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Implications of general lepton mass matrices in the standard model on m_{ee}

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Within the framework of the standard model (SM), using the facility of weak basis (WB) transformations, the general Dirac neutrino mass matrix and the charged lepton mass matrix can essentially be considered as texture two zero mass matrices. Using type I seesaw formula for Majorana neutrino mass matrix, our analysis yields lower bounds $m_{ee} \gtrsim 0.001$ eV for normal mass ordering and $m_{ee} \gtrsim 0.08$ eV for inverted mass ordering, the latter being tantalizingly close to the expected outcome of the ongoing experiments. Interestingly, for inverted mass ordering, m_{ee} is largely independent of variation of mass m_3, whereas, for normal mass ordering with m_1 in the range 0.0001 eV–0.01 eV, the bound on parameter m_{ee} gets further sharpened and one obtains m_{ee} within the band 0.014–0.042 eV. Further, noting that a particular set of texture four zero quark mass matrices has been shown to be a unique viable option for the description of quark mixing data, an analysis of similar mass matrices in the lepton sector has also been carried out to obtain bounds for the parameter m_{ee} with interesting consequences.

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I. INTRODUCTION

For the last more than a decade, spectacular advances have taken place in our understanding of the neutrino oscillation phenomenology owing to various solar [1], atmospheric [2], reactor [3], and accelerator [4] neutrino experiments. At present, the neutrino oscillation data is parametrized in terms of two mass squared differences $\Delta m^2_{\mathrm{sol}}$, $\Delta m^2_{\mathrm{atm}}$ and three mixing angles θ_{12}, θ_{23}, θ_{13}, with the latter being measured in the last few years only [5,6]. An analysis of the present data reveals that the absolute neutrino masses, although not determined, are much smaller than their charged counterparts. Similarly, ordering of neutrino mass eigenstates is also not clear, one may have either the normal mass ordering (NO) or the inverted mass ordering (IO) [7].

In the absence of any deep theoretical understanding of fermion masses and mixings, the "smallness" of neutrino masses is best understood in terms of the seesaw mechanism [8]. This can simply be realized in the standard model (SM) by the addition of three heavy right handed neutrinos, e.g., an effective neutrino mass matrix is generated through type I seesaw formula

$$M_\nu = -M^T_{\nu D} M_R^{-1} M_{\nu D}, \tag{1}$$

with M_ν, $M_{\nu D}$, and M_R corresponding to the light Majorana neutrino mass matrix, the Dirac neutrino mass matrix, and the heavy right handed Majorana neutrino mass matrix, respectively. In this context, a smoking gun signature for establishing Majorana nature of neutrinos is expected to be provided by the observation of neutrinoless double beta decay (NDBD) [9], measured in terms of the effective Majorana mass m_{ee} expressed as

$$|m_{ee}| = |m_1 U^2_{e1} + m_2 U^2_{e2} + m_3 U^2_{e3}|, \tag{2}$$

where m_1, m_2, m_3 are the absolute neutrino masses and U_{e1}, U_{e2}, U_{e3} are the elements of the Pontecorvo Maki Nakagawa Sakata (PMNS) matrix [10]. In terms of the standard parametrization [11] of PMNS matrix, the parameter m_{ee} can be rewritten as

$$|m_{ee}| = |m_1 c^2_{12} c^2_{13} + m_2 s^2_{12} c^2_{13} e^{2i\eta_1} + m_3 s^2_{13} e^{2i\eta_2}|, \tag{3}$$

with η_1, η_2 representing the Majorana phases and c_{12}, s_{12}, etc. corresponding to the cosine and sine of the leptonic mixing angles, respectively. Noting that the mixing angles are quite well determined, even in the absence of knowledge of the phases η_1 and η_2, one can obtain constraints on the parameter m_{ee} provided one has information about the absolute neutrino masses. Since, at present, neither the absolute neutrino masses nor the parameter m_{ee} are well determined, therefore any constraints on either of these would have mutual implications.

On both the phenomenological as well as experimental fronts, a good deal of effort has been made to find constraints on m_{ee}. On the phenomenological front, most of the attempts have been model dependent [12] within the "top-down" as well as "bottom-up" approach. In the case of the bottom-up approach, emphasis has mostly been on the texture zero approach [13], with most of the attempts made by considering the mass matrices to be in the "flavor basis" [14], wherein the charged lepton mass matrix M_l is considered to be diagonal while a texture is imposed on the Majorana neutrino matrix M_ν. Along with these, some

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attempts have also been carried out in the "nonflavor basis" [15] wherein it is usual to impose texture on the charged lepton mass matrix M_l and on the Dirac neutrino mass matrix $M_{\nu D}$. Equation (1) can then be used to obtain the Majorana neutrino matrix M_ν which along with the matrix M_l allows the construction of the PMNS matrix for examining the viability of the mass matrices and consequently for obtaining constraints on parameter m_{ee}.

On the experimental front, again large amount of efforts have been devoted in constraining the parameter m_{ee}, in particular, the data provides an upper bound [16], $m_{ee} < 0.1$–0.25 eV, expected to be refined largely by several next generation NDBD experiments [9] aiming to achieve a sensitivity up to 0.01 eV for m_{ee} in the near future. The measurement of m_{ee} would not only establish the Majorana nature of neutrinos, but would also pave the way for obtaining useful constraints on absolute neutrino masses through the relation given in Eq. (2). Therefore, fruitful constraints on absolute neutrino masses may be obtained by combining information from bounds on m_{ee} as well as the results obtained from the Planck Satellite [17] and the direct neutrino mass measurements [18]. In light of the above discussion, one can conclude that determination of m_{ee} would have far reaching implications for having a deeper understanding of the neutrino oscillation phenomenology. In this context, it would be interesting to obtain bounds on m_{ee} starting with general mass matrices within the framework of SM which would be useful for both phenomenologists as well as experimentalists.

It is perhaps desirable to mention that recently [19] in the case of quarks, starting with general quark mass matrices, following the texture zero approach and coupling it with the facility of weak basis (WB) transformations [20,21], a finite set of texture four zero mass matrices was arrived at as a unique viable option for the description of quark mixing data. Keeping in mind the quark lepton universality, as advocated by Smirnov [22] as well as required by most of the grand unified theories (GUTs), it becomes desirable to examine the implications of the corresponding matrices in the leptonic sector in order to obtain bounds on the parameter m_{ee}.

Working in the nonflavor basis, the purpose of the present manuscript, therefore, is to start with general Dirac lepton mass matrices $M_{\nu D}$ and M_l within the framework of SM and consider these to be texture specific mass matrices by using WB transformations without any further input. Using the matrices M_l and M_ν, the latter obtained using Eq. (1), the elements of the PMNS matrix are then constructed, allowing us to obtain bounds for m_{ee}. For both normal and inverted neutrino mass orderings, the dependence of these bounds on the lightest neutrino mass have been examined. Further, as discussed above, an analysis of texture four zero lepton mass matrices has also been carried out to obtain bounds for the parameter m_{ee}.

The plan of the paper is as follows. In Sec. II, within the framework of SM, we first start with general lepton mass matrices $M_{\nu D}$ and M_l and following WB transformations we consider these as texture two zero lepton mass matrices. Some essential details pertaining to the construction of the corresponding PMNS matrix have been presented in Sec. III. Using inputs given in Sec. IV and keeping focus on the parameter m_{ee}, the results pertaining to the analyses of texture two zero and texture four zero lepton mass matrices for the different neutrino mass orderings have been presented in Secs. V A and V B, respectively. Finally, Sec. VI summarizes our conclusions.

II. GENERAL LEPTON MASS MATRICES IN THE SM

In order to construct matrix M_ν using Eq. (1), we begin with the Dirac lepton mass matrices, which, within the framework of SM, arise from the Higgs-fermion couplings characterized by

$$\mathcal{L}_{\text{Yukawa}} = Y^l_{ij}\overline{L_{Li}}Hl_{Rj} + Y^\nu_{ij}\overline{L_{Li}}H^c\nu_{Rj} + \text{H.c.},\quad i,j = 1,2,3, \tag{4}$$

where $L_L \equiv \binom{l_L}{\nu_L}$, H and H^c correspond to the left-handed lepton doublet, the Higgs field, and its charge conjugate, respectively. The charged lepton and the Dirac neutrino mass matrices M_l and $M_{\nu D}$ are related to the Yukawa couplings Y_{ij}'s as

$$M_l = \frac{\upsilon}{\sqrt{2}}Y^l_{ij},\qquad M_{\nu D} = \frac{\upsilon}{\sqrt{2}}Y^{\nu D}_{ij}, \tag{5}$$

with υ corresponding to the vacuum expectation value of the Higgs field. Within the SM and some of its extensions, without loss of parameter space, the general 3×3 complex mass matrices M_l and $M_{\nu D}$ can be considered to be Hermitian [13] and in general expressed as

$$M_k = \begin{pmatrix} C_k & A_k & F_k \\ A^*_k & D_k & B_k \\ F^*_k & B^*_k & E_k \end{pmatrix}\qquad (k = l, \nu D). \tag{6}$$

It may be pointed out that SM and its extensions wherein the right-handed fermions remain singlets have the facility of carrying out transformations, without loss of their generality, in the above mass matrices known as the weak basis (WB) transformations [20]. In particular, this implies making unitary transformations, e.g.,

$$\nu'_L = W_L\nu_L,\qquad l'_L = W_Ll_L,$$
$$l'_R = W_Rl_R,\qquad \nu'_R = W_R\nu_R, \tag{7}$$

where W_L and W_R are unitary matrices. Under these transformations, the gauge currents

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \bar{l}_L \gamma_\mu \nu_L W^\mu + \text{H.c.}, \tag{8}$$

remain real and diagonal, but the matrices M_l and $M_{\nu D}$ transform as

$$M'_l = W_L^\dagger M_l W_R, \qquad M'_{\nu D} = W_L^\dagger M_{\nu D} W_L. \tag{9}$$

Using this facility, the above mass matrices M'_l and $M'_{\nu D}$ can be reduced to [20]

$$M_l = \begin{pmatrix} C_l & A_l & 0 \\ A_l^* & D_l & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \quad M_{\nu D} = \begin{pmatrix} C_{\nu D} & A_{\nu D} & 0 \\ A_{\nu D}^* & D_{\nu D} & B_{\nu D} \\ 0 & B_{\nu D}^* & E_{\nu D} \end{pmatrix}. \tag{10}$$

It may be reemphasized that there is no loss of generality of the mass matrices as we reduce the ones given in Eq. (6) to the ones mentioned above. Apart from the form of the lepton mass matrices given above, other equivalent forms of matrices based on WB transformations have also been proposed in the literature [21], however for the present analysis, we have considered the form mentioned above since it corresponds to a parallel texture structure for the charged lepton and the neutrino sector, in consonance with some classes of family symmetries and grand unified theories (GUTs). In the language of texture specific mass matrices, these matrices are texture one zero type with $A_{l(\nu D)} = |A_{l(\nu D)}| e^{i\alpha_{l(\nu D)}}$ and $B_{l(\nu D)} = |B_{l(\nu D)}| e^{i\beta_{l(\nu D)}}$, together these are considered as texture two zero lepton mass matrices. Further, using the matrix $M_{\nu D}$ and the right handed Majorana neutrino mass matrix M_R, the Majorana neutrino matrix M_ν can also be obtained through Eq. (1). It may be emphasized that the matrices M_l and $M_{\nu D}$ are considered as texture two zero lepton mass matrices, however, no texture has been imposed on the matrix M_ν.

III. TEXTURE TWO ZERO MASS MATRICES AND CONSTRUCTION OF THE PMNS MATRIX

As a next step, construction of the PMNS matrix is carried out in terms of the diagonalization transformations of the matrices M_l and that of M_ν which is expressed in terms of the diagonalizing transformation of $M_{\nu D}$. In this context, the Hermitian mass matrices M_l and $M_{\nu D}$ can be expressed as

$$M_k = P_k^\dagger M_k^r P_k \qquad k = l, \nu D, \tag{11}$$

where M_k^r is a real symmetric matrix with real eigenvalues and P_k is a diagonal phase matrix. In general, the real matrix M_k^r is diagonalized by the orthogonal transformation O_k, yielding

$$M_k = P_k^\dagger O_k \xi_k M_k^{\text{diag}} O_k^T P_k. \tag{12}$$

A diagonal phase matrix ξ_k defined as $\text{diag}(1, e^{i\pi}, 1)$ for the case of normal mass ordering and as $\text{diag}(1, e^{i\pi}, e^{i\pi})$ for the case of inverted mass ordering has been introduced to facilitate the construction of diagonalization transformations for different neutrino mass orderings [13].

The elements of the transformation O_k corresponding to the mass matrices given in Eq. (10) are

$$\begin{pmatrix} \sqrt{\frac{(E_k-m_1)(D_k+E_k-m_1-m_2)(D_k+E_k-m_1-m_3)}{(D_k+2E_k-m_1-m_2-m_3)(m_1-m_2)(m_1-m_3)}} & \sqrt{\frac{(E_k-m_2)(m_3-C_k)(m_1-C_k)}{(E_k-C_k)(m_1-m_2)(m_3-m_2)}} & \sqrt{\frac{(-C_k+m_1)(-E_k+m_3)(C_k-m_2)}{(m_1-m_3)(m_3-m_2)(C_k-E_k)}} \\ \sqrt{\frac{(m_1-C_k)(m_1-E_k)}{(m_1-m_2)(m_1-m_3)}} & -\sqrt{\frac{(E_k-m_2)(C_k-m_2)}{(m_1-m_2)(m_3-m_2)}} & \sqrt{\frac{(-m_3-C_k)(E_k-m_3)}{(m_1-m_3)(m_3-m_2)}} \\ -\sqrt{\frac{(E_k-m_2)(E_k-m_3)(m_1-C_k)}{(m_1-m_2)(m_1-m_3)(E_k-C_k)}} & \sqrt{\frac{(-E_k+m_1)(C_k-m_2)(E_k-m_3)}{(m_1-m_2)(m_2-m_3)(E_k-C_k)}} & \sqrt{\frac{(E_k-m_1)(E_k-m_2)(m_3-C_k)}{(C_k-E_k)(m_1-m_3)(m_3-m_2)}} \end{pmatrix}, \tag{13}$$

with m_1, $-m_2$, m_3 being the eigenvalues of M_k, negative sign with m_2 is to facilitate construction of the transformation O_k. In the case of charged leptons, because of the strong hierarchy $m_e \ll m_\mu \ll m_\tau$, the mass eigenstates can be approximated respectively to the flavor eigenstates, as is usually considered [14].

For the case of neutrinos, in analogy with Eq. (12), we can express $M_{\nu D}$ as

$$M_{\nu D} = P_{\nu D}^\dagger O_{\nu D} \xi_{\nu D} M_{\nu D}^{\text{diag}} O_{\nu D}^T P_{\nu D}. \tag{14}$$

Substituting the above value of $M_{\nu D}$ in Eq. (1), one obtains

$$M_\nu = -(P_{\nu D}^\dagger O_{\nu D} \xi_{\nu D} M_{\nu D}^{\text{diag}} O_{\nu D}^T P_{\nu D})^T (M_R)^{-1} \times (P_{\nu D}^\dagger O_{\nu D} \xi_{\nu D} M_{\nu D}^{\text{diag}} O_{\nu D}^T P_{\nu D}). \tag{15}$$

On using $P_{\nu D}^T = P_{\nu D}$, the above equation can further be written as

$$M_\nu = -P_{\nu D} O_{\nu D} M_{\nu D}^{\text{diag}} \xi_{\nu D} O_{\nu D}^T (P_{\nu D}^\dagger)^T (M_R)^{-1} \times P_{\nu D}^\dagger O_{\nu D} \xi_{\nu D} M_{\nu D}^{\text{diag}} O_{\nu D}^T P_{\nu D}. \tag{16}$$

To simplify calculations, the phase matrices $(P^\dagger_{\nu D})^T$ and $P^\dagger_{\nu D}$ along with $-M_R$ can be taken as $m_R \mathrm{diag}(1,1,1)$ [15] as well as using the unitarity of $\xi_{\nu D}$ and orthogonality of $O_{\nu D}$, the above equation can be expressed as

$$M_\nu = P_{\nu D} O_{\nu D} \frac{(M_{\nu D}^{\mathrm{diag}})^2}{(m_R)} O_{\nu D}^T P_{\nu D}. \tag{17}$$

From the above equation, it is immediately clear that matrix M_ν can be diagonalized in terms of the diagonalizing transformation of $M_{\nu D}$. The corresponding lepton mixing matrix is expressed as

$$U_{\mathrm{PMNS}} = (P_l^\dagger O_l \xi_l)^\dagger (P_{\nu D} O_{\nu D}). \tag{18}$$

Eliminating the phase matrix ξ_l by redefinition of the charged lepton phases, the above equation becomes

$$U_{\mathrm{PMNS}} = O_l^\dagger P_l P_{\nu D} O_{\nu D}, \tag{19}$$

where $P_l P_{\nu D}$, without loss of generality, can be taken as $(e^{i\phi_1}, 1, e^{i\phi_2})$, ϕ_1 and ϕ_2 are related to the phases of mass matrices as $\phi_1 = \alpha_{\nu D} - \alpha_l$, $\phi_2 = \beta_{\nu D} - \beta_l$ and can be treated as free parameters.

IV. INPUTS USED FOR THE ANALYSIS

Summarizing essentials of various inputs, the results of the latest global three neutrino oscillation analyses [23] have been presented in Table I. Further, for ready reference, we present the following 3σ C.L. ranges of the PMNS matrix elements given by Garcia *et al.* [23]

$$U_{\mathrm{PMNS}} = \begin{pmatrix} 0.801-0.845 & 0.514-0.580 & 0.137-0.158 \\ 0.225-0.517 & 0.441-0.699 & 0.614-0.793 \\ 0.246-0.529 & 0.464-0.713 & 0.590-0.776 \end{pmatrix}. \tag{20}$$

While carrying out our analysis, the magnitudes of solar and atmospheric neutrino mass squared differences are given variation within their 3σ ranges mentioned in Table I. The lightest neutrino mass, m_1 for the case of NO and m_3 for the case of IO, is considered as a free parameter while the other two masses are obtained using the following relations

$$\mathrm{NO:}\ m_2^2 = \Delta m_{\mathrm{sol}}^2 + m_1^2, \qquad m_3^2 = \Delta m_{\mathrm{atm}}^2 + \frac{(m_1^2 + m_2^2)}{2}, \tag{21}$$

$$\mathrm{IO:}\ m_2^2 = \frac{2(m_3^2 + \Delta m_{\mathrm{atm}}^2) + \Delta m_{\mathrm{sol}}^2}{2}, \quad m_1^2 = \frac{2(m_3^2 + \Delta m_{\mathrm{atm}}^2) - \Delta m_{\mathrm{sol}}^2}{2}. \tag{22}$$

For both the mass orderings of neutrinos, in the absence of any lower bound on the lightest neutrino mass, its range has been explored from 0 eV $-$ 10^{-1} eV. The phases ϕ_1, ϕ_2 have also been considered to be free parameters and given full variation from 0 to 2π. Further, the mass matrix elements $D_{l,\nu D}$ and $C_{l,\nu D}$ have been constrained such that diagonalizing transformations O_l and O_ν always remain real, ensuring the mass matrices to be "natural" as advocated by Peccei and Wang [24]. Incorporating these constraints on the input parameters and using the usual methodology, detailed in Ref. [13], one can easily reproduce the PMNS matrix elements.

TABLE I. Current data for neutrino mixing parameters from the latest global fits [23].

Parameter	3σ range
$\Delta m_{\mathrm{sol}}^2$ $[10^{-5}\ \mathrm{eV}^2]$	(7.02-8.09)
$\Delta m_{\mathrm{atm}}^2$ $[10^{-3}\ \mathrm{eV}^2]$	(2.317-2.607)(NO); (2.590-2.307)(IO)
$\sin^2\theta_{13}$ $[10^{-2}]$	(1.86-2.50)(NO); (1.88-2.51)(IO)
$\sin^2\theta_{12}$ $[10^{-1}]$	(2.70-3.44)
$\sin^2\theta_{23}$ $[10^{-1}]$	(3.82-6.43)(NO); (3.89-6.44)(IO)

V. RESULTS AND DISCUSSION

A. Texture two zero lepton mass matrices

Coming to the results of the analysis, we have presented the results corresponding to the normal and inverted neutrino mass orderings, those for the degenerate scenario can be derived from these. It may be mentioned that we have focused our attention on the parameter m_{ee} and its implications on the lightest neutrino mass.

1. Inverted ordering of neutrino masses

For the IO case, to begin with, the magnitudes of the PMNS matrix elements are given by

$$U_{\mathrm{PMNS}}^{IO} = \begin{pmatrix} 0.034-0.859 & 0.0867-0.593 & 0.135-0.996 \\ 0.250-0.971 & 0.068-0.812 & 0.043-0.808 \\ 0.103-0.621 & 0.395-0.822 & 0.088-0.810 \end{pmatrix}. \tag{23}$$

It is immediately clear that the ranges of the matrix elements obtained by Garcia *et al.*, given in Eq. (20), are inclusive in the ranges found above, therefore, establishing the viability of texture two zero mass matrices for the IO case. As a next step, we examine the constraints obtained for the parameter m_{ee}. To this end, in Fig. 1 we present the plots showing mass m_{ee} versus the phases ϕ_1 and ϕ_2, these being related to the phases of the mass matrices. While plotting these figures, all the three mixing angles have been constrained by their 3σ experimental bounds given in Table I, while the Majorana phases η_1 and

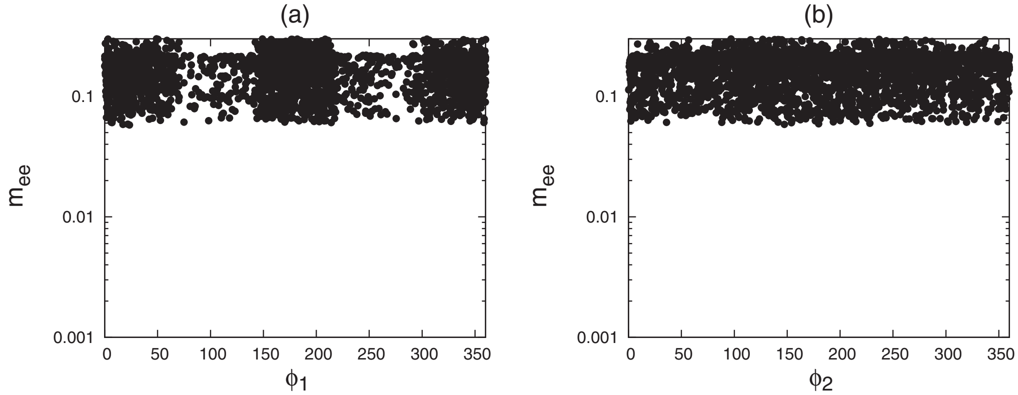

FIG. 1. Parameter m_{ee} (in eV) versus (a) ϕ_1 and (b) ϕ_2 for texture two zero mass matrices (IO).

η_2 as well as the other free parameters have been allowed full variation.

Several interesting points are in order. It is immediately clear from the graphs that we obtain a lower bound of the order of 0.08 eV on m_{ee}, independent of the values of the phases ϕ_1 and ϕ_2. Interestingly, this bound is tantalizingly close to the likely explored range of m_{ee} by the ongoing experiments [9,16]. Therefore, an absence of a signal of NDBD by these experiments would have important implications for the IO scenario.

Further, to examine the dependence of parameter m_{ee} on the lightest neutrino mass m_3, in Fig. 2 we have presented m_{ee} versus m_3, plotted by giving full variation to other parameters. As mentioned earlier, the lightest neutrino mass has been explored within the range $0\,\mathrm{eV}$–$10^{-1}\,\mathrm{eV}$, however, the graph has been plotted for m_3 from $10^{-4}\,\mathrm{eV}$–$10^{-1}\,\mathrm{eV}$, our conclusions remain unaffected even if the range is extended below 10^{-4} eV. From the graph, one finds that the above mentioned bound on parameter m_{ee} looks to be independent of the range of mass m_3 considered here.

2. Normal ordering of neutrino masses

For the NO case, as a first step, we again reproduce the magnitudes of the PMNS matrix elements, e.g.,

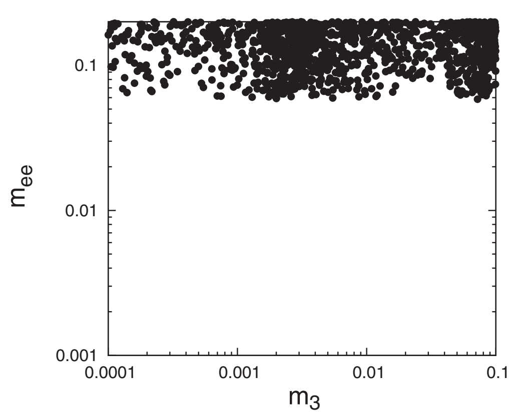

FIG. 2. Parameter m_{ee} (in eV) versus the lightest neutrino mass m_3 for texture two zero mass matrices (IO).

$$U_{\mathrm{PMNS}}^{\mathrm{NO}} = \begin{pmatrix} 0.444-0.993 & 0.123-0.837 & 0.004-0.288 \\ 0.061-0.816 & 0.410-0.941 & 0.047-0.872 \\ 0.012-0.848 & 0.049-0.779 & 0.460-0.992 \end{pmatrix}, \tag{24}$$

these again being compatible with the ones obtained by Garcia *et al.* given in Eq. (20). As a next step, we obtain the bounds on parameter m_{ee} by plotting m_{ee} versus phases ϕ_1 and ϕ_2, shown in Figs. 3(a) and 3(b). While plotting these figures, the neutrino oscillation parameters and the other free parameters have been varied in a manner similar to the IO case. From the graphs, it is clear that parameter m_{ee}, contrary to the IO case, shows substantial dependence on the phases ϕ_1 and ϕ_2. Also, it is interesting to note that now one obtains a lower bound of the order of 0.001 eV for the parameter m_{ee}, this being considerably lower compared with the bound obtained for IO case.

Further, interesting conclusions can be derived by studying the variation of m_{ee} with respect to the lightest neutrino mass m_1, shown in Fig. 4. In particular, one notices that for m_1 from 0.0001 eV–0.01 eV, the bound on parameter m_{ee} gets further sharpened and one obtains m_{ee} within the band 0.014–0.042 eV, whereas for $m_1 > 0.01$ eV, the parameter m_{ee} does not remain constrained to the above mentioned band but instead there is a considerable spreading of the m_{ee} values outside the band. This observation has interesting implications for the orderings of neutrino masses. For example, in case the range of parameter m_{ee} settles around values outside the band, which is possible in the near future as several ongoing experiments like GERDA, CUORE, MAJORANA and EXO are already aiming to approach sensitivity on m_{ee} around these values, then the allowed range of m_1 would correspond to the degenerate scenario of neutrino masses [13].

B. Texture four zero lepton mass matrices

As mentioned earlier, recently, for the case of quarks it has been shown [19] that a particular type of texture structure, i.e. texture four zero mass matrices, emerges as a unique possibility for the up as well as down sector

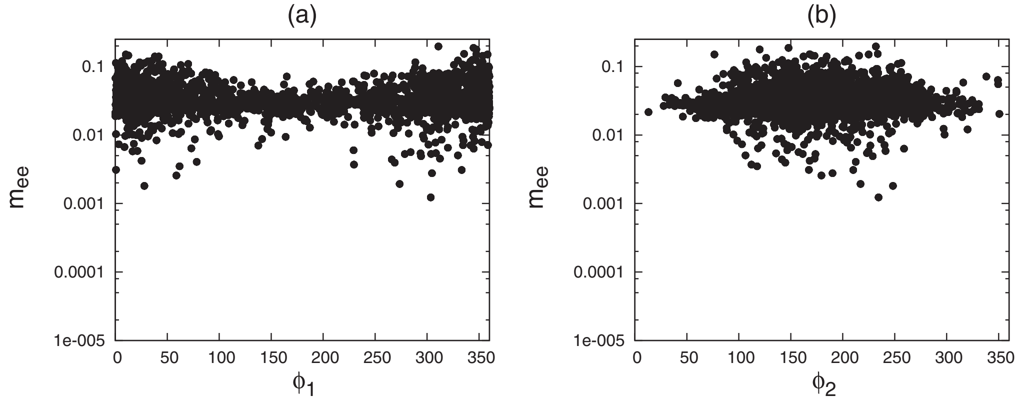

FIG. 3. Parameter m_{ee} (in eV) versus (a) ϕ_1 and (b) ϕ_2 for texture two zero mass matrices (NO).

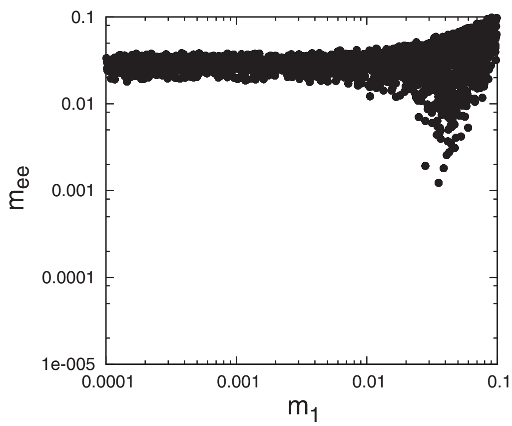

FIG. 4. Parameter m_{ee} (in eV) versus the lightest neutrino mass m_1 for texture two zero mass matrices (NO).

mass matrices. Keeping in mind the quark lepton unification, as advocated by Smirnov [22] as well as required by most of the GUTs, it therefore becomes interesting to investigate the implications of similar type of mass matrices in the leptonic sector as well, e.g.,

$$M_l = \begin{pmatrix} 0 & A_l & 0 \\ A_l^* & D_l & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \quad M_{\nu D} = \begin{pmatrix} 0 & A_{\nu D} & 0 \\ A_{\nu D}^* & D_{\nu D} & B_{\nu D} \\ 0 & B_{\nu D}^* & E_{\nu D} \end{pmatrix}. \tag{25}$$

Following the methodology discussed earlier for the case of texture two zero mass matrices, the texture four zero lepton mass matrices have also been analyzed for the normal as well as inverted ordering of neutrino masses.

1. Inverted ordering of neutrino masses

Interestingly, the present well-defined data rules out the mass matrices given in Eq. (25) for the IO case as the PMNS matrix constructed using these mass matrices is not compatible with the one constructed by Garcia *et al.* presented in Eq. (20). To confirm this conclusion, in Fig. 5, we have presented the plot showing the parameter space corresponding to the mixing angles s_{13} and s_{23}. The blank rectangular region indicates the experimentally allowed 3σ region of the plotted angles. The graph clearly shows that the plotted parameter space does not include simultaneously the experimental bounds of the plotted angles, therefore, ruling out the texture four zero lepton mass matrices for the IO case.

2. Normal ordering of neutrino masses

For the NO case, the viability of texture four zero lepton mass matrices is quite well established in the literature [25]. For the sake of completion, we present below the magnitudes of the corresponding PMNS matrix elements

$$U_{\text{PMNS}}^{\text{NO}} = \begin{pmatrix} 0.692-0.995 & 0.074-0.711 & 0.028-0.199 \\ 0.074-0.701 & 0.417-0.892 & 0.185-0.829 \\ 0.051-0.593 & 0.164-0.758 & 0.554-0.976 \end{pmatrix}. \tag{26}$$

A comparison of this matrix with the one given in Eq. (20) establishes the viability of mass matrices given in Eq. (25) for the normal ordering of neutrino masses.

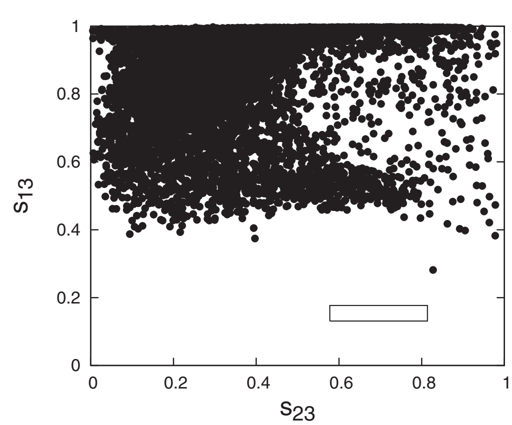

FIG. 5. Plot showing the parameter space corresponding to s_{13} and s_{23} for texture four zero mass matrices (IO).

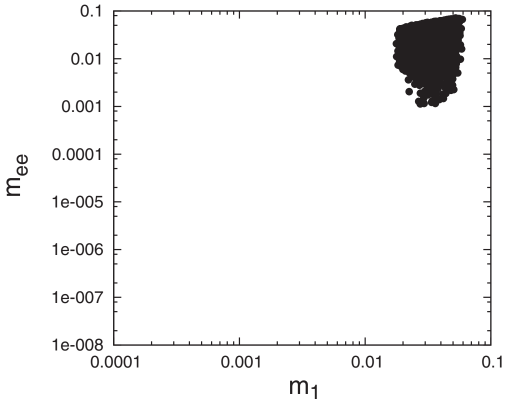

FIG. 6. Parameter m_{ee} (in eV) versus the lightest neutrino mass m_1 for texture four zero mass matrices (NO).

Regarding the predictions for bounds on the parameter m_{ee}, the plots of m_{ee} versus phases ϕ_1 and ϕ_2 yield results similar to the case of texture two zero mass matrices, i.e., one obtains a lower bound of the order of 0.001 eV. However, a plot depicting mass m_{ee} versus the lightest neutrino mass m_1, given in Fig. 6, yields results considerably different from those obtained in texture two zero case. In particular, a careful comparison of the plots showing parameter m_{ee} versus mass m_1 for the texture two zero and texture four zero NO cases, shown in Figs. 4 and 6, respectively, reveals that in the case of latter one obtains a very limited region of viability. Therefore, nonzero values of the (1,1) elements in the charged lepton and neutrino mass matrices as given in Eq. (10) leads to significantly different predictions as compared to the case when both of these elements are zero as given in Eq. (25). This, interestingly, is contrary to the observation in the quark sector [19] wherein the (1,1) element seems to be essentially redundant. In particular, the mass matrices given in Eq. (25) lead to an upper bound of the order of 0.09 eV for the parameter m_{ee}. Further, the range of the lightest neutrino mass gets severely constrained too, viz. 0.02–0.08 eV. Therefore if, by any theoretical considerations, texture four zero structure turns out to be the only viable possibility then it would be very easy to rule out or establish the Majorana nature of neutrinos within the next few years.

VI. SUMMARY AND CONCLUSIONS

To summarize, without loss of generality, within the framework of SM, using WB transformations, general Dirac neutrino mass matrix $M_{\nu D}$ and the charged lepton mass matrix M_l, can be considered as texture two zero mass matrices. The Majorana neutrino matrix M_ν, with no texture imposed on it, has then been expressed through the seesaw formula given in Eq. (1). The construction of the corresponding PMNS matrix elements allows us to obtain bounds for m_{ee}. For both normal and inverted neutrino mass orderings, the implications of these bounds have been examined for the lightest neutrino mass. Further, considering quark-lepton universality and taking clues from a particular set of texture four zero quark mass matrices, shown to be a unique viable option for the description of quark mixing data, analysis of similar lepton mass matrices has also been carried out to obtain bounds for the parameter m_{ee}.

It is interesting to note that the bounds for m_{ee} obtained from our analysis are well within the reach of the ongoing experiments. For example, considering texture two zero lepton mass matrices, for the inverted ordering of neutrino masses, we find a lower bound of around 0.08 eV for m_{ee}, therefore, an absence of a signal of NDBD by these experiments would have important implications for this mass ordering scenario. One also finds that variation of the lightest neutrino mass in this case m_3 has no implications for the parameter m_{ee}.

For the normal mass ordering case, one obtains a lower bound of the order of 0.001 eV for the parameter m_{ee}, this being quite lower compared to the bound obtained for the IO case. Further, on examining the implications for the lightest neutrino mass, one finds that higher values of m_1 are allowed for lower values of m_{ee} and vice versa. If in the ongoing experiments NO is not ruled out, it would narrow down the window for the lightest neutrino mass and lead to the possibility of neutrino masses being degenerate. For the case of texture four zero matrices, interestingly, the present data rules out IO, while for the NO case, the lightest neutrino mass gets constrained in the range 0.02–0.08 eV.

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