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Constraining alternative theories of gravity using GW150914 and GW151226

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The recently reported gravitational wave events GW150914 and GW151226 caused by the mergers of binary black holes [Abbott *et al.*, Phys. Rev. Lett. **116**, 221101 (2016); Phys. Rev. Lett. **116**, 241103 (2016); Phys. Rev. X **6**, 041015] provide a formidable way to set constraints on alternative metric theories of gravity in the strong field regime. In this paper, we develop an approach where an arbitrary theory of gravity can be parametrized by an effective coupling $G_{\rm eff}$ and an effective gravitational potential $\Phi(r)$. The standard Newtonian limit of general relativity is recovered as soon as $G_{\rm eff} \to G_N$ and $\Phi(r) \to \Phi_N$. The upper bound on the graviton mass and the gravitational interaction length, reported by the LIGO-VIRGO Collaboration, can be directly recast in terms of the parameters of the theory that allows an analysis where the gravitational wave frequency modulation sets constraints on the range of possible alternative models of gravity. Numerical results based on published parameters for the binary black hole mergers are also reported. The comparison of the observed phases of GW150914 and GW151226 with the modulated phase in alternative theories of gravity does not give reasonable constraints due to the large uncertainties in the estimated parameters for the coalescing black holes. In addition to these general considerations, we obtain limits for the frequency dependence of the α parameter in scalar tensor theories of gravity.

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I. INTRODUCTION

In September and December 2015, the LIGO-VIRGO Collaboration reported on the direct detection of gravitational-wave (GW) signals from coalescing binary black hole (BH) systems [1,4]. This has opened new opportunities in gravity research and has begun the era of gravitational wave astronomy. In particular, this achievement can be considered as the first direct probe of metric theories of gravity in the regime of strong fields and relativistic velocities. The individual masses of the merging BHs at the beginning of the collision were $29^{+6}_{-4}\ M_\odot$ and $36^{+4}_{-5}\ M_\odot$ for the September signal and $14.2^{+8.3}_{-4.2}\ M_\odot$ and $7.5^{+2.3}_{-2.3}\ M_\odot$ for the December signal. Specifically, the GW150914 signal was emitted by a rapidly evolving dynamical binary that merged in a fraction of a second with an observed variation of the period $\dot{P}_b$ ranging from ~ -0.1 at $f_{\rm GW} \sim 30$ Hz to ~ -1 at $f_{\rm GW} \sim 132$ Hz. The frequency and amplitude of the GW151226 signal was observed over 55 cycles spanning a range in frequency from 35 to 450 Hz. Using the templates created from numerical relativity, the data are consistent with the merger of two compact objects into a merged black hole with masses of $\sim 65.3^{+4.1}_{-3.4}\ M_\odot$ and $\sim 21.8^{+5.9}_{-1.7}\ M_\odot$, respectively. In this process, the energy emitted in the form of GW amounts to $3.0^{+0.5}_{-0.4}\ M_\odot$ and $1.0^{+0.1}_{-0.2}\ M_\odot$, and the velocity v reached the value $\sim 0.5c$ at the time of the merger. In particular, the signal from GW150914 exhibits the typical behavior predicted by the coalescence of compact systems where inspiral, merger, and ringdown phases are traversed [5]. The LIGO-VIRGO Collaboration has analyzed the three regimes adopting a parametrized analytical family of inspiral-merger-ringdown waveforms [6–11]. The signal is divided in terms of frequency: the early to late inspiral regime from ~20 Hz to ~55 Hz; the intermediate region from ~55 Hz to ~130 Hz; and the merger-ringdown region from ~130 Hz until the end of the waveform. The simplest and fastest parametrized waveform model that is currently

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available [12] sets bounds on the physical effects based on the inspiral phase only, where a calibrated post-Newtonian (PN) treatment is sufficient. For the later phases, phenomenological coefficients fitted to numerical relativity waveforms are used. In this paper, we discuss the possibility to set constraints on extended theories of gravity via the modified inspiral phase.

It is worth noting that the existence of GWs confirms metric theories of gravity, among them general relativity (GR), but there is ample room for other possibilities (see [1] for a detailed discussion). Any extended theory of gravity can be parametrized by means of a suitable post-Newtonian parametrization where the governing parameter is the effective gravitational coupling constant $G_{\rm eff}$ and the effective gravitational potential $\Phi(r)$. Both these quantities are functions of the radial coordinate that influence the phase of the GW signal. In other words, the GW waveform could, in principle, single out the range of possible gravitational metric theories that are in agreement with the data.

The paper is organized as follows. In Sec. II, we discuss how different theories of gravity can be parametrized by the coupling constant and the gravitational potential. The main differences of these theories with respect to GR can be reduced to the effective dependence on the radial coordinate. It is then straightforward to obtain the corresponding phase modulation, and we will exemplarily do so in Sec. III and compare with the observed data. Section IV is devoted to the discussion of the Shapiro delay that can be modulated according to the parameters of the given theory. Discussion and conclusions are drawn in Sec. V.

II. EFFECTIVE GRAVITATIONAL CONSTANT IN EXTENDED THEORIES OF GRAVITY

Alternative theories of gravity are extensions of GR where higher order curvature invariants and/or additional scalar fields are taken into account in the Hilbert-Einstein gravitational action (see [13–16] for a comprehensive review on the subject). If the gravitational Lagrangian is nonlinear in the Ricci scalar or, more generally, in the curvature invariants, the field equations become higher than second order in the derivatives; it is for this reason that such theories are often called *higher-order gravitational theories*. In principle, one can take into account wide classes of higher-order-scalar-tensor theories of gravity in four dimensions [13].

With the emergence of the inflationary paradigm, these theories have gained heightened attention as they can provide solutions to the shortcomings of the standard cosmological model. These are, for example, the horizon problem, the density fluctuation problem, the dark matter problem, the exotic relics problem, the thermal state problem, the cosmological constant problem, the singularity problem, and the time scale problem [17–21]. Furthermore, the presence of scalar fields is important also in multidimensional gravity, such as Kaluza-Klein theories and in the effective action of string theory. In this framework, the strength of gravity, given by the local value of the gravitational coupling, depends on time and location. For example, the Brans-Dicke theory that is the most used scalar-tensor theory of gravity [22] includes the hypothesis suggested by Dirac of the variation of the gravitational coupling with time [23]. As a consequence, scalar-tensor theories do not satisfy the strong equivalence principle (EP) as the variation of the gravitational constant $G_{\rm eff}$—which is, in general, different from G_N, the standard Newton gravitational constant—implies that local gravitational physics depends on the scalar field strength. Theories that present such a feature are called *nonminimally coupled theories*.

In these theories, the gravitational coupling is determined by the form of the Lagrangian. We can have two physically interesting situations which could be tested by experiments:

(1) When $G_{\rm eff}(r)_{r\to\infty} \to G_N$, the Newton gravitational constant and GR are recovered.
(2) The possibility that gravitational coupling is not asymptotically constant; i.e., $G_{\rm eff}$ is always varying with the epoch and $\dot{G}_{\rm eff}/G_{\rm eff}|_{\rm now} \neq 0$.

The variability of the gravitational coupling can be tested by three classes of experiments:

(i) Through observations of Solar System dynamics. In fact, several weak-field tests of GR are based on planetary motion and dynamics of self-gravitating objects nearby the Sun. Deviations from classical tests are possible probes for the variation of the gravitational coupling.
(ii) Through binary pulsar systems. To obtain information from these systems, it has been necessary to extend the post-Newtonian approximation, which can be used only in the presence of a weakly gravitationally interacting n-body system, to strong gravitationally interacting systems. The estimation of $\dot{G}/G$ is 2×10^{-11} per year [24,25].
(iii) Through gravitational lensing observations of distant galaxies [26].

Concerning the solar system tests, the most stringent limits are obtained by Lunar Laser Ranging (LLR) combined with accurate ephemeris of the solar system. LLR consists of measuring the round-trip travel time of photons that are reflected back to Earth from mirrors located on the Moon; the change of round-trip time contains information about the Earth-Moon system. The round-trip travel time has been investigated for many years, and the best estimates for $\dot{G}/G$ range from 0.4×10^{-11} yr^{-1} to 10^{-11} yr^{-1} [24,27]. However, none of these tests probes the strong field regime which, up to now, could not be investigated at all.

Besides the variation of the gravitational coupling, it is well known that a wide class of these theories gives rise to Yukawa-like corrections, $r^{-1}e^{-mr}$ in the gravitational potential [15]. Here, the parameter m is an effective mass related to the additional degrees of freedom in the gravitational action. Specifically, an additional scalar field is introduced by the corresponding Klein-Gordon equation of the form $\Box\phi - dV(\phi)/d\phi = 0$ that has to be added to the standard set of Einstein field equations. In the static case, the Klein-Gordon equation reduces to

$$(\nabla^2 - m^2)\phi = 0, \quad (1)$$

where the effective mass m is given by the minimum of the potential $V(\phi)$. The solution of Eq. (1) is a Newtonian potential corrected by a Yukawa-like term that, as in the Klein-Gordon case, disappears at infinity, allowing one to recover the Newtonian limit and Minkowski flat spacetime.

In general, most alternative gravities have a weak field limit that can be expressed in the form (see also [28,29])

$$\Phi(r) = -\frac{G_N M}{r}\left[1 + \sum_{k=1}^{n} \alpha_k e^{-r/r_k}\right] = -\frac{G_{\rm eff} M}{r}, \quad (2)$$

where G_N is the value of the gravitational constant as measured at infinity and r_k is the interaction length of the kth component of the non-Newtonian corrections (see also [30,31]). See Refs. [30,31], for a general discussion of this last equation containing the non-Newtonian corrections.

Clearly, the standard Newtonian potential is restored as soon as $G_{\rm eff} \to G_N$, which means $e^{-r/r_k} \to 0$ at infinity. The amplitude α_k of each component is normalized to the standard Newtonian term, and the signs of the α_k coefficients indicate whether the corrections are attractive or repulsive [32].

For the simplicity of the estimations, one can truncate to the first term of the expansion series in Eq. (2).[1]

One then obtains a potential for the form

$$\Phi(r) = -\frac{G_N M}{r}[1 + \alpha_1 e^{-r/r_1}], \quad (3)$$

where the influence of non-Newtonian terms can be parametrized through the constants (α_1, r_1). For asymptotically large distances, where $r \gg r_1$, the exponential term tends to 0 and consequently the gravitational coupling tends to the limiting value G_N. In the opposite case when $r \ll r_1$, the exponential term tends to unity; consequently, by differentiating Eq. (3) and comparing with the gravitational force measured in laboratory experiments, one can get

$$G_{\rm lab} = G_N\left[1 + \alpha_1\left(1 - \frac{r}{r_1}\right)\right] \simeq G_N(1 + \alpha_1), \quad (4)$$

where $G_{\rm lab} = 6.67 \times 10^{-8}\ {\rm g^{-1}\,cm^3\,s^{-2}}$ is the standard Newton gravitation constant precisely measured in Cavendish-like experiments and where G_N and $G_{\rm lab}$ are identically the same in the standard gravity. However, the inverse square law is asymptotically valid, but the measured coupling constant is different by a factor of $(1 + \alpha_1)$.

For self-gravitating systems, any correction involves a characteristic length that acts at a certain scale. The range of the characteristic scale r_k corresponds to Compton's length

$$r_k = \frac{\hbar}{m_k c} \quad (5)$$

and is identified through the mass m_k of a pseudoparticle. Accordingly, in the weak energy limit, fundamental theories attempting to unify gravity with other forces introduce extra particles *with mass* which may carry the further degrees of freedom of the gravitational force [33].

There have been several attempts to constrain r_k and α_k (and hence m_k) by experiments on scales in the range $1\ {\rm cm} < r < 10^8\ {\rm cm}$, using a variety of independent and different techniques [34–36]. The expected masses for particles which should carry the additional gravitational force are in the range $10^{-13}\ {\rm eV} < m_k < 10^{-5}\ {\rm eV}$. Given these, one can obtain the following estimates for the parameters:

$$|\alpha_1| \sim 10^{-2}, \qquad r_1 \sim 10^4\text{–}10^5\ {\rm cm}. \quad (6)$$

Assuming that the dilaton is an ultrasoft boson that carries the scalar mode of gravitational field, one obtains a length scale of $\sim 10^{22}$–10^{23} cm, if the mass range is $m \sim 10^{-27}$–10^{-28} eV. This length scale is necessary to explain the flat rotation curves of the spiral galaxies. Furthermore, very long baseline interferometry observations impose a limit of $\alpha \sim 1.4 \times 10^{-2}$ [37]. On the other hand, binary-pulsar data place a limit from 10^{-2} to 10^{-4} on α [38–41].

However, new limits from GW150914, reported in [1], give as an upper limit for the graviton mass $m_g \le 10^{-22}$ eV and $r_g \ge 10^{18}$ cm for the related Compton length. We obtain the same limit also for GW151226. These experimental numbers open new interesting perspectives in the present debate as soon as the above m_k and r_k are interpreted. Below, we will discuss how $G_{\rm eff}$ and $\Phi(r)$ could be constrained according to the GW150914 and GW151226 data. As we will see, such constraints can be interpreted, at fundamental level, as the above effective mass m_k and interaction length r_k.

III. CONSTRAINING $G_{\rm eff}$ AND $\Phi(r)$ BY GW150914 AND GW151226

Starting from the above considerations, it is possible to constrain $G_{\rm eff}$ and $\Phi(r)$ by the GW parameters reported for the events GW150914 and GW151226. Before this, let us review the post-Newtonian approximation required to perform this kind of analysis. Specifically, let us compute the 3.5PN approximation that is relevant for our analysis [42,43]. In particular, PN waveform models at the 3.5PN order are developed, e.g., in [44].

[1]This assumption is not applicable in some cases where additional corrections are taken into account.

To compare the theoretical waveforms with experimental sensitivities, we write the Fourier transform of the two GW strains h_+, $h_\times$ as

$$h_+ = Ae^{i\phi_+(f)}\frac{c}{r}\left(\frac{G_{\rm eff}M}{c^3}\right)^{\frac{5}{6}}\frac{1}{f^{\frac{7}{6}}}\left(\frac{1+\cos^2 i}{2}\right), \tag{7}$$

$$h_\times = Ae^{i\phi_\times(f)}\frac{c}{r}\left(\frac{G_{\rm eff}M}{c^3}\right)^{\frac{5}{6}}\frac{1}{f^{\frac{7}{6}}}\cos i, \tag{8}$$

where i is the inclination angle of the line of sight and the constant A has the value

$$A = \frac{1}{\pi^{\frac{2}{3}}}\left(\frac{5}{24}\right)^{\frac{1}{2}}. \tag{9}$$

The phase ϕ_+ is given as

$$\phi_+(f) = 2\pi f\left(t_c + \frac{r}{c}\right) - \varphi_c - \frac{\pi}{4} + \frac{3}{4}\left(\frac{G_{\rm eff}M}{c^3}8\pi f\right)^{-\frac{5}{3}}, \tag{10}$$

where φ_c and t_c are the value of the phase and the time at coalescence, respectively. Furthermore the phases of the two strains are directly related, $\phi_\times = \phi_+ + \frac{\pi}{2}$.

An accurate computation of the phase going well beyond the Newtonian approximation is crucial for discriminating the signal of a coalescing binary from the noise. Therefore one has to give the PN correction to the phase (10). To exploit the signal present in the detector, and thus detect sources at a further distance, an accurate theoretical prediction on the time evolution of the waveform is required.

To calculate the PN corrections, we write the equation of motion in a more general form,

$$\frac{dv^i}{dt} = -\frac{G_{\rm eff}M}{r^2}\left[(1+\mathcal{A})\frac{x^i}{r} + \mathcal{B}v^i\right] + \mathcal{O}\left(\frac{1}{c^8}\right), \tag{11}$$

such that it has a term proportional to the relative separation x^i and a term proportional to the relative velocity v^i in the center of mass frame. Here, the effective gravitational constant is not given by the standard Newton constant, but by $G_{\rm eff} = G_N(1+\alpha)$.

Explicit expressions for the functions $\mathcal{A}$ and $\mathcal{B}$ are extremely long and are given in Ref. [45]. We proceed by considering the following relation for the frequency-domain phase:

$$\phi = 2\pi f t_c - \varphi_c - \frac{\pi}{4} + \frac{3}{128\eta}\left(\pi\frac{MfG_{\rm eff}}{c^3}\right)^{-\frac{5}{3}}\sum_{i=0}^{7}\varphi_i(\Theta)\left(\pi\frac{MfG_{\rm eff}}{c^3}\right)^{\frac{i}{3}}, \tag{12}$$

where $\varphi_i(\Theta)$ are the PN expansion coefficients that are functions of the intrinsic binary parameters. The information on the spin χ_i (with $i = 1, 2$) is incorporated via the relations

$$\chi_s = \frac{(\chi_1+\chi_2)}{2}, \tag{13}$$

$$\chi_a = \frac{(\chi_1-\chi_2)}{2} \tag{14}$$

that appear in the functions $\varphi_i(\Theta)$.

The 3.5PN expansion coefficients are

$$\varphi_0 = 1, \tag{15}$$

$$\varphi_1 = 0, \tag{16}$$

$$\varphi_2 = \frac{3715}{756} + \frac{55\eta}{9}, \tag{17}$$

$$\varphi_3 = -16\pi + \frac{113\delta\chi_a}{3} + \left(\frac{113}{3} - \frac{76\eta}{3}\right)\chi_s, \tag{18}$$

$$\varphi_4 = \frac{15293365}{508032} + \frac{27145\eta}{504} + \frac{3085\eta^2}{72} + \left(-\frac{405}{8} + 200\eta\right)\chi_a^2 - \frac{405}{4}\delta\chi_a\chi_s + \left(-\frac{405}{8} + \frac{5\eta}{2}\right)\chi_s^2, \tag{19}$$

$$\varphi_5 = \left[1 + \log\left(\pi\frac{G_{\rm eff}Mf}{c^3}\right)\right]\left[\frac{38645\pi}{756} - \frac{65\pi\eta}{9} + \delta\left(-\frac{732985}{2268} - \frac{140\eta}{9}\right)\chi_a + \left(-\frac{732985}{2268} + \frac{24260\eta}{81} + \frac{340\eta^2}{9}\right)\chi_s\right], \tag{20}$$

$$\varphi_6 = \frac{11583231236531}{4694215680} - \frac{6848\gamma_E}{21} - \frac{640\pi^2}{3} + \left(-\frac{15737765635}{3048192} + \frac{2255\pi^2}{12}\right)\eta + \frac{76055\eta^2}{1728} - \frac{127825\eta^3}{1296} - \frac{6848}{63}\log\left(64\pi\frac{G_{\rm eff}Mf}{c^3}\right) + \frac{2270}{3}\pi\delta\chi_a + \left(\frac{2270\pi}{3} - 520\pi\eta\right)\chi_s, \tag{21}$$

$$\varphi_7 = \frac{77096675\pi}{254016} + \frac{378515\pi\eta}{1512} - \frac{74045\pi\eta^2}{756} + \delta\left(-\frac{25150083775}{3048192} + \frac{26804935\eta}{6048} - \frac{1985\eta^2}{48}\right)\chi_a + \left(-\frac{25150083775}{3048192} + \frac{10566655595\eta}{762048} - \frac{1042165\eta^2}{3024} + \frac{5345\eta^3}{36}\right)\chi_s. \tag{22}$$

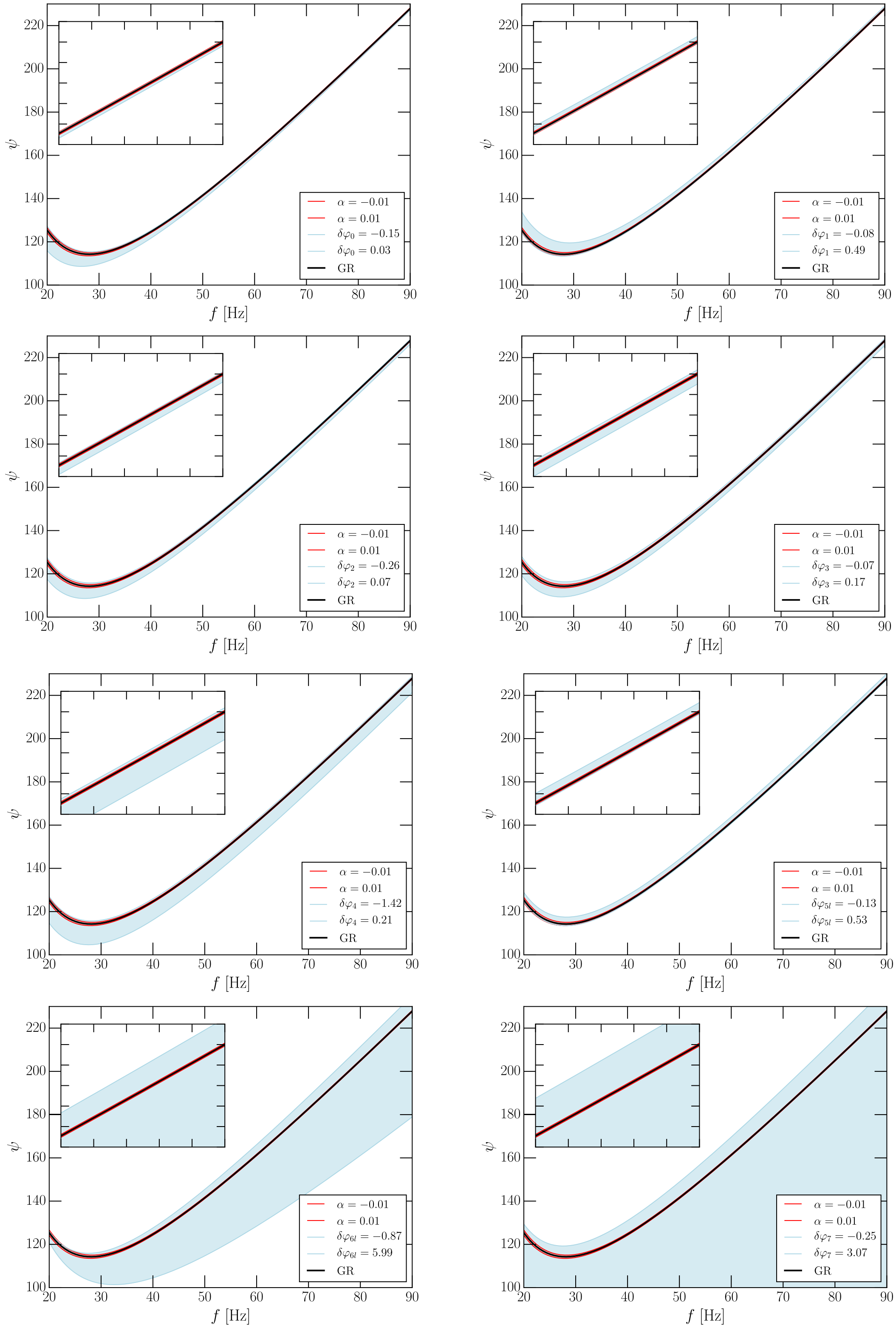

FIG. 1. Frequency-domain phase representation for GW150914 with masses $m_1 = 36.2\ M_\odot$, $m_2 = 29.1\ M_\odot$ and initial spins $\chi_1 < 0.7, \chi_2 < 0.8$ [4]. The solid black curve is the GR prediction where $\alpha = 0$ and $\delta\varphi_i = 0$. The red lines are $\alpha = \pm 0.01$. The shaded blue area is the range allowed for the $\delta\varphi_i$ parameter in accordance with Table 1 of [5]. From left to right: the first on the left column shows the phase at 0PN order, and the right one is for the 0.5PN order. The second on the left column is 1PN, while on the right there is the 1.5PN. The third on the left column represents the phase at 2PN. On the right column, the phase at 2.5PN is shown. Finally, the fourth on the left column shows the 3PN and on the right 3.5PN. Note that the error on the $\delta\varphi_7$ is so large that it falls outside the scale. The inset frequency ranges from 80 Hz to 90 Hz to illustrate the curves at these frequencies.

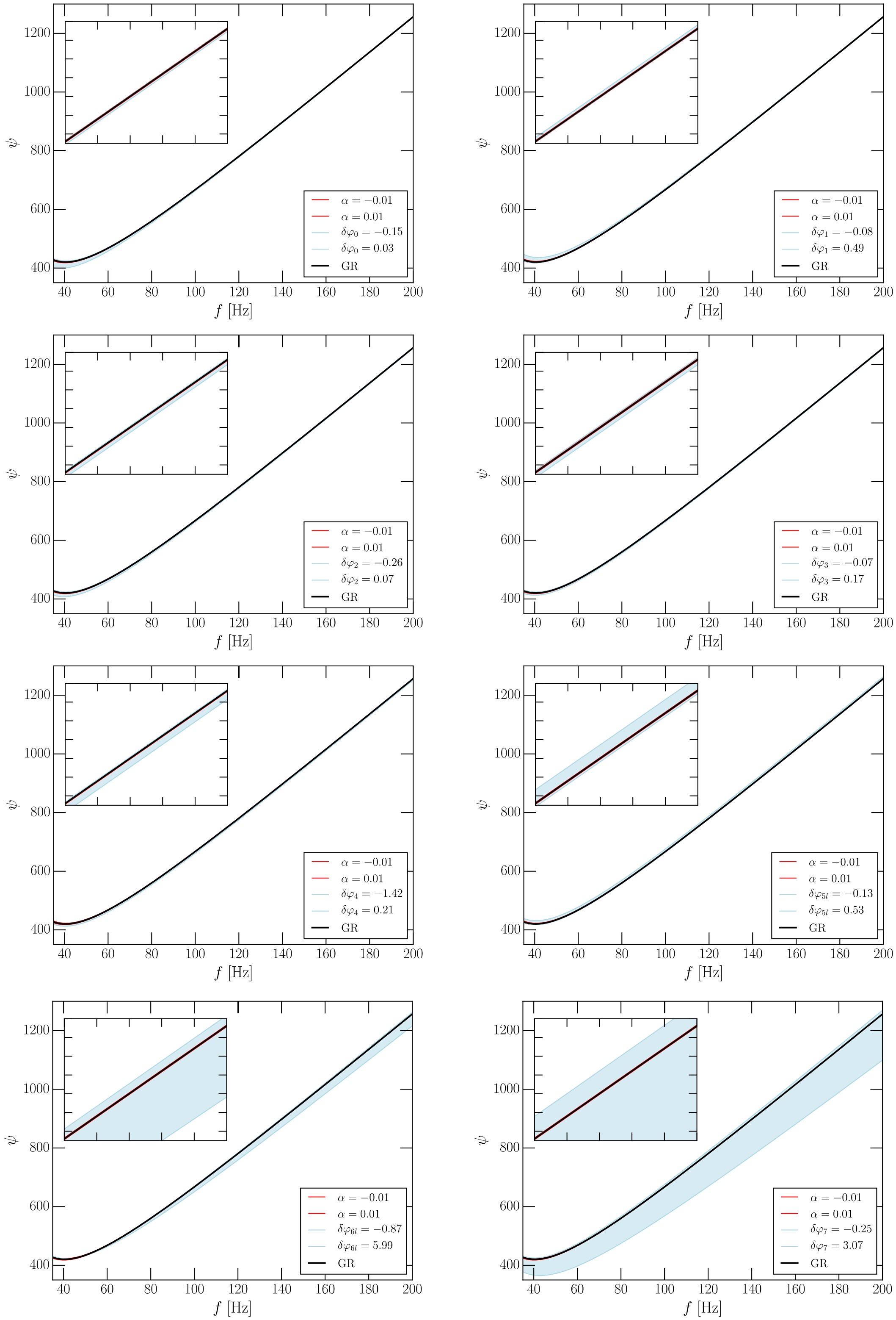

FIG. 2. Frequency-domain phase representation for GW151226 with masses $m_1 = 14.2\ M_\odot$, $m_2 = 7.5\ M_\odot$ and initial spins $\chi_1 < 0.7$, $\chi_2 < 0.8$ [2,3]. The solid black curve is the GR prediction where $\alpha = 0$ and $\delta\varphi_i = 0$. The red lines are $\alpha = \pm 0.01$. The shaded blue area is the range allowed for the $\delta\varphi_i$ parameter in accordance with Table 1 of [5]. From left to right: the first on the left column shows the phase at 0PN order, and the right one is for the 0.5PN order. The second on the left column is 1PN, while on the right there is the 1.5PN. The third on the left column represents the phase at 2PN. On the right column, the phase at 2.5PN is shown. Finally, the fourth on the left column show the 3PN and on the right 3.5PN. The inset shows the frequency from 180 to 190 Hz.

TABLE I. We report the frequency dependence of each parameter of Fig. 6 in [3], median, and 90% credible regions. For each parameter we report the corresponding quantities for the combined signals of GW150914 and GW151226 analyses as in [3,5].

Waveform regime	Parameter	f-dependence	Median GW150914 + GW151226
Early-inspiral regime	$\delta\varphi_0$	$f^{-5/3}$	$-0.05^{+0.08}_{-0.1}$
	$\delta\varphi_1$	$f^{-4/3}$	$0.18^{+0.31}_{-0.26}$
	$\delta\varphi_3$	$f^{-2/3}$	$0.11^{+0.06}_{-0.18}$
	$\delta\varphi_2$	f^{-1}	$-0.05^{+0.12}_{-0.21}$
	$\delta\varphi_4$	$f^{-1/3}$	$-0.6^{+0.81}_{-0.82}$
	$\delta\varphi_{5l}$	$\log(f)$	$0.27^{+0.26}_{-0.4}$
	$\delta\varphi_6$	$f^{1/3}$	$-0.38^{+0.49}_{-0.72}$
	$\delta\varphi_{6l}$	$f^{1/3}\log(f)$	$2.66^{+3.33}_{-3.53}$
	$\delta\varphi_7$	$f^{2/3}$	$1.48^{+1.59}_{-1.73}$

where $\varphi_0, \ldots, \varphi_7$ indicate the $0, \ldots, 3.5$PN approximation, respectively, $\gamma_E = 0.577$ is the Euler-Mascheroni constant [10], and we have used the common definitions

$$\delta = \frac{(m_1 - m_2)}{M}, \quad (23)$$

$$\eta = \frac{(m_1 m_2)}{M}, \quad (24)$$

where m_1, m_2 are the masses of the two compact objects.

In Figs. 1 and 2 we have plotted the frequency domain phase representation of GW150914 and GW151226 and show the effect of varying the $\delta\varphi_i$ parameters as provided by the single parameter analysis of [2,3,5]. Note that we follow their naming convention in introducing the quantities φ_{5l} and φ_{6l} that contain the logarithmic dependence with frequency. In addition, the variation with the leading order deviation $\alpha = \pm 10^{-2}$ is shown. The single parameter analysis of [2] was performed by setting all but the considered $\delta\varphi_i$ to 0. In contrast, the multiple parameter analysis was done by allowing all $\delta\varphi_i$ to vary freely. The latter leads to an error that is almost 1 order of magnitude larger due to the additional degrees of freedom. We do not consider GW150914 and GW151226 data for the multiple parameter analysis performed in [2,5] due to the large error bars in the $\delta\varphi_i$ parameters.

For the masses, we have used the values as given in [3] with $m_1 = 36.2\,M_\odot$, $m_2 = 29.1\,M_\odot$ for GW150914, while $m_1 = 14.2\,M_\odot$, $m_2 = 7.5\,M_\odot$ for GW151226. The initial spins were only constrained to be less than $\chi_1 < 0.7$, $\chi_2 < 0.8$ [3,4], and for our analysis we have taken the values of $\chi_1 = 0.7$, $\chi_2 = 0.8$. Additionally, we adopted the values $t_c = 0.43s$ for GW150914 and $t_c = 1s$ for GW151226 [1,2,5] and set $\varphi_c = 0$ for both.

Furthermore, we studied the sensitivity of our results by varying the initial masses m_1, m_2 and initial spins χ_1, χ_2 within the errors and ranges obtained by [4]. We found that the resulting changes were mostly quantitative, such as altering the slopes of the curves, and that the qualitative behaviors of the curves, such as the width of the constraints given by different α, remained unchanged. Thus the results plotted in Figs. 1 and 2 are representative of the possible physical parameters reported in [4].

In Fig. 1, we show the frequency-domain phase representation for GW150914 and as the shaded blue area the constraints due to the $\delta\varphi_i$ of the combined events, as provided by [3] and given in Table I. The GR evolution is shown as the black solid line, while the extended theory is marked in the range of $\alpha \in [-10^{-2}, +10^{-2}]$ as red curves. We report on the early inspiral range $f \in [20, 90]$ Hz and zoom in on the range $f \in [80, 90]$ Hz in the inset. As one can observe, at all the parameter orders the single parameter analysis does not rule out $|\alpha| < 10^{-2}$.

The data of the second event, GW151226, are shown in Fig. 2; due to the lower masses involved in the merger, the frequency is much higher. Consequently, the inspiral regime lasts until 450 Hz, and we show the phase in the range $f \in [40, 200]$ Hz. Inspecting Eqs. (20) and (22), we see that the variation with α is more pronounced for objects with lower total mass M. Hence, in principle this event could yield tighter constraints on the value of the allowed α.

To investigate the required tolerances, in Fig. 3 we plot two PN orders, 0 and 4, where, for GW151226, we have decreased the variations by factors of 2,5, and 10. For the PN terms of order 0, an increase by an order of magnitude would be sufficient to set constraints on α. More promising still are the higher order terms which, even with a factor of 5 improvement, would be able to set constraints on $|\alpha| < 10^{-3}$.

We stress that these results are obtained from the single parameter analysis, while a more correct treatment would have to adopt the uncertainties of the multiple parameter analysis. Furthermore, we expect that as more GWs are detected, the statistics on the combined posterior density distributions [3] will improve, and we will be able to set stronger constraints on these types of alternative theories of gravity in the near future.

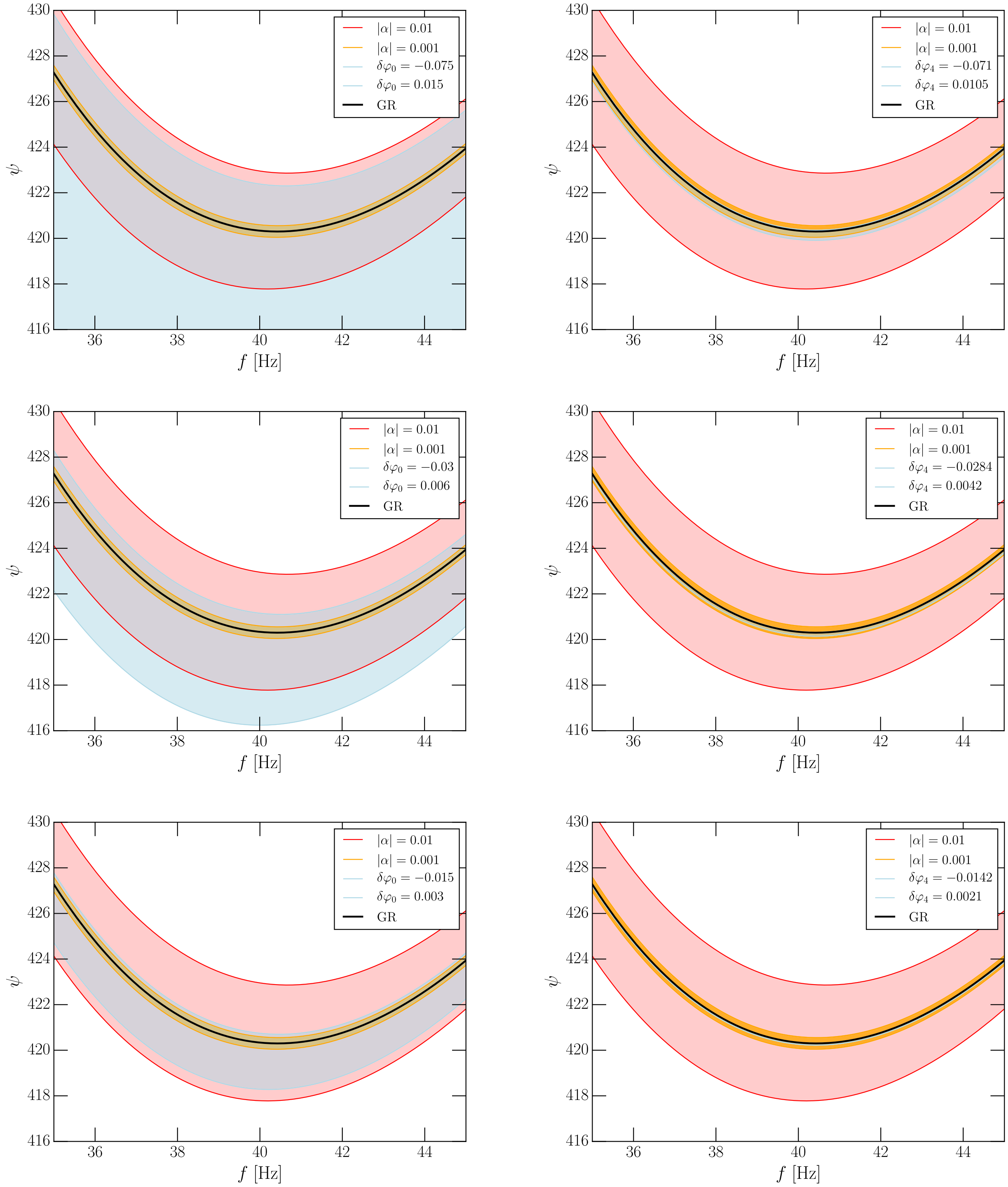

FIG. 3. Two PN orders for GW151226, 0 (left) and 4 (right), with errors in $\delta\varphi_i$ improved (from top to bottom) by factors of 2,5, and 10. The scales have increased in order to more clearly show the curves.

IV. CONSTRAINTS FROM THE SHAPIRO DELAY

In this section we obtain constraints from the Shapiro delay using the relative time difference between observations at multiple frequencies. This allows one to infer violations of the EP using the observed time delay from astrophysical particle messengers like photons, gravitons, or neutrinos [46–48]. To date, the strongest constraints on the frequency dependence of the parametrized post Newtonian (PPN)-γ parameter are obtained by observations of fast radio bursts (FRBs) yielding $\Delta\gamma(f) \sim 10^{-9}$. In the case of FRBs, the largest uncertainty is the signal dispersion due to the poorly understood line-of-sight free electron population [48]. The fact that this uncertainty is completely avoided for gravitational waves makes them an appealing messenger to test EP violations.

The Shapiro gravitational time delay is caused by the slowing passage of light as it moves through a gravitational potential (3) as

$$\Delta t_{\rm grav} = -\frac{1+\gamma}{c^3}\int_{r_e}^{r_o}\Phi(r)dr, \qquad (25)$$

where γ is the (theory dependent) PPN parameter and r_o and r_e are the positions of the observer and the source of emission. Let us conservatively assume a short burst of emission; that is, all wave frequencies are emitted at the same instant. Now given the observed signal duration for GW150914 of $\sim 0.2s$, we can obtain an estimate for the frequency dependence of γ and α, respectively. In the absence of other dispersive propagation effects, e.g., due to Lorentz invariance violation (see also [49,50]), we obtain an upper limit for $\Delta\alpha/\Delta f$.

For example, in scalar tensor theories the γ PPN parameter is expressed in terms of the nonminimal coupling function of a scalar field, equivalently, in terms of the α parameter, that is (for more details see [51]),

$$\gamma - 1 = -\frac{(f'(\varphi))^2}{f(\varphi)+2[f'(\varphi)]^2} = -2\frac{\alpha^2}{1+\alpha^2}. \qquad (26)$$

In this case, the delay (25) takes the form

$$\Delta t_{\rm grav} = -\left(2 - \frac{2\alpha^2}{1+\alpha^2}\right)/c^3\int_{r_e}^{r_o}\Phi_{\rm N}(r)(1+\alpha e^{-r/r_1})dr, \qquad (27)$$

of which the most important contribution comes from the term linear in α,

$$\Delta t_{\rm grav} \simeq -2\alpha/c^3\int_{r_e}^{r_o}\Phi_{\rm N}(r)e^{-r/r_1}dr. \qquad (28)$$

It is evident that for $r_1 \gg r_e$, r_o the value for $\Delta\alpha/\Delta f$ is just half the constraint that can be set on $\Delta\gamma/\Delta f$ using the usual Shapiro delay. Thus with the same assumptions for the potential encountered by the gravitons as [47] (corresponding to a Shapiro delay of 1800 days), we can set the limit $|\alpha(250~{\rm Hz}) - \alpha(35~{\rm Hz})| < 1.3\times 10^{-9}$.

It is important to note that this seemingly tight limit relates to the frequency dependence only; e.g., if we have $\alpha = \alpha_0 + \alpha(f)$, the constant term α_0 is entirely unconstrained by this experiment. However, knowledge of $\Delta\alpha/\Delta f$ can be used to extrapolate measurements of the absolute value of α across the spectrum and thus extend their range of validity.

V. DISCUSSION AND CONCLUSIONS

The GW150914 and GW151226 signals [1,3] show the inspiral and merger regimes and GW150914 is also observed in the ringdown phase. Here we have analyzed the inspiral data for GW150914 and GW151226, using an extended post-Newtonian approximation. We would like to underline that corrections coming from alternative gravity to the standard relativistic equations and waveforms describing binary black hole systems are negligible up to 2.5PN order (see, e.g., [45]).

However, since, as we have demonstrated, extended theories of gravity give rise to an effective gravitational coupling constant $G_{\rm eff}$, the post-Newtonian dynamics of any metric formalism can be obtained straightforwardly for the lowest order deviation parameter $\alpha = {\rm const}$. The recently detected gravitational waveforms of GW150914 and GW151226 can thus give constraints on the theory.

Using the fact that the gravitational wave frequencies are modulated through $G_{\rm eff}$, we have shown that this modulation will change the phase of the detected gravitational signal. Our conclusions are in agreement with [52] who found that GW150914 and GW151226 do not place strong constraints on the theory of gravity, since the parameters of the merging black holes are not measured with high enough precision. However, improved statistics on the deviations $\delta\phi_i$ could remedy this shortcoming in the future.

Moreover, we have used the Shapiro delay of GW150914 to set an upper limit $|\alpha(250~{\rm Hz}) - \alpha(35~{\rm Hz})| < 1.3\times 10^{-9}$. Although this result was obtained for scalar tensor theories, this applies for all theories where the PPN-γ is at least quadratic in α.

The constraints provided by GW150914 and GW151226 on GR and, in general, metric theories of gravity, are unprecedented due to the nature of the sources and the strong field regime. However, they have not reached high enough precision to definitively discriminate among concurring theories. Furthermore, in order to extract new physical effects, one would need a wide range of GW waveforms beyond the standard forms adopted for GR and allow for polarizations beyond the standard $\times$ and $+$ modes [53].

Finally, more stringent bounds could be obtained by combining results from multiple GW observations [11,54–56]. Given the rate of coalescence of binary black holes as inferred in Ref. [57,58], we are looking forward to the upcoming joint observation surveys from advanced LIGO and VIRGO experiments.

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