Flavor violating leptonic decays of the Higgs boson

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Recent data from the ATLAS and CMS detectors at the Large Hadron Collider at CERN give a hint of possible violation of flavor in the leptonic decays of the Higgs boson. In this work we analyze the flavor violating leptonic decays $H_1^0 \to l_i \bar{l}_j$ $(i \neq j)$ within the framework of a minimal supersymmetric standard model extension with a vectorlike leptonic generation. Specifically we focus on the decay mode $H_1^0 \to \mu\tau$. The analysis is done including tree and loop contributions involving exchange of W, Z, charged and neutral Higgs bosons and leptons and mirror leptons, charginos and neutralinos and sleptons and mirror sleptons. It is found that a substantial branching ratio of $H_1^0 \to \mu\tau$, i.e., of as much as $\mathcal{O}(1)\%$, can be achieved in this model, the size hinted by the ATLAS and CMS data. The flavor violating decays $H_1^0 \to e\mu$, $e\tau$ are also analyzed and found to be consistent with the current experimental limits. An analysis of the dependence of flavor violating decays on CP phases is given. The analysis is extended to include flavor decays of the heavier Higgs bosons. A confirmation of the flavor violation in Higgs boson decays with more data that is expected from LHC at $\sqrt{s} = 13$ TeV will be evidence of new physics beyond the standard model.

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I. INTRODUCTION

Recently the ATLAS [1] and the CMS [2] Collaborations at CERN have observed some possible hints of flavor violating decays of the Higgs boson H_1^0. Thus the ATLAS Collaboration finds [1]

$$\mathrm{BR}(H_1^0 \to \mu\tau) = \mathrm{BR}(H_1^0 \to \mu^+\tau^-) + \mathrm{BR}(H_1^0 \to \mu^-\tau^+) = (0.77 \pm 0.62)\% \tag{1}$$

while the CMS Collaboration finds [2]

$$\mathrm{BR}(H_1^0 \to \mu\tau) = \mathrm{BR}(H_1^0 \to \mu^+\tau^-) + \mathrm{BR}(H_1^0 \to \mu^-\tau^+) = (0.84^{+0.39}_{-0.37})\%. \tag{2}$$

For the $e\mu$ and $e\tau$ modes the experiments find a 95% C.L. bounds so that

$$\mathrm{BR}(H_1^0 \to e\mu) < 0.036\%, \quad \mathrm{BR}(H_1^0 \to e\tau) < 0.70\%. \tag{3}$$

More data are expected in the near future, which makes an investigation of the lepton flavor violation in Higgs decays a timely topic of investigation. Thus in the standard model there is no explanation of flavor violating leptonic decays of the Higgs boson and if they are confirmed that would be direct evidence for new physics beyond the standard model. In this work we explain the flavor violating leptonic decays of the Higgs boson in the framework of an extended minimal supersymmetric standard model (MSSM) with a vectorlike leptonic generation following the techniques discussed in [3–5]. Flavor changing Higgs decays are of significant theoretical interest and for some previous works see, e.g., [6–28].

In the analysis of this work the three leptonic generations mix with the vectorlike generation, which leads to flavor violation for the Higgs interactions. The analysis is carried out at the tree (see Fig. 1) and loop level where loop diagrams involving W, Z, leptons and mirror leptons (see Figs. 2 and 4), charginos, neutralinos, sleptons and mirror sleptons (see Figs. 3 and 5), and charged Higgs, neutral Higgs, sleptons and mirror sleptons (see Figs. 6 and 7) are taken account of. It is shown that flavor violating decays of the Higgs boson of the size hinted by the ATLAS and CMS data can be achieved consistent with the Higgs boson mass constraint. The dependence of the branching ratio of the

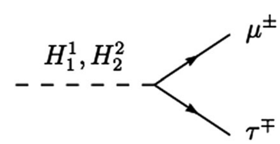

FIG. 1. Tree level contribution to the flavor violating $\mu^\pm\tau^\mp$ decay of the neutral Higgs bosons.

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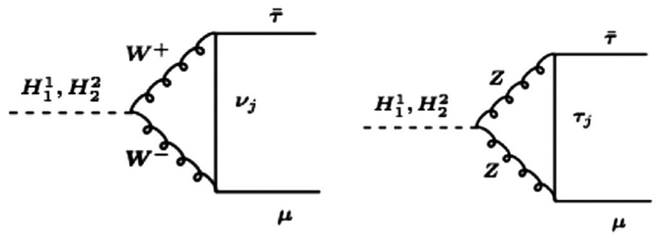

FIG. 2. Left panel: The W loop diagram involving the exchange of sequential and vectorlike neutrinos and mirror neutrinos. Right panel: The Z loop diagram involving the exchange of sequential and vectorlike leptons and mirror leptons.

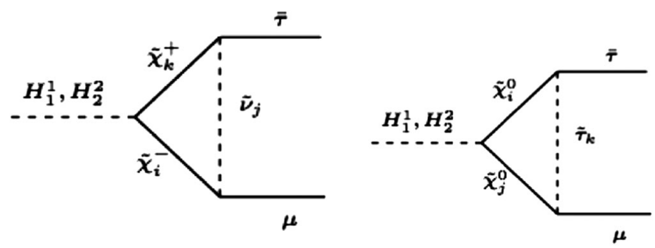

FIG. 3. Left panel: The chargino loop diagram involving the exchange of sequential and vectorlike sneutrinos and mirror sneutrinos. Right panel: The neutralino loop diagram involving the exchange of sequential and vectorlike sleptons and mirror sleptons.

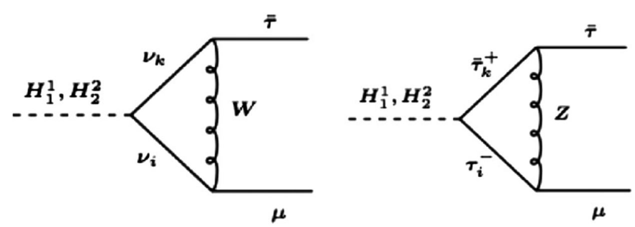

FIG. 4. Left panel: The W loop diagram involving the exchange of neutrinos and mirror neutrinos. Right panel: The Z loop diagram involving the exchange of charged leptons and charged mirror leptons.

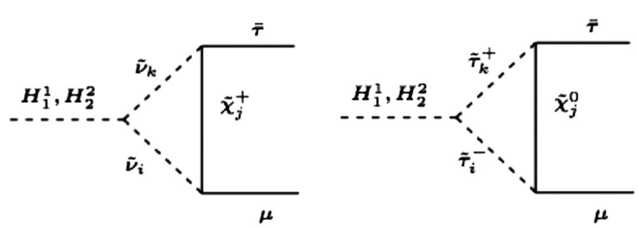

FIG. 5. Left panel: Chargino loop diagram involving the exchange of sneutrinos and mirror sneutrinos. Right panel: Neutralino loop diagram involving the exchange of sleptons and mirror sleptons.

flavor violating decay $\mu\tau$ as well as the dependence of the Higgs boson mass on CP phases is analyzed.

The outline of the rest of the paper is as follows. In Sec. II we give a description of the extended MSSM model. In

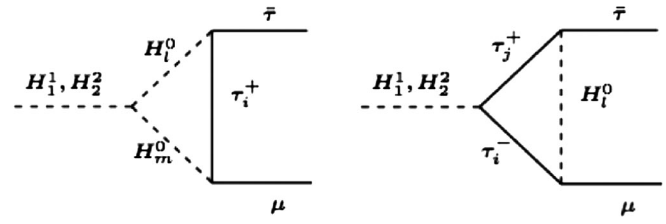

FIG. 6. Loop diagrams with neutral Higgs, charged leptons and mirror charged leptons.

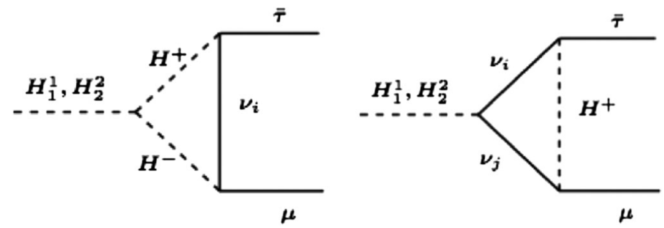

FIG. 7. Loops with charged Higgs, neutrinos and mirror neutrinos.

Sec. III an analytic analysis of the triangle loops Figs. 2–7 that contribute to the flavor changing processes is given. Numerical analysis is given in Sec. IV. Here we also study the dependence of the flavor violation on CP phases. Conclusions are given in Sec. V. Further details of the analysis are given in the appendix.

II. THE MODEL

As mentioned in Sec. I the model we use for the computation of the flavor violating leptonic decays of the Higgs boson is an extended MSSM which includes a vectorlike leptonic generation. As is well known, vectorlike multiplets appear in a variety of unified models including string and D brane models [29–32]. Many applications of these vectorlike multiplets exist in the literature [3–5,33–35]. In our analysis we include one vectorlike matter multiplet along with the three generations of matter. We begin by defining the notation for the matter content of the model and their properties under $SU(3)_C \times SU(2)_L \times U(1)_Y$. For the four sequential leptonic families we use the notation

$$\psi_{iL} \equiv \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix} \sim \left(1, 2, -\frac{1}{2}\right), \ell^c_{iL} \sim (1,1,1), \nu^c_{iL} \sim (1,1,0), \tag{4}$$

where the last entry on the right-hand side of each $\sim$ is the value of the hypercharge Y defined so that $Q = T_3 + Y$ and we have included in our analysis the singlet field ν^c_i, with i runs from 1–4. For the mirrors we use the notation

$$\chi^c \equiv \begin{pmatrix} E^c_{\mu L} \\ N^c_L \end{pmatrix} \sim \left(1, 2, \frac{1}{2}\right), E_{\mu L} \sim (1, 1, -1), N_L \sim (1, 1, 0). \tag{5}$$

The main difference between the leptons and the mirrors is that while the leptons have $V - A$ type interactions with $SU(2)_L \times U(1)_Y$ gauge bosons the mirrors have $V + A$ interactions. Further details of the model including the superpotential, Lagrangian, and mass matrices are given below.

As discussed above the analysis is based on the assumption that there is a vectorlike leptonic generation that lies at low scales. Including this vectorlike generation we discuss the superpotential, soft terms, mass matrices and particle and sparticle spectrum that enters in the analysis in this section. Thus the superpotential of the model for the lepton part is taken to be of the form

$$\begin{aligned} W = &-\mu\epsilon_{ij}\hat{H}^i_1\hat{H}^j_2 + \epsilon_{ij}[f_1\hat{H}^i_1\hat{\psi}^j_L\hat{\tau}^c_L + f'_1\hat{H}^j_2\hat{\psi}^i_L\hat{\nu}^c_{\tau L} + f_2\hat{H}^i_1\hat{\chi}^{cj}\hat{N}_L + f'_2\hat{H}^j_2\hat{\chi}^{ci}\hat{E}_L \\ &+ h_1\hat{H}^i_1\hat{\psi}^j_{\mu L}\hat{\mu}^c_L + h'_1\hat{H}^j_2\hat{\psi}^i_{\mu L}\hat{\nu}^c_{\mu L} + h_2\hat{H}^i_1\hat{\psi}^j_{eL}\hat{e}^c_L + h'_2\hat{H}^j_2\hat{\psi}^i_{eL}\hat{\nu}^c_{eL} + y_5\hat{H}^i_1\hat{\psi}^j_{4L}\hat{\ell}^c_{4L} + y'_5\hat{H}^j_2\hat{\psi}^i_{4L}\hat{\nu}^c_{4L}] \\ &+ f_3\epsilon_{ij}\hat{\chi}^{ci}\hat{\psi}^j_L + f'_3\epsilon_{ij}\hat{\chi}^{ci}\hat{\psi}^j_{\mu L} + f_4\hat{\tau}^c_L\hat{E}_L + f_5\hat{\nu}^c_{\tau L}\hat{N}_L + f'_4\hat{\mu}^c_L\hat{E}_L + f'_5\hat{\nu}^c_{\mu L}\hat{N}_L \\ &+ f''_3\epsilon_{ij}\hat{\chi}^{ci}\hat{\psi}^j_{eL} + f''_4\hat{e}^c_L\hat{E}_L + f''_5\hat{\nu}^c_{eL}\hat{N}_L + h_6\epsilon_{ij}\hat{\chi}^{ci}\hat{\psi}^j_{4L} + h_7\hat{\ell}^c_{4L}\hat{E}_L + h_8\hat{\nu}^c_{4L}\hat{N}_L, \end{aligned} \tag{6}$$

where $\hat{}$ implies superfields, $\hat{\psi}_L$ stands for $\hat{\psi}_{3L}$, $\hat{\psi}_{\mu L}$ stands for $\hat{\psi}_{2L}$ and $\hat{\psi}_{eL}$ stands for $\hat{\psi}_{1L}$. Mixings of the above type can arise via nonrenormalizable interactions. Consider, for example, a term such as $1/M_{\rm Pl}\nu^c_L N_L \Phi_1\Phi_2$. If Φ_1 and Φ_2 develop VEVs of size 10^{9-10}, a mixing term of the right size can be generated. We assume that the couplings in Eq. (6) are complex and we define their phases so that

$$\begin{aligned} f_k &= |f_k|e^{i\chi_k}, \quad f'_k = |f'_k|e^{i\chi'_k}, \quad f''_k = |f''_k|e^{i\chi''_k}, \\ h_i &= |h_i|e^{i\theta_{h_i}}, \quad h'_i = |h'_i|e^{i\theta'_{h_i}}, \quad h''_i = |h''_i|e^{i\theta_{h''_i}}, \end{aligned} \tag{7}$$

where k, i take on the appropriate values that appear in Eq. (6).

The mass terms for the neutrinos, mirror neutrinos, leptons and mirror leptons arise from the term

$$\mathcal{L} = -\frac{1}{2}\frac{\partial^2 W}{\partial A_i \partial A_j}\psi_i\psi_j + \text{H.c.} \tag{8}$$

where ψ and A stand for generic two-component fermion and scalar fields. After spontaneous breaking of the electroweak symmetry, $(\langle H^1_1\rangle = v_1/\sqrt{2}$ and $\langle H^2_2\rangle = v_2/\sqrt{2})$, we have the following set of mass terms written in the four-component spinor notation so that

$$-\mathcal{L}_m = \bar{\xi}^T_R(M_f)\xi_L + \bar{\eta}^T_R(M_\ell)\eta_L + \text{H.c.}, \tag{9}$$

where the basis vectors in which the mass matrix is written are given by

$$\begin{aligned} \bar{\xi}^T_R &= (\bar{\nu}_{\tau R} \quad \bar{N}_R \quad \bar{\nu}_{\mu R} \quad \bar{\nu}_{eR} \quad \bar{\nu}_{4R}), \\ \xi^T_L &= (\nu_{\tau L} \quad N_L \quad \nu_{\mu L} \quad \nu_{eL} \quad \nu_{4L}), \\ \bar{\eta}^T_R &= (\overline{\tau_R} \quad \bar{E}_R \quad \overline{\mu_R} \quad \overline{e_R} \quad \bar{\ell}_{4R}), \\ \eta^T_L &= (\tau_L \quad E_L \quad \mu_L \quad e_L \quad \ell_{4L}), \end{aligned} \tag{10}$$

and the mass matrix M_f of neutrinos is given by

$$M_f = \begin{pmatrix} f'_1 v_2/\sqrt{2} & f_5 & 0 & 0 & 0 \\ -f_3 & f_2 v_1/\sqrt{2} & -f'_3 & -f_3'' & -h_6 \\ 0 & f'_5 & h'_1 v_2/\sqrt{2} & 0 & 0 \\ 0 & f_5'' & 0 & h'_2 v_2/\sqrt{2} & 0 \\ 0 & h_8 & 0 & 0 & y'_5 v_2/\sqrt{2} \end{pmatrix}. \tag{11}$$

We define the matrix element (22) of the mass matrix as m_N so that

$$m_N = f_2 v_1/\sqrt{2}. \tag{12}$$

The mass matrix is not Hermitian and thus one needs biunitary transformations to diagonalize it. We define the biunitary transformation so that

$$D_R^{\nu\dagger}(M_f)D_L^{\nu} = \mathrm{diag}(m_{\psi_1}, m_{\psi_2}, m_{\psi_3}, m_{\psi_4}, m_{\psi_5}). \tag{13}$$

In $\psi_1, \psi_2, \psi_3, \psi_4, \psi_5$ are the mass eigenstates for the neutrinos, where in the limit of no mixing we identify ψ_1 as the light tau neutrino, ψ_2 as the heavier mass mirror eigenstate, ψ_3 as the muon neutrino, ψ_4 as the electron neutrino and ψ_5 as the other heavy four-sequential generation neutrino. A similar analysis goes to the lepton mass matrix M_ℓ where

$$M_\ell = \begin{pmatrix} f_1 v_1/\sqrt{2} & f_4 & 0 & 0 & 0 \\ f_3 & f_2' v_2/\sqrt{2} & f_3' & f_3'' & h_6 \\ 0 & f_4' & h_1 v_1/\sqrt{2} & 0 & 0 \\ 0 & f_4'' & 0 & h_2 v_1/\sqrt{2} & 0 \\ 0 & h_7 & 0 & 0 & y_5 v_1/\sqrt{2} \end{pmatrix}. \tag{14}$$

We introduce now the mass parameter m_E defined by the (22) element of the mass matrix above so that

$$m_E = f_2' v_2/\sqrt{2}. \tag{15}$$

The mass squared matrices of the slepton-mirror slepton and sneutrino-mirror sneutrino come from three sources: the F term, the D term of the potential and the soft supersymmetry (SUSY) breaking terms. After spontaneous breaking of the electroweak symmetry the Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_D + \mathcal{L}_{\rm soft}, \tag{16}$$

where $\mathcal{L}_F$ is deduced from $-\mathcal{L}_F = F_i F_i^*$, while the $\mathcal{L}_D$ is given by

$$\begin{aligned} -\mathcal{L}_D =& \frac{1}{2} m_Z^2 \cos^2\theta_W \cos 2\beta \{\tilde\nu_{\tau L}\tilde\nu_{\tau L}^* - \tilde\tau_L\tilde\tau_L^* + \tilde\nu_{\mu L}\tilde\nu_{\mu L}^* - \tilde\mu_L\tilde\mu_L^* + \tilde\nu_{eL}\tilde\nu_{eL}^* - \tilde e_L\tilde e_L^* + \tilde E_R\tilde E_R^* - \tilde N_R\tilde N_R^* + \tilde\nu_{4L}\tilde\nu_{4L}^* - \tilde\ell_{4L}\tilde\ell_{4L}^*\} \\ &+ \frac{1}{2} m_Z^2 \sin^2\theta_W \cos 2\beta \{\tilde\nu_{\tau L}\tilde\nu_{\tau L}^* + \tilde\tau_L\tilde\tau_L^* + \tilde\nu_{\mu L}\tilde\nu_{\mu L}^* + \tilde\mu_L\tilde\mu_L^* + \tilde\nu_{eL}\tilde\nu_{eL}^* + \tilde e_L\tilde e_L^* + \tilde\nu_{4L}\tilde\nu_{4L}^* + \tilde\ell_{4L}\tilde\ell_{4L}^* \\ &- \tilde E_R\tilde E_R^* - \tilde N_R\tilde N_R^* + 2\tilde E_L\tilde E_L^* - 2\tilde\tau_R\tilde\tau_R^* - 2\tilde\mu_R\tilde\mu_R^* - 2\tilde e_R\tilde e_R^* - 2\tilde\ell_{4R}\tilde\ell_{4R}^*\}. \end{aligned} \tag{17}$$

For $\mathcal{L}_{\rm soft}$ we assume the following form:

$$\begin{aligned} -\mathcal{L}_{\rm soft} =& \tilde M_{\tau L}^2 \tilde\psi_{\tau L}^{i*}\tilde\psi_{\tau L}^i + \tilde M_\chi^2 \tilde\chi^{ci*}\tilde\chi^{ci} + \tilde M_{\mu L}^2 \tilde\psi_{\mu L}^{i*}\tilde\psi_{\mu L}^i + \tilde M_{eL}^2 \tilde\psi_{eL}^{i*}\tilde\psi_{eL}^i + \tilde M_{\nu_\tau}^2 \tilde\nu_{\tau L}^{c*}\tilde\nu_{\tau L}^c + \tilde M_{\nu_\mu}^2 \tilde\nu_{\mu L}^{c*}\tilde\nu_{\mu L}^c + \tilde M_{4L}^2 \tilde\psi_{4L}^{i*}\tilde\psi_{4L}^i + \tilde M_{\nu_4}^2 \tilde\nu_{4L}^{c*}\tilde\nu_{4L}^c \\ &+ \tilde M_{\nu_e}^2 \tilde\nu_{eL}^{c*}\tilde\nu_{eL}^c + \tilde M_\tau^2 \tilde\tau_L^{c*}\tilde\tau_L^c + \tilde M_\mu^2 \tilde\mu_L^{c*}\tilde\mu_L^c + \tilde M_e^2 \tilde e_L^{c*}\tilde e_L^c + \tilde M_E^2 \tilde E_L^*\tilde E_L + \tilde M_N^2 \tilde N_L^*\tilde N_L + \tilde M_4^2 \tilde\ell_{4L}^{c*}\tilde\ell_{4L}^c \\ &+ \epsilon_{ij}\{f_1 A_\tau H_1^i \tilde\psi_{\tau L}^j \tilde\tau_L^c - f_1' A_{\nu_\tau} H_2^i \tilde\psi_{\tau L}^j \tilde\nu_{\tau L}^c + h_1 A_\mu H_1^i \tilde\psi_{\mu L}^j \tilde\mu_L^c - h_1' A_{\nu_\mu} H_2^i \tilde\psi_{\mu L}^j \tilde\nu_{\mu L}^c + h_2 A_e H_1^i \tilde\psi_{eL}^j \tilde e_L^c - h_2' A_{\nu_e} H_2^i \tilde\psi_{eL}^j \tilde\nu_{eL}^c \\ &+ f_2 A_N H_1^i \tilde\chi^{cj}\tilde N_L - f_2' A_E H_2^i \tilde\chi^{cj}\tilde E_L + y_5 A_{4\ell} H_1^i \tilde\psi_{4L}^j \tilde\ell_{4L}^c - y_5' A_{4\nu} H_2^i \tilde\psi_{4L}^j \tilde\nu_{4L}^c + \text{H.c.}\}. \end{aligned} \tag{18}$$

The trilinear couplings A_i are also complex and we define their phases so that

$$A_i = |A_i| e^{\theta_{A_i}}. \tag{19}$$

We define the scalar mass squared matrix $M_{\tilde\tau}^2$ in the basis

$$(\tilde\tau_L, \tilde E_L, \tilde\tau_R, \tilde E_R, \tilde\mu_L, \tilde\mu_R, \tilde e_L, \tilde e_R, \tilde\ell_{4L}, \tilde\ell_{4R}). \tag{20}$$

We label the matrix elements of these as $(M_{\tilde\tau}^2)_{ij} = M_{ij}^2$ where the elements of the matrix are given in [36]. We assume that all the masses are of the electroweak size so all the terms enter in the mass squared matrix. We diagonalize this Hermitian mass squared matrix by the unitary transformation

$$\begin{aligned} \tilde D^{\tau\dagger} M_{\tilde\tau}^2 \tilde D^\tau = \mathrm{diag}(&M_{\tilde\tau_1}^2, M_{\tilde\tau_2}^2, M_{\tilde\tau_3}^2, M_{\tilde\tau_4}^2, M_{\tilde\tau_5}^2, \\ &M_{\tilde\tau_6}^2, M_{\tilde\tau_7}^2, M_{\tilde\tau_8}^2 M_{\tilde\tau_9}^2, M_{\tilde\tau_{10}}^2). \end{aligned} \tag{21}$$

The mass^2 matrix in the sneutrino sector has a similar structure. We work in the basis

$$(\tilde{\nu}_{\tau L}, \tilde{N}_L, \tilde{\nu}_{\tau R}, \tilde{N}_R, \tilde{\nu}_{\mu L}, \tilde{\nu}_{\mu R}, \tilde{\nu}_{eL}, \tilde{\nu}_{eR}, \tilde{\nu}_{4L}, \tilde{\nu}_{4R}) \quad (22)$$

and write the sneutrino mass^2 matrix in the form $(M^2_{\tilde{\nu}})_{ij} = m^2_{ij}$ where the elements are given in [36]. As in the charged lepton sector we assume that all the masses are of the electroweak size so all the terms enter in the mass^2 matrix. This mass^2 matrix can be diagonalized by the unitary transformation

$$\tilde{D}^{\nu\dagger} M^2_{\tilde{\nu}} \tilde{D}^{\nu} = \mathrm{diag}(M^2_{\tilde{\nu}_1}, M^2_{\tilde{\nu}_2}, M^2_{\tilde{\nu}_3}, M^2_{\tilde{\nu}_4}, M^2_{\tilde{\nu}_5}, M^2_{\tilde{\nu}_6}, M^2_{\tilde{\nu}_7}, M^2_{\tilde{\nu}_8}, M^2_{\tilde{\nu}_9}, M^2_{\tilde{\nu}_{10}}). \quad (23)$$

III. ANALYSIS OF FLAVOR VIOLATING LEPTONIC DECAYS OF THE HIGGS BOSON

Flavor changing decays of this extended MSSM model arise at both the tree level due to lepton and mirror lepton mass mixing and at the loop level. There are several diagrams that contribute to the decays. These include the exchange of the charged W bosons and neutrinos and mirror neutrinos (see the left panel of Fig. 2), exchange of Z bosons and leptons and mirror leptons (see the right panel of Fig. 2), exchange of charginos, sneutrinos and mirror sneutrinos (see the left panel of Fig. 3) and the exchange of neutralinos, charged sleptons and mirror charged sleptons (see the right panel of Fig. 3). Additional diagrams which involve Higgs-neutrino-neutrino, Higgs-lepton-lepton, Higgs-sneutrino-sneutrino and Higgs-slepton-slepton vertices are given in Figs. 4 and 5. Other diagrams involving neutral and charged Higgs running in the loops are given in Figs. 6 and 7. So at the tree level, there is a coupling between the fields H^1_1, H^2_2, μ and τ due to mixing given by (see Appendix)

$$-\mathcal{L}_{\mathrm{eff}} = \bar{\mu}\chi_{31} P_L \tau H^1_1 + \bar{\mu}\eta_{31} P_L \tau H^2_2 + \bar{\tau}\chi_{13} P_L \mu H^1_1 + \bar{\tau}\eta_{13} P_L \mu H^2_2 + \mathrm{H.c.} \quad (24)$$

The loop corrections produce the effective Lagrangian

$$\mathcal{L}_{\mathrm{eff}} = \bar{\mu}\delta\xi_{\mu\tau} P_R \tau H^1_1 + \bar{\mu}\Delta\xi_{\mu\tau} P_L \tau H^1_1 + \bar{\mu}\delta\xi'_{\mu\tau} P_R \tau H^2_2 + \bar{\mu}\Delta\xi'_{\mu\tau} P_L \tau H^2_2 + \mathrm{H.c.} \quad (25)$$

This effective Lagrangian written in terms of the mass eigenstates of the neutral Higgs H^0_i with $i = 1, 2, 3$ reads

$$\mathcal{L}_{\mathrm{eff}} = \bar{\mu}(\{-\alpha^s_{31i} + \alpha^s_i\} + \gamma_5\{-\alpha^p_{31i} + \alpha^p_i\})\tau H^0_i + \bar{\tau}(\{-\alpha^s_{13i} + \alpha'^s_i\} + \gamma_5\{-\alpha^p_{13i} + \alpha'^p_i\})\mu H^0_i \quad (26)$$

where the couplings are given by

$$\alpha^s_{kji} = \frac{1}{2\sqrt{2}}(\chi_{kj}\{Y_{i1} + iY_{i3}\sin\beta\} + \eta_{kj}\{Y_{i2} + iY_{i3}\cos\beta\} + \chi^*_{jk}\{Y_{i1} - iY_{i3}\sin\beta\} + \eta^*_{jk}\{Y_{i2} - iY_{i3}\cos\beta\}),$$

$$\alpha^p_{kji} = \frac{1}{2\sqrt{2}}(-\chi_{kj}\{Y_{i1} + iY_{i3}\sin\beta\} - \eta_{kj}\{Y_{i2} + iY_{i3}\cos\beta\} + \chi^*_{jk}\{Y_{i1} - iY_{i3}\sin\beta\} + \eta^*_{jk}\{Y_{i2} - iY_{i3}\cos\beta\}),$$

$$\alpha^s_i = \frac{1}{2\sqrt{2}}(\{\delta\xi_{\mu\tau} + \Delta\xi_{\mu\tau}\}\{Y_{i1} + iY_{i3}\sin\beta\} + \{\delta\xi'_{\mu\tau} + \Delta\xi'_{\mu\tau}\}\{Y_{i2} + iY_{i3}\cos\beta\}),$$

$$\alpha^p_i = \frac{1}{2\sqrt{2}}(\{\delta\xi_{\mu\tau} - \Delta\xi_{\mu\tau}\}\{Y_{i1} + iY_{i3}\sin\beta\} + \{\delta\xi'_{\mu\tau} - \Delta\xi'_{\mu\tau}\}\{Y_{i2} + iY_{i3}\cos\beta\}),$$

$$\alpha'^s_i = \frac{1}{2\sqrt{2}}(\{\delta\xi^*_{\mu\tau} + \Delta\xi^*_{\mu\tau}\}\{Y_{i1} - iY_{i3}\sin\beta\} + \{\delta\xi'^*_{\mu\tau} + \Delta\xi'^*_{\mu\tau}\}\{Y_{i2} - iY_{i3}\cos\beta\}),$$

$$\alpha'^p_i = \frac{1}{2\sqrt{2}}(\{\Delta\xi^*_{\mu\tau} - \delta\xi^*_{\mu\tau}\}\{Y_{i1} - iY_{i3}\sin\beta\} + \{\Delta\xi'^*_{\mu\tau} - \delta\xi'^*_{\mu\tau}\}\{Y_{i2} - iY_{i3}\cos\beta\}), \quad (27)$$

where the matrix elements Y are defined by

$$Y M^2_{\mathrm{Higgs}} Y^T = \mathrm{diag}(m^2_{H^0_1}, m^2_{H^0_2}, m^2_{H^0_3}) \quad (28)$$

and χ_{ij} and η_{ij} are given in Eq. (A27). The decay of the neutral Higgs H^0_i into an antitau and a muon is given by

$$\Gamma_i(H_i^0 \to \bar{\tau}\mu) = \frac{1}{4\pi m^3_{H_i^0}}\sqrt{[(m_\tau^2+m_\mu^2-m^2_{H_i^0})^2-4m_\tau^2 m_\mu^2]}\left\{\frac{1}{2}(|-\alpha^s_{31i}+\alpha^s_i|^2+|-\alpha^p_{31i}+\alpha^p_i|^2)(m^2_{H_i^0}-m_\tau^2-m_\mu^2)\right.$$
$$\left.-\frac{1}{2}(|-\alpha^s_{31i}+\alpha^s_i|^2-|-\alpha^p_{31i}+\alpha^p_i|^2)(2m_\tau m_\mu)\right\},$$
$$\Gamma_i(H_i^0 \to \bar{\mu}\tau) = \frac{1}{4\pi m^3_{H_i^0}}\sqrt{[(m_\tau^2+m_\mu^2-m^2_{H_i^0})^2-4m_\tau^2 m_\mu^2]}\left\{\frac{1}{2}(|-\alpha^s_{13i}+\alpha'^s_i|^2+|-\alpha^p_{13i}+\alpha'^p_i|^2)(m^2_{H_i^0}-m_\tau^2-m_\mu^2)\right.$$
$$\left.-\frac{1}{2}(|-\alpha^s_{13i}+\alpha'^s_i|^2-|-\alpha^p_{13i}+\alpha'^p_i|^2)(2m_\tau m_\mu)\right\}. \quad (29)$$

We give a computation of each of the different loop contributions to $\delta\xi_{\mu\tau}$, $\Delta\xi_{\mu\tau}$, $\delta\xi'_{\mu\tau}$ and $\Delta\xi'_{\mu\tau}$ in the appendix.

IV. NUMERICAL ANALYSIS

As discussed in the introduction, the promising Higgs boson decays for the observation of flavor violation are $\mu\tau$, i.e., $\bar{\tau}\mu$, $\tau\bar{\mu}$. In the MSSM one has three neutral Higgs bosons H_1^0, H_2^0, H_3^0 with H_1^0 being the lightest which is the observed Higgs boson. As is well known in the presence of CP phases the CP even and CP odd Higgs bosons mix [37] (for a recent analysis see [38]). Thus the mass eigenstates in general have dependence on CP phases. We investigate the dependence of the flavor violating decays as well as of the Higgs boson mass on the CP phases in the analysis. We also note that one may allow large CP phases consistent with the current limits on electric dipole moment (EDM) constraints due the cancellation mechanism discussed in many works [39–41]. Thus the flavor violating branching ratios of H_1 into $\bar{\tau}\mu$, $\tau\bar{\mu}$ are given by

TABLE I. The light Higgs boson H_1 decay branching ratios into flavor violating decay modes $\tau\mu$, $e\mu$, $e\tau$. Column 2 gives the contribution at the tree level while column 3 gives the result with tree plus loop contributions. The results of the table are consistent with the experimental data of Eqs. (1)–(3). The mass for the Higgs boson is $m_{H_1^0}=125$ GeV. The parameters used are $\tan(\beta)=15$, $m_0=12\times10^3$, $m_0^{\tilde{\nu}}=12\times10^3$, $|\mu|=600$, $\theta\mu=2.5$, $|A_0|=8000$, $|A_0^{\tilde{\nu}}|=8000$, $\theta_{A_0}=2$, $\theta_{A_0^{\tilde{\nu}}}=3$, $|m_1|=320$, $|m_2|=400$, $\theta_{m_1}=1\times10^{-1}$, $\theta_{m_2}=0.2$, $m_A=300$, $m_0^u=1500$, $m_0^d=1500$, $|A_0^u|=5400$, $|A_0^d|=6000$, $\theta_{A_0^u}=0.3$, $\theta_{A_0^d}=0.6$, $|h_6|=2600$, $|h_7|=30$, $|h_8|=7500$, $|h_6^q|=6500$, $|h_7^q|=6500$, $|h_8^q|=6500$, $\theta_{h_6}=0.2$, $\theta_{h_7}=0.1$, $\theta_{h_8}=1$, $\theta_{h_6^q}=-3$, $\theta_{h_7^q}=-3$, $\theta_{h_8^q}=-3$, $|f_3|=20$, $|f_3'|=0.2$, $|f_3''|=0.003$, $|f_4|=0.8$, $|f_4'|=0.3$, $|f_4''|=0.1$, $|f_5|=0.004$, $|f_5'|=0.002$, $|f_5''|=0.002$, $|h_3|=15$, $|h_3'|=0.2$, $|h_3''|=0.003$, $|h_4|=60$, $|h_4'|=1.5$, $|h_4''|=0.1$, $|h_5|=60$, $|h_5'|=1.5$, $|h_5''|=0.1$, $\theta_{h_3}=1.05$, $\theta_{h_3'}=-4\times10^{-1}$, $\theta_{h_3''}=1.1$, $\theta_{h_4}=-1$, $\theta_{h_4'}=-0.9$, $\theta_{h_4''}=-2.4$, $\theta_{h_5}=-1$, $\theta_{h_5'}=-9\times10^{-1}$, $\theta_{h_5''}=-2.4$, $\chi_3=1.05$, $\chi_3'=-0.4$, $\chi_3''=1.1$, $\chi_4=-1$, $\chi_4'=0.3$, $\chi_4''=-1.4$, $\chi_5=1.5$, $\chi_5'=1.5$, $\chi_5''=1.5$. The mirror and the fourth sequential generation masses are $m_E=210$, $m_N=300$, $m_G=440$, and $m_{G_\nu}=100$ and the Yukawa couplings are $y_2=6.39$, $y_2'=0.432$, $y_5'=0.426$, and $y_5=5.7$. The parameters m_A, h_3, h_3', h_3'', h_4, h_4', h_4'', h_5, h_5', h_5'', y_2, y_2' are as defined in [38]. All masses are in GeV and angles are in rad.

Higgs decay	Treel level	Tree plus loop
$\mathrm{BR}(H_1^0\to\tau\mu)$	0.325	0.321
$\mathrm{BR}(H_1^0\to e\mu)$	3.386×10^{-6}	3.350×10^{-6}
$\mathrm{BR}(H_1^0\to e\tau)$	3.613×10^{-2}	3.572×10^{-2}

TABLE II. W and Z loop contributions to $\delta\xi_{\mu\tau}$, $\Delta\xi_{\mu\tau}$, $\delta\xi'_{\mu\tau}$, $\Delta\xi'_{\mu\tau}$ arising from the exchange diagrams of Figs. 2 and 4 for two points (i) and (ii) on two curves of Fig. 8. (i) is on the dashed green curve for $m_A=300$ at $\tan(\beta)=15$ which is the parameter point of Table I and (ii) is on the dashed blue curve for $m_A=400$ at $\tan(\beta)=20$. Changes in these loop contributions are solely due to the change in $\tan(\beta)$. The light Higgs mass eigenstate for (i) is $m_{H_0^1}=124.98$ and for (ii) is $m_{H_0^1}=124.96$ GeV.

Quantity	Loop contribution
(i) $\delta\xi_{\mu\tau}$	$6.82\times10^{-9}-3.02\times10^{-9}i$
(i) $\Delta\xi_{\mu\tau}$	$-7.41\times10^{-10}-2.63\times10^{-9}i$
(i) $\delta\xi'_{\mu\tau}$	$-3.39\times10^{-10}-1.30\times10^{-9}i$
(i) $\Delta\xi'_{\mu\tau}$	$-1.11\times10^{-8}-3.94\times10^{-8}i$
(ii) $\delta\xi_{\mu\tau}$	$9.09\times10^{-9}-3.99\times10^{-9}i$
(ii) $\Delta\xi_{\mu\tau}$	$-5.56\times10^{-10}-1.97\times10^{-9}i$
(ii) $\delta\xi'_{\mu\tau}$	$-3.39\times10^{-10}-1.30\times10^{-9}i$
(ii) $\Delta\xi'_{\mu\tau}$	$-1.11\times10^{-8}-3.95\times10^{-8}i$

TABLE III. SUSY, neutral Higgs boson, and charged Higgs loop corrections to $\delta\xi_{\mu\tau}$, $\Delta\xi_{\mu\tau}$, $\delta\xi'_{\mu\tau}$, $\Delta\xi'_{\mu\tau}$ arising from the exchange diagrams of Figs. 3 and 5–7 for the same two parameter points as discussed in Table II. Changes in the SUSY loop contributions are solely due to the change in $\tan(\beta)$ while changes in the neutral and charged Higgs loop corrections are due to both of the changes in $\tan(\beta)$ and m_A where m_A enters the theory through the Higgs mass matrix only.

Quantity	SUSY	Neutral and charged Higgs boson	Total
(i) $\delta\xi_{\mu\tau}$	$5.9\times10^{-8}-6.0\times10^{-8}i$	$7.0\times10^{-10}-7.4\times10^{-9}i$	$6.6\times10^{-8}-7.0\times10^{-8}i$
(i) $\Delta\xi_{\mu\tau}$	$4.9\times10^{-5}+6.7\times10^{-6}i$	$9.1\times10^{-5}-2.1\times10^{-4}i$	$1.4\times10^{-4}-2.0\times10^{-4}i$
(i) $\delta\xi'_{\mu\tau}$	$-7.9\times10^{-7}+8.8\times10^{-7}i$	$8.6\times10^{-9}-1.1\times10^{-8}i$	$-7.9\times10^{-7}+8.7\times10^{-7}i$
(i) $\Delta\xi'_{\mu\tau}$	$-2.4\times10^{-6}+2.6\times10^{-6}i$	$5.4\times10^{-6}+1.3\times10^{-5}i$	$2.9\times10^{-6}+1.6\times10^{-5}i$
(ii) $\delta\xi_{\mu\tau}$	$-1.1\times10^{-7}-2.0\times10^{-7}i$	$1.6\times10^{-10}-3.0\times10^{-9}i$	$-9.6\times10^{-8}-2.0\times10^{-7}i$
(ii) $\Delta\xi_{\mu\tau}$	$1.1\times10^{-4}+1.9\times10^{-5}i$	$3.6\times10^{-5}-1.5\times10^{-4}i$	$1.4\times10^{-4}-1.3\times10^{-4}i$
(ii) $\delta\xi'_{\mu\tau}$	$-1.5\times10^{-6}+2.9\times10^{-6}i$	$9.7\times10^{-9}+1.7\times10^{-9}i$	$-1.5\times10^{-6}+2.9\times10^{-6}i$
(ii) $\Delta\xi'_{\mu\tau}$	$-5.6\times10^{-6}+5.6\times10^{-6}i$	$5.8\times10^{-6}+2.7\times10^{-5}i$	$1.5\times10^{-7}+3.2\times10^{-5}i$

$$\mathrm{BR}(H_1^0\to\bar{\tau}\mu)=\frac{\Gamma(H_1^0\to\bar{\tau}\mu)}{\Gamma(H_1^0\to\bar{\mu}\tau)+\Gamma(H_1^0\to\bar{\tau}\mu)+\sum_i\Gamma(H_1^0\to\bar{f}_if_i)+\Gamma_{H_1DB}},$$
$$\mathrm{BR}(H_1^0\to\tau\bar{\mu})=\frac{\Gamma(H_1^0\to\bar{\mu}\tau)}{\Gamma(H_1^0\to\bar{\mu}\tau)+\Gamma(H_1^0\to\bar{\tau}\mu)+\sum_i\Gamma(H_1^0\to\bar{f}_if_i)+\Gamma_{H_1DB}},\tag{30}$$

where f_i stand for fermionic particles that have coupling with the Higgs boson and have a mass less than half the Higgs boson mass and Γ_{H_1DB} is the decay width into diboson states which include gg, $\gamma\gamma$, γZ, ZZ, WW. Thus the computation of the branching ratios of Eq. (30) involves the decay widths

$$\Gamma_i(H_i^0\to\bar{f}f)_{f=b,d,s}=\frac{3g^2m_f^2}{32\pi m_W^2\cos^2\beta}M_i\left\{|Y_{i1}|^2\left(1-\frac{4m_f^2}{M_i^2}\right)^{3/2}+|Y_{i3}|^2\sin^2\beta\left(1-\frac{4m_f^2}{M_i^2}\right)^{1/2}\right\},$$
$$\Gamma_i(H_i^0\to\bar{f}f)_{f=\tau,\mu,e}=\frac{g^2m_f^2}{32\pi m_W^2\cos^2\beta}M_i\left\{|Y_{i1}|^2\left(1-\frac{4m_f^2}{M_i^2}\right)^{3/2}+|Y_{i3}|^2\sin^2\beta\left(1-\frac{4m_f^2}{M_i^2}\right)^{1/2}\right\},$$
$$\Gamma_i(H_i^0\to\bar{f}f)_{f=u,c}=\frac{3g^2m_f^2}{32\pi m_W^2\sin^2\beta}M_i\left\{|Y_{i2}|^2\left(1-\frac{4m_f^2}{M_i^2}\right)^{3/2}+|Y_{i3}|^2\cos^2\beta\left(1-\frac{4m_f^2}{M_i^2}\right)^{1/2}\right\}.\tag{31}$$

The decays into ZZ and WW final states are off shell with the final states being dominantly four fermions. We note that $\tau\mu$ final states do not originate from any of the diboson decay modes of the Higgs boson. Further, at a mass of 125 GeV the Higgs boson is effectively in the decoupling limit. Thus we approximate the diboson decay widths as given by the standard model.

Although the flavor violating $\tau\mu$ mode of the Higgs boson in the model we consider arises already at the tree level as shown in Fig. 1 here we give an analysis of this decay by inclusion of both the tree as well as the loop contributions arising from the exchange diagrams of Figs. 2–7 which are computed in Sec. III. In this extended MSSM model one has one vectorlike generation of leptons which consists of a sequential fourth generation and a mirror generation. It is the mixing of the normal three generations with the vector generation that leads to the flavor violating decays. The flavor mixing arises via the mass matrices, the details of which can be found in Sec. II. To show that such mixings can indeed produce flavor violating Higgs decays of significant size, i.e., $\mathcal{O}(1)\%$, we give in Table I a numerical analysis for flavor violating decays for a specific point in the parameter space. In Table I m_A is the mass of the CP odd Higgs boson before loop corrections are taken into account. We use m_A as

TABLE IV. Tree level couplings of τ and μ with the neutral Higgs boson χ_{13}, η_{31}, χ_{31}, η_{13} for the same two parameter points as discussed in Table II. Some changes are smaller than the order given and thus small to appear here.

Quantity	Value
(i) χ_{13}	$-9.46\times10^{-5}-6.25\times10^{-4}i$
(i) η_{31}	$-3.68\times10^{-4}+1.27\times10^{-3}i$
(i) χ_{31}	$-5.53\times10^{-3}+1.91\times10^{-2}i$
(i) η_{13}	$-5.52\times10^{-6}-3.44\times10^{-5}i$
(ii) χ_{13}	$-1.26\times10^{-5}-8.33\times10^{-4}i$
(ii) η_{31}	$-3.68\times10^{-4}-1.27\times10^{-3}i$
(ii) χ_{31}	$-7.37\times10^{-3}+2.54\times10^{-2}i$
(ii) η_{13}	$-5.52\times10^{-6}-3.43\times10^{-5}i$

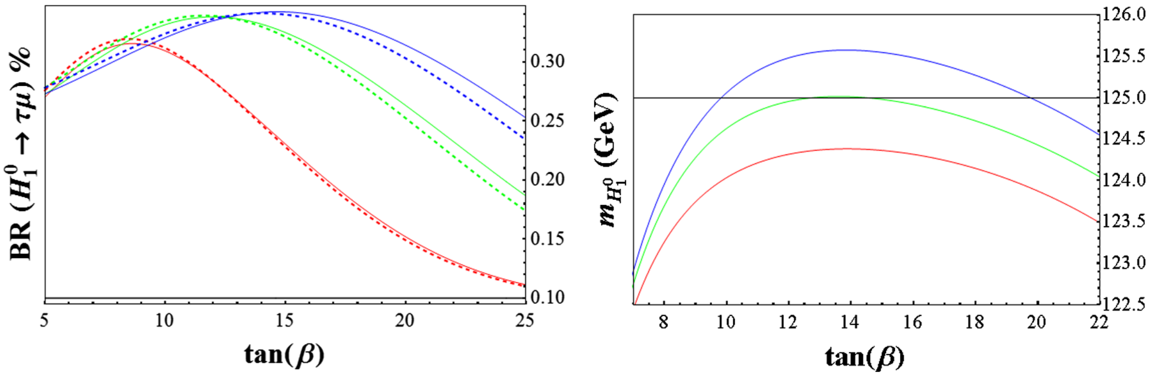

FIG. 8. Left panel: BR$(H_1^0 \to \mu\tau)$ versus $\tan\beta$ when $m_A = 200$ (red), 300 (green), and 400 (blue) where the solid lines are tree contributions and the dashed lines are tree plus loop contributions. Right panel: $m_{H_1^0}$ vs $\tan\beta$ when $m_A = 200$ (red), 300 (green), and 400 (blue) where $m_{H_1^0}$ includes tree and loop contribution.

a free parameter in the analysis. In the analysis of Table I we find that a branching ratio of ~0.33% is achieved which is consistent with the size hinted by the ATLAS and the CMS experiments [see Eqs. (1) and (2)]. In Table I we also give the relative contribution of the loop vs the tree as well as the branching ratios for the flavor violating decays $H_1 \to \mu e$ and $H_1 \to e\tau$. In Table II we give the relative loop contribution from W and Z, and from lepton and mirror lepton exchange, and in Table III we give the loop contribution arising from the MSSM sector, i.e., from the chargino and neutralino exchange, and from the charged Higgs and neutral Higgs, and from slepton and mirror slepton exchange. In Table IV we give the tree level couplings of the Higgs decay to τ and μ in comparison to the loop corrections to couplings given in Tables II and III as defined in Eqs. (24) and (25).

We discuss now further details of the analysis which includes both tree and loop contributions. In the left panel of Fig. 8 we exhibit the dependence of BR$(H_1^0 \to \tau\mu)$ on $\tan\beta$ and the branching ratio is seen to be sensitive to it. The

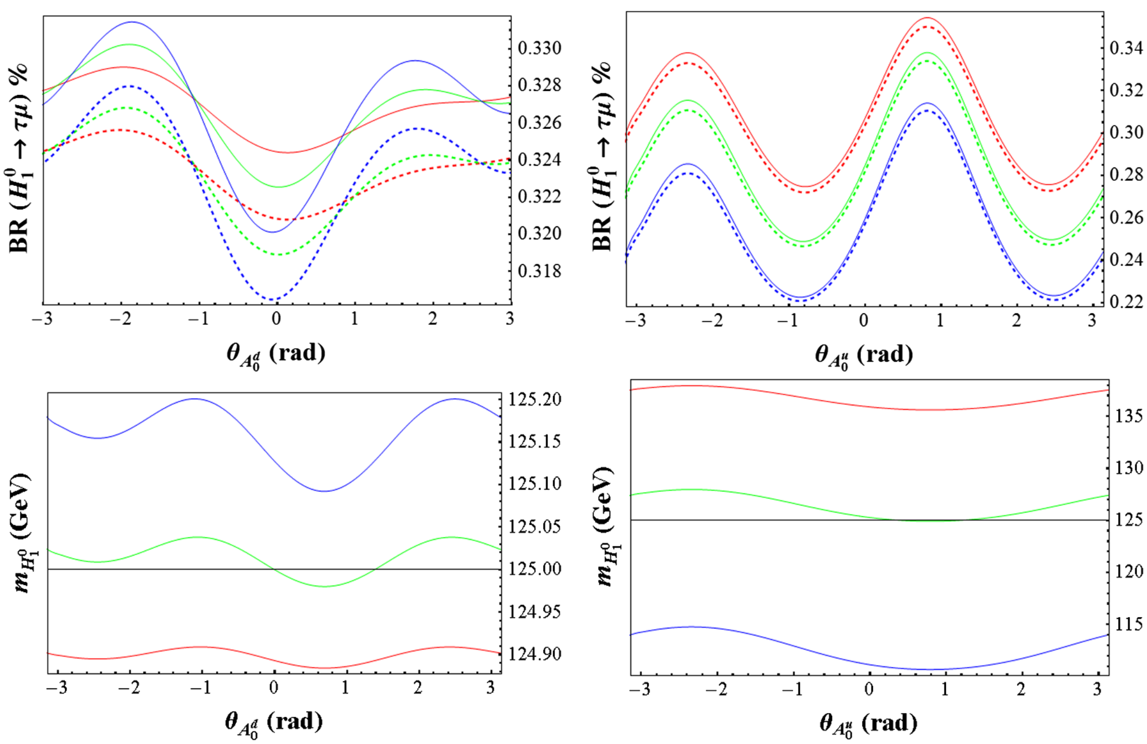

FIG. 9. Top panels: BR$(H_1^0 \to \mu\tau)$ as a function of $\theta_{A_0^d}$ (left panel) and as a function of $\theta_{A_0^u}$ (right panel) when $|A_0^d| = 4000$ (red), 6000 (green), and 8000 (blue) (left panel) and $|A_0^u| = 5200, 5400, 5600$ (right panel). The rest of the parameters are as in Table I. The solid curves are the tree while the dashed curves are the tree and the loop. Bottom panels: $m_{H_1^0}$ as a function of $\theta_{A_0^d}$ (left panel) and $\theta_{A_0^u}$ (right panel) corresponding to each of the curves of the top panels.

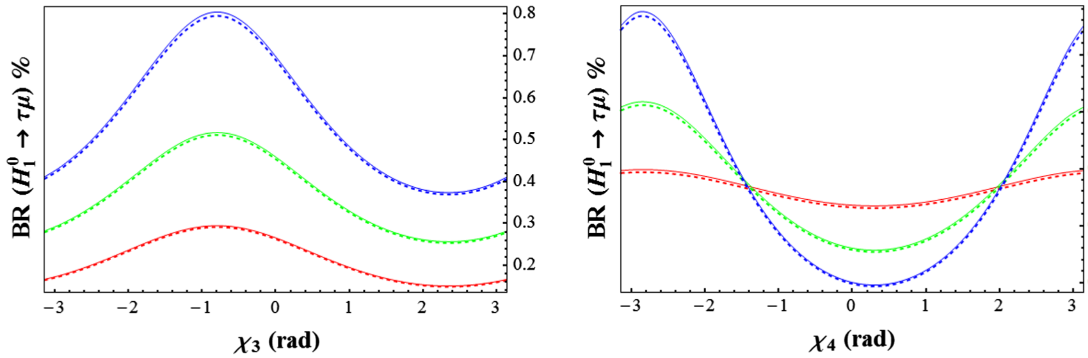

FIG. 10. Left panel: $\mathrm{BR}(H_1^0 \to \mu\tau)$ as a function of the CP phase χ_3 when $|f_3| = 15$ (red), 20 (green), and 25 (blue). Right panel: $\mathrm{BR}(H_1^0 \to \mu\tau)$ as a function of the CP phase χ_4 when $|f_4| = 0.2$ (red), 0.8 (green), and 1.4 (blue). The solid curves are tree level while the dashed curves include the loop contributions of Figs. 3–7.

sensitivity of the light Higgs boson mass on $\tan\beta$ is exhibited in the right panel of Fig. 8 and one finds that a shift in the Higgs boson mass in the range 1–2 GeV can arise from variations in $\tan\beta$. The rest of the analysis relates to the dependence of the flavor violating decays and of the Higgs boson mass on CP phases. Thus Fig. 9 exhibits the dependence of $\mathrm{BR}(H_1^0 \to \mu\tau)$ on $\theta_{A_0^d}$ (top left panel) and on $\theta_{A_0^u}$ (top right panel) and the dependence of the Higgs boson mass m_{H_1} on $\theta_{A_0^d}$ (bottom left panel) and on $\theta_{A_0^u}$ (bottom right panel). In Fig. 10 we exhibit the dependence of $\mathrm{BR}(H_1^0 \to \tau\mu)$ on χ_3 (left panel) and on χ_4 (right panel). Figure 11 exhibits the dependence of $\mathrm{BR}(H_1^0 \to \tau\mu)$ on θ_{h_8} which enters through the neutrino mass matrix and thus enters the loop contributions arising from the W and Z exchange diagrams of Figs. 2 and 4. Figure 12 exhibits the dependence of $\mathrm{BR}(H_1^0 \to \tau\mu)$ on $A_0 = A_0^{\tilde{\nu}}$ which enter through the slepton and sneutrino mass squared matrices which affect the loop corrections arising from the SUSY exchange diagrams of Figs. 3 and 5. Finally in Fig. 13 we exhibit the dependence of $\mathrm{BR}(H_1^0 \to \tau\mu)$ (left panel) and of m_{H_1} (right panel) on θ_μ. The dependence on θ_μ arises since it enters the chargino, neutralino, and slepton mass matrices and thus affects the loop corrections given by the exchange diagrams of Figs. 3 and 5 and the exchange diagrams of Figs. 6 and 7.

In summary one finds that a sizable branching ratio for the flavor violating Higgs decay $H_1^0 \to \mu\tau$ can arise in the extended MSSM model with a vectorlike generation. The branching ratios for the $e\mu$ and $e\tau$ decays are found to be much smaller. While the assumed model is a low energy model, it appears possible to embed it in a UV complete model. However, an analysis of it is outside the framework of this work.

Aside from $h \to \tau\mu$ there are other flavor violating decays such as $\tau \to \mu\gamma$ on which the *BABAR* Collaboration [42] and Bell Collaboration [43] have put significant limits on the branching ratio. The current experimental limit on the branching ratio of this process from the *BABAR* Collaboration [42] and the Belle Collaboration [43] is

$$\mathcal{B}(\tau \to \mu + \gamma) < 4.4 \times 10^{-8} \quad \text{at } 90\%\ \text{C.L.}(BABAR)$$
$$\mathcal{B}(\tau \to \mu + \gamma) < 4.5 \times 10^{-8} \quad \text{at } 90\%\ \text{C.L.}(\text{Belle}). \qquad (32)$$

Because of gauge invariance the decay of $\tau \to \mu\gamma$ can occur only at the loop level. In the MSSM flavor violation can be

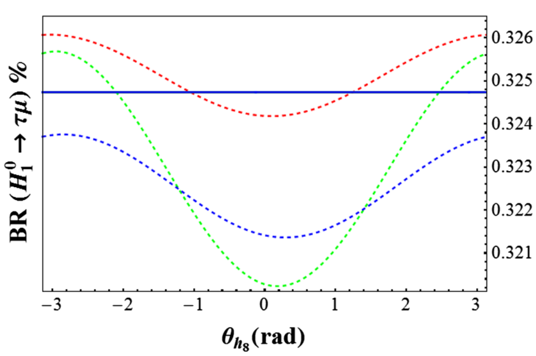

FIG. 11. $\mathrm{BR}(H_1^0 \to \mu\tau)$ vs θ_{h_8} (the phase of h_8) when $|h_8| = 750$ (red), 7500 (green), 75000 (blue) where the horizontal solid line gives the tree value and the dashed curves show tree and loop contributions.The rest of the parameters are common with Table I.

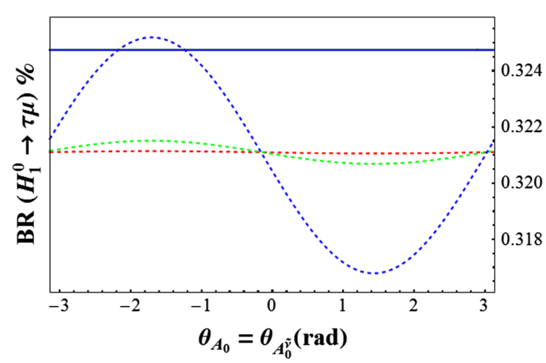

FIG. 12. $\mathrm{BR}(H_1^0 \to \mu\tau)$ vs $\theta_{A_0} = \theta_{A_0^{\tilde{\nu}}}$ when $|A_0| = |A_0^{\tilde{\nu}}| = 800$ (red), 8000 (green), and 80000 (red) where the horizontal solid line at the top gives the tree contributions and the dashed curves give the tree plus loop contributions. The rest of the parameters are common with Table I.

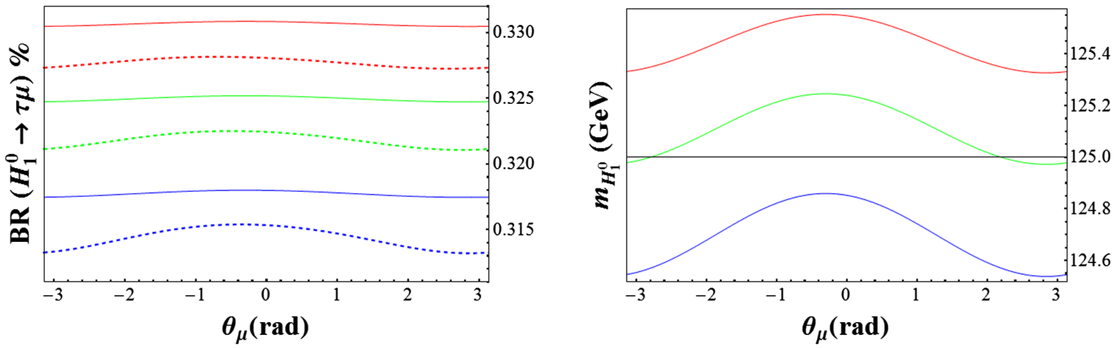

FIG. 13. Left panel: $\mathrm{BR}(H_1^0 \to \mu\tau)$ versus θ_μ when $|\mu| = 500$ (red), 600 (green), and 700 (blue) where the horizontal solid line at the top is the tree and the dashed lines are the tree plus loop. Right panel: m_{H_1} vs θ_μ for the same $|\mu|$ values as the left panel where the horizontal solid line gives the tree and the dashed curves give the tree plus loop. The rest of the parameters are common with Table I.

generated from the off-diagonal elements of the slepton mass squared matrix. The off-diagonal slepton mass squared matrix leads to flavor violating decays $h \to \tau\mu$ and $\tau \to \mu\gamma$. Since both $h \to \tau\mu$ and $\tau \to \mu\gamma$ occur only at the loop level and because $\tau \to \mu\gamma$ is severely constrained by Eq. (32), it is difficult to generate a sizable branching ratio for $h \to \tau\mu$ indicated by Eq. (3). The situation in the extended MSSM with a vector generation we consider here is very different. Here the flavor violating decay of the Higgs boson $H_1 \to \tau\mu$ already occurs at the tree level and the loop correction is a negligible correction while $\tau \to \mu\gamma$ occurs only at the loop level. Indeed two of the authors (T. I. and P. N.) analyzed the $\tau \to \mu\gamma$ decay in the extended MSSM with a vector generation in [44]. There this decay was found to have a significant model dependence because of the much larger parameter space of the extended MSSM relative to the MSSM case. However, as discussed above because of the fact that $H_1 \to \tau\mu$ already occurs at the tree level while $\tau \to \mu\gamma$ occurs only at the loop level and further because of the large parameter space of our model relative to the MSSM the $\tau \to \mu\gamma$ can be suppressed (see, for example, Fig. 3 of [44] where the $\tau \to \mu\gamma$ branching ratio varies over a wide range.) Further, the formalism given here allows one to compute the flavor violating decay $Z \to \mu^\pm\tau^\mp$. Interestingly unlike the process $\tau \to \mu\gamma$ which can occur only at the loop level because at the tree level this decay is forbidden, the decay $Z \to \mu^\pm\tau^\mp$ can occur at the tree level. Currently the experiment gives an upper limit on the branching ratio for this process of 1.2×10^{-5} [45]. We have checked that for the parameter space considered in this model the branching ratio for $Z \to \mu^\pm\tau^\mp$ lies lower than the experimental upper limit stated above. The analysis of the branching ratio $\tau \to 3\mu$ (which experimentally has an upper limit of 2.1×10^{-8} [45]) is more involved and requires a separate treatment. However, based on our previous analysis of $\tau \to \mu\gamma$ we expect that the branching ratio of this process will be consistent with experiment. In summary our analysis of $h \to \tau\mu$ presented here is robust.

V. CONCLUSIONS

Recent data from the ATLAS and CMS detectors at CERN hint at the possible violation of flavor in the leptonic decays of the Higgs boson. Such a violation can occur only in models beyond the standard model of electroweak interactions. In this work we investigate such violations in an extension of the MSSM with a vectorlike leptonic generation consisting of a fourth generation and a mirror generation. Within this framework we first give a general analysis of leptonic decays of $H_i^0 \to \ell_j\ell_k$ $(i, j, k = 1–3)$. The analysis is carried out including tree and loop contributions where the loop contributions include diagrams with exchanges of W, Z, charged and neutral Higgs bosons, and of charginos and neutralinos. It is shown that for the light Higgs boson H_1^0 the flavor violating decay branching ratio for $H_1^0 \to \mu\tau$ can be as much as $\mathcal{O}(1)\%$ which is the size hinted at by the ATLAS and CMS data. We analyze the $H_1^0 \to e\mu, e\tau$ modes and show that the branching ratios for these are consistent with the current data. Analysis of the dependence of the $\mu\tau$ branching ratio on CP phases is given and it is shown that the flavor violating decays are sensitively dependent on the phases. A small variation of the Higgs boson mass on CP phases is found and exhibited. The analysis is then extended to the flavor violating decays of the heavier Higgs bosons. The analysis is carried out including tree and loop contributions where the loop contributions include diagrams with exchanges of W, Z, charged and neutral Higgs bosons, and of charginos and neutralinos. A confirmation of flavor violating decays will provide direct evidence for new physics beyond the standard model. Such a possibility exists with more data that are expected from the LHC at $\sqrt{s} = 13$ TeV.

ACKNOWLEDGMENTS

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APPENDIX ANALYSIS OF LOOP CORRECTIONS TO FLAVOR VIOLATING LEPTONIC DECAYS OF THE HIGGS BOSON

In this appendix we give a computation of each of the different loop contributions to $\delta\xi_{\mu\tau}$, $\Delta\xi_{\mu\tau}$, $\delta\xi'_{\mu\tau}$ and $\Delta\xi'_{\mu\tau}$ discussed in Sec. III. We put the results in the same order as the Feynman diagrams of Figs. 2–7.

$$\begin{aligned}\delta\xi_{\mu\tau} = {} & -\frac{4gm_W\cos\beta}{\sqrt{2}}\sum_{i=1}^{5}C^W_{Li3}C^{W*}_{Ri1}\frac{m_{\nu_i}}{16\pi^2}f(m^2_{\nu_i},m^2_W,m^2_W) - \frac{\sqrt{2}gm_Z\cos\beta}{\cos\theta_W}\sum_{i=1}^{5}C^Z_{L3i}C^Z_{Ri1}\frac{m_{\tau_i}}{16\pi^2}f(m^2_{\tau_i},m^2_Z,m^2_Z)\\ & + g\sum_{i=1}^{2}\sum_{k=1}^{2}\sum_{j=1}^{10}U_{k2}V_{i1}C^R_{3ij}C^{L*}_{1kj}\frac{m_{\chi^-_k}m_{\chi^-_i}}{16\pi^2}f(m^2_{\tilde\nu_j},m^2_{\chi^-_k},m^2_{\chi^-_i}) + g\sum_{i=1}^{4}\sum_{j=1}^{4}\sum_{k=1}^{10}Q'_{ij}C'^R_{3jk}C'^{L*}_{1ik}\frac{m_{\chi^0_i}m_{\chi^0_j}}{16\pi^2}f(m^2_{\tilde\tau_k},m^2_{\chi^0_i},m^2_{\chi^0_j})\\ & - \sum_{i=1}^{5}\sum_{k=1}^{5}4\chi'_{ik}C^W_{Li3}C^{W*}_{Rk1}\frac{m_{\nu_i}m_{\nu_k}}{16\pi^2}f(m^2_W,m^2_{\nu_i},m^2_{\nu_k}) - \sum_{i=1}^{5}\sum_{k=1}^{5}4\chi_{ik}C^Z_{L3i}C^Z_{Rk1}\frac{m_{\tau_i}m_{\tau_k}}{16\pi^2}f(m^2_Z,m^2_{\tau_i},m^2_{\tau_k})\\ & + \sum_{i=1}^{10}\sum_{k=1}^{10}\sum_{j=1}^{2}G_{ik}C^R_{3ji}C^{L*}_{1jk}\frac{m_{\chi^-_j}}{16\pi^2}f(m^2_{\chi^-_j},m^2_{\tilde\nu_i},m^2_{\tilde\nu_k}) + \sum_{i=1}^{10}\sum_{k=1}^{10}\sum_{j=1}^{4}M_{ik}C'^R_{3ji}C'^{L*}_{1jk}\frac{m_{\chi^0_j}}{16\pi^2}f(m^2_{\chi^0_j},m^2_{\tilde\tau_i},m^2_{\tilde\tau_k})\\ & + \sum_{i=1}^{5}\sum_{\ell}^{3}\sum_{m=1}^{3}K_{\ell m}\psi^*_{1im}\psi^*_{i3\ell}\frac{m_{\tau_i}}{16\pi^2}f(m^2_{\tau_i},m^2_{H^0_\ell},m^2_{H^0_m}) + \frac{gm_W\cos\beta}{2\sqrt{2}}\sum_{i=1}^{5}\{1+2\sin^2\beta-\cos2\beta\tan^2\theta_W\}R^{H^+}_{i1}R^{H^-}_{3i}\\ & \times\frac{m_{\nu_i}}{16\pi^2}f(m^2_{\nu_i},m^2_{H^-},m^2_{H^-}),\end{aligned}\tag{A1}$$

$$\begin{aligned}\Delta\xi_{\mu\tau} = {} & -\frac{4gm_W\cos\beta}{\sqrt{2}}\sum_{i=1}^{5}C^W_{Ri3}C^{W*}_{Li1}\frac{m_{\nu_i}}{16\pi^2}f(m^2_{\nu_i},m^2_W,m^2_W) - \frac{\sqrt{2}gm_Z\cos\beta}{\cos\theta_W}\sum_{i=1}^{5}C^Z_{R3i}C^Z_{Li1}\frac{m_{\tau_i}}{16\pi^2}f(m^2_{\tau_i},m^2_Z,m^2_Z)\\ & + \sum_{i=1}^{10}\sum_{k=1}^{10}\sum_{j=1}^{2}G_{ik}C^L_{3ji}C^{R*}_{1jk}\frac{m_{\chi^-_j}}{16\pi^2}f(m^2_{\chi^-_j},m^2_{\tilde\nu_i},m^2_{\tilde\nu_k}) + \sum_{i=1}^{10}\sum_{k=1}^{10}\sum_{j=1}^{4}M_{ik}C'^L_{3ji}C'^{R*}_{1jk}\frac{m_{\chi^0_j}}{16\pi^2}f(m^2_{\chi^0_j},m^2_{\tilde\tau_i},m^2_{\tilde\tau_k})\\ & + \sum_{i=1}^{5}\sum_{\ell}^{3}\sum_{m=1}^{3}K_{\ell m}\psi_{i1m}\psi_{3i\ell}\frac{m_{\tau_i}}{16\pi^2}f(m^2_{\tau_i},m^2_{H^0_\ell},m^2_{H^0_m}) + \sum_{i=1}^{5}\sum_{j=1}^{5}\sum_{\ell=1}^{3}\chi_{ij}\psi_{j1\ell}\psi_{3i\ell}\frac{m_{\tau_i}m_{\tau_j}}{16\pi^2}f(m^2_{H^0_\ell},m^2_{\tau_i},m^2_{\tau_j})\\ & + \frac{gm_W\cos\beta}{2\sqrt{2}}\sum_{i=1}^{5}\{1+2\sin^2\beta-\cos2\beta\tan^2\theta_W\}L^{H^+}_{i1}L^{H^-}_{3i}\frac{m_{\nu_i}}{16\pi^2}f(m^2_{\nu_i},m^2_{H^-},m^2_{H^-})\\ & + \sum_{i}^{5}\sum_{j}^{5}\chi'_{ij}L^{H^-}_{3i}L^{H^+}_{j1}\frac{m_{\nu_i}m_{\nu_j}}{16\pi^2}f(m^2_{H^-},m^2_{\nu_i},m^2_{\nu_j}),\end{aligned}\tag{A2}$$

$$\begin{aligned}\delta\xi'_{\mu\tau} = {} & -\frac{4gm_W\sin\beta}{\sqrt{2}}\sum_{i=1}^{5}C^W_{Li3}C^{W*}_{Ri1}\frac{m_{\nu_i}}{16\pi^2}f(m^2_{\nu_i},m^2_W,m^2_W) - \frac{\sqrt{2}gm_Z\sin\beta}{\cos\theta_W}\sum_{i=1}^{5}C^Z_{L3i}C^Z_{Ri1}\frac{m_{\tau_i}}{16\pi^2}f(m^2_{\tau_i},m^2_Z,m^2_Z)\\ & + g\sum_{i=1}^{2}\sum_{k=1}^{2}\sum_{j=1}^{10}U_{k1}V_{i2}C^R_{3ij}C^{L*}_{1kj}\frac{m_{\chi^-_k}m_{\chi^-_i}}{16\pi^2}f(m^2_{\tilde\nu_j},m^2_{\chi^-_k},m^2_{\chi^-_i}) - g\sum_{i=1}^{4}\sum_{j=1}^{4}\sum_{k=1}^{10}S'_{ij}C'^R_{3jk}C'^{L*}_{1ik}\frac{m_{\chi^0_i}m_{\chi^0_j}}{16\pi^2}f(m^2_{\tilde\tau_k},m^2_{\chi^0_i},m^2_{\chi^0_j})\\ & - \sum_{i=1}^{5}\sum_{k=1}^{5}4\eta'_{ik}C^W_{Li3}C^{W*}_{Rk1}\frac{m_{\nu_i}m_{\nu_k}}{16\pi^2}f(m^2_W,m^2_{\nu_i},m^2_{\nu_k}) - \sum_{i=1}^{5}\sum_{k=1}^{5}4\eta_{ik}C^Z_{L3i}C^Z_{Rk1}\frac{m_{\tau_i}m_{\tau_k}}{16\pi^2}f(m^2_Z,m^2_{\tau_i},m^2_{\tau_k})\\ & + \sum_{i=1}^{10}\sum_{k=1}^{10}\sum_{j=1}^{2}H_{ik}C^R_{3ji}C^{L*}_{1jk}\frac{m_{\chi^-_j}}{16\pi^2}f(m^2_{\chi^-_j},m^2_{\tilde\nu_i},m^2_{\tilde\nu_k}) + \sum_{i=1}^{10}\sum_{k=1}^{10}\sum_{j=1}^{4}L_{ik}C'^R_{3ji}C'^{L*}_{1jk}\frac{m_{\chi^0_j}}{16\pi^2}f(m^2_{\chi^0_j},m^2_{\tilde\tau_i},m^2_{\tilde\tau_k})\\ & + \sum_{i=1}^{5}\sum_{\ell}^{3}\sum_{m=1}^{3}J_{\ell m}\psi^*_{1im}\psi^*_{i3\ell}\frac{m_{\tau_i}}{16\pi^2}f(m^2_{\tau_i},m^2_{H^0_\ell},m^2_{H^0_m}) + \frac{gm_W\sin\beta}{2\sqrt{2}}\sum_{i=1}^{5}\{1+2\cos^2\beta+\cos2\beta\tan^2\theta_W\}R^{H^+}_{i1}R^{H^-}_{3i}\\ & \times\frac{m_{\nu_i}}{16\pi^2}f(m^2_{\nu_i},m^2_{H^-},m^2_{H^-}),\end{aligned}\tag{A3}$$

$$\Delta\xi'_{\mu\tau} = -\frac{4gm_W\sin\beta}{\sqrt{2}}\sum_{i=1}^{5}C^W_{Ri3}C^{W*}_{Li1}\frac{m_{\nu_i}}{16\pi^2}f(m^2_{\nu_i},m^2_W,m^2_W) - \frac{\sqrt{2}gm_Z\sin\beta}{\cos\theta_W}\sum_{i=1}^{5}C^Z_{R3i}C^Z_{Li1}\frac{m_{\tau_i}}{16\pi^2}f(m^2_{\tau_i},m^2_Z,m^2_Z)$$
$$+\sum_{i=1}^{10}\sum_{k=1}^{10}\sum_{j=1}^{2}H_{ik}C^L_{3ji}C^{R*}_{1jk}\frac{m_{\chi^-_j}}{16\pi^2}f(m^2_{\chi^-_j},m^2_{\tilde\nu_i},m^2_{\tilde\nu_k}) + \sum_{i=1}^{10}\sum_{k=1}^{10}\sum_{j=1}^{4}L_{ik}C'^L_{3ji}C'^{R*}_{1jk}\frac{m_{\chi^0_j}}{16\pi^2}f(m^2_{\chi^0_j},m^2_{\tilde\tau_i},m^2_{\tilde\tau_k})$$
$$+\sum_{i=1}^{5}\sum_{\ell}^{3}\sum_{m=1}^{3}J_{\ell m}\psi_{i1m}\psi_{3i\ell}\frac{m_{\tau_i}}{16\pi^2}f(m^2_{\tau_i},m^2_{H^0_\ell},m^2_{H^0_m}) + \sum_{i=1}^{5}\sum_{j=1}^{5}\sum_{\ell=1}^{3}\eta_{ij}\psi_{j1\ell}\psi_{3i\ell}\frac{m_{\tau_i}m_{\tau_j}}{16\pi^2}f(m^2_{H^0_\ell},m^2_{\tau_i},m^2_{\tau_j})$$
$$+\frac{gm_W\sin\beta}{2\sqrt{2}}\sum_{i=1}^{5}\{1+2\cos^2\beta+\cos2\beta\tan^2\theta_W\}L^{H^+}_{i1}L^{H^-}_{3i}\frac{m_{\nu_i}}{16\pi^2}f(m^2_{\nu_i},m^2_{H^-},m^2_{H^-})$$
$$+\sum_{i}^{5}\sum_{j}^{5}\eta'_{ij}L^{H^-}_{3i}L^{H^+}_{j1}\frac{m_{\nu_i}m_{\nu_j}}{16\pi^2}f(m^2_{H^-},m^2_{\nu_i},m^2_{\nu_j}), \tag{A4}$$

where the form factors are given by

$$f(x,y,z) = \frac{1}{(x-y)(x-z)(z-y)}\times\left(zx\ln\frac{z}{x}+xy\ln\frac{x}{y}+yz\ln\frac{y}{z}\right) \qquad f(x,y,y) = \frac{1}{(y-x)^2}\times\left(x\ln\frac{y}{x}+x-y\right) \tag{A5}$$

and the couplings are given by

$$C^W_{L_{i\alpha}} = \frac{g}{\sqrt{2}}[D^{\nu*}_{L1i}D^\tau_{L1\alpha}+D^{\nu*}_{L3i}D^\tau_{L3\alpha}+D^{\nu*}_{L4i}D^\tau_{L4\alpha}+D^{\nu*}_{L5i}D^\tau_{L5\alpha}], \tag{A6}$$

$$C^W_{R_{i\alpha}} = \frac{g}{\sqrt{2}}[D^{\nu*}_{R2i}D^\tau_{R2\alpha}], \tag{A7}$$

$$C^Z_{L_{\alpha\beta}} = \frac{g}{\cos\theta_W}[x(D^{\tau\dagger}_{L\alpha1}D^\tau_{L1\beta}+D^{\tau\dagger}_{L\alpha2}D^\tau_{L2\beta}+D^{\tau\dagger}_{L\alpha3}D^\tau_{L3\beta}+D^{\tau\dagger}_{L\alpha4}D^\tau_{L4\beta}+D^{\tau\dagger}_{L\alpha5}D^\tau_{L5\beta})-\frac{1}{2}(D^{\tau\dagger}_{L\alpha1}D^\tau_{L1\beta}+D^{\tau\dagger}_{L\alpha3}D^\tau_{L3\beta}$$
$$+D^{\tau\dagger}_{L\alpha4}D^\tau_{L4\beta}+D^{\tau\dagger}_{L\alpha5}D^\tau_{L5\beta})], \tag{A8}$$

$$C^Z_{R_{\alpha\beta}} = \frac{g}{\cos\theta_W}[x(D^{\tau\dagger}_{R\alpha1}D^\tau_{R1\beta}+D^{\tau\dagger}_{R\alpha2}D^\tau_{R2\beta}+D^{\tau\dagger}_{R\alpha3}D^\tau_{R3\beta}+D^{\tau\dagger}_{R\alpha4}D^\tau_{R4\beta}+D^{\tau\dagger}_{R\alpha5}D^\tau_{R5\beta})-\frac{1}{2}(D^{\tau\dagger}_{R\alpha2}D^\tau_{R2\beta})], \tag{A9}$$

where $x=\sin^2\theta_W$. The couplings are given by

$$C^L_{\alpha ij} = g(-\kappa_\tau U^*_{i2}D^{\tau*}_{R1\alpha}\tilde D^\nu_{1j}-\kappa_\mu U^*_{i2}D^{\tau*}_{R3\alpha}\tilde D^\nu_{5j}-\kappa_e U^*_{i2}D^{\tau*}_{R4\alpha}\tilde D^\nu_{7j}-\kappa_{4\ell}U^*_{i2}D^{\tau*}_{R5\alpha}\tilde D^\nu_{9j}+U^*_{i1}D^{\tau*}_{R2\alpha}\tilde D^\nu_{4j}-\kappa_N U^*_{i2}D^{\tau*}_{R2\alpha}\tilde D^\nu_{2j}), \tag{A10}$$

$$C^R_{\alpha ij} = g(-\kappa_{\nu_\tau}V_{i2}D^{\tau*}_{L1\alpha}\tilde D^\nu_{3j}-\kappa_{\nu_\mu}V_{i2}D^{\tau*}_{L3\alpha}\tilde D^\nu_{6j}-\kappa_{\nu_e}V_{i2}D^{\tau*}_{L4\alpha}\tilde D^\nu_{8j}+V_{i1}D^{\tau*}_{L1\alpha}\tilde D^\nu_{1j}+V_{i1}D^{\tau*}_{L3\alpha}\tilde D^\nu_{5j}$$
$$-\kappa_{\nu_4}V_{i2}D^{\tau*}_{L5\alpha}\tilde D^\nu_{10j}+V_{i1}D^{\tau*}_{L4\alpha}\tilde D^\nu_{7j}-\kappa_E V_{i2}D^{\tau*}_{L2\alpha}\tilde D^\nu_{4j}), \tag{A11}$$

with

$$(\kappa_N,\kappa_\tau,\kappa_\mu,\kappa_e,\kappa_{4\ell}) = \frac{(m_N,m_\tau,m_\mu,m_e,m_{4\ell})}{\sqrt{2}m_W\cos\beta}, \tag{A12}$$

$$(\kappa_E,\kappa_{\nu_\tau},\kappa_{\nu_\mu},\kappa_{\nu_e},\kappa_{\nu_4}) = \frac{(m_E,m_{\nu_\tau},m_{\nu_\mu},m_{\nu_e},m_{\nu_4})}{\sqrt{2}m_W\sin\beta}, \tag{A13}$$

and

$$U^*M_CV^{-1} = \mathrm{diag}(m_{\tilde\chi^-_1},m_{\tilde\chi^-_2}). \tag{A14}$$

$$C'^{L}_{\alpha ij} = \sqrt{2}(\alpha_{\tau i} D^{\tau *}_{R1\alpha} \tilde{D}^{\tau}_{1j} - \delta_{Ei} D^{\tau *}_{R2\alpha} \tilde{D}^{\tau}_{2j} - \gamma_{\tau i} D^{\tau *}_{R1\alpha} \tilde{D}^{\tau}_{3j} + \beta_{Ei} D^{\tau *}_{R2\alpha} \tilde{D}^{\tau}_{4j} + \alpha_{\mu i} D^{\tau *}_{R3\alpha} \tilde{D}^{\tau}_{5j} - \gamma_{\mu i} D^{\tau *}_{R3\alpha} \tilde{D}^{\tau}_{6j} + \alpha_{ei} D^{\tau *}_{R4\alpha} \tilde{D}^{\tau}_{7j} - \gamma_{ei} D^{\tau *}_{R4\alpha} \tilde{D}^{\tau}_{8j} + \alpha_{4\ell i} D^{\tau *}_{R5\alpha} \tilde{D}^{\tau}_{9j} - \gamma_{4\ell i} D^{\tau *}_{R5\alpha} \tilde{D}^{\tau}_{10j}), \tag{A15}$$

$$C'^{R}_{\alpha ij} = \sqrt{2}(\beta_{\tau i} D^{\tau *}_{L1\alpha} \tilde{D}^{\tau}_{1j} - \gamma_{Ei} D^{\tau *}_{L2\alpha} \tilde{D}^{\tau}_{2j} - \delta_{\tau i} D^{\tau *}_{L1\alpha} \tilde{D}^{\tau}_{3j} + \alpha_{Ei} D^{\tau *}_{L2\alpha} \tilde{D}^{\tau}_{4j} + \beta_{\mu i} D^{\tau *}_{L3\alpha} \tilde{D}^{\tau}_{5j} - \delta_{\mu i} D^{\tau *}_{L3\alpha} \tilde{D}^{\tau}_{6j} + \beta_{ei} D^{\tau *}_{L4\alpha} \tilde{D}^{\tau}_{7j} - \delta_{ei} D^{\tau *}_{L4\alpha} \tilde{D}^{\tau}_{8j} + \beta_{4\ell i} D^{\tau *}_{L5\alpha} \tilde{D}^{\tau}_{9j} - \delta_{4\ell i} D^{\tau *}_{L5\alpha} \tilde{D}^{\tau}_{10j}), \tag{A16}$$

where

$$\alpha_{Ei} = \frac{g m_E X^*_{4i}}{2 m_W \sin\beta}; \qquad \beta_{Ei} = e X'_{1i} + \frac{g}{\cos\theta_W} X'_{2i} \left(\frac{1}{2} - \sin^2\theta_W\right) \tag{A17}$$

$$\gamma_{Ei} = e X'^*_{1i} - \frac{g \sin^2\theta_W}{\cos\theta_W} X'^*_{2i}; \qquad \delta_{Ei} = -\frac{g m_E X_{4i}}{2 m_W \sin\beta}, \tag{A18}$$

and

$$\alpha_{\tau i} = \frac{g m_\tau X_{3i}}{2 m_W \cos\beta}; \qquad \alpha_{\mu i} = \frac{g m_\mu X_{3i}}{2 m_W \cos\beta}; \qquad \alpha_{ei} = \frac{g m_e X_{3i}}{2 m_W \cos\beta}; \qquad \alpha_{4\ell i} = \frac{g m_{4\ell} X_{3i}}{2 m_W \cos\beta} \tag{A19}$$

$$\delta_{\tau i} = -\frac{g m_\tau X^*_{3i}}{2 m_W \cos\beta}; \qquad \delta_{\mu i} = -\frac{g m_\mu X^*_{3i}}{2 m_W \cos\beta}; \qquad \delta_{ei} = -\frac{g m_e X^*_{3i}}{2 m_W \cos\beta}; \qquad \delta_{4\ell i} = -\frac{g m_{4\ell} X^*_{3i}}{2 m_W \cos\beta}, \tag{A20}$$

and where

$$\beta_{\tau i} = \beta_{\mu i} = \beta_{ei} = \beta_{4\ell i} = -e X'^*_{1i} + \frac{g}{\cos\theta_W} X'^*_{2i} \left(-\frac{1}{2} + \sin^2\theta_W\right), \tag{A21}$$

$$\gamma_{\tau i} = \gamma_{\mu i} = \gamma_{ei} = \gamma_{4\ell i} = -e X'_{1i} + \frac{g \sin^2\theta_W}{\cos\theta_W} X'_{2i}. \tag{A22}$$

Here X' are defined by

$$X'_{1i} = X_{1i} \cos\theta_W + X_{2i} \sin\theta_W, \tag{A23}$$

$$X'_{2i} = -X_{1i} \sin\theta_W + X_{2i} \cos\theta_W, \tag{A24}$$

where X diagonalizes the neutralino mass matrix; i.e.,

$$X^T M_{\chi^0} X = \mathrm{diag}(m_{\chi^0_1}, m_{\chi^0_2}, m_{\chi^0_3}, m_{\chi^0_4}), \tag{A25}$$

$$Q'_{ij} = \frac{1}{\sqrt{2}} \{X^*_{3i}(X^*_{2j} - \tan\theta_W X^*_{1j})\}, \qquad S'_{ij} = \frac{1}{\sqrt{2}} \{X^*_{4j}(X^*_{2i} - \tan\theta_W X^*_{1i})\}, \tag{A26}$$

$$\begin{aligned}
\chi'_{ij} &= f_2 D^{\nu *}_{R2i} D^{\nu}_{L2j}, \\
\eta'_{ij} &= f'_1 D^{\nu *}_{R1i} D^{\nu}_{L1j} + h'_1 D^{\nu *}_{R3i} D^{\nu}_{L3j} + h'_2 D^{\nu *}_{R4i} D^{\nu}_{L4j} + y'_5 D^{\nu *}_{R5i} D^{\nu}_{L5j}, \\
\chi_{ij} &= f_1 D^{\tau *}_{R1i} D^{\tau}_{L1j} + h_1 D^{\tau *}_{R3i} D^{\tau}_{L3j} + h_2 D^{\tau *}_{R4i} D^{\tau}_{L4j} + y_5 D^{\tau *}_{R5i} D^{\tau}_{L5j}, \\
\eta_{ij} &= f'_2 D^{\tau *}_{R2i} D^{\tau}_{L2j},
\end{aligned} \tag{A27}$$

$$G_{ji} = -f_2 f_3^* \tilde{D}^{\nu*}_{ij} \tilde{D}^{\nu}_{2i} - f_2^* f_3' \tilde{D}^{\nu*}_{2j} \tilde{D}^{\nu}_{5i} - f_2^* f_3'' \tilde{D}^{\nu*}_{2j} \tilde{D}^{\nu}_{7i} + f_2^* f_5 \tilde{D}^{\nu*}_{3j} \tilde{D}^{\nu}_{4i} + f_2 f_5'^* \tilde{D}^{\nu*}_{4j} \tilde{D}^{\nu}_{6i} + f_2 f_5''^* \tilde{D}^{\nu*}_{4j} \tilde{D}^{\nu}_{8i} - f_2 h_6 \tilde{D}^{\nu*}_{2j} \tilde{D}^{\nu}_{9i}$$
$$+ f_2 h_8^* \tilde{D}^{\nu*}_{4j} \tilde{D}^{\nu}_{10i} + f_2^* A_N^* \tilde{D}^{\nu*}_{2j} \tilde{D}^{\nu}_{4i} - \mu f_1'^* \tilde{D}^{\nu*}_{ij} \tilde{D}^{\nu}_{3i} - \mu h_1'^* \tilde{D}^{\nu*}_{5j} \tilde{D}^{\nu}_{6i} - \mu h_2'^* \tilde{D}^{\nu*}_{7j} \tilde{D}^{\nu}_{8i} - \mu y_5' \tilde{D}^{\nu*}_{9j} \tilde{D}^{\nu}_{10i}$$
$$+ \frac{g m_Z \cos\beta}{4\cos\theta_W} \{ \tilde{D}^{\nu*}_{ij} \tilde{D}^{\nu}_{1i} + \tilde{D}^{\nu*}_{5j} \tilde{D}^{\nu}_{5i} + \tilde{D}^{\nu*}_{7j} \tilde{D}^{\nu}_{7i} + \tilde{D}^{\nu*}_{9j} \tilde{D}^{\nu}_{9i} - \tilde{D}^{\nu*}_{4j} \tilde{D}^{\nu}_{4i} \}, \tag{A28}$$

$$H_{ji} = f_5 f_1'^* \tilde{D}^{\nu*}_{ij} \tilde{D}^{\nu}_{2i} + h_1' f_5'^* \tilde{D}^{\nu*}_{2j} \tilde{D}^{\nu}_{5i} + h_2' f_5''^* \tilde{D}^{\nu*}_{2j} \tilde{D}^{\nu}_{7i} - f_1' f_3^* \tilde{D}^{\nu*}_{3j} \tilde{D}^{\nu}_{4i} - h_1'^* f_3' \tilde{D}^{\nu*}_{4j} \tilde{D}^{\nu}_{6i} - h_2'^* f_3'' \tilde{D}^{\nu*}_{4j} \tilde{D}^{\nu}_{8i} + h_8 y_5^* \tilde{D}^{\nu*}_{2j} \tilde{D}^{\nu}_{9i}$$
$$- y_5' h_6 \tilde{D}^{\nu*}_{4j} \tilde{D}^{\nu}_{10i} - \mu f_2^* \tilde{D}^{\nu*}_{2j} \tilde{D}^{\nu}_{4i} + f_1'^* A^*_{\nu_\tau} \tilde{D}^{\nu*}_{ij} \tilde{D}^{\nu}_{3i} + h_1'^* A^*_{\nu_\mu} \tilde{D}^{\nu*}_{5j} \tilde{D}^{\nu}_{6i} + h_2'^* A^*_{\nu_e} \tilde{D}^{\nu*}_{7j} \tilde{D}^{\nu}_{8i} + y_5' A^*_{4\nu} \tilde{D}^{\nu*}_{9j} \tilde{D}^{\nu}_{10i}$$
$$- \frac{g m_Z \sin\beta}{4\cos\theta_W} \{ \tilde{D}^{\nu*}_{ij} \tilde{D}^{\nu}_{1i} + \tilde{D}^{\nu*}_{5j} \tilde{D}^{\nu}_{5i} + \tilde{D}^{\nu*}_{7j} \tilde{D}^{\nu}_{7i} + \tilde{D}^{\nu*}_{9j} \tilde{D}^{\nu}_{9i} - \tilde{D}^{\nu*}_{4j} \tilde{D}^{\nu}_{4i} \}, \tag{A29}$$

$$M_{ji} = f_4 f_1^* \tilde{D}^{\tau*}_{ij} \tilde{D}^{\tau}_{2i} + h_1 f_4^* \tilde{D}^{\tau*}_{2j} \tilde{D}^{\tau}_{5i} + h_1 f_4'^* \tilde{D}^{\tau*}_{2j} \tilde{D}^{\tau}_{7i} + f_1 f_3^* \tilde{D}^{\tau*}_{3j} \tilde{D}^{\tau}_{4i} + f_3' h_1^* \tilde{D}^{\tau*}_{4j} \tilde{D}^{\tau}_{6i} + f_3'' h_2^* \tilde{D}^{\tau*}_{4j} \tilde{D}^{\tau}_{8i} + y_5 h_7^* \tilde{D}^{\tau*}_{2j} \tilde{D}^{\tau}_{9i}$$
$$+ h_6 y_5^* \tilde{D}^{\tau*}_{4j} \tilde{D}^{\tau}_{10i} - \mu f_2'^* \tilde{D}^{\tau*}_{2j} \tilde{D}^{\tau}_{4i} + f_1^* A_\tau^* \tilde{D}^{\tau*}_{ij} \tilde{D}^{\tau}_{3i} + h_1^* A_\mu^* \tilde{D}^{\tau*}_{5j} \tilde{D}^{\tau}_{6i} + h_2^* A_e^* \tilde{D}^{\tau*}_{7j} \tilde{D}^{\tau}_{8i} + y_5^* A_{4\ell}^* \tilde{D}^{\tau*}_{9j} \tilde{D}^{\tau}_{10i}$$
$$+ \frac{g m_Z \cos\beta}{4\cos\theta_W} \{ \cos 2\theta_W (\tilde{D}^{\tau*}_{4j} \tilde{D}^{\tau}_{4i} - \tilde{D}^{\tau*}_{1j} \tilde{D}^{\tau}_{1i} - \tilde{D}^{\tau*}_{5j} \tilde{D}^{\tau}_{5i} - \tilde{D}^{\tau*}_{7j} \tilde{D}^{\tau}_{7i} - \tilde{D}^{\tau*}_{9j} \tilde{D}^{\tau}_{9i}) + 2\sin^2\theta_W (\tilde{D}^{\tau*}_{2j} \tilde{D}^{\tau}_{2i}$$
$$- \tilde{D}^{\tau*}_{3j} \tilde{D}^{\tau}_{3i} - \tilde{D}^{\tau*}_{6j} \tilde{D}^{\tau}_{6i} - \tilde{D}^{\tau*}_{8j} \tilde{D}^{\tau}_{8i} - \tilde{D}^{\tau*}_{10j} \tilde{D}^{\tau}_{10i}) \}, \tag{A30}$$

$$L_{ji} = f_2' f_3^* \tilde{D}^{\tau*}_{ij} \tilde{D}^{\tau}_{2i} + f_3' f_2'^* \tilde{D}^{\tau*}_{2j} \tilde{D}^{\tau}_{5i} + f_3'' f_2'^* \tilde{D}^{\tau*}_{2j} \tilde{D}^{\tau}_{7i} + f_4 f_2'^* \tilde{D}^{\tau*}_{3j} \tilde{D}^{\tau}_{4i} + f_2' f_4'^* \tilde{D}^{\tau*}_{4j} \tilde{D}^{\tau}_{6i} + f_2' f_4''^* \tilde{D}^{\tau*}_{4j} \tilde{D}^{\tau}_{8i} + h_6 f_2'^* \tilde{D}^{\tau*}_{2j} \tilde{D}^{\tau}_{9i}$$
$$+ f_2' h_7^* \tilde{D}^{\tau*}_{4j} \tilde{D}^{\tau}_{10i} + f_2'^* A_E^* \tilde{D}^{\tau*}_{2j} \tilde{D}^{\tau}_{4i} - \mu f_1^* \tilde{D}^{\tau*}_{ij} \tilde{D}^{\tau}_{3i} - \mu h_1^* \tilde{D}^{\tau*}_{5j} \tilde{D}^{\tau}_{6i} - \mu h_2^* \tilde{D}^{\tau*}_{7j} \tilde{D}^{\tau}_{8i} - \mu y_5^* \tilde{D}^{\tau*}_{9j} \tilde{D}^{\tau}_{10i}$$
$$- \frac{g m_Z \sin\beta}{4\cos\theta_W} \{ \cos 2\theta_W (\tilde{D}^{\tau*}_{4j} \tilde{D}^{\tau}_{4i} - \tilde{D}^{\tau*}_{1j} \tilde{D}^{\tau}_{1i} - \tilde{D}^{\tau*}_{5j} \tilde{D}^{\tau}_{5i} - \tilde{D}^{\tau*}_{7j} \tilde{D}^{\tau}_{7i} - \tilde{D}^{\tau*}_{9j} \tilde{D}^{\tau}_{9i}) + 2\sin^2\theta_W (\tilde{D}^{\tau*}_{2j} \tilde{D}^{\tau}_{2i}$$
$$- \tilde{D}^{\tau*}_{3j} \tilde{D}^{\tau}_{3i} - \tilde{D}^{\tau*}_{6j} \tilde{D}^{\tau}_{6i} - \tilde{D}^{\tau*}_{8j} \tilde{D}^{\tau}_{8i} - \tilde{D}^{\tau*}_{10j} \tilde{D}^{\tau}_{10i}) \}, \tag{A31}$$

$$K_{\ell m} = \frac{g m_Z \cos\beta}{8\sqrt{2}\cos\theta_W} \{ (Y_{m1} - iY_{m3}\sin\beta)(3Y_{\ell 1} + iY_{\ell 3}\sin\beta) - 2(Y_{m2} - iY_{m3}\cos\beta)(Y_{\ell 2} + iY_{\ell 3}\cos\beta)$$
$$- 4Y_{m2}(Y_{\ell 1} - iY_{\ell 3}\sin\beta)\tan\beta \},$$
$$J_{\ell m} = \frac{g m_Z \cos\beta}{8\sqrt{2}\cos\theta_W} \{ \tan\beta (Y_{\ell 2} - iY_{\ell 3}\cos\beta)(3Y_{m2} + iY_{m3}\cos\beta)$$
$$- 4Y_{\ell 1}(Y_{m2} - iY_{m3}\cos\beta) - 2\tan\beta (Y_{m1} - iY_{m3}\sin\beta)(Y_{\ell 1} + iY_{\ell 3}\sin\beta) \}, \tag{A32}$$

$$\psi_{ijk} = \frac{1}{\sqrt{2}} \{ \chi_{ij}(Y_{k1} + iY_{k3}\sin\beta) + \eta_{ij}(Y_{k2} + iY_{k3}\cos\beta) \}, \tag{A33}$$

$$L^{H^-}_{ij} = f_1 \sin\beta D^{\tau*}_{R1i} D^{\nu}_{L1j} + f_2 \sin\beta D^{\tau*}_{R2i} D^{\nu}_{L2j} + h_1 \sin\beta D^{\tau*}_{R3i} D^{\nu}_{L3j} + h_2 \sin\beta D^{\tau*}_{R4i} D^{\nu}_{L4j} + y_5 \sin\beta D^{\tau*}_{R5i} D^{\nu}_{L5j},$$
$$R^{H^-}_{ij} = f_1'^* \cos\beta D^{\tau*}_{L1i} D^{\nu}_{R1j} + f_2'^* \cos\beta D^{\tau*}_{L2i} D^{\nu}_{R2j} + h_1'^* \cos\beta D^{\tau*}_{L3i} D^{\nu}_{R3j} + h_2'^* \cos\beta D^{\tau*}_{L4i} D^{\nu}_{R4j} + y_5'^* \cos\beta D^{\tau*}_{L5i} D^{\nu}_{R5j}$$
$$L^{H^+}_{ij} = (R^{H^-}_{ji})^*$$
$$R^{H^+}_{ij} = (L^{H^-}_{ji})^*. \tag{A34}$$

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