{z_*^2}{d(d-1)(z_* - z_0)}$, and the corresponding temperature $T = \frac{1}{2\pi\zeta_0}$. For $T \approx \mu$, this nice structure breaks down.

III. BROWNIAN MOTION AT ZERO TEMPERATURE

To understand Brownian motion of a heavy charged particle in some strongly coupled field theory in d dimensions which has a gravity dual, one needs to study the dynamics of a long string in the dual gravity background [6,7]. Therefore, to explore the same Brownian motion at zero temperature and finite density, one needs to study a string in an extremal charged black hole. This section contains the main analysis and results of the paper.

A. Green's function by matching solutions

In this Einstein-Maxwell theory, an elementary string cannot couple to the gauge field, A_M. It can only couple to the background metric G_{MN}. We consider geometries with vanishing Kalb-Ramond field, B_{MN}. For this zero temperature case, G_{MN} can be read off from the extremal BH background (2.2) with $f(z)$ given in (2.7).

The string dynamics is given by the standard Nambu-Goto action

$$S_{\mathrm{NG}} = -\frac{1}{2\pi l_s^2}\int d\tau d\sigma \sqrt{-h}, \tag{3.1}$$

where l_s is the string length and h_{ab} is the induced metric on the world sheet,

$$h_{ab} = G_{MN}\partial_a X^M \partial_b X^N.$$

We choose to work in static gauge,

$$\tau \equiv t \quad \text{and} \quad \sigma \equiv \zeta.$$

Also, we can choose one particular direction, say, x_1 (we call this simply x for brevity), along which the world sheet fluctuates,

$$x \equiv x(\tau, \sigma) = x(t, \zeta).$$

To understand the dynamics of the string, we need to use the full background metric (2.2) with the "blackening factor" given in (2.7). Varying the Nambu-Goto action

$$S_{\mathrm{NG}} = -\frac{1}{2\pi l_s^2}\int dt dz \frac{L_{d+1}}{z^2}\left[1 + \frac{1}{2}f(z)x'^2 - \frac{1}{2f(z)}\dot{x}^2\right], \tag{3.2}$$

we obtain the equation of motion (EOM) in frequency space,

$$x''_\omega(z) + \frac{\frac{d}{dz}\left(\frac{f(z)}{z^2}\right)}{\frac{f(z)}{z^2}} x'_\omega(z) + \frac{\omega^2}{[f(z)]^2} x_\omega(z) = 0, \tag{3.3}$$

where we have used $x(z,t) = \int \frac{d\omega}{2\pi} e^{-i\omega t} x_\omega(z)$

Now, to obtain $G_R(\omega)$, the standard procedure would be to solve this equation and obtain it from the on-shell action. But this procedure involves a few subtleties [20]. First, this differential equation is not exactly solvable. Even if we are interested in $G_R(\omega)$ for very small frequencies ($\omega \ll \mu$), we cannot directly perform a perturbation expansion in small ω since at zero temperature the $f(z)$ has a double-zero at the horizon (in the extremal limit) and consequently ω^2 term in the equation of motion generates a double-pole at the horizon. Thus, this singular term dominates at the horizon irrespective of however small we choose ω to be.

To get around this difficulty, we closely follow the matching technique in Ref. [20]. At first, we isolate the "singular" near horizon region from the original background. We already know that the near horizon geometry is given by $\mathrm{AdS}_2 \times \mathbb{R}^{d-1}$ (2.8). This is referred to as the IR/inner region, and the rest of the space time is referred to as the UV/outer region. We can solve the string EOM exactly in this IR region, and therefore the treatment will be nonperturbative in ω. On the other hand, the ω dependence in the UV region can be treated perturbatively as there is no more ω-sensitivity. The main task is to match the solutions over these two regions. The overlap between these to regions is near the boundary ($\zeta \to 0$) of the AdS_2 where

$$\frac{1}{\mu} \ll \zeta \ll \frac{1}{\omega}$$

$$\text{with} \quad z_*^2 \frac{\omega^2}{f(z)} \sim \omega^2\zeta^2 \text{ is very small}$$

$$\text{and} \quad \mu\zeta \sim \frac{z_*}{z_* - z} \text{ remains large.}$$

The last two expressions ensure that the ω-dependent term becomes small in EOM and we are still near the horizon, respectively.

1. Inner region

For the string in $\mathrm{AdS}_2 \times \mathbb{R}^{d-1}$ (2.8),

$$\sqrt{-h} = \frac{L_2^2}{\zeta^2}\sqrt{1 + \frac{L_{d+5}^2}{L_2^2}\mu_*^2\zeta^2(x'^2 - \dot{x}^2)}$$
$$\approx \frac{L_2^2}{\zeta^2}\left[1 + \frac{1}{2}d(d-1)\mu_*^2\zeta^2(x'^2 - \dot{x}^2)\right]. \tag{3.4}$$

The Nambu-Goto action

$$S_{\rm NG} = -\frac{L_2^2}{2\pi l_s^2}\int dt d\zeta\left[\frac{1}{\zeta^2}+\frac{1}{2}d(d-1)\mu_*^2(x'^2-\dot{x}^2)\right]. \tag{3.5}$$

Varying this action, we get a very simple EOM for the string which is that of a free wave equation,

$$x'' - \ddot{x} = 0. \tag{3.6}$$

To solve this linear EOM, the standard way is to go to the Fourier space,

$$x(\zeta,t) = \int\frac{d\omega}{2\pi}e^{-i\omega t}x_\omega(\zeta). \tag{3.7}$$

The equation of motion reduces to

$$x''_\omega(\zeta)+\omega^2 x_\omega(\zeta) = 0. \tag{3.8}$$

This is a very well-known differential equation with two independent solutions,

$$x_\omega(\zeta) = e^{\pm i\omega\zeta}.$$

As we are interested in calculating the retarded Green function, we need to pick the one which is *ingoing* at the horizon ($\zeta\to\infty$). It is easy to see that $e^{+i\omega\zeta}$ is ingoing at the horizon.

Once we pick the right solution at the horizon, we need to expand that near the boundary($\zeta = 0$) of the IR geometry i.e, $\mathrm{AdS}_2\times\mathbb{R}^{d-1}$

$$x_\omega(\zeta) = 1 + i\omega\zeta, \qquad \text{near} \quad \zeta = 0. \tag{3.9}$$

The ratio of normalizable to non-normalizable modes fixes the Green function for the IR geometry

$$\mathcal{G}_R(\omega) = i\omega. \tag{3.10}$$

[We will see in (3.14) that for a string in AdS_{d+1} non-normalizable and normalizable modes go as z^0 and z^3, respectively, whereas in $\mathrm{AdS}_2\times\mathbb{R}^{d-1}$ (3.9), they go as z^0 and z^1.]

2. Outer region

For the outer region, we need to solve the full EOM (3.3). But now, as we are away from the "dangerous" near horizon region, we can do a small frequency expansion. At the leading order, we can put $\omega = 0$. Let us say that the (3.3) has two independent solutions $\eta_+^{(0)}$ and $\eta_-^{(0)}$ for $\omega = 0$. We can fix their behavior near the horizon ($z = z_*$) and near the boundary ($z = 0$) by solving this equation near those regions.

Near horizon.—Near $z = z_*$,

$$f(z) \approx d(d-1)\frac{(z_*-z)^2}{z_*^2}.$$

The EOM reduces to

$$x''_\omega(z) - \frac{2}{z_*-z}x'_\omega(z) = 0. \tag{3.11}$$

The two independent solutions are

$$c \quad \text{and} \quad \frac{z_*}{z_*-z}.$$

Here, c is some constant which can be chosen to be 1. Since we need to match the inner and the outer solutions near $z = z_*$ let us express these independent solutions in terms of ζ,

$$\eta_+^{(0)} \to \left(\frac{\zeta}{z_*}\right)^0, \qquad \eta_-^{(0)} \to \left(\frac{\zeta}{z_*}\right)^1. \tag{3.12}$$

Near boundary.—Near the boundary, $z = 0$, we can approximate $f(z)\approx 1$ and consequently the EOM,

$$x''_\omega(z) - \frac{2}{z}x'_\omega(z) = 0. \tag{3.13}$$

The solutions near $z = 0$ will behave as

$$\eta_+^{(0)} \approx a_+^{(0)}\left(\frac{z}{z_*}\right)^0 + b_+^{(0)}\left(\frac{z}{z_*}\right)^3 \tag{3.14}$$

$$\eta_-^{(0)} \approx a_-^{(0)}\left(\frac{z}{z_*}\right)^0 + b_-^{(0)}\left(\frac{z}{z_*}\right)^3. \tag{3.15}$$

Notice that $a_\pm^{(0)}$ and $b_\pm^{(0)}$ are not independent but related by the Wronskian. We will use this information below to fix one of those coefficients.

Matching the solutions.—We have some solutions to the full EOM in patches. All we need to do to obtain the Green function is to determine the outer solution by matching it to the inner solution in the overlap region. Then, expand that solution near $z = 0$ to compute the ratio of the normalizable to the non-normalizable mode.

Let us do the matching first. From (3.9) and (3.12), we can express the outer solution as

$$x_\omega(z) = \eta_+^{(0)}(z) + \mathcal{G}_R(\omega)z_*\eta_-^{(0)}(z). \tag{3.16}$$

Notice that so far we have been using solutions to the UV equation which are zeroth order in ω (as we have put $\omega = 0$). But in principle, we can systematically add higher-order corrections in ω. In that improved version, the outer solution will be given by

$$x_\omega(z) = \eta_+(z) + \mathcal{G}_R(\omega)z_*\eta_-(z) \tag{3.17}$$

where $\eta_\pm(z) = \eta_\pm^{(0)}(z) + \omega^2\eta_\pm^{(2)}(z) + \omega^4\eta_\pm^{(4)}(z) + \cdots$ (3.18)

And as discussed before (Eqs. 3.14 and 3.15) near boundary (i.e. $z = 0$)

$$\eta_\pm \approx a_\pm \left(\frac{z}{z_*}\right)^0 + b_\pm \left(\frac{z}{z_*}\right)^3 \tag{3.19}$$

$$\text{where } a_\pm = a_\pm^{(0)} + \omega^2 a_\pm^{(2)} + \omega^4 a_\pm^{(4)} + \cdots \tag{3.20}$$

$$b_\pm = b_\pm^{(0)} + \omega^2 b_\pm^{(2)} + \omega^4 b_\pm^{(4)} + \cdots \tag{3.21}$$

Note that $a_\pm$ and $b_\pm$ are all *real* coefficients because the UV equation (3.3) and the boundary condition (3.12) at $z = z_*$ are both real. Also, the perturbations in the frequency are in even powers in ω as (3.3) contains only ω^2.

Finally, to obtain the retarded Green function, we expand the outer solution (3.17) near the boundary ($z = 0$),

$$x_\omega(z) = A(\omega)\left(\frac{z}{z_*}\right)^0 + B(\omega)\left(\frac{z}{z_*}\right)^3 \tag{3.22}$$

$$A(\omega) = a_+ + \mathcal{G}_R(\omega) z_* a_-$$
$$B(\omega) = b_+ + \mathcal{G}_R(\omega) z_* b_-.$$

The Green function of the boundary theory is given by (see Refs. [7,9,18])

$$G_R(\omega) \coloneqq \lim_{z\to 0} T_0(z)\left(-\frac{z^2}{L_{d+1}^2}\right)\frac{x_\omega'(z)}{x_\omega(z)}, \tag{3.23}$$

where

$$T_0(z) = \frac{1}{2\pi l_s^2}\frac{L_{d+1}^4}{z^4}\left[1 + \frac{d}{d-2}\left(\frac{z}{z_*}\right)^{2d-2} - \frac{2(d-1)}{d-2}\left(\frac{z}{z_*}\right)^d\right]$$

is identified as local string tension which comes from the z-dependent normalization of the boundary action. Since we are interested in the boundary Green function,

$$T_0(z) \approx \frac{1}{2\pi l_s^2}\frac{L_{d+1}^4}{z^4},$$

and consequently the retarded Green function

$$\begin{aligned} G_R(\omega) &\approx \lim_{z\to 0} -\frac{1}{2\pi l_s^2}\frac{L_{d+1}^2}{z^2}\frac{\eta_+'(z) + \mathcal{G}_R(\omega) z_* \eta_-'(z)}{\eta_+(z) + \mathcal{G}_R(\omega) z_* \eta_-(z)} \\ &= \lim_{z\to 0} -\frac{L_{d+1}^2}{2\pi l_s^2}\frac{1}{z^2}\frac{3b_+(\frac{z}{z_*})^2\frac{1}{z_*} + \mathcal{G}_R(\omega) 3b_-(\frac{z}{z_*})^2}{[a_+ + \mathcal{G}_R(\omega) z_* a_-] + [b_+ + \mathcal{G}_R(\omega) z_* b_-](\frac{z}{z_*})^3} \\ &= -\frac{\sqrt{\lambda}}{2\pi}\frac{3}{z_*^3}\left[\frac{b_+ + \mathcal{G}_R(\omega) z_* b_-}{a_+ + \mathcal{G}_R(\omega) z_* a_-}\right]. \end{aligned} \tag{3.24}$$

We have introduced a dimensionless quantity $\lambda \coloneqq \frac{L_{d+1}^4}{l_s^4}$ which behaves like a coupling constant in the boundary field theory. Since we are working in the supergravity limit in the bulk, $L_{d+1} \gg l_s$, and therefore $\lambda \gg 1$; i.e, the boundary theory is strongly coupled. The expression (3.24) is the main result of this paper. Below, we analyze this in detail.

B. Properties of the Green function

The following are properties of the Green function:

(1) The expression (3.24) depends on two sets of data:
  (a) $\{a_\pm, b_\pm\}$: These constants come from solving the EOM in the outer region. Therefore, they depend on the geometry of the outer region. In this sense, they are *nonuniversal* UV data.
  (b) $\mathcal{G}_R(\omega)$: This depends only on the IR region which always contains AdS_2 independent of the full UV theory. This is *universal* IR data.

(2) As we have already pointed out, the UV data $(a_\pm, b_\pm)$ are always real, whereas the IR data $[\mathcal{G}_R(\omega)]$ are in general *complex*. Therefore, the dissipation is always controlled by the IR data. Actually, all nonanalytic[11] behavior enters into $G_R(\omega)$ from $\mathcal{G}_R(\omega)$.

(3) In principle, $a_\pm^{(2n)}$ and $b_\pm^{(2n)}$ are fixed by (numerically) solving the EOM in UV region in ω^2 perturbation.

(4) The interesting thing to notice is that the (3.3) with $\omega = 0$ allows a constant solution. From the boundary condition (3.12) at $z = z_*$, we see that

$$\eta_+^{(0)} = 1.$$

This value of $\eta_+^{(0)}$ will continue to solve the EOM (3.3) with $\omega = 0$ for the outer region $z_* \geq z \geq 0$. So, near the boundary ($z = 0$), we have [from (3.19)]

$$\eta_+ \approx a_+^{(0)}\left(\frac{z}{z_*}\right)^0 + b_+^{(0)}\left(\frac{z}{z_*}\right)^3 = 1. \tag{3.25}$$

[11] There are no noninteger powers of ω for the system we are considering. Therefore, there are no branch cuts, but $G_R(\omega)$ can only have poles at particular ω-values.

This fixes

$$a_+^{(0)} = 1$$
$$b_+^{(0)} = 0.$$

Actually, we can fix one more coefficient by equating the generalized Wronskian[12] at the boundary and at the horizon. We get (see Appendix for details)

$$b_-^{(0)} = \frac{1}{3}.$$

The zeroth-order Green function reduces[13] to

$$\begin{aligned} G_R^{(0)}(\omega) &= -\frac{L_{d+1}^2}{2\pi l_s^2}\frac{3}{z_*^3}\frac{b_+^{(0)} + \mathcal{G}_R(\omega) z_* b_-^{(0)}}{a_+^{(0)} + \mathcal{G}_R(\omega) z_* a_-^{(0)}} \\ &= -\frac{\sqrt{\lambda}}{2\pi}\frac{i\mu_*^2\omega}{(1 + i\frac{\omega}{\mu_*}a_-^{(0)})}. \end{aligned} \tag{3.26}$$

The form of $G_R^{(0)}(\omega)$ ensures[14]

$$G_R^{(0)}(\omega) = 0 \quad \text{as } \omega \to 0.$$

The real and imaginary parts of $G_R^{(0)}(\omega)$ are plotted (see Figs. 2 and 3) for particular values of the parameters: $\lambda = 50, \mu_* = 5$ and $a_-^{(0)} = 10$.

For small frequency,

$$\begin{aligned} G_R^{(0)}(\omega) &= -i\frac{\sqrt{\lambda}}{2\pi}\mu_*^2\omega\left(1 - i\frac{\omega}{\mu_*}a_-^{(0)} + \cdots\right) \\ &\approx -i\frac{\sqrt{\lambda}}{2\pi}\mu_*^2\omega - a_-^{(0)}\frac{\sqrt{\lambda}}{2\pi}\mu_*\omega^2. \end{aligned} \tag{3.27}$$

This is also consistent with the Langevin equation (1.5) as $G_R(\omega)$ expansion starts with $-i\omega$. Note that, for small frequencies, the zero temperature dissipation goes linear in ω (see Fig. 3) unlike the $\mu = 0$ case [9,18] where this goes as ω^3. The fact that $G_R(\omega)$ is linear in ω comes from the fact that the effective AdS_2 dimension [which can be read off from (3.12)] of the "quark operator" is 1 (i.e., $\Delta = 1$).

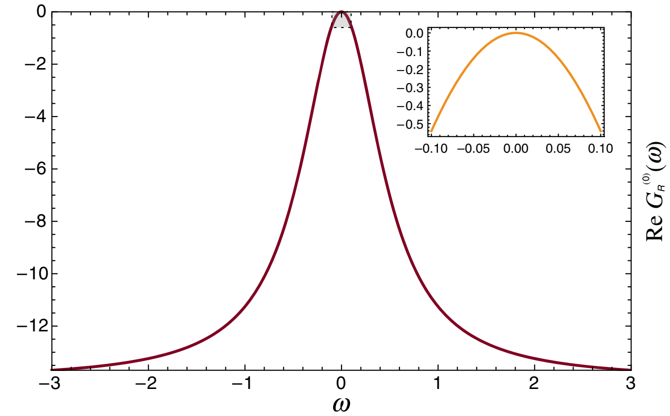

FIG. 2. Real part of $G_R^{(0)}(\omega)$ with $\mu_* = 5$.

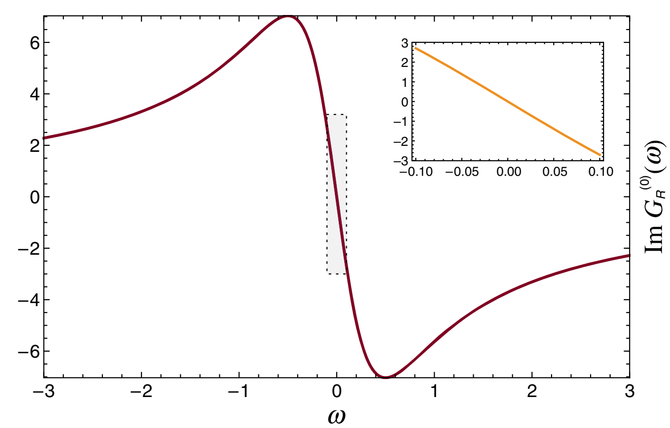

FIG. 3. Imaginary part of $G_R^{(0)}(\omega)$ with $\mu_* = 5$.

The leading dissipative force is proportional to μ_*^2 which indicates that energy loss for the charged Brownian particle is more for medium with higher charge density.

IV. BROWNIAN MOTION AT FINITE TEMPERATURE

To study Brownian motion at finite but very small temperature, we need to follow the same steps as before. But now the inner region will become a nonextremal (rather near extremal) black hole (2.2) background.

A. Green's function by matching solutions

In this section, we will go through the same procedure of the matching functional form of the solutions in the inner and outer regions. There will be a few modifications to the zero temperature $G_R(\omega)$.

[12]The generalized Wronskian of a second-order homogeneous ordinary differential equation with two independent solutions ϕ_1 and ϕ_2 is defined as

$$\begin{aligned} W(z) &\equiv e^{\int^z P(t)dt}[\phi_1\partial_z\phi_2 - \phi_2\partial_z\phi_1] \\ &= \sqrt{-g}g^{zz}[\phi_1\partial_z\phi_2 - \phi_2\partial_z\phi_1]. \end{aligned}$$

[13]There is no principle that tells us that the all the coefficients of Green function (even in zeroth order in ω) should be determined by analytic methods. Due to the simplicity of this particular differential equation, we can fix a few of them analytically. In general, one needs numerical techniques to fix all of them.

[14]Instead of a fluctuating string, if one considers the bulk Fermionic field (not the world sheet field) in the same geometry, $a_\pm^{(n)}$ and $b_\pm^{(n)}$ are functions of momentum k. For a certain value of $k = k_F$, say, $a_+^{(0)} = 0$. At this value of momentum, $G_R^{(0)}(\omega, k_F)$ becomes singular at $\omega = 0$. This indicates the *Fermi surface*.

1. Inner region

The metric in this region is a black hole[15] in $AdS_2 \times \mathbb{R}^{d-1}$ (2.10).

The Nambu-Goto action

$$S_{\rm NG} = -\frac{L_2^2}{2\pi l_s^2}\int dt d\zeta \times\left[\frac{1}{\zeta^2}+\frac{1}{2}d(d-1)\mu_*^2\left(g(\zeta)x'^2-\frac{1}{g(\zeta)}\dot{x}^2\right)\right]. \quad (4.1)$$

Varying this action, we obtain the EOM in frequency space,

$$x''_\omega(\zeta)+\frac{2\zeta}{\zeta^2-\zeta_0^2}x'_\omega(\zeta)+\frac{\zeta_0^4\omega^2}{(\zeta^2-\zeta_0^2)^2}x_\omega(\zeta)=0. \quad (4.2)$$

This EOM can be solved exactly. The two independent solutions[16] are

$$x_\omega(\zeta)=e^{\pm i\zeta_0\omega\tanh^{-1}(\frac{\zeta}{\zeta_0})}.$$

Again, we are interested in the retarded Green function, so we pick the solution which is ingoing at the horizon ($\zeta=\zeta_0$),

$$e^{+i\zeta_0\omega\tanh^{-1}(\frac{\zeta}{\zeta_0})}.$$

Once we have the ingoing solution, we need to expand it near the boundary($\zeta=0$) of near horizon geometry,

$$x_\omega^R(\zeta)=1+i\omega\zeta. \quad (4.3)$$

We can now read off the Green function in the IR region:

$$\mathcal{G}_{R,T}(\omega)=i\omega. \quad (4.4)$$

This is identical to the zero temperature case (3.10).

We discussed earlier that the dissipative part of $G_R(\omega)$ comes solely from the IR Green function. For this particular problem, $\mathcal{G}_{R,T}(\omega)=\mathcal{G}_R(\omega)=i\omega$. Therefore, T-dependence can only creep in via the expansion coefficients ($a_\pm, b_\pm$).

2. Outer region

This outer region analysis will be almost identical to that of the zero temperature case. One just has to be careful about the coefficients ($a_\pm$ and $b_\pm$), which are now *temperature dependent*, in general. Therefore, we can skip repeating the analysis and directly write down the Green function at finite temperature following the zero temperature case (see Sec. III),

$$\begin{aligned}G_{R,T}(\omega)&=-\frac{\sqrt{\lambda}}{2\pi}\frac{3}{z_*^3}\left[\frac{b_+(\omega,T)+\mathcal{G}_{R,T}(\omega)z_*b_-(\omega,T)}{a_+(\omega,T)+\mathcal{G}_{R,T}(\omega)z_*a_-(\omega,T)}\right]\\&=-\frac{\sqrt{\lambda}}{2\pi}\frac{3}{z_*^3}\left[\frac{b_+(\omega,T)+i\omega z_*b_-(\omega,T)}{a_+(\omega,T)+i\omega z_*a_-(\omega,T)}\right].\end{aligned} \quad (4.5)$$

If we consider only the leading order in ω (i.e, putting $\omega=0$ in the EOM), even for the nonextremal case, $x_\omega=\text{const}$ is again a solution. As before, we can normalize it to 1. By the same argument as in the zero temperature case,

$$a_+^{(0)}=1$$
$$b_+^{(0)}=0.$$

Therefore, the leading-order Green function is identical to that of the zero temperature case (3.26),

$$G_{R,T}^{(0)}(\omega)=-\frac{\sqrt{\lambda}}{2\pi}\frac{i\mu_*^2\omega}{(1+i\frac{\omega}{\mu_*}a_-^{(0)})}. \quad (4.6)$$

This Green's function can be improved by solving the (3.3) perturbatively in ω and T. Actually, the corrections will be in powers of $\frac{\omega}{\mu_*}$ and $\frac{T}{\mu_*}$. The corresponding real coefficients can also be obtained numerically in a systematic fashion.

V. DISCUSSIONS

We have studied in detail the important properties of the retarded Green function we obtained from the matching technique. It has a nice structure in terms of frequency (and also in temperature). We discussed that the dissipative (in general nonanalytic) part of the system is determined by the near horizon behavior, i.e, the IR data of the system. On the other hand, the near boundary behavior, i.e., the UV data, is always some analytic expansion in nature. Actually, these facts are compatible with our field theoretic and geometric intuitions.

For generic many-body systems, we know that IR physics can show nonanalytic behavior but UV physics can only give analytic corrections to that. From the renormalization group point of view, this matching technique can be thought of as matching UV to IR physics at some intermediate energy scale fixed by the chemical potential (μ_*).

Geometrically, also this is expected. Dissipation is caused due to energy or "modes" disappearing into "something." In the bulk picture, this can only happen near the horizon of a black hole where the modes fall into the black hole and never come back, whereas near boundary

[15]This black hole is related to AdS_2 geometry by a coordinate transformation [42,43] (combined with a gauge transformation) that acts as a conformal transformation on the boundary of AdS_2. So, correlators can be obtained directly from AdS_2 correlators via conformal transformation.

[16]Notice the same $\zeta_0\tanh^{-1}(\frac{\zeta}{\zeta_0})$ factor appears in the conformal transformation from AdS_2 to AdS_2-BH (see Ref. [43]).

geometry is very smooth, and therefore no nonanalytic behavior can be expected from that UV region.

It is worth mentioning that the leading-order dissipative term at zero temperature is linear[17] in frequency unlike the zero density situations [9,18] where it starts with the cubic term (ω^3). Therefore, this is actually the *drag* term associated to the velocity of the charged particle rather than the acceleration of the same. A particle moving at constant velocity at zero temperature can dissipate energy for this setup since the presence of finite charge density breaks Lorentz symmetry of the boundary theory explicitly. Nevertheless, there will be dissipation due to acceleration of the charged particle as radiation at the subleading order. Expanding $G_R^{(0)}(\omega)$ (3.26) in small frequency, one can obtain the Bremsstrahlung function $B(\lambda)$ by collecting the coefficient of ω^3,

$$B(\lambda) = \frac{\sqrt{\lambda}}{2\pi}(a_-^{(0)})^2.$$

$a_-^{(0)}$ can be fixed by numerical technique. But this is obtained solving the string EOM (3.3) only up to $\mathcal{O}(\omega^0)$. It will get corrections for higher orders in ω^2 that can be taken into account systematically.

For a particular theory at *finite density* but at *zero temperature*, if one can independently compute the Bremsstrahlung function, then that can be compared with the result obtained in our method. The standard and well-known method of computing the Bremsstrahlung function is using the supersymmetric localization technique (see, e.g, Refs. [24,36,37]). But one would face the following challenges[18] to apply this technique at finite density. First, one needs to, if possible, turn on background fields corresponding to finite density while preserving enough supersymmetry. Second, and more specific to the computation of the Bremsstrahlung function, finite density breaks conformal invariance. Some of the steps in computing the Bremsstrahlung function use explicitly conformal symmetry. Although the Bremsstrahlung function must exist for nonconformal theories, it may no longer be controlled by localization.

The method we have used to obtain the Green function only required an AdS_2 factor near the horizon. Therefore, it should work even if the UV theory is nonconformal (not asymptotically AdS) but the IR geometry has an AdS_2 factor. For example, instead of D3 branes, one can look at D2 or D4 brane geometries. They are nonconformal [44]. If for some charge density they flow to some AdS_2, then this procedure can be applied. Also, by the same argument, it can possibly work for some rotating extremal black hole backgrounds, too.

All our results are valid for large chemical potential and small temperature. If one is interested in studying Brownian motion in the opposite regime, this technique cannot be used. The reason is that for $\mu \sim T$ the "nice" inner region structure breaks down. In that case, the small μ corrections can be computed using the same techniques used in Refs. [7,9] but for a charged black hole background in AdS with very small charge.

VI. CONCLUSIONS

In this paper, we have used the holographic technique to study Langevin dynamics of a heavy particle moving at finite charge density. We have studied this by computing the retarded Green function via a solution matching technique. Here are the main results:

(i) An analytic form of the retarded Green function for small frequencies has been obtained at zero temperature.
(ii) The drag force at zero temperature shows up as the leading contribution at small frequencies.
(iii) How the retarded Green function gets corrections due to small (but finite) temperature has also been sketched. The leading dissipative part (drag) remains identical to that in the zero temperature case.
(iv) The drag force on the heavy particle grows quadratically with the chemical potential i.e, loss in energy of the Brownian particle is more for medium with higher charge density.
(v) The leading contribution to the Bremsstrahlung function $B(\lambda)$ is obtained with an unknown coefficient $a_-^{(0)}$ which can be fixed by a numerical method. Its corrections in $\frac{\omega}{\mu_*}$ and $\frac{T}{\mu_*}$ can be computed systematically.

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APPENDIX: FIXING COEFFICIENT USING GENERALIZED WRONSKIAN

A field $\psi(z)$ satisfying a second-order linear homogeneous differential equation

[17] This linear dependence in frequency comes from the fact that effective AdS_2 dimension [see (3.12)] of the quark operator is 1 (i.e., $\Delta = 1$) and is very crucial. Due to this particular low frequency behavior, the dissipative structure is qualitatively the same at zero and finite temperatures. If the dimension has been different from 1, the small ω expansion in (3.27) at zero temperature would have started with a different power (i.e., not linear), and the story would have been different from the $T \neq 0$ result.

[18] The author would like to thank Tomeu Fiol for pointing out this possibility and also the possible challenges.

$$\psi''(z) + P(z)\psi'(z) + Q(z)\psi(z) = 0, \tag{A1}$$

where $P(z)$ and $Q(z)$ are real the generalized Wronskian, is defined as

$$W(\psi_1, \psi_2; z) \coloneqq e^{\int^z P(t)dt}[\psi_1 \partial_z \psi_2 - \psi_2 \partial_z \psi_1] \tag{A2}$$

$$= \sqrt{-g} g^{zz}[\psi_1 \partial_z \psi_2 - \psi_2 \partial_z \psi_1], \tag{A3}$$

where ψ_1 and ψ_2 are two solutions of (A1). The interesting fact about this $W(z)$ is it is independent of z,

$$\partial_z W(\psi_1, \psi_2; z) = 0.$$

Therefore, we can write

$$W(\psi_1, \psi_2; z) \equiv W(\psi_1, \psi_2).$$

Equation (3.3) is exactly of the form (A1). We know how its two independent solutions behave at the horizon ($z = z_*$) and at the boundary ($z = 0$). The generalized Wronskian

$$W(\psi_1, \psi_2) = \left(\frac{L_{d+1}^2}{z^2}\right)^{\frac{d+1}{2}} \frac{f(z)}{L_{d+1}^2} z^2 (\psi_1 \partial_z \psi_2 - \psi_2 \partial_z \psi_1) \tag{A4}$$

$$= \left(\frac{L_{d+1}}{z}\right)^{d-1} (\psi_1 \partial_z \psi_2 - \psi_2 \partial_z \psi_1) \tag{A5}$$

is independent of z. Therefore,

$$W(\psi_1, \psi_2)|_{z=0} = W(\psi_1, \psi_2)|_{z=z_*} \tag{A6}$$

Now, for extremal case,

$$f(z) = 1 + \frac{d}{d-2}\left(\frac{z}{z_*}\right)^2 - \frac{2d-2}{d-2}\left(\frac{z}{z_*}\right)^d$$

(i) $f(z)|_{z=0} = 1$
(ii) $f(z)|_{z=z_*} \approx d(d-1)\frac{(z_*-z)^2}{z_*^2}$

$$\text{lhs of (A.6)}: \; W(\psi_1, \psi_2)|_{z=0} = \frac{1}{z^2}(\eta_+^{(0)} \partial_z \eta_-^{(0)} - \eta_-^{(0)} \partial_z \eta_+^{(0)}) \\ = \frac{3}{z_*^3}(a_+^{(0)} b_-^{(0)} - a_-^{(0)} b_+^{(0)}) \tag{A7}$$

$$\text{rhs of (A.6)}: \; W(\psi_1, \psi_2)|_{z=z_*} \\ = \frac{d(d-1)(z_* - z)^2}{z^2 z_*^2}(\eta_+^{(0)} \partial_z \eta_-^{(0)} - \eta_-^{(0)} \partial_z \eta_+^{(0)}) \\ = \frac{d(d-1)(z_* - z)^2}{z^2 z_*^2}\left[1.\partial_z\left(\frac{\zeta}{z_*}\right) - \left(\frac{\zeta}{z_*}\right)\partial_z(1)\right]. \\ = \frac{1}{z_*^3} \tag{A8}$$

Equating (A7) and (A8),

$$a_+^{(0)} b_-^{(0)} - a_-^{(0)} b_+^{(0)} = \frac{1}{3},$$

and substituting $b_+^{(0)} = 0$ and $a_+^{(0)} = 1$, we get

$$b_-^{(0)} = \frac{1}{3}.$$

[1] J. M. Maldacena, The Large N limit of superconformal field theories and supergravity, Int. J. Theor. Phys. **38**, 1113 (1999); Adv. Theor. Math. Phys. **2**, 231 (1998).

[2] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Gauge theory correlators from noncritical string theory, Phys. Lett. B **428**, 105 (1998).

[3] E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. **2**, 253 (1998).

[4] E. Witten, Anti-de Sitter space, thermal phase transition, and confinement in gauge theories, Adv. Theor. Math. Phys. **2**, 505 (1998).

[5] G. Policastro, D. T. Son, and A. O. Starinets, The Shear Viscosity of Strongly Coupled $N = 4$ Supersymmetric Yang-Mills Plasma, Phys. Rev. Lett. **87**, 081601 (2001).

[6] J. de Boer, V. E. Hubeny, M. Rangamani, and M. Shigemori, Brownian motion in AdS/CFT, J. High Energy Phys. 07 (2009) 094.

[7] D. T. Son and D. Teaney, Thermal noise and stochastic strings in AdS/CFT, J. High Energy Phys. 07 (2009) 021.

[8] C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, and L. G. Yaffe, Energy loss of a heavy quark moving through $N = 4$ supersymmetric Yang-Mills plasma, J. High Energy Phys. 07 (2006) 013.

[9] P. Banerjee and B. Sathiapalan, Holographic Brownian motion in $1 + 1$ dimensions, Nucl. Phys. **B884**, 74 (2014).

[10] E. Caceres, M. Chernicoff, A. Guijosa, and J. F. Pedraza, Quantum fluctuations and the Unruh effect in strongly coupled conformal field theories, J. High Energy Phys. 06 (2010) 078.

[11] S. Chakrabortty, S. Chakraborty, and N. Haque, Brownian motion in strongly coupled, anisotropic Yang-Mills plasma: A holographic approach, Phys. Rev. D **89**, 066013 (2014).
[12] S. S. Gubser, Momentum fluctuations of heavy quarks in the gauge-string duality, Nucl. Phys. **B790**, 175 (2008).
[13] J. Casalderrey-Solana and D. Teaney, Transverse momentum broadening of a fast quark in a $N = 4$ Yang Mills plasma, J. High Energy Phys. 04 (2007) 039.
[14] W. Fischler, P. H. Nguyen, J. F. Pedraza, and W. Tangarife, Fluctuation and dissipation in de Sitter space, J. High Energy Phys. 08 (2014) 028.
[15] D. Giataganas and H. Soltanpanahi, Universal properties of the Langevin diffusion coefficients, Phys. Rev. D **89**, 026011 (2014).
[16] D. Giataganas and H. Soltanpanahi, Heavy quark diffusion in strongly coupled anisotropic plasmas, J. High Energy Phys. 06 (2014) 047.
[17] D. Roychowdhury, Quantum fluctuations and thermal dissipation in higher derivative gravity, Nucl. Phys. **B897**, 678 (2015).
[18] P. Banerjee and B. Sathiapalan, Zero temperature dissipation and holography, J. High Energy Phys. 04 (2016) 089.
[19] D. T. Son and A. O. Starinets, Minkowski space correlators in AdS/CFT correspondence: Recipe and applications, J. High Energy Phys. 09 (2002) 042.
[20] T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, Emergent quantum criticality, Fermi surfaces, and AdS(2), Phys. Rev. D **83**, 125002 (2011).
[21] C. P. Herzog and D. T. Son, Schwinger-Keldysh propagators from AdS/CFT correspondence, J. High Energy Phys. 03 (2003) 046.
[22] D. J. Griffiths, *Introduction to Electrodynamics*, 3rd ed. (Dorling Kindersley Pvt. Ltd., India, 2007).
[23] A. Mikhailov, Nonlinear waves in AdS/CFT correspondence, arXiv:hep-th/0305196.
[24] A. Lewkowycz and J. Maldacena, Exact results for the entanglement entropy and the energy radiated by a quark, J. High Energy Phys. 05 (2014) 025.
[25] A. M. Polyakov, Gauge fields as rings of glue, Nucl. Phys. **B164**, 171 (1980).
[26] M. Chernicoff and A. Guijosa, Acceleration, energy loss and screening in strongly coupled gauge theories, J. High Energy Phys. 06 (2008) 005.
[27] M. Chernicoff, J. A. Garcia, and A. Guijosa, Generalized Lorentz-Dirac Equation for a Strongly-Coupled Gauge Theory, Phys. Rev. Lett. **102**, 241601 (2009).
[28] M. Chernicoff, J. A. Garcia, and A. Guijosa, A Tail of a Quark in $N = 4$ SYM, J. High Energy Phys. 09 (2009) 080.
[29] M. Chernicoff, A. Guijosa, and J. F. Pedraza, The gluonic field of a heavy quark in conformal field theories at strong coupling, J. High Energy Phys. 10 (2011) 041.
[30] M. Chernicoff, J. A. Garcia, A. Guijosa, and J. F. Pedraza, Holographic lessons for quark dynamics, J. Phys. G **39**, 054002 (2012).
[31] B.-W. Xiao, On the exact solution of the accelerating string in AdS(5) space, Phys. Lett. B **665**, 173 (2008).
[32] Y. Hatta, E. Iancu, A. H. Mueller, and D. N. Triantafyllopoulos, Radiation by a heavy quark in $N = 4$ SYM at strong coupling, Nucl. Phys. **B850**, 31 (2011).
[33] T. Fulton and F. Rohrlich, Classical radiation from a uniformly accelerated charge, Ann. Phys. (N.Y.) **9**, 499 (1960).
[34] D. G. Boulware, Radiation from a uniformly accelerated charge, Ann. Phys. (N.Y.) **124** (1980) 169.
[35] D. Correa, J. Henn, J. Maldacena, and A. Sever, An exact formula for the radiation of a moving quark in $N = 4$ super Yang Mills, J. High Energy Phys. 06 (2012) 048.
[36] B. Fiol, B. Garolera, and A. Lewkowycz, Exact results for static and radiative fields of a quark in $N = 4$ super Yang-Mills, J. High Energy Phys. 05 (2012) 093.
[37] B. Fiol, E. Gerchkovitz, and Z. Komargodski, Exact Bremsstrahlung Function in $N = 2$ Superconformal Field Theories, Phys. Rev. Lett. **116**, 081601 (2016).
[38] M. Edalati, J. F. Pedraza, and W. Tangarife Garcia, Quantum fluctuations in holographic theories with hyperscaling violation, Phys. Rev. D **87**, 046001 (2013).
[39] M. Ahmadvand and K. B. Fadafan, Energy loss at zero temperature from extremal black holes, arXiv:1512.05290.
[40] I. R. Klebanov, World volume approach to absorption by nondilatonic branes, Nucl. Phys. **B496**, 231 (1997).
[41] J. M. Maldacena and A. Strominger, Black hole grey body factors and d-brane spectroscopy, Phys. Rev. D **55**, 861 (1997).
[42] M. Spradlin and A. Strominger, Vacuum states for AdS(2) black holes, J. High Energy Phys. 11 (1999) 021.
[43] T. Faulkner, N. Iqbal, H. Liu, J. McGreevy, and D. Vegh, Holographic non-Fermi liquid fixed points, Phil. Trans. R. Soc. A **369**, 1640 (2011).
[44] N. Itzhaki, J. M. Maldacena, J. Sonnenschein, and S. Yankielowicz, Supergravity and the large N limit of theories with sixteen supercharges, Phys. Rev. D **58**, 046004 (1998).