

Small deformation of a simple $\mathcal{N} = 2$ superconformal theoryMatthew Buican^{1,2} and Takahiro Nishinaka³¹*CRST and School of Physics and Astronomy, Queen Mary University of London, London E1 4NS, United Kingdom*²*Enrico Fermi Institute and Department of Physics, The University of Chicago, Chicago, Illinois 60637, USA*³*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*
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We study an interesting relevant deformation of the simplest interacting $\mathcal{N} = 2$ superconformal field theory (SCFT)—the original Argyres-Douglas (AD) theory. We argue that, although this deformation is not strictly speaking Banks-Zaks-like (certain operator dimensions change macroscopically), there are senses in which it constitutes a mild deformation of the parent AD theory: the exact change in the a anomaly is small and is essentially saturated at one loop. Moreover, contributions from IR operators that have a simple description in the UV theory reproduce a particular limit of the IR index to a remarkably high order. These results lead us to conclude that the IR theory is an interacting $\mathcal{N} = 1$ SCFT with particularly small a and c central charges and that this theory sheds some interesting light on the spectrum of its AD parent. Our results also lead us to the conclusion that the theory spaces emanating from some of the simplest $\mathcal{N} = 1$ gauge theories may be richer than anticipated.

DOI: [10.1103/PhysRevD.94.125002](https://doi.org/10.1103/PhysRevD.94.125002)**I. INTRODUCTION**

Argyres-Douglas (AD) theories [1–3] are considered to be relatively mysterious superconformal field theories (SCFTs). One reason for this view is the way they were initially constructed as special points in the moduli space of $\mathcal{N} = 2$ gauge theories where mutually nonlocal Bogomol’nyi-Prasad-Sommerfeld states become simultaneously massless.

On the other hand, there is evidence that AD theories are particularly simple: their conformal anomalies scale linearly with the dimensions of their Coulomb branches [4,5], and their superconformal indices take a particularly simple form [6–11].

This simplicity manifests itself in many ways. For example, even though the Schur limit of the index [12] does not receive direct “single letter” (SL) contributions from $\mathcal{N} = 2$ chiral operators whose vacuum expectation values parametrize the Coulomb branch, the AD Schur index still “nonperturbatively” encodes the spectrum of these operators in its pole structure [7] (we can therefore think of these theories as dominated by the few degrees of freedom (DOF) parametrizing the Coulomb branch).

One consequence of this paper will be to see how to make contributions from $\mathcal{N} = 2$ chiral operators more manifest by performing certain small deformations of the parent AD theory. The price we will pay for making these operators more visible is that we will break $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$.

Another striking fact about AD theories is that the simplest AD theory—the so-called (A_1, A_2) theory—saturates a universal lower bound for the c central charge of a unitary interacting $\mathcal{N} = 2$ SCFT [13]. Moreover, the

(A_1, A_2) theory has the smallest-known value of a for an interacting $\mathcal{N} = 2$ theory. As a result, one can think of it as the simplest member of the simplest class of $\mathcal{N} = 2$ SCFTs.

Therefore, it is interesting to deform this theory, since renormalization group (RG) intuition tells us that the resulting IR theory should be simpler. However, the above discussion suggests that we should, at best, find a free theory if we deform the (A_1, A_2) SCFT while preserving $\mathcal{N} = 2$. Indeed, this is true [5,14]. On the other hand, we find a more interesting IR theory if we deform the UV SCFT in such a way as to break $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ via

$$\delta W = \lambda \mathcal{O}^2, \quad (1)$$

where \mathcal{O} is the dimension $6/5$ chiral primary of the (A_1, A_2) theory. One reason to study (1) is that, among the available deformations, it is the lowest-dimensional deformation that gives rise to a stable vacuum with an interacting theory at long distance (another deformation has been recently studied in [15]) [16].

In what follows, we will analyze the IR theory, \mathcal{T} , resulting from the deformation in (1). We will see that

- (i) \mathcal{T} is interacting.
- (ii) In \mathcal{T} ’s chiral ring

$$\mathcal{O}^2 = \mathcal{O} \cdot \mathcal{O}_\alpha = 0, \quad (2)$$

where \mathcal{O}_α is a spin-half chiral primary related to \mathcal{O} by $\mathcal{N} = 2$ supersymmetry (SUSY) in the UV.

- (iii) There is strong evidence suggesting our flow does not have accidental symmetries.

(iv) \mathcal{T} has

$$a_{\mathcal{T}} = \frac{263}{768}, \quad c_{\mathcal{T}} = \frac{271}{768}, \quad (3)$$

where a free chiral superfield has $c = 1/24$.

The first point might come as a surprise, since \mathcal{T} can be reached by a flow from one of the simplest-known gauge theories: $SU(2)$ $\mathcal{N} = 1$ adjoint SQCD with one flavor. The advantage of starting at the AD point is that we find simple variables to describe \mathcal{T} .

In addition, we will use the index to argue that in the IR

(i) There is a semishort multiplet with a spin half primary, J_α , of dimension $D(J_\alpha) = \frac{11}{4}$ satisfying

$$D^\alpha J_\alpha = 0. \quad (4)$$

We will also find evidence that

(i) \mathcal{O} , \mathcal{O}_α , and a third chiral primary, \mathcal{O}' (also related to the other two by $\mathcal{N} = 2$ SUSY in the UV), exist as flavor-singlet chiral operators in \mathcal{T} with scaling dimensions $D(\mathcal{O}) = \frac{3}{2}$, $D(\mathcal{O}_\alpha) = \frac{7}{4}$, and $D(\mathcal{O}') = 2$, respectively.

The apparent existence of a flavor singlet chiral primary, \mathcal{O} , satisfying $\mathcal{O}^2 = 0$ with a scaling dimension that is within 5% of the extrapolated dimension for the flavor-singlet chiral primary ϕ operator (satisfying $\phi^2 = 0$) in [18] begs the question of whether \mathcal{T} is the minimal $\mathcal{N} = 1$ SCFT discussed in [18] and if $\mathcal{O} = \phi$. While these points give some reason to suspect this identification of theories might be correct, the value of the extrapolated c central charge in [18] is roughly a factor of 3 smaller than the central charge in (3). Therefore, we are not sure if \mathcal{T} is the theory in [18,19].

On the other hand, our study of this SCFT will shed new light on the (A_1, A_2) theory and on aspects of $\mathcal{N} = 1$ dynamics. Moreover, the values of the central charges in (3) are particularly small for an interacting $\mathcal{N} = 1$ SCFT in four dimensions [20]. Therefore, \mathcal{T} clearly deserves to be studied in its own right.

Our plan is as follows. Next, we will construct our theory and establish (2) and (3). In the following section we will use recent insights into the superconformal indices of AD theories to argue that \mathcal{T} is interacting. We will then discuss constraints on accidental symmetries. In the following section, we use the index to find evidence for the existence of the primaries \mathcal{O} , \mathcal{O}_α , \mathcal{O}' , and J_α in the IR. Finally, we conclude with some brief comments on the implications of our results.

II. THE MINIMAL $\mathcal{N} = 1$ DEFORMATION

We will make one assumption in studying the deformation (1): there are no accidental flavor symmetries along the corresponding RG flow. In Sec. III, we will give some justifications for this assumption.

From this starting point, we compute $a_{\mathcal{T}}$ and $c_{\mathcal{T}}$ using anomaly matching and [4,22]

$$a_{(A_1, A_2)} = \frac{43}{120}, \quad c_{(A_1, A_2)} = \frac{11}{30}. \quad (5)$$

Indeed, since the (A_1, A_2) theory has no $\mathcal{N} = 2$ flavor symmetries, there is a unique R symmetry along the RG flow

$$\hat{R} = -2 \left(r - \frac{7}{12} J \right) = \frac{1}{6} (-5r + 7R), \quad (6)$$

where r is the overall $\mathcal{N} = 2$ $U(1)_R$ charge, R is the $SU(2)_R$ Cartan, and J is the $\mathcal{N} = 1$ flavor symmetry

$$J = r + R. \quad (7)$$

We adopt the conventions $r(Q_\alpha^2) = -R(Q_\alpha^2) = 1/2$ so that $J(Q_\alpha^2) = 0$ [we are integrating the deformation (1) over the half of superspace corresponding to $\tilde{Q}_{2\dot{\alpha}}$ and Q_α^2 [23]]. Therefore

$$r(\mathcal{O}) = J(\mathcal{O}) = -\frac{6}{5}, \quad R(\mathcal{O}) = 0, \quad (8)$$

from which (6) follows.

Next, using the fact that the 't Hooft anomalies are [4] (our conventions are $r = -\frac{1}{2}R_{\mathcal{N}=2}$, where $R_{\mathcal{N}=2}$ is defined in [4])

$$\begin{aligned} \mathcal{A}(r^3) &= -6(a - c), & \mathcal{A}(rR^2) &= -(2a - c), \\ \mathcal{A}(r) &= -24(a - c), \end{aligned} \quad (9)$$

with all other R anomalies vanishing, we get

$$\mathcal{A}(\hat{R}) = -\frac{1}{6}, \quad \mathcal{A}(\hat{R}^3) = \frac{251}{216}. \quad (10)$$

Therefore we obtain (3) (which is compatible with [24]).

Moreover, in the IR, \mathcal{O}^2 is a descendant since (1) breaks the J symmetry in (7)

$$\tilde{D}^2 J \sim \lambda \mathcal{O}^2. \quad (11)$$

As promised in (2), \mathcal{O}^2 vanishes in \mathcal{T} 's chiral ring.

In fact, we get more information by studying the $\mathcal{N} = 2$ supercurrent multiplet. This multiplet (see [25]) contains an $\mathcal{N} = 1$ submultiplet, J_α , with a primary of dimension $5/2$ and the (broken) second supersymmetry current. In the absence of supersymmetry breaking, it satisfies $\tilde{D}^2 J_\alpha = 0$. However, in the presence of the SUSY breaking deformation (1), we find

$$\tilde{D}^2 J_\alpha \sim \lambda \mathcal{O} \cdot \mathcal{O}_\alpha. \quad (12)$$

Therefore, as promised in (2), $\mathcal{O} \cdot \mathcal{O}_\alpha$ vanishes in \mathcal{T} 's chiral ring.

III. \mathcal{T} IS INTERACTING

To gain further insight into \mathcal{T} , it is useful to study the superconformal index of the (A_1, A_2) theory. Recall that the $\mathcal{N} = 2$ index can be defined as

$$\mathcal{I}(p, q, t) = \text{Tr}(-1)^F t^{R+r} p^{j_2-j_1-r} q^{j_2+j_1-r} e^{-\beta\Delta}, \quad (13)$$

where R , r , and $j_{1,2}$ are the $SU(2)_R$ Cartan, the overall superconformal $U(1)_R$ generator, and the two Cartans of the rotation group, respectively. Note that the contributions to the trace come from states that are annihilated by \tilde{Q}_{2-} [i.e., states that have $\Delta = \frac{1}{2}\{\tilde{Q}_{2-}, \tilde{Q}_{2-}^\dagger\} = \frac{1}{2}(E - 2j_2 - 2R + r) = 0$], that the fugacities p, q, t satisfy $|p|, |q|, |t|, |pq/t| < 1$, and that the corresponding charges also commute with \tilde{Q}_{2-} (for simplicity, we have dropped the dependence on potential flavor fugacities). While the full indices of AD theories are not presently known, results are known for various special limits [6–11]. In particular, we will use the Schur limit of the (A_1, A_2) index [8].

This limit is defined by taking $t = q$ in (13). As a result, all contributing states are annihilated by both \tilde{Q}_{2-} and Q_-^\dagger . Using $\{Q_-^\dagger, Q_-^\dagger\} = \frac{1}{2}(E - 2j_1 - 2R - r)$ and recalling that contributions to the index satisfy $E = 2j_2 + 2R - r$, we see that for the contributing states in (13), $\{Q_-^\dagger, Q_-^\dagger\} = j_2 - j_1 - r$. Therefore, we conclude that the Schur index is independent of p .

Using this freedom, take $p = q^{\frac{5}{7}}$ and obtain

$$\mathcal{I}_S(q) = \mathcal{I}(q^{\frac{5}{7}}, q, q) = \text{Tr}(-1)^F q^{\frac{1}{7}(12j_2+2j_1+6\hat{R})} e^{-\beta\Delta}. \quad (14)$$

In particular, we see that this index is explicitly preserved when we turn on our $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ breaking deformation in (1). Moreover, from [8] we know that

$$\mathcal{I}_{S(A_1, A_2)}(q) = 1 + \sum_{\ell=1}^{\infty} \frac{q^{\ell(\ell+1)}}{\prod_{k=1}^{\ell} (1 - q^k)} = 1 + q^2 + \dots, \quad (15)$$

where the right-hand side is the Rogers-Ramanujan H function.

After we deform our theory, we should think of the index as corresponding (up to a prefactor) to a twisted partition function for the massive theory on $S^1 \times S^3$. This partition function does not depend on the RG scale. In the deep IR, after flowing to \mathcal{T} , we interpret the resulting partition function as an index that counts states annihilated by \tilde{Q}_{2-} (i.e., those states satisfying $\Delta_{\text{IR}} = E - 2j_2 - \frac{3}{2}\tilde{R} = 0$, where \tilde{R} is the IR superconformal R symmetry). More precisely, since we start from a well-defined index in the UV, and our relevant deformation leads to a stable vacuum, then the partition function should interpolate to the IR index or to a suitable continuation of the IR index.

We now use this logic to rule out the possibility that \mathcal{T} is a collection of free fields. In particular, the a -theorem [26] guarantees that the IR SCFT can at most consist of (a) 17 free chiral multiplets and no vector multiplets or (b) at most 8 free chiral multiplets and an Abelian vector multiplet. Neither of these possibilities reproduce (15).

To understand this claim, consider (a). We have a collection of free chiral multiplets, ϕ_i , with \hat{R} charges \hat{R}_i . In our conventions, contributions to the IR index come from operators satisfying $\Delta_{\text{IR}} = E - 2j_2 - \frac{3}{2}\tilde{R} = 0$, where \tilde{R} is the free superconformal R symmetry. These contributions can only come from states built out of bosonic chiral primaries, ϕ_i , antichiral fermions, $\tilde{\psi}_{i+}$, and their derivatives.

Thinking in terms of the partition function, it is natural to consider theories with $\hat{R}_i \in (0, 2)$ since the curved space potential is bounded from below (moreover, the index is absolutely convergent) [27,28]. However, it is easy to see that such a theory cannot reproduce (15) in the IR.

Indeed,

$$\mathcal{I}_{\text{IR}}(q) = \prod_{i=1}^N \prod_{m, \ell \geq 0} \frac{1 - q^{\frac{5}{7}(2-\hat{R}_i) + \frac{5}{7}m + \ell}}{1 - q^{\frac{5}{7}\hat{R}_i + \frac{5}{7}m + \ell}}, \quad (16)$$

where $N \leq 17$. We have boson(s), ϕ_a , of lowest R charge, $\hat{R}_{\text{min}} \in (0, 2)$. In order to match (15), we see that the zero-derivative single-letter contributions of the ϕ_a must be canceled by fermionic contributions from some $\tilde{\Psi}_a$ [since the bosonic contributions appear at order less than $\mathcal{O}(q^2)$ in the index]. If the $\tilde{\Psi}_a$ are composites (in $\tilde{\psi}_{i+}$, ϕ_i , and derivatives), then there are contributions of lower order than the index contributions of the ϕ_a , and these contributions cannot be canceled, which is in contradiction with (15). On the other hand, if $\tilde{\Psi}_a = \tilde{\psi}_{i_a+}$, then we have an exact pairing $\phi_a \oplus \tilde{\psi}_{i_a}$ and $\phi_{i_a} \oplus \tilde{\psi}_i$. Therefore, the corresponding contributions to the index cancel pairwise. We proceed iteratively through the remaining DOF and find that the IR index is unity. In particular, we see that (16) cannot match the UV index.

More generally, we can ask if \mathcal{T} can be free if we allow some $\hat{R}_i \in (0, 2)$. In this case we can try to define the index by a suitable continuation. More precisely, starting from the index of free chiral superfields

$$\mathcal{I}_{\text{IR}}(q) = \prod_{i=1}^N \prod_{m, \ell \geq 0} \frac{1 - q^{\frac{5}{7}(2-\tilde{R}_i) + \frac{5}{7}m + \ell} u_i^{-1}}{1 - q^{\frac{5}{7}\tilde{R}_i + \frac{5}{7}m + \ell} u_i}, \quad (17)$$

with fugacities u_i for the symmetries that act on ϕ_i with charge one and leave the other primaries invariant. In particular, taking $u_i \rightarrow q^{\frac{5}{7}\alpha_i}$ so that $\hat{R}_i = \tilde{R}_i + \alpha_i$, we can obtain

$$\tilde{\mathcal{I}}_{\text{IR}}(q) = \prod_{i=1}^N \prod_{m, \ell \geq 0} \frac{1 - q^{\frac{5}{7}(2-\hat{R}_i) + \frac{5}{7}m + \ell}}{1 - q^{\frac{5}{7}\hat{R}_i + \frac{5}{7}m + \ell}}, \quad (18)$$

with some of the $\hat{R}_i \in (0, 2)$. In (18), we have added a tilde over \mathcal{I}_{IR} to remind ourselves that this is a continued expression for the index. This continuation is well defined and nonvanishing so long as $\hat{R}_i \neq -\frac{5}{6}m_i - \frac{7}{6}\ell_i$ and $\hat{R}_i \neq 2 + \frac{5}{6}m'_i + \frac{7}{6}\ell'_i$ for all non-negative integers $m_i, m'_i, \ell_i, \ell'_i$.

Now, we can rewrite (18) as

$$\begin{aligned} \tilde{\mathcal{I}}_{\text{IR}} &= \prod_{a=1}^{N_-} \prod_{m, \ell \geq 0} \left(\frac{1}{1 - q^{\frac{6}{7}\hat{R}_a + \frac{5}{7}m + \ell}} \right) \\ &\times \prod_{A=1}^{N_+} \prod_{m, \ell \geq 0} (1 - q^{\frac{6}{7}(2-\hat{R}_A) + \frac{5}{7}m + \ell}) \times \tilde{\mathcal{I}}'_{\text{IR}}, \end{aligned} \quad (19)$$

where the first factor contains the contributions of the bosons with $\hat{R}_a < 0$, the second factor contains the contributions of the fermions coming from superfields conjugate to chiral multiplets with $\hat{R}_A > 2$, and $\tilde{\mathcal{I}}'_{\text{IR}}$ contains contributions from the remaining DOF. Moreover, we can rewrite the products over the \hat{R}_a and \hat{R}_A in (19) as

$$\begin{aligned} &\prod_{a=1}^{N_-} \prod_{m, \ell \geq 0} \left(\frac{1}{1 - q^{\frac{6}{7}\hat{R}_a + \frac{5}{7}m + \ell}} \right) \times \prod_{A=1}^{N_+} \prod_{m, \ell \geq 0} (1 - q^{\frac{6}{7}(2-\hat{R}_A) + \frac{5}{7}m_A + \ell_A}) \\ &= \prod_{a=1}^{N_-} \prod_{m_a=0}^{M_a} \prod_{\ell_a=0}^{L_a(m_a)} \left(\frac{1}{1 - q^{\frac{6}{7}\hat{R}_a + \frac{5}{7}m_a + \ell_a}} \right) \\ &\times \prod_{A=1}^{N_+} \prod_{m_A=0}^{M_A} \prod_{\ell_A=0}^{L_A(m_A)} (1 - q^{\frac{6}{7}(2-\hat{R}_A) + \frac{5}{7}m_A + \ell_A}) \dots, \end{aligned} \quad (20)$$

where we have separated contributions with $\frac{6}{7}\hat{R}_a + \frac{5}{7}m_a + \ell_a < 0$ in the product over the \hat{R}_a and fermionic contributions with $\frac{6}{7}(2-\hat{R}_A) + \frac{5}{7}m_A + \ell_A < 0$ in the product over the \hat{R}_A (all other terms, with sufficiently many derivatives so that they give rise to contributions with positive powers of q , appear in the ellipsis).

Note that none of the contributions to the IR index can come from contributions appearing explicitly in (20). Indeed, if this statement did not hold, then, by acting with sufficiently many derivatives, we would get contributions that render the IR index vanishing or ill defined. Therefore, the bosonic and fermionic factors with the most negative q exponents in (20) must cancel. Such terms necessarily come from contributions of the ϕ_a with the most negative $\hat{R}_a < 0$ and the $\tilde{\psi}_{A+}$ with the most negative $2 - \hat{R}_A$. In particular, we see that $\hat{R}_a = 2 - \hat{R}_A$ and that therefore the ϕ_a pair up with the $\tilde{\psi}_{A+}$ and cancel in the index (similarly, the $\tilde{\psi}_{a+}$ pair up with the ϕ_A and cancel). We can proceed this way iteratively through all the DOF having $\hat{R}_a < 0$ and $\hat{R}_A > 2$. In particular, we are back to the previous case with $\hat{R}_i \in (0, 2)$, and so we see that the IR theory cannot consist solely of free chiral superfields. Case (b) can be understood similarly.

IV. CONSTRAINTS ON ACCIDENTAL SYMMETRIES

One reason to be skeptical about the appearance of accidental symmetries is that there are no apparent unitarity bound violations. For example, if \mathcal{O} , \mathcal{O}_a , and \mathcal{O}' exist in the IR chiral ring, their dimensions are above the relevant unitarity bounds. Moreover, none of the UV DOF in the Schur sector have any apparent unitarity bound violations in the IR. For example, we will argue that the nonchiral IR $\mathcal{N} = 1$ operator, J_a , which descends from the UV $\mathcal{N} = 2$ stress tensor multiplet, has dimension $11/4 > 3/2$.

Another reason to doubt the existence of accidental symmetries in the IR is because the one-loop change in a is close to the value we compute using \hat{R} . In particular,

$$\begin{aligned} \delta a_{1\text{-loop}} &= -2\pi^4 \int_0^{\lambda_*} d\lambda \times \beta = \frac{1}{8} \tau_U = \frac{11}{640} \sim \frac{61}{3840} \\ &= a_{(A_1, A_2)} - a_{\mathcal{T}}, \end{aligned} \quad (21)$$

where $\beta = \frac{3}{25} \lambda (-5 + 12\pi^4 \tau_U^{-1} \lambda^2 + \dots)$, we have taken \mathcal{O}^2 to have unit normalization in the UV, $a_{\mathcal{T}}$ is defined in (3), and [29]

$$\tau_U = -\frac{27}{4} \mathcal{A}(\tilde{R}_{\text{UV}}(\tilde{R}_{\text{UV}} - \hat{R})^2) = \frac{11}{80}. \quad (22)$$

Therefore $\delta a_{1\text{-loop}} \sim a_{(A_1, A_2)} - a_{\mathcal{T}} \ll a_{(A_1, A_2)}$. As a result, the one-loop fixed point seems to yield a consistent and surprisingly good approximation of \mathcal{T} [32].

V. COMMENTS ON IR OPERATORS

Now we would like to motivate the existence of the \mathcal{O} , \mathcal{O}_a , and \mathcal{O}' chiral primaries in the IR SCFT, \mathcal{T} . If these operators exist, we can reproduce the superconformal index to a very nontrivial order in q .

To that end, the IR SL contributions are

$$\begin{aligned} \mathcal{I}_S^{\text{SL}}(\mathcal{O}) &= \frac{q^{\frac{6}{7}}}{(1-q)(1-q^{\frac{5}{7}})}, \\ \mathcal{I}_S^{\text{SL}}(\mathcal{O}_a) &= -\frac{q^{\frac{6}{7}} + q^{\frac{8}{7}}}{(1-q)(1-q^{\frac{5}{7}})}, \\ \mathcal{I}_S^{\text{SL}}(\mathcal{O}') &= \frac{q^{\frac{8}{7}}}{(1-q)(1-q^{\frac{5}{7}})}. \end{aligned} \quad (23)$$

These contributions cancel since the Coulomb branch sector does not contribute to the Schur limit of the UV index.

On the other hand, we have broken $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ by turning on (1). At leading order in the Coulomb branch sector, this breaking is encoded in (2). These relations give [33]

$$\begin{aligned} \mathcal{I}_S^{\text{SL}}(\mathcal{O}^2 = 0) &= -\frac{q^{\frac{12}{7}}}{(1-q)(1-q^{\frac{5}{7}})}, \\ \mathcal{I}_S^{\text{SL}}(\mathcal{O} \times \mathcal{O}_\alpha = 0) &= \frac{q^{\frac{12}{7}} + q^2}{(1-q)(1-q^{\frac{5}{7}})}. \end{aligned} \quad (24)$$

Now, we know that we must also have a short $\mathcal{N} = 1$ supercurrent, $J_{\dot{\alpha}\dot{\alpha}}$, in the IR contributing

$$\mathcal{I}_S^{\text{SL}} = -\frac{q^{\frac{17}{7}} + q^{\frac{19}{7}}}{(1-q)(1-q^{\frac{5}{7}})}. \quad (25)$$

Assuming these are the only low-order contributions, then

$$\mathcal{I}_{\text{IR}}(q) = 1 + q^2 - q^{\frac{17}{7}} + \dots, \quad (26)$$

and we reproduce the IR index up to order less than $\mathcal{O}(q^{\frac{17}{7}})$. We can do better.

Indeed, recall from (the conjugate of) (12) that

$$D^2 \tilde{J}_{\dot{\alpha}} \sim \tilde{\lambda} \tilde{\mathcal{O}} \times \tilde{\mathcal{O}}_{\dot{\alpha}}. \quad (27)$$

Therefore, we might naively conclude this is a long multiplet. However, in conformal perturbation theory

$$\tilde{D}_{\dot{\alpha}} \tilde{J}^{\dot{\alpha}} = 0. \quad (28)$$

At a fixed point, (28) is the shortening condition for a $\tilde{\mathcal{C}}_{-\frac{1}{6}(0, \frac{1}{2})}$ multiplet [34]. Therefore, it is reasonable to believe that $\tilde{J}_{\dot{\alpha}}$ exists as a short multiplet in the IR theory with dimension $11/4$. It is straightforward to check that the corresponding index contribution is

$$\mathcal{I}_S^{\text{SL}}(\tilde{J}_{\dot{\alpha}}) = \frac{q^{\frac{17}{7}}}{(1-q)(1-q^{\frac{5}{7}})}. \quad (29)$$

Taking (29) into account gives

$$\mathcal{I}_{\text{IR}}(q) = 1 + q^2 + q^3 + \mathcal{O}(q^4), \quad (30)$$

and we reproduce the H function to a high order using just the $\mathcal{N} = 2$ stress tensor multiplet and operators from the Coulomb branch sector.

Finally, one sees that

- (i) There are no additional SL contributions from the IR $\mathcal{N} = 1$ multiplets that descend from the $\mathcal{N} = 2$ supercurrent multiplet.
- (ii) The IR contributions due to operators that are annihilated in the UV by \tilde{Q}_{2-} and sit in the remaining $\mathcal{N} = 2$ Schur multiplets cannot arise at order smaller than $\mathcal{O}(q^{\frac{29}{7}})$.

VI. CONCLUSIONS

We have learned a surprising amount by studying a simple deformation of the minimal Argyres-Douglas theory. At the level of the parent theory, we have seen evidence that the full low-lying spectrum of short multiplets is likely simpler than one might expect. Indeed, we were able to reproduce (30) simply from the IR descendants of the $\mathcal{N} = 2$ Coulomb branch and stress tensor multiplets (the existence of the semishort J_α multiplet in the IR suggests that our deformation of the parent AD theory is particularly mild). Moreover, we saw that we could trade UV index contributions from the $SU(2)_R$ current with contributions from constrained chiral operators in the IR. This result points to deeper connections between the physics of chiral algebras and $\mathcal{N} = 2$ chiral rings upon $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ breaking that we will return to soon.

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Note added.—Recently, Ref. [35] appeared, which has overlap with our Sec. I (our calculations agree with theirs). On the other hand, our papers are largely complementary. Indeed, [35] motivates additional conjectures regarding $\mathcal{N} = 2$ -preserving chiral ring relations [their Eq. (11)] that are compatible with our results, while our paper discusses aspects of nonchiral operators, the superconformal index, accidental symmetries, and absence of free fields.

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