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Quarkonium production in an improved color evaporation model

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We propose an improved version of the color evaporation model to describe heavy quarkonium production. In contrast to the traditional color evaporation model, we impose the constraint that the invariant mass of the intermediate heavy quark-antiquark pair be larger than the mass of produced quarkonium. We also introduce a momentum shift between the heavy quark-antiquark pair and the quarkonium. Numerical calculations show that our model can describe the charmonium yields as well as the ratio of ψ' over J/ψ better than the traditional color evaporation model.

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I. INTRODUCTION

The study of heavy quarkonium production is one of the best ways to understand hadronization in QCD. Currently, the most widely used theory for heavy quarkonium production is the nonrelativistic QCD (NRQCD) approach [1] proposed in 1994. By introducing a systematic velocity expansion, this theory can naturally solve the infrared divergence problem encountered in the color singlet model (CSM) [2–4]. In this sense, NRQCD factorization can be thought of as a generalized version of CSM. Furthermore, it also successfully explains the ψ' surplus found at the Tevatron [5] by including color octet contributions.

Nevertheless, recent studies have shown that NRQCD factorization encounters serious difficulties [6]. First, naive power counting implies that $\psi(nS)$ and $\Upsilon(nS)$ productions at hadron colliders are dominated by the ${}^3S_1$ color octet channel which results in transverse polarization at high transverse momentum, p_T. However, experimental measurements found these states to be almost unpolarized. Current explanations of J/ψ polarization include ${}^1S_0$ color octet dominance [7–9] and cancelation of transverse polarization between the ${}^3S_1$ and ${}^3P_J$ color octet channels [7,10,11]. Whether these explanations can be generalized to other quarkonium states is still in question. Second, the nonperturbative color octet long-distance matrix elements (LDMEs) extracted from hadron colliders [12–14] are inconsistent with the upper bound set by e^+e^- collisions [15]. Thus, the LDMEs are not universal. Finally, there is still no convincing proof of NRQCD factorization to all orders in α_s. The state-of-the-art proof is only to next-to-next-to-leading order for special cases [16].

Considering the above difficulties, one should definitely study NRQCD factorization in more detail, but, at the same time, one may need to turn to other theories of quarkonium production. A theory which is known to satisfy all-order factorization is the color evaporation model (CEM) [17,18]. In this model, to produce a charmonium state ψ, one first produces a charm quark-antiquark pair $c\bar{c}$ with invariant mass smaller than the D-meson threshold. The pair then hadronizes to the ψ by randomly emitting soft particles.[1] The production cross section is expressed as

$$\frac{d\sigma_\psi(P)}{d^3P} = F_\psi \int_{2m_c}^{2M_D} dM \frac{d\sigma_{c\bar{c}}(M,P)}{dMd^3P}, \tag{1}$$

where m_c (M_D) is the mass of the charm quark (D meson) and M is the invariant mass of the $c\bar{c}$ pair. In this model, it is assumed that the ψ momentum, P, is approximately the same as the momentum of the $c\bar{c}$ pair. The predictive power of the CEM is based on the assumption that the hadronization factor F_ψ is universal and, thus, independent of the kinematics and spin of the ψ, as well as the production process.

Although CEM is intuitive, simple, and successful to explain J/ψ production data, it has a very fatal flaw. A straightforward conclusion from the CEM is that the ratio of differential cross sections of two charmonia states is independent of the kinematics and independent of the colliding species. However, it has long been known that the experimental results of the ratio of production cross section of ψ' over J/ψ depend on their transverse

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[1] We refer to them as soft gluons here.

momentum (for recent experimental data, see Refs. [19,20]). This disagreement is regarded as the main evidence that CEM is wrong.

Considering the advantages of CEM mentioned above, we may need to study whether a modification of CEM can provide a correct theory for quarkonium production. In this paper, by taking into account the physical effects overlooked in the original CEM, we propose an improved color-evaporation model (ICEM). On the one hand, the nice features of CEM are retained in the ICEM, including having only one parameter for each quarkonium state and satisfying all-order factorization. On the other hand, the ICEM can correctly describe charmonium production cross section ratios.

II. THE IMPROVED COLOR-EVAPORATION MODEL

Our picture of heavy quarkonium (for example, charmonium) production is as follows. To produce a charmonium state ψ, it is necessary to produce a $c\bar{c}$ pair in the hard collision, because the mass of the $c\bar{c}$ pair is much larger than the QCD nonperturbative scale $\Lambda_{\rm QCD}$. Before the $c\bar{c}$ pair hadronizes to charmonium, it will exchange many soft gluons between various color sources, as well as emit soft gluons. An illustration of this picture is given in Fig. 1. In this figure, the blob marked by "H" denotes the hard collision kernel, the blob marked by "S" denotes soft interactions, and the thick double lines denote the $c\bar{c}$ pair with momentum P. To separate the hard part from the other parts, we introduce a scale λ with $m_c \gg \lambda \gg \Lambda_{\rm QCD}$ and define the hard part as all particles that are off shell by more than λ^2.

We emphasize that we distinguish soft gluons exchanged between the $c\bar{c}$ pair and other color sources (with momentum denoted by P_S) from soft gluons emitted by the $c\bar{c}$ pair (with momentum denoted by P_X). Indeed, these two kinds of soft gluons are significantly different. The total energy of exchanged gluons can be either positive or negative. However, the emitted gluons will eventually evolve to experimentally observable particles. Thus, their total momentum must be timelike, and their total energy must be positive.

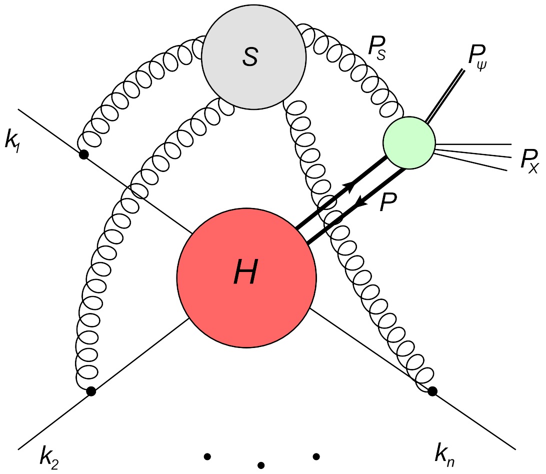

FIG. 1. An illustration of charmonium production in a high-energy collision. See text for details.

In our model, we construct a relationship between P and $\langle P_\psi \rangle$, the average momentum of ψ that has hadronized from a $c\bar{c}$ pair with fixed momentum P. The relationship is easy to obtain in the rest frame of P, with $P = (M, 0, 0, 0)$. For each event, we have

$$P = P_\psi + P_S + P_X. \tag{2}$$

In the spirit of the traditional CEM, we assume the distributions of P_S and P_X are rotation invariant in this frame, which implies $\langle P_S \rangle = (m_S, 0, 0, 0)$ and $\langle P_X \rangle = (m_X, 0, 0, 0)$. Because exchanged gluons can flow in either direction, we may expect $m_S \approx 0$. Thus, $\langle P_\psi \rangle = (M - m_X, 0, 0, 0)$ with $m_X > 0$. Therefore,

$$M_\psi < M - m_X < M, \tag{3}$$

where we use the fact that $\langle P_\psi^0 \rangle$ must be larger than M_ψ. Equation (3) sets a lower limit on M that is significantly different from the lower limit $2m_c$ of the traditional CEM.

As both P_S and P_X are order of λ, power counting of P_ψ gives $(O(m_c), O(\lambda), O(\lambda), O(\lambda))$. Combining with the on-shell condition $P_\psi^2 = M_\psi^2$, we arrive at $P_\psi^0 = M_\psi + O(\lambda^2/m_c)$. Thus, we have

$$\langle P_\psi \rangle = \frac{M_\psi}{M} P + O(\lambda^2/m_c), \tag{4}$$

which again differs from the relation used in the traditional CEM where P_ψ is identified with P. Note that the proportionality between the momenta of the mother and daughter particles in Eq. (4) was first proposed in Ref. [21] to relate the momentum of the χ_{cJ} and the J/ψ produced by its decay. It has since been used in many calculations of quarkonium production in the NRQCD framework. In this paper, we prove the relation rigorously with clear assumptions. By combining Eqs. (3) and (4), we arrive at the improved color evaporation model (ICEM),

$$\begin{aligned} \frac{d\sigma_\psi(P)}{d^3P} &= F_\psi \int_{M_\psi}^{2M_D} d^3P' dM \frac{d\sigma_{c\bar{c}}(M, P')}{dM d^3P'} \delta^3\left(P - \frac{M_\psi}{M} P'\right) \\ &= F_\psi \int_{M_\psi}^{2M_D} dM \frac{d\sigma_{c\bar{c}}(M, P' = (M/M_\psi)P)}{dM d^3P}, \end{aligned} \tag{5}$$

with correction at $O(\lambda^2/m_c^2)$. If one is only interested in the transverse momentum distribution, we have

$$\frac{d\sigma_\psi(P)}{dp_T} = F_\psi \int_{M_\psi}^{2M_D} dM \frac{M}{M_\psi} \frac{d\sigma_{c\bar{c}}(M,P')}{dM dp'_T}\bigg|_{p'_T=(M/M_\psi)p_T}. \tag{6}$$

Before performing any numerical calculations, we can already expect some advantages from the ICEM. First, because there is an explicit charmonium mass dependence in Eq. (5), the ratio of differential cross sections of two charmonia is no longer p_T independent in the ICEM. Thus, it is possible to explain data such as $d\sigma_{\psi(2S)}/d\sigma_{J/\psi}$. Second, by making a distinction between the momentum of the $c\bar{c}$ pair and that of charmonium, the predicted p_T spectra will be softer and, thus, may explain the high-p_T data better.

We emphasize that the ICEM Eq. (5) does not mean that the $c\bar{c}$ pair with invariant mass smaller than M_ψ has no possibility to hadronize to ψ. In fact, this kind of $c\bar{c}$ pair can absorb energy by interacting with other color source, and thus can have larger invariant mass and hadronize to ψ. At the same time, even if the invariant mass of the $c\bar{c}$ pair is larger than M_ψ, it may lose energy by interacting with other color sources and eventually not hadronize to ψ because of the invariant mass being too small. By assuming $m_S \approx 0$, we effectively approximate that the two effects cancel each other. As a result, Eq. (5) should only be interpreted at the integration level.

An exception for the above argument is for the ground state particle production, say η_c for charmonium. Based on the quark-hadron duality, the $c\bar{c}$ pair with invariant mass smaller than the D-meson threshold must hadronize to charmonium; therefore, it is not possible for a $c\bar{c}$ pair with $M_{c\bar{c}} > M_{\eta_c}$ to emit too much energy so that its invariant mass becomes smaller than M_{η_c}. This means that the approximation $m_S \approx 0$ is not reasonable here and, thus, ICEM is not good for η_c production. However, for η_c production, as the condition Eq. (3) is not needed, the original CEM should be good.

III. NUMERICAL RESULTS

To confront our model with experimental data, we updated the CEM parameters determined in Ref. [22]. In that work, in an attempt to reduce the uncertainty on the total charm cross section, the charm mass was fixed at 1.27 ± 0.09 GeV while the factorization and renormalization scales were fit to a subset of the measured total charm cross section data. The values found were $\mu_F/m = 2.1^{+2.55}_{-0.85}$ and $\mu_R/m = 1.6^{+0.11}_{-0.12}$ employing the CT10 proton parton densities [23].

The central open charm parameter set $(m, \mu_F/m, \mu_R/m) = (1.27, 2.1, 1.6)$ was used to calculate the energy dependence of the forward J/ψ cross section, $\sigma(x_F > 0)$, in the CEM using the exclusive $c\bar{c}$ production code described in Ref. [24]. Because the NLO $c\bar{c}$ code is an exclusive calculation, the mass cut is on the invariant average over kinematic variables of the c and $\bar{c}$. Thus, in this calculation μ_F and μ_R are defined relative to the transverse mass of the charm quark, $\mu_{F,R} \propto m_T = \sqrt{m^2 + p_T^2}$ where $p_T^2 = 0.5(p_{T_c}^2 + p_{T_{\bar{c}}}^2)$. The normalization F_ψ is the scale factor that adjusted the fraction of the total charm cross section in the mass range $2m < M < 2m_D$ to the forward cross section data.

To determine the uncertainty on the J/ψ calculation, the charm mass was varied between the upper and lower limits, 1.36 and 1.18 GeV, respectively, for the central values of μ_F/m and μ_R/m, and the scales were varied around their central values while the charm mass was held fixed at its central value of 1.27 GeV: $(\mu_F/m, \mu_R/m) = (C, L)$, (L, C), (L, L), (C, H), (H, C), (H, H) where $H(L)$ is the upper (lower) limit of the factorization and renormalization scales determined from the charm fits. Using the same value of F_ψ in all cases, the uncertainty band on the J/ψ cross section was calculated by finding the upper and lower limits of the mass and scale variations and adding them in quadrature, as discussed in Refs. [22,25].

To calculate the charmonium p_T dependence, a Gaussian transverse momentum broadening is added to the final state. The value of the average k_T kick applied was taken to be $\langle k_T^2 \rangle = 1 + (1/12)\ln(\sqrt{s}/20)$ GeV^2 [22], giving 1.19 GeV^2 at RHIC and 1.49 GeV^2 at 7 TeV. Alternatively, one can use color glass condensation to give a correct small p_T behavior [26].

Since the ICEM calculation discussed here reduces the cross section relative to the calculation in Ref. [22], the value of F_ψ had to be increased by 40% to retain agreement with the data. The ψ' cross section and its uncertainty was calculated with the same parameters but with a value of $F_{\psi'}$ scaled to the ψ' data.

To obtain the uncertainty on the ψ'/ψ ratio, the mass and scale uncertainties were assumed to be correlated. The resulting uncertainty band is dominated by the scale uncertainty, the mass uncertainty is small.

Our results for J/ψ production cross section as a function of p_T are shown in Fig. 2, where we compare with data at hadron colliders for center of mass energies of 0.2 and 7 TeV. The 0.2 TeV RHIC data are measured by the PHENIX Collaboration [19] at central rapidities, $|y| < 0.35$, and the 7 TeV LHC data are measured by the LHCb Collaboration [27] at forward rapidity, $2.5 < y < 4$. The largest discrepancy between the model and the data is in the RHIC data at intermediate p_T, $4 < p_T < 7$ GeV. However, since the experimental uncertainty is rather large in this region, our results are in general agreement with the data.

We now turn to the ψ' production cross section as a function of p_T in Fig. 3. We again compare with the midrapidity PHENIX data [19] at 0.2 TeV and the forward LHCb data [20] at 7 TeV. Since the ψ' rates are generally lower, the measured uncertainty is larger. Given this, the agreement of the calculation with the data is also good.

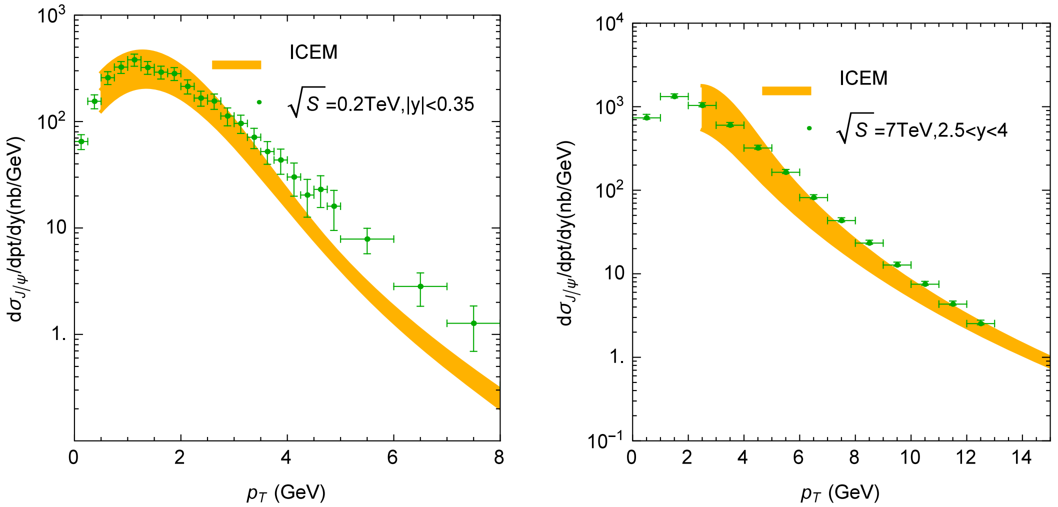

FIG. 2. Results for J/ψ production. The 0.2 TeV PHENIX data and 7 TeV LHCb data are taken from Ref. [19] and Ref. [27], respectively.

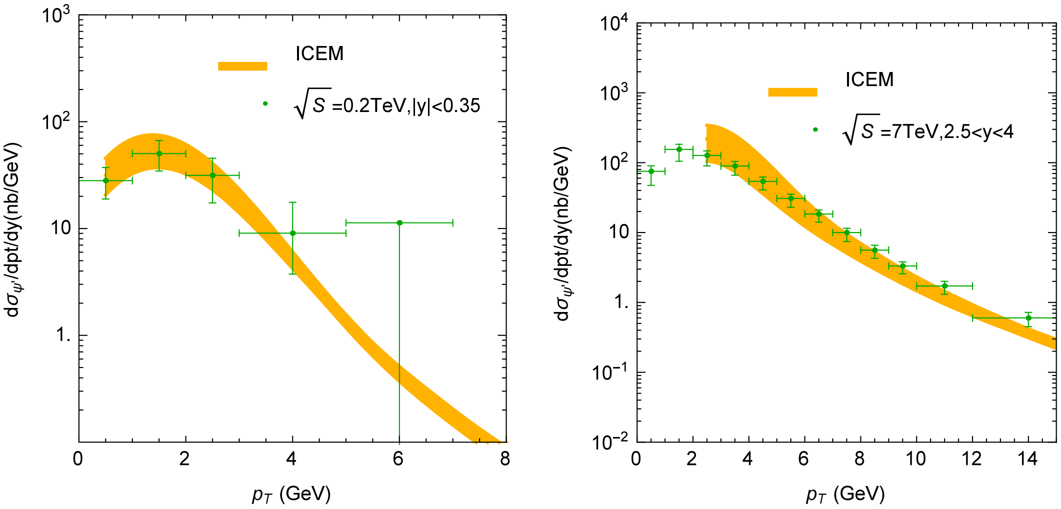

FIG. 3. Results for ψ' production. The 0.2 TeV PHENIX data and 7 TeV LHCb data are taken from Ref. [19] and Ref. [20], respectively.

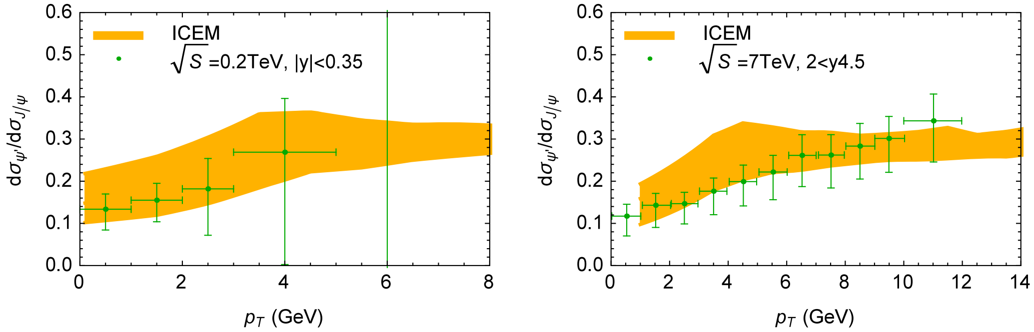

FIG. 4. Results for ratio of the ψ' production cross section to that of J/ψ. The 0.2 TeV PHENIX data and 7 TeV LHCb data are taken from Ref. [19] and Ref. [20], respectively.

The ratio of the production cross sections of ψ' to that of J/ψ as a function of p_T is given in Fig. 4. The 0.2 TeV RHIC data and 7 TeV LHC data are taken from Ref. [19] and Ref. [20], respectively. Although the original CEM predicts a constant for this ratio, in contradiction with the data, our ICEM calculations are in good agreement with all data.

IV. SUMMARY AND DISCUSSION

By distinguishing between exchanged and emitted soft gluons and considering some physical constraints, we propose an improved color evaporation model for charmonium production. Comparison with data shows that the ICEM can nicely reproduce the p_T dependence of the ratio of the ψ' to J/ψ production cross sections. Thus, this improved model overcomes one of the main obstacles of the original CEM. The success of the ICEM calculation confirms our picture of charmonium production.

We note that the question of polarization in the ICEM as well as the original CEM has not yet been addressed. As seen in the NRQCD approach, the polarization is an important test of models. The prediction of the final-state charmonium polarization depends on whether soft gluons change spin and angular momentum of the $c\bar{c}$ pair. A preliminary study of charmonium polarization in the CEM will be presented elsewhere [28].

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