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Lepton number violation in 331 models

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Different models based on the extended $SU(3)_C \times SU(3)_L \times U(1)_X$ (331) gauge group have been proposed over the past four decades. Yet, despite being an active research topic, the status of lepton number in 331 models has not been fully addressed in the literature, and furthermore many of the original proposals can not explain the observed neutrino masses. In this paper we review the basic features of various 331 models, focusing on potential sources of lepton number violation. We then describe different modifications which can be made to the original models in order to accommodate neutrino (and charged lepton) masses.

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I. INTRODUCTION

It is conceivable that the Standard Model gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ (321) is just a remnant of a larger one. Indeed, such scenarios are attractive as they are able to unify the three gauge couplings, provided that the extended gauge group is simple [1–4]. However, one should not exclude the possibility that the enlarged group is a product of simple factors. This could happen as an intermediate step towards a grand unified group. A famous example is the left-right symmetric group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [5–7], which fits neatly into $SO(10)$. Another possibility is $SU(3)_C \times SU(3)_L$, yet with such models one cannot get the correct fermion masses [8]. On the other hand, it was realized long ago [9–11] that with an extra $U(1)_X$ it is possible to construct viable models.

These $SU(3)_C \times SU(3)_L \times U(1)_X$ (331) models have received considerable attention in connection with various topics: neutrino mass generation [12–31], flavor symmetries [32–43], quark flavor observables [44–51] or the recent LHC diphoton excess [52–57], among others. Underpinning this interest is the fact that the 331 to 321 symmetry breaking energy scale can be of the TeV order; hence, it could possibly be explored at the LHC; see for example [58–65].

However, despite the large list of papers on 331 models, the issue of lepton number violation (LNV) has not been fully addressed in the literature and, in fact, many misleading statements on the subject can be found in papers on 331 models. It turns out that models based on this extended symmetry can be quite different from one another since the way the 321 group is embedded in the 331 group is not unique. In particular (a) the existence of neutrino masses, (b) the nature of neutrinos and (c) the status of lepton number varies markedly among 331 models. As such, with this work we intend to collect and summarize the relevant information concerning lepton number and neutrino mass generation in this class of models.

We have found that several of the originally proposed 331 models can not explain correctly the observed neutrino masses (nor charged lepton masses, in one case). Thus, it is necessary to extend these models, and we present several possible modifications that can bring these models in agreement with experimental data, some of which have already been considered before [14,66–70]. We focus here (mostly) on neutrino masses and mixings and leave aside other LNV processes, which we mention only briefly when it is relevant.

The rest of this paper is organized as follows. Section II describes the basic features of six different 331 models. Four of these fall into a particular subclass since they have a common structure (they all follow what we call the SVS framework, after its prototype model [9]). To cover the full variety of 331 models, we then discuss two more models, which do not follow the SVS scheme, and clarify LNV related issues in them as well. None of the basic models in the SVS class generates lepton masses and mixings in a fully satisfactory way, hence modifications are required. A list of simple improvements is discussed in Sec. III. For each of the possibilities in our list we give a brief description on how the modified versions of the original models can be brought into agreement with experimental neutrino (and charged lepton) data. Finally, in Sec. IV we summarize the most important points in this manuscript. An appendix at the end of the text provides supplementary information.

II. THE $SU(3)_L \times U(1)_X$ GROUP AND BASIC 331 MODELS

One can build different 331 models, not just by changing the field content, but also by varying the way in which the SM electroweak gauge group is embedded in $SU(3)_L \times U(1)_X$. This can be encoded in a continuous parameter β which controls the relation between the hypercharge Y, X, and the T_8 generator of $SU(3)_L$

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$$Y = \beta T_8 + X. \tag{1}$$

From here one can derive that $SU(3)_L \times U(1)_X$ representations break as follows into $SU(2)_L \times U(1)_Y$ representations (more details can be found in the Appendix)[1]:

$$(\mathbf{3}, x) \to \left(\hat{\mathbf{2}}, x + \frac{1}{2\sqrt{3}}\beta\right) + \left(\hat{\mathbf{1}}, x - \frac{1}{\sqrt{3}}\beta\right), \tag{2}$$

$$(\bar{\mathbf{6}}, x) \to \left(\hat{\mathbf{3}}, x - \frac{1}{\sqrt{3}}\beta\right) + \left(\hat{\mathbf{2}}, x + \frac{1}{2\sqrt{3}}\beta\right) + \left(\hat{\mathbf{1}}, x + \frac{2}{\sqrt{3}}\beta\right), \tag{3}$$

$$(\mathbf{8}, 0) \to (\hat{\mathbf{3}}, 0) + (\hat{\mathbf{1}}, 0) + \left(\hat{\mathbf{2}}, -\frac{\sqrt{3}}{2}\beta\right) + \left(\hat{\mathbf{2}}, \frac{\sqrt{3}}{2}\beta\right). \tag{4}$$

Together with the requirement $|\beta| < \tan^{-1}\theta_W \approx 1.8$ [obtainable from equation (A11) and the fact that g_X^2 must be positive], these equations show that there are only four values of β for which it is possible to avoid colorless, fractionally charged fermions. Bearing this constraint in mind, we can then describe six different 331 models

(i) In the first four models, the three lepton families are in equal representations, but the quarks are not. The structure of these models is similar, with the main difference between them being the value of β: $-1/\sqrt{3}$ in the Singer-Valle-Schechter (SVS) model [9], $-\sqrt{3}$ in the Pisano-Pleitez-Frampton (PPF) model [10,11], $1/\sqrt{3}$ in the Pleitez-Özer model [71,72], and $\sqrt{3}$ in what we call the model X. They all share a common structure, which we call the SVS framework below.

(ii) The flipped model [73], where quark families are all in the same representations, but leptons are not.

(iii) The E_6 model [74], where complete family replication is true for both the lepton and quark sectors.

A. The Singer-Valle-Schechter (SVS) model

The first 331 model with three generations of quarks and leptons was proposed in [9], using $\beta = -1/\sqrt{3}$. As stated previously, this model can be considered the prototype model for what might be called the SVS framework. All four models in this class have in common the following features:

(i) The SM lepton doublets are placed in three triplets of $SU(3)_L$.[2]

(ii) Two families of left-handed quarks are placed inside antitriplets of $SU(3)_L$ while the third one is placed in a triplet.

(iii) Extra $SU(3)_L$ fermion singlets are necessary in order to include some of the SM $SU(2)_L$ singlets, and also to provide the necessary vector partners to some extra fermions contained in the triplets of $SU(3)_L$.

(iv) Three scalar triplets of $SU(3)_L$ are used to generate the necessary Yukawa interactions with fermions.

These conditions guarantee that models in this class recover correctly the SM fermion content in the limit where 331 is first broken to 321, and they also have a sufficiently large scalar sector to achieve both 331 symmetry breaking and a realistic quark spectrum.

In the specific case of the SVS model where $\beta = -1/\sqrt{3}$, right-handed neutrinos, here denoted N^c, are included in the same extended gauge multiplet ψ_ℓ as the SM left-handed leptons. The full field content of the original SVS model is shown in Table I. In addition to the SM fermions, extra vectorlike quarks appear, which are a common feature of all 331 models.

To determine whether or not there is lepton number conservation in a given model, one can simply attempt to build diagrams describing processes where the number of leptons changes. Finding one such diagram would prove conclusively that there is LNV. On the other hand, if one is able to show that no such diagram exists, then lepton number is preserved (perturbatively at least). The latter, however, can be quite cumbersome, when worked out with the language of Feynman diagrams.

In practice, thus, it is better to replace this pragmatic approach by the following simpler one: show whether or not the total Lagrangian of the model has a global $U(1)_L$ symmetry under which the SM (anti)leptons have $+1(-1)$ charge, and (anti)quarks as well as the SM gauge bosons have no charge.[3] Lepton number is violated if and only if no such symmetry exists.

If there is LNV, then usually there is no single coupling which is responsible for it—rather, it is the existence of several interactions which gives rise to the phenomenon. Nevertheless, in practice only a few of the couplings in a given model are relevant for LNV and in their absence, the Lagrangian gains a $U(1)_L$ symmetry with the characteristics previously described. However, this means that one can have situations where the removal of either of two sets of interactions—$\{I_i\}$, $\{I'_i\}$—both lead to a lepton number conserving scenario, hence the procedure of labeling LNV interactions is not unique, see below.

Finally, one has to bear in mind that, even if the Lagrangian is $U(1)_L$ preserving, it is still possible for lepton number to be broken spontaneously by the vacuum expectation value (VEV) of scalars which carry a nonzero $U(1)_L$ charge.

[1]Hats are added to $SU(2)_L$ representations to avoid confusion between 331 and 321 representations.

[2]The original model in [9] also contained right-handed neutrinos in $SU(3)_L$ singlets, which were latter removed [75]. Here, we call this variation of the original proposal "the SVS model".

[3]We stress here that this $U(1)_L$ does not need to commute with the remaining symmetries of the model [in particular, the $SU(3)_L \times U(1)_X$ gauge symmetry in 331 models].

TABLE I. Field content of the Singer-Valle-Schechter (SVS) model [9]. The indices α and i denote different flavors.

Field	331 Representation	G_{SM} decomposition	# Flavors	Components	Lepton number
$\psi_{\ell,\alpha}$	$(\mathbf{1},\mathbf{3},-\frac{1}{3})$	$(\mathbf{1},\hat{\mathbf{2}},-\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},0)$	3	$((\nu_\alpha,\ell_\alpha),N^c_\alpha)^T$	$(1,1,-1)^T$
ℓ^c_α	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\hat{\mathbf{1}},1)$	3	ℓ^c_α	-1
$Q_{\alpha=1,2}$	$(\mathbf{3},\bar{\mathbf{3}},0)$	$(\mathbf{3},\hat{\mathbf{2}},\frac{1}{6})+(\mathbf{3},\hat{\mathbf{1}},-\frac{1}{3})$	2	$((d_\alpha,-u_\alpha),D_\alpha)^T$	$(0,0,2)^T$
Q_3	$(\mathbf{3},\mathbf{3},\frac{1}{3})$	$(\mathbf{3},\hat{\mathbf{2}},\frac{1}{6})+(\mathbf{3},\hat{\mathbf{1}},\frac{2}{3})$	1	$((t,b),U)^T$	$(0,0,-2)^T$
u^c_α	$(\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})$	$(\bar{\mathbf{3}},\hat{\mathbf{1}},-\frac{2}{3})$	4	u^c_α	0
d^c_α	$(\bar{\mathbf{3}},\mathbf{1},\frac{1}{3})$	$(\bar{\mathbf{3}},\hat{\mathbf{1}},\frac{1}{3})$	5	d^c_α	0
ϕ_1	$(\mathbf{1},\mathbf{3},\frac{2}{3})$	$(\mathbf{1},\hat{\mathbf{2}},\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},1)$	1	$((\phi_1^+,\phi_1^0),\tilde{\phi}_1^+)^T$	$(0,0,-2)^T$
$\phi_{i=2,3}$	$(\mathbf{1},\mathbf{3},-\frac{1}{3})$	$(\mathbf{1},\hat{\mathbf{2}},-\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},0)$	2	$((\phi_i^0,\phi_i^-),\tilde{\phi}_i^0)^T$	$(0,0,-2)^T$

We now exemplify once the application of these well-known (but often neglected) comments, and derive the $U(1)_L$ charges in the last column of Table I, which correspond to the SVS model. For reasons which will become obvious later, we first put the coefficient of the term $\phi_1\phi_2\phi_3$ to zero. Using the field notation in that table, we then may start from the lepton Yukawa interactions $\psi_\ell\psi_\ell\phi_1$ and $\psi_\ell\ell^c\phi_1^*$: from the first one it follows that $L(\phi_1^+)=L(\phi_1^0)=-1-L(N^c_\alpha)$ and $L(\tilde{\phi}_1^+)=-2$, while from the second interaction we conclude that $L(\phi_1^+)=L(\phi_1^0)=0$ and $L(\tilde{\phi}_1^+)=-1-L(N^c_\alpha)$. Hence $L(N^c_\alpha)=-1$ and therefore $L(\psi_{\ell,\alpha})=(1,1,-1)^T$ and $L(\phi_1)=(0,0,-2)^T$. Moving along to the quark sector, we do not know the lepton number of the third component of the multiplets $Q_{1,2}$ and Q_3 (which we call $D_{1,2}$ and U respectively), but these can be inferred from the interactions $Q_{1,2}u^c\phi_1$ and $Q_3d^c\phi_1^*$. Indeed, from the first interaction it follows that $L(D_{1,2})=2$, while the second one yields $L(U)=-2$. At this point, the only $U(1)_L$ fermion/scalar charges yet to be found are those of the components of the scalar triplets $\phi_{2,3}$. But from the interactions $Q_{1,2}d^c\phi_{2,3}$ one readily obtains that $L(\phi_{2,3})=(0,0,-2)^T$. Note that the extra Yukawa coupling $Q_3u^c\phi_{2,3}^*$ does preserve this lepton number assignment.

It is clear that the constrains on the $U(1)_L$ charges discussed above form a linear system of equations, which can be solved at once

$$
\begin{array}{r}
\psi_\ell\psi_\ell\phi_1\left\{\begin{array}{c}\\ \\ \\ \end{array}\right.\\
\psi_\ell\ell^c\phi_1^*\left\{\begin{array}{c}\\ \\ \\ \end{array}\right.\\
Q_{1,2}u^c\phi_1\left\{\begin{array}{c}\\ \\ \\ \end{array}\right.\\
Q_3d^c\phi_1^*\left\{\begin{array}{c}\\ \\ \\ \end{array}\right.\\
Q_{1,2}d^c\phi_{2,3}\left\{\begin{array}{c}\\ \\ \\ \end{array}\right.\\
Q_3u^c\phi_{2,3}^*\left\{\begin{array}{c}\\ \\ \\ \end{array}\right.
\end{array}
\begin{pmatrix}
0&0&0&0&0&1&0&0&0\\
1&0&0&0&1&0&0&0&0\\
1&0&0&1&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0\\
1&0&0&0&0&-1&0&0&0\\
0&0&0&1&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0\\
0&1&0&0&0&1&0&0&0\\
0&0&0&1&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0\\
0&0&1&0&0&-1&0&0&0\\
0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&1&0\\
0&1&0&0&0&0&0&0&1\\
0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&1&0\\
0&0&1&0&0&0&0&0&-1
\end{pmatrix}
\cdot
\begin{pmatrix}
L(N^c_\alpha)\\
L(D_{1,2})\\
L(U)\\
L(\phi_1^+)\\
L(\phi_1^0)\\
L(\tilde{\phi}_1^+)\\
L(\phi_{2,3}^0)\\
L(\phi_{2,3}^-)\\
L(\tilde{\phi}_{2,3}^0)
\end{pmatrix}
=
\begin{pmatrix}
-2\\-1\\-1\\0\\0\\1\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0\\0
\end{pmatrix}.
\tag{5}
$$

We now turn to gauge bosons. The $SU(3)_L$ gauge interactions for triplets $T = (T_1, T_2, T_3)^T$ and antitriplets $A = (A_1, A_2, A_3)^T$ are of the forms $-ig_L \bar{T}\gamma^\mu \mathcal{M}_\mu T$ and $ig_L \bar{A}\gamma^\mu \mathcal{M}_\mu^T A$ with

$$\mathcal{M}_\mu = \frac{1}{2}\begin{pmatrix} W^3_{L,\mu} + \frac{W^8_{L,\mu}}{\sqrt{3}} & W^1_{L,\mu} - iW^2_{L,\mu} & W^4_{L,\mu} - iW^5_{L,\mu} \\ W^1_{L,\mu} + iW^2_{L,\mu} & \frac{W^8_{L,\mu}}{\sqrt{3}} - W^3_{L,\mu} & W^6_{L,\mu} - iW^7_{L,\mu} \\ W^4_{L,\mu} + iW^5_{L,\mu} & W^6_{L,\mu} + iW^7_{L,\mu} & -\frac{2W^8_{L,\mu}}{\sqrt{3}} \end{pmatrix}. \quad (6)$$

So, given that the lepton number of the components of triplets and antitriplets are always of the form of either $(x, x, x-2)$ or $(y, y, y+2)$ for some arbitrary values of x and y, it is clear that gauge interactions preserve the $U(1)_L$ we have been discussing, with $L(W^{1,2,3,8}_{L,\mu}) = 0$ while $\frac{1}{\sqrt{2}}(W^4_{L,\mu} \pm iW^5_{L,\mu})$ and $\frac{1}{\sqrt{2}}(W^6_{L,\mu} \pm iW^7_{L,\mu})$ carry ∓ 2 units of lepton number.

One can easily check that with these assignments all terms in the scalar potential—except one—conserve the $U(1)_L$. This particular term is identified as $\phi_1\phi_2\phi_3$ and with the assignments given in Table I it violates $U(1)_L$ by two units. If we had switched off the interactions $\psi_\ell\psi_\ell\phi_1$ or $\psi_\ell\ell^c\phi_1^*$ instead of following the procedure above, different $U(1)_L$ symmetries could be defined. Thus, as discussed previously, it is the simultaneous presence of various couplings which violates explicitly lepton number. We remind, however, that even in the absence of the trilinear term $\phi_1\phi_2\phi_3$ the SVS model does break $U(1)_L$ spontaneously through nonzero VEVs in the third component of the scalars $\phi_{2,3}$. In the form just presented, the SVS model is not viable as it cannot accommodate the known neutrino oscillation data. This can be understood as follows. The $\psi_\ell\psi_\ell\phi_1$ interaction is completely antisymmetric in the flavor indices. This leads to the tree-level prediction of a degenerate light neutrino mass spectrum with eigenvalues $(0, m, m)$. Since lepton number is violated in the SVS model, one expects that radiative corrections to this tree-level result will generate Majorana neutrino masses and lead to a nonzero splitting of the degenerate states. Figure 1 shows an example. However, in the original SVS model all loop corrections to neutrino masses are necessarily themselves proportional to the $\psi_\ell\psi_\ell\phi_1$ interaction, which is the coupling responsible for the generation of neutrino masses at tree level. (Indeed, any loop contributing to neutrino masses must have an odd number of $\psi_\ell\psi_\ell\phi_1$ interactions.) The 1-loop corrections are then related to the tree-level mass and the relative size of $\delta m_\nu^{1-\mathrm{loop}}/m_\nu^{\mathrm{tree}}$ can be estimated to be at most $\sim\frac{1}{16\pi^2}h_\tau^2 \times \cdots < 10^{-6}$, where h_τ is the tau Yukawa coupling and the dots stand for other factors which are at most one. We will return to a more explicit calculation of this loop in the next section. For now it suffices to say that neutrino oscillation data requires that the smaller mass splitting in the neutrino sector relative to the larger one must be larger than very roughly 1/6, in gross contradiction to the above estimate for the original SVS model.

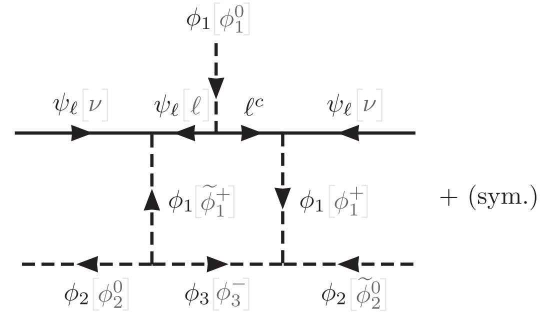

FIG. 1. One loop contribution to neutrino masses in the original SVS model. There are in total four diagrams, since (on top of exchanging the internal ψ_L and l^c) one can exchange everywhere ϕ_2 with ϕ_3.

Just for completeness, note that in the diagram of Fig. 1 the LNV interaction $\phi_1\phi_2\phi_3$ and its conjugate are present; hence, the real source of LNV in this case are the $\tilde{\phi}_2^0$ and $\tilde{\phi}_3^0$ VEVs. This does not, however, mean that the LNV in the trilinear interaction is irrelevant in general. In fact, it is easy to built up diagrams containing $\psi_\ell\psi_\ell\phi_1$, the SM charged current and this trilinear interaction to generate processes such as $e^-e^- \to 4j$ (or $6j$) at loop-level (tree-level).

B. The Pisano-Pleitez-Frampton (PPF) model

Following the generic framework of the SVS model, in 1992 a different 331 model was presented [10,11]. This model chooses $\beta = -\sqrt{3}$, and thus the third component of the lepton triplet field ψ_ℓ has charge $+1$; hence, it is identifiable as a right-handed charged lepton—see Table II.

A central assertion in [10] is that lepton number is violated by charged scalars and gauge bosons. However, we want to stress here that this is not the case. Using the procedure outlined above for the SVS model, the PPF model with the interactions described in [10] preserves the $U(1)_L$ symmetry under which the various fields have the charges indicated in Table II, so there is no explicit lepton number violation in the model as written down in [10]. Moreover, unlike the SVS model, here all neutral scalar components have $L = 0$; hence, there cannot be spontaneous lepton number violation either. Thus the original model of [10] is lepton number conserving. It is important to note, however, that PPF neglected some quartic scalar interactions which are allowed by the gauge symmetry. Most notably it can be shown that the coupling $\phi_1\phi_2\phi_3^*\phi_3^*$, missing in the original paper, violates lepton number by two units.

From now on, we will call the version of this model with the most general gauge invariant Lagrangian the Pisano-Pleitez-Frampton model. This PPF model is indeed lepton

TABLE II. Field content of the Pisano-Pleitez-Frampton (PPF) model [10,11].

Field	331 Representation	$G_{\rm SM}$ decomposition	# Flavors	Components	Lepton number
$\psi_{\ell,\alpha}$	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\hat{\mathbf{2}},-\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},1)$	3	$((\nu_\alpha,\ell_\alpha),\ell^c_\alpha)^T$	$(1,1,-1)^T$
$Q_{\alpha=1,2}$	$(\mathbf{3},\bar{\mathbf{3}},-\frac{1}{3})$	$(\mathbf{3},\hat{\mathbf{2}},\frac{1}{6})+(\mathbf{3},\hat{\mathbf{1}},-\frac{4}{3})$	2	$((d_\alpha,-u_\alpha),J^c_\alpha)^T$	$(0,0,2)^T$
Q_3	$(\mathbf{3},\mathbf{3},\frac{2}{3})$	$(\mathbf{3},\hat{\mathbf{2}},\frac{1}{6})+(\mathbf{3},\hat{\mathbf{1}},\frac{5}{3})$	1	$((t,b),J^c_3)^T$	$(0,0,-2)^T$
u^c_α	$(\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})$	$(\bar{\mathbf{3}},\hat{\mathbf{1}},-\frac{2}{3})$	3	u^c_α	0
d^c_α	$(\bar{\mathbf{3}},\mathbf{1},\frac{1}{3})$	$(\bar{\mathbf{3}},\hat{\mathbf{1}},\frac{1}{3})$	3	d^c_α	0
$J_{\alpha=1,2}$	$(\bar{\mathbf{3}},\mathbf{1},\frac{4}{3})$	$(\bar{\mathbf{3}},\hat{\mathbf{1}},\frac{4}{3})$	2	J_α	−2
J_3	$(\bar{\mathbf{3}},\mathbf{1},-\frac{5}{3})$	$(\bar{\mathbf{3}},\hat{\mathbf{1}},-\frac{5}{3})$	1	J_3	2
ϕ_1	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\hat{\mathbf{2}},\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},2)$	1	$((\phi_1^+,\phi_1^0),\tilde{\phi}_1^{++})^T$	$(0,0,-2)^T$
ϕ_2	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\hat{\mathbf{2}},-\frac{3}{2})+(\mathbf{1},\hat{\mathbf{1}},0)$	1	$((\phi_2^-,\phi_2^{--}),\tilde{\phi}_2^0)^T$	$(2,2,0)^T$
ϕ_3	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\hat{\mathbf{2}},-\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},1)$	1	$((\phi_3^0,\phi_3^-),\tilde{\phi}_3^+)^T$	$(0,0,-2)^T$

number violating. Thus, LNV processes such as neutrinoless double beta decay, will occur. Interestingly, the PPF model, however, does not generate a nonzero neutrino mass.[4] This can be understood by following the possible interactions of the ψ_ℓ triplet, which contains the SM leptons. Apart from gauge interactions, there is only the $y_\ell\psi_\ell\psi_\ell\phi_3$ Yukawa interaction where gauge indices are contracted antisymmetrically. Hence y_ℓ must be an antisymmetric matrix (in flavor space). Yet, one must have an odd number of y_ℓ matrices along the ψ_ℓ fermion line in any diagram contributing to a neutrino mass matrix (see Fig. 2). Hence the flavor matrix $\mathcal{O}$ associated to the effective operator $\mathcal{O}_{\alpha\beta}\psi_{\ell,\alpha}\psi_{\ell,\beta}\times(\text{scalars})$ will always be antisymmetric (note that the gauge interactions do not change flavor). Thus, no $\nu\nu$ term will be generated at any order of a perturbative expansion.

That a LNV model can have zero Majorana neutrino masses but a finite half-life for neutrinoless double beta decay, seems to be a contradiction of the well-known "black-box" theorem [76]. However, this apparent contradiction can be traced to another flaw of the PPF model. In it, the $\psi_\ell\psi_\ell\phi_3$ interaction is the only source of charged lepton masses. Its antisymmetry implies that the tree-level charged lepton masses are $(0,m,m)$. (This prediction is analogous to the one for neutrino masses in the SVS model discussed previously.) This is in clear disagreement with the experimentally observed charged lepton masses and thus requires a modification of the PPF model.[5] Moreover, this prediction for the charged lepton spectrum violates the (implicit) assumption in the formulation of the black box theorem [76,77] that the electron has a nonzero mass. If one follows the procedure given in the original papers on the black box theorem of completing the $0\nu\beta\beta$ decay diagram with charged current interactions, in order to form a Majorana neutrino mass term, one finds that mass insertions are necessary to convert right-handed electrons into left-handed ones. For the PPF model all contributions to $0\nu\beta\beta$ decay produce final states with e_Le_R. The particular prediction for the charged lepton spectrum in this model then leads to an exact zero of the $e_R\to e_L$ insertions, independent of the flavor compositions of the three mass eigenstates. This is most easily seen for the case where the only nonzero entry in y_ℓ is $y_\ell^{\mu\tau}$. In this case, the massless state is the electron, and it is obvious that $e_R\to e_L$ conversion is impossible. For other cases, the two contributions from the degenerate leptons cancel each other exactly. However, one expects that once that the PPF model has been modified to correct for the unrealistic charged lepton spectrum, nonzero Majorana masses will also automatically appear and the standard form of the black-box theorem is recovered. A discussion of modified PPF models is given below in Sec. III.

[4] In the absence of right-handed neutrinos, it would necessarily be Majorana-like.

[5] A modified version of the PPF model, which can accommodate a realistic charged lepton spectrum, was presented shortly after the original one [66]. We will come back to this in the next section.

C. The Pleitez-Özer (PÖ) model

The generic SVS framework with $\beta=1/\sqrt{3}$ gives rise to the Pleitez-Özer model [71,72].[6] In it, the third component of ψ_ℓ has charge −1, so it can be interpreted as the vector partner of the SM right-handed charged leptons ℓ^c. Since there are three flavors of ψ_ℓ, six copies of ℓ^c are then necessary to account for the SM right-handed charged leptons as well as three extra vector fermion pairs (ℓ^c,E). There are no right-handed neutrinos and it can be checked that there is an unbroken global $U(1)_L$ (see Table III). Furthermore, none of the neutral scalars carries lepton number, thus there is also no spontaneous violation of lepton number. Neutrinos are therefore massless and the model is not satisfactory from this point of view.

[6] A basic sketch of this model also appears in [78].

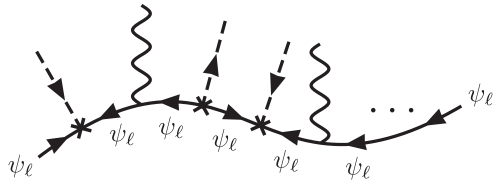

FIG. 2. The only interactions of the ψ_ℓ multiplet in the PPF model are the ones with the gauge bosons (which do not change flavor) and those of the form $\psi_\ell\psi_\ell\phi_3$, which are antisymmetric in flavor space. Since an odd number of these latter interactions are needed to build a mass diagram for ψ_ℓ, such a mass must necessarily be flavor antisymmetric, and hence it cannot generate (Majorana) neutrino masses.

D. Model X

Finally, in the generic SVS framework it is also possible to have $\beta = \sqrt{3}$—we call this the model X. For this value of β, the third component of ψ_ℓ has charge -2. Hence we need the ℓ_X representation (charge $+2$) shown in Table IV to form a massive, vector fermion pair with this state once the 331 symmetry is broken. The SM right-handed charged leptons are then in a separate representation ℓ^c. It is straightforward to check that this model preserves lepton number, just like the Pleitez-Özer model. So, in the absence of right-handed neutrinos, it predicts massless neutrinos.

E. The flipped model

All previous four models follow the SVS framework of placing SM lepton doublets in triplets of $SU(3)_L$, while quark doublets are spread over one triplet and two antitriplets. In other words, the extended gauge symmetry discriminates quark families, but not lepton families. Recently [73] we proposed a new model which reverts this scheme: all three quark families are in equal representations, while lepton families are not. To achieve gauge anomaly cancellation and acceptable fermion masses, one of the SM lepton doublets is placed in a sextet of $SU(3)_L$, while the rest of the fermions are in singlets, triplets and antitriplets. As for the scalars, on top of three triplets $\phi_{1,2,3}$, we have introduced a sextet S which plays an important role in the generation of lepton masses, through both tree and loop diagrams. The full field content of the model is

TABLE III. Field content of the Pleitez-Özer model [71,72], which conserves lepton number.

Field	331 Representation	$G_{\rm SM}$ decomposition	# Flavors	Components	Lepton number
$\psi_{\ell,\alpha}$	$(\mathbf{1},\mathbf{3},-\frac{2}{3})$	$(\mathbf{1},\hat{\mathbf{2}},-\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},-1)$	3	$((\nu_\alpha,\ell_\alpha),E_\alpha)^T$	$(1,1,1)^T$
ℓ^c_α	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\hat{\mathbf{1}},1)$	6	ℓ^c_α	-1
$Q_{\alpha=1,2}$	$(\mathbf{3},\bar{\mathbf{3}},\frac{1}{3})$	$(\mathbf{3},\hat{\mathbf{2}},\frac{1}{6})+(\mathbf{3},\hat{\mathbf{1}},\frac{2}{3})$	2	$((d_\alpha,-u_\alpha),U_\alpha)^T$	$(0,0,0)^T$
Q_3	$(\mathbf{3},\mathbf{3},0)$	$(\mathbf{3},\hat{\mathbf{2}},\frac{1}{6})+(\mathbf{3},\hat{\mathbf{1}},-\frac{1}{3})$	1	$((t,b),D)^T$	$(0,0,0)^T$
u^c_α	$(\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})$	$(\bar{\mathbf{3}},\hat{\mathbf{1}},-\frac{2}{3})$	5	u^c_α	0
d^c_α	$(\bar{\mathbf{3}},\mathbf{1},\frac{1}{3})$	$(\bar{\mathbf{3}},\hat{\mathbf{1}},\frac{1}{3})$	4	d^c_α	0
$\phi_{i=1,2}$	$(\mathbf{1},\mathbf{3},\frac{1}{3})$	$(\mathbf{1},\hat{\mathbf{2}},\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},0)$	2	$((\phi_i^+,\phi_i^0),\tilde{\phi}_i^0)^T$	$(0,0,0)^T$
ϕ_3	$(\mathbf{1},\mathbf{3},-\frac{2}{3})$	$(\mathbf{1},\hat{\mathbf{2}},-\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},-1)$	1	$((\phi_3^0,\phi_3^-),\tilde{\phi}_3^-)^T$	$(0,0,0)^T$

TABLE IV. Field content of the model X, which conserves lepton number.

Field	331 Representation	$G_{\rm SM}$ decomposition	# Flavors	Components	Lepton number
$\psi_{\ell,\alpha}$	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\hat{\mathbf{2}},-\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},-2)$	3	$((\nu_\alpha,\ell_\alpha),\ell^c_{X,\alpha})^T$	$(1,1,1)^T$
ℓ^c_α	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\hat{\mathbf{1}},1)$	3	ℓ^c_α	-1
$\ell_{X,\alpha}$	$(\mathbf{1},\mathbf{1},2)$	$(\mathbf{1},\hat{\mathbf{1}},2)$	3	$\ell_{X,\alpha}$	-1
$Q_{\alpha=1,2}$	$(\mathbf{3},\bar{\mathbf{3}},\frac{2}{3})$	$(\mathbf{3},\hat{\mathbf{2}},\frac{1}{6})+(\mathbf{3},\hat{\mathbf{1}},\frac{5}{3})$	2	$((d_\alpha,-u_\alpha),J^c_\alpha)^T$	$(0,0,0)^T$
Q_3	$(\mathbf{3},\mathbf{3},-\frac{1}{3})$	$(\mathbf{3},\hat{\mathbf{2}},\frac{1}{6})+(\mathbf{3},\hat{\mathbf{1}},-\frac{4}{3})$	1	$((t,b),J^c_3)^T$	$(0,0,0)^T$
u^c_α	$(\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})$	$(\bar{\mathbf{3}},\hat{\mathbf{1}},-\frac{2}{3})$	3	u^c_α	0
d^c_α	$(\bar{\mathbf{3}},\mathbf{1},\frac{1}{3})$	$(\bar{\mathbf{3}},\hat{\mathbf{1}},\frac{1}{3})$	3	d^c_α	0
$J_{\alpha=1,2}$	$(\bar{\mathbf{3}},\mathbf{1},-\frac{5}{3})$	$(\bar{\mathbf{3}},\hat{\mathbf{1}},-\frac{5}{3})$	2	J_α	0
J_3	$(\bar{\mathbf{3}},\mathbf{1},\frac{4}{3})$	$(\bar{\mathbf{3}},\hat{\mathbf{1}},\frac{4}{3})$	1	J_3	0
ϕ_1	$(\mathbf{1},\mathbf{3},0)$	$(\mathbf{1},\hat{\mathbf{2}},\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},-1)$	1	$((\phi_1^+,\phi_1^0),\tilde{\phi}_1^-)^T$	$(0,0,0)^T$
ϕ_2	$(\mathbf{1},\mathbf{3},1)$	$(\mathbf{1},\hat{\mathbf{2}},\frac{3}{2})+(\mathbf{1},\hat{\mathbf{1}},0)$	1	$((\phi_2^{++},\phi_2^+),\tilde{\phi}_2^0)^T$	$(0,0,0)^T$
ϕ_3	$(\mathbf{1},\mathbf{3},-1)$	$(\mathbf{1},\hat{\mathbf{2}},-\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},-2)$	1	$((\phi_3^0,\phi_3^-),\tilde{\phi}_3^{--})^T$	$(0,0,0)^T$

TABLE V. Field content of the flipped 331 model [73].

Field	331 Representation	$G_{\rm SM}$ decomposition	# Flavor	Components	Lepton number
L_e	$(\mathbf{1},\mathbf{6},-\frac{1}{3})$	$(\mathbf{1},\hat{\mathbf{3}},0)+(\mathbf{1},\hat{\mathbf{2}},-\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},-1)$	1	$\begin{pmatrix}\Sigma^+ & \frac{1}{\sqrt{2}}\Sigma^0 & \frac{1}{\sqrt{2}}\nu_e \\ \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^- & \frac{1}{\sqrt{2}}\ell_e \\ \frac{1}{\sqrt{2}}\nu_e & \frac{1}{\sqrt{2}}\ell_e & E_e\end{pmatrix}$	$\begin{pmatrix}-1 & -1 & 1\\ -1 & -1 & 1\\ 1 & 1 & 3\end{pmatrix}$
$L_{\alpha=\mu,\tau}$	$(\mathbf{1},\mathbf{3},-\frac{2}{3})$	$(\mathbf{1},\hat{\mathbf{2}},-\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},-1)$	2	$(\nu_\alpha,\ell_\alpha,E_\alpha)^T$	$(1,1,3)^T$
ℓ^c_α	$(\mathbf{1},\mathbf{1},1)$	$(\mathbf{1},\hat{\mathbf{1}},1)$	6	ℓ^c_α	-1
Q_α	$(\mathbf{3},\bar{\mathbf{3}},\frac{1}{3})$	$(\mathbf{3},\hat{\mathbf{2}},\frac{1}{6})+(\mathbf{3},\hat{\mathbf{1}},\frac{2}{3})$	3	$(d_\alpha,-u_\alpha,U_\alpha)^T$	$(0,0,-2)^T$
u^c_α	$(\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})$	$(\bar{\mathbf{3}},\hat{\mathbf{1}},-\frac{2}{3})$	6	u^c_α	0
d^c_α	$(\bar{\mathbf{3}},\mathbf{1},\frac{1}{3})$	$(\bar{\mathbf{3}},\hat{\mathbf{1}},\frac{1}{3})$	3	d^c_α	0
$\phi_{i=1,2}$	$(\mathbf{1},\mathbf{3},\frac{1}{3})$	$(\mathbf{1},\hat{\mathbf{2}},\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},0)$	2	$(H^+_i,H^0_i,\sigma^0_i)^T$	$(0,0,2)^T$
ϕ_3	$(\mathbf{1},\mathbf{3},-\frac{2}{3})$	$(\mathbf{1},\hat{\mathbf{2}},-\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},-1)$	1	$(H^0_3,H^-_3,\sigma^-_3)^T$	$(0,0,2)^T$
S	$(\mathbf{1},\mathbf{6},\frac{2}{3})$	$(\mathbf{1},\hat{\mathbf{3}},1)+(\mathbf{1},\hat{\mathbf{2}},\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},0)$	1	$\begin{pmatrix}\Delta^{++} & \frac{1}{\sqrt{2}}\Delta^+ & \frac{1}{\sqrt{2}}H^+_S \\ \frac{1}{\sqrt{2}}\Delta^+ & \Delta^0 & \frac{1}{\sqrt{2}}H^0_S \\ \frac{1}{\sqrt{2}}H^+_S & \frac{1}{\sqrt{2}}H^0_S & \sigma^0_S\end{pmatrix}$	$\begin{pmatrix}-2 & -2 & 0\\ -2 & -2 & 0\\ 0 & 0 & 2\end{pmatrix}$

reproduced in Table V. Note that this construction requires $\beta = 1/\sqrt{3}$.

Without making a rigorous fit, we showed in [73] that the model is able to reproduce the observed fermion masses and mixing angles, hence modifications of the model are not mandatory. Here, neutrinos are Majorana particles, and therefore lepton number is obviously not conserved by the full Lagrangian. However, if one were to keep only the gauge interactions as well as all the Yukawa interactions allowed by the gauge symmetry, one finds a preserved $U(1)_L$ lepton number symmetry, with the associated charges shown in the last column of Table V. As for scalar couplings, most of them also preserve this $U(1)_L$, including $\phi_i^*\phi_j\phi_3 S$ and $\phi_3\phi_3 SS$ which are not self-conjugate. There are only two interactions allowed by the gauge symmetry which break lepton number (by two units): $\phi_1\phi_2\phi_3$ and $\phi_i\phi_j S^*$ $(i,j=1,2)$. In fact, this last one was mentioned in [73] as being important to achieve a realistic neutrino mass matrix. Apart from these two sources of LNV, one also has to consider the VEVs $\langle\sigma_i^0\rangle$, $\langle\Delta^{0*}\rangle$ and $\langle\sigma_S^0\rangle$ which all break $U(1)_L$ by two units.

F. The E_6 inspired model

The $SU(3)\times SU(3)\times U(1)$ is contained in $SU(3)^3$ which in turn is a subgroup of the exceptional E_6 group. This group has been used in grand unified model building [79]. In these models, fermions are in three copies of the fundamental representation **27**; hence, upon breaking the group down to the $SU(3)\times SU(3)\times U(1)$ subgroup, one ought to obtain a 331 model with family replication in both the lepton and quark sectors. Apart from a possible flip between the $SU(3)$'s representations with their anti-representations, the E_6 fundamental representation branches as follows:

$$\mathbf{27}\to\left(\mathbf{1},\mathbf{3},\begin{bmatrix}2a\\-a+b\\-a-b\end{bmatrix}\right)+\left(\bar{\mathbf{3}},\mathbf{1},\begin{bmatrix}-2a\\a-b\\a+b\end{bmatrix}\right)+(\mathbf{3},\bar{\mathbf{3}},0), \tag{7}$$

where the square brackets indicate in an economical way different states with different $U(1)_X$ charges, while a and b are parameters describing the linear combination of two $U(1)$'s which form $U(1)_X$. So, in order to place the left-handed quarks in $(\mathbf{3},\bar{\mathbf{3}},0)$, this branching rule implies that one must have $\beta=-1/\sqrt{3}$. In this case, $(\mathbf{3},\bar{\mathbf{3}},0)$ will also contain the $(\mathbf{3},\hat{\mathbf{1}},-\frac{1}{3})$ SM representation. Hence, from the second term in (7) one must get two d^c-like states, $(\bar{\mathbf{3}},\hat{\mathbf{1}},\frac{1}{3})$, and one u^c like state, $(\bar{\mathbf{3}},\hat{\mathbf{1}},-\frac{2}{3})$. Since the leptons [first term in (7)] have the opposite X charges to these colored states, we shall have two $(\mathbf{1},\mathbf{3},-\frac{1}{3})$ representations plus one $(\mathbf{1},\mathbf{3},\frac{2}{3})$. Note that any E_6 model is anomaly free, hence this list of 331 fields is so too. Table VI contains the overall picture. This model cleary dispels the claim that 331 models predict that the number of generations has to be necessarily equal to the number of colors.

This 331 model was first studied by Sánchez, Ponce and Martinéz in [74]. They considered a scalar sector with three triplet fields with the same quantum numbers as the scalars in the SVS model. With this field content, there are no sources of explicit lepton number violation, but the electroweak singlet components $\tilde{\phi}^0_{2,3}$ inside the two $\phi_{2,3}$ triplets do lead to spontaneous LNV (see Table VI).

While we will not do a complete flavor fit of this model to all experimental data, we shall briefly describe how it is possible to obtain realistic lepton masses. To start, consider the notation $\langle\phi_1\rangle=(0,k_1,0)^T$, $\langle\phi_{2,3}\rangle=(k_{2,3},0,n_{2,3})^T$ and

TABLE VI. Field content of the E_6 inspired 331 model [74].

Field	331 Representation	G_{SM} decomposition	# Flavors	Components	Lepton number
$\psi_{\ell,\alpha}$	$(\mathbf{1},\mathbf{3},-\frac{1}{3})$	$(\mathbf{1},\hat{\mathbf{2}},-\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},0)$	6	$((\nu_\alpha,\ell_\alpha),N_\alpha^c)^T$	$(1,1,-1)^T$
$\psi_{X,\alpha}$	$(\mathbf{1},\mathbf{3},\frac{2}{3})$	$(\mathbf{1},\hat{\mathbf{2}},\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},1)$	3	$((E_{X,\alpha},\nu_{X,\alpha}),\ell_\alpha^c)^T$	$(1,1,-1)^T$
Q_α	$(\mathbf{3},\bar{\mathbf{3}},0)$	$(\mathbf{3},\hat{\mathbf{2}},\frac{1}{6})+(\mathbf{3},\hat{\mathbf{1}},-\frac{1}{3})$	3	$((d_\alpha,-u_\alpha),D_\alpha)^T$	$(0,0,2)^T$
u_α^c	$(\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})$	$(\bar{\mathbf{3}},\hat{\mathbf{1}},-\frac{2}{3})$	3	u_α^c	0
d_α^c	$(\bar{\mathbf{3}},\mathbf{1},\frac{1}{3})$	$(\bar{\mathbf{3}},\hat{\mathbf{1}},\frac{1}{3})$	6	d_α^c	0
ϕ_1	$(\mathbf{1},\mathbf{3},\frac{2}{3})$	$(\mathbf{1},\hat{\mathbf{2}},\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},1)$	1	$((\phi_1^+,\phi_1^0),\tilde{\phi}_1^+)^T$	$(0,0,-2)^T$
$\phi_{i=2,3}$	$(\mathbf{1},\mathbf{3},-\frac{1}{3})$	$(\mathbf{1},\hat{\mathbf{2}},-\frac{1}{2})+(\mathbf{1},\hat{\mathbf{1}},0)$	2	$((\phi_i^0,\phi_i^-),\tilde{\phi}_i^0)^T$	$(0,0,-2)^T$

note that the only allowed interactions between ψ_ℓ, ψ_X and the scalars are

$$\mathcal{L}=\cdots+y_{\ell\ell}\psi_\ell\psi_\ell\phi_1+y_{\ell X}^{(2)}\psi_\ell\psi_X\phi_2 + y_{\ell X}^{(3)}\psi_\ell\psi_X\phi_3+\text{H.c.}+\ldots. \quad (8)$$

Here, $y_{\ell\ell}$ stands for a square 6×6 matrix, while $y_{\ell X}^{(i)}$ are two rectangular 6×3 matrices. Hence, considering only the colorless leptons,

$$\langle\mathcal{L}\rangle_{\text{lepton mass}}=m_{\ell,\alpha\beta}\Psi_\alpha^\ell\Psi_\beta^{\ell^c}+m_{\nu,\alpha\beta}\Psi_\alpha^\nu\Psi_\beta^\nu$$

with

$$M_\nu=2\begin{pmatrix}0 & k_1 y_{\ell\ell} & -n_2 y_{\ell X}^{(2)}-n_3 y_{\ell X}^{(3)}\\ \cdot & 0 & k_2 y_{\ell X}^{(2)}+k_3 y_{\ell X}^{(3)}\\ \cdot & \cdot & 0\end{pmatrix}, \quad (9)$$

$$M_\ell=(\,n_2 y_{\ell X}^{(2)}+n_3 y_{\ell X}^{(3)} \quad ,-k_2 y_{\ell X}^{(2)}-k_3 y_{\ell X}^{(3)}\,), \quad (10)$$

in the basis $\Psi^\nu=(\nu_\alpha,N_\alpha^c,\nu_{X,\beta})^T$, $\Psi^\ell=(\ell_\alpha)$ and $\Psi^{\ell^c}=(E_{X,\beta},\ell_\beta^c)^T$ ($\alpha=1,\ldots,6$; $\beta=1,\ldots,3$).

A careful analysis of the neutrino mass matrix reveals that, baring the existence of special alignments and/or cancellations, one expects the following mass eigenstates:

(i) Three light Majorana neutrino states ν_M with seesaw masses $\mathcal{O}(y_{\ell\ell}k_1k_i/n_i)$, $i=2$, 3, and composed almost entirely of N^c states;

(ii) Three quasi-Dirac neutrino pairs ν_{lQD} with masses $\mathcal{O}(y_{\ell\ell}k_1)$ composed of a ~50%/50% admixture of N^c and ν states;

(iii) Three quasi-Dirac heavy neutrino pairs ν_{hQD} with masses $\mathcal{O}(y_{\ell X}^{(i)}n_i)$ composed of a ~50%/50% admixture of ν and ν_X states.

This rough estimation holds true only if there is a clear hierarchy between these sets of neutrino masses: $y_{\ell X}^{(i)}n_i \gg y_{\ell\ell}k_1 \gg y_{\ell\ell}k_1k_i/n_i$ ($i=2$, 3). However, in this limit the three seesawed neutrinos are mostly singlets under the SM gauge group; hence, they cannot play the role of the observed active neutrinos. That role must then be played by the three quasi-Dirac neutrino pairs ν_{lQD} with masses proportional to the value of the coupling matrix $y_{\ell\ell}$ and the VEV k_1. Even though we will not write down the precise expressions for the neutrino masses and lepton mixing angles, it is possible to have sub-eV active neutrinos ν_{lQD}, at the price of choosing small $\mathcal{O}(10^{-12})$ entries in the matrix $y_{\ell\ell}$. This choice does not affect the mass of the remaining active neutrinos ν_{hQD}, which must have masses above the SM Z^0 mass, in order not to be in conflict with the measured invisible width of the Z^0 boson. One must then additionally ensure that the light Majorana neutrino states ν_M do not mix significantly with the ν_{lQD} states.

As for charged leptons, the M_ℓ matrix will have rank 3 if the matrix $n_2y_{\ell X}^{(2)}+n_3y_{\ell X}^{(3)}$ is proportional to $k_2y_{\ell X}^{(2)}+k_3y_{\ell X}^{(3)}$, and this is an interesting limit as it would imply that 3 of the charged leptons are massless (e, μ, τ), so a small departure from this scenario can actually be used to explain the ratio $m_\tau/m_{W,Z}$. Finally, since the quark Yukawa coupling matrices are free parameters, the quark masses and mixing parameters can easily be fitted in this model. Since the E_6-inspired model can, in principle, explain the observed fermion masses, we will not discuss extended versions of this 331 model.

III. SIMPLE EXTENSIONS OF THE SVS, PPF, PÖ AND X MODELS

Four of the basic 331 models discussed above fail to produce a viable neutrino mass spectrum. These four follow the basic framework of the SVS model, and we called them SVS, PPF, PÖ and X in the previous section. The PPF model moreover predicts a charged lepton spectrum in disagreement with experimental data, see Table VII for a summary. The table also recalls, as discussed above, that lepton number is actually *conserved* in models PÖ and X.

To fix these problems, in the following we consider four simple extensions of the field content for these basic models:

(i) Add a fermionic particle, N'^c, singlet under the 331 symmetry group.

TABLE VII. A summary of lepton number violation and problems with the lepton sector in four of the basic 331 models discussed in Sec. II.

Issue	SVS	PPF	PÖ	X
$U(1)_L$ violation?	✓	✓	✗	✗
ν masses?	✓	✗	✗	✗
Correct ν masses?	✗	✗	✗	✗
Correct ℓ masses?	✓	✗	✓	✓

(ii) Add a scalar sextet S such that $\psi_\ell \psi_\ell S$ provides a symmetric contribution to the neutrino and charged leptons mass matrix.
(iii) Add a vectorlike pair (E^c, E) of charged leptons.
(iv) Add a ϕ_X triplet scalar field in order to generate the interaction $\psi_\ell \psi_\ell \phi_X$.

However, not all of these extensions work equally well for all models, see Table VIII. Here, extensions which will fix the problems with the lepton spectra for a particular model are marked with (✓), while those that do not work are marked with (✗). The cases which fail can be understood as follows:

(i) Adding right-handed neutrinos to the PPF model leads to the generation of neutrino masses, but it does not fix the charged lepton mass problem.
(ii) Both the PPF and SVS models already contain a $\psi_\ell \psi_\ell \phi_i$ interaction, so adding another ϕ_X scalar does not lead to a qualitative change of these models.
(iii) Models PÖ and X already contain vectorlike leptons; hence, adding another pair is again unhelpful. Furthermore, adding the vector fermions (E^c, E) to the SVS model is also unsatisfactory.

Having said this, we now turn to a detailed discussion of the effects of the model extensions which do work.

A. Extended PPF models

We start our discussion with the PPF model. Adding a fermion singlet N'^c to the PPF model allows an interaction term $\psi_\ell N'^c \phi_3^*$ (and a mass term $N'^c N'^c$). Since this addition does not affect the charged lepton spectrum, by itself such extension of the PPF model is insufficient, and we will thus not discuss it here (but see below for other models).

TABLE VIII. Simple extensions of the four models (PPF, SVS, PÖ, X) which will fix (✓) the problems with the lepton spectra summarized in Table VII. Cases that will not work are marked with (✗). For explanation see text.

Modification	PPF	SVS	PÖ	X
$+N'^c$	✗	✓	✓	✓
$+S$	✓	✓	✓	✓
$+E^c, E$	✓	✗	✗	✗
$+\phi_X$	✗	✗	✓	✓

Adding a scalar sextet, on the other hand, provides a valid fix for the PPF model. Consider $S = (\mathbf{1}, \mathbf{6}, 0)$

$$S = \begin{pmatrix} \Delta^0 & \frac{1}{\sqrt{2}}\Delta^- & \frac{1}{\sqrt{2}}H^+ \\ \frac{1}{\sqrt{2}}\Delta^- & \Delta^{--} & \frac{1}{\sqrt{2}}H^0 \\ \frac{1}{\sqrt{2}}H^+ & \frac{1}{\sqrt{2}}H^0 & \sigma^{++} \end{pmatrix}. \quad (11)$$

The components denoted as Δ, H, and σ form a triplet, a doublet, and a singlet respectively under the $SU(2)_L$ group. The interaction of the lepton triplet with this sextet contains the terms

$$y_S \psi_{\ell,\alpha} S^* \psi_{\ell,\beta} = y_S[\nu_\alpha \nu_\beta \Delta^{0*} + \frac{1}{\sqrt{2}}(\ell_\alpha \ell_\beta^c + \ell_\alpha^c \ell_\beta) H^{0*} + \ldots]. \quad (12)$$

The term proportional to Δ^{0*} will give a type-II seesaw contribution to the neutrino masses, once Δ^0 acquires a VEV, proportional to $m^\nu_{\alpha\beta} = (y_S)_{\alpha\beta} \langle \Delta^0 \rangle$, while the charged lepton mass matrix is now the sum of two terms: $m^\ell = y_\ell \langle \phi_3^0 \rangle + y_S \langle H^0 \rangle$. It is easy to see that in the absence of y_S the mass spectrum for the charged leptons has the eigenvalues (0, m, m). Thus, in order to achieve the correct hierarchies for e, μ and τ, the second term in m^ℓ must dominate. This puts a lower limit on the largest entries in $y_S \langle H^0 \rangle$ of the order of the τ mass.

Since the same y_S appears in neutrino masses, one must have $\langle \Delta^0 \rangle / \langle H^0 \rangle \lesssim 10^{-10}$ for a correct explanation of neutrino data. Adding a sextet to the original PPF model was already proposed in [66]. These authors, however, argued that such a small ratio calls for a protecting symmetry. The proposed symmetry eliminates all lepton number violating scalar interactions from the model: in addition to the original term $\phi_1 \phi_2 \phi_3^* \phi_3^*$, these are $\phi_3 \phi_3 S^*$ and SSS. Since under this condition lepton number is conserved, neutrinos are massless again. Thus, with the addition of only a sextet to the original PPF model, we have to accept the fine-tuning between the triplet and doublet VEVs if we are to explain neutrino data. We note in passing that such a small ratio of VEVs might be due to a small parameter in the scalar potential, such as the coefficient of $\phi_3 \phi_3 S^*$.

We now turn to the third possibility in our list. Both problems, neutrino and charged lepton masses, can be cured in the PPF model by the introduction of a pair of vectorlike charged leptons, E and E^c, in the representations $(\mathbf{1}, \mathbf{1}, \mp 1)$.[7] In the original PPF model, the Lagrangian contains the interaction terms

[7]That the wrong prediction for the charged lepton spectrum in the PPF model can be cured using vectorlike leptons was noted already in [69,70].

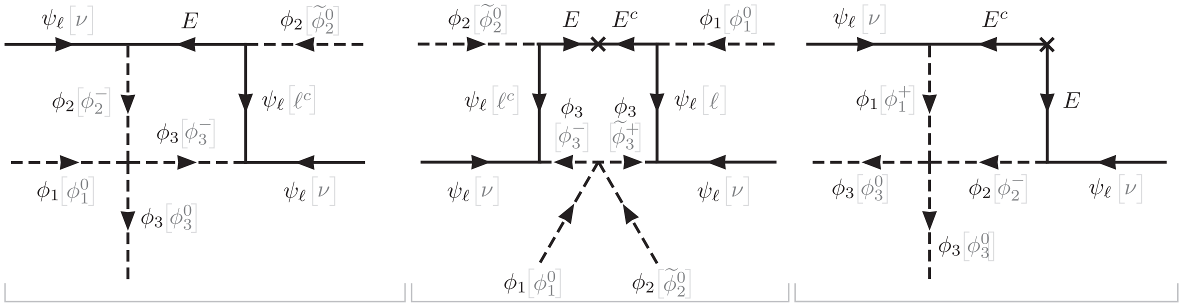

FIG. 3. One-loop diagrams in the PPF model, extended with a vectorlike lepton.

$$\mathcal{L} = \cdots + \frac{1}{2} y_{\ell,\alpha\beta} \psi_{\ell,\alpha} \psi_{\ell,\beta} \phi_3 + \lambda_7 \phi_1 \phi_2 \phi_3^* \phi_3^* + \text{H.c.}, \quad (13)$$

and introducing the left-handed Weyl spinors E and E^c makes it possible to write down the following also:

$$\mathcal{L}_{EE^c} = h_{E^c,\alpha} \psi_{\ell,\alpha} E^c \phi_1^* + h_{E,\alpha} \psi_{\ell,\alpha} E \phi_2^* + m_{EE^c} E E^c. \quad (14)$$

The charged lepton mass matrix, after symmetry breaking, becomes

$$M_{\ell E} = \begin{pmatrix} 0 & y_{\ell,e\mu} k_3 & y_{\ell,e\tau} k_3 & h_{E^c,e} k_1 \\ -y_{\ell,e\mu} k_3 & 0 & y_{\ell,\mu\tau} k_3 & h_{E^c,\mu} k_1 \\ -y_{\ell,e\tau} k_3 & -y_{\ell,\mu\tau} k_3 & 0 & h_{E^c,\tau} k_1 \\ h_{E,e} n_2 & h_{E,\mu} n_2 & h_{E,\tau} n_2 & m_{EE^c} \end{pmatrix}. \quad (15)$$

Here, k_1, k_3 and n_2 are the VEVs of ϕ_1^0, ϕ_3^0 and $\tilde{\phi}_2^0$, respectively. We now define $|y_\ell| = \sqrt{y_{\ell,e\mu}^2 + y_{\ell,e\tau}^2 + y_{\ell,\mu\tau}^2}$, $|h_{E^c}| = \sqrt{\sum_\alpha (h_{E^c,\alpha})^2}$ and $|h_E| = \sqrt{\sum_\alpha (h_{E,\alpha})^2}$. Then, in the limit $|h_{E^c}| = |h_E| \to 0$ we obtain the original result for the charged lepton masses, given by: $m_{1,2,3} = (-|y_\ell| k_3, 0, |y_\ell| k_3)$. Note that the massless state has the eigenvector $e_2 = \frac{1}{|y_\ell|}(y_{\ell,\mu\tau}, y_{\ell,e\tau}, 1)^T$. In other words, a good starting point to have the electron as the lightest state corresponds to the choice $y_{\ell,e\mu}$, $y_{\ell,e\tau} \ll y_{\ell,\mu\tau}$. Note that $|y_\ell| \to 0$ is not allowed, since in this case the matrix $M_{\ell E}$ in Eq. (15) only has two nonzero eigenvalues. In this limit, the lighter of the two nonzero mass states is given by $m = |h_E||h_{E^c}| k_1 n_2 / m_{EE^c}$. For values of $m_{EE^c} = \mathcal{O}(\text{TeV})$ and $n_2 = \mathcal{O}(\text{TeV})$, fitting m_τ then requires $|h_E| \times |h_{E^c}| \sim \mathcal{O}(10^{-2})$. For nonzero values of $|h_E|$, $|h_{E^c}|$ and y_ℓ the mass degeneracy is broken and a realistic charged lepton spectrum can be easily obtained for $|y_\ell| \lesssim \mathcal{O}(10^{-2}\text{–}10^{-3})$ together with $|y_\ell| < |h_E||h_{E^c}|$ (and $k_1 \sim k_3$).

Extending the PPF model with a vectorlike lepton does not only solve the charged lepton mass problem, but it also leads to the generation of 1-loop neutrino masses—see Fig. 3.[8] For these loops, in addition to the terms given in Eq. (14), the two interactions terms in Eq. (13) are needed. As explained in the previous section, in the minimal PPF model lepton number violation is proportional to λ_7 and, thus, all loops that generate a Majorana neutrino mass must contain this particular quartic vertex. Such statement is still true once E and E^c are added to the model, hence this important scalar interaction is present in all diagrams show in Fig. 3.

We will give a rough estimate of the size of these loops. A complete calculation would require rotating all internal states in the diagrams to the mass eigenstate basis and then summing over all states. However, since (i) the mass of the vectorlike lepton has to be much larger than the mass of the tau, and (ii) λ_7 has to be small, as shown below, we can estimate the relative contributions of each diagram in Fig. 3 to the neutrino mass individually. Let us concentrate on diagram (c) first. It's contribution to the neutrino mass matrix is estimated to be

$$(m_\nu)_{\alpha\beta} = \frac{1}{16\pi^2} \sin 2\theta_S m_{EE^c} \Delta B_0 [h_{E,\alpha} h_{E^c,\beta} + (\alpha \leftrightarrow \beta)]. \quad (16)$$

Here, θ_S is the angle that diagonalizes the (2,2) submatrix of the charged scalars,

$$M_{\phi_1\phi_2}^2 = \begin{pmatrix} m_{\phi_1}^2 & \lambda_7 k_3^2 \\ \lambda_7 k_3^2 & m_{\phi_2}^2 \end{pmatrix}, \quad (17)$$

and is given by

$$\sin 2\theta_S = \frac{2\lambda_7 k_3^2}{m_1^2 - m_2^2}, \quad (18)$$

[8]We show the loops with the internal scalars as propagating degrees of freedom. In a full calculation one should take into account that the same scalars are used to break the 331 symmetry, i.e. some components of these scalars become the Goldstone bosons and are "eaten" by the massive vectors. This will lead to the generation of equivalent diagrams, but now with vector bosons. We will omit this (irrelevant) complication in our discussion here.

with $m_{1,2}^2$ being the eigenvalues of $M^2_{\phi_1\phi_2}$. In Eq. (16) ΔB_0 stands for the difference between the two 1-loop B_0 functions for the two scalar mass eigenstates, and it reads

$$\Delta B_0 = \frac{m_1^2 \log(m_1^2/m_{EE^c}^2)}{m_1^2 - m_{EE^c}^2} - \frac{m_2^2 \log(m_2^2/m_{EE^c}^2)}{m_2^2 - m_{EE^c}^2}. \quad (19)$$

Since we know experimentally that neutrino masses are small, while the mass of the vectorlike lepton should be larger than several 100's of GeV, either the Yukawa couplings or the factor $\sin 2\theta_S \Delta B_0$ should be small. The former is not an option in the present model, since only for $|h_E||h_{E^c}| \sim \mathcal{O}(10^{-2})$ a realistic charged lepton spectrum can be obtained, as discussed above.

We define

$$\bar{M} = \frac{1}{2}(m_1 + m_2), \quad (20)$$

$$\Delta M = (m_2 - m_1). \quad (21)$$

Then, in the limit of $\Delta M \ll \bar{M}$ and for $m_{EE^c} < \bar{M}$, $\sin 2\theta_S \Delta B_0$ becomes simply $\sim(\lambda_7 k_3^2)/\bar{M}^2$, so the neutrino mass is roughly given by the expression

$$m_\nu \sim \left(\frac{m_{EE^c}}{\text{TeV}}\right)\left(\frac{\text{TeV}}{\bar{M}^2}\right)^2\left(\frac{k_3}{100\ \text{GeV}}\right)^2 \times \left(\frac{|h_{E^c}||h_{\bar{E}^c}|}{10^{-2}}\right)\left(\frac{\lambda_7}{10^{-7}}\right)10^{-1}\ \text{eV}. \quad (22)$$

We now turn to a brief discussion of the relative importance of the diagrams (a)–(c) in Fig. 3. Diagrams (a) and (b) contain the same parameters as diagram (c) just discussed and, furthermore, they also depend on the other doublet VEV in the model (k_1) as well as the couplings $y_{\ell,\alpha\beta}$. Assuming that the Yukawas $h_{E,\alpha}$ and $h_{E^c,\beta}$ are very roughly of the same order of magnitude numerically, the relative importance of the three diagrams can then be estimated to be

$$(c):(a):(b) = 1 : |y_\ell|\frac{k_1}{k_3} : \left(|y_\ell|\frac{k_1}{k_3}\right)^2. \quad (23)$$

The ratio $\frac{k_1}{k_3}$ is not fixed in this model; therefore, k_3 can be smaller than k_1. Only the combination $\sqrt{k_1^2 + k_3^2} = v = 174$ GeV is fixed. However, as discussed above, the tau mass constrains the combination $|h_E||h_{E^c}|k_3$ to be of the GeV order, for $n_2 \simeq m_{EE^c}$ of the TeV order. For Yukawa couplings in the perturbative regime, this means that k_3 can not be much smaller than 1 GeV. Thus, diagram (c) is usually the dominant one, and the other diagrams can be at most equally important, if k_3 is pushed to its lower limit.

Before moving on, we recall that adding additional triplet scalars, without adding (E, E^c), does not provide a valid solution for the PPF model, see Table VIII.

B. Extended SVS models

For the SVS model, two of the four possibilities listed in Table VIII will be valid solutions: (i) adding a fermion singlet N'^c and (ii) adding a scalar sextet.

Adding (three copies) of N'^c, in addition to the term $y_\ell \psi_\ell \psi_\ell \phi_1$, one can write down three new Lagrangian terms for the (extended) SVS model

$$\mathcal{L} = \sum_{j=2,3} y_{N'}^{(j)} \psi_\ell N'^c \phi_j^* + \mu N'^c N'^c. \quad (24)$$

In the basis (ν, N^c, N'^c), the neutrino mass matrix becomes

$$M^\nu = \begin{pmatrix} 0 & m_D & m_L \\ m_D^T & 0 & M_R \\ m_L^T & M_R^T & \mu \end{pmatrix}. \quad (25)$$

Here, $m_D = y_\ell k_1$, $m_L = \sum_j y_{N'}^{(j)} k_j$, $M_R = \sum_j y_{N'}^{(j)} n_j$ and μ are 3×3 matrices.[9] There are two limits for μ. For $M_R \ll \mu$ the matrix in Eq. (25) will lead to a double seesaw, in other words, integrating out N'^c would give a Majorana mass entry in the (2,2) position of the above matrix of the order of $M_R \mu^{-1} M_R^T$. If $\mu \ll M_R$, the matrix gives neutrinos a mass via the inverse seesaw mechanism

$$m_\nu = m_D (M_R^T)^{-1} \mu M_R^{-1} m_D^T. \quad (26)$$

The fit to neutrino masses can easily be done. This case has been studied in [80]. Note that if $\mu = 0$ there is no linear seesaw contribution proportional to m_L. Indeed, in a model such as this one where $m_L \propto M_R$ and $m_D^T = -m_D$, one has

$$m_D (M_R^T)^{-1} m_L + [m_D (M_R^T)^{-1} m_L]^T = 0. \quad (27)$$

Such limit $\mu = 0$ can be achieved with some additional symmetry, as discussed in [14]. However, neutrinos will still acquire mass at 1-loop level via, for example, the diagram shown in Fig. 1, and also via the gauge loops discussed in [14]. Consider first the loop shown in Fig. 1. The loop will vanish in the limit where the coefficient ρ of the term $\phi_1\phi_2\phi_3$ vanishes. The calculation is very similar to the loop discussed for the PPF model, with some modifications: m_{EE^c} has to be replaced by the SM charged lepton masses, and the Yukawa matrices appearing at the vertices are y_ℓ and $y_{\ell\ell^c}$, where the latter is the matrix entering the charged lepton mass matrix. If $\rho/\bar{M}$ is a small

[9] We keep following here the convention that $k_i(n_i)$ is the $SU(2)_L$ doublet(singlet) VEV of the scalar triplet ϕ_i.

number, where $\bar{M}$ is some average mass of the scalars, we very roughly estimate that

$$m_\nu \sim \frac{2}{16\pi^2} m_\tau \frac{k}{n} \left(\frac{\rho}{\bar{M}}\right)^2 \left(\frac{n}{\bar{M}}\right)^2 |y_\ell||y_{\ell\ell^c}| \tag{28}$$

$$\sim 0.05 \left(\frac{\rho/\bar{M}}{10^{-2}}\right)^2 \frac{|y_\ell|}{10^{-2}} \frac{|y_{\ell\ell^c}|}{10^{-2}} \text{ eV} \tag{29}$$

for $k_i \simeq k \sim 100$ GeV and $n_i \simeq n \sim 1$ TeV.

The gauge loops discussed in [14] are more subtle. In the SVS model, in addition to the trilinear coupling ρ, the VEVs n_2 and n_3 also violate lepton number. Thus, once the 331 symmetry is broken, there exists a mixing between gauge bosons that leads to lepton number violating processes. In particular, one can draw the diagrams shown in Fig. 4. Note that the VEV insertions indicated at the top of these diagrams always are in the combination $k_2 n_2$ and/or $k_3 n_3$, i.e. they correspond to a $\Delta(L) = 2$ effect.

The diagram on the left shows the contribution to the neutrino mass in the basis where the internal fermions are mass eigenstates. One can understand this propagator as an infinite series of mass insertions, as indicated by the diagrams to the right. The first term in this expansion is proportional to m_D, which is completely antisymmetric, and thus does not give any contribution. However, higher order terms will come proportional to powers of $f = (m_D M_R^* M_R - M_R M_R^\dagger m_D)$, which in general is nonzero. It is interesting to note that, for the special case where the heavy Dirac-pairs start out degenerate ($M_R \propto \mathbb{1}$), the commutator f vanishes and the gauge loops go to zero. In the general case, where f is not much smaller than $M_R M_R^\dagger m_D$ this gauge loop will dominate over the scalar loop and put a constraint on $m_D M_R^{-1}$ to be typically below 10^{-8} or so.

Adding a sextet S with the quantum numbers $(\mathbf{1}, \mathbf{6}, 2/3)$ also may solve the neutrino mass problem. The components of such a field can be written as in Eq. (11), with the only difference being the electric charges. The part of the Lagrangian involving S contains the following important terms:

$$\mathcal{L} = \cdots + y_S(\Delta^0 \nu\nu + \sqrt{2} H^0 \nu N^c + \sigma^0 N^c N^c) + \text{H.c.}. \tag{30}$$

If all the VEVs of Δ^0, H^0 and σ^0 are nonzero, the light neutrino masses have both seesaw type-I and type-II contributions. One just needs to ensure that $\langle \Delta^0 \rangle \sim$ eV $\ll \langle H^0 \rangle \sim 100$ GeV $\ll \langle \sigma^0 \rangle \sim$ TeV.

C. Extending the PÖ and X models

The situation is rather simpler in models PÖ and X, which both conserve lepton number. They also do not have neutrino singlets, hence they predict that neutrinos are massless. Here, we will very briefly discuss the different extended versions of these models, commenting also on the differences with respect to the models SVS and PPF. Since models PÖ and X are very similar in this respect, we discuss both at the same time.

Adding three copies of fermion singlets N^c makes it possible to write down the terms

$$\mathcal{L} = y_\nu \psi_\ell N^c \phi_3^* + M_N N^c N^c. \tag{31}$$

Note that, since the triplet ψ_ℓ does not contain a N^c in neither model PÖ nor model X, this will give an ordinary seesaw mechanism of type-I (to be compared with the inverse or double seesaw in the SVS model) which is sufficient to explain neutrino data.

Adding a sextet S, with the quantum numbers $(\mathbf{1}, \mathbf{6}, -4/3)$ in the case of model PÖ and $(\mathbf{1}, \mathbf{6}, 2)$ in case of model X, gives rise to Majorana neutrino masses once the neutral component Δ^0 of the $SU(2)_L$ scalar triplet contained in S acquires a VEV. This is a pure seesaw type-II contribution since Δ^0 is the only neutral component of these sextets.

Finally, neutrino masses can be generated at the 1-loop level also in the models PÖ and X, by introducing an additional triplet scalar ϕ_X. The required quantum numbers are $(1, \mathbf{3}, 4/3)$ (model PÖ) and $(1, \mathbf{3}, 2)$ (model X). The resulting Feynman diagram, in the Pleitez-Özer model, is shown in Fig. 5. In both models the calculation of the loop and the resulting constraints on model parameters are very similar to the results discussed above for models PPF and SVS, with some obvious replacements.

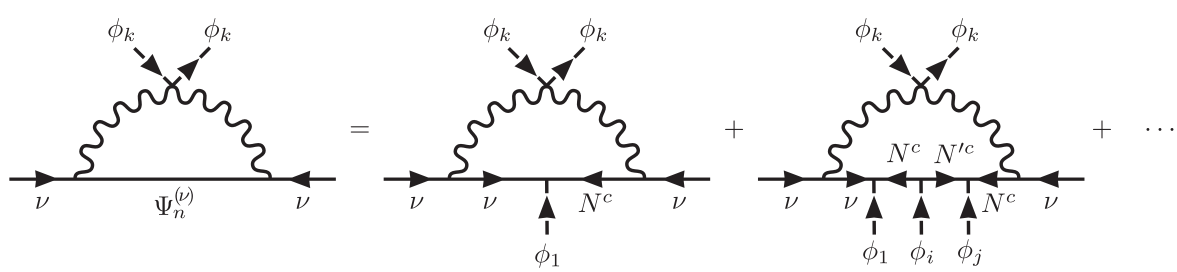

FIG. 4. Gauge loops in the extended SVS model. The full neutrino propagator with the mass eigenstates $\Psi_n^{(\nu)}$ can be expanded in a series of mass insertions. The first nonzero term involves three mass insertions (see text).

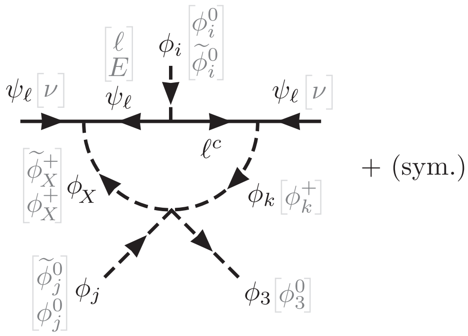

FIG. 5. Scalar loop for neutrino masses in the Pleitez-Özer model extended with a scalar triplet ϕ_X. In fact, two distinct diagrams are possible, depending on which components inside the square brackets are picked (either the ones on top, or the ones at the bottom). Analogous loops can be made for model X with an added ϕ_X scalar triplet.

IV. CONCLUSIONS

We have studied in a systematic way the status of lepton number in 331 models. The fact that lepton number often does not commute with the extended $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge group makes this an interesting topic, leading to the existence of gauge bosons and colored fermions with a nonzero $U(1)_L$ charge and, potentially, to lepton number violation. Note also that the 331 symmetry may break to the Standard Model gauge group at a relatively low energy scale (~TeV), in which case the LHC would be able to probe the sources of lepton number violation.

However, as we have made clear in this work, there is a large diversity of 331 models, and in some of them lepton number not only commutes with the gauge group, but it is also preserved by the full Lagrangian and VEVs of the scalars. These are nevertheless exceptional cases; in general it is possible to (a) write down sets of gauge invariant interactions which do not preserve any global $U(1)_L$ and/or (b) have neutral scalar components with a nonzero lepton number which break spontaneously this symmetry.

Most of the models we discuss, in their original form, are unable to explain the observed lepton masses and neutrino oscillation data. For these models we have listed several simple extensions which can accommodate all lepton data (some of them had already been proposed previously by other authors). As such, any of these extended models can be used for further study.

We have focused mainly on the generation of acceptable neutrino masses (and mixing angles), having mentioned lepton number violating processes, such as neutrinoless double beta decay, only in passing when it was most relevant. Elsewhere [81], we shall provide a more detailed analysis of this process, both in 331 models as well as in other models with an extended gauge groups.

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APPENDIX: DECOMPOSITION OF 331 REPRESENTATIONS

The decomposition of the most relevant $SU(3)_L \times U(1)_X$ representations into $SU(2)_L \times U(1)_Y$ representations has already been provided in Eq. (2)–(4). As such, in this appendix we simply clarify how the components of these representations are related.

A triplet $\mathbf{3}$ of $SU(3)_L$ breaks into a doublet $\hat{\mathbf{2}}$ plus a singlet $\hat{\mathbf{1}}$ of $SU(2)_L$. Noting that the electric charge of each component depends on the $U(1)_X$ charge of the triplet, as well as the β parameter as shown in Eq. (2), we may simply label the components of $\hat{\mathbf{2}}$ by their isopin (1/2 and $-1/2$). We can then settle with the following identification:

$$\mathbf{3} = \begin{pmatrix} \hat{\mathbf{2}}_{1/2} \\ \hat{\mathbf{2}}_{-1/2} \\ \hat{\mathbf{1}} \end{pmatrix}. \tag{A1}$$

From here we infer that an antitriplet $\bar{\mathbf{3}}$ of $SU(3)_L$, which also decomposes into a doublet $\hat{\mathbf{2}}$ plus a singlet $\hat{\mathbf{1}}$ must be written as

$$\bar{\mathbf{3}} = \begin{pmatrix} \hat{\mathbf{2}}_{-1/2} \\ -\hat{\mathbf{2}}_{1/2} \\ \hat{\mathbf{1}} \end{pmatrix}. \tag{A2}$$

A (anti)sextuplet $\mathbf{6}$ of $SU(3)_L$ breaks into a triplet $\hat{\mathbf{3}}$, a doublet $\hat{\mathbf{2}}$ and a singlet $\hat{\mathbf{1}}$ of $SU(2)_L$. These representations (6 and $\bar{\mathbf{6}}$) are often pictured as matrices instead of vectors, since that makes their contraction with triplets more intuitive. For example, if $\bar{\mathbf{6}}_{ij}\mathbf{3}_i\mathbf{3}'_j$ is gauge invariant, one must have the following identification:

$$\mathbf{6} = \begin{pmatrix} \hat{\mathbf{3}}_1 & \frac{1}{\sqrt{2}}\hat{\mathbf{3}}_0 & \frac{1}{\sqrt{2}}\hat{\mathbf{2}}_{1/2} \\ \frac{1}{\sqrt{2}}\hat{\mathbf{3}}_0 & \hat{\mathbf{3}}_{-1} & \frac{1}{\sqrt{2}}\hat{\mathbf{2}}_{-1/2} \\ \frac{1}{\sqrt{2}}\hat{\mathbf{2}}_{1/2} & \frac{1}{\sqrt{2}}\hat{\mathbf{2}}_{-1/2} & \hat{\mathbf{1}} \end{pmatrix},$$

$$\bar{\mathbf{6}} = \begin{pmatrix} \hat{\mathbf{3}}_{-1} & -\frac{1}{\sqrt{2}}\hat{\mathbf{3}}_0 & \frac{1}{\sqrt{2}}\hat{\mathbf{2}}_{-1/2} \\ -\frac{1}{\sqrt{2}}\hat{\mathbf{3}}_0 & \hat{\mathbf{3}}_1 & -\frac{1}{\sqrt{2}}\hat{\mathbf{2}}_{1/2} \\ \frac{1}{\sqrt{2}}\hat{\mathbf{2}}_{-1/2} & -\frac{1}{\sqrt{2}}\hat{\mathbf{2}}_{1/2} & \hat{\mathbf{1}} \end{pmatrix}. \tag{A3}$$

A mass terms $\mathrm{Tr}(\bar{\mathbf{6}}'\cdot\mathbf{6})$ then translates into $\mathrm{Tr}(\bar{\Delta}'\cdot\Delta)+\hat{\mathbf{2}}'_{-1/2}\hat{\mathbf{2}}_{1/2}-\hat{\mathbf{2}}'_{1/2}\hat{\mathbf{2}}_{-1/2}+\hat{\mathbf{1}}'\hat{\mathbf{1}}$ for two $SU(2)_L$ triplets Δ and $\bar{\Delta}'$ which we can write in terms of isospin components as

$$\bar{\Delta}'=\begin{pmatrix}\hat{\mathbf{3}}'_{-1} & -\frac{1}{\sqrt{2}}\hat{\mathbf{3}}'_0\\ -\frac{1}{\sqrt{2}}\hat{\mathbf{3}}'_0 & \hat{\mathbf{3}}'_1\end{pmatrix},$$

$$\Delta=\begin{pmatrix}\hat{\mathbf{3}}_1 & \frac{1}{\sqrt{2}}\hat{\mathbf{3}}_0\\ \frac{1}{\sqrt{2}}\hat{\mathbf{3}}_0 & \hat{\mathbf{3}}_{-1}\end{pmatrix}. \tag{A4}$$

Note that conventions in the literature vary regarding the signs in front of some of the triplet components $\hat{\mathbf{3}}_i$, since these might change with a rephasing of fields components. However, the $\frac{1}{\sqrt{2}}$ factors cannot be absorbed, so the expression in Eq. (A3) for the sextet differs in a material way from the one used in [13,17,66], for example, agreeing instead with [16].[10]

Finally, we consider what happens to gauge bosons $W_{L,i}$ ($i=1,\cdots 8$) which are in the adjoint representation ($\mathbf{8}$) of $SU(3)_L$. The representation $(\mathbf{8},0)$ of $SU(3)_L\times U(1)_X$ breaks into one $SU(2)_L$ triplet $\hat{\mathbf{3}}$, one singlet $\hat{\mathbf{1}}$ and two doublets $\hat{\mathbf{2}}$ and $\hat{\mathbf{2}}'$ with opposite hypercharges; for definiteness let us consider $\hat{\mathbf{2}}$ to be the one with $y=\frac{\sqrt{3}}{2}\beta$—see Eq. (4). Contractions with (anti)triplets are done in the standard way ($\mathbf{8}_{ij}\bar{\mathbf{3}}_i\mathbf{3}_j$), resulting in the following identification of the octet components:

$$\mathbf{8}=\begin{pmatrix}\frac{1}{\sqrt{6}}\hat{\mathbf{1}}-\frac{1}{\sqrt{2}}\hat{\mathbf{3}}_0 & \hat{\mathbf{3}}_1 & -\hat{\mathbf{2}}_{1/2}\\ -\hat{\mathbf{3}}_{-1} & \frac{1}{\sqrt{6}}\hat{\mathbf{1}}+\frac{1}{\sqrt{2}}\hat{\mathbf{3}}_0 & -\hat{\mathbf{2}}_{-1/2}\\ -\hat{\mathbf{2}}'_{-1/2} & \hat{\mathbf{2}}'_{1/2} & -\sqrt{\frac{2}{3}}\hat{\mathbf{1}}\end{pmatrix}. \tag{A5}$$

In the case of gauge bosons, we are dealing with a *real* field transforming as $\mathbf{8}$ hence the $SU(2)_L$ $\hat{\mathbf{2}}$ and $\hat{\mathbf{2}}'$ doublets are not independent. Indeed, one can alternatively write $\mathbf{8}=\frac{1}{\sqrt{2}}W^a\lambda^a$, where $\lambda^{1,\ldots,8}$ are the Gell-Mann matrices

$$\mathbf{W}_L=\frac{1}{\sqrt{2}}\begin{pmatrix}W_L^3+\frac{1}{\sqrt{3}}W_L^8 & W_L^1-iW_L^2 & W_L^4-iW_L^5\\ W_L^1+iW_L^2 & -W_L^3+\frac{1}{\sqrt{3}}W_L^8 & W_L^6-iW_L^7\\ W_L^4+iW_L^5 & W_L^6+iW_L^7 & -\frac{2}{\sqrt{3}}W_L^8\end{pmatrix}. \tag{A6}$$

Equating the expressions in Eqs. (A5) and (A6), we get the identification

$$\begin{pmatrix}\hat{\mathbf{3}}_1\\ \hat{\mathbf{3}}_0\\ \hat{\mathbf{3}}_{-1}\end{pmatrix}=\begin{pmatrix}\hat{\mathbf{3}}^*_{-1}\\ \hat{\mathbf{3}}^*_0\\ \hat{\mathbf{3}}^*_1\end{pmatrix}=\begin{pmatrix}(W_L^1-iW_L^2)/\sqrt{2}\\ -W_L^3\\ (W_L^1+iW_L^2)/\sqrt{2}\end{pmatrix}, \tag{A7}$$

$$\begin{pmatrix}\hat{\mathbf{2}}_{1/2}\\ \hat{\mathbf{2}}_{-1/2}\end{pmatrix}=\begin{pmatrix}\hat{\mathbf{2}}'^*_{-1/2}\\ -\hat{\mathbf{2}}'^*_{1/2}\end{pmatrix}=\begin{pmatrix}(-W_L^4+iW_L^5)/\sqrt{2}\\ (-W_L^6+iW_L^7)/\sqrt{2}\end{pmatrix}, \tag{A8}$$

$$\hat{\mathbf{1}}=W_L^8. \tag{A9}$$

It is then obvious that the Standard Model $SU(2)_L$ gauge bosons correspond to the triplet $\hat{\mathbf{3}}$ (i.e., $W_L^{1,2,3}$) while the singlet $\hat{\mathbf{1}}$ (i.e., W_L^8) mixes with the $U(1)_X$ gauge boson W_X to form the $U(1)_Y$ gauge boson B

$$B=\frac{1}{\sqrt{g_L^2+g_X^2\beta^2}}(g_X\beta W_L^8+g_LW_X). \tag{A10}$$

In this expression, g_L and g_X stand for the gauge coupling constants of $SU(3)_L$ and $U(1)_X$, which are related to g_Y through the relation[11]

$$g_Y^{-2}=\beta^2g_L^{-2}+g_X^{-2}. \tag{A11}$$

Finally, note that the charge of the various components of $\mathbf{W}_L$ depend only on β

$$Q(\mathbf{W}_L)_\beta=\begin{pmatrix}0 & + & 0\\ - & 0 & -\\ 0 & + & 0\end{pmatrix}_{-\frac{1}{\sqrt{3}}},\quad \begin{pmatrix}0 & + & +\\ - & 0 & 0\\ - & 0 & 0\end{pmatrix}_{\frac{1}{\sqrt{3}}},$$
$$\begin{pmatrix}0 & + & -\\ - & 0 & --\\ + & ++ & 0\end{pmatrix}_{-\sqrt{3}},\quad \begin{pmatrix}0 & + & ++\\ - & 0 & +\\ -- & - & 0\end{pmatrix}_{\sqrt{3}}. \tag{A12}$$

[10]Without these $\frac{1}{\sqrt{2}}$ factors, it is easy to check that a mass term $\mathrm{Tr}(\mathbf{6}^\dagger\mathbf{6})$ will not correspond to the sum of the norm-squared of all six components.

[11]The relation changes if we choose instead to normalize the X and Y charges in a different way [15].

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