Natural generalized mirage mediation

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In the supersymmetric scenario known as mirage mediation (MM), the soft supersymmetry (SUSY) breaking terms receive comparable anomaly-mediation and moduli-mediation contributions leading to the phenomenon of *mirage unification*. The simplest MM SUSY breaking models which are consistent with the measured Higgs mass and sparticle mass constraints are strongly disfavored by fine-tuning considerations. However, while MM makes robust predictions for gaugino masses, the scalar sector is quite sensitive to specific mechanisms for moduli stabilization and potential uplifting. We suggest here a broader setup of generalized mirage mediation (GMM), where heretofore discrete parameters are allowed as continuous to better parametrize these other schemes. We find that natural SUSY spectra consistent with both the measured value of m_h as well as LHC lower bounds on superpartner masses are then possible. We explicitly show that models generated from natural GMM may be beyond the reach of even high-luminosity LHC searches. In such a case, the proposed International Linear e^+e^- Collider will be required for natural SUSY discovery via higgsino pair production reactions. We also outline prospects for detection of higgsino-like WIMPs from natural GMM.

I. NATURALNESS IN MIRAGE MEDIATION

Superstring theory yields a consistent quantum theory of gravity and appears to have the required ingredients to potentially unify all four forces of nature. However, in order to gain predictivity, it is necessary to understand how the degeneracy associated with the many flat directions in the space of scalar fields (the moduli) is lifted to yield the true ground state, since many quantities relevant for physics at low energy are determined by the ground state values of these fields. The implementation of a class of compactifications where the extra spatial dimensions are curled up to small sizes with fluxes of additional fields trapped along these extra dimensions was used by Kachru, Kallosh, Linde and Trivedi (KKLT) [1] to construct models with a stable, calculable ground state with a positive cosmological constant and broken supersymmetry. The KKLT toy model is based on type-IIB superstrings including compactification with fluxes to a Calabi-Yau orientifold. While the background fluxes serve to stabilize the dilaton and the moduli that determine the shape of the compact manifold, it is necessary to invoke a nonperturbative mechanism such as gaugino condensation [2] on a $D7$-brane to stabilize the size of the compact manifold. Finally, a nonsupersymmetric antibrane ($\overline{D3}$) was included in order to break supersymmetry completely and obtain a de Sitter universe

as required by observations. The resulting low energy theory thus has no unwanted light moduli, has a broken supersymmetry, and a positive cosmological constant. The existence of these flux compactifications with stable calculable minima having many desired properties may be viewed as a starting point for the program of discovering a string ground state that may lead to a phenomenologically viable low energy theory of SM particles and their superpartners, with $N = 1$ supersymmetry softly broken just above the weak scale.

The KKLT picture motivated several groups to analyze the structure of the soft supersymmetry (SUSY) breaking (SSB) terms in models based on a generalization of the KKLT setup [3]. The key observation is that because of the mass hierarchy,

$$m_{\text{moduli}} \gg m_{3/2} \gg m_{\text{SUSY}}, \qquad (1)$$

that develops in these models, the soft terms receive comparable contributions from both modulus (gravity) [4] and anomaly mediation of SUSY breaking [5], with their relative size parametrized by an additional parameter α. Moreover, the hierarchy (1) that leads to this mixed modulus-anomaly mediated SUSY breaking (also known as mirage mediation or MM as discussed shortly) automatically alleviates phenomenological problems from late decaying moduli and gravitinos that could disrupt, for instance, the predictions of light element abundances from big bang nucleosynthesis. Upon integrating out the heavy dilaton field and the shape moduli, one is left with an effective broken supergravity theory of the observable

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sector fields denoted by $\hat{Q}$ and the size modulus field $\hat{T}$. The Kähler potential depends on the location of matter and Higgs superfields in the extra dimensions via their modular weights $n_i = 0(1)$ for matter fields located on $D7$- ($D3$)-branes, or $n_i = 1/2$ for chiral multiplets on brane intersections, while the gauge kinetic function $f_a = \hat{T}^{l_a}$, where a labels the gauge group, is determined by the corresponding location of the gauge supermultiplets, since the power $l_a = 1(0)$ for gauge fields on $D7$- ($D3$)-branes [6].

Within the MM model, the SSB gaugino mass parameters, trilinear SSB parameters and sfermion mass parameters, all renormalized just below the unification scale (taken to be $Q = M_{\text{GUT}}$), are given by

$$M_a = M_s(l_a\alpha + b_a g_a^2), \tag{2}$$

$$A_{ijk} = M_s(-a_{ijk}\alpha + \gamma_i + \gamma_j + \gamma_k), \tag{3}$$

$$m_i^2 = M_s^2(c_i\alpha^2 + 4\alpha\xi_i - \dot{\gamma}_i), \tag{4}$$

where $M_s \equiv \frac{m_{3/2}}{16\pi^2}$, b_a are the gauge β function coefficients for gauge group a and g_a are the corresponding gauge couplings. The coefficients that appear in (2)–(4) are given by $c_i = 1 - n_i$, $a_{ijk} = 3 - n_i - n_j - n_k$ and $\xi_i = \sum_{j,k} a_{ijk}\frac{y_{ijk}^2}{4} - \sum_a l_a g_a^2 C_2^a(f_i)$. Finally, y_{ijk} are the superpotential Yukawa couplings, C_2^a is the quadratic Casimir for the ath gauge group corresponding to the representation to which the sfermion $\tilde{f}_i$ belongs, γ_i is the anomalous dimension and $\dot{\gamma}_i = 8\pi^2 \frac{\partial \gamma_i}{\partial \log \mu}$. Expressions for the last two quantities involving the anomalous dimensions can be found in the Appendix of Refs. [7,8].

The MM model is then specified by the parameters

$$m_{3/2}, \quad \alpha, \quad \tan\beta, \quad sign(\mu), \quad n_i, \quad l_a. \tag{5}$$

The mass scale for the SSB parameters is dictated by the gravitino mass $m_{3/2}$. The phenomenological parameter α, which could be of either sign, determines the relative contributions of anomaly mediation and gravity mediation to the soft terms, and is expected to be $|\alpha| \sim \mathcal{O}(1)$. Grand unification implies matter particles within the same grand unification theory (GUT) multiplet have common modular weights, and that the l_a are universal. We will assume here that all $l_a = 1$ and, for simplicity, there is a common modular weight for all matter scalars c_m but we will allow for different modular weights c_{H_u} and c_{H_d} for each of the two Higgs doublets of the MSSM. Such choices for the scalar field modular weights are motivated for instance by $SO(10)$ SUSY GUT models where the MSSM Higgs doublets may live in different 10-dimensional Higgs reps.

Various aspects of MM phenomenology have been examined in Refs. [6,7,9–11]. The universality of the l_a leads to the phenomenon of *mirage unification* [6,7] of gaugino mass parameters (and also corresponding matter

scalar mass parameters of first and second generation sfermions whose Yukawa couplings are negligible). Here, for reasons that will become clear later, we focus on the gaugino mass parameters M_i: when extrapolated to high energies using one loop renormalization group equations (RGEs), these will unify at a scale $Q = \mu_{\text{mir}} \neq M_{\text{GUT}}$, where M_{GUT} is the unification scale for gauge couplings. Indeed, the observation of gaugino mass unification at the mirage unification scale,

$$\mu_{\text{mir}} = M_{\text{GUT}} e^{-8\pi^2/\alpha}, \tag{6}$$

is the smoking gun of such a scenario [12]. If $\alpha < 0$, then $\mu_{\text{mir}} > M_{\text{GUT}}$ and one finds *virtual* mirage unification at super-GUT energy scales. We stress that there is no physical threshold at $Q = \mu_{\text{mir}}$, and the evolution can be continued to $Q = M_{\text{GUT}}$ where the gaugino mass parameters would take on the values close to (2). The determination of the mirage unification scale also determines α, the parameter that governs the relative moduli- versus anomaly-mediation contribution to the soft-SUSY breaking terms. Once α is known, then further extrapolation of the gaugino masses to $Q = m_{\text{GUT}}$ allows for a determination of the *gravitino mass* $m_{3/2}$.

Alas, this attractive MM scenario has recently been confronted by the twin constraints of LHC searches on the one hand and a clarified understanding of SUSY naturalness on the other. One important LHC constraint comes from the newfound Higgs mass $m_h \simeq 125$ GeV which in the context of the MSSM requires highly mixed TeV-scale top squarks [13]. The other LHC constraint is that the gluino mass, based on LHC13 searches with ~ 10 fb^{-1} of data, require $m_{\tilde{g}} \gtrsim 1.9$ TeV (within the context of various simplified models) [14].

For the case of naturalness, it has been emphasized [15–17] that previous studies—that lead to the conclusion that naturalness requires light top squarks—neglect the fact that one must evaluate the sensitivity of m_h or m_Z only with respect to the *independent* parameters of the theory, as embodied for instance in the frequently used EENZ/BG measure [18], $\Delta_{\text{BG}} \equiv max_i |\frac{\partial \log m_Z^2}{\partial \log p_i}|$. Here i labels the various independent, fundamental parameters p_i of the theory. Historically, this measure has been applied to multi-soft-parameter effective SUSY theories where the additional parameters are introduced to parametrize our ignorance of the source of soft terms. However, in any more fundamental theory the various soft terms are derived in terms of more fundamental entities, such as the gravitino mass in gravity mediation [19], or via Eqs. (2)–(4) for mirage mediation. In this case, the soft-SUSY breaking parameters are *correlated* and not independent: then, neglecting these correlations will lead to an *overestimate* of the fine-tuning in these theories [15–17]. In MM, where α takes on a predetermined value, the soft parameters are all

determined by $m_{3/2}$ and Δ_{BG} reduces to the model-independent electroweak measure Δ_{EW}.[1]

The electroweak fine-tuning parameter [20,21], Δ_{EW}, is a measure of the degree of cancellation between various contributions on the right-hand side (rhs) in the well-known expression for the Z mass:

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \Sigma_u^u - \mu^2 \tag{7}$$

which results from the minimization of the Higgs potential in the MSSM. Here, $\tan\beta = v_u/v_d$ is the ratio of Higgs field vacuum-expectation values and the Σ_u^u and Σ_d^d contain an assortment of radiative corrections, the largest of which typically arise from the top squarks. Expressions for the Σ_u^u and Σ_d^d are given in the Appendix of Ref. [21]. If the rhs terms in Eq. (7) are individually comparable to $m_Z^2/2$, then no unnatural fine-tunings are required to generate $m_Z = 91.2$ GeV. Δ_{EW} is defined to be the largest of these terms, scaled by $m_Z^2/2$. Clearly, low electroweak fine-tuning requires that μ be close to m_Z^2 and that $m_{H_u}^2$ be radiatively driven to *small* negative values close to the weak scale. This scenario has been dubbed radiatively driven natural supersymmetry or RNS [20,21].

The main requirements for low electroweak fine-tuning $(\Delta_{EW} \lesssim 30)$[2] are the following.

(i) $|\mu| \sim 100 - 300$ GeV [23] where $\mu \gtrsim 100$ GeV is required to accommodate LEP2 limits from chargino pair production searches.

(ii) $m_{H_u}^2$ is driven radiatively to small, and not large, negative values at the weak scale [20,21].

(iii) The top-squark contributions to the radiative corrections $\Sigma_u^u(\tilde{t}_{1,2})$ are minimized for TeV-scale highly mixed top squarks [20]. This latter condition also lifts the Higgs mass to $m_h \sim 125$ GeV. For $\Delta_{EW} \lesssim 30$, the lighter top squarks are bounded by $m_{\tilde{t}_1} \lesssim 3$ TeV [21,22].

(iv) The gluino mass, which feeds into the stop masses and hence the $\Sigma_u^u(\tilde{t}_{1,2})$, is bounded by $m_{\tilde{g}} \lesssim 4$ TeV [21,22].

Detailed scans over MM parameter space for various choices of matter and Higgs field modular weights found all models consistent with LHC8 sparticle and Higgs mass constraints were in fact highly fine-tuned with $\Delta_{EW} > 100$ (for a summary, see Fig. 13 of Ref. [16]). This means these

models give a poor prediction for the weak scale as typified by $m_{weak} \sim m_{W,Z,h} \sim 100$ GeV, i.e. the weak scale of 100 GeV is only generated by excessive fine-tuning of the μ parameter. One may thus ask: Are mirage mediation models on their way to the dustbin of failed SUSY models?[3,4]

II. NATURAL GENERALIZED MIRAGE MEDIATION

The evident failure of naturalness in MM mentioned at the end of the last section leads us to re-examine the phenomenological implications of moving from discrete choices of the parameters a_{ijk} and c_i in Eqs. (3) and (4) to a continuous range, and also to allow c_i values greater than 1. While the discrete parameter choices occur in a wide range of KKLT-type compactifications (for some discussion, see Ref. [27]), a continuous range of these parameters may be expected if one allows for more generic methods of moduli stabilization and potential uplifting. For instance, if the soft terms scan as in the string landscape picture, then their moduli-mediated contributions may be expected to be parametrized by a continuous value. For models which generate a small μ term ~ 100 GeV from multi-TeV soft terms, such as radiative Peccei-Quinn breaking [28], it has been suggested that the statistical pull by the landscape towards large soft terms, coupled with the anthropic requirement of $m_{weak} \sim 100$ GeV, acts as an attractor towards natural SUSY soft term boundary conditions [29].

Note that the phenomenological modification that we suggest will not affect the result (2) for gaugino mass parameters, which has been stressed [12] to be the most robust prediction of the MM mechanism. In this paper, we allow for the more *general* mirage mediation (GMM) parameters, thus adopting a parameter space given by

$$m_{3/2}, \quad \alpha, \quad \tan\beta, \quad a_3, \quad c_m, \quad c_{H_u}, \quad c_{H_d}\text{(GMM)}, \tag{8}$$

where a_3 is short for $a_{Q_3 H_u U_3}$. The independent values of c_{H_u} and c_{H_d} which set the moduli-mediated contribution to the soft Higgs mass terms may conveniently be traded for weak scale values of μ and m_A as is done in the two-parameter nonuniversal Higgs model [30]:

$$m_{3/2}, \quad \alpha, \quad \tan\beta, \quad a_3, \quad c_m, \quad \mu, \quad m_A\text{(GMM')}. \tag{9}$$

This trick allows for more direct exploration of natural SUSY parameter space which requires $\mu \sim 100 - 300$ GeV.

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[1]More generally, we advocate the use of Δ_{EW} in the discussion of naturalness of models with a given superpartner spectrum since discarding any high scale model with a (seemingly) large value of Δ_{BG} and a low value of Δ_{EW} may be prematurely discarding an effective theory because (unincorporated) correlations among the high scale parameters could well lower the value of Δ_{BG} all the way to Δ_{EW}.

[2]The onset of fine-tuning for $\Delta_{EW} \gtrsim 30$ is visually displayed in Ref. [22].

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[3]The models of deflected mirage mediation [24] which combine gauge, moduli and anomaly mediation, still seem viable and may allow for naturalness [25].

[4]A phenomenological anomaly-mediated supersymmetry breaking model has been proposed which can reconcile $(g-2)_\mu$ with the value $m_h \simeq 125$ GeV [26].
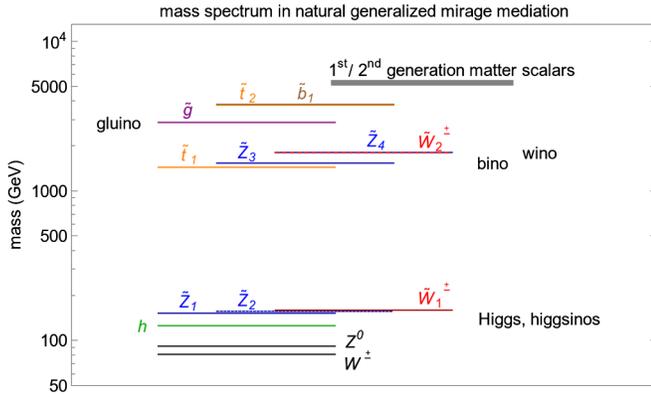
FIG. 1. A typical superparticle mass spectrum generated from nGMM as in Table I.

TABLE I. Input parameters and masses in GeV units for a natural generalized mirage mediation SUSY benchmark point with $m_t = 173.2$ GeV.

Parameter	nGMM
$m_{3/2}$	75000
α	4
$\tan\beta$	10
c_{H_u}	11.3
c_{H_d}	1.15
c_m	6.9
μ	150
m_A	2000
$m_{\tilde{g}}$	2856.5
$m_{\tilde{u}_L}$	5266.7
$m_{\tilde{u}_R}$	5398.2
$m_{\tilde{e}_R}$	4824.6
$m_{\tilde{t}_1}$	1433.1
$m_{\tilde{t}_2}$	3732.0
$m_{\tilde{b}_1}$	3770.5
$m_{\tilde{b}_2}$	5124.5
$m_{\tilde{\tau}_1}$	4749.5
$m_{\tilde{\tau}_2}$	5093.9
$m_{\tilde{\nu}_\tau}$	5103.1
$m_{\tilde{W}_2}$	1791.6
$m_{\tilde{W}_1}$	158.7
$m_{\tilde{Z}_4}$	1799.4
$m_{\tilde{Z}_3}$	1526.9
$m_{\tilde{Z}_2}$	155.8
$m_{\tilde{Z}_1}$	151.4
m_h	124.9
$\Omega^{\rm std}_{\tilde{Z}_1} h^2$	0.005
$BF(b \to s\gamma) \times 10^4$	3.1
$BF(B_s \to \mu^+\mu^-) \times 10^9$	3.9
$\sigma^{SI}(\tilde{Z}_1, p)$ (pb)	3.0×10^{-10}
$\sigma^{SD}(\tilde{Z}_1 p)$ (pb)	9.6×10^{-6}
$\langle\sigma v\rangle\vert_{v\to 0}({\rm cm}^3/\sec)$	3.1×10^{-25}
$\Delta_{\rm EW}$	15.5

In Fig. 1, we show the SUSY spectrum generated from one such parameter space point in the *natural* GMM model (nGMM), with the corresponding data shown in Table I. This benchmark point was generated using the Isajet/Isasugra computer code [31] with nonuniversal soft term inputs. The specific input parameters are $m_{3/2} = 75$ TeV, $\alpha = 4$, $\tan\beta = 10$, $a_3 = 3$, $c_m = 6.9$ and with $\mu = 150$ GeV and $m_A = 2$ TeV. The latter two choices end up corresponding to $c_{H_u} = 11.3$ and $c_{H_d} = 1.15$. From Table I, we see the gluino mass is $m_{\tilde{g}} = 2856$ GeV, which is just beyond the 5σ projected reach of HL-LHC with $\sqrt{s} = 14$ TeV and 3000 fb^{-1} of integrated luminosity [32], at least without tagged b s to further enhance the signal. The Higgs mass $m_h = 124.9$ GeV agrees well with measurements from LHC. The squarks and sleptons of the first/second generation lie in the 5 TeV range while third generation squarks can be lighter, with $m_{\tilde{t}_1} \simeq 1433$ GeV. This latter value appears beyond the reach of HL-LHC where a 95% exclusion reach with 3000 fb^{-1} extends out to $m_{\tilde{t}_1} \sim 1100$ GeV for $m_{\tilde{Z}_1} \sim 100$ GeV [33]. Note that this benchmark point has $\Delta_{\rm EW} = 15.5$ and so relatively low electroweak fine-tuning. A high scale theory with $\alpha = 4$ which led to the assumed values of c_i and a_3 would have $\Delta_{\rm BG} \simeq 15$ and would not be fine-tuned.

In Fig. 2, we show the running of the three gaugino masses for the nGMM benchmark model. In this case, we see the most robust feature of GMM: the celebrated mirage unification of gaugino masses at the intermediate scale $\mu_{mir} \sim 10^{7.5}$ GeV consistent with $\alpha = 4$, as can be seen from Eq. (6).

In Fig. 3, we show the renormalization group evolution of the various scalar soft mass terms for the nGMM benchmark model. First/second generation matter scalar mass parameters remain close to 5 TeV. Unlike for the model with a common modular weight for the two Higgs doublets, these do not unify at $Q = \mu_{\rm mir}$ because for the nGMM model, the hypercharge D-term contribution to the evolution no longer vanishes. In contrast, third generation and Higgs mass

square parameters evolve considerably more because of large Yukawa interactions. In particular, $m^2_{U_3}$, runs to much lower values ~1.5 TeV at the weak scale. The up-Higgs soft mass $m^2_{H_u}$ begins about 20% higher in value than matter scalar masses at $Q = M_{\rm GUT}$, but then evolves to small negative values at the weak scale, so that the requirement for electroweak naturalness, $|m^2_{H_u}| \sim m^2_Z$ is satisfied. The soft term $m^2_{H_d}$ which sets the heavy Higgs mass scale can be adjusted up or down with not-to-much cost to naturalness $\Delta_{\rm EW}$. We remark here that because the matter scalars are essentially decoupled, our spectra for phenomenological purposes is similar to what may be derived from the NUHM2 model but with gaugino mass parameters fixed by the MM values rather than by universality.
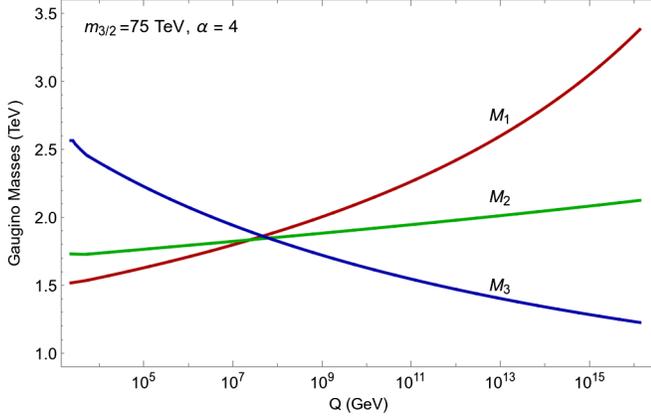
FIG. 2. Evolution of gaugino masses from the nGMM benchmark point with $m_{3/2} = 75$ TeV, $\alpha = 4$.
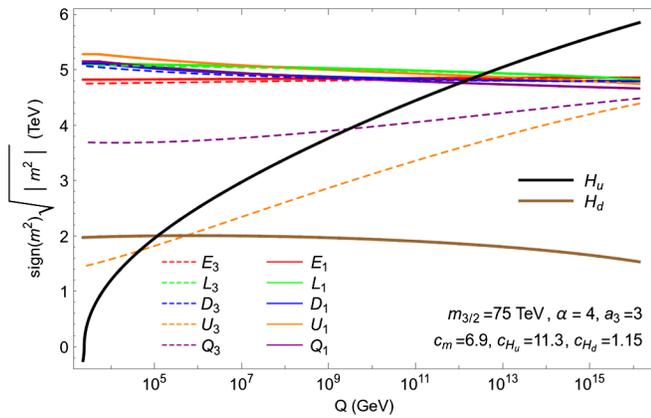
FIG. 3. Plot of running scalar masses from the nGMM benchmark point with $m_{3/2} = 75$ TeV, $\alpha = 4$, $\tan\beta = 10$ and $c_m = 6.9$, $a_3 = 3$ with $c_{H_u} = 11.3$ and $c_{H_d} = 1.15$ (corresponding to $\mu = 150$ GeV and $m_A = 2$ TeV at the weak scale).
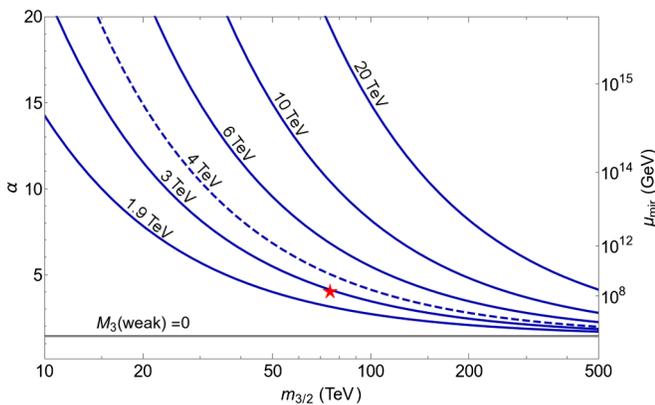
FIG. 4. Contours of $M_3(\text{weak})$ in the $m_{3/2}$ vs α plane of the nGMM model with other parameters as fixed in Table I. The region below $M_3 \sim 1.9$ is roughly excluded by LHC gluino pair searches. The location of our benchmark point is shown with a red star. The region below the dashed $m_{\tilde{g}} = 4$ TeV contour has the capacity to be natural. On the right side, some corresponding values of μ_{mir} are shown.

In Fig. 4, we show a larger set of GMM parameter space by contours of gaugino mass $M_3(\text{weak})$ in the $m_{3/2}$ vs α plane. At tree level, then $m_{\tilde{g}} \sim M_3(\text{weak})$. Thus, the region below $M_3(\text{weak}) \lesssim 1.9$ TeV is excluded by LHC13 gluino pair searches. The location of our benchmark point is noted with a red star. The region below the dashed $m_{\tilde{g}} = 4$ TeV contour has the capacity to be natural. On the right side, some corresponding values of μ_{mir} are shown.

III. CONSEQUENCES FOR COLLIDERS

A. LHC

It has been pointed out in Ref. [34] that in natural SUSY models such as RNS with gaugino mass unification, additional signatures for SUSY with light higgsinos are present at the LHC even though gluinos and also top squarks may be too heavy to be detectable. The first of these, labeled same-sign diboson production [35], arises from wino pair production $pp \to \widetilde{W}_4^{\pm} \widetilde{Z}_4$ where, for instance, $\widetilde{W}_2^+ \to W^+ \widetilde{Z}_{1,2}$ while $\widetilde{Z}_4 \to W^+ \widetilde{W}_1^-$. This leads to a robust $W^{\pm} W^{\pm} + \not{E}_T$ signature consisting of two acollinear same-sign dilepton $+ \not{E}_T$ events with jet activity only from QCD radiation. These event topologies have very low backgrounds. For large integrated luminosity $\sim 300 - 3000$ fb^{-1}—anticipated at the high-luminosity LHC—this channel yields the greatest LHC14 reach.

A second robust signature expected in RNS-type models is higgsino pair production $\widetilde{Z}_1 \widetilde{Z}_2 j$ in association with a hard monojet from QCD radiation, followed by $\widetilde{Z}_2 \to \widetilde{Z}_1 \ell^+ \ell^-$ decay. The leptons in the OS/SF pair emerging from $\widetilde{Z}_2$ decay are quite soft (due to the small $m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1} \sim 10 - 20$ GeV mass gap expected in models with universal gaugino masses) and would frequently fail detector trigger requirements. However, the hard ISR jet or the associated large $\not{E}_T$ could serve as a trigger. After suitable cuts, it appears this signature gives a good reach in the μ direction of the $\mu - m_{1/2}$ parameter plane of the model. The calculations of Ref. [34] indicate that essentially all of the RNS parameter space with $\Delta_{EW} \leq 30$ is covered by these two channels assuming ~ 3000 fb^{-1} of integrated luminosity at LHC14.

In contrast, for the nGMM model, both these signatures appear much more challenging for LHC SUSY searches. The reason is the compressed spectrum of gauginos which occurs in nGMM. In NUHM2 with gaugino mass unification at $Q = M_{GUT}$, then the weak scale gauginos after RG running are expected to occur in a ratio $M_3 : M_2 : M_1 \sim 7 : 2 : 1$. Naturalness considerations require gluinos not much heavier than ~ 4 TeV in NUHM2 for $\Delta_{EW} < 30$ [21,22]; if they do become heavy, they increase the top-squark masses which increases the $\Sigma_u^u(\tilde{t}_{1,2})$ contributions so that again one must fine-tune against these contributions. This naturalness condition, together with gaugino mass universality, then guarantees that the winos

are almost always accessible to LHC14 searches for NUHM2 if $\Delta_{EW} \leq 30$. Also, in this case, the $\tilde{Z}_2 - \tilde{Z}_1$ mass gap is always larger than ~ 10 GeV. In contrast, compressed gaugino spectra with $M_1 \sim M_2 \sim M_3$ at an intermediate scale are the hallmark of MM models with a low α and concomitantly low mirage unification scale. This means that–with $m_{\tilde{g}} \sim 3 - 4$ TeV–wino pairs (with mass $m(\text{wino}) \sim m_{\tilde{g}}$) may well be too heavy to be produced at detectable rates at LHC14. Moreover, these larger values of M_1 and M_2 from nGMM result in an even more compressed spectrum of neutral higgsinos, as exemplified by the benchmark in Table I for which the mass gap $m_{\tilde{Z}_2} - m_{\tilde{Z}_1} \sim 5.4$ GeV. Such a small mass gap makes the LHC monojet plus soft dilepton search much more difficult—in fact, in a recent CMS search for this channel [36], they indeed required $m(\ell^+\ell^-) > 4$ GeV to stay away from the J/ψ and γ^* poles with a cut around 9–10.5 GeV to stay away from the Υ pole. Such cuts would veto much of the signal region expected from our nGMM benchmark.

B. Linear electron-positron colliders

In Ref. [37], a variety of measurements were proposed for MM models at the LHC and International Linear e^+e^- Collider or ILC which could determine the modular weights associated with matter scalars and measure the relative moduli-/anomaly-mediated contributions to the soft terms and the gravitino mass $m_{3/2}$. The ILC would initially be operating with $\sqrt{s} = 0.5$ TeV but is upgradable to 1 TeV. In Ref. [38], it was pointed out that for SUSY models with radiatively-driven naturalness, the ILC would be a *higgsino factory* for $\sqrt{s} > 2m(\text{higgsino}) \sim 2\mu$. The two reactions $e^+e^- \rightarrow \tilde{W}_1^+\tilde{W}_1^-$ and $\tilde{Z}_1\tilde{Z}_2$ occur at rates comparable to muon pair production once the kinematic production threshold is passed. Moreover, the higgsino pair production cross section exceeds that for Higgs boson production unless higgsino poduction is kinematically suppressed. In Ref. [38], it was shown that the clean environment of ILC detector events and the adjustable beam energy and polarization can easily allow for both discovery as well as a suite of precision measurements, at least for $\tilde{Z}_1 - \tilde{Z}_2$ mass gaps expected in the RNS framework with $\Delta_{EW} < 30$. Direct measurement of the $E(\ell^+\ell^-)$ and $m(\ell^+\ell^-)$ distributions from $\tilde{Z}_1\tilde{Z}_2$ production followed by $\tilde{Z}_2 \rightarrow \tilde{Z}_1\ell^+\ell^-$ decay allows for measurement of $m_{\tilde{Z}_2}$ and $m_{\tilde{Z}_1}$ to subpercent precision [38,39]. Measurement of the $E(jj)$ and $m(jj)$ distributions from $\tilde{W}_1\tilde{W}_1 \rightarrow (q\bar{q}'\tilde{Z}_1) + (\ell\nu_\ell\tilde{Z}_1)$ production allow for subpercent measurements of $m_{\tilde{W}_1}$ and $m_{\tilde{Z}_1}$ if the mass gap is sufficiently large. Moreover, the mass gaps are sensitive to $\tan\beta$ and gaugino masses M_1 and M_2. In the RNS case with $m_{\tilde{Z}_2} - m_{\tilde{Z}_1} \sim 20$ GeV, it was shown that the gaugino mass parameters can be extracted with a precision of 5–10%, and examination of the more difficult case of the 10 GeV mass

gap is in progress [39]. Clearly, prospects for the detection of the higgsinos of nGMM models (where the mass gap is even smaller) and corresponding measurements of gaugino masses will be even more challenging but worthy of investigation.[5] A positive outcome would mean that the ILC would be a *discovery machine* for a scenario that would likely be beyond the reach of even a high-luminosity LHC. We emphasize that if the extraction of gaugino masses turns out to be feasible, then extrapolation of these masses via RGEs to high energies would indicate mirage unification and allow extraction of the parameter α, and also the associated gravitino mass $m_{3/2}$.

IV. WIMP SIGNALS FROM NGMM

We now turn to prospects for dark matter detection in the natural generalized mirage mediation scenario. Since electroweak naturalness requires a low μ parameter, $\mu \sim 100 - 300$ GeV, the LSP is expected to be mainly higgsino-like with a non-negligible gaugino component. However, comparing nGMM to natural models with gaugino mass unification like RNS, it is clear that for nGMM, the electroweak gauginos are much heavier because the gaugino spectrum is more compressed. As a result, both $\tilde{Z}_1$ and $\tilde{Z}_2$ are considerably more higgsino-like than in RNS, and further, the inter-higgsino mass gaps are also smaller. This, in turn, means the higgsino co-annihilation rate is enhancd in nGMM relative to RNS. Consequently the thermally produced higgsino-like neutralino abundance can be as low as $\Omega_{\tilde{Z}_1}^{TP}h^2 \sim 0.12/40$, i.e. thermally produced higgsinos make up just a few percent of the observed DM, an even lower relic abundance than in natural NUHM2 models. The possibility that the deficit in dark matter abundance is made up by nonthermal processes such as moduli production and subsequent decay to higgsinos is excluded as we will see below. In Ref. [40] it is suggested that if one insists on naturalness in the electroweak sector, one ought to have naturalness in the QCD sector: this brings into the discussion axion superfields, mixed axion-higgsino dark matter and production of neutralinos via axino/saxion production and decay. In this latter case, then axions may make up the bulk of dark matter with only a small fraction of the abundance consisting of higgsino-like WIMPs.

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[5]The nGMM model is not the only scenario with a compressed higgsino spectrum and very heavy gauginos that suggests that the ILC could be a discovery machine. If the vacuum-expectation value of the auxiliary field that breaks supersymmetry transforms as a **75**-dimensional representation of $SU(5)$ (rather than a singlet as is usually assumed), the resulting nonuniversal pattern of GUT scale gaugino masses leads to $M_3 : M_2 : M_1 = 6, 6, -5$ at the weak scale, so winos and binos would be even heavier than for our nGMM case study, and the higgsinos even more compressed. Such a scenario would be even more challenging to detect.

In Fig. 5(a), we show the WIMP spin-independent (SI) direct detection rates expected from nGMM in the $m_{\tilde{Z}_1}$ vs $\xi\sigma^{SI}(\tilde{Z}_1, p)$ plane. The vertical axis includes a factor of $\xi \equiv \Omega^{TP}_{\tilde{Z}_1} h^2/0.12$ to account for the possibility of a depleted local abundance of target WIMPs. Here, we adopt matter scalar soft terms ~ 5 TeV with $a_3 = 3$ and the $m_A = 2$ TeV and then scan over $m_{3/2}: 10 - 200$ TeV, $\alpha: 0 - 20$ and $\mu: 100 - 400$ GeV. We show only the points with $\Delta_{EW} < 30$. The upper black points assume that higgsinos produced by an additional non-thermal $\tilde{Z}_1$ production saturate the observed dark matter density, so $\xi = 1$, while for the lower green points we assume the higgsino abundance is given by its thermal value so that the bulk of dark matter is axions. Nonthermal production of higgsinos from axino/saxion decays would increase $\Omega_{\tilde{Z}_1} h^2$ resulting in an increase to ξ of the green points. Of course, the density of neutralinos could be diluted if there was additional entropy production [41] during the history of the Universe. The current reach of the LUX experiment [42] is shown as red solid while the XENON1T reach [43] is purple dashed. We see that the current LUX experiment has just started to probe the parameter space with $\xi = 1$ while all of this space will be probed by XENON1T. Multiton noble liquid detectors such as LZ [44], XENONnT (20Y exposure) [43], DarkSide-20K [45], DEAP-50T [46] and DARWIN [47] will be required to probe the entire parameter space with $\xi < 1$. We note these detection rates are lower than expected from natural NUHM2 models [48,49] since both ξ is reduced and also with heavier electroweak-ino masses, the LSP is more pure higgsino-like in nGMM. In this case, the Higgs exchange amplitude, which depends on a product of higgsino times gaugino couplings, is reduced in nGMM compared to NUHM2.

In Fig. 5(b), we show the spin-dependent cross sections for the same scan as in frame (a) with $\xi = 1$ and $\xi < 1$ (fixed by the thermal abundance of higgsinos), along with the current bound from the IceCube experiment (red solid line) [50] and projected reaches of the XENON1T (dashed purple line) and PICO500 (dashed-blue line) [51]. We see that the nGMM points, even with $\xi = 1$, satisfy all current bounds. This situation is quite different from the case of the well-tempered neutralino where the higgsino-rich neutralino branch is solidly excluded by the IceCube data. The reason is that though higgsinos couple with full gauge strength to the Z, in the case of the (nearly) pure higgsino-LSP of the nGMM, the coupling of Z to *identical* neutralinos (which determines the SD cross section) vanishes when the $\tilde{Z}_i \simeq \frac{\tilde{h}_u \pm \tilde{h}_d}{\sqrt{2}}$. We see that the XENON1T experiment will detect a signal even via spin-dependent scattering for $m_{\tilde{Z}_1} \lesssim 200$ GeV if neutralinos make up all the local DM. Experiments like PICO-500 will be needed to probe yet higher mass values. Finally, we remark that if the neutralino density

(a) SI direct detection rate

(b) SD direct detection rate
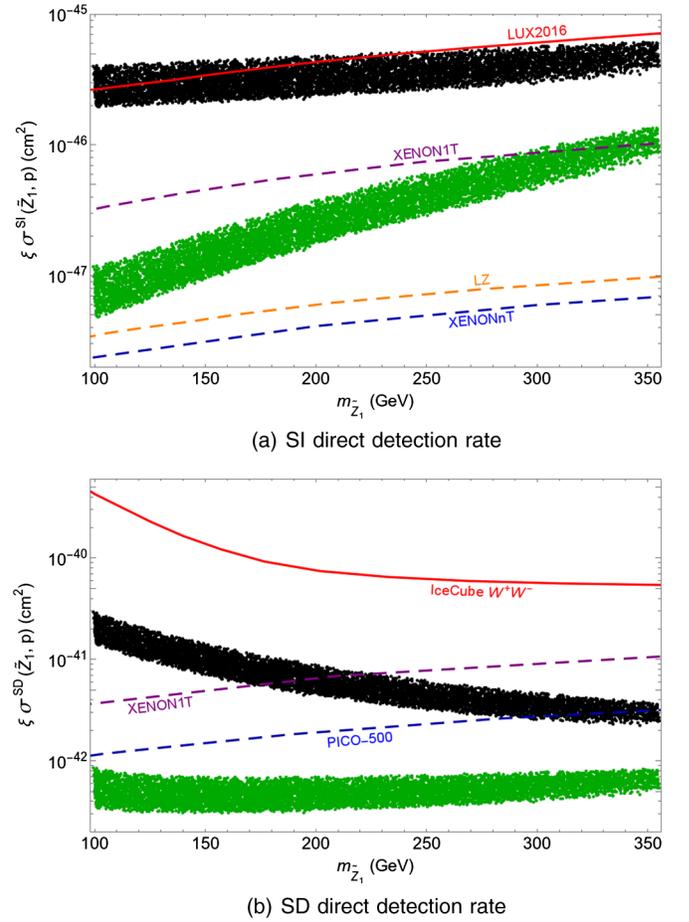
FIG. 5. (a) The spin-independent, and (b) the spin-dependent neutralino-nucleon direct detection rates multiplied by fractional dark matter abundance $\xi \equiv \Omega^{TP}_{\tilde{Z}_1} h^2/0.12$ in the $m_{\tilde{Z}_1}$ vs $\xi\sigma^{SI}(\tilde{Z}_1, p)$ plane from a scan over $m_{3/2}$, α and μ, with other parameters fixed as in the benchmark model. The black points have $\xi = 1$ while the green points have $\xi < 1$ corresponding to the fraction given by thermally produced higgsinos. The current LUX bound is denoted by the solid line, while the projected reaches of several noble liquid direct detection experiments are shown by the dashed lines in frame (a). In frame (b), we show the current IceCube limit by the red-solid line and projected reaches of future detectors XENON1T (dashed-purple) and PICO-500 (dashed-blue).

is determined by its thermal value, it will escape detection via SD neutralino-nucleon scattering in the case of the nGMM.

In Fig. 6 we plot the values of $\xi^2\langle\sigma v\rangle$, the thermally averaged neutralino annihilation cross section times velocity, versus the lightest neutralino mass for the same scan as in Fig. 5. Higgsino-like neutralinos in the range of interest dominantly annihilate to W^+W^- pairs. As before, we show results for $\xi = 1$ by black dots, and for ξ determined assuming the neutralino relic density is given by its thermal value by green dots. The solid red line shows the upper bound on the neutralino cross section, assuming

V. CONCLUDING REMARKS
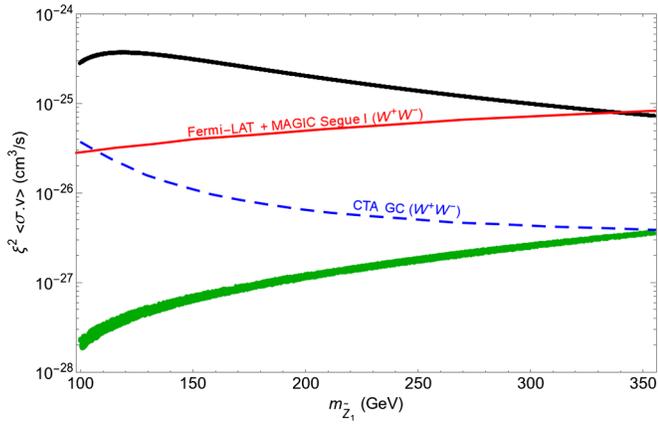
FIG. 6. The neutralino annihilation cross section times velocity scaled by ξ^2 in the $m_{\tilde{Z}_1}$ vs $\xi^2\langle\sigma v\rangle$ plane from a scan over $m_{3/2}$, α and μ. The black points have $\xi = 1$ while the green points have $\xi < 1$. The solid red line shows the upper bound from FERMI-LAT Collaboration (assuming neutralino annihilation to W^+W^- pairs) while the dashed blue line shows the corresponding projected reach of Cherenkov Telescope Array.

annihilation to W boson pairs, obtained in Ref. [52] by combining the dwarf-spheroidal data from the Fermi-LAT collaboration and the MAGIC Collaboration.[6] Taken at face value, this analysis excludes the possibility that higgsino relics dominate the CDM density over almost the entire mass range favored by electroweak naturalness.[7] In contrast, if we assume that the higgsino contribution to the DM density is given by its thermal expectation, it appears that in nGMM dark matter indirect detection via the gamma ray signal would be very difficult even at the proposed ground-based Cherenkov Telescope Array, projections for which are shown by the dashed blue line in the figure [53].

The simplest renditions of the very intriguing model of mirage mediation seem to be strongly disfavored by naturalness considerations, when combined with the measured value of the Higgs boson mass and lower limits from the LHC on superparticle masses. However, several groups have observed that while MM gaugino mass predictions are very robust, the scalar sector is quite sensitive to the mechanisms for moduli stabilization and potential uplifting. Here, we advocated a generalized version of MM where discrete parameters depending on modular weights are elevated to continuous ones to parametrize more general possibilities for moduli stabilization and potential uplifting. The added flexibility of general mirage mediation allows for construction of natural GMM models which are consistent with LHC Higgs mass measurements and sparticle search constraints. We exhibit a benchmark point with a natural superpartner spectrum which maintains mirage unification in the gaugino sector. The resulting spectrum, while highly natural, will likely elude LHC searches even at very high luminosity. In the nGMM, prospects for dark matter detection are also modified significantly from expectations in natural scenarios with GUT scale gaugino mass unification. The possibility that (nonthermally produced) higgsinos comprise all the DM appears to be excluded by the combined FERMI-LAT-MAGIC analysis. If instead the WIMP density is given by its thermal value, with the remainder being composed for instance of axions, then multiton noble liquid detectors such as LZ or XENONnT or others will be required for detection. For the nGMM scenario, the resolving power of ILC may well offer the best hope to unearth the predicted light higgsinos signal. If ILC finds such a signal, it is possible that fits to the gaugino masses may allow for measurements of the relative moduli/anomaly mixing (α) parameter and the gravitino mass $m_{3/2}$.

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[6]For the mass range of our interest the limit is mainly dominated by FERMI-LAT observations.

[7]For the nGMM model scan that we are discussing, we have checked that the Fermi-LAT constraint restricts the higgsino component of the dark matter to no more than $\sim$35% (50%) [85%] for $m_{\tilde{Z}_1} = 150$ GeV (200 GeV) [300 GeV].

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[1] S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, De Sitter vacua in string theory, Phys. Rev. D **68**, 046005 (2003).

[2] S. Ferrara, L. Girardello, and H. P. Nilles, Breakdown of local supersymmetry through gauge fermion condensates, Phys. Lett. **125B**, 457 (1983); H. P. Nilles, Gaugino condensation and SUSY breakdown, arXiv:hep-th/0402022.

[3] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski, and S. Pokorski, Stability of flux compactifications and the pattern

of supersymmetry breaking, J. High Energy Phys. 11 (2004) 076; , Soft supersymmetry breaking in KKLT flux compactification, Nucl. Phys. **B718**, 113 (2005).J. P. Conlon, F. Quevedo, and K. Suruliz, Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking, J. High Energy Phys. 08 (2005) 007.

 [4] A. Chamseddine, R. Arnowitt, and P. Nath, Locally Supersymmetric Grand Unification, Phys. Rev. Lett. **49**, 970 (1982); R. Barbieri, S. Ferrara, and C. Savoy, Gauge models with spontaneously broken local supersymmetry, Phys. Lett. **119B**, 343 (1982); N. Ohta, Grand unified theories based on local supersymmetry, Prog. Theor. Phys. **70**, 542 (1983); L. Hall, J. Lykken, and S. Weinberg, Supergravity as the messenger of supersymmetry breaking, Phys. Rev. D **27**, 2359 (1983).

 [5] L. Randall and R. Sundrum, Out of this world supersymmetry breaking, Nucl. Phys. **B557**, 79 (1999); G. F. Giudice, M. Luty, H. Murayama, and R. Rattazzi, Gaugino mass without singlets, J. High Energy Phys. 12 (1998) 027; J. Bagger, T. Moroi, and E. Poppitz, Anomaly mediation in supergravity theories, J. High Energy Phys. 04 (2000) 009; P. Binetruy, M. K. Gaillard, and B. Nelson, One loop soft supersymmetry breaking terms in superstring effective theories, Nucl. Phys. **B604**, 32 (2001).

 [6] K. Choi, K-S. Jeong, and K. Okumura, Phenomenology of mixed modulus-anomaly mediation in fluxed string compactifications and brane models, J. High Energy Phys. 09 (2005) 039.

 [7] A. Falkowski, O. Lebedev, and Y. Mambrini, SUSY phenomenology of KKLT flux compactifications, J. High Energy Phys. 11 (2005) 034.

 [8] K. Choi, K. S. Jeong, T. Kobayashi, and K.-I. Okumura, TeV scale mirage mediation and natural little SUSY hierarchy, Phys. Rev. D **75**, 095012 (2007).

 [9] M. Endo, M. Yamaguchi, and K. Yoshioka, A Bottom-up approach to moduli dynamics in heavy gravitino scenario: Superpotential, soft terms and sparticle mass spectrum, Phys. Rev. D **72**, 015004 (2005).

 [10] R. Kitano and Y. Nomura, Dark matter before the LHC in a natural supersymmetric standard model, Phys. Lett. B **632**, 162 (2006).

 [11] H. Baer, E. Park, X. Tata, and T. T. Wang, Collider and dark matter searches in models with mixed modulus-anomaly mediated SUSY breaking, J. High Energy Phys. 08 (2006) 041.

 [12] K. Choi and H. P. Nilles, The Gaugino code, J. High Energy Phys. 04 (2007) 006 stress the robustness of the gaugino mass relation in MM models. See also O. Lebedev, V. Löwen, Y. Mambrini, H. P. Nilles, and M. Ratz, Metastable Vacua in Flux Compactifications and Their Phenomenology, J. High Energy Phys. 02 (2007) 063; E. Dudas, C. Papineau, and S. Pokorski, Moduli stabilization and uplifting with dynamically generated F-terms, J. High Energy Phys. 02 (2007) 028; H. Abe, T. Higaki, and Y. Omura, Moduli stabilization, F-term uplifting and soft supersymmetry breaking terms, Phys. Rev. D **75**, 025019 (2007) for earlier work leading up to this.

 [13] H. Baer, V. Barger, and A. Mustafayev, Implications of a 125 GeV Higgs scalar for LHC SUSY and neutralino dark matter searches, Phys. Rev. D **85**, 075010 (2012).

 [14] ATLAS Collaboration, Report No. ATLAS-CONF-2016-052.

 [15] H. Baer, V. Barger, and D. Mickelson, How conventional measures overestimate electroweak fine-tuning in supersymmetric theory, Phys. Rev. D **88**, 095013 (2013).

 [16] H. Baer, V. Barger, D. Mickelson, and M. Padeffke-Kirkland, SUSY models under siege: LHC constraints and electroweak fine-tuning, Phys. Rev. D **89**, 115019 (2014).

 [17] A. Mustafayev and X. Tata, Supersymmetry, Naturalness, and Light Higgsinos, Indian J. Phys. **88**, 991 (2014).

 [18] J. Ellis, K. Enqvist, D. Nanopoulos, and F. Zwirner, Observables in Low-Energy Superstring Models, Mod. Phys. Lett. A **01**, 57 (1986); R. Barbieri and G. Giudice, Upper Bounds on Supersymmetric Particle Masses, Nucl. Phys. **B306**, 63 (1988).

 [19] S. K. Soni and H. A. Weldon, Analysis of the supersymmetry breaking induced by $N = 1$ supergravity theories, Phys. Lett. **126B**, 215 (1983); V. S. Kaplunovsky and J. Louis, Model independent analysis of soft terms in effective supergravity and in string theory, Phys. Lett. B **306**, 269 (1993); A. Brignole, L. E. Ibanez, and C. Munoz, Towards a theory of soft terms for the supersymmetric Standard model, Nucl. Phys. **B422**, 125 (1994); Erratum: Towards a theory of soft terms for the supersymmetric Standard model, Nucl. Phys. **B436**, E747 (1995).

 [20] H. Baer, V. Barger, P. Huang, A. Mustafayev, and X. Tata, Radiative Natural SUSY with a 125 GeV Higgs Boson, Phys. Rev. Lett. **109**, 161802 (2012).

 [21] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, and X. Tata, Radiative natural supersymmetry: Reconciling electroweak fine-tuning and the Higgs boson mass, Phys. Rev. D **87**, 115028 (2013).

 [22] H. Baer, V. Barger, and M. Savoy, Upper bounds on sparticle masses from naturalness or how to disprove weak scale supersymmetry, Phys. Rev. D **93**, 035016 (2016).

 [23] K. L. Chan, U. Chattopadhyay, and P. Nath, Naturalness, weak scale supersymmetry and the prospect for the observation of supersymmetry at the Tevatron and at the CERN LHC, Phys. Rev. D **58**, 096004 (1998); R. Kitano and Y. Nomura, Supersymmetry, naturalness, and signatures at the LHC, Phys. Rev. D **73**, 095004 (2006); R. Barbieri and D. Pappadopulo, S-particles at their naturalness limits, J. High Energy Phys. 10 (2009) 061; H. Baer, V. Barger, and P. Huang, Hidden SUSY at the LHC: the light higgsino-world scenario and the role of a lepton collider, J. High Energy Phys. 11 (2011) 031; M. Papucci, J. T. Ruderman, and A. Weiler, Natural SUSY endures, J. High Energy Phys. 09 (2012) 035; C. Brust, A. Katz, S. Lawrence, and R. Sundrum, SUSY, the Third Generation and the LHC, J. High Energy Phys. 03 (2012) 103.

 [24] L. L. Everett, I. W. Kim, P. Ouyang, and K. M. Zurek, Deflected Mirage Mediation: A Framework for Generalized Supersymmetry Breaking, Phys. Rev. Lett. **101**, 101803 (2008); Moduli Stabilization and supersymmetry breaking in deflected mirage mediation, J. High Energy Phys. 08 (2008) 102; K. Choi, K. S. Jeong, S. Nakamura, K. I. Okumura, and M. Yamaguchi, Sparticle masses in deflected mirage mediation, J. High Energy Phys. 04 (2009) 107; B.

Altunkaynak, B. D. Nelson, L. L. Everett, I. W. Kim, and Y. Rao, Phenomenological implications of deflected mirage mediation: Comparison with mirage mediation, J. High Energy Phys. 05 (2010) 054; B. Altunkaynak, B. D. Nelson, L. L. Everett, Y. Rao, and I. W. Kim, Landscape of supersymmetric particle mass hierarchies in deflected mirage mediation, Eur. Phys. J. Plus **127**, 2 (2012); L. L. Everett, T. Garon, B. L. Kaufman, and B. D. Nelson, Mirage models confront the LHC: III. Deflected mirage mediation, Phys. Rev. D **93**, 055031 (2016).

[25] V. Barger, L. L. Everett, and T. S. Garon, Electroweak naturalness and deflected mirage mediation, Phys. Rev. D **93**, 075024 (2016).

[26] D. Chowdhury and N. Yokozaki, Muon $g - 2$ in anomaly mediated SUSY breaking, J. High Energy Phys. 08 (2015) 111.

[27] K. Choi and K. S. Jeong, String theoretic QCD axion with stabilized saxion and the pattern of supersymmetry breaking, J. High Energy Phys. 01 (2007) 103.

[28] H. Murayama, H. Suzuki, and T. Yanagida, Radiative breaking of Peccei-Quinn symmetry at the intermediate mass scale, Phys. Lett. B **291**, 418 (1992); K. J. Bae, H. Baer, and H. Serce, Natural little hierarchy for SUSY from radiative breaking of the Peccei-Quinn symmetry, Phys. Rev. D **91**, 015003 (2015).

[29] H. Baer, V. Barger, M. Savoy, and H. Serce, The Higgs mass and natural supersymmetric spectrum from the landscape, Phys. Lett. B **758**, 113 (2016).

[30] D. Matalliotakis and H. P. Nilles, Implications of non-universality of soft terms in supersymmetric grand unified theories, Nucl. Phys. **B435**, 115 (1995); P. Nath and R. L. Arnowitt, Nonuniversal soft SUSY breaking and dark matter, Phys. Rev. D **56**, 2820 (1997); J. Ellis, K. Olive, and Y. Santoso, The MSSM parameter space with non-universal Higgs masses, Phys. Lett. B **539**, 107 (2002); J. Ellis, T. Falk, K. Olive, and Y. Santoso, Exploration of the MSSM with non-universal Higgs masses, Nucl. Phys. **B652**, 259 (2003); H. Baer, A. Mustafayev, S. Profumo, A. Belyaev, and X. Tata, Direct, indirect and collider detection of neutralino dark matter in SUSY models with non-universal Higgs masses, J. High Energy Phys. 07 (2005) 065.

[31] H. Baer, F. Paige, S. Protopopescu, and X. Tata, ISAJET 7.85, arXiv:hep-ph/0312045; Isasugra, by H. Baer, C. H. Chen, R. B. Munroe, F. E. Paige, and X. Tata, Multichannel search for minimal supergravity at $p\bar{p}$ and e^+e^- colliders, Phys. Rev. D **51**, 1046 (1995).

[32] H. Baer, V. Barger, A. Lessa, and X. Tata, Discovery potential for SUSY at a high luminosity upgrade of LHC14, Phys. Rev. D **86**, 117701 (2012).

[33] ATLAS Collaboration, Physics at a high-luminosity LHC with ATLAS, arXiv:1307.7292.

[34] H. Baer, V. Barger, M. Savoy, and X. Tata, Multichannel assault on natural supersymmetry at the high luminosity LHC, Phys. Rev. D **94**, 035025 (2016).

[35] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, W. Sreethawong, and X. Tata, Same sign diboson signature from supersymmetry models with light higgsinos at the LHC, Phys. Rev. Lett. **110**, 151801 (2013).

[36] CMS Collaboration, Report No. CMS-PAS-SUS-16-025.

[37] H. Baer, E. K. Park, X. Tata, and T. T. Wang, Measuring modular weights in mirage unification models at the LHC and ILC, Phys. Lett. B **641**, 447 (2006).

[38] H. Baer, V. Barger, D. Mickelson, A. Mustafayev, and X. Tata, Physics at a Higgsino factory, J. High Energy Phys. 06 (2014) 172.

[39] J. List, in *38th International Conference on High Energy Physics: ICHEP2016*, Chicago, USA, 2016 (unpublished); H. Baer, M. Berggren, K. Fujii, S. L. Lehtinen, J. List, T. Tanabe, and J. Yan, Naturalness and light higgsinos: A powerful reason to build the ILC, arXiv:1611.02846.

[40] K. J. Bae, H. Baer, and E. J. Chun, Mainly axion cold dark matter from natural supersymmetry, Phys. Rev. D **89**, 031701 (2014).

[41] K. J. Bae, H. Baer, and A. Lessa, Dark radiation constraints on mixed axion/neutralino dark matter, J. Cosmol. Astropart. Phys. 04 (2013) 041; K. J. Bae, H. Baer, A. Lessa, and H. Serce, Coupled Boltzmann computation of mixed axion neutralino dark matter in the SUSY DFSZ axion model, J. Cosmol. Astropart. Phys. 10 (2014) 082.

[42] D. Akerib *et al.* (LUX Colloboaration), Results from a search for dark matter in the complete LUX exposure, arXiv:1608.07648.

[43] E. Aprile *et al.*, Physics reach of the XENON1T dark matter experiment, J. Cosmol. Astropart. Phys. 04 (2016) 027.

[44] D. S. Akerib *et al.* (LZ Collaboration), LUX-ZEPLIN (LZ) conceptual design report, arXiv:1509.02910.

[45] P. Agnes *et al.* (DarkSide Collaboration), Direct search for dark matter with DarkSide, J. Phys. Conf. Ser. **650**, 012006 (2015).

[46] P.-A. Amaudruz *et al.* (DEAP Collaboration), DEAP-3600 dark matter search, Nucl. Part. Phys. Proc. 273–275, 340 (2016).

[47] J. Aalbers *et al.* (DARWIN Collaboration), DARWIN: Towards the ultimate dark matter detector, J. Cosmol. Astropart. Phys. 11 (2016) 017.

[48] H. Baer, V. Barger, and D. Mickelson, Direct and indirect detection of Higgsino-like WIMPs: Concluding the story of electroweak naturalness, Phys. Lett. B **726**, 330 (2013); K. J. Bae, H. Baer, V. Barger, M. R. Savoy, and H. Serce, Supersymmetry with radiatively driven naturalness: Implications for WIMP and axion searches, Symmetry **7**, 788 (2015).

[49] H. Baer, V. Barger, and H. Serce, SUSY under siege from direct and indirect WIMP detection experiments, arXiv:1609.06735.

[50] M. Aartsen *et al.*, Improved limits on dark matter annihilation in the Sun with the 79-string IceCube detector and implications for supersymmetry, J. Cosmol. Astropart. Phys. 04 (2016) 022.

[51] C. Kraus, in *38th International Conference on High Energy Physics: ICHEP2016* Chicago, 2016 (unpublished).

[52] M. Ahlen *et al.*, Limits to dark matter annihilation cross-section from a combined analysis of MAGIC and Fermi-LAT observations of dwarf satellite galaxies, J. Cosmol. Astropart. Phys. 02 (2016) 039.

[53] M. Wood, J. Buckley, S. Digel, D. Nieto, and M. Sanchez-Conde, Prospects for Indirect Detection of Dark Matter with CTA, arXiv:1305.0302.