Restrained dark $U(1)_d$ at low energies

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We investigate a spontaneously broken $U(1)_d$ gauge symmetry with a muon-specific dark Higgs. Our first goal is to verify how the presence of a new dark Higgs, ϕ, and a dark gauge boson, V, can simultaneously face the anomalies from the muon magnetic moment and the proton charge radius. Second, by assuming that V must decay to an electron-positron pair, we explore the corresponding parameter space determined with the low-energy constraints coming from $K \to \mu X$, electron $(g-2)_e$, $K \to \mu\nu_\mu e^+ e^-$, $K \to \mu\nu_\mu \mu^+\mu^-$, and $\tau \to \nu_\tau \mu\nu_\mu e^+ e^-$. We focus on the scenario where the V mass is below $\sim 2m_\mu$ and the ϕ mass runs from few MeV to 250 MeV, with V-photon mixing of the order $\sim \mathcal{O}(10^{-3})$. Among weak process at low energies, we check the influence of the new light vector on kaon decays as well as on the scattering $e^+ e^- \to \mu^+\mu^- e^+ e^-$ and discuss the impact of the dark Higgs on $e^+ e^- \to \mu^+\mu^-\mu^+\mu^-$. Finally, we consider contributions of the V-photon mixing in the decays $\pi^0 \to \gamma e^+ e^-$, $\eta \to \gamma e^+ e^-$, $\rho \to \pi e^+ e^-$, $K^* \to K e^+ e^-$, and $\phi(1020) \to \eta e^+ e^-$.

I. INTRODUCTION

There is a number of discrepancies between standard model (SM) theoretical predictions and experimental results at energies below the kaon mass which might be signatures of new physics. The long-lasting muon anomalous magnetic moment $(g-2)_\mu$ puzzle, for instance, is still present at 3.6σ level, and, as claimed by many authors [1–9], it can be explained by new dark bosons with the masses below 200 MeV. One more recent example is the discrepancy between the proton charge radius measured via the Lamb shift in atomic and muonic hydrogen (see, e.g., Refs. [10–13]).

One of the proposals to explain the origin of such low-energy puzzles is based on the spontaneously broken $U(1)_d$ gauge symmetry [1–9], introduced in the context of astrophysical anomalies (see, e.g., Refs. [14,15]). Its basic mechanism allows the gauge coupling to be $\sim O(10^{-3})$ and implies a kinetic mixing amplitude between the new gauge boson V and the photon field.

In decays with particles identified through the missing energy, one might expect that some set of the invisible states is due to the existence of the coupled dark sector. Many experiments are devoted to searching for weakly interacting particles [16–18], and one pioneering work in this direction was done by the authors of Ref. [19], who succeeded to put limits on the decay BR($K \to \mu$ missing energy). By applying these bounds, the authors in Ref. [20] have found, for example, that the leptonic decay $K \to \mu\nu V$ is already very constraining on the V parameters.

In this paper, we reinvestigate a spontaneously broken $U(1)_d$ gauge model, following the ideas presented in Ref. [2] and implementing additional constraints. First, we assume that both the dark gauge boson, V, and the dark Higgs, ϕ, cannot be directly detected and assume that they both are present in the explanation of the proton size anomaly and kaon leptonic decays. We find a tension between the upper bounds on the decay width of the kaon leptonic decay and the proton size band for a specific range of relevant parameters. Second, we lose this prior restriction. Since we are mainly interested in the low-mass region, we continue to treat the dark Higgs as the muon-specific scalar contributing to the missing mass and work in the scenario where V must decay to $e^+ e^-$. The model will be further constrained by the *BABAR* additional observables: the uncertainty in $\Gamma(K \to \mu\nu_\mu e^+ e^-)$ for $m_{ee} > 145$ MeV, the upper bound for $\tau \to \nu_\tau \mu\nu_\mu e^+ e^-$, and the electron anomalous magnetic moment $(g-2)_e$. When the dark Higgs mass is in the range $2m_\mu \leq m_\phi \leq (m_K - m_\mu)$, we derive bounds from the experimental upper bound on $\Gamma(K \to \mu\nu_\mu \mu^+\mu^-)$.

The analysis described above can be considered complementary to the recent *BABAR* result [17] on the search for a new neutral vector boson in the process

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$e^+e^- \to \mu^+\mu^-V$. Their result has placed very strong limits on the coupling constant of V, indicating that the presence of a massive vector state can be excluded in the range 0.212–10 GeV.

Section I contains the description of the model we explore in our study. Section II is devoted to the derivation of bounds from low-energy phenomenology. In Sec. III, we discuss implications of this proposal on the low-energy processes, and Sec. IV contains the short summary of our results.

II. FRAMEWORK: DARK U(1)$_d$

The $U(1)_d$ gauge-invariant Lagrangian under consideration is written by [2]

$$\mathcal{L} = -\frac{1}{4}V_{\alpha\beta}V^{\alpha\beta} + |D_\mu\phi|^2 + \bar{\mu}_R i\slashed{D}\mu_R$$
$$- \frac{\kappa}{2}V_{\alpha\beta}F^{\alpha\beta} - \bar{L}\mu_R H_{SM}\frac{\phi}{\Lambda} + \text{H.c.} \quad (1)$$

Here, V is the gauge boson, neutral under the SM gauge group and charged under $U(1)_d$. The field ϕ is the dark Higgs with a condensate $\langle\phi\rangle = \frac{v_R}{\sqrt{2}}$. The covariant derivative $D_\alpha = \partial_\alpha + ig_R V_\alpha + ieQ_{EM}A_\alpha$, and κ is the mixing angle.

The muon mass is then introduced as $vv_R/(2\Lambda)$, while the SM-like Yukawa coupling is given by $v_R/(\sqrt{2}\Lambda)$. As asserted by the authors of Ref. [2], the proton charge radius phenomenology will favor the range of the new parameters such that the scale Λ can be at the weak scale. Moreover, the model given in Eq. (1) leads to gauge anomalies involving the photon and the vector V, and in order to restore gauge invariance, it is mandatory to introduce new dynamical scalar degrees of freedom.

There are different ways to make this theory UV complete. For instance, a number of SM extensions with new vectorlike fermions were constructed for this purpose [21–26]. Reference [27] suggested extending the SM ⊗ $U(1)_d$ by three right-handed neutrinos in order to generate neutrino masses. One last example was recently offered in Ref. [9] by the "lepton-specific" representation of a generic two-Higgs-doublet model in which the scalar sector contains the SM Higgs, an additional doublet, and the dark ϕ.

The Lagrangian given in Eq. (1) leads to the couplings of the new vector and scalar to fermions,

$$V \to -i\gamma^\mu(\bar{g}^\mu_V + \bar{g}^\mu_A\gamma_5), \qquad \phi \to -ig_\phi, \quad (2)$$

with the definitions

$$\bar{g}^\mu_V = e\kappa + \frac{g_R}{2}, \quad \bar{g}^\mu_A = \frac{g_R}{2}, \quad g_\phi = g_R\frac{m_\mu}{M_V}. \quad (3)$$

In our analysis, we choose to work with the set of parameters $(g_\phi, m_\phi, \kappa, M_V)$, by assuming $g_R = 2\lambda\kappa$, which, from the relation (3), leave us with

$$\lambda = \frac{M_V}{m_\mu}\frac{g_\phi}{2\kappa}. \quad (4)$$

In the next section, we will preferably consider the specific choice of (λ, m_ϕ) since in the literature bounds and predictions are often presented for the space (M_V, κ). We will also mention this combination in Sec. III.

III. LOW-ENERGY PHENOMENOLOGY BOUNDS

One of main goals of the model presented in Eq. (1) was to explain the proton size discrepancy [2]. It was first noticed by the authors of Ref. [20] that the $K \to \mu X$ decay, with X being a set of states seen only as missing energy, can give very strong constraints on the parameters of V [19]. Nevertheless, it was assumed that only an invisible vector state gives a new contribution to this process. We first try to establish parameter space of (M_V, κ), which is allowed by the proton charge radius and the leptonic kaon decay, including the contributions of both vector and scalar dark bosons as missing mass.

The procedure described above can be summarized as:
(i) *Proton charge radius.*—The measurement of the Lamb shift in muonic and atomic hydrogen [28,29] has indicated a difference for the proton radius square, r_p^2, which can be abbreviated to (for details, see Ref. [2])

$$\Delta r_p^2 = (r_p)_{e-p}^2 - (r_p)_{\mu-p}^2 = 0.060(12) \text{ fm}^2. \quad (5)$$

As discussed in Ref. [2], this discrepancy can be properly approached by the model of Eq. (1) due to the mixing with the photon. Moreover, since the dark Higgs couples to muons only, the mass m_ϕ will remain free to adjust additional limits. Here, we rewrite the theoretical corrections to the difference in Eq. (5), following the notation of Eq. (3),

$$\Delta r^2|_{e-H} = -\frac{6\kappa^2}{M_V^2},$$
$$\Delta r^2|_{\mu-H} = -\frac{6\kappa^2(1+\frac{\lambda}{e})}{M_V^2}f(aM_V), \quad (6)$$

where $a = (\alpha m_\mu m_p)^{-1}(m_p + m_\mu)$ is the $\mu - H$ Bohr radius, α is the fine-structure constant, and $f(z) = (z/(1+z))^4$. Therefore, for $aM_V \gg 1$, one can obtain the 2σ favorable region for the parameter κ, using the proton radius discrepancy given in Eq. (6):

$$\kappa^2 = \frac{eM_V^2}{6\lambda}(\Delta r_p^2 \pm 2\sigma). \quad (7)$$

(ii) *Muonic kaon decay $K \to \mu X$.*—In the context of V and ϕ bremsstrahlung from μ, the result of analysis in Ref. [19] can be converted to the upper bound,

$$\frac{\Gamma_{K \to \mu X}}{\Gamma_{K \to \mu \nu}} < 3.5 \times 10^{-6},$$

$$227.6 < m_X(\text{MeV}) < 302.2 \quad 90\% \text{C.L.}, \quad (8)$$

with

$$\Gamma_{K \to \mu X} = \Gamma_{K \to \mu \nu V} + \Gamma_{K \to \mu \nu \phi}. \quad (9)$$

Note that in Eq. (8) there is an experimental acceptance on the missing mass, m_X. Equation (9) can be written as a function of $(\kappa, M_V, \lambda, m_\phi)$.

In Fig. 1, we present the allowed parameter space (M_V, κ^2) obtained when the constraints from Eqs. (7) and (8) are applied, for fixed values of (λ, m_ϕ). On these particular examples, the gray color denotes the region excluded at the 90% C.L. by the bound on BR($\Gamma(K \to \mu X)$), while the pink one denotes the region allowed by the proton size anomaly up to 2σ. We have checked if there is any region which can satisfy both conditions. For a large set of the (λ, m_ϕ) points, we could not find any positive solution.

In this section, we would like to illustrate how the muonic kaon decay itself is very restrictive. Once the dark Higgs is muon specific, and we are mainly interested in the regime of small masses (below $2m_\mu$), we have to relax our first assumption and assume that the gauge boson V decays to electron-positron pair $V \to e^+ e^-$. However, such $V e^+ e^-$ interaction creates additional effects in a number of processes.

A. Muon anomalous magnetic moment

The discrepancy between experimental results and the SM prediction for $(g - 2)_\mu$ persists as an intriguing low-energy puzzle in particle physics, currently being $\delta a_\mu^{\text{exp}-\text{SM}} = \Delta(g - 2)_\mu / 2 = 288(80) \times 10^{-11}$ [30].

The model of Eq. (1) contributes at one-loop level with the three different contributions to $(g - 2)_\mu$—vector, axial-vector, and scalar. The authors of Ref. [2] have noticed that within this framework there will be an enhancement of opposite sign to δa_μ, if compared to the pure vector case. Such a feature might allow an overlap with the proton anomaly allowed region, since in the Eq. (7) there is no dependence on m_ϕ. We can clearly see this feature through writing the complete expression below (see Refs. [31] and [32]):

$$\begin{aligned}
\delta a_\mu &= (g_V)^2 I_V\left(\frac{M_V^2}{m_\mu^2}\right) + (g_A)^2 I_A\left(\frac{M_V^2}{m_\mu^2}\right) + (g_\phi)^2 I_\phi\left(\frac{m_\phi^2}{m_\mu^2}\right), \\
&= \kappa^2 \left[(e + \lambda)^2 I_V\left(\frac{M_V^2}{m_\mu^2}\right) + (\lambda)^2 I_A\left(\frac{M_V^2}{m_\mu^2}\right) \right. \\
&\quad \left. + \left(\frac{2m_\mu \lambda}{M_V}\right)^2 I_\phi\left(\frac{m_\phi^2}{m_\mu^2}\right) \right] \\
&\equiv \kappa^2 F(\lambda, M_V, m_\phi).
\end{aligned} \quad (10)$$

The full expression for the one-loop integrals $I_{V,A,\phi}$ can be found in Ref. [31]. We note that in the regime where the function $F(\lambda, M_V, m_\phi)$ is small the coupling κ can reach arbitrarily large values. This behavior will be tested in Sec. III F.

B. Leptonic kaon decays

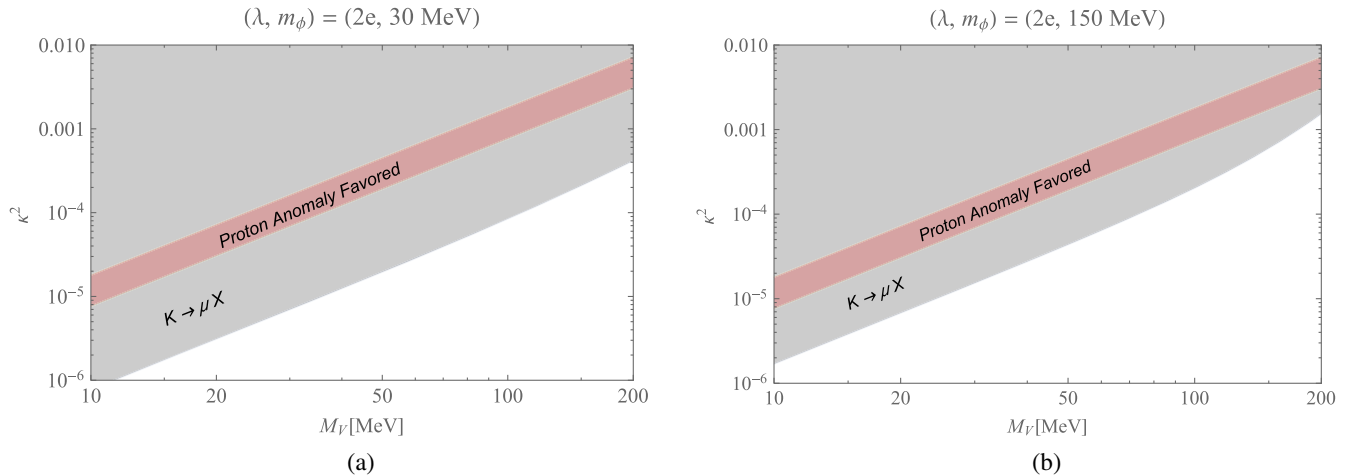The first conclusion of our analysis is that the dark photon must decay to an electron-positron pair. This,

$(\lambda, m_\phi) = (2e, 30 \text{ MeV})$ $(\lambda, m_\phi) = (2e, 150 \text{ MeV})$

(a)   (b)

FIG. 1. The (M_V, κ^2) parameter space fixed by the bounds from $K \to \mu X$ with the muonic V and ϕ bremsstrahlung (gray) along with the allowed area of the proton size anomaly (pink). The gray color marks the excluded region at 90% C.L.

however, does not mean that the $K \to \mu X$ bound is not important anymore, since the scalar ϕ still takes a role as a missing mass. The relation (3) accompanied by $g_R = 2\lambda\kappa$ will again produce the excluded region for (M_V, κ^2), even being dependent on m_ϕ. Moreover, given the richness of kaon phenomenology, the new requirement applied on the additional channels involving V can produce even stronger bounds on the parameter space:

(i) $K^+ \to \mu^+\nu_\mu e^+e^-$.—The branching ratio for this process is given in Ref. [30]:

$$\frac{\Gamma(K^+ \to \mu^+\nu_\mu e^+e^-)}{\Gamma_K}$$
$$= 7.06(31) \times 10^{-8} (m_{ee} > 145 \text{ MeV}). \quad (11)$$

The authors of Ref. [4] considered contributions of V via kinetic mixing with a radiated SM photon, as in Refs. [12,33,34]. They also made a comparison between $K^+ \to \mu^+\nu_\mu V \to \mu^+\nu_\mu e^+e^-$ and the QED background [35], having found that a new light vector boson, if it decays before leaving the detector, might produce bumps in the electron-positron invariant mass spectrum. The model presented in Eq. (1), which we use, leads to a signal a few orders of magnitude larger than the proposal of Ref. [4].

We assume that these corrections by themselves should not be larger than 1σ of the result given in Eq. (11). Using the narrow-width approximation and anticipating that the range for M_V is 145 MeV $< M_V < 2m_\mu$, where V can decay only to e^+e^-, we can impose the following upper bound:

$$\Gamma(K^+ \to \mu^+\nu_\mu V, V \to e^+e^-)$$
$$= \Gamma(K^+ \to \mu^+\nu_\mu V) \times \text{Br}(V \to e^+e^-). \quad (12)$$

Finally, since $\text{BR}(V \to e^+e^-) = 1$, the Eq. (12) becomes

$$\frac{\Gamma(K^+ \to \mu^+\nu_\mu V)}{\Gamma_K} < 3.1 \times 10^{-9}. \quad (13)$$

(ii) $K^+ \to \mu^+\nu_\mu\mu^+\mu^-$.—If the muon-specific dark Higgs has a mass larger than $2m_\mu$, the ϕ bremsstrahlung will be then followed by the decay $\phi \to \mu^+\mu^-$. In this case, the bound from $K \to \mu X$ cannot be applied and should be replaced by the existing upper bound [30]:

$$\frac{\Gamma(K^+ \to \mu^+\nu_\mu\mu^+\mu^-)}{\Gamma_K} < 4.1 \times 10^{-7}, \quad 90\% \text{ C.L.} \quad (14)$$

The above expression can be useful in the region $2m_\mu < m_\phi < (m_K - m_\mu)$, and as we will find out in
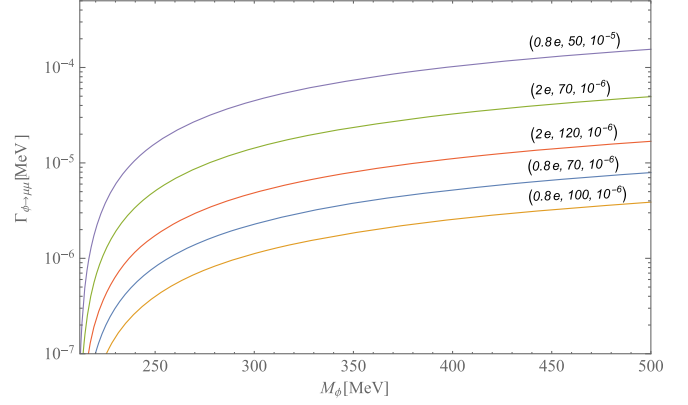

FIG. 2. Above the muon threshold, the dark Higgs ϕ will decay and might show up through muonic channels. Here, the lines correspond to the total width on different parameters $(\lambda, M_V [\text{MeV}], \kappa^2)$.

Sec. III F, if m_ϕ is close to $2m_\mu$, this constraint is equally as powerful as one coming from $K \to \mu X$ [see Eq. (8)]. Since the dark Higgs interacts with muons only, its decay to $\mu^+\mu^-$ is now allowed, and $\text{Br}(\phi \to \mu^+\mu^-) = 1$. We see from the total width presented in Fig. 2 that the narrow-width approximation is fully justified. Finally,

$$\frac{\Gamma(K^+ \to \mu^+\nu_\mu\phi)}{\Gamma_K} \times \text{BR}(\phi \to \mu^+\mu^-)$$
$$= \frac{\Gamma(K^+ \to \mu^+\nu_\mu\phi)}{\Gamma_K} < 4.1 \times 10^{-7}, \quad 90\% \text{ C.L.}$$
$$(15)$$

C. Constraints from $\tau^+ \to \nu_\tau\mu^+\nu_\mu e^+e^-$

Within the SM, the e^+e^- pair in the process $\tau^+ \to \nu_\tau\mu^+\nu_\mu e^+e^-$ originates from the virtual photon or Z emission in the decay $\tau^+ \to \nu_\tau\mu^+\nu_\mu$. Analogously to the previous case with $V \to e^+e^-$, we can consider the upper bound to the ratio of this process by assuming that one can safely use the narrow-width approximation:

$$\frac{\Gamma(\tau \to \nu_\tau\mu\bar{\nu}_\mu V)}{\Gamma_\tau} \text{BR}(V \to e^+e^-)$$
$$= \frac{\Gamma(\tau \to \nu_\tau\mu\bar{\nu}_\mu V)}{\Gamma_\tau} < 3.6 \times 10^{-5}, \quad 90\% \text{ C.L.} \quad (16)$$

The differential decay rate for $\tau^+ \to \nu_\tau\mu^+\nu_\mu e^+e^-$ is given by

$$d\Gamma_{\tau\to\nu_\tau\bar{\nu}_\mu V\mu} = \frac{m_\tau^3}{256(2\pi)^6} |\mathcal{M}_{\tau\to\nu_\tau\bar{\nu}_\mu V\mu}|^2$$

$$\times \sqrt{\lambda(1,\delta_3,0)}\sqrt{\lambda(\delta_2,\delta_\mu,\delta_V)}$$

$$\times \frac{\sqrt{\lambda(\delta_3,\delta_2,0)}}{\delta_2\delta_3} d\delta_2 d\delta_3 dC_{\theta_2} dC_{\theta_3} d\phi, \qquad (17)$$

where we have assigned the momenta $\tau(k)$, $\mu(p_1)$, $V(p_2)$, $\nu_\mu(p_3)$, $\nu_\tau(p_4)$. C_{θ_2} is the angle between μ and τ momenta in the rest frame of $k_2 \equiv p_1 + p_2$, and C_{θ_3} is the angle between k_2 and k in the rest frame of $k_3 \equiv p_1 + p_2 + p_3$. Besides, $\delta_i \equiv \frac{M_i^2}{m_\tau^2}$, $i = \mu, V, 2, 3$ and $M_j^2 \equiv k_j^2$, $j = 2, 3$, ϕ is the angle between the planes composed by $\vec{k}_1 \times \vec{k}_2$ and $\vec{k}_2 \times \vec{k}_3$, and $\lambda(a,b,c) = (a - (\sqrt{b} + \sqrt{c})^2)(a - (\sqrt{b} - \sqrt{c})^2)$.

We point out that the same analysis could be done for $\mu \to \nu_\mu e\bar{\nu}_e e^+ e^-$, but this is not as restrictive as the constraint (16), due to the smaller phase space.

D. Electron anomalous magnetic moment

The standard model contribution to $a_e = \frac{(g-2)_e}{2}$ has been recently improved up to the tenth order, corresponding to $\Delta a_e = 1159652181.13(84) \times 10^{-12}$ [36], facing the experimental value $a_e = (g - 2)/2 = (1159.65218076 \pm 0.00000027) \times 10^{-6}$ [30]. In Ref. [37], the author argues that the one-loop correction to this quantity must be reinterpreted as an effective shift of the fine-structure constant, or

$$d\alpha = 2\pi a_e \to \frac{d\alpha^{-1}}{\alpha^{-1}} = -\frac{2\pi a_e}{\alpha}. \qquad (18)$$

From the last measurements of α^{-1} (see, e.g., Refs. [36–39]), we can claim that the relative uncertainty must not exceed 0.5 ppb, leading to the constraint

$$\left(\frac{2}{3}\frac{m_e^2}{M_V^2}\right)\kappa^2 < 5 \times 10^{-10}, \qquad (19)$$

where we have used $8\pi^2 I_V(\frac{M_V^2}{m_e^2}) \approx \frac{2}{3}\frac{m_e^2}{M_V^2}$ in the limit of $m_e \ll M_V$. The above relation will be considered along with all the bounds presented in the previous subsections.

E. Experimental bounds

There are many experimental searches for the dark sector (see, e.g., Refs. [16,18,40–47]). We mention here only the most recent bounds. The NA48/2 Collaboration [43] has searched for bounds in $\pi^0 \to \gamma e^+ e^-$ decay and obtained that $\kappa^2 = (0.8 - 1.11) \times 10^{-5}$ at 90% C.L. for the mass of the vector gauge boson in the range $2m_e < M_V < 140$ MeV. The Kloe-2 Collaboration determined the bound on the mass of dark photon and

photon-dark photon mixing parameter from the study of dark photon contribution in the $\phi \to \eta V \to \eta e^+ e^-$ decay, by measuring the cross sections $e^+ e^- \to V\gamma \to \mu^+\mu^-\gamma$ and $e^+ e^- \to V\gamma \to e^+ e^-\gamma$. They found that κ^2 has to be smaller than 5×10^{-5} [42,48].

The *BABAR* Collaboration obtained the very restrictive bounds on the dark Z' boson (corresponding to V in our case) [17] from the cross section for the $e^+ e^- \to \mu^-\mu^+ Z' \to \mu^+\mu^-\mu^+\mu^-$ process relying on the model described in Refs. [49,50]. A basic feature of this model is the absence of the Z' coupling to the first lepton generation. The *BABAR* search is based on 514 fb^{-1} of data collected at the PEP-II $e^+ e^-$ storage ring, predominantly taken at the $\Upsilon(4s)$ resonance, and their result is applicable also on the models in which the gauge bosons are coupled exclusively to right-handed muons. They obtained a strong bound on the coupling and the mass of Z' in the region 0.2 GeV $< m_{Z'} < 4$ GeV. Since our model contains dark Higgs and dark gauge bosons, we also expect that the inclusion of the dark Higgs contribution in $e^+ e^- \to \mu^+\mu^-\mu^+\mu^-$ might only slightly modify the phenomenology of this channel. Thus, we combine the results of the NA48/2, Kloe-2, and *BABAR* analyses on our plots, which we present in the next section.

F. Discussion

The constraints derived in the previous subsections are presented in Fig. 3. The colored areas are excluded, while the yellow and red bands correspond to the allowed region of the proton charge radius and muon anomalous magnetic moment at 2σ level, respectively. The regions excluded by NA48/2 [43], Kloe-2 [42,48], and *BABAR* [17] are gray.

To derive a more general conclusion on the proton anomaly explanation, we can combine the definition in (4) along with the constraint in (7) to obtain

$$\lambda = \frac{3}{2e}\frac{g_\phi^2}{\Delta r_p^2 m_\mu^2}. \qquad (20)$$

In addition, by writing the amplitude of $K \to \mu\nu\phi$ as a function of (g_ϕ, m_ϕ), we have concluded that the parameter space for $g_\phi > 0.03$, with $m_\phi < 2m_\mu$, will be ruled out at 90% C.L. Equation (20) translates this assertion to $\lambda > 0.86e$, a condition that can necessarily exclude the central value of the proton radius discrepancy. We can verify this, for instance, in Fig. 3. In Fig. 3(a), even if the above-mentioned limit is respected, the dependence on the small m_ϕ results in the exclusion of the yellow region from the kaon muonic decay. In Fig. 3(b), the $\lambda = 0.8e$ accompanied with a large $m_\phi = 150$ MeV will loosen the $K \to \mu X$ bound. However, in both cases, the bound from $(g-2)_e$ will disfavor this sector. In Figs. 3(c) and 3(d), since $\lambda = 2e$, the proton band is necessarily excluded.
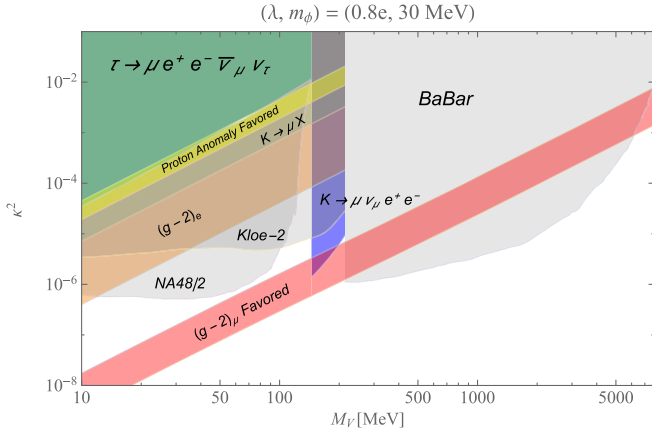
(a) For $(\lambda, m_\phi) = (0.8e, 30\text{MeV})$ the search for missing mass in $K \to \mu X$ excludes the $\pm 2\sigma$ favored region for the proton anomaly.
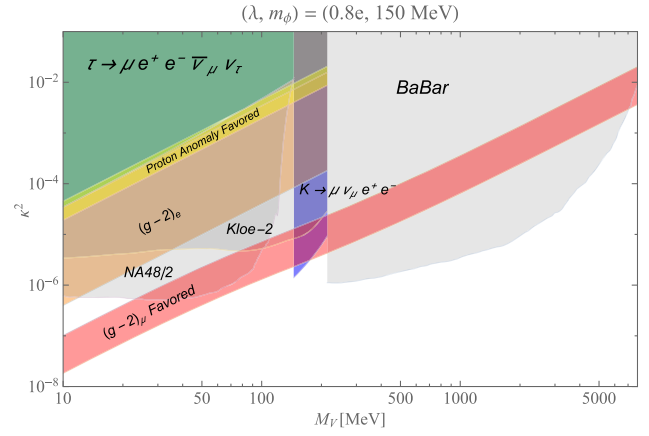
(b) For $(\lambda, m_\phi) = (0.8e, 150\text{MeV})$ the constraint from $BR(K \to \mu X)$ can be relaxed, allowing a small $\pm 2\sigma$ region to accommodate the proton anomaly, which is however excluded by $(g-2)_e$ region. There is no overlap between regions allowed by the proton and $(g-2)_\mu$ discrepancies.
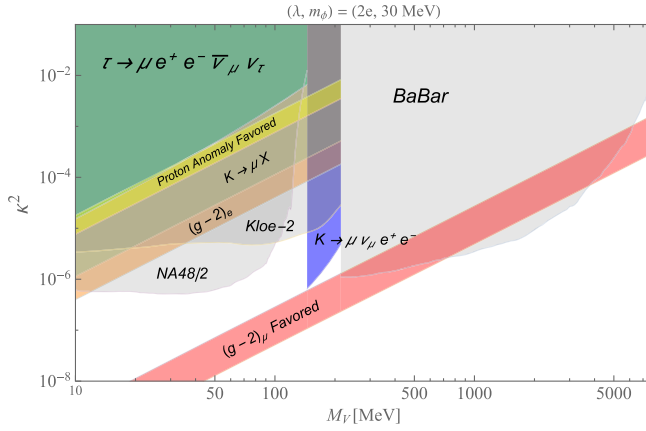
(c) For $(\lambda, m_\phi) = (2e, 30\text{MeV})$ the bound for $BR(K \to \mu X)$ will exclude, at $\pm 2\sigma$ level, the favored region for proton anomaly explanation.
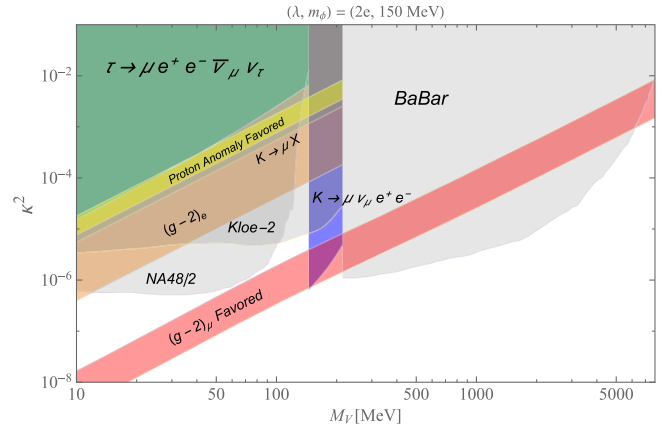
(d) For $(\lambda, m_\phi) = (2e, 150\text{MeV})$ the leptonic kaon channel and the 2016 BaBar data start to exclude the $\pm 2\sigma$ favored region for $(g-2)_\mu$.

FIG. 3. Parameter space for (M_V, κ^2). The colored regions are excluded by the respective processes, and the region favored by $(g-2)_\mu$ at $\pm 2\sigma$ is marked by pink, while the region allowed by the proton size anomaly is yellow.

We stress that the only areas in Fig. 3 dependent on m_ϕ are those related to $K \to \mu X$ and $(g-2)_\mu$. The four plots are pointing out that the proton charge radius cannot be explained by the spontaneously broken dark $U(1)_d$ gauge symmetry. The whole region which allows us to explain proton radius puzzle is being excluded by the constraints from $(g-2)_e$, $K \to \mu X$, $K \to \mu \nu_\mu e^+ e^-$, and $\tau \to \mu \nu_\mu e^+ e^-$. In any of the cases we analyze, the proton charge radius anomaly and the muon anomalous magnetic moment cannot be simultaneously explained. Apart from that, the constraint coming from $K \to \mu \nu_\mu e^+ e^-$ can almost extrapolate the *BABAR* bound on the mass of the vector gauge boson down to $M_V \sim 145$ MeV.

If the size of λ and a large scalar mass are such that the function $F(\lambda, M_V, m_\phi)$ in Eq. (10) becomes very small, approaching zero, the coupling κ tied to the muon anomaly

might be arbitrarily large. In Fig. 4, for example, the dark Higgs mass is $m_\phi = 250$ MeV, and we can find a tiny overlap between the pink and yellow bands. Nevertheless, for $m_\phi > 2m_\mu$ using the constraint $\text{BR}(K \to \mu \nu \mu \mu) < 4.1 \times 10^{-7}$, we find out that it again leads to the exclusion of the proton favored region at 90% C.L.

We finally note that the contribution of both the dark Higgs and the dark V can enable $(g-2)_\mu$ to be explained. If, for example, only the dark gauge boson is present, there would be no region on the parameter space allowed by experimental results—or by the bounds we have shown here—that could explain the respective anomaly.

IV. PREDICTIONS AT LOW ENERGIES

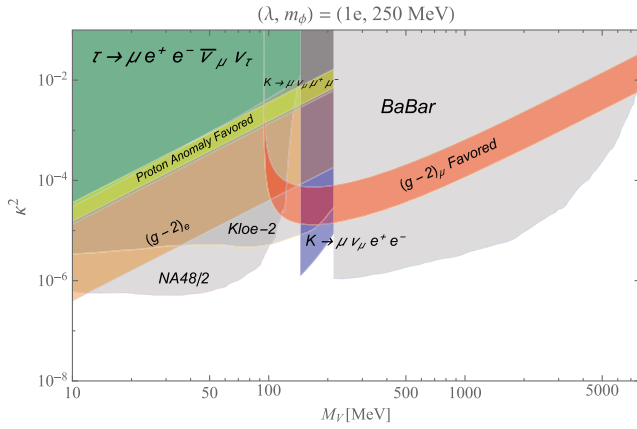Our analyses of the dark $U(1)_d$ gauge sector allows the mass of M_V to be in the region around

FIG. 4. Parameter space for (M_V, κ^2). The colored regions are excluded by the processes denoted on the respective areas. The region favored by $(g-2)_\mu$ at $\pm 2\sigma$ is marked by red, and the proton radius anomaly is marked by yellow. The size of λ and the large scalar mass are such that $F(\lambda, M_V, m_\phi)$ in Eq. (10) goes to zero, requiring large values of κ. For $m_\phi > 2m_\mu$, we use the constraint from $BR(K \to \mu\nu\mu\mu) < 4.1 \times 10^{-7}$, which excludes the proton favored region at 90% C.L.

$50 < M_V(\mathrm{MeV}) < 150$ with the parameter $\kappa \sim 10^{-3}$. One would expect that the weak decays are more likely to offer a good testing ground for the dark sector [51]. Particularly, the flavor-changing neutral current processes occurring in meson decays were most favorable for such searches. For example, in Ref. [39], the rare decay of K and B mesons to $\pi e^+ e^-$ were suggested as interesting candidates for the dark boson searches, mainly due to the low rate of $BR(K^+ \to \pi^+ e^+ e^-)_{\mathrm{exp}} = (3.00 \pm 0.09) \times 10^{-7}$ in the SM. By relying on the reanalysis of $K \to \pi\gamma^*$ in Ref. [6], we calculate the branching ratio for $K \to \pi V$ and present our result in Fig. 5. The V dark boson promptly decays to $V \to e^+ e^-$, and the narrow-width approximation will then give that $BR(K \to \pi V \to \pi e^+ e^-) = BR(K \to \pi V)$.
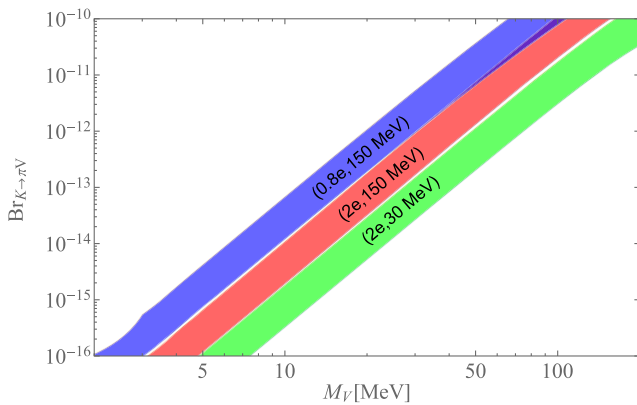

FIG. 5. The branching ratio of $K \to \pi V$ for some specific parameters (λ, m_ϕ). The values of the coupling κ are given by the favored region of $(g-2)_\mu$ up to 2σ. In this model, V decays promptly to $V \to e^+ e^-$.

A. $K \to \pi V$

In Eq. (12) of Ref. [37], the author presented a general formula for the branching ratio of $K \to \pi V$ valid for M_V below 200 MeV and given by

$$\mathrm{Br}_{K \to \pi V} \simeq 8 \times 10^{-5} \times \kappa^2 \left(\frac{M_V}{100 \, \mathrm{MeV}}\right)^2. \quad (21)$$

If we replace κ^2 by the value which explains the $(g-2)_\mu$ anomaly in Eq. (10), we can derive the branching ratio as a function of M_V for a specific choice of the parameters (λ, m_ϕ). Some examples are presented in Fig. 5.

The NA48/2 Collaboration has commented in Ref. [43] that sensitivity on this process is not competitive with the existing bounds. Namely, they found that $\kappa^2 = (0.8\text{--}1.11) \times 10^{-5}$ at 90% C.L. for the vector gauge boson mass in the range $2m_e < M_V < 140$ MeV.

## B. $e^+ e^- \to \mu^+ \mu^- (\phi \to \mu^+ \mu^-)$ and $e^+ e^- \to \mu^+ \mu^- (V \to e^+ e^-)$

In the work of Ref. [17], the search for a direct production of muonic dark forces in a model-independent method was done. The results were presented as the measured $e^+ e^- \to \mu^+ \mu^- Z'$, $Z' \to \mu^+ \mu^-$ cross section being a function of the Z' mass. Within the dark $U(1)_d$ model we consider in this paper, the only contribution to the process $e^+ e^- \to \mu^+ \mu^- \mu^+ \mu^-$ is from the dark Higgs scalar ϕ in the region $M_V < 2m_\mu$, and it is presented in Fig. 6(a). Nevertheless, a complementary search at a low-mass region for V will be in the process $e^+ e^- \to \mu^+ \mu^- V$, $V \to e^+ e^-$. The theoretical results are presented in Fig. 6(b) for the center-of-mass energy equal to $\sqrt{10}$ GeV.

C. $\pi^0 \to \gamma e^+ e^-$ and $\eta \to \gamma e^+ e^-$

The dark $U(1)_d$ sector might affect low-energy observables due to the mixing of the SM photon with the part of the dark gauge boson. That means in all processes where this conversion $\gamma \leftrightarrow V$ occurs one can search for the dark boson. Because of the long-lived vector, the use of the narrow-width approximation is fully justified, and we use $Br(V \to e^+ e^-) = 1$. As already suggested in Ref. [52], the search for the presence of the dark sector in electromagnetic decays seems to be possible. We make predictions for the decays of $P \to \gamma V \to \gamma e^+ e^-$ for $P = \pi^0, \eta$ by noticing that our V can have mass in the region $50 \, \mathrm{MeV} < M_V < 150 \, \mathrm{MeV}$ and can decay only to the electron-positron pair. In Ref. [52], $\pi^0(\eta) \to \gamma V \to \gamma e^+ e^-$ were approached by relying on the result of model-independent study given in Ref. [53]. In this approach, the branching ratio of $\pi^0 \to \gamma e^+ e^-$ agrees very well with the experimental one. The decay width for $\pi^0 \to \gamma V \to \gamma e^+ e^-$ with the use of the narrow-width approximation can be written as

$(\sqrt{s}, g_\phi) = (10 \text{ GeV}, 10^{-3})$
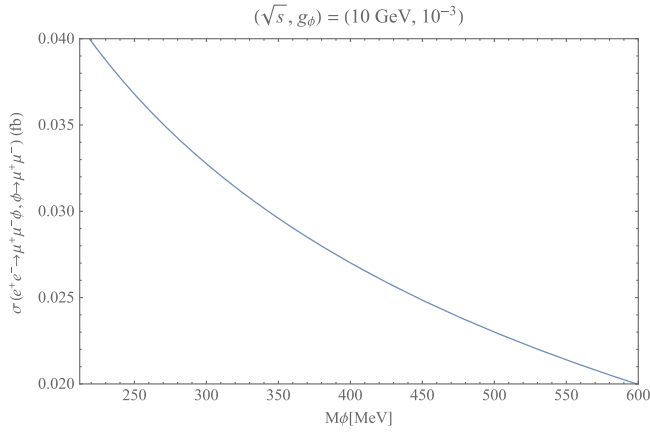
(a) The total cross section for $e^+e^- \to \mu^+\mu^-\phi, \phi \to \mu^+\mu^-$ as function of the scalar mass m_ϕ. The energy $\sqrt{s}$ and the constant coupling g_ϕ were chosen, respectively, as 10 GeV and 10^{-3}.

$(\sqrt{s}, \kappa) = (10 \text{ GeV}, 10^{-3})$

$\lambda = 2e$

$\lambda = 0.8e$
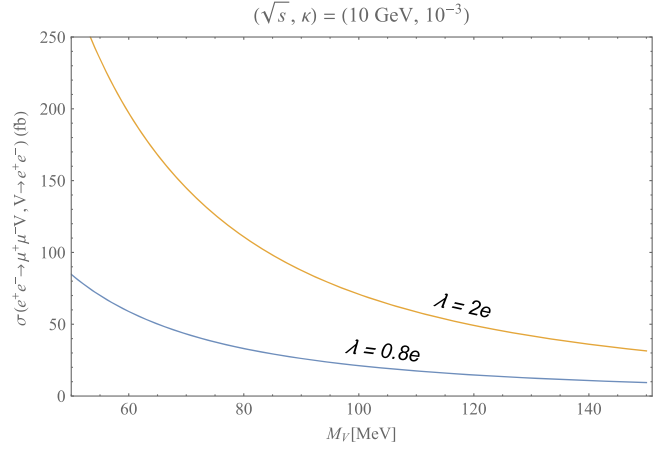
(b) The total cross section for $e^+e^- \to \mu^+\mu^-V, V \to \mu^+\mu^-$ as function of the vector mass, M_V for two values of λ.

FIG. 6. The total cross section for $e^+e^- \to \mu^+\mu^-\mu^+\mu^-$ in the framework of ϕ and V emission. The results are complementary to Fig. 3 of Ref. [17].

$$\Gamma(\pi^0 \to \gamma V \to \gamma e^+ e^-)$$
$$= |f(1,0,x_V)|^2 \Gamma(\pi^0 \to \gamma V) Br(V \to e^+ e^-), \quad (22)$$

with $\Gamma(\pi^0 \to \gamma V) = 2\kappa^2 (1 - M_V^2/m_\pi^2)^3 \Gamma_0$, $\Gamma_0 \equiv \Gamma(\pi^0 \to \gamma\gamma)$, and $f(1,0,x) = 1 + a_\pi x + b_\pi x^2 + \mathcal{O}(x^3)$, with $a_\pi = -0.0324(12)_{\text{stat}}(19)_{\text{sys}}$, $b_\pi = 1.06(9))_{\text{stat}}(25)_{\text{sys}} \times 10^{-3}$, and $x_V = M_V^2/m_{\pi^0}^2$, as explained in detail in Ref. [52]. For the value $M_V = 50$ MeV, we obtained BR($\pi^0 \to \gamma V \to \gamma e^+ e^-) = 1.3 \times 10^{-6} (\kappa/10^{-3})^2$, while for the $M_V = 100$ MeV, we calculate BR($\pi^0 \to \gamma V \to \gamma e^+ e^-) = 1.9 \times 10^{-7} (\kappa/10^{-3})^2$.

For the decay of $\eta \to \gamma V \to \gamma e^+ e^-$ following Ref. [52] and using the transition form factor from the same reference $f_\eta(1, 0, x) = 1 + b_\eta x + c_\eta x^2 + d_\eta x^3 \mathcal{O}(x^4)$ ($b_\eta = 0.576(11)_{\text{stat}}(4)_{\text{sys}}$, $c_\eta = 0.339(15)_{\text{stat}}(5)_{\text{sys}}$, $d_\eta = 0.200(14)_{\text{stat}}(18)_{\text{sys}}$, and $x_V = M_V^2/m_\eta^2$ [54]), we obtain for $M_V = 50$ MeV the branching ratio BR($\eta \to \gamma V \to \gamma e^+ e^-) = 1.5 \times 10^{-6} (\kappa/10^{-3})^2$, while for $M_V = 100$ MeV, we calculate BR($\eta \to \gamma V \to \gamma e^+ e^-) = 1.4 \times 10^{-6} (\kappa/10^{-3})^2$. There are few experimental studies of the dark matter contributions in $\pi^0(\eta) \to \gamma e^+ e^-$ as described in Ref. [52], starting with the beam-dump experiments E141 [55], CHARM [56], and NA48/2 [43]. Also, there are plans for the future facilities APEX [16], HPS [57], DarkLight [58], and LHCb [59]. The NA48/2 experiment almost reached sensitivity on the mixing parameter $\kappa \sim 10^{-3}$ [43] in their search for the dark photons in $\pi^0(\eta) \to \gamma e^+ e^-$. In Ref. [56], based on NOMAD and PS191, it was claimed that the bound on BR($\pi^0 \to \gamma X \to \gamma e^+ e^-) \leq 10^{-15}$ can be reached and for the decay BR($\eta \to X\gamma \to \gamma e^+ e^-) \leq 10^{-14}$ [60].

D. $\rho \to \pi e^+ e^-$, $K^* \to K e^+ e^-$ and $\phi(1020) \to \eta e^+ e^-$

The amplitude for the decays $P^* \to PV$, where $P^* = \rho^{+,0}, K^{*+,0}, \phi$ and $P = \pi^{+,0}, K^{+,0}, \eta$ can be written as

$$\mathcal{M}(P^*(p_{P^*}, \epsilon_{P^*}) \to P(p_P)V(p_V, \epsilon_V)$$
$$= \kappa g_{P^*PV} \epsilon_{\mu\nu\alpha\beta} p_{P^*}^\mu \epsilon_{P^*}^\nu p_V^\alpha \epsilon_V^\beta, \quad (23)$$

with p_P, p_{P^*}, and P_V being the momenta of the corresponding mesons and ϵ_{P^*} and ϵ_V being polarization vectors of P^* and V, respectively. To determine decay widths, we assume that to a good approximation $g_{P^*PV} \simeq g_{P^*P\gamma}$. We expect that this approximation is satisfied as long as the dark vector boson mass is relatively small. There are numerous attempts within the lattice QCD community to calculate $\rho\pi\gamma^*$ form factors [61], which will help in more precise studies of g_{P^*PV}. The transition coefficient $g_{P^*P\gamma}$ can be extracted from the decay width for $P^* \to P\gamma$. Knowing that $\Gamma(P^* \to P\gamma) = |g_{P^*P\gamma}|^2 (m_{P^*}^2 - m_P^2)^3/(96\pi m_{P^*}^3)$, one can determine $g_{P^*P\gamma}$. It was found by the authors of Ref. [62] that $g_{\rho^+\pi^+\gamma} = 2.19 \times 10^{-4}$ MeV^{-1}, $g_{\rho^0\pi^0\gamma} = 2.52 \times 10^{-4}$ MeV^{-1}, $g_{K^{*+}K^+\gamma} = 2.53 \times 10^{-4}$ MeV^{-1}, and $g_{K^{*0}K^0\gamma} = 2.19 \times 10^{-4}$ MeV^{-1}, and we obtain the value $g_{\Phi(1020)\eta\gamma} = 1.26 \times 10^{-4}$ MeV^{-1}, using data given by the Particle Data Group [30]. The decay width for $P^* \to PV$ can be written as

$$\Gamma(P^* \to PV) = \frac{|\kappa g_{P^*PV}|^2}{96\pi} \frac{\lambda(m_{P^*}^2, m_P^2, M_V^2)^{3/2}}{m_{P^*}^3}, \quad (24)$$

and finally

TABLE I. Predicted branching ratios for BR($P^* \to PV \to Pe^+e^-$) for the dark gauge boson mass $M_V = 50, 100$ MeV and $\kappa = 0.001$. For other values of the photon-V mixing parameter κ, one should rescale these results by $(\kappa/10^{-3})^2$.

P^*, P, V	$M_V = 50$ MeV	$M_V = 100$ MeV
ρ^0, π^0, V	6.3×10^{-10}	6.1×10^{-10}
ρ^+, π^+, V	4.8×10^{-10}	4.6×10^{-10}
K^{*0}, K^0, V	7.6×10^{-10}	7.0×10^{-10}
K^{*+}, K^+, V	1.0×10^{-9}	9.5×10^{-10}
$\Phi(1020), \eta, V$	9.1×10^{-10}	8.9×10^{-10}

$$\Gamma(P^* \to PV \to Pe^+e^-) = \Gamma(P^* \to PV)\text{BR}(V \to e^+e^-). \tag{25}$$

Some predicted branching ratios for two values of the V mass are presented at Table I.

The above mentioned processes might be relevant for a number of planned experimental searches as APEX, HPS, DarkLight and LHCb. The KLOE-2 experiment has already searched for the dark photon contribution in $\phi \to \eta V \to Ve^+e^-$ decay [46], not finding any bump in the differential distribution.

V. SUMMARY

The $U(1)_d$ gauge model of Eq. (1) introduces a new dark gauge boson and a dark Higgs. The model was first proposed to explain the proton charge radius discrepancy as well as the $(g-2)_\mu$ anomaly.

We have explored the phenomenology of these two dark bosons—the dark Higgs ϕ and the vector V—through a set of low-energy processes, focusing on the parameter space (M_V, κ). In our approach, the muon magnetic moment receives the contribution of both particles, and the ϕ mass provides an additional freedom to adjust the allowed band of $(g-2)_\mu$ within 2σ. We find out that V has to decay to e^+e^- in order to explain $K \to \mu X$, where X refers to a missing energy, implying that in this process any signature of the dark Higgs could be detected. Further, we concluded that the allowed band for the proton radius anomaly is strongly constrained by a set of well-established bounds, namely, the bounds from $(g-2)_e$ and $\tau \to \nu_\tau \mu \nu_\mu e^+e^-$ decay. This feature, for instance, will enable different ranges for M_V which were at first excluded in the context of a generic vector coupling by the *BABAR* searches [17], as presented in Fig. 3(c). The bounds from $K \to \mu\nu_\mu e^+e^-$, $K \to \mu\nu_\mu \mu^+\mu^-$, and $\tau \to \nu_\tau \mu \nu_\mu e^+e^-$ when combined with the above-mentioned bounds allow the mass of M_V to be in the region around 50 MeV $< M_V < 150$ MeV with the parameter $\kappa \sim 10^{-3}$, while the mass of the dark Higgs can be from a few MeV to ~ 200 MeV, for a particular choice of the remaining parameter λ. We finally mention that the bound from $K \to \mu\nu_\mu e^+e^-$ leads to constraints as strong as those experimentally achieved by the *BABAR* and NA48/2 collaborations on the correspondent sector.

We have also presented a set of predictions. The very small branching ratios of the processes $K \to \pi V, V \to e^+e^-$, as it was pointed in Ref. [37], makes the search for the dark gauge boson rather difficult. The electromagnetic decays of $\pi^0 \to \gamma e^+e^-$ and $\eta \to \gamma e^+e^-$, $\rho \to \pi e^+e^-$, $K^* \to Ke^+e^-$, and $\phi(1020) \to \eta e^+e^-$ might also proceed through the dark gauge boson. Some of these processes are already the subjects of experimental studies. The small mixing parameter between the photon and dark photon suppresses the branching ratios for these processes, but hopefully future experiments for the dark matter search would shed more light on dark bosons at low energies.

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