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Impact of finite density on spectroscopic parameters of decuplet baryons

K. Azizi,[1] N. Er,[2] and H. Sundu[3]

[1]*Department of Physics, Doğuş University, Acıbadem-Kadıköy, 34722 Istanbul, Turkey*
[2]*Department of Physics, Abant İzzet Baysal University, Gölköy Kampüsü, 14980 Bolu, Turkey*
[3]*Department of Physics, Kocaeli University, 41380 İzmit, Turkey*

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The decuplet baryons, Δ, Σ^*, Ξ^*, and Ω^-, are studied in nuclear matter by using the in-medium QCD sum rules. By fixing the three-momentum of the particles under consideration at the rest frame of the medium, the negative energy contributions are removed. It is obtained that the parameters of the Δ baryon are more affected by the medium against the Ω^- state, containing three strange quarks, whose mass and residue are not considerably affected by the medium. We also find the vector and scalar self-energies of these baryons in nuclear matter. By the recent progresses at the $\overline{P}$ANDA experiment at the FAIR and NICA facility, it may be possible to study the in-medium properties of such states, even the multistrange Ξ^* and Ω^- systems, in the near future.

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I. INTRODUCTION

The investigations of the properties of hadrons under extreme conditions have been the focus of much attention for many years. Such investigations are very important in the study of the internal structure of dense astrophysical objects like neutron stars. The formation of neutron stars is influenced by all four known fundamental interactions. Hence, understanding of their nature can help us in the course of unification of all fundamental forces within a common theoretical framework, which is one of the biggest challenges for physics. The recent observation of massive neutron stars with roughly twice the solar mass [1,2] has stimulated the focuses on the equation of state of the dense nuclear matter (see for instance [3–6]). However, the expected appearance of hyperons at about two times nuclear density, called the “hyperon puzzle,” remains an unresolved mystery in neutron stars (concerning the appearance of hyperons in neutron stars, see, for example, [7,8]). It has also been found that Δ isobars appear at a density of the order of 2–3 times nuclear matter saturation density, and a “Δ puzzle” exists, similar to the hyperon puzzle if the potential of the Δ in nuclear matter is close to the one indicated by the experimental data [9]. More theoretical and experimental investigations on the properties of strange and nonstrange light baryons in a dense medium are needed to solve such puzzles.

From the experimental side, the bound nuclear systems with one, two, or three units of strangeness are poorly known compared to that of the nonstrange states like nucleons. The large production probability of various hyperon-antihyperon pairs in antiproton collisions will provide opportunities for a series of new studies on the behavior of the systems containing two or even more units of strangeness at the $\overline{P}$ANDA experiment at FAIR. By the progresses made, it will be possible to study the in-medium properties of the doubly strange $\Lambda\Lambda$-hypernuclei as well as the multistrange Ξ^-, $\overline{\Xi}^+$, and Ω^- systems in the near future [10].

From the theoretical side, the effects of a nuclear medium on the physical parameters of the nucleon have been widely investigated in the literature (see for instance [11–15] and references therein). But, we have only a few studies dedicated to the in-medium properties of hyperons and decuplet baryons in the literature (for instance, see [16–22]). In the present study, we investigate the impact of nuclear matter on some spectroscopic parameters of the Δ, Σ^*, Ξ^*, and Ω^- decuplet baryons. In particular, we calculate the mass and residue as well as the scalar and vector self-energies of these baryons using the well-established in-medium QCD sum rule approach. We compare the in-medium results with those obtained at $\rho = 0$ or vacuum and find the corresponding shifts. To remove the contributions of the negative energy particles, we work at the rest frame of the nuclear matter and fix the three-momentum of the particles under consideration.

II. Δ, Σ^*, Ξ^*, AND Ω^- BARYONS IN NUCLEAR MATTER

In this section we aim to construct sum rules for the mass, residue, and vector self-energy of the decuplet baryons and numerically analyze the obtained results. To this end and in accordance with the general philosophy of the QCD sum rule approach,we start with a correlation function as the building block of the method:

$$\Pi_{\mu\nu}(p) = i\int d^4x e^{ip\cdot x}\langle\psi_0|T[\eta_{\mu,D}(x)\bar{\eta}_{\nu,D}(0)]|\psi_0\rangle, \quad (1)$$

where p is the four-momentum of the decuplet (D) baryon, $|\psi_0\rangle$ is the ground state of the nuclear matter, and $\eta_{\mu,D}$ is the

TABLE I. The value of the normalization constant A_D and the quark flavors q_1, q_2, q_3 for the decuplet baryons.

	A_D	q_1	q_2	q_3
Σ^*	$\sqrt{2/3}$	u	d	s
Δ^0	$\sqrt{1/3}$	d	d	u
Ξ^*	$\sqrt{1/3}$	s	s	u
Ω^-	$1/3$	s	s	s

interpolating current of the D baryon. The general form of the interpolating current for decuplet baryons in a compact form reads

$$\eta_{\mu,D} = A_D \epsilon^{abc}\{(q_1^{aT} C\gamma_\mu q_2^b)q_3^c + (q_2^{aT} C\gamma_\mu q_3^b)q_1^c + (q_3^{aT} C\gamma_\mu q_1^b)q_2^c\}, \tag{2}$$

where a, b, c are color indices; C is the charge conjugation operator; and A_D is the normalization constant. The quark content and value of A_D for different members are given in Table I [23]. We will calculate the aforementioned correlation function in two representations: hadronic and operator product expansion (OPE). By equating these two representations, one can get the QCD sum rules for the aimed physical quantities.

A. Hadronic representation

The correlation function in the hadronic side is obtained by inserting a complete set of baryonic states with the same quantum numbers as the interpolating current. After performing the integral over four-x, we get

$$\Pi_{\mu\nu}^{\mathrm{Had}}(p) = -\frac{\langle\psi_0|\eta_{\mu,D}(0)|D(p^*,s)\rangle\langle D(p^*,s)|\bar{\eta}_{\nu,D}(0)|\psi_0\rangle}{p^{*2}-m_D^{*2}} + \cdots, \tag{3}$$

where $|D(p^*,s)\rangle$ is the decuplet baryon state with spin s and in-medium four-momentum p^*, m_D^* is the modified mass of the decuplet baryon in a medium, and ... indicates the contributions of the higher states and continuum. The matrix elements in Eq. (3) can be represented as

$$\langle\psi_0|\eta_{\mu,D}(0)|D(p^*,s)\rangle = \lambda_D^* u_\mu(p^*,s),$$
$$\langle D(p^*,s)|\bar{\eta}_{\nu,D}(0)|\psi_0\rangle = \bar{\lambda}_D^* \bar{u}_\nu(p^*,s), \tag{4}$$

where $u_\mu(p^*,s)$ is the in-medium Rarita-Schwinger spinor and λ_D^* is the modified residue or the coupling strength of the decuplet baryon to the nuclear medium. Inserting Eq. (4) into Eq. (3) and summing over the spins of the D baryon, one can, in principle, find the hadronic side of the correlation function. Before that, it should be stated that the current $\eta_{\mu,D}$ couples to both the spin-$1/2$ octet states and the spin-$3/2$ decuplet states. In order to get only the contributions of the decuplet baryons, the contributions of the unwanted spin-$1/2$ states must be removed from the correlation function. For this aim, we come next with the following procedure. The matrix element of $\eta_{\mu,D}$ between the spin-$1/2$ and in-medium states can be decomposed as

$$\langle\psi_0|\eta_{\mu,D}(0)|\frac{1}{2}(p^*)\rangle = (C_1 p_\mu^* + C_2\gamma_\mu)u(p^*), \tag{5}$$

where C_1 and C_2 are constants and $u(p^*)$ is the in-medium Dirac spinor of momentum p^*. By multiplying both sides of the above equation with γ^μ and using the condition $\eta_{\mu,D}\gamma^\mu = 0$, we immediately find the constant C_1 in terms of C_2. Hence,

$$\langle\psi_0|\eta_{\mu,D}(0)|\frac{1}{2}(p^*)\rangle = C_2\left(-\frac{4}{m_{1/2}^*}p_\mu^* + \gamma_\mu\right)u(p^*), \tag{6}$$

where $m_{1/2}^*$ is the modified mass of the spin-$1/2$ baryons. It can be easily seen that the unwanted contributions of the spin-$1/2$ states are proportional to p_μ^* and γ_μ. By ordering the Dirac matrices as $\gamma_\mu \not{p}^* \gamma_\nu$ and setting to zero the terms with γ_μ in the beginning and γ_ν at the end and those proportional to p_μ^* and p_ν^*, the contributions from the unwanted spin-$1/2$ states can be easily eliminated.

Now, we insert Eq. (4) into Eq. (3) and use the summation over spins of the Rarita-Schwinger spinor as

$$\sum_s u_\mu(p^*,s)\bar{u}_\nu(p^*,s) = -(\not{p}^* + m_D^*)\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2p_\mu^* p_\nu^*}{3m_D^{*2}} + \frac{p_\mu^*\gamma_\nu - p_\nu^*\gamma_\mu}{3m_D^*}\right], \tag{7}$$

as a result of which we get

$$\Pi_{\mu\nu}^{\mathrm{Had}}(p) = \frac{\lambda_D^*\bar{\lambda}_D^*(\not{p}^* + m_D^*)}{p^{*2}-m_D^{*2}}\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2p_\mu^* p_\nu^*}{3m_D^{*2}} + \frac{p_\mu^*\gamma_\nu - p_\nu^*\gamma_\mu}{3m_D^*}\right] + \cdots. \tag{8}$$

To proceed, we would like to mention that the in-medium momentum and the modified mass can be written in terms of the self-energies $\Sigma_{\mu,\nu}$ and Σ^S as $p_\mu^* = p_\mu - \Sigma_{\mu,v}$ and $m_D^* = m_D + \Sigma^S$, where Σ^S is the scalar self-energy. The self-energy $\Sigma_{\mu,v}$ can also be written in a general form as

$$\Sigma_{\mu,v} = \Sigma_v u_\mu + \Sigma_v' p_\mu, \tag{9}$$

where Σ_v is called the vector self-energy and u_μ is the four-velocity of the nuclear medium. In the mean-field approximation, the scalar and vector self-energies are obtained to be real and independent of momentum and the Σ_ν' is taken to be identically zero [11,24]. In this context, particles of any three-momentum appear as stable quasiparticles with self-energies that are roughly linear in the density up to nuclear matter density [11,25]. We perform the calculations in the rest frame of the nuclear medium, i.e., $u_\mu = (1, 0)$, and at the fixed three-momentum of the D baryon, $|\vec{p}|$. We get

$$\Pi^{\rm Had}_{\mu\nu}(p_0,\vec{p}) = \lambda^*_D\bar{\lambda}^*_D \frac{(\not{p} - \Sigma_v\not{u} + m^*_D)}{p^2 + \Sigma_v^2 - 2p_0\Sigma_v - m^{*2}_D} \times \Big[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2}{3m^{*2}_D}(p_\mu p_\nu - \Sigma_v p_\mu u_\nu - \Sigma_v u_\mu p_\nu + \Sigma_v^2 u_\mu u_\nu) + \frac{1}{3m^*_D}(p_\mu\gamma_\nu - \Sigma_v u_\mu\gamma_\nu - p_\nu\gamma_\mu + \Sigma_v u_\nu\gamma_\mu)\Big] + \cdots, \quad (10)$$

where $p_0 = p\cdot u$ is the energy of the quasiparticle. After ordering the Dirac matrices and eliminating the unwanted spin-$1/2$ contributions, we get

$$\Pi^{Had}_{\mu\nu}(p_0,\vec{p}) = \frac{\lambda^*_D\bar{\lambda}^*_D}{(p_0 - E_p)(p_0 - \bar{E}_p)}[m^*_D g_{\mu\nu} + g_{\mu\nu}\not{p} - \Sigma_v g_{\mu\nu}\not{u}] + \cdots, \quad (11)$$

where $E_p = \Sigma_v + \sqrt{|\vec{p}|^2 + m^{*2}_D}$ and $\bar{E}_p = \Sigma_v - \sqrt{|\vec{p}|^2 + m^{*2}_D}$ are the positions of the positive- and negative-energy poles, respectively. One can write the above equation as an integral representation in terms of the spectral density,

$$\Pi^{\rm Had}_{\mu\nu}(p_0,\vec{p}) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{\Delta\rho^{\rm Had}_{\mu\nu}(p_0,\vec{p})}{\omega - p_0}, \quad (12)$$

where the spectral density $\Delta\rho^{\rm Had}_{\mu\nu}(p_0,\vec{p})$, defined by

$$\Delta\rho^{\rm Had}_{\mu\nu}(p_0,\vec{p}) = {\rm Lim}_{\epsilon\to 0^+}[\Pi^{\rm Had}_{\mu\nu}(\omega + i\epsilon,\vec{p}) - \Pi^{\rm Had}_{\mu\nu}(\omega - i\epsilon,\vec{p})], \quad (13)$$

is given as

$$\Delta\rho^{\rm Had}_{\mu\nu}(p_0,\vec{p}) = -\frac{1}{2\sqrt{m^{*2}_D + |\vec{p}|^2}}\lambda^*_D\bar{\lambda}^*_D[m^*_D g_{\mu\nu} + g_{\mu\nu}\not{p} - \Sigma_v g_{\mu\nu}\not{u}][\delta(\omega - E_p) - \delta(\omega - \bar{E}_p)]. \quad (14)$$

The next step is to exclude the negative-energy pole contribution by multiplying the correlation function with the weight function $(\omega - \bar{E}_p)e^{\frac{-\omega^2}{M^2}}$ and performing the integral over ω from $-\omega_0$ to ω_0, i.e.,

$$\Pi^{\rm Had}_{\mu\nu}(p_0,\vec{p}) = \int_{-\omega_0}^{\omega_0} d\omega\Delta\rho^{\rm Had}_{\mu\nu}(\omega,\vec{p})(\omega - \bar{E}_p)e^{-\frac{\omega^2}{M^2}}, \quad (15)$$

where ω_0 is the threshold parameter and M^2 is the Borel mass parameter that shall be fixed later. After performing the integral in Eq. (15), the hadronic side of the correlation function takes its final form in terms of the corresponding structures,

$$\Pi^{\rm Had}_{\mu\nu}(p_0,\vec{p}) = \lambda^{*2}_D e^{-E_p^2/M^2}[m^*_D g_{\mu\nu} + g_{\mu\nu}\not{p} - \Sigma_v g_{\mu\nu}\not{u}]. \quad (16)$$

B. OPE representation

The OPE side of the correlation function is calculated at the large spacelike region $p^2 \ll 0$ in terms of QCD degrees of freedom. One can write the OPE side of the correlation function, in terms of the involved structures, as

$$\Pi^{\rm OPE}_{\mu\nu}(p_0,\vec{p}) = \Pi_1(p_0,\vec{p})g_{\mu\nu} + \Pi_2(p_0,\vec{p})\not{p}g_{\mu\nu} + \Pi_3(p_0,\vec{p})\not{u}g_{\mu\nu}, \quad (17)$$

where the $\Pi_i(p_0,\vec{p})$ functions, with $i = 1$, 2, or 3, can be written in terms of the spectral densities $\Delta\rho_i(p_0,\vec{p})$ in the OPE side as

$$\Pi_i(p_0,\vec{p}) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} dw \frac{\Delta\rho_i(p_0,\vec{p})}{w - p_0}, \quad (18)$$

where $\Delta\rho_i(p_0,\vec{p})$ are the imaginary parts of $\Pi_i(p_0,\vec{p})$ functions obtained from the OPE version of Eq. (13). The main aim, in the present subsection, is to find the $\Delta\rho_i(p_0,\vec{p})$ spectral densities, by use of which we can find the $\Pi_i(p_0,\vec{p})$ functions in the OPE side. To proceed, we start with the correlation function in Eq. (1). By substituting the explicit form of the interpolating current for the decuplet baryons under consideration into the correlation function in Eq. (1) and after contracting out all the quark pairs using Wick's theorem, we get

$$\Pi^{\rm OPE,\Delta}_{\mu\nu}(p) = \frac{i}{3}\epsilon_{abc}\epsilon_{a'b'c'}\int d^4x e^{ipx}\langle\{2S^{ca'}_d(x)\gamma_\nu S'^{ab'}_d(x)\gamma_\mu S^{bc'}_u(x) - 2S^{cb'}_d(x)\gamma_\nu S'^{aa'}_d(x)\gamma_\mu S^{bc'}_u(x) + 4S^{cb'}_d(x)\gamma_\nu S'^{ba'}_u(x)\gamma_\mu S^{ac'}_d(x) + 2S^{ca'}_u(x)\gamma_\nu S'^{ab'}_d(x)\gamma_\mu S^{bc'}_d(x) - 2S^{ca'}_u(x)\gamma_\nu S'^{bb'}_d(x)\gamma_\mu S^{ac'}_d(x) - S^{cc'}_u(x){\rm Tr}[S^{ba'}_d(x)\gamma_\nu S'^{ab'}_d(x)\gamma_\mu] + S^{cc'}_u(x){\rm Tr}[S^{bb'}_d(x)\gamma_\nu S'^{aa'}_d(x)\gamma_\mu] - 4S^{cc'}_d(x){\rm Tr}[S^{ba'}_u(x)\gamma_\nu S'^{ab'}_d(x)\gamma_\mu]\}\rangle_{\psi_0}, \quad (19)$$

$$\Pi_{\mu\nu}^{\mathrm{OPE},\Sigma^*}(p) = -\frac{2i}{3}\epsilon_{abc}\epsilon_{a'b'c'}\int d^4x e^{ipx}\langle\{S_d^{ca'}(x)\gamma_\nu S_u'^{bb'}(x)\gamma_\mu S_s^{ac'}(x) + S_d^{cb'}(x)\gamma_\nu S_s'^{aa'}(x)\gamma_\mu S_u^{bc'}(x) + S_s^{ca'}(x)\gamma_\nu S_d'^{bb'}(x)\gamma_\mu S_u^{ac'}(x) + S_s^{cb'}(x)\gamma_\nu S_u'^{aa'}(x)\gamma_\mu S_d^{bc'}(x) + S_u^{ca'}(x)\gamma_\nu S_s'^{bb'}(x)\gamma_\mu S_d^{ac'}(x) + S_u^{cb'}(x)\gamma_\nu S_d'^{aa'}(x)\gamma_\mu S_s^{bc'}(x) + S_s^{cc'}(x)\mathrm{Tr}[S_d^{ba'}(x)\gamma_\nu S_u'^{ab'}(x)\gamma_\mu] + S_u^{cc'}(x)\mathrm{Tr}[S_s^{ba'}(x)\gamma_\nu S_d'^{ab'}(x)\gamma_\mu] + S_d^{cc'}(x)\mathrm{Tr}[S_u^{ba'}(x)\gamma_\nu S_s'^{ab'}(x)\gamma_\mu]\}\rangle_{\psi_0}, \quad (20)$$

$$\Pi_{\mu\nu}^{\mathrm{OPE},\Xi^*}(p) = \frac{i}{3}\epsilon_{abc}\epsilon_{a'b'c'}\int d^4x e^{ipx}\langle\{2S_s^{ca'}(x)\gamma_\nu S_s'^{ab'}(x)\gamma_\mu S_u^{bc'}(x) - 2S_s^{cb'}(x)\gamma_\nu S_s'^{aa'}(x)\gamma_\mu S_u^{bc'}(x) + 4S_s^{cb'}(x)\gamma_\nu S_u'^{ba'}(x)\gamma_\mu S_s^{ac'}(x) + 2S_u^{ca'}(x)\gamma_\nu S_s'^{ab'}(x)\gamma_\mu S_s^{bc'}(x) - 2S_u^{ca'}(x)\gamma_\nu S_s'^{bb'}(x)\gamma_\mu S_s^{ac'}(x) - S_u^{cc'}(x)\mathrm{Tr}[S_s^{ba'}(x)\gamma_\nu S_s'^{ab'}(x)\gamma_\mu] + S_u^{cc'}(x)\mathrm{Tr}[S_s^{bb'}(x)\gamma_\nu S_s'^{aa'}(x)\gamma_\mu] - 4S_s^{cc'}(x)\mathrm{Tr}[S_u^{ba'}(x)\gamma_\nu S_s'^{ab'}(x)\gamma_\mu]\}\rangle_{\psi_0}, \quad (21)$$

and

$$\Pi_{\mu\nu}^{\mathrm{OPE},\Omega^-}(p) = \epsilon_{abc}\epsilon_{a'b'c'}\int d^4x e^{ipx}\langle\{S_s^{ca'}(x)\gamma_\nu S_s'^{ab'}(x)\gamma_\mu S_s^{bc'}(x) - S_s^{ca'}(x)\gamma_\nu S_s'^{bb'}(x)\gamma_\mu S_s^{ac'}(x) - S_s^{cb'}(x)\gamma_\nu S_s'^{aa'}(x)\gamma_\mu S_s^{bc'}(x) + S_s^{cb'}(x)\gamma_\nu S_s'^{ba'}(x)\gamma_\mu S_s^{ac'}(x) - S_s^{cc'}(x)\mathrm{Tr}[S_s^{ba'}(x)\gamma_\nu S_s'^{ab'}(x)\gamma_\mu] + S_s^{cc'}(x)\mathrm{Tr}[S_s^{bb'}(x)\gamma_\nu S_s'^{aa'}(x)\gamma_\mu]\}\rangle_{\psi_0}, \quad (22)$$

where $S' = CS^TC$. Here, $S_{u,d,s}$ denotes the light quark propagator and it is given at the nuclear medium in the fixed-point gauge as [11]

$$\begin{aligned} S_q^{ab}(x) &\equiv \langle\psi_0|T[q^a(x)\bar{q}^b(0)]|\psi_0\rangle_{\rho_N} \\ &= \frac{i}{2\pi^2}\delta^{ab}\frac{1}{(x^2)^2}\not{x} - \frac{m_q}{4\pi^2}\delta^{ab}\frac{1}{x^2} + \chi_q^a(x)\bar{\chi}_q^b(0) \\ &\quad - \frac{ig_s}{32\pi^2}F_{\mu\nu}^A(0)t^{ab,A}\frac{1}{x^2}[\not{x}\sigma^{\mu\nu} + \sigma^{\mu\nu}\not{x}] + \cdots, \end{aligned} \quad (23)$$

where ρ_N is the nuclear matter density, m_q is the light quark mass, χ_q^a and $\bar{\chi}_q^b$ are the Grassmann background quark fields, and $F_{\mu\nu}^A$ are classical background gluon fields. After inserting Eq. (23) into Eqs. (19)–(22), we obtain the products of the Grassmann background quark fields and classical background gluon fields that correspond to the ground-state matrix elements of the corresponding quark and gluon operators [11]:

$$\begin{aligned} \chi_{a\alpha}^q(x)\bar{\chi}_{b\beta}^q(0) &= \langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_{\rho_N}, \\ F_{\kappa\lambda}^A F_{\mu\nu}^B &= \langle G_{\kappa\lambda}^A G_{\mu\nu}^B\rangle_{\rho_N}, \\ \chi_{a\alpha}^q\bar{\chi}_{b\beta}^q F_{\mu\nu}^A &= \langle q_{a\alpha}\bar{q}_{b\beta}G_{\mu\nu}^A\rangle_{\rho_N}, \\ &\text{and} \\ \chi_{a\alpha}^q\bar{\chi}_{b\beta}^q\chi_{c\gamma}^q\bar{\chi}_{d\delta}^q &= \langle q_{a\alpha}\bar{q}_{b\beta}q_{c\gamma}\bar{q}_{d\delta}\rangle_{\rho_N}. \end{aligned} \quad (24)$$

Now, we need to define the quark, gluon, and mixed condensates in nuclear matter. The matrix element $\langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_{\rho_N}$ is parametrized as [11]

$$\begin{aligned} \langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_{\rho_N} = -\frac{\delta_{ab}}{12}\Bigg[&\Bigg(\langle\bar{q}q\rangle_{\rho_N} + x^\mu\langle\bar{q}D_\mu q\rangle_{\rho_N} + \frac{1}{2}x^\mu x^\nu\langle\bar{q}D_\mu D_\nu q\rangle_{\rho_N} + \cdots\Bigg)\delta_{\alpha\beta} \\ &+ \Bigg(\langle\bar{q}\gamma_\lambda q\rangle_{\rho_N} + x^\mu\langle\bar{q}\gamma_\lambda D_\mu q\rangle_{\rho_N} + \frac{1}{2}x^\mu x^\nu\langle\bar{q}\gamma_\lambda D_\mu D_\nu q\rangle_{\rho_N} + \cdots\Bigg)\gamma_{\alpha\beta}^\lambda\Bigg]. \end{aligned} \quad (25)$$

The quark-gluon mixed condensate in nuclear matter is written as

$$\begin{aligned} &\langle g_s q_{a\alpha}\bar{q}_{b\beta}G_{\mu\nu}^A\rangle_{\rho_N} \\ &= -\frac{t_{ab}^A}{96}\{\langle g_s\bar{q}\sigma\cdot Gq\rangle_{\rho_N}[\sigma_{\mu\nu} + i(u_\mu\gamma_\nu - u_\nu\gamma_\mu)\not{u}]_{\alpha\beta} \\ &\quad + \langle g_s\bar{q}\not{u}\sigma\cdot Gq\rangle_{\rho_N}[\sigma_{\mu\nu}\not{u} + i(u_\mu\gamma_\nu - u_\nu\gamma_\mu)]_{\alpha\beta} \\ &\quad - 4(\langle\bar{q}u\cdot Du\cdot Dq\rangle_{\rho_N} + im_q\langle\bar{q}\not{u}u\cdot Dq\rangle_{\rho_N}) \\ &\quad \times[\sigma_{\mu\nu} + 2i(u_\mu\gamma_\nu - u_\nu\gamma_\mu)\not{u}]_{\alpha\beta}\}, \end{aligned} \quad (26)$$

where t_{ab}^A are Gell-Mann matrices and $D_\mu = \frac{1}{2}(\gamma_\mu\not{D} + \not{D}\gamma_\mu)$. The matrix element of the four-dimensional gluon condensate can also be parametrized as

$$\langle G^A_{\kappa\lambda} G^B_{\mu\nu}\rangle_{\rho_N} = \frac{\delta^{AB}}{96}[\langle G^2\rangle_{\rho_N}(g_{\kappa\mu}g_{\lambda\nu} - g_{\kappa\nu}g_{\lambda\mu}) + O(\langle \mathbf{E}^2 + \mathbf{B}^2\rangle_{\rho_N})], \tag{27}$$

where we ignore the last term in this equation because of its negligible contribution. The different condensates in the above equations are defined in the following way [11,26]:

$$\begin{aligned}
\langle \bar{q}\gamma_\mu q\rangle_{\rho_N} &= \langle \bar{q}\not{u} q\rangle_{\rho_N} u_\mu, \\
\langle \bar{q}D_\mu q\rangle_{\rho_N} &= \langle \bar{q}u\cdot Dq\rangle_{\rho_N} u_\mu = -im_q\langle \bar{q}\not{u} q\rangle_{\rho_N} u_\mu, \\
\langle \bar{q}\gamma_\mu D_\nu q\rangle_{\rho_N} &= \frac{4}{3}\langle \bar{q}\not{u} u\cdot Dq\rangle_{\rho_N}\left(u_\mu u_\nu - \frac{1}{4}g_{\mu\nu}\right) + \frac{i}{3}m_q\langle \bar{q}q\rangle_{\rho_N}(u_\mu u_\nu - g_{\mu\nu}), \\
\langle \bar{q}D_\mu D_\nu q\rangle_{\rho_N} &= \frac{4}{3}\langle \bar{q}u\cdot Du\cdot Dq\rangle_{\rho_N}\left(u_\mu u_\nu - \frac{1}{4}g_{\mu\nu}\right) - \frac{1}{6}\langle g_s\bar{q}\sigma\cdot Gq\rangle_{\rho_N}(u_\mu u_\nu - g_{\mu\nu}), \\
\langle \bar{q}\gamma_\lambda D_\mu D_\nu q\rangle_{\rho_N} &= 2\langle \bar{q}\not{u} u\cdot Du\cdot Dq\rangle_{\rho_N}\left[u_\lambda u_\mu u_\nu - \frac{1}{6}(u_\lambda g_{\mu\nu} + u_\mu g_{\lambda\nu} + u_\nu g_{\lambda\mu})\right] \\
&\quad - \frac{1}{6}\langle g_s\bar{q}\not{u}\sigma\cdot Gq\rangle_{\rho_N}(u_\lambda u_\mu u_\nu - u_\lambda g_{\mu\nu}),
\end{aligned} \tag{28}$$

where, in their derivations, the equation of motion has been used and the terms $O(m_q^2)$ have been neglected due to their ignorable contributions [11].

By substituting the above matrix elements and the in-medium condensates, after lengthy calculations, we find the expression of the correlation function in coordinate space. Using the relation

$$\frac{1}{(x^2)^n} = \int \frac{d^D t}{(2\pi)^D} e^{-it\cdot x} i(-1)^{n+1} 2^{D-2n}\pi^{D/2} \times \frac{\Gamma(D/2-n)}{\Gamma(n)}\left(-\frac{1}{t^2}\right)^{D/2-n}, \tag{29}$$

we transform the calculations to the momentum space. Then, with the help of the replacement

$$\Gamma\left(\frac{D}{2}-n\right)\left(-\frac{1}{L}\right)^{\frac{D}{2}-n} \to \frac{(-1)^{n-1}}{(n-2)!}(-L)^{n-2}\ln(-L), \tag{30}$$

we find the imaginary parts of the obtained results for different structures called the spectral densities $\Delta\rho_i(p_0, \vec{p})$ in the OPE side in terms of $(p^2)^n$. After ordering the Dirac matrices like the physical side, we set $p^2 = p_0^2 - |\vec{p}|^2$ and replace p_0 with w. In order to remove the contributions of the negative-energy particles, we multiply the OPE side by the weight function $(w - \bar{E}_p)e^{-\frac{w^2}{M^2}}$ like the physical side and perform the integral

$$\Pi_i(w_0, \vec{p}) = \int_{-w_0}^{w_0} dw \Delta\rho_i(w, \vec{p})(w - \bar{E}_p)e^{-\frac{w^2}{M^2}}. \tag{31}$$

By carrying out the integration over w, one can find the $\Pi_i(w_0, \vec{p})$ functions in the Borel scheme. By using $w_0 = \sqrt{s_0^*}$, with s_0^* being the continuum threshold in nuclear matter, and making some variable changes, we find the final expressions of the $\Pi_i(s_0^*, M^2)$ functions. As an example, we present the functions $\Pi_i(s_0^*, M^2)$ for Σ^*, which are obtained as

$$\Pi_i(s_0^*, M^2) = \Pi_i^{\rm pert}(s_0^*, M^2) + \sum_{k=3}^{k=6}\Pi_i^k(s_0^*, M^2), \tag{32}$$

where $pert$ denotes the perturbative contributions and the upper indices 3, 4, 5, and 6 stand for the nonperturbative contributions. These functions are obtained as

$$\begin{aligned}
\Pi_1^{\rm pert}(s_0^*, M^2) &= \frac{1}{512\pi^4}[3\bar{E}_p M^2\sqrt{s_0^*}(m_d + m_u + m_s)(3M^2 - 4\vec{p}^2 + 2s_0^*)]e^{-\frac{s_0^*}{M^2}} \\
&\quad - \frac{1}{1024\pi^4}\int_0^{s_0^*} ds \frac{3\bar{E}_p(m_d + m_u + m_s)(3M^4 - 4M^2\vec{p}^2 + 4\vec{p}^4)}{\sqrt{s}} e^{-\frac{s}{M^2}}, \\
\Pi_2^{\rm pert}(s_0^*, M^2) &= \frac{1}{640\pi^4}[\bar{E}_p M^2\sqrt{s_0^*}(3M^2 - 4\vec{p}^2 + 2s_0^*)]e^{-\frac{s_0^*}{M^2}} - \frac{1}{1280\pi^4}\int_0^{s_0^*} ds \frac{\bar{E}_p(3M^4 - 4M^2\vec{p}^2 + 4\vec{p}^4)}{\sqrt{s}} e^{-\frac{s}{M^2}}, \\
\Pi_3^{\rm pert}(s_0^*, M^2) &= 0,
\end{aligned} \tag{33}$$

$$
\begin{aligned}
\Pi_1^3(s_0^*,M^2) &= \frac{M^2\sqrt{s_0^*}}{24\pi^2}[(3m_s+3m_d-4m_q)\langle u^\dagger u\rangle_{\rho_N}+(3m_u+3m_s-4m_q)\langle d^\dagger d\rangle_{\rho_N}+(3m_u+3m_d-4m_s)\langle s^\dagger s\rangle_{\rho_N}\\
&\quad-2\overline{E}_p(\langle\bar{s}s\rangle_{\rho_N}+\langle\bar{u}u\rangle_{\rho_N}+\langle\bar{d}d\rangle_{\rho_N})]e^{-\frac{s_0^*}{M^2}}\\
&\quad+\frac{1}{144\pi^2}\int_0^{s_0^*}ds\frac{1}{\sqrt{s}}[4\overline{E}_p(\langle\bar{d}iD_0iD_0d\rangle_{\rho_N}+\langle\bar{u}iD_0iD_0u\rangle_{\rho_N}+\langle\bar{s}iD_0iD_0s\rangle_{\rho_N})-4\overline{E}_p(\langle\bar{d}g_s\sigma Gd\rangle_{\rho_N}\\
&\quad+\langle\bar{u}g_s\sigma Gu\rangle_{\rho_N}+\langle\bar{s}g_s\sigma Gs\rangle_{\rho_N})-12\overline{E}_p(m_u+m_s)\langle d^\dagger iD_0d\rangle_{\rho_N}-12\overline{E}_p(m_d+m_s)\langle u^\dagger iD_0u\rangle_{\rho_N}\\
&\quad-12\overline{E}_p(m_d+m_u)\langle s^\dagger iD_0s\rangle_{\rho_N}-6\overline{E}_p(m_qm_s+m_qm_u-M^2+2\vec{p}^2)\langle\bar{d}d\rangle_{\rho_N}\\
&\quad-6\overline{E}_p(m_qm_s+m_qm_d-M^2+2\vec{p}^2)\langle\bar{u}u\rangle_{\rho_N}-6\overline{E}_p(m_qm_u+m_qm_d-M^2+2\vec{p}^2)\langle\bar{s}s\rangle_{\rho_N}\\
&\quad+(12m_q-9m_s-9m_u)\langle d^\dagger d\rangle_{\rho_N}+(12m_q-9m_s-9m_d)\langle u^\dagger u\rangle_{\rho_N}+(12m_q-9m_d-9m_u)\langle s^\dagger s\rangle_{\rho_N}]e^{-\frac{s}{M^2}},\\
\Pi_2^3(s_0^*,M^2) &= \frac{M^2\sqrt{s_0^*}}{36\pi^2}(\langle u^\dagger u\rangle_{\rho_N}+\langle d^\dagger d\rangle_{\rho_N}+\langle s^\dagger s\rangle_{\rho_N})\\
&\quad+\frac{1}{216\pi^2}\int_0^{s_0^*}ds\frac{1}{\sqrt{s}}[4\overline{E}_p(\langle d^\dagger iD_0d\rangle_{\rho_N}+\langle u^\dagger iD_0u\rangle_{\rho_N}+\langle s^\dagger iD_0s\rangle_{\rho_N})+\overline{E}_p(27m_s+27m_u-10m_q)\langle\bar{d}d\rangle_{\rho_N}\\
&\quad+\overline{E}_p(27m_s+27m_d-10m_q)\langle\bar{u}u\rangle_{\rho_N}+\overline{E}_p(27m_u+27m_d-10m_q)\langle\bar{s}s\rangle_{\rho_N}\\
&\quad-3M^2(\langle u^\dagger u\rangle_{\rho_N}+\langle d^\dagger d\rangle_{\rho_N}\langle+s^\dagger s\rangle_{\rho_N})]e^{-\frac{s}{M^2}},\\
\Pi_3^3(s_0^*,M^2) &= \frac{M^2\sqrt{s_0^*}}{216\pi^2}[-32(\langle u^\dagger iD_0u\rangle_{\rho_N}+\langle d^\dagger iD_0d\rangle_{\rho_N}+\langle s^\dagger iD_0s\rangle_{\rho_N})-9\overline{E}_p(\langle u^\dagger u\rangle_{\rho_N}+\langle d^\dagger d\rangle_{\rho_N}+\langle s^\dagger s\rangle_{\rho_N})\\
&\quad+8m_q(\langle\bar{u}u\rangle_{\rho_N}+\langle\bar{d}d\rangle_{\rho_N}+\langle\bar{s}s\rangle_{\rho_N})]\\
&\quad+\frac{1}{432\pi^2}\int_0^{s_0^*}ds\frac{1}{\sqrt{s}}[12\overline{E}_p(\langle d^\dagger iD_0iD_0d\rangle_{\rho_N}+\langle u^\dagger iD_0iD_0u\rangle_{\rho_N}+\langle s^\dagger iD_0iD_0s\rangle_{\rho_N})\\
&\quad+32M^2(\langle d^\dagger iD_0d\rangle_{\rho_N}+\langle u^\dagger iD_0u\rangle_{\rho_N}+\langle s^\dagger iD_0s\rangle_{\rho_N})-8M^2mq(\langle\bar{d}d\rangle_{\rho_N}+\langle\bar{u}u\rangle_{\rho_N}+\langle\bar{s}s\rangle_{\rho_N})\\
&\quad-7\overline{E}_p(\langle d^\dagger g_s\sigma Gd\rangle_{\rho_N}+\langle u^\dagger g_s\sigma Gu\rangle_{\rho_N}+\langle s^\dagger g_s\sigma Gs\rangle_{\rho_N})-\overline{E}_p(54m_qm_s+54m_qm_u-9M^2+18\vec{p}^2)\langle d^\dagger d\rangle_{\rho_N}\\
&\quad-\overline{E}_p(54m_qm_d+54m_qm_s-9M^2+18\vec{p}^2)\langle u^\dagger u\rangle_{\rho_N}-\overline{E}_p(54m_qm_d+54m_qm_u-9M^2+18\vec{p}^2)\langle s^\dagger s\rangle_{\rho_N}]e^{-\frac{s}{M^2}},\\
\Pi_1^4(s_0^*,M^2) &= \frac{1}{128\pi^2}\left\langle\frac{\alpha_sG^2}{\pi}\right\rangle_{\rho_N}\int_0^{s_0^*}dw\frac{\overline{E}_p(m_d+m_u+m_s)}{\sqrt{w}}e^{-\frac{s}{M^2}},\\
\Pi_2^4(s_0^*,M^2) &= \frac{1}{576\pi^2}\left\langle\frac{\alpha_sG^2}{\pi}\right\rangle_{\rho_N}\int_0^{s_0^*}ds\frac{\overline{E}_p}{\sqrt{s}}e^{-\frac{s}{M^2}},\\
\Pi_3^4(s_0^*,M^2) &= 0,
\end{aligned}
\tag{34}
$$

$$
\begin{aligned}
\Pi_1^5(s_0^*,M^2) &= \frac{1}{48\pi^2}[4m_q\langle s^\dagger iD_0s\rangle_{\rho_N}+4m_q\langle d^\dagger iD_0d\rangle_{\rho_N}+4m_q\langle u^\dagger iD_0u\rangle_{\rho_N}-4\langle\bar{d}iD_0iD_0d\rangle_{\rho_N}-4\langle\bar{s}iD_0iD_0s\rangle_{\rho_N}\\
&\quad-\langle\bar{u}iD_0iD_0u\rangle_{\rho_N}-\langle\bar{d}g_s\sigma Gd\rangle_{\rho_N}-\langle\bar{s}g_s\sigma Gs\rangle_{\rho_N}-\langle\bar{u}g_s\sigma Gu\rangle_{\rho_N}]\int_0^{s_0^*}ds\frac{\overline{E}_p}{\sqrt{s}}e^{-\frac{s}{M^2}},\\
\Pi_2^5(s_0^*,M^2) &= 0,\\
\Pi_3^5(s_0^*,M^2) &= -\frac{1}{72\pi^2}[\langle d^\dagger g_s\sigma Gd\rangle_{\rho_N}+\langle u^\dagger g_s\sigma Gu\rangle_{\rho_N}+\langle s^\dagger g_s\sigma Gs\rangle_{\rho_N}]\int_0^{s_0^*}ds\frac{\overline{E}_p}{\sqrt{s}}e^{-\frac{s}{M^2}},
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
\Pi_1^6(s_0^*,M^2) &= 0,\\
\Pi_2^6(s_0^*,M^2) &= 0,\\
\Pi_3^6(s_0^*,M^2) &= 0.
\end{aligned}
\tag{36}
$$

TABLE II. Numerical values of input parameters.

Input parameters	Values
$\lvert\vec{p}\rvert$	270 MeV [11]
m_u; m_d; m_s	$2.2^{0.6}_{-0.4}$ MeV; $4.7^{0.5}_{-0.4}$ MeV; 96^{+8}_{-4} MeV [27]
ρ_N	$(0.11)^3$ GeV^3 [11,26,28]
$\langle q^\dagger q\rangle_{\rho_N}$; $\langle s^\dagger s\rangle_{\rho_N}$	$\frac{3}{2}\rho_N$; 0 [11,26,28,29]
$\langle\bar{q}q\rangle_0$; $\langle\bar{s}s\rangle_0$	$(-0.241)^3$ GeV^3; 0.8 $\langle\bar{q}q\rangle_0$ [30]
m_q	$0.5(m_u+m_d)$ [11,26,28]
σ_N	0.059 GeV [31]
y	0.04 ± 0.02 [32]; $0.066\pm0.011\pm0.002$ [33]; 0.02(13)(10) [34]
$\langle\bar{q}q\rangle_{\rho_N}$; $\langle\bar{s}s\rangle_{\rho_N}$	$\langle\bar{q}q\rangle_0+\frac{\sigma_N}{2m_q}\rho_N$; $\langle\bar{s}s\rangle_0+y\frac{\sigma_N}{2m_q}\rho_N$ [11,26,28,29,35]
$\langle q^\dagger g_s\sigma Gq\rangle_{\rho_N}$; $\langle s^\dagger g_s\sigma Gs\rangle_{\rho_N}$	-0.33 GeV$^2\rho_N$; $-y0.33$ GeV$^2\rho_N$ [11,26,28,29,35]
$\langle q^\dagger iD_0q\rangle_{\rho_N}$; $\langle s^\dagger iD_0s\rangle_{\rho_N}$	0.18 GeVρ_N; $\frac{m_s\langle\bar{s}s\rangle_{\rho_N}}{4}+0.02$ GeVρ_N [11,26,28,29,35]
$\langle\bar{q}iD_0q\rangle_{\rho_N}$; $\langle\bar{s}iD_0s\rangle_{\rho_N}$	$\frac{3}{2}m_q\rho_N\simeq0$; 0 [11,26,28,29,35]
m_0^2	0.8 GeV^2 [30]
$\langle\bar{q}g_s\sigma Gq\rangle_0$; $\langle\bar{s}g_s\sigma Gs\rangle_0$	$m_0^2\langle\bar{q}q\rangle_0$; $m_0^2\langle\bar{s}s\rangle_0$
$\langle\bar{q}g_s\sigma Gq\rangle_{\rho_N}$; $\langle\bar{s}g_s\sigma Gs\rangle_{\rho_N}$	$\langle\bar{q}g_s\sigma Gq\rangle_0+3$ GeV$^2\rho_N$; $\langle\bar{s}g_s\sigma Gs\rangle_0+3y$ GeV$^2\rho_N$ [11,26,28,29,35]
$\langle\bar{q}iD_0iD_0q\rangle_{\rho_N}$; $\langle\bar{s}iD_0iD_0s\rangle_{\rho_N}$	0.3 GeV$^2\rho_N-\frac{1}{8}\langle\bar{q}g_s\sigma Gq\rangle_{\rho_N}$; 0.3$y$ GeV$^2\rho_N-\frac{1}{8}\langle\bar{s}g_s\sigma Gs\rangle_{\rho_N}$ [11,26,28,29,35]
$\langle q^\dagger iD_0iD_0q\rangle_{\rho_N}$; $\langle s^\dagger iD_0iD_0s\rangle_{\rho_N}$	0.031 GeV$^2\rho_N-\frac{1}{12}\langle q^\dagger g_s\sigma Gq\rangle_{\rho_N}$; 0.031$y$ GeV$^2\rho_N-\frac{1}{12}\langle s^\dagger g_s\sigma Gs\rangle_{\rho_N}$ [11,26,28,29,35]
$\langle\frac{\alpha_s}{\pi}G^2\rangle_0$	$(0.33\pm0.04)^4$ GeV^4 [30]
$\langle\frac{\alpha_s}{\pi}G^2\rangle_{\rho_N}$	$\langle\frac{\alpha_s}{\pi}G^2\rangle_0-0.65$ GeVρ_N [11,26,28]

C. Sum rules for physical observables: Numerical results

Having obtained the hadronic and OPE sides of the correlation function, we match them to find QCD sum rules for the mass, residue, and self-energies of the considered decuplet baryons:

$$\lambda_D^{*2}m_D^*e^{-\frac{E_p^2}{M^2}}=\Pi_1(s_0^*,M^2),$$
$$\lambda_D^{*2}e^{-\frac{E_p^2}{M^2}}=\Pi_2(s_0^*,M^2),$$
$$\lambda_D^{*2}\Sigma_\nu e^{-\frac{E_p^2}{M^2}}=\Pi_3(s_0^*,M^2). \quad (37)$$

Now, we proceed to numerically analyze the above sum rules in the $\Delta^0, \Sigma^*, \Xi^*$, and Ω^- channels, both in vacuum and nuclear medium. The sum rules contain numerous parameters, numerical values of which are collected in Table II.

Besides the above input parameters, the QCD sum rules depend also on two auxiliary parameters that should be fixed: the Borel parameter M^2 and the continuum threshold s_0^*. The continuum threshold is not totally arbitrary and it is correlated with the energy of the first excited state with the same quantum numbers as the interpolating currents for decuplet baryons. According to the standard prescriptions, we take the interval $(m_D+0.4)^2$ GeV$^2\le s_0^*\le(m_D+0.6)^2$ GeV^2. The standard criteria in calculating the working window of the Borel parameter is that not only the contributions of the higher resonances and continuum should be adequately suppressed, but the contributions of the higher-dimensional condensates should be small and the perturbative contributions should exceed the nonperturbative ones. These criteria lead to the following intervals:

$$1.1\ \mathrm{GeV}^2\le M^2\le1.4\ \mathrm{GeV}^2\quad\text{for }\Delta^0$$
$$1.5\ \mathrm{GeV}^2\le M^2\le1.9\ \mathrm{GeV}^2\quad\text{for }\Sigma^{*0}$$
$$2.2\ \mathrm{GeV}^2\le M^2\le2.5\ \mathrm{GeV}^2\quad\text{for }\Xi^{*}$$
$$2.6\ \mathrm{GeV}^2\le M^2\le3.0\ \mathrm{GeV}^2\quad\text{for }\Omega^{-}.$$

Making use of the working windows of the auxiliary parameters and the values of other inputs, as examples, we

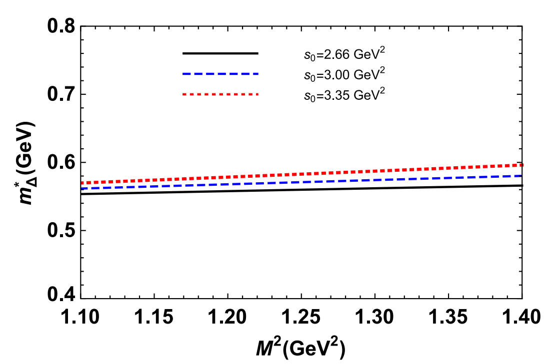

FIG. 1. The in-medium mass of the Δ baryon as a function of M^2 at different fixed values of the threshold parameter s_0 and central values of other input parameters.

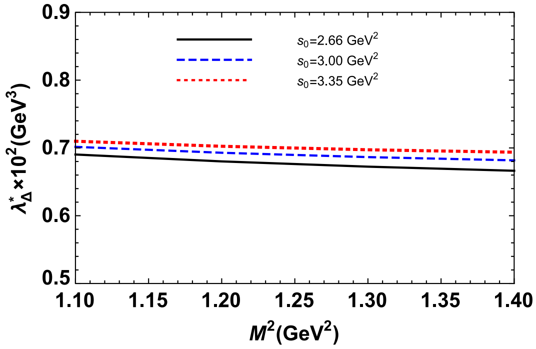

FIG. 2. The in-medium residue of the Δ baryon as a function of M^2 at different fixed values of the threshold parameter s_0 and central values of other input parameters.

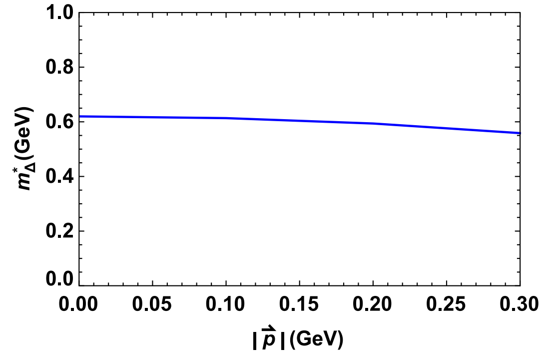

FIG. 4. The in-medium mass of the Δ baryon as a function of $|\vec{p}|$ at central values of all auxiliary and input parameters.

plot the in-medium mass m^*_Δ, residue λ^*_Δ, and vector self-energy Σ^ν_Δ of the Δ baryon as functions of M^2 at different fixed values of the threshold parameter s_0 and central values of other input parameters in Figs. 1–3. From these figures we see that the in-medium mass and residue as well as the vector self-energy demonstrate good stability with respect to M^2 in its working region. It is also clear that the results very weakly depend on the threshold parameter s_0 in its working window.

In this part, we would like to briefly discuss the dependence of the results on the values of the three-momentum of the particles under consideration and the density of the nuclear matter. We work at zero temperature and, as is seen from Table II, we take the external three-momentum of the quasiparticles approximately equal to Fermi momentum, $|\vec{p}| = 270$ MeV, in the numerical analysis. However, our numerical results show that the physical quantities overall do not considerably depend on this parameter in the interval [0, 0.27] MeV (see Figs. 4–6). This is an expected result. In the case of nucleons in nuclear matter, each quasinucleon has its own quasi-Fermi sea; hence, the external three-momentum of the quasinucleon is set at the Fermi momentum at $\rho_N = 0.16\ \mathrm{fm}^{-3} = (110\ \mathrm{MeV})^3$ [11,15]. For a similar reason, the external three-momentum for the quasidecuplet baryons, especially the strange members, can be easily set to zero. To see how the results behave with respect to the nuclear matter density, we show the dependence of the ratio of the mass and residue of, for instance, the Δ baryon in nuclear matter (m^*_Δ, λ^*_Δ) to the mass and residue in vacuum (m_Δ, λ_Δ), as well as $\Sigma^\nu_\Delta/m^*_\Delta$ on $\rho_N/\rho_N^{\mathrm{sat}}$, with $\rho_N^{\mathrm{sat}} = (0.11)^3\ \mathrm{GeV}^3$ being the saturation density used in the analysis, in Figs. 7–9. From these figures we see that the results depend linearly on the nuclear matter density.

After numerical analyses of the results for all baryons, using the values presented in Table II, we find the values of the masses and residues both in nuclear matter and vacuum. We also obtain the vector and scalar self-energies of the baryons under consideration in the nuclear medium. Note that the vacuum results are obtained from those of the in-medium when $\rho_N \to 0$. The average values for the considered physical quantities are presented in Table III. The

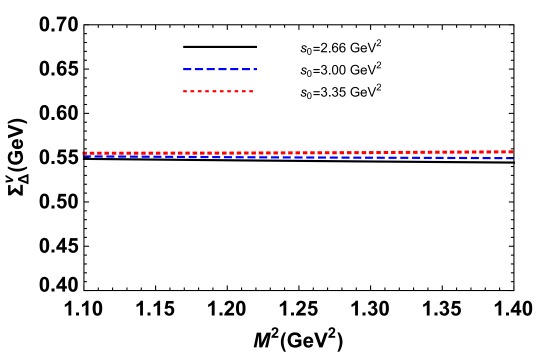

FIG. 3. The vector self-energy of the Δ baryon as a function of M^2 at different fixed values of the threshold parameter s_0 and central values of the other input parameters.

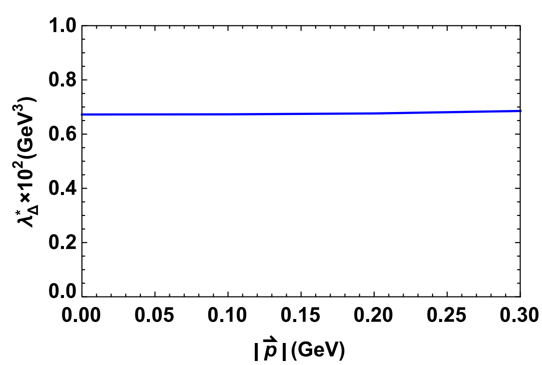

FIG. 5. The in-medium residue of the Δ baryon as a function of $|\vec{p}|$ at central values of all auxiliary and input parameters.

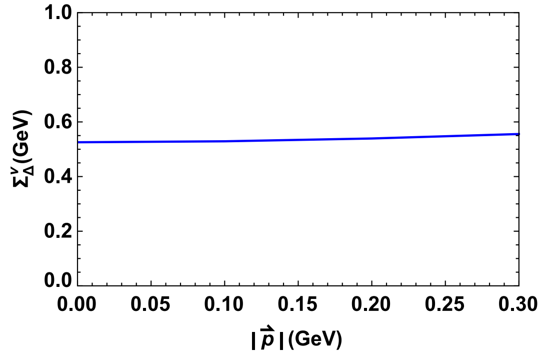

FIG. 6. The vector self-energy of the Δ baryon as a function of $|\vec{p}|$ at central values of all auxiliary and input parameters.

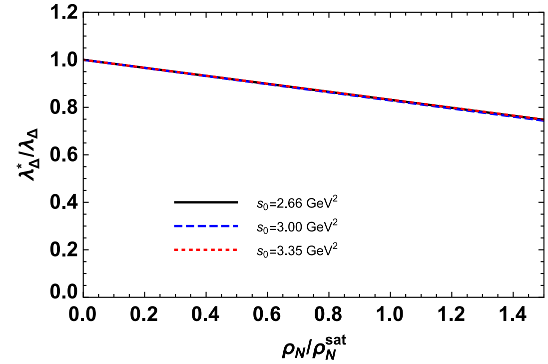

FIG. 8. $\lambda_\Delta^*/\lambda_\Delta$ versus $\rho_N/\rho_N^{\mathrm{sat}}$ at central values of M^2 and other input parameters.

errors quoted in this table correspond to the uncertainties in the calculations of the working regions for the auxiliary parameters as well as those coming from the errors of other input parameters.

From this table, first of all, we see that our predictions on the masses in vacuum are in good consistency with the average experimental data presented by PDG [27]. The masses obtained in the nuclear medium show negative shifts for all decuplet baryons. From the values of the scalar self-energy (Σ_D^S), demonstrating the shifts in the masses due to finite density, we deduce that the maximum shift in the masses, due to the nuclear medium, with the amount of 56% belongs to the Δ baryon and its minimum, 2%, corresponds to the Ω^- state. This is an expected result since the Δ state has the same quark content as the nuclear medium and is more affected by the nuclear matter. When going from Δ to Ω^-, the up and down quarks are replaced with the strange quark. The Ω^- state, having three s quarks, is less affected by the medium. The small shifts in the parameters of Ω^- may be attributed to the intrinsic strangeness in the nucleons.

In the case of the residues, our predictions in vacuum are overall comparable with those obtained in [36,37] within the errors. The small differences may be linked to different input parameters used in these works. The values of residues are also considerably affected by the medium. The shift in the residue of Δ with an amount of 46% is the maximum. The residue of Ω^- again is minimally affected by the medium with an amount of roughly 5%.

The value of vector self-energy is considerably large in all decuplet channels. It is again systematically reduced when going from the Δ to the Ω^- baryon. Our results may be confronted with the experimental data of the $\overline{P}$ANDA Collaboration at the FAIR and NICA facility. However, we should state that those experiments correspond to heavy ion collisions and not exactly to a nuclear medium. Hence, the appropriate way to make such a comparison would be to present sum rules at finite density but where the density is introduced through the baryonic chemical potential. This offers the possibility of exploring a wide range of densities. We worked with the nuclear matter density since the in-medium condensates are available as functions of nuclear matter density, not chemical potential, and we extracted the zero-density (vacuum) sum rules, as a means of normalizing the finite density sum rules, to compare the results with the available experimental data and other theoretical predictions in vacuum.

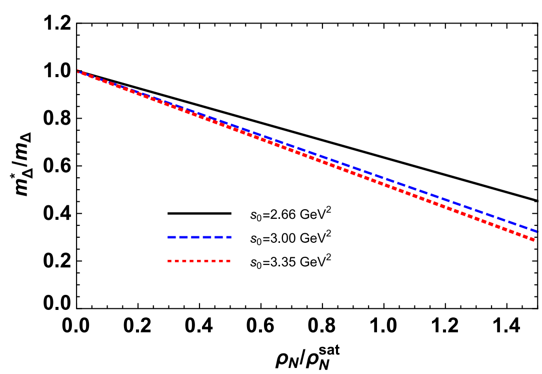

FIG. 7. m_Δ^*/m_Δ versus $\rho_N/\rho_N^{\mathrm{sat}}$ at central values of M^2 and other input parameters.

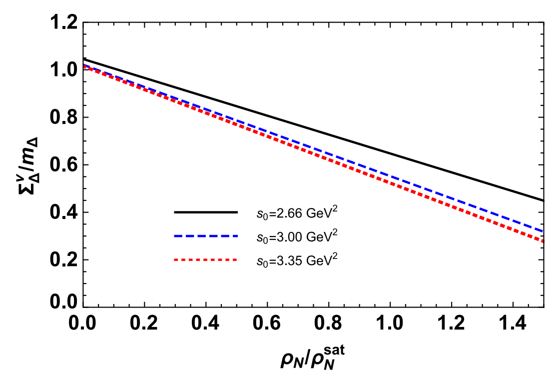

FIG. 9. $\Sigma_\Delta^\nu/m_\Delta$ versus $\rho_N/\rho_N^{\mathrm{sat}}$ at central values of M^2 and other input parameters.

TABLE III. The numerical values of masses, residues, and self-energies of Δ, Σ^*, Ξ^*, and Ω^- baryons.

	λ_Δ (GeV^3)	λ^*_Δ (GeV^3)	m_Δ (GeV)	m^*_Δ (GeV)	Σ^ν_Δ (MeV)	Σ^S_Δ (MeV)
Present study	0.013 ± 0.004	0.007 ± 0.002	1.297 ± 0.364	0.571 ± 0.159	550 ± 51	-726
	λ_{Σ^*} (GeV^3)	$\lambda^*_{\Sigma^*}$ (GeV^3)	m_{Σ^*} (GeV)	$m^*_{\Sigma^*}$ (GeV)	$\Sigma^\nu_{\Sigma^*}$ (MeV)	$\Sigma^S_{\Sigma^*}$ (MeV)
Present study	0.024 ± 0.007	0.016 ± 0.005	1.385 ± 0.387	0.927 ± 0.259	409 ± 41	-458
	λ_{Ξ^*} (GeV^3)	$\lambda^*_{\Xi^*}$ (GeV^3)	m_{Ξ^*} (GeV)	$m^*_{\Xi^*}$ (GeV)	$\Sigma^\nu_{\Xi^*}$ (MeV)	$\Sigma^S_{\Xi^*}$ (MeV)
Present study	0.035 ± 0.011	0.027 ± 0.008	1.523 ± 0.426	1.399 ± 0.392	148 ± 15	-124
	λ_{Ω^-} (GeV^3)	$\lambda^*_{\Omega^-}$ (GeV^3)	m_{Ω^-} (GeV)	$m^*_{\Omega^-}$ (GeV)	$\Sigma^\nu_{\Omega^-}$ (MeV)	$\Sigma^S_{\Omega^-}$ (MeV)
Present study	0.044 ± 0.013	0.042 ± 0.013	1.668 ± 0.467	1.634 ± 0.457	46 ± 5	-34

ACKNOWLEDGMENTS

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