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Inflationary field excursion in broad classes of scalar field models

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In single-field slow-roll inflation models, the height and slope of the potential are made to satisfy certain conditions to match with observations. This in turn translates into bounds on the number of e-foldings and the excursion of the scalar field during inflation. In this work we consider broad classes of inflationary models to study how much the field excursion starting from the horizon exit to the end of inflation, $\Delta\phi$, can vary for the set of inflationary parameters given by Planck. We also derive an upper bound on the number of e-foldings between the horizon exit of a cosmologically interesting mode and the end of inflation. We comment on the possibility of having super-Planckian and sub-Planckian field excursions within the framework of single-field slow-roll inflation.

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I. INTRODUCTION

The standard big bang cosmology has been proved to be successful in explaining the observed evolution of the Universe, albeit with some extremely fine-tuned initial conditions. The era of cosmological inflation [1,2] was introduced to take care of such initial conditions, and it provides a very nice proposal for the solution to the horizon problem, the flatness problem and a very good explanation for the nonexistence of unwanted relics. The most salient feature of inflation is the quantum fluctuations that render seeds for the large scale structure, together with a possible gravitational wave contribution, for the cosmic microwave background (CMB) anisotropy [3,4]. In its simplest form, inflation is best realized by means of a minimally coupled scalar field in the framework of Einstein gravity. Recent CMB data by Planck 2015 [5] indicate that the power spectrum of density perturbations of the scalar field is nearly scale invariant, which is apparent from the value of the scalar spectral index, $n_s = 0.968 \pm 0.006$. Planck has also taken the cosmologists by surprise by predicting an almost Gaussian nature of the power spectrum and putting only an upper bound, $r < 0.11$, on the amplitude of primordial gravity waves by considering a tensor amplitude as a one-parameter extension to the ΛCDM model. An even tighter bound, $r < 0.09$, has been obtained by combining Planck with BICEP2/Keck likelihoods [5]. The BICEP2/Keck array VI reports an even more tighter bound on the tensor-to-scalar ratio, $r < 0.07$, when the above-mentioned constraints from the Planck analysis of CMB temperature are combined with BAO (Baryon Acoustic Oscillation) and other data [6]. The height and the slope of the inflaton potential must maintain a delicate balance for the compatibility with observations. This in turn translates into the excursion of the scalar field during the horizon exit to the end of inflation. In this work we want to address the question of how much the field excursion can vary for a given set of inflationary observables.

The magnitude of the stochastic background gravitational waves, for single-field slow-roll inflation, is related to the energy scale of inflation and more importantly, it is linked to the inflaton excursion. In the standard single-field slow-roll inflationary scenario, according to the Lyth bound [7], a sizable detection of tensors would mean a super-Planckian excursion of the inflaton via the constraint $r \lesssim 0.01(\Delta\phi/M_{\rm Pl})^2$, where $M_{\rm Pl}$ is the reduced Planck mass. This is definitely interesting both from a model building standpoint and from an observational one. The original Lyth bound can be evaded if one considers non-slow-roll inflation [8] or simply considers extra sources for density perturbations [9], or has additional light degrees of freedom contributing to the production of perturbations [10]. Other theoretical bounds on the tensor fraction as a generalization of the original Lyth bound have been discussed in [11]. The amount of inflation between the horizon exit of a cosmologically relevant mode and the end of inflation is given by

$$N = N_* - N_f = \int_{a_*}^{a_f} d\ln a, \qquad (1.1)$$

where the subscript $*$ means that the quantities are evaluated at the horizon exit, f means that the quantities are evaluated at the end of inflation, and a is the scale factor. To address the horizon problem and subsequently all

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others, it is necessary to have at least 50 e-foldings in this period in the conventional inflationary scenario. So observationally we can only fix a lower limit for N but there is no compelling evidence of any upper limit on the total amount of inflation. In fact it may be extended a long way further into the past than the present horizon size. This possibility was explored in [12] by using the phase space analysis to foliate a FRW (Friedmann-Robertson-Walker) universe.

Our aim, in this work, is to determine how much the field excursion $\Delta\phi = |\phi_* - \phi_f|$ can vary given an inflaton reproducing observed cosmological parameters such as n_s, α_s, and r. To this end we consider the classification of single-field slow-roll inflationary models as demonstrated in [13,14]. All those models whose slow-roll parameters scale with 1/N or a higher power can be classified into two broad categories characterized by a single parameter $\Delta\phi$, the field excursion. By expressing the inflationary observables in terms of N, one can also group the models of inflation into broad classes like constant, perturbative, nonperturbative, and logarithmic [13,14]. This large-N formalism is a more effective way of studying the inflationary models instead of doing the case-by-case analysis.

Now as the benchmark we choose a model of inflation with a strong field theoretical background, which passes successfully through the observational constraints set by recent CMB data. There can be many viable phenomenological models that fit well with observations. In this article the choice for the benchmark model has been made by giving stress on a high energy theoretical background. We select a model that arises in the context of type IIB string theory via Calabi-Yau flux compactification. One such example occurs when one of the Kähler moduli plays the role of an inflaton when internal spaces are weighted as projective spaces in type IIB string theories [15]. The version with the canonically normalized inflaton field known as the Kähler moduli II (KM II) inflaton [16] has been chosen as the benchmark in our case. Most importantly, this model can be understood in the context of supergravity, viewed as an effective theory. It has been the general practice earlier [17] to choose the chaotic inflationary scenario [18] as the benchmark. However the minimal chaotic models are almost ruled out after Planck 2015 for not satisfying the bound on the stochastic gravity wave amplitude (for the chaotic model, $r > 0.09$). In addition to that, the BICEP2 results giving a large value of r have also been discredited; therefore one cannot be sure about the benchmark status of the chaotic model. We are interested to explore the effect of n_s and r on the field excursion by considering the observational bounds set by the recent Planck data. Now the KM II model of inflation gives a very low value of r, thus giving a sub-Planckian field excursion. Given the fact that the benchmark model passes all observational tests, we find it to be a pertinent question to ask whether the field excursion of inflation should be in the same range of the benchmark or not? To this end we would like to explore the effect of n_s and r on the field excursion by considering the observational bounds set by Planck [5].

II. ASYMPTOTIC HUBBLE FLOW FUNCTIONS IN KM II INFLATION

We start by recalling the basics of the KM II model [15,16] of inflation and finding the Hubble flow functions in the large N formalism. The potential is given by

$$V(\phi) = V_0\left[1 - \alpha\left(\frac{\phi}{M_{\rm Pl}}\right)^{4/3}\exp\left(-\beta\left(\frac{\phi}{M_{\rm Pl}}\right)^{4/3}\right)\right]. \quad (2.1)$$

Making use of the typical orders of magnitude one can write the parameters α and β as

$$\alpha = \mathcal{O}(\mathcal{V}^{5/3}), \qquad \beta = \mathcal{O}(\mathcal{V}^{2/3}), \quad (2.2)$$

where the quantity $\mathcal{V}$ represents the Calabi-Yau volume. The potential starts from a maximum, $V = V_0$ at $\phi = 0$, then reaches the minimum at $\frac{\phi}{M_{\rm Pl}} = \beta^{-3/4}$, and finally asymptotes to $V = V_0$ for $\frac{\phi}{M_{\rm Pl}}$ approaching ∞. Maintaining the consistency with reheating, the slow-roll predictions for the KM II model can be achieved for $\mathcal{V} \in [10^5, 10^7]$ and thus the parameters α and β can have values in the range $\alpha \in [2.15 \times 10^8, 4.64 \times 10^{11}]$ and $\beta \in [2.15 \times 10^3, 4.64 \times 10^4]$. It can be shown that the Hubble slow-roll predictions do not depend significantly on the vales of α and β [19]. We now intend to find the Hubble flow functions ϵ_n, defined as [20]

$$\epsilon_{n+1} = \frac{d}{dN}\log|\epsilon_n|, \qquad n \geq 0, \quad (2.3)$$

for large N in the case of the KM II model of inflation. The above functions basically play the role of slow-roll parameters in the standard formulation in terms of ϕ. Here ϵ_0 is nothing but the Hubble parameter and the range of N runs starting from the horizon exit to the end of inflation. As N depends on derivatives of $V(\phi)$ it is apparent from Eq. (2.3) that the successive Hubble flow functions are related to the derivatives of the potential $V(\phi)$. Consequently the slow-roll parameters can also be expressed in terms of the Hubble flow parameters varying as $1/N^p$ for some values of p at leading order in the limit of large N. This will become evident from the following calculations. Further at first order in ϵ_n, one can represent the CMB observables of inflation as

$$n_s = 1 - 2\epsilon_1 + \epsilon_2, \quad (2.4)$$

$$r = 16\epsilon_1. \quad (2.5)$$

To set up a connection with the observables, one needs to calculate these quantities at the time of the horizon crossing of a cosmologically relevant scale. It has been noticed by Lyth [7] that the tensor-to-scalar ratio of temperature fluctuations, i.e., the first slow-roll parameter, can be related to the field excursion via the relation

$$\frac{1}{M_{\rm Pl}}\frac{d\phi}{dN}\sim\sqrt{\frac{r(N)}{8}}=\sqrt{2\epsilon_1}. \tag{2.6}$$

Assuming $r(N)$ to be invariant throughout the phase of inflation it can be shown that [7,11] the field excursion is

$$\Delta\phi\approx\left(\frac{r}{0.002}\right)^{1/2}\left(\frac{N_*}{58}\right)M_{\rm Pl}, \tag{2.7}$$

where N_* is set to 58, which falls within the range of N_* allowed by the Planck [5] pivot scale. However, this particular value has been chosen arbitrarily within the permissible range. It is apparent from the above equation (2.6) that for $r < 0.002$ we have $\Delta\phi < M_{\rm Pl}$, leading to a sub-Planckian field excursion, while for $r > 0.002$ we get a super-Planckian field excursion. As a result one can distinguish the inflationary models in terms of the field excursion variable.

Now we are all set to calculate the Hubble flow functions for the KM II model given in (2.3). From the observational point of view, one needs N to be large and thus these parameters are of singular importance for the rest of the analysis as we will see that there are large classes of models that agree on the large N limit. The first-order Hubble flow function is given by

$$\epsilon_1=\frac{b}{2N^2\sqrt{\ln N}}, \tag{2.8}$$

where $b=\frac{9}{16}\frac{1}{\beta^{3/2}}$, and the second Hubble flow function is as follows:

$$\epsilon_2=-\frac{2}{N}. \tag{2.9}$$

The basic features of the inflationary model under consideration have been encoded by the above functions at the leading order of N. Subsequent correction terms have a very insignificant role to play with the observational parameters. Let us consider that from now on the quantities of the benchmark model will be denoted by an overhead bar to differentiate them from the other classes of inflation and choose to work with $M_{\rm Pl}=1$.

We take three values of $\mathcal{V}=10^5, 10^6$, and 10^7 for our analysis, leading to the values of $b=5.63\times10^{-6}$, 5.63×10^{-7}, and 5.63×10^{-8} respectively. As most of the inflation takes place at large values of N we can consider N_f to be negligibly small and thus $\bar{N}_*\approx 58$ is justified. This particular choice for the number of e-folds remaining after the exit of the horizon to the end of inflation is in agreement with the Planck pivot scale. Other allowed values of $\bar{N}_*$ may be chosen but that will only enable us to infer a similar output for the analysis. Let us now define a quantity as follows:

$$\epsilon_{1*}=\frac{b}{2\bar{N}_*^2\sqrt{\ln\bar{N}_*}}, \tag{2.10}$$

which is the value of the first Hubble flow parameter at horizon crossing and $\bar{N}_*$ is the number of e-folds at that point in time. As the benchmark matches very well with observational parameters, we set our aim to learn how compatible these predictions are with universality classes of inflationary models that agree in the large N limit. We are also curious to know what happens to the field excursion variable $\Delta\phi$ for the broad classes of models mentioned earlier in comparison with the KM II model.

III. COMPARISON OF FIELD EXCURSIONS IN DIFFERENT CLASSES OF INFLATON

We now intend to look at how the field excursions of the inflationary models vary for a given set of values of the CMB observables n_s and r. It will be interesting to explore whether the field excursion $\Delta\phi$ and the number of e-folds N remain the same or change. The large N behavior of wide classes of inflationary models has been discussed rigorously in [13,14] by finding the dependence on N of the Hubble flow parameters. At leading order the $1/N^p$ behavior for the slow-roll parameter ϵ is considered the perturbative class. In addition to that, there are constant, nonperturbative, and logarithmic classes [13,14]. For these three classes we will find the leading-order contribution of the first and second Hubble flow parameters and by equating those to the respective values for the KM II model we will compare the field excursion for a given set of spectral tilt n_s and tensor-to-scalar ratio r.

A. Perturbative class

An attractive feature of the *perturbative* class of models is that the $1/N$ term provides a natural explanation for the percent variation from the scale invariance of the CMB power spectrum. Chaotic, hilltop, inverse hilltop, and Whitt potentials are typical examples of this particular class. The first two Hubble flow parameters of the perturbative class are given by

$$\epsilon_1=\frac{\mu}{N^p}$$
$$\epsilon_2=-\frac{p}{N}, \tag{3.1}$$

where μ and $p(\geq 1)$ are the parameters. For different values of these parameters, one gets different models within this class. Now by imposing the requirement that the above Hubble flow parameter values should fall in the same range as those of the benchmark model as given in Eq. (2.10), i.e., the same scalar spectral index and tensor-to-scalar ratio will be produced by the perturbative class of models as those of the benchmark, we obtain the following relationship to be followed by the model parameters. Let us consider first the parameter μ, which should follow the restriction given below to reconcile with the above -mentioned demand:

$$\mu = \frac{bN_*^p}{2\bar{N}_*^2\sqrt{\ln \bar{N}_*}}, \tag{3.2}$$

where

$$N_* = \frac{p\bar{N}_*}{2}. \tag{3.3}$$

Terms with an overbar correspond to the values associated with the benchmark model. Now substituting Eq. (3.3) into Eq. (3.2), we obtain

$$\mu = \frac{b}{2}\left(\frac{p\bar{N}_*}{2}\right)^p \times \frac{1}{\bar{N}_*^2\sqrt{\ln \bar{N}_*}} = \epsilon_{1*}\left(\frac{p\bar{N}_*}{2}\right)^p. \tag{3.4}$$

Equation (3.4) explicitly indicates how the parameters should be finely adjusted to guarantee the correct prediction of observational parameters coming from up-to-date CMB observations in several models in the perturbative class. A careful investigation of how the parameter μ behaves for a wide range of p values reveals that one can get the same values of b for diverse values of μ and p. This in turn says that as we fix the values of the slow-roll parameters of the perturbative model with those of the KM II model, the slow-roll parameters of the perturbative model and thus subsequently the values of the spectral index n_s and tensor-to-scalar ratio r are fixed, while those of μ and p change.

Let us explore the number of e-folds N from the horizon exit to the end of inflation and the field excursion $\Delta\phi$ for large classes of perturbative models characterized by different values of p. It is apparent from the definition in Eq. (2.3), the end of inflation can be associated to the first Hubble flow parameter, $\epsilon_1 = 1$. The number of e-folds, N_f, at the end of inflation can thus be determined from the above-mentioned condition. From Eq. (3.1) we get

$$N_f = \mu^{1/p}. \tag{3.5}$$

Therefore the number of e-folds N for the perturbative class in terms of p and benchmark model parameters is obtained by using the above value of N_f and Eqs. (3.3) and (3.4) as given below:

$$N = \frac{p\bar{N}_*}{2}[1 - \epsilon_{1*}^{1/p}]. \tag{3.6}$$

For a given b, the value of ϵ_{1*} being very small, it becomes apparent from the above relation that N increases linearly

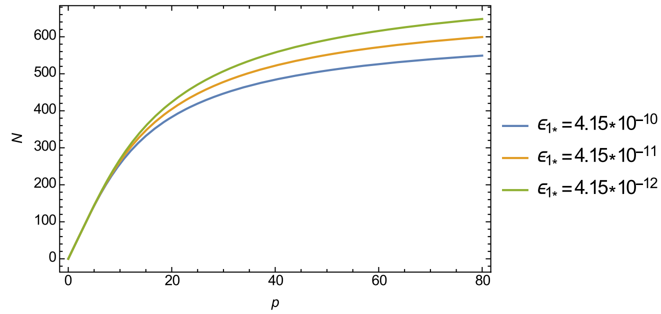

FIG. 1. N increases linearly for low values of p. Three values of ϵ_{1*} are used for three values of $\beta = 2.15 \times 10^3$, 1.00×10^4, 4.64×10^5.

with p for low values, which is evident from Fig. 1. One may find it interesting to allow N to vary for a large range that may be dependent on the postinflationary physics of the model. Curiously, we have noted (in Fig. 2) that a maximum limit on the value of N is reached asymptotically with p and this seems to be a generic feature for this class of models. The consequences of this finding will be explored further by studying the field excursion. Using the definition given in Eq. (2.6) we get the excursion of the inflaton as follows:

$$\Delta\phi = \frac{2\sqrt{2\beta}}{2-p}(N_*^{1-p/2} - N_f^{1-p/2}) \tag{3.7}$$

for the perturbative class. Putting in the values of N_f and N_*, the above expression reduces to the elegant form to the elegant form

$$\Delta\phi = \frac{\sqrt{2}}{2-p}(p\bar{N}_*)\epsilon_{1*}^{1/2}\left[1 - \epsilon_{1*}^{\frac{2-p}{2p}}\right]. \tag{3.8}$$

Let us depict the results graphically in Figs. 3 and 4, which show the variation of $\Delta\phi$ with respect to p. Interestingly, $\Delta\phi$ starts out as sub-Planckian ($\Delta\phi < M_{\rm Pl}$) for small values of p before it becomes equal to 1 (we choose to work with $M_{\rm Pl} = 1$) at a certain value of p. Beyond that a continuous increase is seen in $\Delta\phi$ as p increases, finally saturating at high values of p. This also shows an upper bound on the

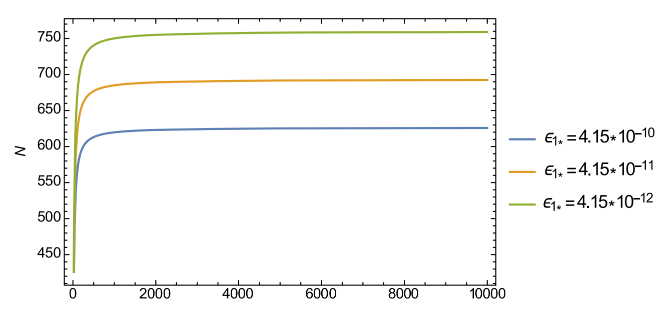

FIG. 2. N approaches a maximum value for high values of p.

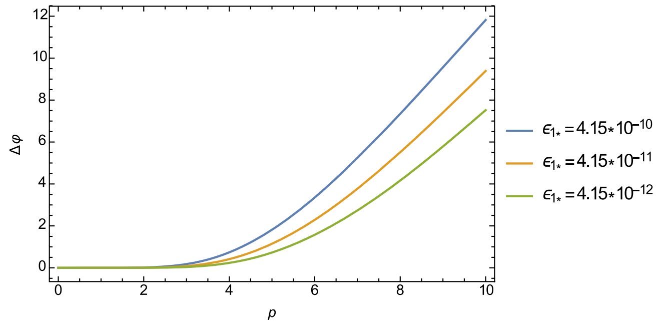

FIG. 3. $\Delta\phi$ increases linearly for low values of p and is sub-Planckian up to a certain value of p, beyond which it becomes super-Planckian. Three values of ϵ_{1*} are used to span the entire range of β by choosing $\beta = 2.15 \times 10^3$, 1.00×10^4, 4.64×10^5.

value of the field excursion similar to what is found in the number of e-folds.

One can easily find the maximum values of the number of e-folds and the field excursion by looking at the limiting tendencies as p goes to infinity. Let us discuss one example by choosing a typical value of $\epsilon_{1*} \approx 10^{-10}$. We find that

$$N_{\rm max} = \lim_{p\to\infty} N = \lim_{p\to\infty} \frac{p\bar{N}_*}{2}[1-\epsilon_{1*}^{1/p}] = -\frac{\bar{N}_*}{2}\ln\epsilon_{1*} = 625.99 \tag{3.9}$$

$$\Delta\phi_{\rm max} = \lim_{p\to\infty}\Delta\phi = \lim_{p\to\infty}\frac{\sqrt{2}}{1-2/p}\bar{N}_*\epsilon_{1*}[1-\epsilon_{1*}^{1-p/2}] = \sqrt{2}\bar{N}_*[1-\epsilon_{1*}^{1/2}] = 82.02. \tag{3.10}$$

However, such large values of N are not necessarily realistic; from a theoretical viewpoint it is interesting to explore such large ranges. Considering the variation of ϵ with respect to N one can show that it is impossible to keep ϵ constant for a large range of e-foldings. As a result there appears an upper bound on N that translates into a limit on the field excursion.

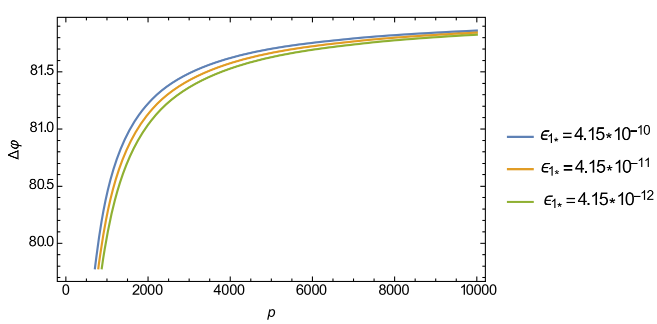

FIG. 4. $\Delta\phi$ becomes super-Planckian for $p > 4$ with the given parameter choice and approaches a maximum value for high values of p. Here we have taken $\beta = 2.15 \times 10^3$, 1.00×10^4, 4.64×10^5, which lead to the values of ϵ_{1*} shown above.

People have been curious for long about how deep the inflation can be in specific classes of models. The Lyth bound gives a guideline for the minimal single-field slow-roll scenario. Depending on whether one considers ϵ to vary or not during the horizon exit to the end of inflation, more stringent constraints of the Lyth bound can be imposed [21]. In this analysis we retain the considerations originally used to define the field excursion. From the main results obtained in the perturbative class we see that both the field excursion and the number of e-folds increase with an increase in p, even as n_s and r remain the same. Most remarkably an upper limit on both $\Delta\phi$ and N has been achieved asymptotically. A careful inspection points towards a degeneracy in the field excursion with different values of N for the models predicting the same values of observational parameters. We have also explored the possibility of having sub-Planckian and super-Planckian field excursions. This is very interesting from both a theoretical and an observational point of view. We only have a bound on the tensor-to-scalar ratio from the observations till today. A definite detection of r will definitely solve the above puzzle, and also an independent detection of r and ϵ_1 is necessary to prove the validity of the Lyth bound. Note that the bound given in Eq. (2.7) implies that for sub-Planckian inflaton excursion, and thus consistent field theory description, r should be less than 0.002, implying that it was beyond the reach of Planck but within the reach of future missions like various ground-based experiments (AdvACT, CLASS, Keck/BICEP3, Simons Array, SPT-3G); balloons (EBEX 10k and Spider); and satellites (CMBPol, COrE, and LiteBIRD) [22,23].

B. Nonperturbative class

The next class that we would like to consider is the nonperturbative models of inflation [13,14] characterized by the Hubble flow parameters that are nonperturbative around $1/N \to 0$. In this case

$$\epsilon_1 = \exp(-2cN)$$
$$\epsilon_2 = -2c, \tag{3.11}$$

where c is a constant. Equating this with the same parameters of the benchmark model we obtain

$$\frac{b}{2\bar{N}_*^2\sqrt{\ln\bar{N}_*}} = \exp[-2cN_*], \tag{3.12}$$

leading to an expression for the constant $c = 1/\bar{N}_*$. We next consider the number of e-folds N between horizon exit and end of inflation. The number of e-folds at the end of inflation, N_f, is obtained by the fact that the first Hubble flow parameter is equal to 1 when inflation ends. Thus for the nonperturbative case we have $N \approx N_*$. Using Eq. (3.12) we can calculate the number e-foldings remaining at the point of horizon exit as

$$N_* = -\frac{\bar{N}_*}{2} \ln \epsilon_{1*}. \tag{3.13}$$

This in turn gives the number of e-foldings in the nonperturbative class from the horizon exit to the end of inflation as $N = -\frac{\bar{N}_*}{2} \ln \epsilon_{1*}$. We get a startling result for the number of e-folds. The number of e-folds N is the same as the form for the maximum number of e-folds for the perturbative class [Eq. (3.9)]. Apparently the N of the nonperturbative class hits the maximum limit of the number of e-folds for the perturbative class.

The field excursion $\Delta\phi$ as obtained using Eqs. (2.6) and (3.11) has the following form:

$$\Delta\phi = -\sqrt{2}\bar{N}_*[\exp(-cN_f) - \exp(-cN_*)]. \tag{3.14}$$

Inserting the value of N_f and using Eq. (3.13), we get

$$\Delta\phi = \sqrt{2}\bar{N}_*[1 - \sqrt{\epsilon_{1*}}], \tag{3.15}$$

which is the same as that for the maximum limit of $\Delta\phi$ for the perturbative class [Eq. (3.10)]. Most significantly N and $\Delta\phi$ in the nonperturbative class are similar to those in the large p limit of the perturbative class. Furthermore, there is not much variation over the different parameters; instead there is one particular value of $\Delta\phi$ and N each for different values of ϵ_{1*}, corresponding to the benchmark model KM II. Curiously, for the nonperturbative class we get super-Planckian field excursion only. This is actually a very strong constraint because first of all it is difficult if not impossible to have one inflationary theory where we have a good control over a Planckian field range. It again establishes the necessity for the independent detection of the first Hubble flow function and r that will tell us about the existence of the Lyth bound.

C. Logarithmic class

The other class of model that we intend to explore is the logarithmic one [14], in which the Hubble flow parameters are

$$\epsilon_1 = \kappa \frac{\ln^q N}{N^p}, \tag{3.16}$$

$$\epsilon_2 = -\frac{p}{N} + \frac{q}{N \ln N}, \tag{3.17}$$

where p and the power coefficient q are model-specific parameters, different values of which lead to different models. We have retained logarithmic correction terms in the generic class. However, as we are working at large N limits, we can readily see that the second term of the above equation for ϵ_2 dies down rapidly and we can ignore its effects compared to the first term of $1/N$ at leading order. Note that the benchmark can be easily retrieved by choosing the parameter $p = 2$ and keeping leading-order contributions of $1/N$. Executing similar techniques as discussed in previous sections, we obtain κ by equating the above parameters with that of the benchmark model as

$$\kappa = \frac{b}{2} \frac{N_*^p}{\bar{N}_*^2 \sqrt{\ln \bar{N}_*} \ln^q N_*}, \tag{3.18}$$

and also the following relationship,

$$\frac{2}{\bar{N}_*} = \frac{p}{N} - \frac{q}{N_* \ln N_*}. \tag{3.19}$$

The field excursion in this context comes out to be

$$\Delta\phi = \sqrt{2}\kappa \int \frac{\ln^{q/2} N}{N^{p/2}} dN. \tag{3.20}$$

For specific choices of p and q one can infer the implications of the above expression. In the large N limit, the second slow-roll parameter is given by

$$\epsilon_2 = -\frac{p}{N}. \tag{3.21}$$

Keeping p fixed, we vary the variable q and see how the inflationary field excursion changes. It is to be noted, from the various models conforming to the logarithmic class of models and from working within our approximation of neglecting the second term of Eq. (3.17) that only values of q running less than 10 are physically acceptable. The value of p is chosen as 2, which is not only the case for the KM II model but also well motivated from the literature [14,24,25]. An intensive study of the variation in the inflationary field range with changing q shows that the field excursion remains sub-Planckian for the parameter range chosen above. Therefore, considering all the results obtained in this and in the previous sections, what we can conclude is that the degeneracy in various pictures may be lifted from future observations aiming at a finer value of tensor-to-scalar ratio r, and also an independent detection of ϵ_1 will help.

One may wonder why the first Hubble flow function has only been chosen to specify the end of inflation. Note that the large N formalism and subsequently the Hubble flow functions considered here are based on the primary quantity $H(\phi)$. The dynamics has been used to define the slow-roll parameters instead of the field potentials. One can show that the first two Hubble flow functions are linked with corresponding potential slow-roll parameters via the relations $\epsilon_1 = \epsilon_V$ and $\epsilon_2 = -4\epsilon_V + 2\eta_V$. Liddle *et al.* in [26] have pointed out that the true end point of inflation gauged by the Hubble flow functions occurs exactly at $\epsilon_1 = 1$. For potential slow-roll parameters, this is a first-order approximation. Now the types of models encompassed by large N formalism [14] exhibit such a dynamics that generically one can assume the end inflation by $\epsilon_1 = 1$. This is also the case for models of inflation that consider the existence of flat directions. On the other hand if one still becomes interested in exploring the possibility of ending inflation via alternative methods, one may look for the possibility $\epsilon_2 = 1$ (note that this is not same as setting $\eta_V = 1$). This possibility gives rise to a decreasing $\Delta\phi$ with respect to p in the logarithmic class for a given value of q. It may be a topic of interest to explore in future endeavors. In those cases one may also go beyond the regime of slow-roll approximation and look for alternatives like ending inflation by introducing a second field potential.

IV. SUMMARY AND DISCUSSIONS

Let us conclude with a few comments. We have emphasized the Planck 2015 data and the strong underlying theoretical background in choosing the benchmark model for our analysis. Considering the span of inflaton field profile $\Delta\phi$ for the KM II model as a reference, we have studied how the range of the field excursion varies in different universality classes of inflationary models corresponding to a chosen point in the $n_s$$-$$r$ plane. The value for n_s satisfies the value found by different experiments and also the most recent values given by Planck 2015 [5]. The value for the tensor-to-scalar ratio r for our benchmark model is well within the allowed upper bound for the value of r as found in recent experiments (unlike the chaotic inflation benchmark case). Thus our choice of the chosen point in the $n_s$$-$$r$ plane is quite well motivated and future experiments which probe with greater accuracy the value of r can comment on its viability. At present it is in excellent agreement with the experimental results. The technique followed in this work has been proposed in the context of chaotic inflation as the benchmark model [17]. However, the recent Planck data release rules out values of $r > 0.09$, while for the quadratic potential in the chaotic class, $r = 0.16$. The KM II model predicts a spectral index n_s well within the 2σ contour of Planck. This also predicts a value of r that gives a sub-Planckian field excursion according to the Lyth bound.

Comparing this with other universality classes of models, we found that it is possible to have super-Planckian as well as sub-Planckian field excursions, for example, for different ranges of parameter p in the perturbative class of models. While equating the slow-roll parameters at the horizon crossing, one not only changes ϕ_*, but also ϕ_f, which are the values of the field at the horizon crossing and at the end of inflation, respectively. This also changes the value of N_f, the number of e-folds at the end of inflation. Fixing the Hubble flow parameters at the horizon crossing for a model amounts to fixing the value of the field variable ϕ_*, which in turn changes N_*, the number of e-folds at the horizon crossing.

Basically the demand to get the same n_s and r as the benchmark model puts a constraint on the theory via the change of N_* and ϕ_* from the original value. This is why we get a range of values for N and $\Delta\phi$ for the same n_s and r. One can also go ahead and constrain the value of the running of the spectral tilt α_s for the benchmark model and the various classes of inflation. We have checked that to infer that it does not introduce any significant constraint for the inflationary field range. For the perturbative and the logarithmic class, the third slow-roll parameter has a similar $1/N$ dependence as the second slow-roll parameter for the two classes of inflation and therefore adds nothing new to the discussion. For the nonperturbative class, the third slow-roll parameter comes out to be zero.

In this analysis with KM II as the benchmark, most interestingly, we have found that one can get a sub-Planckian field excursion in the regime of single-field slow-roll inflation. There appears to be a maximum value for the field excursion variable and the number of e-folds. Owing to the different geometric form of the potential in the benchmark model, we get a distinct limit on the above-mentioned parameters. The chaotic model is much steeper, so the rolling-down velocity of the field is greater than that for the KM II, whose slope is much flatter, leading to a much slower rolling speed. Thus in this case there is a greater number of e-folds but a smaller value of the maximum field excursion. Interestingly, results similar to those in the perturbative class have been observed in the nonperturbative class. Finally we see that for the perturbative class the value of $\Delta\phi_{\max}$ is almost the same for different initial parameters for the benchmark models while the maximum number of e-folds changes appreciably. The degeneracy we have observed in different forms may be lifted by future observations [23].

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