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Electromagnetic form factors of heavy flavored vector mesons

M. Priyadarsini,[1] P. C. Dash,[1] Susmita Kar,[2,*] Sweta P. Patra,[2] and N. Barik[3]

[1]*Department of Physics, Siksha 'O' Anusandhan University, Bhubaneswar 751030, India*
[2]*Department of Physics, North Orissa University, Baripada 757003, India*
[3]*Department of Physics, Utkal University, Bhubaneswar 751004, India*

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We study the electromagnetic form factors of heavy flavored vector mesons such as $(D^*, D_s^*, J/\Psi)$, (B^*, B_s^*, Υ) via one photon radiative decays $(V \to P\gamma)$ in the relativistic independent quark (RIQ) model based on a flavor independent average interaction potential in the scalar vector harmonic form. The momentum dependent spacelike $(q^2 < 0)$ form factors calculated in this model are analytically continued to the physical timelike region $0 \le q^2 \le (M_V - M_P)^2$. The predicted coupling constant $g_{VP\gamma} = F_{VP}(q^2 = 0)$ for real photon case in the limit $q^2 \to 0$ and decay widths $\Gamma(V \to P\gamma)$ are found in reasonable agreement with experimental data and other model predictions.

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I. INTRODUCTION

Electromagnetic form factors regarded as important tools that encode information about the shape of hadrons and give valuable insight into their internal structure in terms of constituent quarks and gluon degree of freedom. Though one photon radiative decays from the low-lying heavy vector(V) to heavy pseudoscalar(P) mesons transition have been investigated by several theoretical approaches such as the quark model QM [1–5], light cone QCD sum rule [6,7], heavy quark effective theory (HQET) [8,9], cloud bag model (CBM) [10], light front quark model (LFQM) [11], lattice QCD (LQCD) [12] and single quark transition (SQT) formalism [13], not much attention has been paid to study their momentum dependent transition form factors. We have predicted the decay widths of several M1 transitions ($V \to P\gamma$ and $P \to V\gamma$), in the relativistic independent quark (RIQ) model, using a static approximation [14], in reasonable agreement with the available data for most decay modes except for those cases which involve a light flavored meson (especially a pion). The noticeable discrepancy in the prediction for such decay modes was due to the recoil effect arising out of large momentum transfer involved, which was not taken into consideration [14]. This was considered in our subsequent analysis of radiative decay modes $[V \to P\gamma, P \to V\gamma]$ [2] by introducing momentum eigenstates of participating mesons into the analysis. The momentum eigenstates of the participating mesons are taken as appropriate wave packets reflecting momentum distribution of constituent quark and antiquark within the meson bound state. In going beyond the static approximation we found a significant improvement in the model predictions for decay modes involving light mesons especially pions in good agreement with the experimental data. In the same analysis our results for $V \to P\gamma$ in the heavy flavor sector stand almost unaffected from the recoil effect and are also found to be in good agreement with other model predictions and experimental data. However we have not yet shown the momentum dependence of the relevant electromagnetic form factors as has been done in other models [1–3] including the light front quark model [11].

The purpose of this paper is to predict the space- and timelike transition form factors for energetically possible electromagnetic decays of heavy flavored mesons $(D^*, D_s^*, J/\Psi)$ and (B^*, B_s^*, Υ) in the framework of RIQ model; hence calculate the decay widths $\Gamma(V \to P\gamma)$ and compare our results with other model predictions as well as available experimental data. The experimental data in the heavy flavored sector are scant. The model predictions in this sector would not only justify the applicability of the model but pin down the RIQ model as one of the suitable phenomenological models for hadrons.

II. TRANSITION FORM FACTOR AND RADIATIVE DECAY WIDTHS IN RIQ MODEL

The transition form factor $F_{VP}(q^2)$ for radiative decay of vector mesons $V(p) \to P(p')\gamma^*(q)$ is defined through a covariant expansion of the hadronic matrix elements as

$$\langle P(p')|J^{\mu}_{em}|V(p,h)\rangle = ie\epsilon^{\mu\nu\rho\sigma}\epsilon_{\nu}(p,h)q_{\rho}p_{\sigma}F_{VP}(q^2) \qquad (1)$$

where, $q = (p - p')$ is the four momentum transfer, $\epsilon_{\nu}(p,h)$ is the polarization vector of the vector meson (V) with four momentum p and helicity h, and p' is the four momentum of the pseudoscalar meson (P). The kinematically allowed momentum transfer squared q^2 ranges from 0 to $q^2_{\rm max} = (M_V - M_P)^2$. The expression for the form factor $F_{VP}(q^2)$ can be obtained from the transition matrix elements in the frame work of RIQ model. In the RIQ model it is assumed that the constituent quarks in the meson core are independently confined by an average flavor

*skar09.sk@gmail.com

independent potential in the scalar-vector harmonic form: [2,14,15]

$$U(r) = \frac{1}{2}(1+\gamma^0)(ar^2 + V_0), \qquad a > 0,$$

with (a, V_0) as the potential parameters. The confining potential in this form is believed to provide the zeroth order quark dynamics inside the meson core. The meson state in this model is represented by an appropriate momentum wave packet as [2,15]:

$$|M(\vec{p}, S_M)\rangle = \frac{1}{\sqrt{N_M(\vec{p})}} \sum_{\lambda_1,\lambda_2 \in S_M} \zeta^M_{q_1,\bar{q}_2}(\lambda_1,\lambda_2) \times \int d^3\vec{p}_{q_1} d^3\vec{p}_{q_2} \delta^{(3)}(\vec{p}_{q_1} + \vec{p}_{q_2} - \vec{p}) \times \mathcal{G}_M(\vec{p}_{q_1}, \vec{p}_{q_2}) \hat{b}^\dagger_{q_1}(\vec{p}_{q_1}, \lambda_1) \hat{\tilde{b}}^\dagger_{q_2}(\vec{p}_{q_2}, \lambda_2)|0\rangle \tag{2}$$

where, $\hat{b}^\dagger_{q_1}(\vec{p}_{q_1}, \lambda_1)$ and $\hat{\tilde{b}}^\dagger_{q_2}(\vec{p}_{q_2}, \lambda_2)$ are, respectively, the quark and antiquark creation operator, and $\zeta^M_{q_1,\bar{q}_2}(\lambda_1,\lambda_2)$ is the SU(6)-spin flavor coefficients. $N_M(\vec{p})$ is the meson normalization factor of the wave packet which is obtained in the form

$$N_M(\vec{p}) = \int d^3\vec{p}_{q_1} |\mathcal{G}_M(\vec{p}_{q_1}, \vec{p} - \vec{p}_{q_1})|^2. \tag{3}$$

Finally, $\mathcal{G}_M(\vec{p}_{q_1}, \vec{p} - \vec{p}_{q_1})$ which represents the effective momentum distribution function for the quark q_1 with the momentum $\vec{p}_{q_1}$ and antiquark $\bar{q}_2$ with momentum $\vec{p}_{q_2} = \vec{p} - \vec{p}_{q_1}$ is taken in the form

$$\mathcal{G}_M(\vec{p}_{q_1}, \vec{p} - \vec{p}_{q_1}) = \sqrt{G_{q_1}(\vec{p}_{q_1}) \tilde{G}_{\bar{q}_2}(\vec{p} - \vec{p}_{q_1})} \tag{4}$$

in a straightforward extension of the ansatz used by Margolis and Mendel in their bag model analysis [16]. Here $G_{q_1}(\vec{p}_{q_1})$ and $\tilde{G}_{\bar{q}_2}(\vec{p} - \vec{p}_{q_1})$ refer to the momentum probability amplitude of the bound quark q_1 and of the antiquark $\bar{q}_2$, respectively in the meson bound state $|M(\vec{p}, S_M)\rangle$, which have been derived via momentum projection of the quark orbitals obtained in this model by solving the Dirac equation. Using appropriate meson states for the initial and final state mesons as in Eq. (2), the transition matrix element in the left-hand side of Eq. (1) is evaluated in the parent meson rest frame. Here the timelike component of the hadronic matrix element being zero, the nonvanishing spacelike component is calculated in the parent meson (V) rest frame for spin projectons $S_V = \pm 1, 0$. The resulting expressions are then compared with the corresponding expressions from the covariant expansion in the right-hand side of Eq. (1) yielding to the momentum dependent form factor $F_{VP}(q^2)$ in the form:

$$F_{VP}(q^2) = e_{q_1} I_{q_1}(m_{q_1}, m_{\bar{q}_2}, q^2) + e_{\bar{q}_2} I_{\bar{q}_2}(m_{\bar{q}_2}, m_{q_1}, q^2), \tag{5}$$

where, model expression for $I_{q,\bar{q}}$ is obtained in the form:

$$I_{q_i}(m_{q_1}, m_{\bar{q}_2}, q^2) = \sqrt{\frac{1}{\bar{N}(0)\bar{N}(\vec{k})}} \int d\vec{p}_{q_i} \mathcal{G}_V(\vec{p}_{q_i}, -\vec{p}_{q_i}) \times \mathcal{G}_P(\vec{p}_{q_i} - \vec{k}, -\vec{p}_{q_i}) \times \sqrt{\frac{(E_{p_i} + m_{q_i})(E_{p_i k} + E_{-p_i})}{4E_{p_i}E_{p_i k}(E_{p_i k} + m_{q_i})(E_{p_i} + E_{-p_i})}} \tag{6}$$

Starting from S-matrix element for $V \to P\gamma$ decay processes one can also obtain the same expression for transition form factor $F_{VP}(q^2)$ using usual Feynman technique. The relevant steps leading to the model expressions of the form factor $F_{VP}(q^2)$ and partial decay width $\Gamma(V \to P\gamma)$ are shown in the Appendix. The general expression for $\Gamma(V \to P\gamma)$ so obtained in the parent meson rest frame is

$$\Gamma(V \to P\gamma) = \frac{\alpha}{3} g^2_{VP\gamma} k^3_\gamma \tag{7}$$

where α is the fine-structure constant, $k_\gamma = (M_V^2 - M_P^2)/2M_V$ is the kinematically allowed energy of the outgoing photon. The coupling constant $g_{VP\gamma}$ for a real photon is determined from $F_{VP}(q^2)$ in the limit $q^2 \to 0$. We use the transverse $(h = \pm 1)$ polarization to extract the coupling constant $g_{VP\gamma}$ since the longitudinal component of the vector meson does not convert into a real photon.

III. NUMERICAL RESULTS

To evaluate the coupling constant $g_{VP\gamma}$, decay width $\Gamma(V \to P\gamma)$ and study q^2-dependence of the transition factor $F_{VP}(q^2)$, we take the input parameters: [2,14,15] as

$$\begin{aligned}(a, V_0) &\equiv (0.017166\ \mathrm{GeV}^3, -0.1375\ \mathrm{GeV}) \\ (m_u = m_d, m_s, m_c, m_b) &\equiv (0.07875, 0.31575, 1.49276, 4.77659)\ \mathrm{GeV} \\ (E_u = E_d, E_s, E_c, E_b) &\equiv (0.47125, 0.59100, 1.57951, 4.76633)\ \mathrm{GeV}.\end{aligned} \tag{8}$$

TABLE I. Coupling constants $g_{VP\gamma}[\mathrm{GeV}^{-1}]$ for radiative $V \to P\gamma$ decays in RIQ model in comparison with the available Experimental data (Expt. [17]).

Coupling constants	Our work	[11]	[3]	[2]	[1]	Expt. [17]
$g_{J/\Psi\eta_c\gamma}$	0.832	0.68 [0.673]	···	···	0.69	0.687 ± 0.45
$g_{D^{*\pm}D^{\pm}\gamma}$	−0.391	−0.384 [−0.398]	−0.3	−0.37	−0.35	-0.466 ± 0.3
$g_{D^{*0}D^0\gamma}$	2.056	1.783 [1.826]	1.85	1.94	1.78	···
$\lvert\frac{g_{D^{*0}D^0\gamma}}{g_{D^{*\pm}D^{\pm}\gamma}}\rvert$	5.26	4.64 [4.59]	6.17	5.24	5.08	···
$g_{D_s^{*\pm}D_s^{\pm}\gamma}$	−0.181	−0.1.67 [−0.161]	···	−0.17	−0.13	···
$g_{B^{*\pm}B^{\pm}\gamma}$	1.574	1.311 [1.313]	1.4	1.5	1.37	···
$g_{B^{*0}B^0\gamma}$	−0.891	−0.749 [0.75]	−0.8	−0.85	−0.78	···
$\lvert\frac{g_{B^{*0}B^0\gamma}}{g_{B^{*\pm}B^{\pm}\gamma}}\rvert$	0.57	0.57 [0.57]	0.57	0.57	···	···
$g_{B_s^{*0}B_s^0\gamma}$	−0.657	−0.553 [−0.536]	···	−0.62	−0.55	···
$g_{\Upsilon\eta_b\gamma}$	−0.138	−0.124 [−0.119]	···	···	−0.13	···

The mass of the participating mesons for different $V \to P\gamma$ transitions are taken to be their average observed values [17].

We present in Table I our results for the coupling constant $g_{VP\gamma}$ (in unit of GeV^{-1}) for radiative $V \to P\gamma$ decays together with other model predictions and the experimental data. The experiment values $(g_{J/\Psi\eta_c\gamma})_{\rm exp} = 0.687 \pm 0.45$ for $J/\Psi \to \eta_c\gamma$ and $(g_{D^{*\pm}D^{\pm}\gamma})_{\rm exp} = -(0.466 \pm 0.3)$ for $D^{*\pm} \to D^{\pm}\gamma$ process are extracted from the branching ratios $\mathrm{Br}(J/\Psi \to \eta_c\gamma)_{\rm exp} = (1.7 \pm 0.4)\%$ and $\mathrm{Br}(D^{*\pm} \to D^{\pm}\gamma)_{\rm exp} = (1.6 \pm 0.4)\%$ together with the full width of $\Gamma_{\rm total}(J/\Psi) = (92.9 \pm 2.8)$ keV and $\Gamma_{\rm total}(D^{\pm}) = (83.4 \pm 1.8)$ keV [17], respectively. The opposite sign of coupling constant for $D^{*\pm}$ and $D_s^{*\pm}$ decays compared to the charmonium J/Ψ decay indicates that charmed quark contribution is largely destructive in the radiative decays of $D^{*\pm}$ and $D_s^{*\pm}$ mesons. Similarly we find that the bottomed quark contribution is largely destructive in the radiative decay of $B^{*\pm}$ meson. Our predicted coupling constant $g_{J/\Psi\eta_c\gamma} = 0.832$ and $g_{D^{*\pm}D^{\pm}\gamma} = -0.391$ are well within the experimental error limit and those for $D^{*0} \to D^0\gamma$ and $D_s^{*\pm} \to D_s^{\pm}\gamma$ decays are comparable with other model predictions [1–3,11]. Our result of coupling constant ratio $|g_{D^{*0}D^0\gamma}/g_{D^{*\pm}D^{\pm}\gamma}| = 5.26$ not only compares well with the model predictions [1–3,11] as shown in Table I but also falls within the limits of theoretical predictions of 6.32 ± 2.97 [6] and 5.54 ± 3.0 [9] from heavy quark effective theory [HQET] and 4.49 ± 0.96 [18] from broken SU(4) symmetry by M1 transition. In the bottom flavor sector, our predicted coupling constants $g_{B^{*\pm}B^{\pm}\gamma}$, $g_{B^{*0}B^0\gamma}$, $g_{B_s^{*\pm}B_s^{\pm}\gamma}$, and $g_{\Upsilon\eta_b\gamma}$ are also found comparable to those of [1–3,11]. The coupling constant ratio as predicted in our model $|g_{B^{*0}B^0\gamma}/g_{B^{*\pm}B^{\pm}\gamma}| = 0.57$ is in agreement with those of [1–3,11] and comparable to 0.64 ± 0.51 [6] and 0.49 ± 0.38 [7] from QCD sum rules and 0.59 ± 0.48 [9] from HQET.

We depict in Fig. 1, the q^2-dependence transition form factors $F_{VP}(q^2)$ for charmed vector meson radiative $V \to P\gamma$ decays. The solid, dotted, dashed, and dot-dashed lines

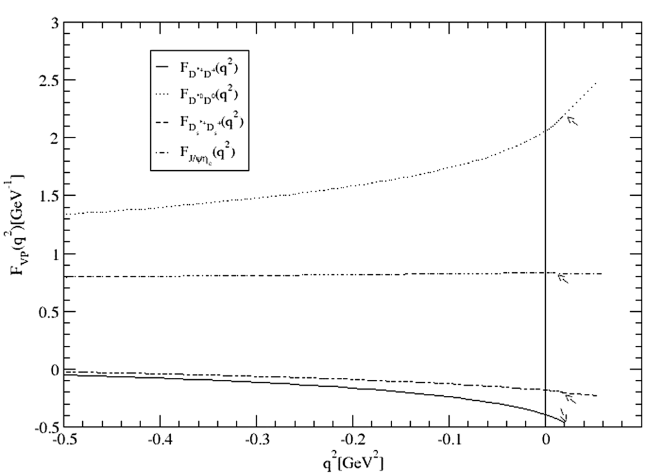

FIG. 1. The q^2 dependence of $F_{VP}(q^2)$ for charmed mesons radiative decay.

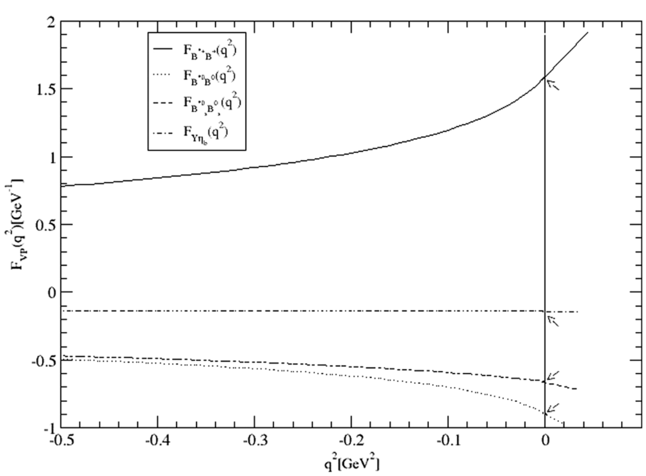

FIG. 2. The q^2 dependence of $F_{VP}(Q^2)$ for bottomed mesons radiative decay.

TABLE II. Decay widths and branching ratios for radiative $V \to P\gamma$ in RIQ model.

Decay mode	$\Gamma(V \to P\gamma)$ in (keV)	$\mathrm{Br}(V \to P\gamma)$	$\mathrm{Br}(V \to P\gamma)_{\mathrm{expt}}$ [17]
$J/\Psi \to \eta_c\gamma$	2.321	$(2.5 \pm 0.1)\%$	$(1.7 \pm 0.4)\%$
$D^{*\pm} \to D^{\pm}\gamma$	0.932	$(1.12 \pm 0.02)\%$	$(1.6 \pm 0.4)\%$
$D^{*0} \to D^0\gamma$	26.509	...	$(38.1 \pm 2.9)\%$
$D_s^{*\pm} \to D_s^{\pm}\gamma$	0.213	...	$(94.2 \pm 0.7)\%$
$B^{*\pm} \to B^{\pm}\gamma$	0.577	...	...
$B^{*0} \to B^0\gamma$	0.181	...	...
$B_s^{*0} \to B_s^0\gamma$	0.119	...	...
$\Upsilon \to \eta_b\gamma$	0.0111	...	...

represent the form factors for $D^{*\pm} \to D^{\pm}\gamma$, $D^{*0} \to D^0\gamma$, $D_s^{*\pm} \to D_s^{\pm}\gamma$, and $J/\Psi \to \eta_c\gamma$, respectively. The arrows in the figure represent the zero recoil points, where $q^2 = q^2_{\mathrm{max}} = (M_V - M_P)^2$. We have performed the analytical continuation of transition form factors from spacelike $(q^2 < 0)$ region to the physical timelike region $(0 \le q^2 \le q^2_{\mathrm{max}})$. The coupling constant $g_{VP\gamma}$ at $q^2 = 0$ corresponds to a final state pseudoscalar meson recoiling with maximum value of three momentum, $|\vec{k}| = \frac{(M_V^2 - M_P^2)}{2M_V}$ in the vector meson rest frame.

Figure 2 depicts our results of transition form factors for the bottomed vector meson radiative $V \to P\gamma$ decays, where the solid, dotted, dashed, and dot-dashed lines represent the form factors for $B^{*\pm} \to B^{\pm}\gamma$, $B^{*0} \to B^0\gamma$, $B_s^{*0} \to B_s^0\gamma$, and $\Upsilon \to \eta_b\gamma$, respectively. Due to small kinematic region $0 \le q^2 \le q^2_{\mathrm{max}}$ for the bottomed and bottomonium meson decays the recoil effects of final state meson are quite negligible i.e., $F_{VP}(q^2_{\mathrm{max}})/g_{VP\gamma} \simeq 1$. Similarly we find $F_{J/\Psi\eta_c\gamma}(q^2_{\mathrm{max}})/g_{J/\Psi\eta_c\gamma} \simeq F_{D_s^{*\pm}D_s^{\pm}\gamma}(q^2_{\mathrm{max}})/g_{D_s^{*\pm}D_s^{\pm}\gamma} \simeq 1$ for $J/\Psi \to \eta_c\gamma$ and $D_s^{*\pm} \to D_s^{\pm}\gamma$ decays. On the other hand, we obtain $F_{D^{*\pm}D^{\pm}\gamma}(q^2_{\mathrm{max}})/g_{D^{*\pm}D^{\pm}\gamma} = 1.18$ and $F_{D^{*0}D^0\gamma}(q^2_{\mathrm{max}})/g_{D^{*0}D^0\gamma} = 1.06$ for $D^{*\pm} \to D^{\pm}\gamma$ and $D^{*0} \to D^0\gamma$ decays, respectively in good comparison with the predicted values of 1.1 and 1.04 from the (LFQM)[11]. This shows that the recoil effect may not be negligible in these decay modes especially in $D^{*\pm} \to D^{\pm}\gamma$ decay. Our predictions: $F_{D_s^{*\pm}D_s^{\pm}}(q^2)/F_{D^{*\pm}D^{\pm}}(q^2) \to 1$ and $F_{B_s^{*0}B_s^0}(q^2)/F_{B^{*0}B^0}(q^2) \to 1$ in the intermediate and deep spacelike $(q^2 < 0)$ region as depicted in Fig. 1 and Fig. 2 indicate that the light quark current contribution in these cases are negligible vindicating restoration of SU(3) flavor symmetry between charmed and charmed-strange mesons as well as bottomed and bottomed-strange mesons.

For a more direct comparison with the experimental data, we calculate the partial decay widths from Eq. (7). Table II presents our results of the partial decay widths and branching fractions together with the available experimental data. Our results for the branching fraction: $Br(J/\Psi \to \eta_c\gamma) = (2.5 \pm 0.1)\%$ is obtained from our predicted decay width: $\Gamma(J/\Psi \to \eta_c\gamma) = 2.321$ keV and experimental full width of $\Gamma(J/\Psi \to \eta_c\gamma) = (92.9 \pm 2.8)$ keV [17]. The errors in our predicted branching fraction come from the uncertainties in the experimental full width. Our result of the branching fraction is of course found larger than $\mathrm{Br}(J/\Psi \to \eta_c\gamma)_{\mathrm{expt}} = (1.7 \pm 0.4)\%$ [17]. Note that there is significant difference between the quark model predictions and the data for the $J/\Psi \to \eta_c\gamma$ decay. Different theoretical predictions including the quark model results as well as the available experimental data for this decay mode, in fact, vary in a wide range. For example, the quark models [19–22] predict a large value of $\Gamma(J/\Psi \to \eta_c\gamma) = 2.85$ keV. On the other hand the effective theory approaches of [23,24] the QCD sum rule analyses [25,26] predict smaller values with large uncertainties. Many years ago the dispersive (model independent) approach of Ref. [27] predicted $\Gamma(J/\Psi \to \eta_c\gamma) = (2.2 \pm 3.2)$ keV. In recent lattice QCD calculations, the decay width $\Gamma(J/\Psi \to \eta_c\gamma)$ has been predicted as (2.64 ± 0.11) keV in [28] and (2.49 ± 0.19) keV in [29]. All these results within errors are found compatible with our result. The experimental data from PDG [17], CLEO [30], and KEDR [31] also differ significantly from one another. The recent result at KEDR suggests the branching fraction: $\mathrm{Br}(J/\Psi \to \eta_c\gamma) = (2.34 \pm 0.15 \pm 0.40)\%$ which would result in $\Gamma(J/\Psi \to \eta_c\gamma) = (2.2 \pm 0.6)$ keV in good agreement with our result. The difference between our result and other theoretical predictions is (mainly) due to charm mass used in different analyses, which varies considerably between models, e.g. $m_c = 1.84$ GeV in [32] while $m_c = 1.628$ GeV in [1] and $m_c = 1.479$ GeV in [33]. In the lattice simulations also while fixing the charm mass to reproduce the mass spectra, it has not been possible to tune it perfectly yielding systematic errors in their predictions for the M1-transition [34]. Determining charm quark mass is therefore a tricky issue. Note that in our case the charm mass is not a free parameter, rather the mass of the charm quark as well as other flavored quark masses have already been fixed at the basic level application of the RIQ-model, reproducing the static hadronic properties such as mass spectra, magnetic moments and charge radii etc. [35]. In view of this we believe that fresh experimental efforts need to be devoted for more precise data in order to clarify the disagreement among experiments in this sector. A study of $J/\Psi \to \eta_c\gamma$ at BSE III is expected to give valuable information.

Our result for the branching fraction $\mathrm{Br}(D^{*\pm} \to D^{\pm}\gamma) = (1.12 \pm 0.02)\%$ is extracted from the predicted $\Gamma(D^{*\pm} \to D^{\pm}\gamma) = 0.932$ keV and the full width $\Gamma_{\mathrm{tot}}(D^{*\pm} \to D^{\pm}\gamma) = (83.4 \pm 1.8)$ keV [17] which agrees with the result of the light-front quark model (LFQM) [11] within the error bars and the experimental data $(1.6 \pm 0.4)\%$ [17]. For neutral charmed meson decays our prediction $\Gamma(D^{*0} \to D^0\gamma) = 26.509$ keV is comparable to other theoretical predictions such as 21.69 keV from RQM [3], (27.0 ± 1.8) keV from the broken-SU(4) symmetry by M1 transition [36] and (20.0 ± 0.3) keV from (LFQM) [11]. For charmed strange meson decay our result $\Gamma(D_s^{*\pm} \to D_s^{\pm}\gamma) = 0.213$ kev is comparable to (0.18 ± 0.01) keV [11], 0.19 keV from RQM [5], and (0.24 ± 0.24) keV from the HQET [9]. Since the D^{*0} life time has not been measured yet, we estimate its full width using the relation

$$\frac{\mathrm{Br}(D^{*\pm} \to D^{\pm}\gamma)_{\mathrm{expt}}}{\mathrm{Br}(D^{*0} \to D^{0}\gamma)_{\mathrm{expt}}} = \frac{\Gamma(D^{*\pm} \to D^{\pm}\gamma)}{\Gamma(D^{*0} \to D^{0}\gamma)} \frac{\Gamma_{\mathrm{tot}}(D^{*0})}{\Gamma_{\mathrm{tot}}(D^{*\pm})}$$

where we use the predicted decay width $\Gamma(D^{*0} \to D^0\gamma)$ to extract the full width for D^{*0}. Similarly the full width for $D_s^{*\pm}$ can be estimated using the same method as in the case of D^{*0}-meson. Our results for the full widths for D^{*0} and $D_s^{*\pm}$ mesons are found to be

$$\Gamma_{\mathrm{tot}}(D^{*0}) = (99.84 \pm 19.51)\ \mathrm{keV}$$
$$\Gamma_{\mathrm{tot}}(D_s^{*\pm}) = (0.32 \pm 0.06)\ \mathrm{keV}$$

respectively, while experimentally only the upper limits: $\Gamma_{\mathrm{tot}}(D^{*0})_{\mathrm{expt}} < 2.1$ MeV and $\Gamma_{\mathrm{tot}}(D_s^{*\pm})_{\mathrm{expt}} < 1.9$ MeV have thus far been reported. Some other theoretical predictions of the full widths for D^{*0} and $D_s^{*\pm}$ mesons have also been reported as $\Gamma_{\mathrm{tot}}(D^{*0}) = (55 \pm 6)$ keV and $\Gamma_{\mathrm{tot}}(D_s^{*\pm}) = (0.19 \pm 0.01)$ keV from LFQM [11]; $\Gamma_{\mathrm{tot}}(D^{*0}) = (36.7 \pm 9.7)$ keV and $\Gamma_{\mathrm{tot}}(D_s^{*\pm}) = (0.24 \pm 0.24)$ keV from HQET [9] and $\Gamma_{\mathrm{tot}}(D^{*0}) = 65.09$ keV from the RQM [3].

For B^* and B_s^* radiative decays, our results for the decay widths $\Gamma(B^{*\pm} \to B^{\pm}\gamma) = 0.577$ keV, $\Gamma(B^{*0} \to B^0\gamma) = 0.181$ keV, and $\Gamma(B_s^{*0} \to B_s^0\gamma) = 0.119$ keV are quite compatible with other theoretical predictions such as $\Gamma(B^{*\pm} \to B^{\pm}\gamma) = 0.429$ keV and $\Gamma(B^{*0} \to B^0\gamma) = 0.142$ keV from the RQM [3]; $\Gamma(B^{*\pm} \to B^{\pm}\gamma) = (0.22 \pm 0.09)$ keV and $\Gamma(B^{*0} \to B^0\gamma) = (0.075 \pm 0.027)$ keV from HQET [9] and $\Gamma(B^{*\pm} \to B^{\pm}\gamma) = 0.14$ keV and $\Gamma(B^{*0} \to B^0\gamma) = 0.09$ keV from the chiral perturbation theory [37]. Finally for the $\Upsilon \to \eta_b\gamma$ decay, our result for the decay width $\Gamma(\Upsilon \to \eta_b\gamma) = 11.1$ eV is to be compared with other model predictions such as $\Gamma(\Upsilon \to \eta_b\gamma) = (15.18 \pm 0.51)$ eV from the effective theory approach [24], (3.6 ± 2.9) eV from nonrelativistic effective theory approach [23], (33.2 ± 0.1) eV [38] and 5.8 eV [39] from the RQM. The prediction of $\Gamma(\Upsilon \to \eta_b\gamma)$ is of special interest as it would help determine the experimentally unmeasured mass of the η_b meson since $\Gamma(\Upsilon \to \eta_b\gamma)$ is very sensitive to the mass difference between two participating mesons $\Delta m = (M_\Upsilon - M_{\eta_b})$ and is proportional to $(\Delta m)^3$.

We estimate the electromagnetic radii of the form factors from the relation $\langle r^2 \rangle = -6 \frac{d}{dq^2} [F_{VP}(q^2)]|_{q^2=0}$. Our results for the charge radii in fm^2: $\langle r^2_{D^{*\pm}D^{\pm}} \rangle - 0.0314$, $\langle r^2_{D^{*0}D^0} \rangle = 0.1622$, $\langle r^2_{D_s^{*\pm}D_s^{\pm}} \rangle = -0.0111$, $\langle r^2_{J/\Psi\eta_c} \rangle = 0.0195$ in the charmed meson sector and $\langle r^2_{B^{*\pm}B^{\pm}} \rangle = 0.0303$, $\langle r^2_{B^{*0}B^0} \rangle - 0.0169$, $\langle r^2_{B_s^{*0}B_s^0} \rangle = -0.0091$, $\langle r^2_{\Upsilon\eta_b} \rangle = -0.0003$ in the bottomed flavor sector are found smaller for the decay involving heavy quarks as expected.

IV. SUMMARY AND CONCLUSION

In this work we investigated the magnetic dipole $V \to P\gamma$ decays of heavy flavored meson such as $(D^*, D; D_s^*, D_s; J/\Psi, \eta_c)$ and $(B^*, B; B_s^*, B_s; \Upsilon, \eta_b)$ in the framework of the relativistic independent quark (RIQ) model. We predicted the momentum dependent transition form factor $F_{VP}(q^2)$ in the spacelike and timelike region. The timelike form factor is obtained by analytical continuation of spacelike $(q^2 < 0)$ form factor to the physical timelike $(0 \le q^2 \le q^2_{\mathrm{max}})$ region. The form factors $F_{D^{*\pm}D^{\pm}}(q^2)$ and $F_{D^{*0}D^0}(q^2)$ seem to have non-negligible recoil effect about 18% and 6%, respectively between zero (q^2_{max}) and maximum $(q^2 = 0)$ recoil point i.e., $F_{D^{*\pm}D^{\pm}}(q^2_{\mathrm{max}})/g_{D^{*\pm}D^{\pm}} \simeq 1.18$ and $F_{D^{*0}D^0}(q^2_{\mathrm{max}})/g_{D^{*0}D^0} \simeq 1.06$. The corresponding ratios in the radiative decays of comparatively heavier vector mesons such as $J/\Psi, D_s^{*\pm}, B^{*\pm}, B^{*0}, B_s^{*0}$, and Υ are found to be unit, indicating negligible recoil effect in such decays. The coupling constants $g_{VP\gamma}$ needed for calculating the decay widths $\Gamma(V \to P\gamma)$ are determined in the limit $q^2 \to 0$, i.e., $g_{VP\gamma} = F_{VP}(q^2 = 0)$. Our predictions for the coupling constants, decay widths and branching ratios of radiative decays of the charmed mesons $(D^{*\pm}, J/\Psi)$ are in reasonable agreement with other theoretical predictions and experimental data within error bars. We also estimated the unmeasured full widths for D^{*0} and $D_s^{*\pm}$ mesons as $\Gamma_{\mathrm{tot}}(D^{*0}) = (99.84 \pm 19.51)$ keV and $\Gamma_{\mathrm{tot}}(D_s^{*\pm}) = (0.32 \pm 0.06)$ keV which can be compared with other theoretical predictions and are well within the experimental limit. Our results of decay widths for the bottomed and bottomed-strange meson decays are found compatible with other theoretical predictions. For the radiative decay of the bottomonium our prediction $\Gamma(\Upsilon \to \eta_b\gamma) = 11.1$ eV can be compared with other theoretical predictions. Since $\Gamma(\Upsilon \to \eta_b\gamma)$ is very sensitive to the mass difference $\Delta m =$

$(M_\Upsilon - M_{\eta_b})$ as it is proportional to $(\Delta m)^3$, the prediction of this decay width is of special interest to determine the unmeasured mass of η_b meson. Our results for the electromagnetic charge radii of the form factors of heavy flavored mesons are also obtained in the same order as expected.

Incorporating recoil effect into the formalism, our results for the q^2-dependence of transition form factors, coupling constants, decay widths and branching ratios of radiative decays of heavy flavored vector mesons are found compatible with other theoretical predictions and available experimental data within error bars. The present model, within the working approximation, thus provides a realistic framework to describe the M1 transition of heavy flavored vector mesons based on the conventional picture of photon emission induced by a quark electromagnetic current. The form factor $F_{VP}(q^2)$ of the vector meson radiative decay $V \to P\gamma$ studied in this work is analogous to the vector current form factor $g(q^2)$ in the weak decay of the ground state pseudoscalar meson to the ground state vector meson. We would like to study the vector current form factor $g(q^2)$ in the RIQ model in our future communication.

APPENDIX: TRANSITION FORM FACTOR AND DECAY WIDTH FROM THE S-MATRIX ELEMENT

The S-matrix element in the configuration space for the decay $V \to P\gamma$ can be written as

$$S_{VP} = \left\langle P\gamma \middle| -ie \int d^4xT\left[\sum_q e_q \bar{\Psi}_q(x)\gamma^\mu \Psi_q(x) A_\mu(x)\right]\middle| V\right\rangle. \tag{A1}$$

Using usual quark field $(\Psi_q(x), \bar{\Psi}_q(x))$ and photon field $A_\mu(x)$ expansions, Eq. (A1) is reduced to

$$S_{VP} = i\sqrt{\alpha/k_0}\langle P|\sum_{q,\lambda,\lambda'} \frac{e_q}{e}\int \frac{dpdp'}{\sqrt{4}E_pE_{p'}}\delta^{(4)}(p'+k-p)\Lambda(p'\lambda'; p\lambda; k\delta)|V\rangle \tag{A2}$$

where,

$$\Lambda(p'\lambda'; p\lambda; k\delta) = \bar{U}(p',\lambda')\gamma.\epsilon(k,\delta)U(p,\lambda)b_q^\dagger(p',\lambda')b_q(p,\lambda) - \bar{V}(p,\lambda)\gamma.\epsilon(k,\delta)V(p',\lambda')\tilde{b}_q^\dagger(p',\lambda')\tilde{b}_q(p,\lambda). \tag{A3}$$

Now incorporating the initial and final meson states as per Eq. (2) the S-matrix element in the parent meson rest frame is obtained in the form

$$S_{VP} = i\sqrt{\alpha/k_0}\delta^{(4)}(p'+k-\hat{O}M_V)[Q(p',\vec{k}) - \tilde{Q}(p',\vec{k})] \tag{A4}$$

where $p' \equiv (E_p, \vec{p}')$; $\hat{O} \equiv (1,0,0,0)$, $\vec{p}' + \vec{k} = 0$

$$Q(p',\vec{k}) = \sum \frac{e_{q_1}}{e}\zeta^V_{q_1q_2}(\lambda_1\lambda_2)\zeta^P_{q_1q_2}(\lambda_1'\lambda_2)\int dp_{q_1}\frac{\mathcal{G}_V(\vec{p}_{q_1},-\vec{p}_{q_1})\mathcal{G}_P(\vec{p}_{q_1}-\vec{k},-\vec{p}_{q_1})}{\sqrt{4E_1E_{1k}\bar{N}_V(0)\bar{N}_P(p')}} \times \bar{U}(-\vec{p}_{q_1},\lambda_1')\gamma.\epsilon(k,\delta)U(\vec{p}_{q_1},\lambda_1)$$

$$\tilde{Q}(p',\vec{k}) = \sum \frac{e_{q_2}}{e}\zeta^V_{q_1q_2}(\lambda_1\lambda_2)\zeta^P_{q_1q_2}(\lambda_1\lambda_2')\int dp_{q_1}\frac{\mathcal{G}_V(\vec{p}_{q_1},-\vec{p}_{q_1})\mathcal{G}_P(\vec{p}_{q_1}-\vec{k},-\vec{p}_{q_1})}{\sqrt{4E_2E_{2k}\bar{N}_V(0)\bar{N}_P(p')}} \times \bar{V}(-\vec{p}_{q_1},\lambda_2)\gamma.\epsilon(k,\delta)V(\vec{p}_{q_1}-\vec{k},\lambda_2') \tag{A5}$$

with $E_i = \sqrt{(p^2_{q_i} + m^2_{q_i})}$ and $E_{ik} = \sqrt{(\vec{p}_{qi} - \vec{k})^2 + m^2_{q_i}}$, $i = 1, 2$. Here the energy conservation at the photon-hadron vertex is ensured by appropriate energy delta function using the usual approximation [2]: $E_1 + E_2 \simeq M_V$ and $E_{1k} + E_2 \simeq E_{2k} + E_1 = E_P$. Now making use of the explicit form of the Dirac spinors $U(\vec{p}_{q_1},\lambda_1)$ and $V(\vec{p}_{q_1},\lambda_2)$ etc. Eq. (A5) can be simplified to the form

$$Q(\vec{k}) = \sum \frac{e_{q_1}}{e}\zeta^V_{q_1q_2}(\lambda_1\lambda_2)\zeta^P_{q_1q_2}(\lambda_1'\lambda_2)\chi^\dagger_{\lambda_1'}(\vec{\sigma}.\vec{K})\chi_{\lambda_1}I_{q_1}(\vec{k})$$

$$\tilde{Q}(\vec{k}) = \sum \frac{e_{q_2}}{e}\zeta^V_{q_1q_2}(\lambda_1\lambda_2)\zeta^P_{q_1q_2}(\lambda_1\lambda_2')\tilde{\chi}^\dagger_{\lambda_2}(\vec{\sigma}.\vec{K})\tilde{\chi}'_{\lambda_2}I_{\bar{q}_2}(\vec{k}) \tag{A6}$$

where, $\vec{K} = \vec{k} \times \vec{\epsilon}(\vec{k}, \delta)$

$$I_{q_1} = \int d\vec{p}_{q_1} \frac{\mathcal{G}_V(\vec{p}_{q_1}, -\vec{p}_{q_1})\mathcal{G}_P(\vec{p}_{q_1} - \vec{k}, -\vec{p}_{q_1})}{\sqrt{\bar{N}(0)\bar{N}(\vec{k})}} \sqrt{\frac{(E_1 + m_{q_1})(E_{1k} + E_2)}{4E_1E_{1k}(E_{1k} + m_{q_1})(E_1 + E_2)}}$$

$$I_{\bar{q}_2} = \int d\vec{p}_{q_1} \frac{\mathcal{G}_V(\vec{p}_{q_1}, -\vec{p}_{q_1})\mathcal{G}_P(\vec{p}_{q_1} - \vec{k}, -\vec{p}_{q_1})}{\sqrt{\bar{N}(0)\bar{N}(\vec{k})}} \sqrt{\frac{(E_2 + m_{q_2})(E_{2k} + E_1)}{4E_2E_{2k}(E_{2k} + m_{q_2})(E_1 + E_2)}}. \quad \text{(A7)}$$

Specifying the appropriate spin-flavor coefficients $\zeta^M_{q_1q_2}(\lambda_1\lambda_2)$ for pseudoscalar meson state and vector meson state of different spin projections $S_V = (\pm 1, 0)$, Eq. (A4) can be further simplified to the form:

$$S_{VP} = i\sqrt{\alpha/k_0}\delta^{(3)}(\vec{p} + \vec{k})\delta(E_P + k_0 + M_V)F_{PV}(q^2)K_{S_V} \quad \text{(A8)}$$

where,

$$F_{VP}(q^2) = e_{q_1}I_{q_1}(m_{q_1}, m_{\bar{q}_2}, q^2) + e_{\bar{q}_2}I_{\bar{q}_2}(m_{\bar{q}_2}, m_{q_1}, q^2), \quad \text{(A9)}$$

from which the model expression for $I_{q_1,\bar{q}_2}$ can be written in the general form:

$$I_{q_i}(m_{q_1}, m_{\bar{q}_2}, q^2) = \sqrt{\frac{1}{\bar{N}(0)\bar{N}(\vec{k})}} \int d\vec{p}_{q_i}\mathcal{G}_V(\vec{p}_{q_i}, -\vec{p}_{q_i})\mathcal{G}_P(\vec{p}_{q_i} - \vec{k}, -\vec{p}_{q_i}) \times \sqrt{\frac{(E_{p_i} + m_{q_i})(E_{p_ik} + E_{-p_i})}{4E_{p_i}E_{p_ik}(E_{p_ik} + m_{q_i})(E_{p_i} + E_{-p_i})}} \quad \text{(A10)}$$

and K_{S_V} for $S_V = (\pm 1, 0)$ stands for

$$K_{S_V} = [\mp (K_1 \pm iK_2)/\sqrt{2}, K_3]. \quad \text{(A11)}$$

The summation over photon polarization index δ and the vector meson spin S_V yields a general relation

$$\sum_{\delta, S_V} |K_{S_V}|^2 = 2k_\gamma^2. \quad \text{(A12)}$$

Finally summing over photon polarization and the daughter(pseudoscalar) meson spin appropriately and averaging over the parent (vector) meson spins, the partial decay widths for the transition $V \to P\gamma$ is realized in the standard form in terms of the outgoing photon energy $k_\gamma = \frac{M_V^2 - M_P^2}{2M_V}$ and the coupling constant $g_{VP\gamma}(k_\gamma)$ as

$$\Gamma(V \to P\gamma) = \frac{\alpha}{3} g^2_{VP\gamma} k^3_\gamma$$

where the coupling constant $g_{VP\gamma}(k_\gamma)$ is obtained from $F_{VP}(q^2)$ in the limit $q^2 \to 0$.

[1] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[2] N. Barik and P. C. Dash, Phys. Rev. D **49**, 299 (1994).
[3] W. Jaus, Phys. Rev. D **53**, 1349 (1996).
[4] N. R. Jones and D. Liu, Phys. Rev. D **53**, 6334 (1996).
[5] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Lett. B **537**, 241 (2002); J. L. Goity and W. Roberts, Phys. Rev. D **64**, 094007 (2001); C.-H. V. Chang, D. Chang, and Wai-Yee-Keung, Phys. Rev. D **61**, 053007 (2000).
[6] H. G. Dosch and S. Narison, Phys. Lett. B **368**, 163 (1996).
[7] T. M. Aliev, D. A. Demir, E. Iltan, and N. K. Pak, Phys. Rev. D **54**, 857 (1996).
[8] H.-Y.Cheng, C.-Y. Cheung, G.-L. Lin, Y. C. Lin, T.-M. Yan, and H.-L. Yu, Phys. Rev. D **47**, 1030 (1993); C.-Y. Cheung and C.-W. Hwang, J. High Energy Phys. 04 (2014) 177.
[9] P. Colangelo, F. De Fazio, and G. Nardulli, Phys. Lett. B **316**, 555 (1993).

[10] P. Singer and G. A. Miller, Phys. Rev. D **33**, 141 (1986); **39**, 825 (1989); G. A. Miller and P. Singer, Phys. Rev. D **37**, 2564 (1988).

[11] H. M. Choi, Phys. Rev. D **75**, 073016 (2007).

[12] D. Becirevic and B. Hass, Eur. Phys. J. C **71**, 1734 (2011); A. AbadaD. Becirevic, Ph. Boucaud, G. Herdoiza, J. P. Leroy, A. Le Yaouanc, O. Pene, and J. Rodriguez-Quintero, Phys. Rev. D **66**, 074504 (2002).

[13] J. L. Rosner, Phys. Rev. D **88**, 034034 (2013).

[14] N. Barik, P. C. Dash, and A. R. Panda, Phys. Rev. D **46**, 3856 (1992).

[15] N. Barik and P. C. Dash, Phys. Rev. D **47**, 2788 (1993); **53**, 1366 (1996); N. Barik, S. K. Tripathy, S. Kar, and P. C. Dash, Phys. Rev. D **56**, 4238 (1997); N. Barik, Sk. Naimuddin, P. C. Dash, and S. Kar, Phys. Rev. D **80**, 074005 (2009); **77**, 014038 (2008); **78**, 114030 (2008); Sk. Naimuddin, S. Kar, M. Priyadarsini, N. Barik, and P. C. Dash, Phys. Rev. D **86**, 094028 (2012); S. Kar, P. C. Dash, M. Priyadarsini, Sk. Naimuddin, and N. Barik, Phys. Rev. D **88**, 094014 (2013).

[16] B. Margolis and R. R. Mendel, Phys. Rev. D **28**, 468 (1983).

[17] K. A. Olive *et al.* (Particle Data Group Collaboration) Chin. Phys. C **38**, 090001 (2014).

[18] R. L. Thews and A. N. Kamal, Phys. Rev. D **32**, 810 (1985).

[19] E. Eichten, S. Godfrey, H. Mahlke, and J. L. Rosner, Rev. Mod. Phys. **80**, 1161 (2008).

[20] M. B. Voloshin, Nucl. Phys. **61**, 455 (2008).

[21] E. S. Swanson, Phys. Rep. **429**, 243 (2006).

[22] N. Brambilla *et al.*, arXiv:hep-ph/0412158.

[23] N. Brambilla, Y. Jia, and A. Vairo, Phys. Rev. D **73**, 054005 (2006).

[24] A. Pineda and J. Segovia, Phys. Rev. D **87**, 074024 (2013).

[25] A. Y. Khodjamirian, Sov. J. Nucl. Phys. **39**, 614 (1984).

[26] V. Beilin and A. Radyushkin, Nucl. Phys. **B260**, 61 (1985).

[27] M. A. Shifman, Z. Phys. C **4**, 345 (1980); **6**, 282(E) (1980).

[28] D. Becirevic and F. Sanfilippo, J. High Energy Phys. 01 (2013) 028.

[29] G. C Donald, C. T. H. Davies, R. J. Dowdall, E. Follanna, K. Hornbostel, J. Koponen, G. P Lepage, and C. McNeile, Phys. Rev. D **86**, 094501 (2012).

[30] J. Gaiser, E. D. Bloom, F. Bulos, G. Godfrey, C. M. Kiesling, W. S. Lockman, M. Oreglia, D. L. Scharre *et al.*, Phys. Rev. D **34**, 711 (1986).

[31] R. E. Mitchell *et al.* (CLEO Collaboration), Phys. Rev. Lett. **102**, 011801 (2009); **106**, 159903(E) (2011).

[32] E. J. Eichten, K. Lane, and C. Quigg, Phys. Rev. Lett. **89**, 162002 (2002).

[33] T. Barnes, S. Godfrey, and E. S. Swanson, Phys. Rev. D **72**, 054026 (2005).

[34] J. J. Dudek, R. G. Edwards, and D. G. Richards, Phys. Rev. D **73**, 074507 (2006).

[35] N. Barik, B. K. Dash, and M. Das, Phys. Rev. D **32**, 1725 (1985); N. Barik and B. K. Dash, Phys. Rev. D **33**, 1925 (1986); **34**, 2803 (1986); **34**, 2092 (1986); N. Barik, B. K. Dash, and P. C. Dash, Pramana **29**, 543 (1987).

[36] R. L. Thews and A. N. Kamal, Phys. Rev. D **32**, 810 (1985).

[37] J. F. Amundson, C. G. Boyd, I. Jenkins, M. Luke, A. V. Manohar, J. L. Rosner, M. J. Savage, and M. B. Wise, Phys. Lett. B **296**, 415 (1992).

[38] C.-W. Hwang and Z.-T Wei, J. Phys. G **34**, 687 (2007).

[39] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D **67**, 014027 (2003).