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# Quark magnetar in three-flavor Nambu–Jona-Lasinio model under strong magnetic fields with two types of vector interactions

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We investigate the properties of strange quark matter (SQM) and quark stars (QSs) in the framework of SU(3) Nambu–Jona-Lasinio (NJL) model with two types of vector interactions under strong magnetic fields: (1) the flavor-dependent repulsion among $u$, $d$, and $s$ quarks with the coupling constant $G_V$, and (2) the universal repulsion and the vector-isovector interaction with the coupling constants $g_V$ and $G_{IV}$. The effects of the two types of vector interactions on the constituent quark mass, vacuum quark mass, quark chemical potential, and quark fraction in SQM under strong magnetic fields are studied, and the results indicate that these physical quantities for SQM are all sensitive to the two types of vector interactions in NJL model under magnetic fields. Using a density-dependent magnetic field profile which is introduced to describe the magnetic field strength distribution inside the magnetars, we calculate the properties of spherical QSs by using two extreme cases for the orientation of the magnetic field inside the stars, i.e., the radial orientation in which the magnetic fields are along the radial direction in stars, and the transverse orientation in which the magnetic fields are randomly oriented in the plane which is perpendicular to the radial direction. Our results indicate that the maximum mass of QSs may dependent on both the strength distribution and the orientation of the magnetic fields inside QSs by using SU(3) NJL model.

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## I. INTRODUCTION

In terrestrial laboratories, heavy ion collisions (HICs) provide unique tools to explore the properties of strong interaction matter, which is one of the main topics of quantum chromodynamics (QCD). The hot quark-gluon plasma (QGP) is expected to be created in heavy ion collisions at the Relativistic Heavy Ion collider (RHIC) and the Large Hadron Collider (LHC). The knowledge of strong interaction matter at high baryon density regions can be further complemented by future experiments planned in heavy ion collisions at FAIR in GSI and the Nuclotron-based Ion Collider Facility (NICA) at JINR, while the cold and dense quark matter may exist in the inner core of the compact stars, which provide another way to explore the strong interaction matter at high baryon density and low temperature. Neutron stars (NSs), which have been shown to provide natural testing grounds of the knowledge on the properties of neutron-rich matter [1,2], are a class of the densest compact stars in the Universe. Theoretically, NSs maybe converted to quark stars (QSs) [3–6], which are made purely by deconfined $u$, $d$, $s$ quark matter and some leptons, and the possible existence of QSs is one of the most intriguing aspects of modern astrophysics. There are also hybrid star conjectures with a phase transition from the nuclear phase to the quark phase at high baryon density [7–13].

In the inner core of compact stars, the baryon density can reach or be larger than around 6 times the normal nuclear matter density $n_0 = 0.16\ \mathrm{fm}^{-3}$, so there might exist hyperons [14–16], meson condensations [17–19], and even strange quark matter (SQM) [20–23]. Theoretically, SQM has been conjectured to be the absolutely stable true ground state of QCD matter (i.e., the strong interaction matter) [20,21], and studying the properties of SQM is helpful to understand and calculate the structure of QSs.

The equation of state (EOS) of dense quark matter at finite density is usually soft due to the asymptotic freedom of QCD feature for the interactions among quarks at ultra high density. In addition, the EOS of SQM can be further softened because of the addition of s quark, which contributes a new freedom for quarks. Recently, by using the general relativistic Shapiro delay, two heaviest neutron stars have been measured with high accuracy. One is the radio pulsar J1614-2230 [24] with a mass of $1.97 \pm 0.04\ M_{\odot}$, and the other is J0348 + 0432 [25] with a mass of $2.01 \pm 0.04\ M_{\odot}$. Even heavier compact stars have been discussed in the works [26,27]. This high mass seems to rule out conventional QS models with soft EOSs. For the NJL model, it has been pointed out that the repulsive vector interaction model can produce a stiff EoS for dense quark

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matter and thus can generate a two-solar-mass compact star [28–33]. The role of the vector interaction in QCD vacuum and medium has been discussed in the literature [28,29,34–50], and the coupling constants are determined by the vector spectra [34,37].

In recent decades, properties of strange quark matter at finite temperature under strong magnetic fields have attracted lots of interest, and as shown in Ref. [51], the presence of external magnetic field can even harden the equation of state of dense quark matter when considering that the compact stars can be endowed with strong magnetic fields, which are called magnetars. In the laboratory, through noncentral heavy ion collisions [52,53] at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC), strong magnetic fields with the strength of $10^{18}$–$10^{20}$ G [which is equivalent to $eB \sim (0.1\text{–}1.0\ \mathrm{GeV})^2$] can be generated. At the surface of the compact star, a large magnetic field strength of $B \sim 10^{14}$ G has been estimated [54–56], and the magnetic field strength may reach as large as $B \sim 10^{18}$ G in the core of the compact stars [57]. In the work by Ferrer *et al.* [58], the estimated magnetic field strength can even reach about $10^{20}$ G in the core of the self-bound QSs. Under such strong magnetic fields, the spatial rotational [$\mathcal{O}(3)$] symmetry will break and one should introduce the pressure anisotropy of the system [58–61]. In order to describe the spatial distribution of the magnetic field strength in compact stars, a density-dependent magnetic field profile [62,63] is introduced, and it is important to investigate the effects of the spatial distribution and the orientation of the magnetic fields on the properties of compact stars. It is still a question whether the inclusion of the strong magnetic fields can reduce or enhance the maximum mass of compact stars [62–73]. For instance, the maximum mass of compact stars increases with the increment of the magnetic fields in the work [65], while in the work [73] the maximum mass of compact stars decreases with magnetic fields. The main motivation of this work is to explore the properties of SQM and QSs under strong magnetic fields. Due to the soft EOSs for quark matter, we introduce two types of vector interactions to stiffen the EOS of quark matter in SU(3) NJL model in order to describe two solar mass QSs under strong magnetic fields.

The paper is organized as follows. In Sec. II, we give a general description of the SU(3) NJL model with two types of vector interactions under magnetic field in $\beta$ equilibrium condition,and then present our numerical results in Sec. III. The conclusion and discussion is given in Sec. IV.

## II. THREE-FLAVOR NJL MODEL WITH VECTOR INTERACTION UNDER MAGNETIC FIELD

We start from the Lagrangian density to investigate the properties of $u$, $d$, and $s$ quark matter under strong magnetic fields $A_\mu^{\mathrm{ext}}$

$$\mathcal{L} = \mathcal{L}_q + \mathcal{L}_l - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \tag{1}$$

where $\mathcal{L}_l$ and $\mathcal{L}_q$ are the Lagrangian densities for leptons (including electrons and muons) and quarks, respectively. The magnetic field $B$ is set as the static magnetic field along z direction, and $A_\mu^{\mathrm{ext}} = \delta_{\mu 2} x_1 B$. $F_{\mu\nu} = \partial_\mu A_\nu^{\mathrm{ext}} - \partial_\nu A_\mu^{\mathrm{ext}}$ is the external electromagnetic field strength tensor.

The lepton Lagrangian density is given as

$$\mathcal{L}_l = \bar{l}[(i\partial_\mu - eA_{\mathrm{ext}}^\mu)\gamma^\mu] l. \tag{2}$$

The Lagrangian density for quarks can be written by the gauged $N_f = 3$ NJL model with vector interaction [35,36]

$$\mathcal{L}_q = \bar{\psi}_f [\gamma_\mu (i\partial^\mu - q_f A_{\mathrm{ext}}^\mu) - \hat{m}_c] \psi_f + \mathcal{L}_4 + \mathcal{L}_{\mathrm{det}}, \tag{3}$$

where $\mathcal{L}_4$ stands for four-fermion interaction compatible with QCD symmetries, and $\mathcal{L}_{\mathrm{det}}$ indicates the six-point interaction which breaks the axial $U(1)_A$ symmetry. $\psi = (u, d, s)^T$ represents the three flavor quark field, $\hat{m}_c = \mathrm{diag}(m_u, m_d, m_s)$ is the current quark mass matrix, and $q_f$ means the quark electric charge. The four-fermion interaction including scalar, pseudoscalar, and the two types of vector interaction takes the form of

$$\mathcal{L}_4 = \mathcal{L}_S + \begin{cases} \mathcal{L}_{Va} \\ \mathcal{L}_{Vb} \end{cases} \tag{4}$$

The scalar part is

$$\mathcal{L}_S = G_S \sum_{a=0}^{8} [(\bar{\psi}_f \lambda_a \psi_f)^2 + (\bar{\psi}_f i\gamma_5 \lambda_a \psi_f)^2], \tag{5}$$

where $G_S$ is the coupling constant in the scalar channel. $\lambda_a (a = 1, \ldots, 8)$ are the Gell-Mann matrices and the generators of the SU(3) flavor groups, and $\lambda_0 = \sqrt{2/3} I$ with $I$ being the unit matrix.

We now consider two types of vector interaction, "type A" ($\mathcal{L}_{Va}$) and "type B" ($\mathcal{L}_{Vb}$), in the Lagrangian as

$$\mathcal{L}_{Va} = -G_V \sum_{a=0}^{8} [(\bar{\psi}\gamma^\mu \lambda^a \psi)^2 + (\bar{\psi} i\gamma^\mu \gamma_5 \lambda^a \psi)^2], \tag{6}$$

$$\mathcal{L}_{Vb} = -g_V (\bar{\psi}\gamma^\mu \psi)^2 - G_{IV} [(\bar{\psi}\gamma^\nu \vec{\tau} \psi)^2 + (\bar{\psi}\gamma^\nu \gamma_5 \vec{\tau} \psi)^2], \tag{7}$$

where $G_V$ and $g_V$ are vector interaction coupling constants, and $G_{IV}$ is the vector-isovector coupling constant. The term proportional to $G_V(>0)$ in Eq. (6) gives a flavor-dependent repulsion for quarks, while the one proportional to $g_V(>0)$ in Eq. (7) contributes the universal repulsion which cannot be distinguished in different flavors. The term proportional to $G_{IV}(>0)$ in Eq. (7) indicates the vector-isovector interaction for quarks, and $\vec{\tau}$ is the Pauli matrices. Since the $G_V$ term interaction is flavor-dependent while $g_V$ term is flavor independent, we add the extra term for the vector-isovector channel ($G_{IV}$ term) in Eq. (7) so as to distinguish the isoscalar and isovector for the vector channel.

The 't Hooft term $\mathcal{L}_{\rm det}$ the takes the form of

$$\mathcal{L}_{\rm det} = -K\{\det_f[\overline{\psi_f}(1+\gamma_5)\psi_f] + \det_f[\overline{\psi_f}(1-\gamma_5)\psi_f]\}, \tag{8}$$

which is to break the $U(1)_A$ symmetry.

### A. The pressure from quark contribution

In the mean-field approximation, the Lagrangian density for the quark part is

$$\begin{aligned}\mathcal{L}_M = \overline{\psi}_f\Big[&\gamma_\mu(i\partial^\mu - q_f A^\mu_{\rm ext}) - \hat{M} - 4G_V\gamma_0\hat{n} \\ &- 2g_V\gamma_0\sum_{i=u,d,s} n_i - 2G_{IV}\gamma_0\tau_{3f}n_f\Big]\psi_f \\ &- 2G_S(\sigma_u^2+\sigma_d^2+\sigma_s^2) + 4K\sigma_u\sigma_d\sigma_s \end{aligned}\tag{9}$$

$$+\begin{cases} 2G_V(n_u^2+n_d^2+n_s^2) \\ g_V\left(\sum_{i=u,d,s} n_i\right)^2 + G_{IV}(n_u-n_d)^2 \end{cases}, \tag{10}$$

where

$$\hat{n} = \begin{pmatrix} n_u & 0 & 0 \\ 0 & n_d & 0 \\ 0 & 0 & n_s \end{pmatrix}$$

is the density matrix for quarks, and

$$\hat{M} = \begin{pmatrix} M_u & 0 & 0 \\ 0 & M_d & 0 \\ 0 & 0 & M_s \end{pmatrix}$$

is the constituent quark mass matrix.

The quark mass can be derived by the gap equation of

$$M_i = m_i - 4G_S\sigma_i + 2K\sigma_j\sigma_k, \tag{11}$$

where $(i,j,k)$ is any permutation of $(u,d,s)$. The chiral condensate is written as

$$\sigma_f = \langle\overline{\psi_f}\psi_f\rangle = -i\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}\frac{1}{(p\!\!/ - M_f + i\epsilon)}. \tag{12}$$

We can obtain the pressure for quarks $p_q = -\Omega_q$ ($\Omega_q$ is the thermodynamical potential for quarks) as

$$\begin{aligned} p_q = &-2G_S(\sigma_u^2+\sigma_d^2+\sigma_s^2) + 4K\sigma_u\sigma_d\sigma_s \\ &+ (\Omega^u_{ln} + \Omega^d_{ln} + \Omega^s_{ln}) \\ &+ \begin{cases} 2G_V(n_u^2+n_d^2+n_s^2) \\ g_V(n_u+n_d+n_s)^2 + G_{IV}(n_u-n_d)^2 \end{cases}, \end{aligned}\tag{13}$$

with the logarithmic contribution

$$\Omega^f_{\rm ln} = -i\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}\ln\left\{\frac{1}{T}[p\!\!/ - \hat{M}_f + \gamma_0\tilde{\mu}_f]\right\}, \tag{14}$$

here $T$ comes from the Matsubara frequencies for fermions, and

$$\tilde{\mu}_f = \mu_f - 4G_V n_f - 2g_V\sum_{i=u,d,s} n_i - 2G_{IV}\tau_{3f}(n_u - n_d), \tag{15}$$

where $\mu_f$ is the chemical potential for quarks and $\tau_{3f}$ indicates the isospin quantum number for quarks: $\tau_{3u} = 1$, $\tau_{3d} = -1$, and $\tau_{3s} = 0$.

The calculation for $\Omega^f_{ln}$ and $\sigma_f$ at zero temperature can be acquired as

$$\Omega^f_{ln} = \Omega^{f,\rm vac}_{ln} + \Omega^{f,\rm mag}_{ln} + \Omega^{f,\rm med}_{ln}. \tag{16}$$

The first term is tahe vacuum contribution to the thermodynamical potential

$$\Omega^{f,\rm vac}_{\rm ln} = -\frac{N_c}{8\pi^2}\left\{M_f^4\ln\left[\frac{\Lambda+\epsilon_\Lambda}{M_f}\right] - \epsilon_\Lambda\Lambda(\Lambda^2+\epsilon_\Lambda^2)\right\}, \tag{17}$$

where $\epsilon_\Lambda^2 = \Lambda^2 + M_f^2$ and $\Lambda$ is the noncovariant cutoff. The magnetic field contribution is calculated as

$$\begin{aligned}\Omega^{f,\rm mag}_{\rm ln} = \frac{N_c}{2\pi^2}(|q_f|B)^2\Big[&\frac{x_f^2}{4} + \zeta'(-1,x_f) \\ &- \frac{1}{2}(x_f^2 - x_f)\ln(x_f)\Big], \end{aligned}\tag{18}$$

where $x_f = \frac{M_f^2}{2|q_f|B}$ and $\zeta(z,x)$ is the Riemann-Hurwitz function. The medium contribution is

$$\begin{aligned}\Omega^{f,\rm med}_{ln} = \sum_{k=0}^{k_{\rm fmax}}\alpha_k\frac{(|q_f|BN_c)}{4\pi^2}\Bigg\{&\tilde{\mu}_f\sqrt{\tilde{\mu}_f^2 - s_f(k,B)^2} \\ &- s_f(k,B)^2\ln\left[\frac{\tilde{\mu}_f + \sqrt{\tilde{\mu}_f^2 - s_f(k,B)^2}}{s_f(k,B)}\right]\Bigg\}, \end{aligned}\tag{19}$$

where

$$s_f(k,B) = \sqrt{M^2 + 2|q_f|Bk}, \tag{20}$$

and

$$k_{\mathrm{fmax}} = \frac{\tilde{\mu}_f^2 - M^2}{2|q_f|B} = \frac{p_{f,F}^2}{2|q_f|B}, \tag{21}$$

is the upper Landau level with $\alpha_k = 2 - \delta_{k0}$.

Then the condensates for each flavor of quarks can be acquired as:

$$\sigma_f = \sigma_f^{\mathrm{vac}} + \sigma_f^{\mathrm{mag}} + \sigma_f^{\mathrm{med}} \tag{22}$$

with

$$\sigma_f^{\mathrm{vac}} = -\frac{M_f N_c}{2\pi^2}\left\{\Lambda\sqrt{\Lambda^2 + M_f^2} - M_f^2 \ln\left[\frac{\left(\Lambda + \sqrt{\Lambda^2 + M_f^2}\right)^2}{(M_f^2)}\right]\right\}, \tag{23}$$

$$\sigma_f^{\mathrm{mag}} = -\frac{M_f N_c}{2\pi^2}(|q_f|B)\left\{\ln[\Gamma(x_f)] - \frac{1}{2}\ln(2\pi) + \frac{\ln(x_f)}{2} - x_f \ln(x_f)\right\}, \tag{24}$$

$$\sigma_f^{\mathrm{med}} = \sum_{k=0}^{k_{\mathrm{fmax}}} \alpha_k \frac{M_f |q_f| B N_c}{\pi^2} \times \left\{\ln\left[\frac{\tilde{\mu}_f + \sqrt{\tilde{\mu}_f^2 - s_f(k,B)^2}}{s_f(k,B)}\right]\right\}. \tag{25}$$

### B. The SQM

For SQM, the weak beta-equilibrium condition can be acquired as

$$\mu_u + \mu_e = \mu_d = \mu_s, \tag{26}$$

where $\mu_i$ ($i = u$, $d$, $s$ and $e^-$) means the chemical potential of the particles in SQM. We can write down the electric charge neutrality condition as

$$\frac{2}{3}n_u = \frac{1}{3}n_d + \frac{1}{3}n_s + n_l. \tag{27}$$

Where

$$n_f = \sum_{k=0}^{k_f,\mathrm{max}} \alpha_k \frac{|q_f|BN_c}{2\pi^2} k_{F,f} \tag{28}$$

is the quark density with $k_{F,f} = \sqrt{\tilde{\mu}_f^2 - s_f(k,B)^2}$, and

$$n_l = \sum_{k=0}^{k_l,\mathrm{max}} \alpha_k \frac{|q_l|B}{2\pi^2} k_{F,l} \tag{29}$$

the lepton number density.

### C. Pressure of SQM under magnetic fields

For SQM under magnetic fields, the $\mathcal{O}(3)$ rotational symmetry is broken and the pressure for SQM will be split into two cases and become anisotropic. The first case is the longitudinal pressure $P_{||}$ case, which is parallel to the magnetic field orientation, and the second case for the anisotropic pressure is the transverse pressure $P_\perp$, which is perpendicular to the orientation of the magnetic field. The expressions for the longitudinal and transverse pressure for the system under strong magnetic fields take the form of [58]

$$p_{||} = p - \frac{1}{2}B^2, \tag{30}$$

$$p_\perp = p + \frac{1}{2}B^2 - \mathcal{M}B, \tag{31}$$

where we have defined

$$p = p_q + p_l - p_0 = -\Omega, \tag{32}$$

with $p_0 = -\Omega_0 = -\Omega(B = 0, \mu = 0)$ the vacuum pressure density, which ensures $p = 0$ in the vacuum. And $\mathcal{M}$ is the system magnetization with the form of

$$\mathcal{M} = -\partial\Omega/\partial B \tag{33}$$

The energy density for SQM at zero temperature under magnetic fields can be given by

$$\epsilon = -p + \sum_{i=u,d,s,l} \mu_i n_i + \frac{1}{2}B^2. \tag{34}$$

It is interesting to see that the longitudinal pressure $p_{||}$ meets the requirement of the Hugenholtz-Van Hove (HVH) theorem [31,74], while the transverse pressure $p_\perp$ has extra contributions terms from the magnetic fields.

### D. The spatial distribution and the orientation for magnetic fields inside QSs

It is generally believed that the magnetic field strength in the core of the magnetars could be much larger than the magnetic field strength at the surface, which suggests that a density-dependent magnetic field profile should be introduced to describe this behavior for the spatial distribution of the magnetic field strength in magnetars [62,63]. Besides the spatial distribution of the magnetic field strength in magnetars, the orientation of the magnetic fields is also very important for determining the structure of the compact stars because the large anisotropy for quark matter pressure density under strong magnetic fields. In the work [31], the authors assume two extremely special cases for the orientation inside quark stars: one is that the local magnetic fields are along the radial direction (denoted as "radial orientation"), and the other is that the magnetic fields are perpendicular to the radial direction but randomly oriented in the plane which is perpendicular to the radial direction (denoted as "transverse orientation"). For these two extreme cases for the orientation of the magnetic fields, one can calculate the structure of the static magnetized quark stars by using the following Tolman-Oppenheimer-Volkoff (TOV) equations [75]:

$$\frac{dM}{dr} = 4\pi r^2 \epsilon(r), \tag{35}$$

$$\frac{dp}{dr} = -\frac{G\epsilon(r)M(r)}{r^2}\left[1+\frac{p(r)}{\epsilon(r)}\right] \times \left[1+\frac{4\pi p(r) r^3}{M(r)}\right]\left[1-\frac{2GM(r)}{r}\right]^{-1}, \tag{36}$$

where $M(r)$ stands for the total mass inside the sphere of radius $r$, $\epsilon(r)$ is the corresponding energy density (which includes the magnetic field contribution), $p(r)$ is the corresponding (radial orientation) pressure (including the magnetic field contribution), and $G$ is Newton's gravitational constant.

## III. NUMERICAL RESULTS AND CONCLUSIONS

For our numerical calculations, following [51], the set of parameters we used is: $\Lambda = 631.4$ MeV, $m_u = m_d = 5.5$ MeV, $m_s = 135.7$ MeV, $G\Lambda^2 = 1.835$, and $K\Lambda^5 = 9.29$.

### A. Properties of SQM under strong magnetic fields

In this subsection, we study the properties of the quark matter within $\beta$−equilibrium condition (i.e., SQM) under strong magnetic fields by considering two types of vector interactions in SU(3) NJL model.

The value of the coupling constant for $G_V$ can be fixed in the vacuum by reproducing the spectrum of the vector mesons. However, it is not clear how large the modification of $G_V$ in the medium for SU(3) NJL model. Instead of fixing it by the meson spectrum in the vacuum, we treat this constant as a free parameter in order to grasp its effects on the properties of SQM at finite density under strong magnetic fields. We discuss the constituent quark masses for $u$, $d$, and $s$ quarks as functions of the magnetic field within four cases first: (1) $G_V = g_V = G_{IV} = 0$, (2) $G_V = 0.5G_S$, $g_V = G_{IV} = 0$, (3) $g_V = 0.5G_S$, $G_V = G_{IV} = 0$, and (4) $G_{IV} = 0.5G_S$, $G_V = g_V = 0$, when baryon density is fixed at $10n_0$ (where $n_0$ is the normal nuclear matter density, and the central baryon density in QSs with different usual phenomenological models is calculated roughly around $10n_0$). As Fig. 1 shown, the constituent quark masses of $u$, $d$, and $s$ quarks at $10n_0$ do not change much until the magnetic field increases to $B = 2 \times 10^{19} G$ for all the parameter sets, and it can be observed that several kinks appear in the mass curves when the magnetic field continues getting larger than $B = 2 \times 10^{19} G$ in all the four cases. This phenomenon is caused by the filling of different Landau levels under strong magnetic fields, which can be found in Eqs. (10), (22), and (25). We can also see that for all the four cases in this figure, the constituent quark mass for d quark can be larger than that for u quark once the magnetic field exceeds $B = 6 \times 10^{19} G$, and the oscillatory behavior for s quark mass is more obvious than that of u and d quarks around $B = 6 \times 10^{19} G$ in each case. For the flavor-dependent repulsion in case(2), one can find that the value of the constituent quark mass of s quark is less than the other three cases at the same magnetic field, which indicates the flavor-dependent repulsion can put a brake on the oscillatory behavior for s quark mass. In case (3), where we only consider the universal repulsion, it can be seen that the lines of the constituent quark mass for quarks are identical to those in case(1), which implies that the vector-isoscalar channel has no effects on the quark constituent mass in the SU(3) NJL model under strong magnetic fields.

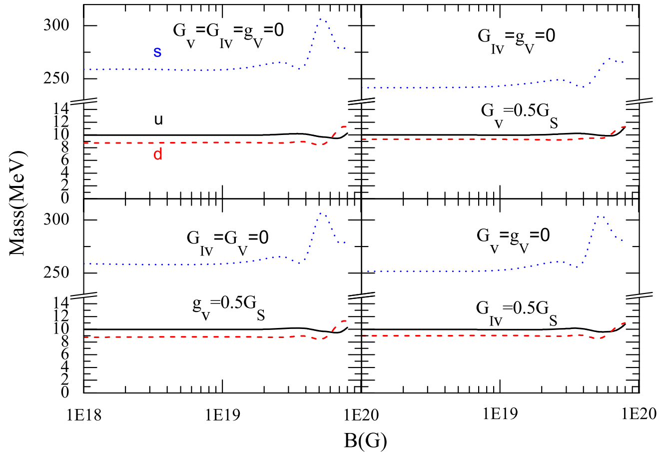


FIG. 1. Constituent quark mass for different flavors of quarks under strong magnetic fields within different vector interaction when baryon density is fixed.

In Fig. 2, we investigate the vacuum constituent mass for $u$, $d$, and $s$ quarks as functions of the magnetic field within the same parameter cases in Fig. 1. One can find in this figure that the vacuum quark mass for three flavors of quarks do not change much when magnetic field is less than $B = 1 \times 10^{19} G$. As the magnetic field reaches $B = 3 \times 10^{19} G$, the quark masses for $u$ and $d$ quarks begin to increase drastically, and the constituent quark mass for u quark even gets larger than the s quark mass when the magnetic field is larger than $B = 1.2 \times 10^{19} G$, which indicates an obvious magnetic catalysis phenomenon. After the calculation in this figure, we find that the vacuum quark mass is independent of the vector interaction for SU(3) NJL model under strong magnetic fields.

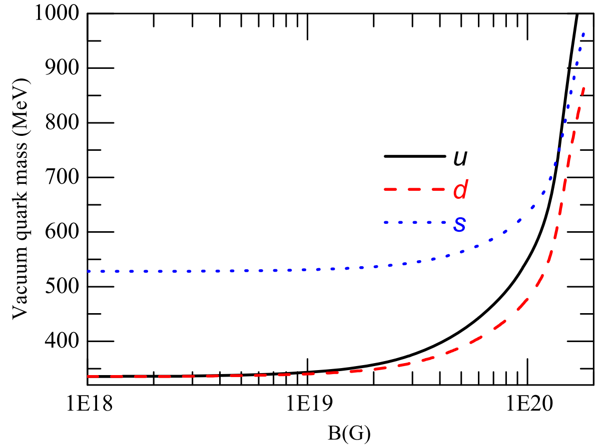


FIG. 2. Vacuum constituent quark masses for $u$, $d$, and $s$ quarks as functions of the magnetic field in SQM within different vector interactions.

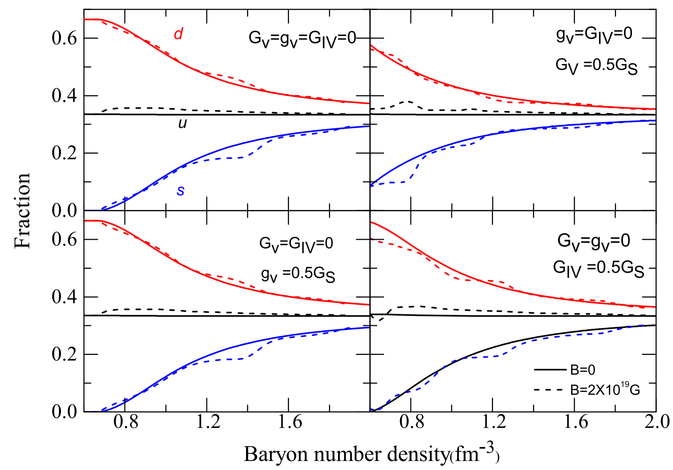


FIG. 4. Chemical potentials for $u$, $d$, $s$ quarks as functions of magnetic field with $G_V = 0$ and $G_V = 0.8G_S$ at baryon density $n_b = 10n_0$ in SQM.

Figure 3 shows the quark chemical potentials for SQM as functions of the magnetic field $B$ within four cases: (1) $G_V = g_V = G_{IV} = 0$, (2) $G_V = 0.5G_S$, $g_V = G_{IV} = 0$, (3) $g_V = 0.5G_S$, $G_V = G_{IV} = 0$, and (4) $G_{IV} = 0.5G_S$, $G_V = g_V = 0$, when baryon density is fixed at $10n_0$. In case (1), one can find that the value of the chemical potential for $u$ quark is less than $d$ and $s$ quarks, and the chemical potential for $d$ and $s$ quark is identical due to $\beta$-equilibrium. The chemical potential for quarks in the four cases all increase with magnetic field at first, then oscillate every time when a different number of Landau level is filled. It can be obviously seen that the chemical potential for each flavor in case (2) and case (3) is magnificently enhanced when comparing case (1), which implies that a stiffer EoS can be generated once the flavor-dependent repulsion or the universal repulsion is considered, and one can also find that the universal repulsion contributes more to enhance the chemical potential in SQM. Comparing case (1) and case (3), we can see that the shapes of the chemical potential lines from this two cases are identical, though the chemical potential in case (3) has a higher value. For case (4), the chemical potential is a little enhanced by considering the vector-isoscalar interaction, which indicates that the vector-isovector interaction may have not much effect on stiffening the EoS in SQM for SU(3) NJL model under strong magnetic fields.

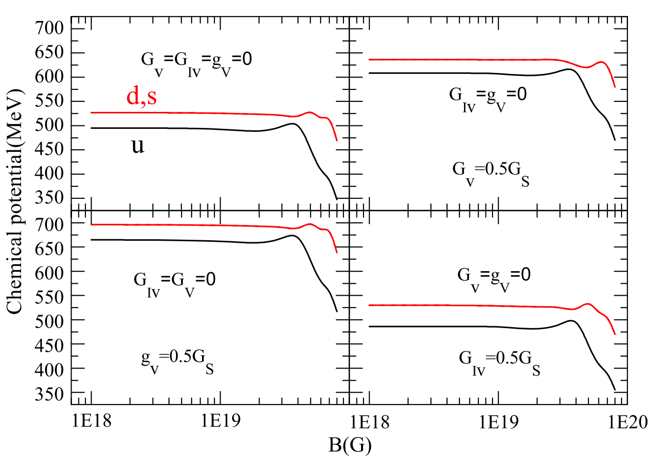


FIG. 3. Chemical potentials for $u$, $d$, $s$ quarks as functions of magnetic field with different vector interaction in NJL model under strong magnetic field.

In Fig. 4, we show the $u$, $d$, and $s$ quark fraction as functions of the baryon density in SQM at $B = 0$ and $B = 2 \times 10^{19}$ G with the four parameter sets we have discussed before. One can observe that in all four cases, the curves for fraction are no longer smooth owing to the filling of the Landau level, when strong magnetic field $B = 2 \times 10^{19}$ G is considered, while the values of the fraction do not vary too much. In case (2) and case (4), one can see that the difference among the fraction of $u$, $d$, and $s$ quark decreases, when the flavor-dependent repulsion or isovector interaction is considered. We can also find that the fraction for different flavors of quarks in case (3) is identical to that in case (1) under magnetic fields, which indicates that the universal repulsion among SQM still has no effects on quark fraction in SU(3) NJL model under strong magnetic fields, either.

### B. Quark stars in density-dependent magnetic fields

As we have mentioned earlier, the magnetic field inside magnetized compact stars is generally believed to be varied with the radius. The density-dependent magnetic field is usually introduced to mimic the magnetic field strength distribution inside the magnetars. Because it is hard to detect the magnetic field strength distribution inside the magnetars, we use the popular parametrization for the density-dependent magnetic field profile in magnetars as in Refs. [63,65–67].

$$B = B_{\rm surf} + B_0[1 - \exp(-\beta_0(n_b/n_0)^\gamma)], \tag{37}$$

where $B_{\rm surf}$ is the magnetic field strength at the surface of compact stars whose value is fixed at $B_{\rm surf} = 10^{15} G$ in this paper, $n_0 = 0.16\ {\rm fm}^{-3}$ is the normal nuclear matter density, $B_0$ is the constant magnetic field used as a parameter, and $\beta_0$ and $\gamma$ are two dimensionless parameters controlling how the magnetic field strength decays from the surface to the center. In order to reproduce the magnetic field that is weak below the nuclear saturation point while getting stronger at higher density, we set $B_0 = 4 \times 10^{18} G$, $\beta = 0.001$, and $\gamma = 3$ as the parameter set in the following calculations, and this magnetic field distribution has been already proved to be as a gentle magnetic field distribution for SQM inside QSs, which leads to a not big pressure anisotropy and small maximum mass splitting for QSs in confined-isospin-density-dependent-mass (CIDDM) model [31].

In order to describe the pressure anisotropy for quark matter under strong magnetic fields quantitatively, one can define the normalized pressure splitting factor as [31]

$$\delta_p = \frac{P_\perp - P_{||}}{(P_\perp + P_{||})/2}. \tag{38}$$

Using this definition, one has $\delta_p = 0$ if there is no pressure splitting between $P_\perp$ and $P_\|$, and $\delta_p = 2$ for the extremely anisotropic case with $P_\| = 0$.

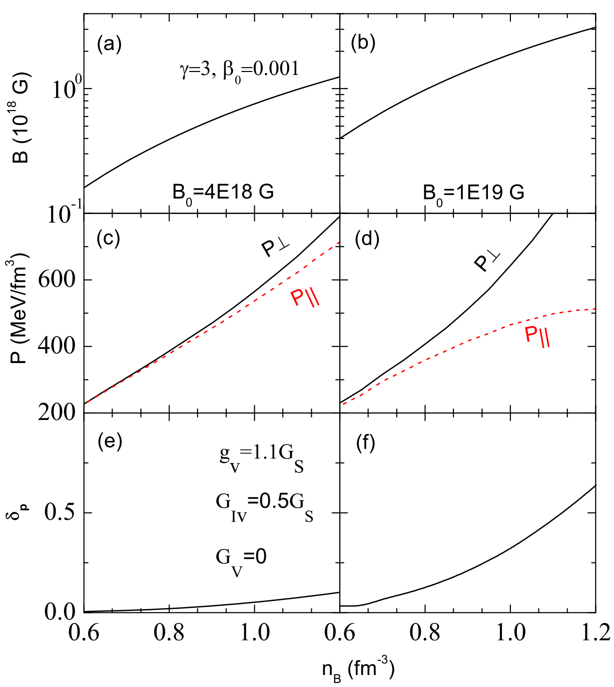


FIG. 5. Density dependence of the magnetic field strength $B$ and the transverse and longitudinal pressures, as well as the pressure anisotropy $\delta_p$ for SQM inside the magnetar by using the density-dependent magnetic field with $B_0 = 4 \times 10^{18}$ G (left panels) and $1 \times 10^{19}$ G (right panels) being considered.

Shown in Fig. 5 are the transverse and longitudinal pressures together with the pressure anisotropy $\delta_p$ as functions of the baryon density. The parameter set we used here is $g_V = 1.1 G_S$, $G_{IV} = 0.5 G_S$, using that which we can obtain the 2.01 $M_\odot$ when magnetic field is zero, which is consistent with the recently discovered large mass pulsar J0348 + 0432(2.01 ± 0.04) $M_\odot$. In the work [33], two solar mass compact stars can be obtained by using this set of parameters, and the symmetry energy which is given by this set of parameters is about twice than that of a free quark gas or normal quark matter within the conventional NJL model, which is consistent with the results given by the CIDDM model [30]. For the smaller value of $B_0(4 \times 10^{18}$ G), one can find that $P_\perp$ is larger than $P_{||}$ at high density, while the pressure anisotropy is very tiny when baryon density is less than 0.8 ${\rm fm}^{-3}$. For this smaller value of $B_0$, the pressure splitting between $P_\perp$ and $P_\|$ is not so big and we have $\delta_p = 0.11$ at 1.2 ${\rm fm}^{-3}$. On the other side, for $B_0 = 1 \times 10^{19}$ G, we can find that $P_\perp$ is much larger than $P_\|$ at higher densities, and while $P_\perp$ increases with baryon density, $P_\|$ begins to decrease with the increment of $n_B$ when $n_B \geq 1.1\ {\rm fm}^{-3}$, which leads to a big pressure splitting, i.e., $\delta_p = 0.65$. Therefore, the pressure could be strongly anisotropic in the core of magnetars for the larger $B_0$ (i.e., $B_0 = 1 \times 10^{19}$ G).

The large pressure anisotropy under strong magnetic fields implies that the orientation of the magnetic fields in magnetars will play an important role in the structure of magnetars [31]. Shown in Fig. 6 is the maximum mass of static QSs using the transverse and radial orientations of the magnetic fields as a function of $B_0$. The parameter set we used here is $g_V = 1.1 G_S$, $G_{IV} = 0.5 G_S$, using which we can obtain a 2.01 $M_\odot$ quark star in SU(3) NJL model, when the magnetic field is zero. We can find that the maximum mass of static QSs increases with the increment of $B_0$ for

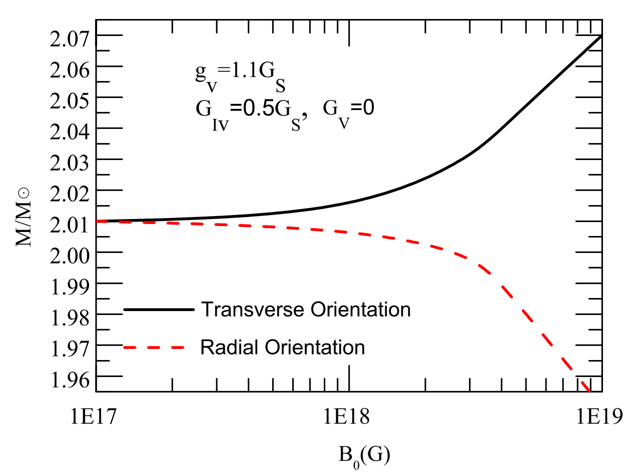


FIG. 6. Maximum mass of static QSs using the transverse and radial orientations of the magnetic fields as a function of $B_0$ within the SU(3) NJL model with $g_V = 1.1 G_S$, $G_{IV} = 0.5 G_S$.

the transverse orientation, while it decreases with $B_0$ for the radial orientation, especially when $B_0$ is larger than about $2 \times 10^{18}$ G. In order to describe the effect of the magnetic field orientation on the maximum mass of magnetars, we follow the normalized mass asymmetry $\delta_m$ for the maximum mass of QSs mass as

$$\delta_m = \frac{M_\perp - M_{||}}{(M_\perp + M_{||})/2}, \tag{39}$$

where $M_\perp$ ($M_{||}$) stands for the maximum mass of QSs by using the transverse (radial) orientation way. Then we acquire the result that the maximum mass of static QSs with the transverse (radial) orientation can reach about 2.07 $M_\odot$(1.95 $M_\odot$) at $B_0 = 1 \times 10^{19}$ G, and the corresponding largest mass asymmetry is $\delta_m = 5.97\%$ at $B_0 = 1 \times 10^{19}$ G. When the magnetic field is set as $B_0 = 4 \times 10^{18}$ G, which is used as a reasonable magnetic field in Ref. [31], the maximum mass of static QSs with the transverse (radial) orientation can reach about 2.04 $M_\odot$(1.99 $M_\odot$) with $\delta_m = 2.48\%$, which indicates that we can approximately use TOV equations to calculate the maximum mass for magnetars, because the pressure and maximum mass splitting is not big. Therefore, our results indicate that the maximum mass of magnetized QSs may be dependent on both the strength distribution and the orientation of the magnetic fields inside the stars by using SU(3) NJL model.

## IV. CONCLUSION AND DISCUSSION

In this work, we study the properties of SQM and QSs in the framework of SU(3) NJL model with two types of vector interactions under strong magnetic field. The constituent quark mass, vacuum quark mass, quark chemical potential, and quark fraction in SQM under strong magnetic fields are studied, and the results indicate that these physical quantities for SQM are all sensitive to the two types of vector interactions in NJL model, and begin to oscillate when the number of Landau level is filled with the increment of magnetic field.

We have studied the properties of static quark stars by using two hypothetical cases for the orientation of the magnetic fields: the radial orientation in which the local magnetic fields are along the radius and the transverse orientation in which the local magnetic fields are perpendicular to the radius but oriented randomly in the plane perpendicular to the radial direction. We first calculate the density dependence of the transverse and longitudinal pressures and the pressure anisotropy for SQM inside the magnetar by using the density-dependent magnetic field, and we find that the transverse pressure has a higher value than the longitudinal pressure at a certain baryon density. The pressure anisotropy gets bigger once larger $B_0$ is considered, which indicates the pressure could be strongly anisotropic in the core of magnetars for the larger $B_0$. Then we demonstrate that the maximum mass of magnetars may significantly depend on the magnetic field orientation inside the stars by using these two extreme cases, and the magnetic fields with radial (transverse) orientation can significantly decrease (increase) the maximum mass of the quark stars.

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