Light propagation in the gravitational field of one arbitrarily moving pointlike body in the 2PN approximation

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An analytical solution for the light trajectory in the near zone of the gravitational field of one pointlike body in arbitrary slow motion in the post-post-Newtonian approximation is presented in harmonic gauge. Expressions for total light deflection and time delay are given. The presented solution is a further step toward high-precision astrometry aiming at nanoarcsecond level of accuracy.

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I. INTRODUCTION

In order to determine the positions and motions of astronomical objects on the sky, astrometry uses light signals (photons) which are emitted by the celestial objects. These light rays propagate from the celestial light source through the gravitational field of the Solar System and do finally arrive at the observer. Therefore, the precise determination of the trajectories of light signals through the warped space-time of the Solar System is a fundamental assignment of a task in relativistic astrometry. According to the theory of general relativity [1,2] light rays propagate along null geodesics governed by the geodesic equation,

$$\frac{d^2x^\alpha(\lambda)}{d\lambda^2}+\Gamma^\alpha_{\mu\nu}\frac{dx^\mu(\lambda)}{d\lambda}\frac{dx^\nu(\lambda)}{d\lambda}=0, \tag{1}$$

$$g_{\alpha\beta}\frac{dx^\alpha(\lambda)}{d\lambda}\frac{dx^\beta(\lambda)}{d\lambda}=0, \tag{2}$$

where (1) represents the geodesic equation and the constraint (2) must be imposed for null geodesics which states that the tangent four-vector along light rays is isotropic. In (1) and (2) the four-coordinates of a light signal $x^\alpha(\lambda)$ depend on affine parameter λ, and the Christoffel symbols in (1) are related to the metric $g_{\alpha\beta}$ of curved space-time,

$$\Gamma^\alpha_{\mu\nu}=\frac{1}{2}g^{\alpha\beta}\left(\frac{\partial g_{\beta\mu}}{\partial x^\nu}+\frac{\partial g_{\beta\nu}}{\partial x^\mu}-\frac{\partial g_{\mu\nu}}{\partial x^\beta}\right), \tag{3}$$

with metric signature $(-,+,+,+)$. The geodesic equation (1) and the isotropic condition (2) are valid in any reference system. With the aid of the zeroth component of (1), the geodesic equation and the isotropic condition can be expressed in terms of coordinate time t rather than the affine parameter λ as follows [3–5],

$$\frac{d^2x^i(t)}{c^2dt^2}+\Gamma^i_{\mu\nu}\frac{dx^\mu(t)}{cdt}\frac{dx^\nu(t)}{cdt}=\Gamma^0_{\mu\nu}\frac{dx^\mu(t)}{cdt}\frac{dx^\nu(t)}{cdt}\frac{dx^i(t)}{cdt}, \tag{4}$$

$$g_{\alpha\beta}\frac{dx^\alpha(t)}{cdt}\frac{dx^\beta(t)}{cdt}=0, \tag{5}$$

while the zeroth component in (4) vanishes identically. The equations in (4) and (5) are more appropriate in order to integrate the geodesic equation and also in view of the fact that real astrometric measurements do by all means imply the use of concrete reference systems. In line with the resolutions of International Astronomical Union (IAU) [6], the Barycentric Celestial Reference System (BCRS) is adopted, which is the standard global chart in modern-day astrometry. The origin of the spatial axes of the BCRS is located at the barycenter of the Solar System, the harmonic coordinates of the BCRS are denoted by (ct, x^i) where t is the BCRS coordinate time and x^i are the three-dimensional coordinates referred to the spatial axes of the BCRS, and obey the harmonic gauge condition (de Donder gauge):

$$\frac{\partial\sqrt{-g}g^{\alpha\beta}}{\partial x^\alpha}=0, \tag{6}$$

where $g=\det(g_{\mu\nu})$ is the determinant of metric tensor.

For a unique solution of the geodesic equation (4) mixed initial-boundary conditions must be imposed [4,7–13]:

$$\boldsymbol{x}_0=\boldsymbol{x}(t_0), \tag{7}$$

$$\boldsymbol{\sigma}=\lim_{t\to-\infty}\frac{\dot{\boldsymbol{x}}(t)}{c}, \tag{8}$$

where the dot in (8) denotes total derivative with respect to coordinate time. The first condition (7) defines the spatial coordinates of the photon at the moment t_0 of emission of light. The second condition (8) defines the unit direction of the light ray at past null infinity, which means the unit-tangent vector along the light path in the infinite past hence at infinite spatial distance from the origin of the global coordinate system. Then, the exact solution of (4) for the trajectory of the light ray, propagating from the light source through the Solar System towards the observer, can formally be written as follows:

$$\boldsymbol{x}(t) = \boldsymbol{x}_0 + c(t - t_0)\boldsymbol{\sigma} + \Delta\boldsymbol{x}, \tag{9}$$

where the term $\Delta\boldsymbol{x}$ denotes gravitational corrections to the unperturbed light ray.

In case of weak gravitational fields it is useful to decompose the metric tensor as follows:

$$g_{\alpha\beta}(t,\boldsymbol{x}) = \eta_{\alpha\beta} + h_{\alpha\beta}(t,\boldsymbol{x}), \tag{10}$$

where $\eta_{\alpha\beta} = \eta^{\alpha\beta} = \mathrm{diag}(-1,+1,+1,+1)$ is the metric of Minkowskian space and for any components of the metric perturbations $|h_{\alpha\beta}| \ll 1$. Because the gravitational fields are weak in the Solar System, the orbital motions of the Solar System bodies are slow (virial theorem), $m_A/P_A \ll 1$ and $v_A/c \ll 1$ (for notations see Appendix A), hence an expansion of the metric in terms of inverse powers of the speed of light can be applied, called post-Newtonian expansion or weak-field slow-motion approximation [4,6,8,14–17], which for the covariant and contravariant components reads

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h^{(2)}_{\alpha\beta} + h^{(3)}_{\alpha\beta} + h^{(4)}_{\alpha\beta} + \mathcal{O}(c^{-5}), \tag{11}$$

$$g^{\alpha\beta} = \eta^{\alpha\beta} - h^{\alpha\beta}_{(2)} - h^{\alpha\beta}_{(3)} - h^{\alpha\beta}_{(4)} + \mathcal{O}(c^{-5}), \tag{12}$$

where $h^{(n)}_{\alpha\beta} = \mathcal{O}(c^{-n})$ with $n = 2,\ 3,\ 4$; e.g. Eqs. (4.17)–(4.19) in [17]. Notice that the post-Newtonian expansion in (11) describes the metric in the near zone of the Solar System defined by $|\boldsymbol{x}| < \lambda_{\mathrm{gr}}$ where λ_{gr} is a characteristic wavelength of gravitational radiation emitted by the Solar System. It should be mentioned that, according to the famous theorem in [18], the post-Newtonian expansion of the metric tensor is, in fact, nonanalytic because it contains logarithmic terms. However, in the near zone the post-Newtonian expansion in inverse powers of the speed of light is valid up to 4PN approximation, which means logarithmic terms in metric coefficients emerge at the order of $\mathcal{O}(c^{-8})$ [19].

The post-Newtonian expansion of the metric in (11) inherits a corresponding post-Newtonian expansion of the light trajectory (9), which up to terms of the order $\mathcal{O}(c^{-5})$ reads

$$\boldsymbol{x}(t) = \boldsymbol{x}_0 + c(t - t_0)\boldsymbol{\sigma} + \Delta\boldsymbol{x}_{\mathrm{1PN}} + \Delta\boldsymbol{x}_{\mathrm{1.5PN}} + \Delta\boldsymbol{x}_{\mathrm{2PN}}, \tag{13}$$

where the labels 1PN, 1.5PN, and 2PN refer to terms of the order $\mathcal{O}(c^{-2})$, $\mathcal{O}(c^{-3})$, and $\mathcal{O}(c^{-4})$, respectively.

The expressions for $\Delta\boldsymbol{x}_{\mathrm{1PN}}$ and $\Delta\boldsymbol{x}_{\mathrm{1.5PN}}$ for a light trajectory in the field of N arbitrarily moving bodies of finite size have recently been determined in [12,13]. In these investigations each individual body $A = 1, 2, \ldots, N$ is allowed to move along its own arbitrary worldline $\boldsymbol{x}_A(t)$ and the global metric of the Solar System has been described in terms of the full set of time-dependent intrinsic mass multipoles $M^L_A(t)$ and full set of time-dependent intrinsic spin multipoles $S^L_A(t)$, allowing for arbitrary shape, inner structure and rotational motion of the massive bodies of the Solar System. About the magnitude of these terms in time delay and light deflection we refer to Tables II and III in [13].

However, rapidly growing accuracy in astrometric measurements demands to account for post-post-Newtonian terms $\Delta\boldsymbol{x}_{\mathrm{2PN}}$ as well. In particular, it is well known that present-day precision in astrometry has reached a level of a few microarcseconds (μas) in angular observations of stars [20,21] and a level of a few nanoseconds (ns) in measurements of time delay [22]. Such extremely high-precision astrometry necessitates to account for 2PN effects in the theory of light propagation [8,23]. On the other side, results about the post-post-Newtonian terms $\Delta\boldsymbol{x}_{\mathrm{2PN}}$ in (13) are fairly rare. So far, 2PN effects in light propagation have mainly been determined for the case of mass monopoles at rest [24–29], which means where the position of the mass monopole remains constant: $\boldsymbol{x}_A = \mathrm{const}$. In this respect, important progress in calculating post-post-Newtonian effects on light propagation in the monopole field has been achieved in [4,7] where an explicit 2PN solution for light trajectories in the Schwarzschild field as a function of coordinate time has been found and later been confirmed within several progressing investigations [8,9,13,30–32]. Also alternative approaches for the calculation of directions of light rays and their propagation time in 2PN approximation have been developed, which avoid the peculiarities of solving the null geodesic equations, based on the eikonal concept [33], on the Synge's world function [34] or on the time transfer function formalism [35,36].

An ambitious goal in astrometric measurements in the near future is to aim at submicroarcsecond (sub-μas) level in angular determination and subnanosecond (sub-ns) level in time delay measurements. For instance, several space-based astrometry missions are under discussion which have been proposed to the European Space Agency (ESA) which aims at precisions on subnanoarcsecond (sub-nas) level in angular determination of celestial objects [37–44]. Such extremely high-precision astrometry needs to account for the impact of the motion of massive bodies on the light propagation in 2PN approximation. The problem, however, of light propagation in the field of moving monopoles in 2PN approximation has not been considered yet, aside from the investigation in [30] which was not intended for the problem of light propagation in the Solar System. For this reason, we will consider the problem of light propagation through the gravitational field of one pointlike body in slow but otherwise arbitrary motion in the 2PN approximation. The article is organized as follows: In Sec. II the geodesic equation in 2PN approximation is presented, in Sec. III the metric of one massive pointlike body in arbitrary motion in 2PN approximation is given, and in Secs. IV and V the first and second integration of geodesic equation is represented. The observable effects of total light deflection and time delay are given in Sec. VI. A summary and outlook can be found in Sec. VII. The notation in use is given in Appendix A.

II. GEODESIC EQUATION IN 2PN APPROXIMATION

The Solar System is composed of N massive bodies of finite size which move according their mutual gravitational interaction. In our investigation we will consider one of these massive bodies and approximate the body as a pointlike object with Newtonian rest mass M_A. By inserting the metric (11) into (4) we obtain the geodesic equation in 2PN approximation, which in terms of global coordinate time reads [4,11,30]

$$\begin{aligned}\frac{\ddot{x}^i(t)}{c^2} = &+\frac{1}{2}h^{(2)}_{00,i} - h^{(2)}_{00,j}\frac{\dot{x}^i(t)}{c}\frac{\dot{x}^j(t)}{c} - h^{(2)}_{ij,k}\frac{\dot{x}^j(t)}{c}\frac{\dot{x}^k(t)}{c} + \frac{1}{2}h^{(2)}_{jk,i}\frac{\dot{x}^j(t)}{c}\frac{\dot{x}^k(t)}{c} - h^{(2)}_{ij,0}\frac{\dot{x}^j(t)}{c}\\ &+\frac{1}{2}h^{(2)}_{jk,0}\frac{\dot{x}^i(t)}{c}\frac{\dot{x}^j(t)}{c}\frac{\dot{x}^k(t)}{c} - \frac{1}{2}h^{(2)}_{00,0}\frac{\dot{x}^i(t)}{c} - h^{(3)}_{0i,j}\frac{\dot{x}^j(t)}{c} + h^{(3)}_{0j,i}\frac{\dot{x}^j(t)}{c} - h^{(3)}_{0j,k}\frac{\dot{x}^i(t)}{c}\frac{\dot{x}^j(t)}{c}\frac{\dot{x}^k(t)}{c}\\ &- h^{(3)}_{0i,0} - \frac{1}{2}h^{(2)}_{ij}h^{(2)}_{00,j} - h^{(2)}_{00}h^{(2)}_{00,j}\frac{\dot{x}^i(t)}{c}\frac{\dot{x}^j(t)}{c} + h^{(2)}_{is}h^{(2)}_{sj,k}\frac{\dot{x}^j(t)}{c}\frac{\dot{x}^k(t)}{c} - \frac{1}{2}h^{(2)}_{is}h^{(2)}_{jk,s}\frac{\dot{x}^j(t)}{c}\frac{\dot{x}^k(t)}{c}\\ &+\frac{1}{2}h^{(4)}_{00,i} - h^{(4)}_{00,j}\frac{\dot{x}^i(t)}{c}\frac{\dot{x}^j(t)}{c} - h^{(4)}_{ij,k}\frac{\dot{x}^j(t)}{c}\frac{\dot{x}^k(t)}{c} + \frac{1}{2}h^{(4)}_{jk,i}\frac{\dot{x}^j(t)}{c}\frac{\dot{x}^k(t)}{c}\\ &+ h^{(4)}_{0j,i}\frac{\dot{x}^j(t)}{c} - h^{(4)}_{0i,j}\frac{\dot{x}^j(t)}{c} - h^{(4)}_{0j,k}\frac{\dot{x}^i(t)}{c}\frac{\dot{x}^j(t)}{c}\frac{\dot{x}^k(t)}{c} - h^{(4)}_{0i,0} + \mathcal{O}(c^{-5}),\end{aligned} \tag{14}$$

where we have taken into account that in general $h^{(2)}_{0i} = h^{(3)}_{00} = h^{(3)}_{ij} = 0$ [3–6,15,16]. The last term in (14), i.e. the term $h^{(4)}_{0i,0}$, is a peculiarity in the sense that this term is seemingly of the order $\mathcal{O}(c^{-5})$ but by inspection of (25) one realizes that the first integration (30) of this term results into $4m_A a_A/c^2$which is of the order $\mathcal{O}(c^{-4})$ and, therefore, cannot be neglected. Furthermore, the following relations have been used:

$$\begin{aligned} &h^{(2)}_{00} = h^{00}_{(2)}, \qquad h^{(2)}_{ij} = h^{ij}_{(2)},\\ &h^{(3)}_{0i} = -h^{0i}_{(3)}, \qquad h^{(4)}_{0i} = -h^{0i}_{(4)},\\ &h^{(4)}_{00} = h^{00}_{(4)} - h^{00}_{(2)}h^{00}_{(2)}, \qquad h^{(4)}_{ij} = h^{ij}_{(4)} + h^{ik}_{(2)}h^{kj}_{(2)}, \end{aligned} \tag{15}$$

which result from $g_{\alpha\mu}g^{\mu\beta} = \delta^\beta_\alpha = \mathrm{diag}(+1,+1,+1,+1)$.

The metric perturbations in (11) are functions of the field points $(t,\boldsymbol{x})$, while in the geodesic equation (14) the metric perturbations are of relevance at the coordinates of the photon $\boldsymbol{x}(t)$. Consequently, the derivatives in (14) are taken along the light ray:

$$h^{(n)}_{\alpha\beta,\mu} = \frac{\partial h^{(n)}_{\alpha\beta}(t,\boldsymbol{x})}{\partial x^\mu}\bigg|_{\boldsymbol{x}=\boldsymbol{x}(t)}, \qquad n = 2,3,4. \tag{16}$$

The geodesic equation in 2PN approximation in (14) can be solved by iteration and allows one to determine the coordinate velocity (first integration) and the light trajectory (second integration) up to terms of the order $\mathcal{O}(c^{-5})$:

$$\dot{\boldsymbol{x}}(t) = c\boldsymbol{\sigma} + \Delta\dot{\boldsymbol{x}}_{\mathrm{1PN}} + \Delta\dot{\boldsymbol{x}}_{\mathrm{1.5PN}} + \Delta\dot{\boldsymbol{x}}_{\mathrm{2PN}}, \tag{17}$$

$$\boldsymbol{x}(t) = \boldsymbol{x}_0 + c(t-t_0)\boldsymbol{\sigma} + \Delta\boldsymbol{x}_{\mathrm{1PN}} + \Delta\boldsymbol{x}_{\mathrm{1.5PN}} + \Delta\boldsymbol{x}_{\mathrm{2PN}}. \tag{18}$$

As mentioned above, the 1PN and 1.5PN terms in (17) and (18) have recently been determined in [12] and [13], respectively, for the case of N bodies in slow but otherwise arbitrary motion and the bodies may have arbitrary shape and inner structure and can be in arbitrary rotational motion. The aim of this investigation is to determine the 2PN terms for the case of one pointlike body in arbitrary slow motion.

III. METRIC IN 2PN APPROXIMATION FOR ONE BODY

We shall assume that the one-body system is isolated (Fock-Sommerfeld boundary conditions), which means flatness of the metric at spatial infinity and the constraint of no-incoming gravitational radiation is imposed at Minkowskian past null infinity $\mathcal{J}^-_M$, which in terms of trace-reversed metric perturbation $\bar{h}^{\mu\nu} = \eta^{\mu\nu} - \sqrt{-g}g^{\mu\nu}$ read as follows [6,8,17,45–47]:

$$\lim_{\substack{r\to\infty\\ t+\frac{r}{c}=\mathrm{const}}} \bar{h}^{\mu\nu}(t,\boldsymbol{x}) = 0, \tag{19}$$

$$\lim_{\substack{r\to\infty\\ t+\frac{r}{c}=\mathrm{const}}}\left(\frac{\partial}{\partial r}r\bar{h}^{\mu\nu}(t,\boldsymbol{x})+\frac{\partial}{\partial ct}r\bar{h}^{\mu\nu}(t,\boldsymbol{x})\right)=0, \quad (20)$$

where $r=|\boldsymbol{x}|$. In addition, $r\partial_\alpha\bar{h}^{\mu\nu}$ should be bounded in this limit [45,46], which means any component of the metric tensor obeys the constraint

$$\lim_{\substack{r\to\infty\\ t+\frac{r}{c}=\mathrm{const}}}\left|\frac{\partial\bar{h}^{\mu\nu}(t,\boldsymbol{x})}{\partial x^\alpha}\right|<\frac{K}{r}, \quad (21)$$

where $K>0$ is some positive number related to the total rest mass of the gravitational system. According to (11) and Eqs. (C17)–(C22), the metric perturbations for the gravitational fields of one pointlike body in slow motion read:

$$h_{00}^{(2)}(t,\boldsymbol{x})=+\frac{2m_A}{r_A(t)}, \quad (22)$$

$$h_{ij}^{(2)}(t,\boldsymbol{x})=+\frac{2m_A}{r_A(t)}\delta_{ij}, \quad (23)$$

$$h_{0i}^{(3)}(t,\boldsymbol{x})=-\frac{4m_A}{r_A(t)}\frac{v_A^i(t)}{c}, \quad (24)$$

$$h_{0i}^{(4)}(t,\boldsymbol{x})=+4m_A\frac{a_A^i(t)}{c^2}, \quad (25)$$

$$h_{00}^{(4)}(t,\boldsymbol{x})=+\frac{4m_A}{r_A(t)}\frac{v_A^2(t)}{c^2}-\frac{m_A}{r_A(t)}\frac{(\boldsymbol{n}_A(t)\cdot\boldsymbol{v}_A(t))^2}{c^2}-m_A\frac{(\boldsymbol{n}_A(t)\cdot\boldsymbol{a}_A(t))}{c^2}-\frac{2m_A^2}{r_A^2(t)}, \quad (26)$$

$$h_{ij}^{(4)}(t,\boldsymbol{x})=-\frac{m_A}{r_A(t)}\frac{(\boldsymbol{n}_A(t)\cdot\boldsymbol{v}_A(t))^2}{c^2}\delta_{ij}+\frac{4m_A}{r_A(t)}\frac{v_A^i(t)}{c}\frac{v_A^j(t)}{c}-m_A\frac{(\boldsymbol{n}_A(t)\cdot\boldsymbol{a}_A(t))}{c^2}\delta_{ij}+\frac{m_A^2}{r_A^2(t)}\delta_{ij}+\frac{m_A^2}{r_A^2(t)}n_A^i(t)n_A^j(t), \quad (27)$$

where $m_A=GM_A/c^2$ and

$$\boldsymbol{r}_A(t)=\boldsymbol{x}-\boldsymbol{x}_A(t), \quad (28)$$

while its absolute value $r_A(t)=|\boldsymbol{r}_A(t)|$, and we introduce the unit vector $\boldsymbol{n}_A(t)=\boldsymbol{r}_A(t)/r_A(t)$. The constraints (19)–(21) restrict the time dependence of the acceleration so that it vanishes at past null infinity: $\lim_{t\to-\infty}\boldsymbol{a}_A(t)=0$.

The BCRS metric coefficients $h_{00}^{(2)}$, $h_{ij}^{(2)}$, $h_{0i}^{(3)}$, $h_{00}^{(4)}$ in the mass-monopole approximation for N slowly moving bodies were given by Eqs. (8) and (51)–(55) in [6]. The same metric coefficients were also given by Eqs. (39.63a)–(39.63c) in [3]. In the limit of one monopole in slow motion they agree with our metric coefficients in Eqs. (22), (23), (24) and (26). We also notice that in the limit of one body A at rest, the metric in (22)–(27) agrees with the metric in Eqs. (25) in [9] if the body is assumed to be located at the origin of the global reference system. For further details consult [14,31,48] and Appendix C.

The three-vector $\boldsymbol{x}$ in (28) is an arbitrary spatial field point. But according to (16), as soon as the partial derivatives in the geodesic equation (14) are performed, the field-point $\boldsymbol{x}$ has to be identified with the exact spatial position of the light signal $\boldsymbol{x}(t)$, which means after all partial derivatives are performed we have

$$\boldsymbol{r}_A(t)=\boldsymbol{x}(t)-\boldsymbol{x}_A(t). \quad (29)$$

Of course, one has strictly to distinguish between (28) and (29), but nevertheless the same notation $\boldsymbol{r}_A(t)$ for these expressions is in use and will certainly not cause any confusion.

IV. FIRST INTEGRATION OF GEODESIC EQUATION IN 2PN APPROXIMATION

The first integration of the geodesic equation yields the coordinate velocity of the photon,

$$\frac{\dot{\boldsymbol{x}}(t)}{c}=\int_{-\infty}^{t}dct\frac{\ddot{\boldsymbol{x}}(t)}{c^2}, \quad (30)$$

where $\ddot{\boldsymbol{x}}(t)$ is given by (14) and the boundary condition (8) must be imposed. The geodesic equation (14) is solved by iteration, which means in first iteration the integration is performed along the unperturbed light ray and in the second iteration the integration proceeds along the light ray in 1PN approximation. Owing to the fact that the metric, thence the geodesic equation, depends on the arbitrary worldline of the body, a solution of geodesic equation is obtained by means of integration by parts with respect to coordinate time. One may show that, after a finite set of partial integrations, the remaining nonintegrated terms of such an approach are terms beyond 2PN approximation. The solution for the coordinate velocity of the photon is, first of all, given in terms of the spatial position of the massive body at coordinate time, $\boldsymbol{x}_A(t)$. Since gravitational action propagates with the finite speed of light, it is meaningful to reexpress this solution in terms of retarded time of the massive body's position $\boldsymbol{x}_A(t^{\mathrm{ret}})$. The retarded time is defined by an implicit relation,

$$t^{\mathrm{ret}}=t-\frac{r_A(t^{\mathrm{ret}})}{c}, \quad (31)$$

where $r_A(t^{\mathrm{ret}})=|\boldsymbol{r}_A(t^{\mathrm{ret}})|$ with

$$\boldsymbol{r}_A(t^{\mathrm{ret}})=\boldsymbol{x}(t)-\boldsymbol{x}_A(t^{\mathrm{ret}}), \quad (32)$$

and $\boldsymbol{x}(t)$ being the exact photon trajectory. Further details are given in Appendix B.

Accordingly, we may define an impact vector of the incident light ray associated with the body's position at retarded instant of time, given by

$$\boldsymbol{d}_A(t^{\rm ret}) = \boldsymbol{\sigma} \times (\boldsymbol{r}_A(t^{\rm ret}) \times \boldsymbol{\sigma}), \tag{33}$$

and weak gravitational field means

$$m_A \ll d_A(t^{\rm ret}), \tag{34}$$

where $d_A(t^{\rm ret}) = |\boldsymbol{d}_A(t^{\rm ret})|$. For grazing light rays the impact vector at the retarded position equals the radius of the massive body, while in general it will be larger: $d_A(t^{\rm ret}) \geq P_A$. It should also be remarked that the solution for the coordinate velocity of the photon takes the most simple form in terms of $\boldsymbol{x}_A(t^{\rm ret})$. In this way one obtains the following 2PN solution for the photon's coordinate velocity in the field of one arbitrarily moving pointlike body:

$$\begin{aligned}\frac{\dot{\boldsymbol{x}}(t)}{c} &= \boldsymbol{\sigma} + m_A \boldsymbol{A}_1(\boldsymbol{r}_A^{\rm 1PN}(t^{\rm ret})) + m_A \boldsymbol{A}_2(\boldsymbol{r}_A^{\rm N}(t^{\rm ret}), \boldsymbol{v}_A(t^{\rm ret}))\\ &\quad + m_A^2 \boldsymbol{A}_3(\boldsymbol{r}_A^{\rm N}(t^{\rm ret})) + \mathcal{O}(c^{-5}),\end{aligned} \tag{35}$$

$$\boldsymbol{A}_1(\boldsymbol{x}) = -2\left(\frac{\boldsymbol{\sigma} \times (\boldsymbol{x} \times \boldsymbol{\sigma})}{x(x - \boldsymbol{\sigma} \cdot \boldsymbol{x})} + \frac{\boldsymbol{\sigma}}{x}\right), \tag{36}$$

$$\begin{aligned}\boldsymbol{A}_2(\boldsymbol{x}, \boldsymbol{v}) &= 2\frac{\boldsymbol{\sigma} \times (\boldsymbol{x} \times \boldsymbol{\sigma})}{x(x - \boldsymbol{\sigma} \cdot \boldsymbol{x})}\frac{\boldsymbol{\sigma} \cdot \boldsymbol{v}}{c} + \frac{4}{x}\frac{\boldsymbol{v}}{c} + 2\frac{\boldsymbol{\sigma} \times (\boldsymbol{x} \times \boldsymbol{\sigma})}{x^2}\frac{\boldsymbol{\sigma} \cdot \boldsymbol{v}}{c}\\ &\quad - 2\frac{\boldsymbol{\sigma}}{x^2}\frac{\boldsymbol{x} \cdot \boldsymbol{v}}{c} - 2\frac{\boldsymbol{\sigma} \times (\boldsymbol{x} \times \boldsymbol{\sigma})}{x^2(x - \boldsymbol{\sigma} \cdot \boldsymbol{x})}\frac{(\boldsymbol{\sigma} \times (\boldsymbol{x} \times \boldsymbol{\sigma})) \cdot \boldsymbol{v}}{c}\\ &\quad + \boldsymbol{\epsilon}_1,\end{aligned} \tag{37}$$

$$\begin{aligned}\boldsymbol{A}_3(\boldsymbol{x}) &= -\frac{1}{2}\frac{\boldsymbol{\sigma} \cdot \boldsymbol{x}}{x^4}\boldsymbol{x} + 8\frac{\boldsymbol{\sigma} \times (\boldsymbol{x} \times \boldsymbol{\sigma})}{x^2(x - \boldsymbol{\sigma} \cdot \boldsymbol{x})} + 4\frac{\boldsymbol{\sigma} \times (\boldsymbol{x} \times \boldsymbol{\sigma})}{x(x - \boldsymbol{\sigma} \cdot \boldsymbol{x})^2}\\ &\quad - 4\frac{\boldsymbol{\sigma}}{x(x - \boldsymbol{\sigma} \cdot \boldsymbol{x})} + \frac{9}{2}\frac{\boldsymbol{\sigma}}{x^2} - \frac{15}{4}(\boldsymbol{\sigma} \cdot \boldsymbol{x})\frac{\boldsymbol{\sigma} \times (\boldsymbol{x} \times \boldsymbol{\sigma})}{x^2|\boldsymbol{\sigma} \times \boldsymbol{x}|^2}\\ &\quad - \frac{15}{4}\frac{\boldsymbol{\sigma} \times (\boldsymbol{x} \times \boldsymbol{\sigma})}{|\boldsymbol{\sigma} \times \boldsymbol{x}|^3}\left(\arctan\frac{\boldsymbol{\sigma} \cdot \boldsymbol{x}}{|\boldsymbol{\sigma} \times \boldsymbol{x}|} + \frac{\pi}{2}\right),\end{aligned} \tag{38}$$

where the arguments of the vectorial functions are given in Appendix D. One may demonstrate that in the case of body at rest (35) agrees with [4,7–9], and the terms up to order $\mathcal{O}(c^{-4})$ agree with [13,49]. In view of the complexity of the 2PN solution, all those terms in (37) have been combined in some small parameter $\boldsymbol{\epsilon}_1$ which can be estimated by

$$|\boldsymbol{\epsilon}_1| \leq \frac{16}{d_A(t^{\rm ret})}\frac{v_A^2(t^{\rm ret})}{c^2}, \tag{39}$$

which amounts to be less than $m_A|\boldsymbol{\epsilon}_1| < 0.14$ nas for grazing light rays at Jupiter and even less for all the other Solar System bodies. We emphasize that the solution in (35) does not depend on the acceleration of the massive body, as long as it is given in terms of the retarded position of the massive body. This fact is related to the case of an arbitrarily moving and radiating electron in classical electrodynamics where the radiation fields do not depend on the acceleration of the electron if the worldline of the electron is expressed in terms of its retarded position [50,51]. Finally, we remark that the vectorial function $\boldsymbol{A}_1$ in Eq. (36) agrees with the vectorial function in Eq. (46) in [9], and the vectorial function $\boldsymbol{A}_3$ in Eq. (38) agrees with the vectorial function in Eq. (48) in [9], recalling that in general theory of relativity the PPN parameter are $\alpha = \beta = \gamma = \epsilon = 1$. But of course, the arguments of the vectorial functions in Eq. (35) (moving monopole) differ from the arguments in Eq. (44) in [9] (monopole at rest).

V. SECOND INTEGRATION OF GEODESIC EQUATION IN 2PN APPROXIMATION

The second integration of geodesic equation yields the trajectory of the light signal,

$$\boldsymbol{x}(t) = \int_{t_0}^{t} dct \frac{\dot{\boldsymbol{x}}(t)}{c}, \tag{40}$$

where $\dot{\boldsymbol{x}}(t)$ is given by (35) and the boundary condition (7) must be imposed. Note that (35) is given in terms of retarded time, but in order to proceed with the integration in (40) all terms in (35) must have to be reexpressed in terms of coordinate time by means of relations (B4)–(B6). Like in the case of the first integration in (30), the second integration in (40) is performed by iteration. Furthermore, since the worldline of the body remains arbitrary, $\boldsymbol{x}_A(t)$, the integration is performed by means of integration by parts. In this way, one obtains the light trajectory in terms of the spatial position of the massive body at coordinate time, $\boldsymbol{x}_A(t)$, but can be rewritten in terms of retarded time of the position of massive body, $\boldsymbol{x}_A(t^{\rm ret})$, which is also from the physical point of view more appropriate because gravitational action travels with the finite speed of light. Altogether, we obtain the following 2PN solution for the photon's trajectory in the field of one arbitrarily moving pointlike body, for an illustration see Fig. 1:

$$\begin{aligned}\boldsymbol{x}(t) &= \boldsymbol{x}_0 + c(t - t_0)\boldsymbol{\sigma} + m_A(\boldsymbol{B}_1(\boldsymbol{r}_A^{\rm 1PN}(t^{\rm ret})) - \boldsymbol{B}_1(\boldsymbol{r}_A^{\rm 1PN}(t_0^{\rm ret})))\\ &\quad + m_A(\boldsymbol{B}_2(\boldsymbol{r}_A^{\rm N}(t^{\rm ret}), \boldsymbol{v}_A(t^{\rm ret})) - \boldsymbol{B}_2(\boldsymbol{r}_A^{\rm N}(t_0^{\rm ret}), \boldsymbol{v}_A(t_0^{\rm ret})))\\ &\quad + m_A^2(\boldsymbol{B}_3(\boldsymbol{r}_A^{\rm N}(t^{\rm ret})) - \boldsymbol{B}_3(\boldsymbol{r}_A^{\rm N}(t_0^{\rm ret}))) + \mathcal{O}(c^{-5}),\end{aligned} \tag{41}$$

$$\boldsymbol{B}_1(\boldsymbol{x}) = -2\frac{\boldsymbol{\sigma} \times (\boldsymbol{x} \times \boldsymbol{\sigma})}{x - \boldsymbol{\sigma} \cdot \boldsymbol{x}} + 2\boldsymbol{\sigma}\ln(x - \boldsymbol{\sigma} \cdot \boldsymbol{x}), \tag{42}$$

$$\begin{aligned}\boldsymbol{B}_2(\boldsymbol{x}, \boldsymbol{v}) &= 2\frac{\boldsymbol{\sigma} \times (\boldsymbol{x} \times \boldsymbol{\sigma})}{x - \boldsymbol{\sigma} \cdot \boldsymbol{x}}\frac{\boldsymbol{\sigma} \cdot \boldsymbol{v}}{c} - 2\frac{\boldsymbol{v}}{c}\ln(x - \boldsymbol{\sigma} \cdot \boldsymbol{x}) + 2\frac{\boldsymbol{v}}{c}\\ &\quad + \boldsymbol{\epsilon}_2,\end{aligned} \tag{43}$$

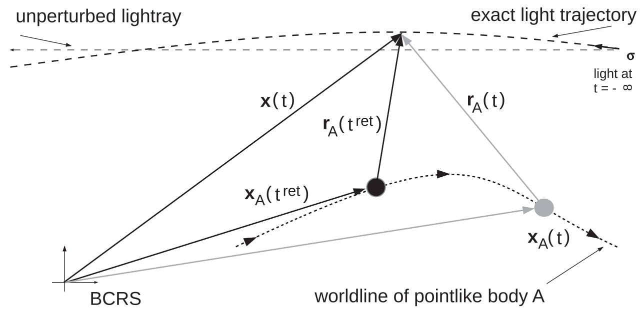

FIG. 1. A geometrical representation of the light trajectory $\boldsymbol{x}(t)$ of Eq. (41) through the gravitational field of one pointlike massive body A moving along an arbitrary worldline in slow motion $v_A \ll c$. At the same instant of coordinate time the body's position is $\boldsymbol{x}_A(t)$ (gray sphere). However, since gravitational action travels with the finite speed of light, the light ray at $\boldsymbol{x}(t)$ is influenced by the gravitational field generated by the body at its retarded position $\boldsymbol{x}_A(t^{\rm ret})$ (black sphere). The spatial vector $\boldsymbol{r}_A(t^{\rm ret})$ is defined by Eq. (32) and points from the massive body A at its retarded position toward the exact photon's position at instant t.

$$\boldsymbol{B}_3(\boldsymbol{x}) = 4\frac{\boldsymbol{\sigma}}{x-\boldsymbol{\sigma}\cdot\boldsymbol{x}} + 4\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma})}{(x-\boldsymbol{\sigma}\cdot\boldsymbol{x})^2} + \frac{1}{4}\frac{\boldsymbol{x}}{x^2} - \frac{15}{4}\frac{\boldsymbol{\sigma}}{|\boldsymbol{\sigma}\times\boldsymbol{x}|}\arctan\frac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{|\boldsymbol{\sigma}\times\boldsymbol{x}|} - \frac{15}{4}(\boldsymbol{\sigma}\cdot\boldsymbol{x})\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma})}{|\boldsymbol{\sigma}\times\boldsymbol{x}|^3}\left(\arctan\frac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{|\boldsymbol{\sigma}\times\boldsymbol{x}|}+\frac{\pi}{2}\right), \tag{44}$$

where the arguments of the vectorial functions are given in Appendix D. One may demonstrate that in the case of body at rest (41) agrees with [4,7–9], and the terms up to order $\mathcal{O}(c^{-4})$ agree with [13,49]. In view of the complexity of the 2PN solution, all those terms in (43) have been combined in some small parameter $\boldsymbol{\epsilon}_2$ which can be estimated by

$$|\boldsymbol{\epsilon}_2| \le \frac{v_A^2(t^{\rm ret})}{c^2}\sqrt{\left(\frac{r_A(t^{\rm ret})+\sigma\cdot r_A(t^{\rm ret})}{d_A(t^{\rm ret})}\right)^2 + \left(\ln(r_A(t^{\rm ret})-\sigma\cdot r_A(t^{\rm ret}))\right)^2}, \tag{45}$$

which yields $m_A|\boldsymbol{\epsilon}_2| < 10^{-4}$ m for grazing light rays at Jupiter and even less for all the other Solar System bodies (assuming an observer nearby the Earth). We emphasize that the solution in (41) does not depend on the acceleration of the massive body, as long as the solution is given in terms of the retarded position of the massive body. This important fact resembles the case of an arbitrarily moving electron, where its radiation field does not depend on the acceleration as long as the worldline of the electron is given in terms of its retarded position [50,51]. We notice that the vectorial function $\boldsymbol{B}_1$ in Eq. (42) agrees with the vectorial function in Eq. (50) in [9], and the vectorial function $\boldsymbol{B}_3$ in Eq. (44) agrees with the vectorial function in Eq. (51) in [9], recalling that in general theory of relativity the PPN parameter are $\alpha=\beta=\gamma=\epsilon=1$. But of course, the arguments of the vectorial functions in Eq. (41) (moving monopole) dier from the arguments in Eq. (45) in [9] (monopole at rest).

VI. OBSERVABLE EFFECTS

In this section we briefly consider the observable effects of total light deflection and time delay which are of upmost relevance for astrometry and belong to the classical tests of relativity.

A. Total light deflection

The total light deflection of a light signal propagating through the gravitational field of one arbitrarily moving body is defined by the angle between the coordinate light velocity at $t\to\pm\infty$. From (35) we first of all obtain up to terms of the order $\mathcal{O}(c^{-5})$:

$$\lim_{t\to-\infty}\frac{\dot{\boldsymbol{x}}(t)}{c} \equiv \boldsymbol{\sigma}, \tag{46}$$

$$\lim_{t\to+\infty}\frac{\dot{\boldsymbol{x}}(t)}{c} \equiv \boldsymbol{\nu} = \boldsymbol{\sigma} - 4m_A\frac{\boldsymbol{d}_A(t^{\rm ret})}{(d_A(t^{\rm ret}))^2}\left(1-\frac{\boldsymbol{\sigma}\cdot\boldsymbol{v}_A(t^{\rm ret})}{c}\right) - 8m_A^2\frac{\boldsymbol{\sigma}}{(d_A^{\rm N}(t^{\rm ret}))^2} - \frac{15}{4}\pi m_A^2\frac{\boldsymbol{d}_A^{\rm N}(t^{\rm ret})}{(d_A^{\rm N}(t^{\rm ret}))^3} + 8m_A^2\frac{\boldsymbol{d}_A^{\rm N}(t^{\rm ret})}{(d_A^{\rm N}(t^{\rm ret}))^4}(r_A^{\rm N}(t^{\rm ret})+\boldsymbol{\sigma}\cdot\boldsymbol{r}_A^{\rm N}(t^{\rm ret})). \tag{47}$$

In the limit of one body at rest and located at the origin of reference system, the expression in (47) agrees with Eq. (64) in [9]. The impact vector $\boldsymbol{d}_A(t^{\rm ret})$ and the impact vector $\boldsymbol{d}_A^{\rm N}(t^{\rm ret})$ are related to each other subject to (D2). Then, from (47) one obtains for the total light deflection up to terms of the order $\mathcal{O}(c^{-5})$:

$$|\boldsymbol{\sigma}\times\boldsymbol{\nu}| = \lim_{t\to+\infty}\frac{4m_A}{d_A^{\rm N}(t^{\rm ret})}\left[1-\frac{\boldsymbol{\sigma}\cdot\boldsymbol{v}_A(t^{\rm ret})}{c} - 2m_A\frac{r_A(t_0^{\rm ret})+\boldsymbol{\sigma}\cdot\boldsymbol{r}_A(t_0^{\rm ret})}{(d_A^{\rm N}(t^{\rm ret}))^2}\frac{\boldsymbol{d}_A^{\rm N}(t_0^{\rm ret})\cdot\boldsymbol{d}_A^{\rm N}(t^{\rm ret})}{(d_A^{\rm N}(t_0^{\rm ret}))^2} + \frac{15}{16}\pi\frac{m_A}{d_A^{\rm N}(t^{\rm ret})}\right], \tag{48}$$

where $t_0^{\rm ret} = t_0 - r_A(t_0^{\rm ret})/c$ and

$$\boldsymbol{r}_A(t_0^{\rm ret}) = \boldsymbol{x}(t_0) - \boldsymbol{x}_A(t_0^{\rm ret}), \tag{49}$$

and terms $|\boldsymbol{\sigma} \times \boldsymbol{\epsilon}_1|$ have been omitted in (48) in view of the estimate in (39). Furthermore, in (48) the impact vector at $t_0^{\rm ret}$ and $t^{\rm ret}$ of the unperturbed light ray has been used:

$$\boldsymbol{d}_A^{\rm N}(t_0^{\rm ret}) = \boldsymbol{\sigma} \times ((\boldsymbol{x}_0 - \boldsymbol{x}_A(t_0^{\rm ret})) \times \boldsymbol{\sigma}), \tag{50}$$

$$\boldsymbol{d}_A^{\rm N}(t^{\rm ret}) = \boldsymbol{\sigma} \times ((\boldsymbol{x}_0 - \boldsymbol{x}_A(t^{\rm ret})) \times \boldsymbol{\sigma}), \tag{51}$$

which in the case of a motionless body at the origin of the coordinate system coincides with the impact vector defined by Eq. (55) in [9]. The expression in (48) depends on the direction of the light ray $\boldsymbol{\sigma}$, on the coordinates of the light source $\boldsymbol{x}_0$, t_0 and on the mass, position and velocity of the massive body $m_A, \boldsymbol{x}_A, \boldsymbol{v}_A$ and it generalizes the corresponding 2PN expression for a body at rest [4,7,9], cf. Eq. (3.2.44) in [4] or Eq. (65) in [9]. The occurrence of the third term in the brackets in (48) is caused by the fact that the total light deflection, which is a coordinate-independent observable, is expressed in terms of coordinate-dependent quantities. This assertion can be shown by introducing a coordinate independent impact vector similar to the one given by Eq. (57) in [9]. But, as emphasized above, the use of concrete reference systems is inevitable in real astrometric data reduction.

B. Time delay

A light signal which propagates through the curved space of a massive body takes a longer time to travel from one space-time point to another space-time point compared to the flat Minkowskian space. Let us assume the light source and the observer to be located at $(\boldsymbol{x}_0, t_0)$ and $(\boldsymbol{x}_1, t_1)$, respectively, and to be at rest with respect to the global reference system, and we may define a spatial distance $R = |\boldsymbol{x}_1 - \boldsymbol{x}_0|$. Then, from (41) one obtains the following expression for the time delay up to terms of the order $\mathcal{O}(c^{-5})$:

$$\begin{aligned} c(t_1 - t_0) = R &- 2m_A\left(\frac{\boldsymbol{\sigma}\cdot\boldsymbol{v}_A(t_1^{\rm ret})}{c} - \frac{\boldsymbol{\sigma}\cdot\boldsymbol{v}_A(t_0^{\rm ret})}{c}\right) \\ &- 2m_A\left(1 - \frac{\boldsymbol{\sigma}\cdot\boldsymbol{v}_A(t_1^{\rm ret})}{c}\right) \\ &\times \ln\left(r_A(t_1^{\rm ret}) - \boldsymbol{\sigma}\cdot\boldsymbol{r}_A(t_1^{\rm ret}) + 2m_A\right) \\ &+ 2m_A\left(1 - \frac{\boldsymbol{\sigma}\cdot\boldsymbol{v}_A(t_0^{\rm ret})}{c}\right) \\ &\times \ln\left(r_A(t_0^{\rm ret}) - \boldsymbol{\sigma}\cdot\boldsymbol{r}_A(t_0^{\rm ret}) + 2m_A\right), \end{aligned} \tag{52}$$

where $t_1^{\rm ret} = t_1 - r_A(t_1^{\rm ret})/c$; the terms $\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}_2$ were neglected in view of the estimate in (45). The expression in (52) generalizes the corresponding 2PN expression for one monopole at rest [4,7,9,52] and it generalizes the expression in Eqs. (146)–(148) in [13] which is valid for arbitrarily moving monopoles but in 1.5PN approximation.

VII. SUMMARY AND OUTLOOK

Present-day astrometry has reached a level of a few microarcseconds in angular determination of celestial objects and prospective astrometry aims at submicroarcsecond or even nanoarcsecond level of precision. Associated therewith is the precise determination of light trajectories through the warped space-time of the Solar System as one central issue in relativistic astrometry. An exact solution for the light ray is, however, not possible because of the involved structure of the metric of the Solar System and one has, therefore, to resort to approximation schemes. The gravitational fields of the Solar System are weak, $m_A/P_A \ll 1$, and the velocities of the bodies are slow, $v_A/c \ll 1$, so that an expansion of the metric tensor of the Solar System in inverse powers of the speed of light becomes meaningful as given by Eq. (11),

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}^{(2)} + h_{\alpha\beta}^{(3)} + h_{\alpha\beta}^{(4)}, \tag{53}$$

up to terms of the order $\mathcal{O}(c^{-5})$. This so-called post-Newtonian expansion (weak-field slow-motion approximation) implicitly assumes that all retardations are small, which is well justified inside the near zone of the Solar System. A corresponding expansion of the light trajectory is given by Eq. (13) and reads

$$\boldsymbol{x}(t) = \boldsymbol{x}_0 + c(t - t_0)\boldsymbol{\sigma} + \Delta\boldsymbol{x}_{\rm 1PN} + \Delta\boldsymbol{x}_{\rm 1.5PN} + \Delta\boldsymbol{x}_{\rm 2PN}, \tag{54}$$

up to terms of the order $\mathcal{O}(c^{-5})$. One of the most intricate problems in the relativistic theory of light propagation concerns the impact of the motion of the massive bodies on light trajectory. In recent investigations [12,13] the 1PN and 1.5PN terms, $\Delta\boldsymbol{x}_{\rm 1PN}$ and $\Delta\boldsymbol{x}_{\rm 1.5PN}$, have been determined for the case of N arbitrarily moving bodies having full mass-multipole and spin-multipole structure. The rapid advance in astrometric measurements enforces one to account for post-post-Newtonian effects $\Delta\boldsymbol{x}_{\rm 2PN}$ in the theory of light propagation as well. The 2PN terms in (54) are only known for the case of one monopole at rest, first determined in [4,7] and later confirmed within several ongoing investigations [8,9,13,31,35,36]. But little is known about these terms in (54) for the case of moving bodies. So far, the only investigation in 2PN approximation regarding light trajectory in the field of moving bodies has been performed in [30] which was, however, not intended for light propagation inside the Solar System.

In our investigation, the problem of light propagation in the field of one arbitrarily moving pointlike monopole has been considered. Especially, an analytical solution in post-post-Newtonian approximation for coordinate velocity $\dot{\boldsymbol{x}}(t)$ and trajectory $\boldsymbol{x}(t)$ of the light ray is presented. According to the recommendations of IAU [6] the metric is given in terms of harmonic coordinates. Because of the fact that the worldline $\boldsymbol{x}_A(t)$ of the massive body is arbitrary, an integration of the geodesic equation in (14) is only possible by means of integration by parts. The first integration (30) and the second integration (40) has been performed in terms of coordinate time. In this respect one has to keep in mind that the post-Newtonian expansion of the metric (53) and of the light ray (54) inherits that all retardations are small, but they are not negligible. Instead, the fact remains that gravitational action travels with the speed of light also inside the near zone of the Solar system. The phrase *smallness of retardation effects* in the Solar System means that a series expansion of the retarded time is meaningful, as given by Eqs. (B2)–(B6). By means of these relations the solution, first of all given in terms of the instantaneous position of the body $\boldsymbol{x}_A(t)$, can be expressed in terms of the retarded position of the body $\boldsymbol{x}_A(t^{\rm ret})$, where the first integration and the second integration of geodesic equation adopt the most simple form, as given by Eqs. (35) and (41), respectively. The expressions for the observables of total light deflection and of Shapiro time delay are given by Eqs. (48) and (52).

The case of N arbitrarily moving pointlike bodies is rather involved and needs special consideration. But in view of the fact that the impact of two-body effects on light deflection is less than 0.1 nas in the Solar System [31], one might assert that the 2PN light trajectory in the field of N pointlike monopoles in arbitrary slow motion can be obtained from our solution just by a summation over N individual bodies, at least for an envisaged accuracy on nas level. These aspects will have to be scrutinized within more detailed prospective analyses.

ACKNOWLEDGMENTS

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APPENDIX A: NOTATION

Throughout the article the following notation is in use.

(1) G is the Newtonian constant of gravitation.
(2) c is the vacuum speed of light.
(3) M_A denotes the rest mass of body A.
(4) $m_A = GM_A/c^2$ is the Schwarzschild radius.
(5) P_A denotes the equatorial radius of body A.
(6) v_A denotes the orbital velocity of massive body A.
(7) $1\ \mu\mathrm{as} = \frac{\pi}{180\times 60\times 60}10^{-6}\ \mathrm{rad} \simeq 4.85\times 10^{-12}$ rad.
(8) $1\ \mathrm{nas} = \frac{\pi}{180\times 60\times 60}10^{-9}\ \mathrm{rad} \simeq 4.85\times 10^{-15}$ rad.
(9) Lower case Latin indices take values 1, 2, 3.
(10) Lower case Greek indices take values 0, 1, 2, 3.
(11) The three-dimensional coordinate quantities (three-vectors) referred to the spatial axes of the reference system are in boldface: $\boldsymbol{a}$.
(12) The contravariant components of three-vectors $a^i = (a^1, a^2, a^3)$.
(13) The contravariant components of four-vectors $a^\mu = (a^0, a^1, a^2, a^3)$.
(14) The absolute value of a three-vector $a = |\boldsymbol{a}| = \sqrt{a^1a^1 + a^2a^2 + a^3a^3}$.
(15) The scalar product of two three-vectors $\boldsymbol{a}\cdot\boldsymbol{b} = \delta_{ij}a^ib^j = a^ib^i$ with Kronecker delta δ_{ij}.
(16) The vector product of two three-vectors reads $(\boldsymbol{a}\times\boldsymbol{b})^i = \varepsilon_{ijk}a^jb^k$ with Levi-Civita symbol ε_{ijk}.

APPENDIX B: RETARDED TIME

Gravitational action travels with the finite speed of light which implies that the gravitational field at some field point $\boldsymbol{x}$ is generated by the pointlike body at its position $\boldsymbol{x}_A(t^{\rm ret})$ at the retarded instant of time defined by

$$t^{\rm ret} = t - \frac{r_A(t^{\rm ret})}{c}, \tag{B1}$$

where $r_A(t^{\rm ret}) = |\boldsymbol{x} - \boldsymbol{x}_A(t^{\rm ret})|$. In the near zone of the Solar System [3,17,53] one may assume that all retardations are small, hence a series expansion of (B1) becomes meaningful,

$$t^{\rm ret} = t - \frac{r_A(t)}{c} - \frac{\boldsymbol{r}_A(t)\cdot\boldsymbol{v}_A(t)}{c^2} + \mathcal{O}(c^{-3}), \tag{B2}$$

which will later be used for the series expansion of the metric tensor. Using (B2) and the series expansion of the retarded position of the body which up to terms of the order $\mathcal{O}(c^{-3})$ reads

$$\boldsymbol{x}_A(t^{\rm ret}) = \boldsymbol{x}_A(t) + \dot{\boldsymbol{x}}_A(t)\frac{(t^{\rm ret}-t)}{1!} + \ddot{\boldsymbol{x}}_A(t)\frac{(t^{\rm ret}-t)^2}{2!}, \tag{B3}$$

we find the following relations:

$$\boldsymbol{r}_A(t^{\rm ret}) = \boldsymbol{r}_A(t) + \frac{\boldsymbol{v}_A(t)}{c}r_A(t) + \frac{\boldsymbol{v}_A(t)}{c}\frac{\boldsymbol{r}_A(t)\cdot\boldsymbol{v}_A(t)}{c} - \frac{1}{2}\frac{\boldsymbol{a}_A(t)}{c}\frac{r_A^2(t)}{c} + \mathcal{O}(c^{-3}), \tag{B4}$$

$$r_A(t^{\rm ret}) = r_A(t)\left(1 + \frac{\boldsymbol{r}_A(t)\cdot\boldsymbol{v}_A(t)}{c\,r_A(t)} + \frac{1}{2}\frac{v_A^2(t)}{c^2} + \frac{1}{2}\frac{(\boldsymbol{v}_A(t)\cdot\boldsymbol{r}_A(t))^2}{c^2r_A^2(t)} - \frac{1}{2}\frac{\boldsymbol{r}_A(t)\cdot\boldsymbol{a}_A(t)}{c^2}\right) + \mathcal{O}(c^{-3}), \tag{B5}$$

$$\frac{\boldsymbol{v}_A(t^{\rm ret})}{c} = \frac{\boldsymbol{v}_A(t)}{c} - \frac{\boldsymbol{a}_A(t)}{c}\frac{r_A(t)}{c} + \mathcal{O}(c^{-3}), \tag{B6}$$

where $\boldsymbol{v}_A(t) = \dot{\boldsymbol{x}}_A(t)$ and $\boldsymbol{a}_A(t) = \ddot{\boldsymbol{x}}_A(t)$ is the velocity and acceleration of the body, respectively. These relations agree with Eqs. (47)–(49) in [54] up to the term proportional to the acceleration $\boldsymbol{a}_A$. These relations have been obtain for $\boldsymbol{r}_A(t^{\rm ret}) = \boldsymbol{x} - \boldsymbol{x}_A(t^{\rm ret})$, but we notice that the relations in

(B2) and (B4)–(B6) remain its validity for $\boldsymbol{r}_A(t^{\rm ret}) = \boldsymbol{x}(t) - \boldsymbol{x}_A(t^{\rm ret})$, because they root on the expansion in (B3).

The series expansions in (B4)–(B6) are useful as long as the retardations are small which is well justified in the near zone of the Solar System. It especially constrains the accelerations,

$$\frac{a_A(t) r_A(t)}{c^2} \ll \frac{v_A(t)}{c} \ll 1, \tag{B7}$$

for any moment of time.

APPENDIX C: 2PN METRIC FOR ONE ARBITRARILY MOVING BODY

For our intention we need the 2PN metric in harmonic gauge (6) for the case of one arbitrarily but slowly moving pointlike monopole. The 2PN metric contains terms proportional to G and terms proportional to G^2, which are considered in what follows.

1. Metric coefficients proportional to G

The terms proportional to G can easily be obtained from the metric for one arbitrarily moving pointlike monopole in post-Minkowskian approximation, which has been given by Eq. (10) in [49], by Eq. (11) in [55], and also by Eq. (43) in [54]:

$$h_{\mu\nu}^{(M)}(t^{\rm ret},\boldsymbol{x}) = \frac{4m_A}{\gamma_A(t^{\rm ret})\left(r_A(t^{\rm ret}) - \frac{\boldsymbol{v}_A(t^{\rm ret})\cdot\boldsymbol{r}_A(t^{\rm ret})}{c}\right)} \times \left(\frac{u_\mu^A(t^{\rm ret})}{c}\frac{u_\nu^A(t^{\rm ret})}{c} + \frac{\eta_{\mu\nu}}{2}\right), \tag{C1}$$

where $\gamma_A^{-1}(t^{\rm ret}) = \sqrt{1 - v_A^2(t^{\rm ret})/c^2}$ is the Lorentz factor and (M) denotes monopole. The vector pointing from the retarded position $\boldsymbol{x}_A(t^{\rm ret})$ of the body A towards the field point $\boldsymbol{x}$ reads

$$\boldsymbol{r}_A(t^{\rm ret}) = \boldsymbol{x} - \boldsymbol{x}_A(t^{\rm ret}). \tag{C2}$$

Let us note that in (C1) we present the covariant components of the metric perturbations, while in [49,54,55] the contravariant components have been used. Accordingly, the covariant components of the four-velocity of the body are $u_\mu^A(t^{\rm ret}) = \gamma(t^{\rm ret})(-c, \boldsymbol{v}_A(t^{\rm ret}))$, and $\boldsymbol{v}_A(t^{\rm ret})$ is the three-velocity of the body in the global system. Let us also draw attention to the fact that the metric tensor in (C1) does not depend on the acceleration but only on the velocity of body A because it is given in terms of retarded time, see also the comment below Eq. (39).

The metric in (C1) is valid for an arbitrarily moving body which could even be in ultrarelativistic motion. We are interested in the case of a slowly moving body, and a corresponding series expansion of (C1) in terms of the small parameter $v_A/c \ll 1$ yields up to order $\mathcal{O}(c^{-5})$:

$$h_{00}^{(M)}(t^{\rm ret},\boldsymbol{x}) = +\frac{2m_A}{r_A(t^{\rm ret})}\left(1 + \frac{\boldsymbol{v}_A(t^{\rm ret})\cdot\boldsymbol{r}_A(t^{\rm ret})}{c\,r_A(t^{\rm ret})} + \frac{(\boldsymbol{v}_A(t^{\rm ret})\cdot\boldsymbol{r}_A(t^{\rm ret}))^2}{c^2 r_A^2(t^{\rm ret})} + \frac{3}{2}\frac{v_A^2(t^{\rm ret})}{c^2}\right), \tag{C3}$$

$$h_{0i}^{(M)}(t^{\rm ret},\boldsymbol{x}) = -\frac{4m_A}{r_A(t^{\rm ret})}\frac{v_A^i(t^{\rm ret})}{c}\left(1 + \frac{\boldsymbol{v}_A(t^{\rm ret})\cdot\boldsymbol{r}_A(t^{\rm ret})}{c\,r_A(t^{\rm ret})}\right), \tag{C4}$$

$$h_{ij}^{(M)}(t^{\rm ret},\boldsymbol{x}) = +\frac{2m_A}{r_A(t^{\rm ret})}\delta_{ij}\left(1 + \frac{\boldsymbol{v}_A(t^{\rm ret})\cdot\boldsymbol{r}_A(t^{\rm ret})}{c\,r_A(t^{\rm ret})} + \frac{(\boldsymbol{v}_A(t^{\rm ret})\cdot\boldsymbol{r}_A(t^{\rm ret}))^2}{c^2 r_A^2(t^{\rm ret})} - \frac{1}{2}\frac{v_A^2(t^{\rm ret})}{c^2}\right) + \frac{4m_A}{r_A(t^{\rm ret})}\frac{v_A^i(t^{\rm ret})v_A^j(t^{\rm ret})}{c^2}. \tag{C5}$$

The retarded time argument in (C3)–(C5) has to be replaced by the global coordinate time using the relations in (B4)–(B6).

Before going further, one has to realize that the acceleration of some body A is proportional to G according to the equations of motion for N pointlike bodies,

$$\boldsymbol{a}_A(t) = -G\sum_{B\neq A}^{N-1} M_B \frac{\boldsymbol{r}_A(t) - \boldsymbol{r}_B(t)}{r_{AB}^3} + \mathcal{O}(c^{-2}), \tag{C6}$$

where the terms of order $\mathcal{O}(c^{-2})$ are given by the Einstein-Infeld-Hoffmann equations [3–5,17,56]. Here, however, we cannot use the equations of motion (C6) because we consider the metric of one body A in arbitrary motion and the physical origin of the motion of the body is not relevant for the moment being. Especially, we do not have some kind of equations of motion like in an N-body system (just imagine accelerating rockets tied to that body). Therefore, we are enforced to keep the acceleration terms explicitly in Eqs. (B4)–(B6). Of course, if one would go back and consider an N-body system under the influence of their mutual gravitational interaction, then one could make use of the equations of motion (C6) and then such an acceleration term would appear as a term of the order G^2 in the metric tensor. According to these considerations, to order G we obtain

$$h_{00}^{(2)}(t,\boldsymbol{x}) = +\frac{2m_A}{r_A(t)}, \tag{C7}$$

$$h_{ij}^{(2)}(t,\boldsymbol{x}) = +\frac{2m_A}{r_A(t)}\delta_{ij}, \tag{C8}$$

$$h_{0i}^{(3)}(t,\boldsymbol{x}) = -\frac{4m_A}{r_A(t)}\frac{v_A^i(t)}{c}, \tag{C9}$$

while $h_{0i}^{(2)} = h_{00}^{(3)} = h_{ij}^{(3)} = 0$ and

$$h_{00}^{(4)G}(t,\boldsymbol{x}) = +\frac{4m_A}{r_A(t)}\frac{v_A^2(t)}{c^2} - \frac{m_A}{r_A(t)}\frac{(\boldsymbol{n}_A(t)\cdot\boldsymbol{v}_A(t))^2}{c^2} - \frac{m_A}{r_A(t)}\frac{(\boldsymbol{r}_A(t)\cdot\boldsymbol{a}_A(t))}{c^2}, \tag{C10}$$

$$h_{0i}^{(4)G}(t,\boldsymbol{x}) = +4m_A\frac{a_A^i(t)}{c^2}, \tag{C11}$$

$$h_{ij}^{(4)G}(t,\boldsymbol{x}) = -\frac{m_A}{r_A(t)}\frac{(\boldsymbol{n}_A(t)\cdot\boldsymbol{v}_A(t))^2}{c^2}\delta_{ij} + \frac{4m_A}{r_A(t)}\frac{v_A^i(t)}{c}\frac{v_A^j(t)}{c} - \frac{m_A}{r_A(t)}\frac{(\boldsymbol{r}_A(t)\cdot\boldsymbol{a}_A(t))}{c^2}\delta_{ij}. \tag{C12}$$

We recognize that in (C10)–(C12) there are terms proportional to the acceleration of the body. The metric in Eqs. (C7)–(C12) agrees with the metric given by Eqs. (7.2a)–(7.2c) in [14] for all terms proportional to G and up to the order $\mathcal{O}(c^{-5})$. But we notice that in Eqs. (7.2a)–(7.2c) in [14] there are no acceleration terms, because they have been rewritten by means of the equations of motion of an N-body system (C6), cf. Eq. (3.11) in [14] and the text above that equation.

Another point to mention concerns the expression in (C11). For an N-body system it is a strict law that there are no terms to power c^{-4} in g_{0i} [3,4,17,53], because in an N-body system, instead of (C11), we would have a summation over all bodies,

$$h_{0i}^{(4)G}(t,\boldsymbol{x}) = +\frac{4G}{c^4}\sum_{A=1}^{N} M_A a_A^i(t) = +\frac{4G}{c^4}\frac{d}{dt}P^i(t) = \mathcal{O}(c^{-6}), \tag{C13}$$

where $P^i(t) = \sum_{A=1}^{N} M_A v_A^i(t)$ is the total Newtonian momentum of the N-body system, which is strictly conserved to order $\mathcal{O}(c^{-2})$, which means $\frac{d}{dt}P^i(t) = \mathcal{O}(c^{-2})$ [53]. Therefore, in an N-body system there is in fact no term to power c^{-4} in g_{0i} [3,4,17,53]. But in our case of one single body which moves along an arbitrary worldline without resorting to the equations of motion (C6), there is no conservation of total Newtonian momentum, hence we have to keep that term in (C11). Nevertheless, there is an important difference regarding the acceleration terms: in 2PN approximation the acceleration term in (C11) would disappear for an N-body system, while the acceleration term in (C10) and (C12) could be, by means of Eq. (C6), rewritten in a form proportional to G^2, but they remain to be of the order $\mathcal{O}(c^{-4})$, hence they would not disappear for an N-body system in 2PN approximation.

2. Metric coefficients proportional to G^2

The metric of a system of two pointlike bodies under the influence of their mutual gravitational interaction has been determined in the 2.5PN approximation in [14], which means g_{00}, g_{i0} and g_{ij} up to terms of the order $\mathcal{O}(c^{-8})$, $\mathcal{O}(c^{-7})$, and $\mathcal{O}(c^{-6})$, respectively. Recently, the metric of N pointlike bodies has been determined in [31] in the 2PN approximation for the light rays, which is $g_{\alpha\beta}$ up to terms of the order $\mathcal{O}(c^{-5})$. In order to find all terms proportional to G^2, we may issue the results from Ref. [31], but have to take the limit $M_B \to 0$ for all bodies except body A. In this way we obtain from Eqs. (47)–(49) in [31] (with $\alpha = \beta = \gamma = 1$)

$$h_{00}^{(4)G^2}(t,\boldsymbol{x}) = -\frac{2m_A^2}{r_A^2(t)}, \tag{C14}$$

$$h_{ij}^{(4)G^2}(t,\boldsymbol{x}) = \frac{m_A^2}{r_A^2(t)}\delta_{ij} + \frac{m_A^2}{r_A^2(t)}n_A^i(t)n_A^j(t). \tag{C15}$$

The metric perturbations $h_{\alpha\beta}^{(4)}$ are given by

$$h_{\alpha\beta}^{(4)} = h_{\alpha\beta}^{(4)G} + h_{\alpha\beta}^{(4)G^2}, \tag{C16}$$

with $h_{\alpha\beta}^{(4)G}$ given by (C10)–(C12) and $h_{\alpha\beta}^{(4)G^2}$ given by (C14) and (C15).

Let us draw attention to the fact that if we insert the equations of motion (C6) into the last term in (C10), then we would get the next-to-last term in Eq. (47) in [31]. Similarly, if we insert the equations of motion (C6) into the last term in (C12), then we would get the next-to-last term in the second line of Eq. (49) in [31]. But we emphasize again that we are not allowed to do such replacements [which in [14] were called the order-reduced form of the metric, cf. text above Eq. (3.11) *ibid.*], because we do not consider an N-body system but a system of one body which moves arbitrarily along its worldline without resorting to the equations of motion in (C6).

3. Collection of all terms

Collecting all metric coefficients in Eqs. (C7)–(C12) and in Eqs. (C14)–(C16), the post-post-Newtonian metric for light rays can also be expressed in terms of so-called potentials (w, w_i, τ_{ij}) in the following form, cf. Eqs. (A1)–(A3) in [57] or Eq. (2) in [58]:

$$g_{00}(t,\boldsymbol{x}) = -1 + 2w(t,\boldsymbol{x}) - 2w^2(t,\boldsymbol{x}) + \mathcal{O}(c^{-5}), \tag{C17}$$

$$g_{0i}(t,\boldsymbol{x}) = -4w_i(t,\boldsymbol{x}) + \mathcal{O}(c^{-5}), \tag{C18}$$

$$g_{ij}(t,\boldsymbol{x}) = (1 + 2w(t,\boldsymbol{x}) + 2w^2(t,\boldsymbol{x}))\delta_{ij} + 4\tau_{ij}(t,\boldsymbol{x}) + \mathcal{O}(c^{-5}), \tag{C19}$$

where the potentials read

$$\begin{aligned} w(t,\boldsymbol{x}) &= \frac{m_A}{r_A(t)} + \frac{3}{2}\frac{m_A}{r_A(t)}\frac{v_A^2(t)}{c^2} + \frac{1}{2}\frac{m_A}{c^2}\frac{d^2}{dt^2}r_A(t) \\ &= \frac{m_A}{r_A(t)} + 2\frac{m_A}{r_A(t)}\frac{v_A^2(t)}{c^2} - \frac{1}{2}\frac{m_A}{r_A(t)}\frac{(\boldsymbol{n}_A(t)\cdot\boldsymbol{v}_A(t))^2}{c^2} \\ &\quad - \frac{1}{2}m_A\frac{(\boldsymbol{n}_A(t)\cdot\boldsymbol{a}_A(t))}{c^2}, \end{aligned} \tag{C20}$$

$$w_i(t,\boldsymbol{x}) = \frac{m_A}{r_A(t)}\frac{v_A^i(t)}{c} - m_A\frac{a_A^i(t)}{c^2}, \tag{C21}$$

$$\begin{aligned} \tau_{ij}(t,\boldsymbol{x}) = &-\frac{m_A}{r_A(t)}\frac{v_A^2(t)}{c^2}\delta_{ij} + \frac{m_A}{r_A(t)}\frac{v_A^i(t)}{c}\frac{v_A^j(t)}{c} \\ &- \frac{1}{4}\frac{m_A^2}{r_A^2(t)}\delta_{ij} + \frac{1}{4}\frac{m_A^2}{r_A^2(t)}n_A^i(t)n_A^j(t), \end{aligned} \tag{C22}$$

recalling $r_A(t) = |\boldsymbol{x} - \boldsymbol{x}_A(t)|$.

APPENDIX D: LIGHT TRAJECTORY IN 1PN APPROXIMATION

The Newtonian and the first post-Newtonian solution for the light ray appears as an argument in the vectorial functions of the 2PN solution in (35) and (41). In this Appendix we will present these expressions. In Newtonian approximation we have

$$\boldsymbol{r}_A^{\rm N}(t^{\rm ret}) = \boldsymbol{x}_0 + c(t - t_0)\boldsymbol{\sigma} - \boldsymbol{x}_A(t^{\rm ret}). \tag{D1}$$

Furthermore, the light trajectory in the field of one arbitrarily moving body in the first post-Newtonian approximation can be obtained from [13] by means of relations (B4)–(B5) and reads

$$\begin{aligned} \boldsymbol{r}_A^{\rm 1PN}(t^{\rm ret}) = &\ \boldsymbol{r}_A^{\rm N}(t^{\rm ret}) + 2m_A\boldsymbol{\sigma}\ln\frac{r_A^{\rm N}(t^{\rm ret}) - \boldsymbol{\sigma}\cdot\boldsymbol{r}_A^{\rm N}(t^{\rm ret})}{r_A^{\rm N}(t_0^{\rm ret}) - \boldsymbol{\sigma}\cdot\boldsymbol{r}_A^{\rm N}(t_0^{\rm ret})} \\ &- 2m_A\left(\frac{\boldsymbol{\sigma}\times(\boldsymbol{r}_A^{\rm N}(t^{\rm ret})\times\boldsymbol{\sigma})}{r_A^{\rm N}(t^{\rm ret}) - \boldsymbol{\sigma}\cdot\boldsymbol{r}_A^{\rm N}(t^{\rm ret})} - \frac{\boldsymbol{\sigma}\times(\boldsymbol{r}_A^{\rm N}(t_0^{\rm ret})\times\boldsymbol{\sigma})}{r_A^{\rm N}(t_0^{\rm ret}) - \boldsymbol{\sigma}\cdot\boldsymbol{r}_A^{\rm N}(t_0^{\rm ret})}\right). \end{aligned} \tag{D2}$$

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