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Off-shell higher spin $\mathcal{N}=2$ supermultiplets in three dimensions

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Off-shell higher spin $\mathcal{N}=2$ supermultiplets in three spacetime dimensions (3D) are presented in this paper. We propose gauge prepotentials for higher spin superconformal gravity and construct the corresponding gauge-invariant field strengths, which are proved to be conformal primary superfields. These field strengths are higher spin generalizations of the (linearized) $\mathcal{N}=2$ super-Cotton tensor, which controls the superspace geometry of conformal supergravity. We also construct the higher spin extensions of the linearized $\mathcal{N}=2$ conformal supergravity action. We provide two dually equivalent off-shell formulations for massless higher spin $\mathcal{N}=2$ supermultiplets. They involve one and the same superconformal prepotential but differ in the compensators used. For the lowest superspin value $3/2$, these higher spin series terminate at the linearized actions for the (1,1) minimal and $w=-1$ nonminimal $\mathcal{N}=2$ Poincaré supergravity theories constructed in S. M. Kuzenko and G. Tartaglino-Mazzucchelli, arXiv:1109.0496. Similar to the pure 3D supergravity actions, their higher spin counterparts propagate no degrees of freedom. However, the massless higher spin supermultiplets are used to construct off-shell massive $\mathcal{N}=2$ supermultiplets by combining the massless actions with those describing higher spin extensions of the linearized $\mathcal{N}=2$ conformal supergravity. We also demonstrate that every higher spin super-Cotton tensor can be represented as a linear superposition of the equations of motion for the corresponding massless higher spin supermultiplet, with the coefficients being higher-derivative linear operators.

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I. INTRODUCTION

In supersymmetric field theory, it is of interest to construct off-shell supersymmetric extensions in diverse dimensions of the (Fang-)Fronsdal actions for massless higher spin fields in Minkowski [1,2] and anti-de Sitter [3,4] spacetimes. In four spacetime dimensions (4D), this problem was solved in the early 1990s. In the $\mathcal{N}=1$ super-Poincaré case, the off-shell formulations for massless higher spin supermultiplets were developed in [5,6]. For each superspin[1] $s \geq 1$, half-integer [5] and integer [6], these publications provided two dually equivalent off-shell realizations in $\mathcal{N}=1$ Minkowski superspace. At the component level, each of the two superspin-s actions [5,6] reduces, *upon* imposing a Wess-Zumino-type gauge and eliminating the auxiliary fields, to a sum of the spin-s and spin-$(s+1/2)$ actions [1,2]. The off-shell higher spin supermultiplets of [5,6] were generalized to the case of anti-de Sitter supersymmetry in [7]. Making use of the $\mathcal{N}=1$ supermultiplets constructed in [5–7], off-shell formulations for 4D $\mathcal{N}=2$ massless higher spin supermultiplets were presented in [8,9].[2] A pedagogical review of the supersymmetric higher spin models proposed in [5,6] is given in Sec. VI. 9 of [11].[3] A comprehensive review of the results of [5–7], including a detailed analysis of the component structure of the models constructed, is given in [12].

In this paper, we present off-shell $\mathcal{N}=2$ supersymmetric generalizations of the 3D (Fang-)Fronsdal actions and derive their massive deformations. In principle, one may construct all 3D $\mathcal{N}=2$ massless higher spin supermultiplets by applying an off-shell version of dimensional reduction $d=4 \to d=3$ to the 4D $\mathcal{N}=1$ supermultiplets 5,6]]. Such a procedure has been carried out in [13] to obtain one of the four off-shell actions (given in [13]) for linearized 3D $\mathcal{N}=2$ supergravity (superspin $s=3/2$). In

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[1]In four dimensions, the massless multiplet of superspin s describes two fields of spin s and $s+\frac{1}{2}$; it is often denoted $(s, s+\frac{1}{2})$.

[2]An important by-product of the higher spin construction given in [9] was the explicit description of the infinite dimensional superalgebra of Killing tensor superfields of 4D $\mathcal{N}=1$ anti-de Sitter superspace. This superalgebra corresponds to the rigid symmetries of the generating action for the massless supermultiplets of arbitrary superspin in 4D $\mathcal{N}=1$ anti-de Sitter superspace, which was constructed in [9]. A generalization of the concept of Killing tensor superfields given in [9] recently appeared in [10].

[3]Section 6. 9 of [11] also contains a pedagogical review of the (Fang-)Fronsdal actions for free massless higher spin fields in 4D Minkowski space [1,2], including a direct proof of the fact that the massless spin-s action describes two helicity states $\pm s$.

practice, however, naive dimensional reduction is not quite efficient to deal with in the case of higher spin supermultiplets. The point is that its application to a 4D superspin-s multiplet, with $s > 3/2$, leads to a superposition of several 3D multiplets, one of which carries superspin s and the others correspond to lower superspin values.[4] Some work is required in order to disentangle the superspin-s multiplet from the lower-superspin ones, which is actually quite nontrivial. It proves to be more efficient to recast the 4D gauge principle of [5,6] in a 3D form and use it to construct gauge-invariant actions. This is our approach in the present paper.

In three dimensions, the massless spin-s actions of [1,2] are known to propagate no local degrees of freedoms for $s > 1$.[5] Of course, this is consistent with the fact that the notion of 3D spin is well defined only in the massive case [14]. When speaking of a 3D massless spin-s theory, we will refer to the kinematic structure of the field variables, their gauge transformation laws, and the gauge-invariant action. One reason to study such a theory is that it may be deformed (say, by including auxiliary lower-spin fields and adding mass terms) to result in a model describing a massive spin-s field.

There have appeared two different constructions of Lagrangian models for 3D massive higher spin fields [15,16]. The approach of [16] has been used to formulate *on-shell* models for massive $\mathcal{N} = 1$ higher spin supermultiplets [17]. In this paper we will pursue an alternative approach to address the problem of constructing *off-shell* massive $\mathcal{N} = 2$ higher spin supermultiplets. Our approach will be based on deriving a higher spin generalization of the $\mathcal{N} = 2$ super-Cotton tensor[6] [18,20] that can be used to write down a topological mass term.

An important feature of 3D gauge theories is the possibility to generate the mass for gauge fields of different spin by adding to the massless action a gauge-invariant Chern-Simons-type term of topological origin. This idea has been used to construct topologically massive electrodynamics [21–23], topologically massive gravity [23], and topologically massive $\mathcal{N} = 1$ supergravity [24,25]. The latter theory admits generalizations with $\mathcal{N} > 1$, including the off-shell topologically massive supergravity theories with $\mathcal{N} = 2$ [26] and $\mathcal{N} = 3$ and $\mathcal{N} = 4$ [27]. In the case of 3D supergravity theories, the topological mass term may be interpreted as an action for conformal supergravity (see [28] for a review of 4D conformal supergravity theories). The off-shell actions for $\mathcal{N}$-extended conformal supergravity theories were constructed in [29] for $\mathcal{N} = 1$, [30] for $\mathcal{N} = 2$, [31] for $\mathcal{N} = 3$, 4, 5, and [32,33] for $\mathcal{N} = 6$.[7] An arbitrary variation of such an action with respect to a supergravity prepotential is given in terms of the $\mathcal{N}$-extended super-Cotton tensor [20]. This means that a linearized supergravity action is determined by the linearized super-Cotton tensor, $W(H)$. The corresponding Lagrangian is symbolically $L_{\rm CSG} = H \cdot W(H)$, where H is the linearized conformal supergravity prepotential. The super-Cotton tensor $W(H)$ is a unique field strength being superconformal primary and invariant under the linearized gauge transformations of conformal supergravity.

Our construction of the linearized higher spin superconformal actions is analogous to that of the 3D higher spin conformal gravity actions derived by Pope and Townsend [34] (the 3D analogs of the conformal higher spin actions pioneered by Fradkin and Tseytlin [28]). The Pope-Townsend conformal action for the spin-s field makes use of the linearized spin-s Cotton tensor (which can be read off from the action (31) in [34]). The $s = 3$ case was studied earlier in [35]. For recent discussions of the linearized higher spin Cotton tensors [34] and their generalizations, see [36,37] and references therein.

This paper is organized as follows. Section II is devoted to general properties of transverse and longitudinal linear superfields. Section III is concerned with on-shell massive fields and $\mathcal{N} = 2$ superfields. Two series of off-shell actions for massless half-integer superspin multiplets are introduced in Sec. IV. Section V is devoted to a brief discussion of the component reduction of the models presented in Sec. IV. In Sec. VI we present $\mathcal{N} = 2$ superconformal higher spin actions and derive a higher spin extension of the linearized $\mathcal{N} = 2$ super-Cotton tensor. Off-shell actions for massive higher spin supermultiplets are presented in Sec. VII. Concluding comments are given in Sec. VIII. The main body of the paper is accompanied by four appendixes. Appendix A summarizes our notation and conventions. Appendix B is devoted to the 3D (Fang-) Fronsdal massless actions in the two-component notation. The component structure of the massless superspin-$(s + \frac{1}{2})$ model (4.9) is studied in Appendix C. Appendix D is devoted to the proof of two fundamental properties of the superconformal field strength (6.22).

II. LINEAR SUPERFIELDS

A symmetric rank-n spinor superfield, $\Gamma_{\alpha_1\cdots\alpha_n} = \Gamma_{(\alpha_1\cdots\alpha_n)}$, is called *transverse linear* if it obeys the constraint

[4] In the case of a half-integer superspin $s = n + 1/2$, with $n = 2, 3, \ldots$, one of the 4D dynamical variables [5] is a real unconstrained superfield $H_{\alpha_1\ldots\alpha_n\dot\alpha_1\ldots\dot\alpha_n} = H_{(\alpha_1\ldots\alpha_n)(\dot\alpha_1\ldots\dot\alpha_n)}$. Its dimensional reduction $d = 4 \to d = 3$ leads to a family of unconstrained symmetric superfields $H_{\alpha_1\ldots\alpha_{2n}}$, $H_{\alpha_1\ldots\alpha_{2n-2}}$, $\cdots$, H, of which only $H_{\alpha_1\ldots\alpha_{2n}}$ is required to describe a massless 3D supermultiplet. In the supergravity case, $s = 3/2$, dimensional reduction $d = 4 \to d = 3$ leads to two multiplets, an off-shell $\mathcal{N} = 2$ supergravity multiplet and an Abelian vector multiplet [13].

[5] See Appendix B for a direct proof.

[6] Upon fixing the super-Weyl and local $\mathsf{U}(1)_R$ symmetries, the super-Cotton tensor derived in [18] reduces to that introduced earlier by Zupnik and Pak [19].

[7] The component actions for $\mathcal{N} = 1$, 2 conformal supergravities [29,30] have been rederived within the universal superspace setting of [31].

$$\bar{D}^\beta\Gamma_{\beta\alpha_1\ldots\alpha_{n-1}}=0, \qquad n>0. \tag{2.1a}$$

A symmetric rank-n spinor superfield, $G_{\alpha_1\cdots\alpha_n}=G_{(\alpha_1\cdots\alpha_n)}$, is called *longitudinal linear* if it obeys the constraint

$$\bar{D}_{(\alpha_1}G_{\alpha_2\ldots\alpha_{n+1})}=0, \tag{2.1b}$$

which for $n=0$ is equivalent to the chirality condition

$$\bar{D}_\alpha G=0. \tag{2.2}$$

The constraints (2.1a) and (2.1b) imply that $\Gamma_{\alpha_1\cdots\alpha_n}$ and $G_{\alpha_1\cdots\alpha_n}$ are linear superfields in the usual sense:

$$\bar{D}^2\Gamma_{\alpha_1\cdots\alpha_n}=0, \tag{2.3a}$$

$$\bar{D}^2G_{\alpha_1\cdots\alpha_n}=0. \tag{2.3b}$$

In the case $n=0$, the transverse constraint (2.1a) is not defined, but its corollary (2.3a) can be used. In four dimensions, the transverse and longitudinal linear superfields were introduced for the first time by Ivanov and Sorin [38] (who built on the earlier results by Salam and Strathdee [39] and Sokatchev [40] in the super-Poincaré case) as a means to realize the irreducible representations of the $\mathcal{N}=1$ anti-de Sitter supersymmetry. As dynamical variables, such superfields were used for the first time in [5–7].

We assume that $\Gamma_{\alpha_1\cdots\alpha_n}$ and $G_{\alpha_1\cdots\alpha_n}$ are complex and the differential conditions (2.1a) and (2.1b) are the only constraints these superfields obey. The constraints (2.1a) and (2.1b) can be solved in terms of complex unconstrained prepotentials $\xi_{\alpha_1\ldots\alpha_{n+1}}=\xi_{(\alpha_1\ldots\alpha_{n+1})}$ and $\zeta_{\alpha_1\ldots\alpha_{n-1}}=\zeta_{(\alpha_1\ldots\alpha_{n-1})}$ according to the rules

$$\Gamma_{\alpha_1\ldots\alpha_n}=\bar{D}^\beta\xi_{\beta\alpha_1\ldots\alpha_n}, \tag{2.4a}$$

$$G_{\alpha_1\ldots\alpha_n}=\bar{D}_{(\alpha_1}\zeta_{\alpha_2\ldots\alpha_n)}. \tag{2.4b}$$

There is a natural arbitrariness in the choice of the prepotentials ξ and ζ, namely,

$$\delta\xi_{\alpha_1\ldots\alpha_{n+1}}=\Gamma_{\alpha_1\ldots\alpha_{n+1}}, \tag{2.5a}$$

$$\delta\zeta_{\alpha_1\ldots\alpha_{n-1}}=G_{\alpha_1\ldots\alpha_{n-1}}. \tag{2.5b}$$

Here, the gauge parameter $\Gamma_{\alpha(n+1)}$ is a transverse linear superfield, and $G_{\alpha(n-1)}$ is a longitudinal linear one. As a result, there emerge the transverse and longitudinal gauge hierarchies:

$$\Gamma_{\alpha(n)}\to\Gamma_{\alpha(n+1)}\to\Gamma_{\alpha(n+2)}\ldots, \tag{2.6a}$$

$$G_{\alpha(n)}\to G_{\alpha(n-1)}\to G_{\alpha(n-2)}\ldots\to G. \tag{2.6b}$$

Thus, in accordance with the terminology of gauge theories with linearly dependent generators [41], any Lagrangian theory described by a transverse (longitudinal) linear superfield $\Gamma_{\alpha_1\ldots\alpha_n}$ ($G_{\alpha_1\ldots\alpha_n}$) can be considered as the theory of an unconstrained prepotential $\xi_{\alpha_1\ldots\alpha_{n+1}}$ ($\zeta_{\alpha_1\ldots\alpha_{n-1}}$) with an additional gauge invariance of an infinite (finite) stage of reducibility.

Suppose we are given a supersymmetric field theory described by a transverse linear superfield $\Gamma_{\alpha(n)}$ and its conjugate $\bar{\Gamma}_{\alpha(n)}$, for $n>0$, with an action functional $S[\Gamma,\bar{\Gamma}]$. Such a theory possesses a dual formulation, $S_{\rm D}[G,\bar{G}]$, described in terms of a longitudinal linear superfield $G_{\alpha(n)}$ and its conjugate $\bar{G}_{\alpha(n)}$. The latter theory is obtained by introducing a first-order action of the form

$$S[V,\bar{V},G,\bar{G}]=S[V,\bar{V}]+\int {\rm d}^3x{\rm d}^2\theta{\rm d}^2\bar{\theta}(V^{\alpha(n)}G_{\alpha(n)} + (-1)^n\bar{V}^{\alpha(n)}\bar{G}_{\alpha(n)}), \tag{2.7}$$

where the symmetric rank-n spinor $V_{\alpha(n)}$ is a complex unconstrained superfield. The first term in the action, $S[V,\bar{V}]$, is obtained from $S[\Gamma,\bar{\Gamma}]$ by the replacement $\Gamma_{\alpha(n)}\to V_{\alpha(n)}$. Varying (2.7) with respect to $G_{\alpha(n)}$ gives $V_{\alpha(n)}=\Gamma_{\alpha(n)}$, and then the second term in (2.7) drops out, due to the identity

$$\int {\rm d}^3x{\rm d}^2\theta{\rm d}^2\bar{\theta}\Gamma^{\alpha(n)}G_{\alpha(n)}=0. \tag{2.8}$$

As a result, the first-order action reduces to the original one, $S[\Gamma,\bar{\Gamma}]$. On the other hand, we can consider the equation of motion for $V^{\alpha(n)}$,

$$\frac{\delta}{\delta V^{\alpha(n)}}S[V,\bar{V}]+G_{\alpha(n)}=0, \tag{2.9}$$

and the conjugate equation. We assume that these equations are uniquely solved to give $V_{\alpha(n)}$ as a functional of $G_{\alpha(n)}$ and $\bar{G}_{\alpha(n)}$. Substituting this solution back into (2.7), we end up with the dual action $S_{\rm D}[G,\bar{G}]$.

A real transverse linear superfield $T_{\alpha_1\cdots\alpha_n}=T_{(\alpha_1\cdots\alpha_n)}$ is characterized by the properties

$$\bar{T}_{\alpha_1\cdots\alpha_n}=T_{\alpha_1\cdots\alpha_n}, \qquad \bar{D}^\beta T_{\beta\alpha_1\cdots\alpha_{n-1}}=0\Leftrightarrow D^\beta T_{\beta\alpha_1\cdots\alpha_{n-1}}=0. \tag{2.10}$$

The second-order differential operator

$$\Delta=\frac{\rm i}{2}D^\alpha\bar{D}_\alpha \tag{2.11}$$

acts on the space of such superfields. Indeed, $\Delta T_{\alpha_1\cdots\alpha_n}$ is real and one may check that

$$\bar{D}^{\beta}\Delta T_{\beta\alpha_1\cdots\alpha_{n-1}} = 0, \qquad D^{\beta}\Delta T_{\beta\alpha_1\cdots\alpha_{n-1}} = 0. \tag{2.12}$$

III. MASSIVE (SUPER)FIELDS

In this section we discuss on-shell (super)fields which realize the massive representations of the (super-) Poincaré group.

A. Massive fields

Let P_a and $J_{ab} = -J_{ba}$ be the generators of the 3D Poincaré group. The Pauli-Lubanski scalar

$$W := \frac{1}{2}\varepsilon^{abc}P_a J_{bc} = -\frac{1}{2}P^{\alpha\beta}J_{\alpha\beta} \tag{3.1}$$

commutes with the generators P_a and J_{ab}. Irreducible unitary representations of the Poincaré group are labeled by two parameters, mass m and helicity λ, which are associated with the Casimir operators,

$$P^a P_a = -m^2\mathbb{1}, \qquad W = m\lambda\mathbb{1}. \tag{3.2}$$

One defines $|\lambda|$ to be the spin.

In the case of field representations, it holds that

$$W = \frac{1}{2}\partial^{\alpha\beta}M_{\alpha\beta}, \tag{3.3}$$

where the action of $M_{\alpha\beta} = M_{\beta\alpha}$ on a field $\phi_{\gamma_1\cdots\gamma_n} = \phi_{(\gamma_1\cdots\gamma_n)}$ is defined by

$$M_{\alpha\beta}\phi_{\gamma_1\cdots\gamma_n} = \sum_{i=1}^{n}\varepsilon_{\gamma_i(\alpha}\phi_{\beta)\gamma_1\cdots\hat{\gamma}_i\ldots\gamma_n}, \tag{3.4}$$

where the hatted index of $\phi_{\beta\gamma_1\cdots\hat{\gamma}_i\ldots\gamma_n}$ is omitted.

For $n > 1$, a massive field, $\phi_{\alpha_1\cdots\alpha_n} = \bar{\phi}_{\alpha_1\ldots\alpha_n} = \phi_{(\alpha_1\cdots\alpha_n)}$, is a real symmetric rank-n spinor field which obeys the differential conditions [15] (see also [42])

$$\partial^{\beta\gamma}\phi_{\beta\gamma\alpha_1\cdots\alpha_{n-2}} = 0, \tag{3.5a}$$

$$\partial^{\beta}{}_{(\alpha_1}\phi_{\alpha_2\ldots\alpha_n)\beta} = m\sigma\phi_{\alpha_1\ldots\alpha_n}, \qquad \sigma = \pm 1. \tag{3.5b}$$

In the spinor case, $n = 1$, Eq. (3.5a) is absent, and it is the Dirac equation (3.5b) which defines a massive field. It is easy to see that (3.5a) and (3.5b) imply the mass-shell equation[8]

$$(\Box - m^2)\phi_{\alpha_1\cdots\alpha_n} = 0, \tag{3.6}$$

which is the first equation in (3.2). In the case $n = 1$, Eq. (3.6) follows from the Dirac equation (3.5b). The second relation in (3.2) also holds, with

$$\lambda = \frac{n}{2}\sigma. \tag{3.7}$$

B. Massive superfields

Let P_a, $J_{ab} = -J_{ba}$, Q_α and $\bar{Q}_\alpha$ be the generators of the 3D $\mathcal{N} = 2$ super-Poincaré group. The supersymmetric extension of the Pauli-Lubanski scalar (3.1) is the following operator [44]

$$Z = W - \frac{i}{4}Q^\alpha\bar{Q}_\alpha = \frac{1}{2}\varepsilon^{abc}P_a J_{bc} - \frac{i}{4}Q^\alpha\bar{Q}_\alpha, \tag{3.8}$$

which commutes with the supercharges,

$$[Z, Q_\alpha] = [Z, \bar{Q}_\alpha] = 0. \tag{3.9}$$

Irreducible unitary representations of the super-Poincaré group are labeled by two parameters, mass m and superhelicity κ, which are associated with the Casimir operators,

$$P^a P_a = -m^2\mathbb{1}, \qquad Z = m\kappa\mathbb{1}. \tag{3.10}$$

Our definition of the superhelicity agrees with [44]. It is instructive to compare the operator Z, Eq. (3.8), with the 4D $\mathcal{N} = 1$ superhelicity operator introduced in [11]. The massive representation of superhelicity κ is a direct sum of four massive representations of the Poincaré group with helicity values $(\kappa - \frac{1}{2}, \kappa, \kappa, \kappa + \frac{1}{2})$. The parameter $|k|$ is referred to as superspin [44].

In the case of superfield representations, the superhelicity operator may be expressed in the following manifestly supersymmetric form:

$$Z = \frac{1}{2}\partial^{\alpha\beta}M_{\alpha\beta} + \frac{1}{2}\Delta, \tag{3.11}$$

where the operator Δ is given by (2.11).

For $n > 0$, a massive superfield, $\mathcal{E}_{\alpha_1\cdots\alpha_n} = \bar{\mathcal{E}}_{\alpha_1\ldots\alpha_n} = \mathcal{E}_{(\alpha_1\cdots\alpha_n)}$, is a real symmetric rank-n spinor which obeys the differential conditions [45]

$$\bar{D}^{\beta}\mathcal{E}_{\beta\alpha_1\cdots\alpha_{n-1}} = D^{\beta}\mathcal{E}_{\beta\alpha_1\cdots\alpha_{n-1}} = 0 \Rightarrow \partial^{\beta\gamma}\mathcal{E}_{\beta\gamma\alpha_1\ldots\alpha_{n-2}} = 0, \tag{3.12a}$$

$$\Delta\mathcal{E}_{\alpha_1\ldots\alpha_n} = m\sigma\mathcal{E}_{\alpha_1\ldots\alpha_n}, \qquad \sigma = \pm 1. \tag{3.12b}$$

Due to the identity

$$\Box = \Delta^2 + \frac{1}{16}\{D^2, \bar{D}^2\}, \tag{3.13}$$

Eqs. (3.12) lead to the mass-shell equation

$$(\Box - m^2)\mathcal{E}_{\alpha_1\ldots\alpha_n} = 0. \tag{3.14}$$

[8]The equations (3.5) and (3.6) proves to be equivalent to the 3D Fierz-Pauli field equations [43].

One may also check that

$$\Delta\mathcal{E}_{\alpha_1\ldots\alpha_n} = \partial^{\beta}{}_{(\alpha_1}\mathcal{E}_{\alpha_2\ldots\alpha_n)\beta}, \qquad (3.15)$$

as a consequence of (3.12a). We conclude that $\mathcal{E}_{\alpha_1\ldots\alpha_n}$ is an eigenvector of the superhelicity operator (3.11),

$$Z\mathcal{E}_{\alpha_1\ldots\alpha_n} = m\kappa\mathcal{E}_{\alpha_1\ldots\alpha_n}, \qquad \kappa = \frac{1}{2}(n+1)\sigma. \qquad (3.16)$$

For completeness, we also consider massive scalar superfields. The massive multiplet of superhelicity $\kappa_\sigma = \pm\frac{1}{2} \equiv \frac{1}{2}\sigma$ is described by a real scalar superfield $\mathcal{G}_\sigma = \bar{\mathcal{G}}_\sigma$, which is constrained by

$$\Delta\mathcal{G}_\sigma = m\sigma\mathcal{G}_\sigma, \qquad (3.17)$$

with $m > 0$ the mass parameter. This equation implies that $\mathcal{G}_\sigma$ is linear, $\bar{D}^2\mathcal{G}_\sigma = D^2\mathcal{G}_\sigma = 0$. It follows from (3.17) that $Z\mathcal{G}_\sigma = m\kappa_\sigma\mathcal{G}_\sigma$.

Constraint (3.17) is the equation of motion for a supersymmetric Chern-Simons theory with action

$$S_{\rm CS}[V] = -\frac{1}{2}\int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\{\mathcal{G}^2 - m\sigma V\mathcal{G}\}, \qquad \mathcal{G} := \Delta V, \qquad (3.18)$$

with V the gauge prepotential of the vector multiplet.

The superhelicity $\kappa = 0$ multiplet is described by a chiral superfield Φ, $\bar{D}_\alpha\Phi = 0$, constrained by

$$-\frac{1}{4}D^2\Phi + m\bar{\Phi} = 0. \qquad (3.19)$$

This is the equation of motion for the model

$$S[\Phi,\bar{\Phi}] = \int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\,\bar{\Phi}\,\Phi + \frac{m}{2}\left\{\int \mathrm{d}^3x\mathrm{d}^2\theta\Phi^2 + \mathrm{c.c.}\right\}. \qquad (3.20)$$

IV. MASSLESS HALF-INTEGER SUPERSPIN MULTIPLETS

We fix an integer $s > 1$ and consider two sets of superfield dynamical variables:

$$\mathcal{V}^{\perp} = \{H_{\alpha(2s)}, \Gamma_{\alpha(2s-2)}, \bar{\Gamma}_{\alpha(2s-2)}\}; \qquad (4.1)$$

$$\mathcal{V}^{\parallel} = \{H_{\alpha(2s)}, G_{\alpha(2s-2)}, \bar{G}_{\alpha(2s-2)}\}. \qquad (4.2)$$

In both cases $H_{\alpha(2s)} = H_{(\alpha_1\ldots\alpha_{2s})}$ is an unconstrained real superfield. The complex superfields $\Gamma_{\alpha(2s-2)} = \Gamma_{(\alpha_1\ldots\alpha_{2s-2})}$ and $G_{\alpha(2s-2)} = G_{(\alpha_1\ldots\alpha_{2s-2})}$ are transverse and longitudinal, respectively.

We postulate the following linearized gauge transformations for the dynamical superfields introduced:

$$\delta H_{\alpha(2s)} = g_{\alpha(2s)} + \bar{g}_{\alpha(2s)} \equiv \bar{D}_{(\alpha_1}L_{\alpha_2\ldots\alpha_{2s})} - D_{(\alpha_1}\bar{L}_{\alpha_2\ldots\alpha_{2s})}, \qquad (4.3)$$

$$\delta\Gamma_{\alpha(2s-2)} = \frac{s}{2s+1}\bar{D}^{\beta_1}D^{\beta_2}\bar{g}_{\alpha(2s-2)\beta(2)} = -\frac{1}{4}\bar{D}^{\beta}D^2\bar{L}_{\beta\alpha(2s-2)}, \qquad (4.4)$$

$$\delta G_{\alpha(2s-2)} = \frac{s}{2s+1}D^{\beta_1}\bar{D}^{\beta_2}g_{\beta(2)\alpha(2s-2)} + \mathrm{i}s\partial^{\beta(2)}g_{\beta(2)\alpha(2s-2)} = -\frac{1}{4}\bar{D}^2D^{\beta}L_{\beta\alpha(2s-2)} + \mathrm{i}(s-1)\partial^{\beta_1\beta_2}\bar{D}_{(\alpha_1}L_{\alpha_2\ldots\alpha_{2s-2})\beta_1\beta_2}. \qquad (4.5)$$

Here the complex gauge parameter $g_{\alpha(2s)} = g_{(\alpha_1\ldots\alpha_{2s})}$ is an arbitrary longitudinal linear superfield. It can be expressed in terms of an unconstrained complex parameter $L_{\alpha(2s-1)} = L_{(\alpha_1\ldots\alpha_{2s-1})}$ by the rule

$$g_{\alpha(2s)} = \bar{D}_{(\alpha_1}L_{\alpha_2\ldots\alpha_{2s})}. \qquad (4.6)$$

The two sets of dynamical variables, $\mathcal{V}^{\perp}$ and $\mathcal{V}^{\parallel}$, give rise to two gauge-invariant actions, transverse and longitudinal ones, which are dual to each other.

Let us introduce unconstrained prepotentials, $\xi_{\alpha(2s-1)}$ and $\zeta_{\alpha(2s-3)}$, for the constrained superfields $\Gamma_{\alpha(2s-2)}$ and $G_{\alpha(2s-2)}$ according to the rule (2.4). The gauge transformations of $\Gamma_{\alpha(2s-2)}$ and $G_{\alpha(2s-2)}$ are induced by the following variations of the prepotentials:

$$\delta\xi_{\alpha(2s-1)} = -\frac{1}{4}D^2\bar{L}_{\alpha(2s-1)}, \qquad (4.7)$$

$$\delta\zeta_{\alpha(2s-3)} = -\frac{1}{2}\bar{D}^{\beta}D^{\gamma}L_{\alpha(2s-3)\beta\gamma} + \mathrm{i}(s-1)\partial^{\beta\gamma}L_{\alpha(2s-3)\beta\gamma}. \qquad (4.8)$$

In what follows, we will use the notation $\mathrm{d}^{3|4}z = \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}$ for the full superspace measure.

A. Transverse formulation

The transverse formulation for a massless superspin-$(s+\frac{1}{2})$ multiplet is described by the action

$$S^{\perp}_{s+\frac{1}{2}}[H,\Gamma,\bar{\Gamma}] = \left(-\frac{1}{2}\right)^s \int \mathrm{d}^{3|4}z\left\{\frac{1}{8}H^{\alpha(2s)}D^\beta\bar{D}^2D_\beta H_{\alpha(2s)} + H^{\alpha(2s)}(D_{\alpha_1}\bar{D}_{\alpha_2}\Gamma_{\alpha_3\ldots\alpha_{2s}} - \bar{D}_{\alpha_1}D_{\alpha_2}\bar{\Gamma}_{\alpha_3\ldots\alpha_{2s}}) + \frac{2s-1}{s}\bar{\Gamma}\cdot\Gamma + \frac{2s+1}{2s}(\Gamma\cdot\Gamma+\bar{\Gamma}\cdot\bar{\Gamma})\right\}. \tag{4.9}$$

It may be shown that the action is invariant under the gauge transformations (4.3) and (4.4). The requirement of gauge invariance fixes this action uniquely up to a constant.

Consider an arbitrary variation of the action

$$\delta S^{\perp}_{s+\frac{1}{2}} = \left(-\frac{1}{2}\right)^s \int \mathrm{d}^{3|4}z\{\delta H^{\alpha(2s)}E^{\perp}_{\alpha(2s)} - \delta\xi^{\alpha(2s-1)}F_{\alpha(2s-1)} - \delta\bar{\xi}_{\alpha(2s-1)}\bar{F}^{\alpha(2s-1)}\}, \tag{4.10}$$

where we have introduced the gauge-invariant field strengths

$$E^{\perp}_{\alpha(2s)} = \frac{1}{4}D^\beta\bar{D}^2D_\beta H_{\alpha(2s)} + D_{(\alpha_1}\bar{D}_{\alpha_2}\Gamma_{\alpha_3\ldots\alpha_{2s})} - \bar{D}_{(\alpha_1}D_{\alpha_2}\bar{\Gamma}_{\alpha_3\ldots\alpha_{2s})}, \tag{4.11a}$$

$$F_{\alpha(2s-1)} = -\frac{1}{2}\bar{D}^2D^\beta H_{\alpha(2s-1)\beta} + \frac{2s-1}{s}\bar{D}_{(\alpha_1}\bar{\Gamma}_{\alpha_2\ldots\alpha_{2s-1})} + \frac{2s+1}{s}\bar{D}_{(\alpha_1}\Gamma_{\alpha_2\ldots\alpha_{2s-1})}. \tag{4.11b}$$

The field strengths are related to each other by the Bianchi identity

$$D^\beta E^{\perp}_{\beta\alpha(2s-1)} = \frac{1}{4}D^2F_{\alpha(2s-1)}. \tag{4.12}$$

The equations of motion for the theory are

$$E^{\perp}_{\alpha(2s)} = 0, \qquad F_{\alpha(2s-1)} = 0. \tag{4.13}$$

B. Longitudinal formulation

In accordance with our general discussion in Sec. II, the theory with action (4.9) possesses a dual formulation. It is obtained by considering the following first-order action:

$$\begin{aligned} S[H,G,\bar{G},V,\bar{V}] &\coloneqq S^{\perp}_{s+\frac{1}{2}}[H,V,\bar{V}] - \frac{2}{s}\left(-\frac{1}{2}\right)^s\int \mathrm{d}^{3|4}z\{G\cdot V+\bar{G}\cdot\bar{V}\} \\ &= \left(-\frac{1}{2}\right)^s\int \mathrm{d}^{3|4}z\left\{\frac{1}{8}H^{\alpha(2s)}D^\gamma\bar{D}^2D_\gamma H_{\alpha(2s)} + H^{\alpha(2s)}(D_{\alpha_1}\bar{D}_{\alpha_2}V_{\alpha_3\ldots\alpha_{2s}} - \bar{D}_{\alpha_1}D_{\alpha_2}\bar{V}_{\alpha_3\ldots\alpha_{2s}}) \right.\\ &\quad\left. + \frac{2s-1}{s}\bar{V}\cdot V + \frac{2s+1}{2s}(V^2+\bar{V}^2) - \frac{2}{s}(G\cdot V+\bar{G}\cdot\bar{V})\right\}. \end{aligned} \tag{4.14}$$

Here $V_{\alpha(2s-2)}$ is an unconstrained complex superfield, while the Lagrange multiplier $G_{\alpha(2s-2)}$ is constrained to be a complex longitudinal linear superfield. With the normalization of the Lagrange multiplier chosen, the action (4.14) proves to be invariant under the gauge transformations (4.3) and (4.5) accompanied by

$$\delta V_{\alpha(2s-2)} = -\frac{1}{4}\bar{D}^\beta D^2\bar{L}_{\beta\alpha(2s-2)}. \tag{4.15}$$

Varying (4.14) with respect to $G_{\alpha(2s-2)}$ gives $V_{\alpha(2s-2)} = \Gamma_{\alpha(2s-2)}$, and then (4.14) reduces to the transverse action (4.9). On the other hand, we can first consider the equation of motion for $V_{\alpha(2s-2)}$ and its conjugate, which imply

$$V_{\alpha(2s-2)} = -\frac{1}{8}[D^\beta,\bar{D}^\gamma]H_{\beta\gamma\alpha_1\ldots\alpha_{2s-2}} - \frac{\mathrm{i}}{2}s\partial^{\beta\gamma}H_{\beta\gamma\alpha_1\ldots\alpha_{2s-2}} + \frac{2s+1}{4s}G_{\alpha(2s-2)} - \frac{2s-1}{4s}\bar{G}_{\alpha(2s-2)}. \tag{4.16}$$

Using this and the conjugate relation, we can express the action (4.14) in terms of the dynamical variables $H_{\alpha(2s)}$, $G_{\alpha(2s-2)}$ and $\bar{G}_{\alpha(2s-2)}$. The result is

$$\begin{aligned} S^{\parallel}_{s+\frac{1}{2}}[H,G,\bar{G}] &= \left(-\frac{1}{2}\right)^s\int \mathrm{d}^{3|4}z\left\{\frac{1}{8}H^{\alpha(2s)}D^\gamma\bar{D}^2D_\gamma H_{\alpha(2s)} \right.\\ &\quad - \frac{1}{16}([D_{\beta_1},\bar{D}_{\beta_2}]H^{\beta_1\beta_2\alpha(2s-2)})[D^{\gamma_1},\bar{D}^{\gamma_2}]H_{\gamma_1\gamma_2\alpha(2s-2)} \\ &\quad + \frac{s}{2}(\partial_{\beta_1\beta_2}H^{\beta_1\beta_2\alpha(2s-2)})\partial^{\gamma_1\gamma_2}H_{\gamma_1\gamma_2\alpha(2s-2)} \\ &\quad + \mathrm{i}\frac{2s-1}{2s}(G-\bar{G})^{\alpha(2s-2)}\partial^{\beta_1\beta_2}H_{\beta_1\beta_2\alpha(2s-2)} \\ &\quad\left. + \frac{2s-1}{2s^2}G\cdot\bar{G} - \frac{2s+1}{4s^2}(G\cdot G+\bar{G}\cdot\bar{G})\right\}. \end{aligned} \tag{4.17}$$

This action is invariant under the gauge transformations (4.3) and (4.5). It defines the longitudinal formulation of the theory. By construction, the transverse and longitudinal formulations, (4.9) and (4.17), are dual to each other.

Computing the first variational derivatives of the action (4.17) with respect to the prepotentials, we obtain the following gauge-invariant field strengths:

$$\begin{aligned} E^{\parallel}_{\alpha(2s)} &:= \frac{1}{4} D^{\beta}\bar{D}^2 D_{\beta} H_{\alpha(2s)} \\ &\quad - \frac{1}{8}[D_{(\alpha_1}, \bar{D}_{\alpha_2}][D^{\beta_1}, \bar{D}^{\beta_2}] H_{\alpha_3\ldots\alpha_{2s})\beta_1\beta_2} \\ &\quad - s\partial^{\beta(2)}\partial_{(\alpha_1\alpha_2} H_{\alpha_3\ldots\alpha_{2s})\beta(2)} \\ &\quad - \mathrm{i}\frac{2s-1}{2s}\partial_{(\alpha_1\alpha_2}(G-\bar{G})_{\alpha_3\ldots\alpha_{2s})}, \\ B_{\alpha(2s-3)} &:= -\mathrm{i}\frac{2s-1}{2s}\bar{D}^{\gamma}\partial^{\beta(2)} H_{\alpha(2s-3)\beta(2)\gamma} \\ &\quad + \frac{1}{s}\frac{2s-1}{2s}\bar{D}^{\gamma}(G-\bar{G})_{\gamma\alpha(2s-3)}. \end{aligned} \tag{4.18}$$

They are related to each other by the Bianchi identity

$$\bar{D}^{\beta} E^{\parallel}_{\beta\alpha(2s-1)} = \frac{1}{2} D_{(\alpha_1}\bar{D}_{\alpha_2} B_{\alpha_3\ldots\alpha_{2s-1})} - \mathrm{i}(s-1)\partial_{(\alpha_1\alpha_2} B_{\alpha_3\ldots\alpha_{2s-1})}. \tag{4.19}$$

The equations of motion are

$$E^{\parallel}_{\alpha(2s)} = 0, \qquad B_{\alpha(2s-3)} = 0. \tag{4.20}$$

C. Linearized supergravity models

For the case $s=1$, the longitudinal action (4.17) takes the form

$$\begin{aligned} S^{\parallel}_{3/2}[H,G,\bar{G}] = -\frac{1}{2}\int \mathrm{d}^{3|4}z \bigg\{ &\frac{1}{8} H^{\alpha\beta} D^{\gamma}\bar{D}^2 D_{\gamma} H_{\alpha\beta} \\ &- \frac{1}{16}([D_{\alpha},\bar{D}_{\beta}]H^{\alpha\beta})^2 + \frac{1}{2}(\partial_{\alpha\beta}H^{\alpha\beta})^2 \\ &+ \frac{\mathrm{i}}{2}(G-\bar{G})\partial_{\alpha\beta}H^{\alpha\beta} + \frac{1}{2}G\bar{G} \bigg\}, \end{aligned} \tag{4.21}$$

where the compensator G is chiral, $\bar{D}_{\alpha}G = 0$. This action proves to coincide with the linearized action for type I supergravity [13] upon rescaling $3G = \sigma$. The action is invariant under the gauge transformations

$$\delta H_{\alpha\beta} = g_{\alpha\beta} + \bar{g}_{\alpha\beta} = \bar{D}_{(\alpha} L_{\beta)} - D_{(\alpha}\bar{L}_{\beta)}, \tag{4.22a}$$

$$\delta G = \frac{1}{3} D^{\alpha}\bar{D}^{\beta} g_{\alpha\beta} + \mathrm{i}\partial^{\beta(2)} g_{\beta(2)} = -\frac{1}{4}\bar{D}^2 D^{\beta} L_{\beta}, \tag{4.22b}$$

where $g_{\alpha\beta} = \bar{D}_{(\alpha}L_{\beta)}$ and the spinor gauge parameter L_{α} is an unconstrained complex superfield.

Varying the action (4.21) with respect to the gravitational superfield $H^{\alpha\beta}$, one obtains the gauge-invariant field strength

$$\begin{aligned} E^{\parallel}_{\alpha\beta} &= \frac{1}{4} D^{\gamma}\bar{D}^2 D_{\gamma} H_{\alpha\beta} - \frac{1}{8}[D_{(\alpha}\bar{D}_{\beta)}][D^{\gamma},\bar{D}^{\delta}] H_{\gamma\delta} \\ &\quad - \partial_{\alpha\beta}\partial^{\gamma\delta} H_{\gamma\delta} - \frac{\mathrm{i}}{2}\partial_{\alpha\beta}(G-\bar{G}). \end{aligned} \tag{4.23}$$

Since every chiral or antichiral superfield is annihilated by the operator Δ, Eq. (2.11), from (4.23) we derive the descendant

$$\begin{aligned} W_{\alpha\beta}(H) := -\Delta E^{\parallel}_{\alpha\beta} = \Delta\bigg\{ &2\Delta^2 H_{\alpha\beta} + \frac{1}{8}[D_{(\alpha}\bar{D}_{\beta)}][D^{\gamma},\bar{D}^{\delta}] H_{\gamma\delta} \\ &+ \partial_{\alpha\beta}\partial^{\gamma\delta} H_{\gamma\delta} \bigg\}, \end{aligned} \tag{4.24}$$

which is constructed solely in terms of the gravitational superfield $H_{\alpha\beta}$. The gauge-invariant superfield $W_{\alpha\beta}$ proves to be a linearized form of the $\mathcal{N}=2$ super-Cotton tensor [18,19]. The linearized expression (4.24) was recently given in [46]. Our analysis shows that the gauge-invariant field strength $W_{\alpha\beta}$ naturally follows from the results of the earlier work [13].

In the supergravity framework, the super-Cotton tensor transforms homogeneously under the super-Weyl transformations [18] (see also [20] for a more general supergravity formulation). A direct consequence of this result is that the linearized version of the super-Cotton tensor $W_{\alpha\beta}$, given by Eq. (4.24), is a primary superfield with respect to the superconformal group.

It is an instructive exercise to show that

$$\Delta E^{\parallel}_{\alpha\beta} = -\frac{1}{2}\Delta\{\Box H_{\alpha\beta} + \partial_{\alpha}{}^{\gamma}\partial_{\beta}{}^{\delta} H_{\gamma\delta} + 2\Delta\partial^{\gamma}{}_{(\alpha}H_{\beta)\gamma}\}. \tag{4.25}$$

Using this relation gives an alternative expression for the field strength (4.24).

Direct calculations show that $W_{\alpha\beta}$ is transverse linear,

$$\bar{D}^{\beta} W_{\alpha\beta} = D^{\beta} W_{\alpha\beta} = 0. \tag{4.26}$$

This relation is a linearized form of the Bianchi identity for the $\mathcal{N}=2$ super-Cotton tensor [20]. It follows from (4.26) that the functional

$$S_{\rm CSG} = \int \mathrm{d}^{3|4}z H^{\alpha\beta} W_{\alpha\beta}(H) \tag{4.27}$$

is invariant under the gauge transformation (4.22a). This functional is a linearized version [46] of the $\mathcal{N}=2$ conformal supergravity action [30,31].

Let us now look at the transverse formulation for the $s=1$ case. It is given by the following action:

$$S^{\perp}_{3/2}[H,\Gamma,\bar{\Gamma}] = -\frac{1}{2}\int \mathrm{d}^{3|4}z\bigg\{\frac{1}{8}H^{\alpha\beta}D^{\gamma}\bar{D}^2D_{\gamma}H_{\alpha\beta} + H^{\alpha\beta}(D_{\alpha}\bar{D}_{\beta}\Gamma - \bar{D}_{\alpha}D_{\beta}\bar{\Gamma}) + \bar{\Gamma}\Gamma + \frac{3}{2}(\Gamma^2+\bar{\Gamma}^2)\bigg\}, \tag{4.28}$$

which is invariant under the following gauge transformations:

$$\delta H_{\alpha\beta} \coloneqq g_{\alpha\beta} + \bar{g}_{\alpha\beta} = \bar{D}_{(\alpha}L_{\beta)} - D_{(\alpha}\bar{L}_{\beta)}, \tag{4.29a}$$

$$\delta\Gamma = \frac{1}{3}\bar{D}^{\alpha}D^{\beta}\bar{g}_{\alpha\beta} = -\frac{1}{4}\bar{D}^{\beta}D^2\bar{L}_{\beta}. \tag{4.29b}$$

The functional (4.28) coincides with the linearized action for $w=-1$ nonminimal $\mathcal{N}=2$ supergravity [13].

Associated with the action (4.28) are the gauge-invariant field strengths

$$E^{\perp}_{\alpha\beta} = \frac{1}{4}D^{\gamma}\bar{D}^2D_{\gamma}H_{\alpha\beta} + D_{(\alpha}\bar{D}_{\beta)}\Gamma - \bar{D}_{(\alpha}D_{\beta)}\bar{\Gamma}, \tag{4.30a}$$

$$F_{\alpha} = -\frac{1}{2}\bar{D}^2D^{\beta}H_{\alpha\beta} + \bar{D}_{\alpha}(\bar{\Gamma}+3\Gamma), \tag{4.30b}$$

in terms of which the equations of motion are $E^{\perp}_{\alpha\beta}=0$ and $F_{\alpha}=0$. The linearized super-Cotton tensor (4.24) can be expressed in terms of the field strengths (4.30) as follows:

$$W_{\alpha\beta} = \frac{1}{2}\Delta E^{\perp}_{\alpha\beta} + \frac{\mathrm{i}}{32}[D_{(\alpha},\bar{D}_{\beta)}](D^{\gamma}F_{\gamma}+\bar{D}^{\gamma}\bar{F}_{\gamma}) - \frac{1}{8}\partial_{\alpha\beta}(D^{\gamma}F_{\gamma}-\bar{D}^{\gamma}\bar{F}_{\gamma}). \tag{4.31}$$

V. COMPONENT ANALYSIS

The linearized gauge transformations (4.3)–(4.5) make use of the longitudinal linear parameter $g_{\alpha(2s)}$, given by Eq. (4.6), and its conjugate $\bar{g}_{\alpha(2s)}$. The most general expression for $g_{\alpha(2s)}$ as a power series in the Grassmann variables θ and $\bar{\theta}$, is

$$g_{\alpha(2s)}(\theta,\bar{\theta}) = \mathrm{e}^{\mathrm{i}\mathcal{H}_0}\{g_{\alpha_1\ldots\alpha_{2s}} + \bar{\theta}_{(\alpha_1}\xi_{\alpha_2\ldots\alpha_{2s})} + \theta^{\beta}v_{\alpha_1\ldots\alpha_{2s},\beta} + \theta^2 f_{\alpha_1\ldots\alpha_{2s}} + \theta^{\beta}\bar{\theta}_{(\alpha_1}\lambda_{\alpha_2\ldots\alpha_{2s}),\beta} + \theta^2\bar{\theta}_{(\alpha_1}\Sigma_{\alpha_2\ldots\alpha_{2s})}\}, \tag{5.1}$$

where

$$\mathcal{H}_0 \coloneqq \theta^{\alpha}(\gamma^m)_{\alpha\beta}\bar{\theta}^{\beta}\partial_m = \theta^{\alpha}\bar{\theta}^{\beta}\partial_{\alpha\beta} \equiv \rho^{\alpha\beta}\partial_{\alpha\beta}, \qquad \rho^{\alpha\beta} \coloneqq \theta^{(\alpha}\bar{\theta}^{\beta)}. \tag{5.2}$$

All component fields in (5.1) are complex and symmetric in their α-indices. The components $\Upsilon_{\alpha_1\ldots\alpha_{2s},\beta}$ and $\lambda_{\alpha_1\ldots\alpha_{2s-1},\beta}$ are not required to have any symmetry property relating their α and β indices, which is indicated by a coma. In other words, $\Upsilon_{\alpha(2s),\beta}$ belongs to the tensor product $(\mathbf{2s}+\mathbf{1})\otimes\mathbf{2}$ of two $\mathsf{SL}(2,\mathbb{R})$ representations.

As follows from the gauge transformation (4.3), the component gauge parameters $g_{\alpha(2s)}$, $\Upsilon_{\alpha(2s),\beta}$ and $f_{\alpha(2s)}$ in (5.1) can be used to choose a Wess-Zumino gauge of the form

$$H_{\alpha_1\ldots\alpha_{2s}}(\theta,\bar{\theta}) = \mathrm{i}\theta\bar{\theta}D_{\alpha_1\ldots\alpha_{2s}} + \theta^{(\beta}\bar{\theta}^{\gamma)}E_{\alpha_1\ldots\alpha_{2s},\beta\gamma} + \bar{\theta}^2\theta^{\beta}\Psi_{\alpha_1\ldots\alpha_{2s},\beta} - \theta^2\bar{\theta}^{\beta}\bar{\Psi}_{\alpha_1\ldots\alpha_{2s},\beta} + \theta^2\bar{\theta}^2A_{\alpha_1\ldots\alpha_{2s}}, \tag{5.3}$$

where the composite scalar $\theta\bar{\theta}=\theta^{\alpha}\bar{\theta}_{\alpha}$ is imaginary. All bosonic fields in (5.3) are real. So far no gauge condition has been imposed on $\Gamma_{\alpha_1\ldots\alpha_{2s-2}}$. To preserve the gauge condition (5.3), some of the gauge parameters contained in (5.1) must be constrained as follows:

$$g_{\alpha_1\ldots\alpha_{2s}} = -\frac{\mathrm{i}}{2}\zeta_{\alpha_1\ldots\alpha_{2s}}, \qquad \bar{\zeta}_{\alpha(2s)} = \zeta_{\alpha(2s)}, \tag{5.4a}$$

$$f_{\alpha_1\ldots\alpha_{2s}} = 0, \tag{5.4b}$$

$$v_{\alpha_1\ldots\alpha_{2s},\beta} = -\varepsilon_{\beta(\alpha_1}\bar{\xi}_{\alpha_2\ldots\alpha_{2s})}. \tag{5.4c}$$

The first term in the third line of (5.1) can be represented as

$$\theta^{\beta}\bar{\theta}_{(\alpha_1}\lambda_{\alpha_2\ldots\alpha_{2s}),\beta} = \frac{1}{2}\theta\bar{\theta}\Lambda_{\alpha_1\ldots\alpha_{2s}} + \frac{2s+1}{2s}\rho^{\beta}{}_{(\alpha_1}\Lambda_{\alpha_2\ldots\alpha_{2s}\beta)} - \frac{2s-1}{2s}\rho_{(\alpha_1\alpha_2}\Lambda_{\alpha_3\ldots\alpha_{2s})}, \tag{5.5}$$

where we have introduced two irreducible components of $\lambda_{\alpha_1\ldots\alpha_{2s-1},\beta}$ by the rule

$$\Lambda_{\alpha_1\ldots\alpha_{2s}} \coloneqq \lambda_{(\alpha_1\ldots\alpha_{2s-1},\alpha_{2s})}, \qquad \Lambda_{\alpha_2\ldots\alpha_{2s-2}} \coloneqq \lambda_{\alpha_2\ldots\alpha_{2s-2}\beta,}{}^{\beta}. \tag{5.6}$$

We recall that the composite $\rho^{\alpha\beta}$ is defined by (5.2). It is clear from (4.3), (5.1), (5.3), and (5.5) that the imaginary part of $\Lambda_{\alpha(2s)}$ can be used to gauge away the component field $D_{\alpha(2s)}$ thus arriving at the stronger Wess-Zumino gauge

$$H_{\alpha_1\ldots\alpha_{2s}}(\theta,\bar{\theta}) = \theta^{(\beta}\bar{\theta}^{\gamma)}E_{\alpha_1\ldots\alpha_{2s},\beta\gamma} + \bar{\theta}^2\theta^{\beta}\Psi_{\alpha_1\ldots\alpha_{2s},\beta} - \theta^2\bar{\theta}^{\beta}\bar{\Psi}_{\alpha_1\ldots\alpha_{2s},\beta} + \theta^2\bar{\theta}^2A_{\alpha_1\ldots\alpha_{2s}}, \tag{5.7}$$

in which the residual Λ-invariance is described by a real parameter,

$$\bar{\Lambda}_{\alpha(2s)} = \Lambda_{\alpha(2s)} \equiv l_{\alpha(2s)}. \quad (5.8)$$

The real bosonic field $E_{\alpha_1\ldots\alpha_{2s},\beta\gamma}$ transforms in the representation $(\mathbf{2s+1})\otimes\mathbf{3}$ of $\mathsf{SL}(2,\mathbb{R})$, while the complex fermionic field $\Psi_{\alpha_1\ldots\alpha_{2s},\beta}$ belongs to the $(\mathbf{2s+1})\otimes\mathbf{2}$. The field $E_{\alpha(2s),\beta\gamma}$ is a higher spin analog of the linearized vielbein (or frame field), which becomes obvious if we convert the spinor indies of $E_{\alpha(2s),\beta\gamma}$ into vector ones by the standard rule

$$E_m{}^{a_1\ldots a_s} := \left(-\frac{1}{2}\right)^{s+1}(\gamma^{a_1})^{\alpha_1\alpha_2}\ldots(\gamma^{a_s})^{\alpha_{2s-1}\alpha_{2s}}(\gamma_m)^{\beta\gamma}E_{\alpha_1\ldots\alpha_{2s},\beta\gamma}. \quad (5.9)$$

Here $E_m{}^{a_1\ldots a_s}$ is symmetric and traceless with respect to the indices $a_1,\ldots,a_s$. This interpretation is confirmed by the fact that the gauge transformation associated with the parameter (5.4a) acts on $E_{\alpha_1\ldots\alpha_{2s},\beta\gamma}$ as follows:

$$\delta E_{\alpha_1\ldots\alpha_{2s},\beta\gamma} = \partial_{\beta\gamma}\zeta_{\alpha_1\ldots\alpha_{2s}} \Leftrightarrow \delta E_m{}^{a_1\ldots a_s} = \partial_m\zeta^{a_1\ldots a_s}. \quad (5.10)$$

The gauge transformation generated by the parameter (5.8) acts on $E_{\alpha_1\ldots\alpha_{2s},\beta\gamma}$ as a higher spin counterpart of the linearized local Lorentz transformation. Therefore the tensor structure of $E_m{}^{a\ldots a_s}$ and its gauge freedom correspond to the 3D massless spin-$(s+1)$ gauge field, see e.g., [47]. In the framelike formulation for massless higher spin fields [47], one introduces two independent gauge fields, one of which is $E_m{}^{a\ldots a_s}$ and the other is a higher spin analogue of the Lorentz connection. The latter is expressed in terms of $E_m{}^{a\ldots a_s}$ on the equations of motion. However, in the off-shell formulations for supergravity, no independent Lorentz connection appears. And its higher spin analog never appears in the framework of the off-shell higher spin supermultiplets introduced in [5–7].

Recalling the gauge transformation law of $\Gamma_{\alpha(2s-2)}$, Eq. (4.4), one may see that the gauge freedom associated with the parameters $\Lambda_{\alpha(2s-2)}$ in (5.5) and $\Sigma_{\alpha(2s-1)}$ in (5.1) allows us to bring $\Gamma_{\alpha(2s-2)}$ to the following form:

$$\begin{aligned}\Gamma_{\alpha_1\ldots\alpha_{2s-2}}(\theta,\bar\theta) = {}& \mathrm{e}^{\mathrm{i}\theta^\lambda\bar\theta^\rho\partial_{\lambda\rho}}[\theta^\beta\omega_{(\beta\alpha_1\ldots\alpha_{2s-2})} + \theta_{(\alpha_1}\bar\Psi_{\alpha_2\ldots\alpha_{2s-2})} \\ & + \theta^2 B_{\alpha_1\ldots\alpha_{2s-2}} + \bar\theta^\beta\theta^\gamma U_{(\beta\gamma\alpha_1\ldots\alpha_{2s-2})} \\ & + \bar\theta^\beta\theta_{(\beta}F_{\alpha_1\ldots\alpha_{2s-2})} + \theta^2\bar\theta^\beta\rho_{(\beta\alpha_1\ldots\alpha_{2s-2})}].\end{aligned} \quad (5.11)$$

The fermionic fields $\Psi_{\alpha_1\ldots\alpha_{2s},\beta}$ in (5.7) and $\Psi_{\alpha_1\ldots\alpha_{2s-3}}$ appearing in $\bar\Gamma_{\alpha_1\ldots\alpha_{2s-2}}$ constitute a complex version of the massless spin-$(s+\frac{1}{2})$ field reviewed in Appendix B 2. The complex fermionic fields $\omega_{\alpha(2s-1)}$ and $\rho_{\alpha(2s-1)}$ in (5.11) turn out to be auxiliary for the theory with action (4.9) in the standard sense that they become functions of the other fermionic fields on the mass shell. The real bosonic field $A_{\alpha(2s)}$ in (5.7) and the complex bosonic fields $B_{\alpha(2s-2)}$, $U_{\alpha(2s)}$ and $F_{\alpha(2s-2)}$ in (5.11) are auxiliary for the theory with action (4.9).

Now we can argue that, upon elimination of the auxiliary fields, the theory with action (4.9) is equivalent to a sum of two massless (Fang-)Fronsdal models, one of which is the bosonic spin-$(s+1)$ model described in Appendix (B1) and the other corresponds to two identical fermionic spin-$(s+1/2)$ models described in Appendix (B2). Equivalently, the fermionic sector describes a complex massless spin-$(s+1/2)$ gauge field. Indeed, consider the frame field in (5.7). It can be represented as the sum of three irreducible components,

$$\begin{aligned}\rho^{\beta\gamma}E_{\alpha_1\ldots\alpha_{2s},\beta\gamma} = {}& \rho^{\beta\gamma}h_{\alpha_1\ldots\alpha_{2s}\beta\gamma} + \rho^\beta{}_{(\alpha_1}m_{\alpha_2\ldots\alpha_{2s})\beta} \\ & + \rho_{(\alpha_1\alpha_2}h_{\alpha_3\ldots\alpha_{2s})},\end{aligned} \quad (5.12)$$

where the irreducible components of $E_{\alpha_1\ldots\alpha_{2s},\beta\gamma}$ are defined by

$$h_{\alpha_1\ldots\alpha_{2s+2}} := E_{(\alpha_1\ldots\alpha_{2s},\alpha_{2s+1}\alpha_{2s+2})}, \quad (5.13a)$$

$$m_{\alpha_1\ldots\alpha_{2s}} := -\frac{2s+1}{s+1}E_{(\alpha_1\ldots\alpha_{2s},\beta)}{}^\beta, \quad (5.13b)$$

$$h_{\alpha_1\ldots\alpha_{2s-2}} := \frac{2s-1}{2s+1}E_{\alpha_1\ldots\alpha_{2s-2}\beta\gamma,}{}^{\beta\gamma}. \quad (5.13c)$$

The field $m_{\alpha(2s)}$ may be algebraically gauged away by the generalized Lorentz transformation described by the parameter (5.8). The remaining bosonic fields $h_{\alpha(2s+2)}$ and $h_{\alpha(2s-2)}$ correspond to the dynamical variables of the Fronsdal spin-$(s+1)$ model reviewed in Appendix (B1). As follows from (5.10), their gauge freedom is equivalent to that of the massless spin-$(s+1)$ gauge field, see Eqs. (B2). Since the requirement of gauge invariance fixes the Fronsdal action modulo an overall numerical factor, we are confident the theory (4.9) leads to the Fronsdal spin-$(s+1)$ model even without explicit calculation of the component bosonic action. Such a calculation may be carried out in complete analogy with the 4D case described in [7]; it is given in Appendix C. Let us turn to the fermionic sector and represent the higher-spin gravitino $\Psi_{\alpha_1\ldots\alpha_{2s},\beta}$ in (5.7) as the sum of two irreducible components,

$$\Psi_{\alpha_1\ldots\alpha_{2s},\beta} = \Psi_{\alpha_1\ldots\alpha_{2s}\beta} + \varepsilon_{\beta(\alpha_1}\Psi_{\alpha_2\ldots\alpha_{2s})}, \quad (5.14)$$

where the irreducible components are defined by

$$\Psi_{\alpha_1\ldots\alpha_{2s}\beta} := \Psi_{(\alpha_1\ldots\alpha_{2s},\beta)}, \quad (5.15a)$$

$$\Psi_{\alpha_1\ldots\alpha_{2s-1}} := \frac{2s}{2s+1}\Psi_{\alpha_1\ldots\alpha_{2s-1}\beta,}{}^\beta. \quad (5.15b)$$

These complex fermionic fields $\Psi_{\alpha(2s+1)}$, $\Psi_{\alpha(2s-1)}$ as well as $\Psi_{\alpha(2s-3)}$ sitting in $\bar\Gamma_{\alpha(2s-2)}$ constitute a complex version of the massless spin-$(s+\frac{1}{2})$ field reviewed in Appendix (B2). Under the fermionic local symmetry generated by the complex parameter $\xi_{\alpha(2s-1)}$ in (5.1), the gauge transformation law of these fields is equivalent to the complex version of the transformation (B13) which corresponds to the massless spin-$(s+1/2)$ field. Since the requirement of gauge invariance fixes the Fang-Fronsdal action modulo an overall numerical factor, we are confident the theory (4.9) leads to the massless spin-$(s+1/2)$ model even without explicit calculation of the component fermionic action. The explicit calculation is carried out in Appendix C.

Instead of dealing with the gauge (5.7) and (5.11), sometimes it is more convenient to work with an alternative Wess-Zumino gauge defined by

$$\begin{aligned} H_{\alpha_1\ldots\alpha_{2s}}(\theta,\bar\theta) &= \theta^\beta\bar\theta^\gamma h_{(\beta\gamma\alpha_1\ldots\alpha_{2s})} + \bar\theta^2\theta^\beta\Psi_{(\beta\alpha_1\ldots\alpha_{2s})} \\ &\quad - \theta^2\bar\theta^\beta\bar\Psi_{(\beta\alpha_1\ldots\alpha_{2s})} + \theta^2\bar\theta^2 A_{\alpha_1\ldots\alpha_{2s}}, \end{aligned} \tag{5.16a}$$

$$\begin{aligned} \Gamma_{\alpha_1\ldots\alpha_{2s-2}}(\theta,\bar\theta) &= \mathrm{e}^{\mathrm{i}\theta^\lambda\bar\theta^\rho\partial_{\lambda\rho}}[h_{\alpha_1\ldots\alpha_{2s-2}} + \theta^\beta\Psi_{(\beta\alpha_1\ldots\alpha_{2s-2})} \\ &\quad + \theta_{(\alpha_1}\bar\Psi_{\alpha_2\ldots\alpha_{2s-2})} + \bar\theta^\beta\Upsilon_{(\beta\alpha_1\ldots\alpha_{2s-2})} \\ &\quad + \theta^2 B_{\alpha_1\ldots\alpha_{2s-2}} + \bar\theta^\beta\theta^\gamma U_{(\beta\gamma\alpha_1\ldots\alpha_{2s-2})} \\ &\quad + \bar\theta^\beta\theta_{(\beta}F_{\alpha_1\ldots\alpha_{2s-2})} + 2\theta^2\bar\theta^\beta\boldsymbol{\rho}_{(\beta\alpha_1\ldots\alpha_{2s-2})}], \end{aligned} \tag{5.16b}$$

with

$$\begin{aligned} \boldsymbol{\rho}_{\alpha_1\ldots\alpha_{2s-1}} &= \rho_{\alpha_1\ldots\alpha_{2s-1}} - \frac{\mathrm{i}}{4}\partial^\beta{}_{(\alpha_1}\psi_{\alpha_2\ldots\alpha_{2s-1})\beta} \\ &\quad - \frac{\mathrm{i}}{4}\partial_{(\alpha_1\alpha_2}\psi_{\alpha_2\ldots\alpha_{2s-1})}. \end{aligned} \tag{5.16c}$$

Here the bosonic fields $h_{\alpha(2s)}$, $h_{\alpha(2s-2)}$ and $A_{\alpha(2s)}$ are real, and the fields $B_{\alpha(2s-2)}$, $U_{\alpha(2s)}$ and $F_{\alpha(2s-2)}$ are complex.

VI. SUPERCONFORMAL HIGHER SPIN MULTIPLETS

In this section we develop a superspace setting for linearized higher spin conformal supergravity. We start with a review of the conformal Killing supervector fields of 3D $\mathcal{N}=2$ Minkowski superspace [48,49], which are defined in complete analogy with the 4D $\mathcal{N}=1$ case [11].

A. Conformal Killing supervector fields

Consider a real supervector field ξ on Minkowski superspace,

$$\begin{aligned} \xi = \xi^B D_B &\coloneqq \xi^b\partial_b + \xi^\beta D_\beta + \bar\xi_\beta\bar D^\beta \\ &= -\frac{1}{2}\xi^{\beta\gamma}\partial_{\beta\gamma} + \xi^\beta D_\beta + \bar\xi_\beta\bar D^\beta. \end{aligned} \tag{6.1}$$

It is called a conformal Killing supervector field if it obeys the equation

$$\begin{aligned} &[\xi + K^{\beta\gamma}M_{\beta\gamma}, D_\alpha] + \delta_\rho D_\alpha = 0 \Leftrightarrow \\ &[\xi + K^{\beta\gamma}M_{\beta\gamma}, \bar D_\alpha] + \delta_\rho \bar D_\alpha = 0, \end{aligned} \tag{6.2}$$

for some Lorentz ($K^{\beta\gamma} = K^{\gamma\beta} = \bar K^{\beta\gamma}$) and super-Weyl ($\rho$) parameters. We recall that the Lorentz generator $M_{\beta\gamma}$ acts on a spinor ψ_α by the rule

$$M_{\beta\gamma}\psi_\alpha = \varepsilon_{\alpha(\beta}\psi_{\gamma)} = \frac{1}{2}(\varepsilon_{\alpha\beta}\psi_\gamma + \varepsilon_{\alpha\gamma}\psi_\beta). \tag{6.3}$$

The super-Weyl transformation of the covariant derivatives is defined according to [13]

$$\delta_\rho D_\alpha = \frac{1}{2}(3\bar\rho - \rho)D_\alpha + (D^\lambda\rho)M_{\lambda\alpha}, \tag{6.4a}$$

$$\delta_\rho \bar D_\alpha = \frac{1}{2}(3\rho - \bar\rho)\bar D_\alpha + (\bar D^\lambda\bar\rho)M_{\lambda\alpha}, \tag{6.4b}$$

where the parameter ρ is chiral,

$$\bar D_\alpha\rho = 0. \tag{6.4c}$$

Equation (6.2) can be rewritten in the form [49]

$$[\xi, D_\alpha] = -K_\alpha{}^\beta D_\beta + \frac{1}{2}(\rho - 3\bar\rho)D_\alpha. \tag{6.5}$$

The equation (6.2), or its equivalent form (6.5), implies

$$D_\alpha\xi^{\beta\gamma} + 4\mathrm{i}\delta_\alpha^{(\beta}\bar\xi^{\gamma)} = 0 \Leftrightarrow \bar D_\alpha\xi^{\beta\gamma} - 4\mathrm{i}\delta_\alpha^{(\beta}\xi^{\gamma)} = 0, \tag{6.6}$$

and therefore the spinor components ξ^α and $\bar\xi_\alpha$ of ξ are determined in terms of the vector ones,

$$\xi^\alpha = -\frac{\mathrm{i}}{6}\bar D_\beta\xi^{\alpha\beta}, \qquad \bar\xi_\alpha = -\frac{\mathrm{i}}{6}D^\beta\xi_{\alpha\beta}, \tag{6.7}$$

and the vector component $\xi^{\alpha\beta} = \xi^{\beta\alpha} = \bar\xi^{\alpha\beta}$ is longitudinal linear,

$$D^{(\alpha}\xi^{\beta\gamma)} = 0, \qquad \bar D^{(\alpha}\xi^{\beta\gamma)} = 0, \tag{6.8}$$

and therefore $\xi^{\alpha\beta}$ is an ordinary conformal Killing vector,

$$\partial^{(\alpha\beta}\xi^{\gamma\delta)} = 0. \tag{6.9}$$

These relations imply that $D^2\xi^{\alpha\beta} = \bar D^2\xi^{\alpha\beta} = 0$, and therefore ξ^α is chiral,

$$\bar D_\alpha\xi^\beta = 0. \tag{6.10}$$

It follows from (6.2), or its equivalent form (6.5), that the Lorentz and super-Weyl parameters, $K_{\alpha\beta}$ and ρ, are uniquely expressed in terms of the components of the conformal Killing supervector field as follows:

$$K_{\alpha\beta}=D_{(\alpha}\xi_{\beta)}=-\bar{D}_{(\alpha}\bar{\xi}_{\beta)}, \tag{6.11a}$$

$$\rho=\frac{1}{8}(D_\alpha\xi^\alpha+3\bar{D}^\alpha\bar{\xi}_\alpha). \tag{6.11b}$$

We also deduce from (6.2) that the Lorentz and super-Weyl parameters are related to each other as

$$D_\alpha K^{\beta\gamma}=\delta_\alpha{}^{(\beta}D^{\gamma)}\rho. \tag{6.12}$$

Using the properties (6.5)–(6.10), one can explicitly check that ρ defined by (6.11b) is chiral.

B. Primary linear superfields

A symmetric rank-n spinor superfield $\Phi_{\alpha_1\ldots\alpha_n}$ is said to be primary of dimension $\frac{1}{2}(x+y)$ if its superconformal transformation is

$$\delta_\xi\Phi_{\alpha_1\ldots\alpha_n}=\xi\Phi_{\alpha_1\ldots\alpha_n}+nK^\beta{}_{(\alpha_1}\Phi_{\alpha_2\ldots\alpha_n)\beta}+(x\rho+y\bar{\rho})\Phi_{\alpha_1\ldots\alpha_n} \tag{6.13}$$

for some real parameters x and y. The R-charge of $\Phi_{\alpha(n)}$ is proportional to $\frac{1}{2}(x-y)$.

Let $G_{\alpha_1\ldots\alpha_n}$ be a longitudinal linear superfield constrained by (2.1b). Requiring $G_{\alpha_1\ldots\alpha_n}$ to be primary fixes one of the superconformal parameters in (6.13),

$$\delta_\xi G_{\alpha_1\ldots\alpha_n}=\xi G_{\alpha_1\ldots\alpha_n}+nK^\beta{}_{(\alpha_1}G_{\alpha_2\ldots\alpha_n)\beta}+\left(x\rho-\frac{n}{2}\bar{\rho}\right)G_{\alpha_1\ldots\alpha_n}. \tag{6.14}$$

Let $\Gamma_{\alpha_1\ldots\alpha_n}$ be a transverse linear superfield constrained by (2.1a). Requiring $\Gamma_{\alpha_1\ldots\alpha_n}$ to be primary fixes one of the superconformal parameters in (6.13),

$$\delta_\xi\Gamma_{\alpha_1\ldots\alpha_n}=\xi\Gamma_{\alpha_1\ldots\alpha_n}+nK^\beta{}_{(\alpha_1}\Gamma_{\alpha_2\ldots\alpha_n)\beta}+\left(x\rho+\left(1+\frac{n}{2}\right)\bar{\rho}\right)\Gamma_{\alpha_1\ldots\alpha_n}. \tag{6.15}$$

An analysis of constrained primary superfields was given by Park [48].

Now, let us come back to the gauge transformation law (4.3). We postulate that the real gauge prepotential $H_{\alpha(2s)}$ and the right-hand side of (4.3) are primary. Then it follows from (6.14) that the superconformal transformation of $H_{\alpha(2s)}$ is

$$\delta H_{\alpha_1\ldots\alpha_{2s}}=\xi H_{\alpha_1\ldots\alpha_{2s}}+2sK^\beta{}_{(\alpha_1}H_{\alpha_2\ldots\alpha_{2s})\beta}-s(\rho+\bar{\rho})H_{\alpha_1\ldots\alpha_{2s}}, \tag{6.16}$$

and the dimension of $H_{\alpha(2s)}$ is equal to $(-s)$.

Let $W_{\alpha_1\ldots\alpha_n}$ be a real transverse linear superfield,

$$D^\beta W_{\beta\alpha_1\ldots\alpha_{n-1}}=\bar{D}^\beta W_{\beta\alpha_1\ldots\alpha_{n-1}}=0. \tag{6.17}$$

Requiring it to be primary, we deduce from (6.15) that the superconformal transformation of $W_{\alpha(n)}$ is

$$\delta W_{\alpha_1\ldots\alpha_n}=\xi W_{\alpha_1\ldots\alpha_n}+nK^\beta{}_{(\alpha_1}W_{\alpha_2\ldots\alpha_n)\beta}+\left(1+\frac{n}{2}\right)(\rho+\bar{\rho})W_{\alpha_1\ldots\alpha_n}, \tag{6.18}$$

and the dimension of $W_{\alpha(n)}$ is equal to $(1+n/2)$.

C. Linearized higher spin conformal supergravity

Consider an action of the form

$$S=\mathrm{i}^n\int\mathrm{d}^{3|4}z H^{\alpha_1\ldots\alpha_n}W_{\alpha_1\ldots\alpha_n}, \tag{6.19}$$

where $H_{\alpha(n)}$ is a real symmetric rank-n spinor superfield with the superconformal transformation

$$\delta H_{\alpha_1\ldots\alpha_n}=\xi H_{\alpha_1\ldots\alpha_n}+K^\beta{}_{(\alpha_1}H_{\alpha_2\ldots\alpha_n)\beta}-\frac{n}{2}(\rho+\bar{\rho})H_{\alpha_1\ldots\alpha_n}, \tag{6.20}$$

which coincides with (6.16) for $n=2s$. The action (6.19) is invariant under the superconformal transformations (6.18) and (6.20). Moreover, it is also invariant under gauge transformations of the form

$$\delta H_{\alpha(n)}=g_{\alpha(n)}+\bar{g}_{\alpha(n)},\qquad\delta W_{\alpha_1\ldots\alpha_n}=0,$$
$$g_{\alpha_1\ldots\alpha_n}=\bar{D}_{(\alpha_1}L_{\alpha_2\ldots\alpha_n)}, \tag{6.21}$$

where the complex gauge parameter $g_{\alpha(n)}$ is an arbitrary longitudinal linear superfield. The gauge invariance follows from (6.17). This gauge transformation law reduces to (4.3) for $n=2s$. We would like to realize $W_{\alpha(n)}$ as a gauge-invariant field strength, $W_{\alpha(n)}(H)$, constructed from the prepotential $H_{\alpha(n)}$. Then (6.19) may be interpreted as a higher spin extension of the linearized conformal supergravity action (4.27).

Given a prepotential $H_{\alpha(n)}=H_{\alpha_1\ldots\alpha_n}$ with the superconformal transformation law (6.16) and the gauge transformation (6.21), we associate with it a gauge-invariant real field strength $W_{\alpha(n)}(H)$ defined by

$$W_{\alpha_1\ldots\alpha_n}(H) := \frac{1}{2^{n-1}}\sum_{J=0}^{\lfloor n/2\rfloor}\Big\{\binom{n}{2J}\Delta\Box^J\partial_{(\alpha_1}{}^{\beta_1}\ldots\partial_{\alpha_{n-2J}}{}^{\beta_{n-2J}}H_{\alpha_{n-2J+1}\ldots\alpha_n)\beta_1\ldots\beta_{n-2J}} + \binom{n}{2J+1}\Delta^2\Box^J\partial_{(\alpha_1}{}^{\beta_1}\ldots\partial_{\alpha_{n-2J-1}}{}^{\beta_{n-2J-1}}H_{\alpha_{n-2J}\ldots\alpha_n)\beta_1\ldots\beta_{n-2J-1}}\Big\}, \tag{6.22}$$

where $\lfloor x\rfloor$ denotes the floor (also known as the integer part) of a number x. One may check that

$$\int d^{3|4}z\tilde{H}^{\alpha_1\ldots\alpha_n}W_{\alpha_1\ldots\alpha_n}(H) = \int d^{3|4}zH^{\alpha_1\ldots\alpha_n}W_{\alpha_1\ldots\alpha_n}(\tilde{H}), \tag{6.23}$$

for arbitrary superfields $H_{\alpha(n)}$ and $\tilde{H}_{\alpha(n)}$ that are bosonic for even n and fermionic for odd n. The field strength obeys the Bianchi identities

$$D^\beta W_{\beta\alpha_1\ldots\alpha_{n-1}} = 0, \qquad \bar{D}^\beta W_{\beta\alpha_1\ldots\alpha_{n-1}} = 0. \tag{6.24}$$

These Bianchi identities are compatible only with the superconformal transformation law (6.18), and thus $W_{\alpha(n)}$ is a primary superfield. In Appendix D, we prove (i) invariance of the field strength (6.22) under the gauge transformation (6.21); and (ii) the Bianchi identies (6.24).

In the case of the half-integer superspin transverse formulation, the superconformal field strength $W_{\alpha(2s)}$ can be expressed in terms of the gauge-invariant field strengths (4.11) as follows:

$$2^{2s}W_{\alpha(2s)} = \frac{1}{2}\partial_{(\alpha_1}{}^{\beta_1}\ldots\partial_{\alpha_{2s-2}}{}^{\beta_{2s-2}}\partial_{\alpha_{2s-1}\alpha_{2s})}(D^\gamma F_{\gamma\beta(2s-2)} - \bar{D}^\gamma\bar{F}_{\gamma\alpha(2s-2)}) - \frac{1}{2}\binom{2s}{2}\partial_{(\alpha_1}{}^{\beta_1}\ldots\partial_{\alpha_{2s-2}}{}^{\beta_{2s-2}}E^\perp_{\alpha_{2s-1}\alpha_{2s})\beta(2s-2)}$$
$$- \sum_{J=1}^{s}\binom{2s}{2J}\Delta\Box^{J-1}\partial_{(\alpha_1}{}^{\beta_1}\ldots\partial_{\alpha_{2s-2J}}{}^{\beta_{2s-2J}}E^\perp_{\alpha_{2s-2J+1}\ldots\alpha_{2s})\beta(2s-2)}$$
$$- 2\sum_{J=0}^{s-1}\binom{2s}{2J+1}\partial_{(\alpha_1}{}^{\beta_1}\ldots\partial_{\alpha_{2s-2J-1}}{}^{\beta_{2s-2J-1}}\Box^J E^\perp_{\alpha_{2s-2J}\ldots\alpha_{2s})\beta(2s-2J-1)}. \tag{6.25}$$

This gauge-invariant field strength is a higher spin extension of the linearized super-Cotton tensor (4.24).

VII. MASSIVE HALF-INTEGER SUPERSPIN MODELS

We now consider a gauge-invariant deformation of the transverse action (4.9)

$$S^\perp = \mu^{2s-1}S^\perp_{s+\frac{1}{2}}[H,\Gamma,\bar{\Gamma}] + \frac{\lambda}{2}S_{\mathrm{CS}}[H], \tag{7.1}$$

where $S_{\mathrm{CS}}[H]$ denotes the Chern-Simons-type superconformal term

$$S_{\mathrm{CS}}[H] = \left(-\frac{1}{2}\right)^s\int d^{3|4}zH^{\alpha(2s)}W_{\alpha(2s)}(H), \tag{7.2}$$

with the field strength $W_{\alpha(2s)}(H)$ given by (6.22). The coupling constant λ in (7.1) is dimensionless. In accordance with (6.16), the dimension of $H_{\alpha(2s)}$ is equal to $(-s)$. To make the action dimensionless, the first term in (7.1) is rescaled by an overall factor μ^{2s-1} with the positive parameter μ of unit mass dimension.

In the $s=1$ case, the action (7.1) coincides with the linearized action for topologically massive $\mathcal{N}=2$ supergravity in the nonminimal $w=-1$ formulation [26].

Since $S_{\mathrm{CS}}[H]$ does not involve the compensating superfield $\Gamma_{\alpha(2s-2)}$ and its conjugate, it follows that the corresponding equations of motion are the same for both the massive (7.1) and massless theories (4.9):

$$F_{\alpha(2s-1)} = 0 \Rightarrow D^\beta E^\perp_{\beta\alpha(2s-1)} = 0. \tag{7.3}$$

However, the addition of the Chern-Simons term results in the following modification to the $H_{\alpha(2s)}$ equation of motion for the massive theory:

$$\mu^{2s-1}E^\perp_{\alpha(2s)} + \lambda W_{\alpha(2s)} = 0. \tag{7.4}$$

This is a gauge-invariant higher-derivative superfield equation.[9]

[9]As demonstrated in [42], the general second-order 3D massive field equations for positive integer spin, and their "self-dual" limit to first-order equations, are equivalent to gauge-invariant higher-derivative equations.

Once the equation of motion (7.3) holds, one can obtain a simplified expression for $W_{\alpha(2s)}$. It is

$$W_{\alpha(2s)} = -\Delta\Box^{s-1}E^{\perp}_{\alpha(2s)}. \tag{7.5}$$

Using this result, we can extract a "higher-superspin" analog of the Klein-Gordon equation from the equation of motion (7.4) as follows. First note that

$$\begin{aligned}0 &= m^{2s-1}E^{\perp}_{\alpha(2s)} + \lambda W_{\alpha(2s)} = \mu^{2s-1}E^{\perp}_{\alpha(2s)} - \lambda\Delta\Box^{s-1}E^{\perp}_{\alpha(2s)} \Rightarrow \\ 0 &= \mu^{2s-1}\Delta E^{\perp}_{\alpha(2s)} - 2\lambda\Box^{s-1}\Delta^2 E^{\perp}_{\alpha(2s)} = (\mu^{2s-1}\Delta - 2\lambda\Box^{s})E^{\perp}_{\alpha(2s)}.\end{aligned} \tag{7.6}$$

This leads to

$$(\Box^{2s-1} - (m^2)^{2s-1})E^{\perp}_{\alpha(2s)} = 0, \qquad m := \frac{\mu}{|\lambda|^{1/(2s-1)}}. \tag{7.7}$$

By making use of the Fourier transform of $E^{\perp}_{\alpha(2s)}$, we deduce from (7.7) the ordinary Klein-Gordon equation

$$(\Box - m^2)E^{\perp}_{\alpha(2s)} = 0. \tag{7.8}$$

It then follows from (7.4) that

$$\Delta E^{\perp}_{\alpha(2s)} = m\sigma E^{\perp}_{\alpha(2s)}, \qquad \sigma = \frac{\lambda}{|\lambda|}. \tag{7.9a}$$

It remains to recall that

$$D^{\beta}E^{\perp}_{\beta\alpha(2s-1)} = \bar{D}^{\beta}E^{\perp}_{\beta\alpha(2s-1)} = 0. \tag{7.9b}$$

The equations (7.9a) and (7.9b) tell us that $E^{\perp}_{\alpha(2s)}$ is a massive superfield of superhelicity $\kappa = (s+\frac{1}{2})\sigma$, in accordance with the analysis given in Sec. III B.

The massive theory (7.1) possesses a dual formulation. It is described by the action

$$S^{\parallel} = \mu^{2s-1}S^{\parallel}_{s+\frac{1}{2}}[H,G,\bar{G}] + \frac{\lambda}{2}S_{\rm CS}[H], \tag{7.10}$$

with the longitudinal action $S^{\parallel}_{s+\frac{1}{2}}[H,G,\bar{G}]$ given by Eq. (4.17). Since $S_{\rm CS}[H]$ does not involve the compensating superfield $G_{\alpha(2s-2)}$ and its conjugate, the equation of motion for $G_{\alpha(2s-2)}$ is the same as in massless theory, Eq. (4.20). Due to the Bianchi identity (4.19), we obtain

$$B_{\alpha(2s-3)} = 0 \Rightarrow D^{\beta}E^{\parallel}_{\beta\alpha(2s-1)} = 0. \tag{7.11}$$

However, the equation of motion for $H_{\alpha(2s)}$ becomes

$$\mu^{2s-1}E^{\parallel}_{\alpha(2s)} + \lambda W_{\alpha(2s)} = 0. \tag{7.12}$$

This demonstrates that the massive models (7.1) and (7.10) possess equivalent dynamics.

In the $s=1$ case, the action (7.10) coincides with the linearized action for topologically massive $\mathcal{N}=2$ supergravity in the minimal (1,1) formulation [26].

VIII. CONCLUDING COMMENTS

The main results of this paper are as follows. In Sec. IV we constructed the two dually equivalent off-shell formulations for the massless superspin-$(s+1/2)$ multiplet, with $s>1$, as 3D analogs of the off-shell 4D $\mathcal{N}=1$ massless multiplets of half-integer superspin [5]. In Sec. VI we presented the linearized higher spin super-Cotton tensors and the linearized actions for higher spin conformal supergravity. In Sec. VII we constructed the off-shell formulation for massive superspin-$(s+1/2)$ multiplets, with $s>1$, as higher-spin extensions of the off-shell topologically massive $\mathcal{N}=2$ supergravity theories [26].

This paper does not include any 3D analogs of the off-shell 4D $\mathcal{N}=1$ massless multiplets of integer superspin [6]. We have constructed such extensions. However, they do not admit massive deformation of the type described in Sec. VII. That is why we will discuss these models elsewhere. This paper does not include any 3D analogs of the off-shell 4D $\mathcal{N}=1$ higher spin supermultiplets in anti-de Sitter space [7]. We have constructed such extensions. They will be discussed elsewhere.

To the best of our knowledge, no off-shell 3D $\mathcal{N}=1$ massive higher spin supermultiplet have appeared in the literature. They can be derived by carrying out the plain superspace reduction $\mathcal{N}=2 \to \mathcal{N}=1$ to the models presented in Sec. VII. This is an interesting technical problem to work out.

Our results on the linearized higher spin super-Cotton tensors provide necessary prerequisites for developing a superspace approach to higher spin $\mathcal{N}=2$ conformal supergravity. We recall that the most general formulation[10] for 3D $\mathcal{N}$-extended conformal supergravity is the conformal superspace of [20], which is a 3D analog of the 4D conformal superspace formulations initiated by Butter [52,53]. In this approach, it is the $\mathcal{N}$-extended super-Cotton tensor which fully determines the superspace geometry of conformal supergravity. It is necessary to mention that the program of constructing a superconformal theory of massless higher spin fields in $(2+1)$ spacetime dimensions was put forward long ago by Fradkin and Linetsky [54] in the component setting. However, it appears that superspace techniques may offer new insights.

[10]As explained in [20], the conventional formulation for 3D $\mathcal{N}$-extended conformal supergravity [50,51] is obtained from that given in [20] by partially fixing the gauge freedom. In this sense, 3D $\mathcal{N}$-extended conformal superspace of [20] is the most general formulation for 3D $\mathcal{N}$-extended conformal supergravity.

Our approach to constructing higher spin massive supermultiplets is a generalization of topologically massive (super)gravity. Recently, there appeared a conceptually different way to generate 3D massive (super)gravity theories—new massive (super)gravity theories [55–58] and their generalizations; see [45,59] and references therein. We believe that our results may be used to construct higher spin analogs of these massive theories.

Our massive transverse supermultiplet (7.1) can be coupled to an external source $\mathcal{J}_{\alpha(2s)}$ using an action functional of the form

$$\mu^{2s-1} S^{\perp}_{s+\frac{1}{2}}[H,\Gamma,\bar{\Gamma}] + \frac{\lambda}{2} S_{\rm CS}[H] + \left(-\frac{1}{2}\right)^{s} \int \mathrm{d}^{3|4}z H^{\alpha(2s)} \mathcal{J}_{\alpha(2s)}. \tag{8.1}$$

In order for such an action to be invariant under the gauge transformations (4.3) and (4.4), the real source $\mathcal{J}_{\alpha(2s)}$ must be conserved, that is

$$D^{\beta} \mathcal{J}_{\beta\alpha_1\ldots\alpha_{2s-1}} = \bar{D}^{\beta} \mathcal{J}_{\beta\alpha_1\ldots\alpha_{2s-1}} = 0. \tag{8.2}$$

Such higher spin conserved current multiplets were considered in [60]. In 3D $\mathcal{N}=2$ superconformal field theory, $\mathcal{J}_{\alpha\beta}$ describes the supercurrent multiplet [13,61].[11] The theory with action (8.1) possesses a dual longitudinal formulation. It is described by the action

$$\mu^{2s-1} S^{\parallel}_{s+\frac{1}{2}}[H,G,\bar{G}] + \frac{\lambda}{2} S_{\rm CS}[H] + \left(-\frac{1}{2}\right)^{s} \int \mathrm{d}^{3|4}z H^{\alpha(2s)} \mathcal{J}_{\alpha(2s)}, \tag{8.3}$$

where the longitudinal action $S^{\parallel}_{s+\frac{1}{2}}[H,G,\bar{G}]$ is given by Eq. (4.17).

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APPENDIX A: NOTATION AND CONVENTIONS

Our 3D notation and conventions correspond to those introduced in [49,51].

The spinor covariant derivatives have the form

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + \mathrm{i}\bar{\theta}^\beta(\gamma^a)_{\alpha\beta}\partial_a = \partial_\alpha + \mathrm{i}\bar{\theta}^\beta\partial_{\alpha\beta}, \tag{A1a}$$

$$\bar{D}_\alpha = -\frac{\partial}{\partial\bar{\theta}^\alpha} - \mathrm{i}\theta^\beta(\gamma^a)_{\alpha\beta}\partial_a = -\bar{\partial}_\alpha - \mathrm{i}\theta^\beta\partial_{\alpha\beta} \tag{A1b}$$

and obey the anti-commutation relations

$$\{D_\alpha, D_\beta\} = \{\bar{D}_\alpha, \bar{D}_\beta\} = 0, \quad \{D_\alpha, \bar{D}_\beta\} = -2\mathrm{i}\partial_{\alpha\beta}. \tag{A2}$$

The generators of supersymmetry transformations are

$$Q_\alpha = \mathrm{i}\partial_\alpha + \bar{\theta}^\beta\partial_{\alpha\beta}, \qquad \bar{Q}_\alpha = -\mathrm{i}\bar{\partial}_\alpha - \theta^\beta\partial_{\alpha\beta}. \tag{A3}$$

We make use of the definitions

$$D^2 = D^\alpha D_\alpha, \qquad \bar{D}^2 = \bar{D}_\alpha \bar{D}^\alpha \tag{A4}$$

such that the complex conjugate of $D^2 V$ is $\bar{D}^2\bar{V}$, for any superfield V.

Most tensor (super)fields encountered in this paper are completely symmetric with respect to their spinor indices. We use the rules introduced in [47] and adopted in [7].

$$V_{\alpha(n)} = V_{\alpha_1\ldots\alpha_n} = V_{(\alpha_1\ldots\alpha_n)}, \tag{A5a}$$

$$V_{\alpha(n)}U_{\alpha(m)} = V_{(\alpha_1\ldots\alpha_n}U_{\alpha_{n+1}\ldots\alpha_{n+m})}, \tag{A5b}$$

$$V\cdot U = V^{\alpha(n)}U_{\alpha(n)} = V^{\alpha_1\ldots\alpha_n}U_{\alpha_1\ldots\alpha_n}, \tag{A5c}$$

$$V_{(\alpha(n)}U_{\beta(m))} = V_{(\alpha_1\ldots\alpha_n}U_{\beta_1\ldots\beta_m)}. \tag{A5d}$$

Parentheses denote symmetrization of indices. Indices sandwiched between vertical bars (e.g., $|\gamma|$) are not subject to symmetrization. Throughout the entire paper, we assume that (super)fields carrying an even number of spinor indices correspond to bosons, whereas (super)fields carrying an odd number of spinor indices correspond to fermions.

APPENDIX B: MASSLESS HIGHER SPIN ACTIONS IN THREE DIMENSIONS

In this appendix we briefly review the (Fang-)Fronsdal actions for massless higher spin fields in three dimensions [1,2]. We also show that these models describe no propagating degrees of freedom, as a simple extension of the 4D analysis in Sec. 6. 9 of [11].

1. Integer spin

Given an integer $s > 1$, we consider the following set of real bosonic fields:

$$\varphi^i = \{h_{\alpha(2s)}, h_{\alpha(2s-4)}\} \tag{B1}$$

[11]The two- and three-point functions of the $\mathcal{N}=2$ supercurrent were computed in [62].

defined modulo gauge transformations

$$\delta h_{\alpha(2s)} = \partial_{(\alpha_1\alpha_2}\zeta_{\alpha_3\ldots\alpha_{2s})}, \tag{B2a}$$

$$\delta h_{\alpha(2s-4)} = \frac{1}{2s-1}\partial^{\beta\gamma}\zeta_{\beta\gamma\alpha_1\ldots\alpha_{2s-4}}, \tag{B2b}$$

where the gauge parameter $\zeta_{\alpha(2s-2)}$ is real. It may be checked that the following action

$$\begin{aligned} S_s = \frac{1}{2}\left(-\frac{1}{2}\right)^s \int \mathrm{d}^3x \Big\{ & h^{\alpha(2s)}\Box h_{\alpha(2s)} - \frac{1}{2}s(\partial^{\beta(2)}h_{\beta(2)\alpha(2s-2)})^2 \\ & -(s-1)(2s-3)\Big[h^{\alpha(2s-4)}\partial^{\beta(2)}\partial^{\gamma(2)}h_{\beta(2)\gamma(2)\alpha(2s-4)} \\ & +4\frac{(s-1)}{s}h^{\alpha(2s-4)}\Box h_{\alpha(2s-4)} \\ & +\frac{1}{2}(s-2)(2s-5)(\partial^{\beta(2)}h_{\alpha(2s-6)\beta(2)})^2\Big]\Big\} \end{aligned} \tag{B3}$$

is gauge invariant. The requirement of gauge invariance determines the action uniquely modulo an overall constant. The theory admits a formal limit to the case of three-dimensional Maxwell's electrodynamics. It is obtained by setting $s=1$, removing the second field in (B1), and switching off all the terms in the second through fourth lines of the action (B3).

The equations of motion are

$$\begin{aligned} &\Box h_{\alpha(2s)} + \frac{1}{2}s\partial^{\beta(2)}\partial_{\alpha(2)}h_{\alpha(2s-2)\beta(2)} \\ &\quad -\frac{1}{2}(s-1)(2s-3)\partial_{\alpha(2)}\partial_{\alpha(2)}h_{\alpha(2s-4)} = 0, \end{aligned} \tag{B4a}$$

$$\begin{aligned} &\partial^{\beta(2)}\partial^{\gamma(2)}h_{\beta(2)\gamma(2)\alpha(2s-4)} + 8\frac{(s-1)}{s}\Box h_{\alpha(2s-4)} \\ &\quad + (s-2)(2s-5)\partial^{\beta(2)}\partial_{\alpha(2)}h_{\alpha(2s-6)\beta(2)} = 0. \end{aligned} \tag{B4b}$$

We now show that the model under consideration has no propagating degrees of freedom.

The gauge freedom (B2) allows us to gauge away the field $h_{\alpha(2s-4)}$,

$$h_{\alpha(2s-4)} = 0. \tag{B5}$$

In this gauge, there still remains a residual gauge freedom. In accordance with (B2b), the gauge parameter is now constrained by

$$\partial^{\beta(2)}\zeta_{\beta(2)\alpha(2s-4)} = 0. \tag{B6}$$

In the gauge (B5), the equation of motion (B4b) reduces to $\partial^{\beta(2)}\partial^{\gamma(2)}h_{\beta(2)\gamma(2)\alpha(2s-4)} = 0$ and tells us that $\partial^{\beta(2)}h_{\alpha(2s-2)\beta(2)}$ is divergenceless. In general, it holds that

$$\begin{aligned} \partial^{\beta(2)}\delta h_{\alpha(2s-2)\beta(2)} = &-\frac{2}{s}\Box\zeta_{\alpha(2s-2)} \\ &+\frac{(s-1)(2s-3)}{s(2s-1)}\partial^{\beta(2)}\partial_{\alpha(2)}\zeta_{\alpha(2s-4)\beta(2)}. \end{aligned} \tag{B7}$$

Under the two conditions that (i) the gauge parameter is constrained as in (B6), and (ii) $\partial^{\beta(2)}h_{\alpha(2s-2)\beta(2)}$ is divergenceless, we are able to impose the gauge condition

$$\partial^{\beta(2)}h_{\alpha(2s-2)\beta(2)} = 0, \tag{B8}$$

in addition to (B5). The residual gauge freedom, which respects the conditions (B5) and (B8), is generated by a gauge parameter constrained by

$$\partial^{\beta(2)}\zeta_{\beta(2)\alpha(2s-4)} = 0, \qquad \Box\zeta_{\alpha(2s-2)} = 0. \tag{B9}$$

Due to (B5) and (B8), the equation of motion (B4a) turns into

$$\Box h_{\alpha(2s)} = 0. \tag{B10}$$

Since both the field $h_{\alpha(2s)}(x)$ and the gauge parameter $\zeta_{\alpha(2s-2)}(x)$ are on-shell, it is useful to switch to momentum space, by replacing $h_{\alpha(2s)}(x)\to h_{\alpha(2s)}(p)$ and $\zeta_{\alpha(2s-2)}(x)\to \zeta_{\alpha(2s-2)}(p)$, where the three-momentum p^a is lightlike, $p^{\alpha\beta}p_{\alpha\beta}=0$. For a given three-momentum, we can choose a frame in which the only nonzero component of $p^{\alpha\beta} = (p^{11}, p^{12}=p^{21}, p^{22})$ is $p^{22}=p_{11}$. Then, the conditions $p^{\beta(2)}h_{\alpha(2s-2)\beta(2)}(p)=0$ and $p^{\beta(2)}\zeta_{\alpha(2s-4)\beta(2)}(p)=0$ are equivalent to

$$h_{\alpha(2s-2)22}(p) = 0, \qquad \zeta_{\alpha(2s-4)22}(p) = 0. \tag{B11}$$

We see that $h_{\alpha(2s)}$ has only two independent components, which are $h_{1\ldots1}$ and $h_{1\ldots12}$, and similar for the gauge parameter $\zeta_{\alpha(2s-4)}$. The gauge transformation law (B2a) now amounts to $\delta h_{1\ldots1}\propto p_{11}\zeta_{1\ldots1}$ and $\delta h_{1\ldots12}\propto p_{11}\zeta_{1\ldots12}$. As a result, the field $h_{\alpha(2s)}$ can be completely gauged away for $s>1$. The case $s=1$ is special. Here the field $h_{\alpha\beta}$ has again two components, h_{11} and h_{12}, while the gauge parameter is a scalar, ζ. The latter allows us to gauge away h_{11}, since its gauge transformation is $\delta h_{11}\propto p_{11}\zeta$. The other component, h_{12}, describes a propagating degree of freedom. In the gauge $h_{11}=0$, it is proportional to a single nonzero component of the gauge-invariant field strength $F^a = \frac{1}{2}\varepsilon^{aba}F_{bc}$, where $F_{ab} = \partial_a h_b - \partial_b h_a$.

2. Half-integer spin

Given an integer $s>1$, we consider the following set of real fermionic fields:

$$\phi^j = \{\psi_{\alpha(2s+1)}, \psi_{\alpha(2s-1)}, \psi_{\alpha(2s-3)}\} \tag{B12}$$

defined modulo gauge transformations of the form

$$\delta\psi_{\alpha(2s+1)} = \partial_{(\alpha_1\alpha_2}\xi_{\alpha_3\ldots\alpha_{2s+1})}, \tag{B13a}$$

$$\delta\psi_{\alpha(2s-1)} = \frac{2s-1}{2s+1}\partial^\beta{}_{(\alpha_1}\xi_{\alpha_2\ldots\alpha_{2s-1})\beta}, \tag{B13b}$$

$$\delta\psi_{\alpha(2s-3)} = \partial^{\beta\gamma}\xi_{\alpha_1\ldots\alpha_{2s-3}\beta\gamma}, \tag{B13c}$$

where the gauge parameter $\xi_{\alpha(2s-1)}$ is real. It may be checked that the following action

$$\begin{aligned} S_{s+\frac{1}{2}} = \frac{\mathrm{i}}{2}\left(-\frac{1}{2}\right)^s \int \mathrm{d}^3x \Big\{ & \psi^{\alpha(2s)\beta}\partial_\beta{}^\gamma\psi_{\gamma\alpha(2s)} \\ & + 2\psi^{\alpha(2s-1)}\partial^{\beta(2)}\psi_{\beta(2)\alpha(2s-1)} \\ & + \frac{4}{2s-1}\psi^{\alpha(2s-2)\beta}\partial_\beta{}^\gamma\psi_{\gamma\alpha(2s-2)} \\ & + \frac{(s-1)(2s+1)}{s(2s-1)}\Big(2\psi^{\alpha(2s-3)}\partial^{\beta(2)}\phi_{\beta(2)\alpha(2s-3)} \\ & - \frac{2s-3}{2s+1}\psi^{\alpha(2s-4)\beta}\partial_\beta{}^\gamma\psi_{\gamma\alpha(2s-4)}\Big)\Big\} \end{aligned} \tag{B14}$$

is gauge invariant. The field $\psi_{\alpha(2s-3)}$ is not defined in the case $s = 1$ which corresponds to the massless gravitino. However, the last two lines in (B14), which contain all the dependence on $\psi_{\alpha(2s-3)}$, does not contribute in the case, due to the overall factor of $(s-1)$. Thus the gravitino action follows from (B14) by deleting the last two lines and then setting $s = 1$.

The equations of motion are

$$\partial^\beta{}_\alpha\psi_{\alpha(2s)\beta} - \partial_{\alpha(2)}\psi_{\alpha(2s-1)} = 0, \tag{B15a}$$

$$\partial^{\beta(2)}\psi_{\alpha(2s-1)\beta(2)} + \frac{4}{2s-1}\left[\partial^\beta{}_\alpha\psi_{\alpha(2s-2)\beta} - \frac{(s-1)(2s+1)}{s}\partial_{\alpha(2)}\psi_{\alpha(2s-3)}\right] = 0, \tag{B15b}$$

$$\partial^{\beta(2)}\psi_{\alpha(2s-3)\beta(2)} - \frac{2s-3}{2s+1}\partial^\beta{}_\alpha\psi_{\alpha(2s-4)\beta} = 0. \tag{B15c}$$

We now show that the model under consideration has no propagating degrees of freedom.

The gauge freedom (B13c) allows us to gauge away the field $\psi_{\alpha(2s-3)}$,

$$\psi_{\alpha(2s-3)} = 0. \tag{B16}$$

In this gauge, there still remains a residual gauge freedom. In accordance with (B13c), the gauge parameter is now constrained by

$$\partial^{\beta(2)}\xi_{\beta(2)\alpha(2s-3)} = 0. \tag{B17}$$

In the gauge (B16), the equation (B15c) reduces to

$$\partial^{\beta(2)}\psi_{\alpha(2s-3)\beta(2)} = 0, \tag{B18}$$

which is preserved by the residual gauge transformations, as a consequence of (B17). Due to (B17) and (B18), it follows from the gauge transformation (B13b) that the field $\psi_{\alpha(2s-1)}$ may be gauged away,

$$\psi_{\alpha(2s-1)} = 0. \tag{B19}$$

Under this gauge condition, there still remains some residual gauge freedom. It is described by an on-shell parameter $\xi_{\alpha(s2-1)}$, which is constrained by

$$\partial^\beta{}_{\alpha_1}\xi_{\alpha_2\ldots\alpha_{2s-1}\beta} = 0 \Rightarrow \Box\xi_{\alpha(2s-1)} = 0, \tag{B20}$$

in addition to (B17). Under the gauge conditions (B16) and (B19), the equations of motion (B15) amount to

$$\begin{aligned} &\partial^\beta{}_{\alpha_1}\psi_{\alpha_2\ldots\alpha_{2s+1}\beta} = 0 \Rightarrow \partial^{\beta(2)}\psi_{\alpha(2s-1)\beta(2)} = 0, \\ &\quad\Box\psi_{\alpha(2s+1)} = 0. \end{aligned} \tag{B21}$$

Since both the field $\psi_{\alpha(2s+1)}(x)$ and the gauge parameter $\xi_{\alpha(2s-1)}(x)$ are on-shell, it is useful to switch to momentum space, by replacing $\psi_{\alpha(2s+1)}(x) \to \psi_{\alpha(2s+1)}(p)$ and $\xi_{\alpha(2s-1)}(x) \to \xi_{\alpha(2s-1)}(p)$, where the three-momentum p^a is lightlike, $p^{\alpha\beta}p_{\alpha\beta} = 0$. As in the bosonic case studied in the previous subsection, we can choose a frame in which the only nonzero component of $p^{\alpha\beta} = (p^{11}, p^{12} = p^{21}, p^{22})$ is $p^{22} = p_{11}$. Then, the conditions $p^\beta{}_{\alpha_1}\psi_{\alpha_2\ldots\alpha_{2s+1}\beta}(p) = 0$ and $p^\beta{}_{\alpha_1}\xi_{\alpha_2\ldots\alpha_{2s-1}\beta}(p) = 0$ are equivalent to

$$\psi_{\alpha(2s)2}(p) = 0, \qquad \xi_{\alpha(2s-2)2}(p) = 0. \tag{B22}$$

Thus the only nonzero components of $\psi_{\alpha(2s+1)}(p)$ and $\xi_{\alpha(2s-1)}(p)$ are $\psi_{1\ldots1}(p)$ and $\xi_{1\ldots1}(p)$. The residual gauge freedom, $\delta\psi_{1\ldots}(p) \propto p_{11}\xi_{1\ldots1}$ allows us to gauge away the field $\psi_{\alpha(2s+1)}$ completely. A minor modification of the above analysis can be used in the case $s = 1$ to show that the massless gravitino action does not describe any propagating degrees of freedom.

APPENDIX C: COMPONENT REDUCTION

Here we shall elaborate on the component structure of the massless superspin-$(s+\frac{1}{2})$ model in the transverse formulation (4.9). The longitudinal action (4.17) can be reduced to components in a similar fashion. Our approach to the component reduction of (4.9) will be similar to that used in [5–7] for the off-shell higher spin $\mathcal{N}=1$ supermultiplets in four dimensions.[12]

It is useful to define the components fields of a superfield using the standard bar-projection

$$U| := U(x,\theta,\bar{\theta})|_{\theta=\bar{\theta}=0}, \tag{C1}$$

for any superfield $U(z)$. Our definition of the component fields of $H_{\alpha(2s)}$ and $\Gamma_{\alpha(2s-2)}$ will be consistent with the Wess-Zumino gauge (5.16).

In the Wess-Zumino gauge (5.16), the component fields of $H_{\alpha(2s)}$ are

$$h_{\alpha(2s+2)} := \frac{1}{2}[D_{(\alpha_1}, \bar{D}_{\alpha_2}]H_{\alpha_3\ldots\alpha_{2s+2})}| = D_{(\alpha_1}\bar{D}_{\alpha_2}H_{\alpha_3\ldots\alpha_{2s+2})}|, \tag{C2a}$$

$$\Psi_{\alpha(2s+1)} := -\frac{1}{4}\bar{D}^2 D_{(\alpha_1}H_{\alpha_2\ldots\alpha_{2s+1})}| = -\frac{1}{4}D_{(\alpha_1}\bar{D}^2 H_{\alpha_2\ldots\alpha_{2s+1})}|, \tag{C2b}$$

$$A_{\alpha(2s)} := \frac{1}{32}\{D^2,\bar{D}^2\}H_{\alpha(2s)}|. \tag{C2c}$$

The component fields of $\Gamma_{\alpha(2s-2)}$ are

$$\gamma_{\alpha(2s-2)} := \Gamma_{\alpha(2s-2)}| = \bar{\gamma}_{\alpha(2s-2)}, \tag{C3a}$$

$$\Psi_{\alpha(2s-1)} := +D_{(\alpha_1}\Gamma_{\alpha_2\ldots\alpha_{2s-1})}|, \tag{C3b}$$

$$\bar{\Psi}_{\alpha(2s-3)} := -\frac{2s-2}{2s-1}D^{\beta}\Gamma_{\alpha(2s-3)\beta}|, \tag{C3c}$$

$$\Upsilon_{\alpha(2s-1)} := -\bar{D}_{(\alpha_1}\Gamma_{\alpha_2\ldots\alpha_{2s-1})}|, \tag{C3d}$$

$$B_{\alpha(2s-2)} := -\frac{1}{4}D^2\Gamma_{\alpha(2s-2)}|, \tag{C3e}$$

$$U_{\alpha(2s)} := +\frac{1}{2}[D_{(\alpha_1},\bar{D}_{\alpha_2}]\Gamma_{\alpha_3\ldots\alpha_{2s})}|, \tag{C3f}$$

$$F_{\alpha(2s-2)} := \frac{2s-1}{2s}D^{\beta}\bar{D}_{\beta}\Gamma_{\alpha(2s-2)}|, \tag{C3g}$$

$$\rho_{\alpha(2s-1)} := \frac{1}{8}D^2\bar{D}_{(\alpha_1}\Gamma_{\alpha_2\ldots\alpha_{2s-1})}|. \tag{C3h}$$

Introducing the superfield Lagrangian $\mathcal{L}^{\perp}_{s+\frac{1}{2}}$ for the transverse action (4.9),

$$S^{\perp}_{s+\frac{1}{2}}[H,\Gamma,\bar{\Gamma}] = \int \mathrm{d}^{3|4}z\mathcal{L}^{\perp}_{s+\frac{1}{2}} \tag{C4}$$

the component Lagrangian L is defined by

$$S^{\perp}_{s+\frac{1}{2}}[H,\Gamma,\bar{\Gamma}] = \frac{1}{16}\int \mathrm{d}^3x D^2\bar{D}^2\mathcal{L}^{\perp}_{s+\frac{1}{2}}| = \int \mathrm{d}^3x L. \tag{C5}$$

The component Lagrangian naturally splits into its bosonic and fermionic parts:

$$L = L_{\mathrm{bos}} + L_{\mathrm{ferm}}. \tag{C6}$$

Below we analyze separately the bosonic and fermionic sectors of L.

1. Bosonic sector

For the bosonic Lagrangian we obtain

$$\begin{aligned} L_{\mathrm{bos}} = \left(-\frac{1}{2}\right)^s \Big\{ & 2A\cdot A + \frac{1}{8}(\partial\cdot h)^2 - \frac{1}{4}h^{\alpha(2s+2)}\Box h_{\alpha(2s+2)} + A\cdot(U+\bar{U}) \\ & + \frac{1}{4}\mathrm{i}(\partial\cdot h)\cdot(U-\bar{U}) - \frac{1}{2}(\partial\cdot\partial\cdot h)^{\alpha(2s-2)}\gamma_{\alpha(2s-2)} \\ & - \frac{2s^2-5s+1}{s^2}\gamma^{\alpha(2s-2)}\cdot\Box\gamma_{\alpha(2s-2)} - \frac{(s-1)(2s-3)(4s-1)}{2s^2(2s-1)}(\partial^{\beta(2)}\gamma_{\alpha(2s-4)\beta(2)})^2 - \frac{1}{2(2s-1)}F\cdot\bar{F} \\ & - \mathrm{i}\frac{s-1}{s}\gamma^{\alpha(2s-3)\rho}\partial_{\rho}{}^{\beta}(F-\bar{F})_{\alpha(2s-3)\beta} + \frac{(2s+1)}{4(2s-1)}(F\cdot F + \bar{F}\cdot\bar{F})\Big\}, \end{aligned} \tag{C7}$$

where the dot notation $F\cdot\bar{F}$ and $\partial\cdot h$ stands for the contraction of spinor indices, for instance: $F^{\alpha(2s-2)}\bar{F}_{\alpha(2s-2)}$ and $\partial^{\beta(2)}h_{\alpha(2s)\beta(2)}$. Integrating out the auxiliary fields $F_{\alpha(2s-2)}$, $U_{\alpha(2s-2)}$, $A_{\alpha(2s)}$ and $B_{\alpha(2s)}$, we arrive at the following Lagrangian:

[12]Further aspects of the component structure of the off-shell higher spin models proposed in [5,6] were studied in [63].

$$\hat{L}_{\rm bos} = \left(-\frac{1}{2}\right)^s \bigg\{ -\frac{1}{4} h^{\alpha(2s+2)} \Box h_{\alpha(2s+2)} + \frac{s+1}{8} (\partial\cdot h)^2 + \frac{2s-1}{2} (\partial\cdot\partial\cdot h)^{\alpha(2s-2)} \gamma_{\alpha(2s-2)} + \frac{4(2s-1)}{s+1} \gamma\cdot\Box\gamma + \frac{(s-1)(2s-1)(2s-3)}{2s(s+1)} (\partial\cdot\gamma)^2 \bigg\}. \tag{C8}$$

The gauge transformations of the component fields $h_{\alpha(2s+2)}$ and $h_{\alpha(2s-2)}$ can be read from the gauge transformations of the $H_{\alpha(2s)}$ and $\Gamma_{\alpha(2s-2)}$ superfields, respectively, in terms of the longitudinal linear gauge parameter $g_{\alpha(2s)}$. In the Wess-Zumino gauge, we have

$$\delta h_{\alpha(2s+2)} = \partial_{(\alpha_1\alpha_2} \zeta_{\alpha_3\ldots\alpha_{2s+2})},$$
$$\delta\gamma_{\alpha(2s-2)} = \frac{s}{2(2s+1)} \partial^{\beta\gamma} \zeta_{\alpha(2s-2)\beta\gamma}. \tag{C9}$$

We recall that the real gauge parameter $\zeta_{\alpha(2s)}(x)$ originates as $g_{\alpha(2s)}| = -\frac{\rm i}{2}\zeta_{\alpha(2s)}$; see Eq. (5.4a).

We now compare (C8) with the Lagrangian corresponding to the massless action given in Sec. B 1 with spin s replaced with $s+1$:

$$S_{s+1} = \frac{1}{2}\left(-\frac{1}{2}\right)^{s+1} \int {\rm d}^3x \bigg\{ h^{\alpha(2s+2)} \Box h_{\alpha(2s+2)} - \frac{s+1}{2} (\partial^{\beta(2)} h_{\beta(2)\alpha(2s)})^2 - s(2s-1)\bigg[h^{\alpha(2s-2)} \partial^{\beta(2)} \partial^{\gamma(2)} h_{\beta(2)\gamma(2)\alpha(2s-4)} + 4\frac{s}{s+1} h^{\alpha(2s-2)} \Box h_{\alpha(2s-2)} + \frac{1}{2}(s-1)(2s-3)(\partial^{\beta(2)} h_{\alpha(2s-4)\beta(2)})^2 \bigg]\bigg\}. \tag{C10}$$

Clearly, the Lagrangians coincide if we make the identification

$$\gamma_{\alpha(2s-2)} = (2s+1) h_{\alpha(2s-2)}. \tag{C11}$$

In this manner, all terms in the bosonic sector agree with Fronsdal's action.

2. Fermionic sector

For the fermionic Lagrangian we obtain

$$\begin{aligned} L_{\rm ferm} = {\rm i}\left(-\frac{1}{2}\right)^s \bigg\{ & \Psi^{\alpha(2s)\beta} \partial_\beta{}^\gamma \bar{\Psi}_{\alpha(2s)\gamma} + (\bar{\Psi}^{\alpha(2s+1)} \partial_{(\alpha_{2s+1}\alpha_{2s}} \Upsilon_{\alpha(2s-1))} + \Psi^{\alpha(2s+1)} \cdot \partial_{\alpha_{2s+1}\alpha_{2s}} \bar{\Upsilon}_{\alpha(2s-1))}) \\ & - \frac{2s-1}{2s{\rm i}} (\rho^{\alpha(2s-1)} \bar{\Psi}_{\alpha(2s-1)} - \bar{\rho}^{\alpha(2s-1)} \Psi_{\alpha(2s-1)}) + \frac{2s-1}{2s} \Psi^{\alpha(2s-2)\beta} \partial_\beta{}^\rho \bar{\Psi}_{\alpha(2s-2)\rho} + \frac{2s-1}{2s} \Psi^{\alpha(2s-1)} \partial_{\alpha_{2s-1}\alpha_{2s-2}} \Psi_{\alpha(2s-3)} \\ & - \frac{2s-1}{2s} \bar{\Psi}^{\alpha(2s-3)} (\partial\cdot\bar{\Psi})_{\alpha(2s-3)} - \frac{2s-3}{2s-2}\frac{2s-1}{2s} \bar{\Psi}^{\alpha(2s-4)\beta} \partial^\rho{}_\beta \Psi_{\alpha(2s-4)\rho} - \frac{2s-1}{2s} \Upsilon^{\alpha(2s-2)\beta} \partial_\beta{}^\gamma \bar{\Upsilon}_{\alpha(2s-2)\gamma} \\ & + \frac{2s+1}{2s{\rm i}} (\rho^{\alpha(2s-2)\beta} \Upsilon_{\alpha(2s-2)\beta} - \bar{\rho}^{\alpha(2s-2)\beta} \bar{\Upsilon}_{\alpha(2s-2)\beta}) \bigg\}. \end{aligned} \tag{C12}$$

The fields $\rho_{\alpha(2s-1)}$ and $\Upsilon_{\alpha(2s-1)}$ are auxiliary. Integrating them out leads to the following Lagrangian involving only the dynamical fields $\Psi_{\alpha(2s+1)}$, $\Psi_{\alpha(2s-1)}$ and $\Psi_{\alpha(2s-3)}$:

$$\begin{aligned} \hat{L}_{\rm ferm} = {\rm i}\left(-\frac{1}{2}\right)^s \bigg\{ & \Psi^{\alpha(2s)\beta} \partial_\beta{}^\gamma \bar{\Psi}_{\alpha(2s)\gamma} - \frac{2s-1}{2s+1} (\bar{\Psi}^{\alpha(2s-1)} \partial^{\beta(2)} \bar{\Psi}_{\alpha(2s-1)\beta(2)} + {\rm c.c.}) + \frac{4(2s-1)}{(2s+1)^2} \Psi^{\alpha(2s-2)\beta} \partial_\beta{}^\rho \bar{\Psi}_{\alpha(2s-2)\rho} \\ & - \frac{2s-1}{2s} (\bar{\Psi}^{\alpha(2s-3)} (\partial\cdot\bar{\Psi})_{\alpha(2s-3)} + \Psi^{\alpha(2s-3)} (\partial\cdot\Psi)_{\alpha(2s-3)}) - \frac{2s-3}{2s-2}\frac{2s-1}{2s} \bar{\Psi}^{\alpha(2s-4)\beta} \partial^\rho{}_\beta \Psi_{\alpha(2s-4)\rho} \bigg\}. \end{aligned} \tag{C13}$$

It is useful to perform the rescaling

$$\{\Psi_{\alpha(2s+1)},\Psi_{\alpha(2s-1)},\Psi_{\alpha(2s-3)}\}\to\left\{\Psi_{\alpha(2s+1)},-\Psi_{\alpha(2s-1)},\frac{2(s-1)}{2s-1}\Psi_{\alpha(2s-3)}\right\},$$

which leads to a massless fermionic action

$$\tilde{S}_{\rm ferm}=\mathrm{i}\left(-\frac{1}{2}\right)^s\int \mathrm{d}^3x\bigg\{\bar{\Psi}^{\alpha(2s)\beta}\partial^\gamma{}_\beta\Psi_{\alpha(2s)\gamma}+\frac{2s-1}{2s+1}(\bar{\Psi}^{\alpha(2s-1)}\partial^{\beta(2)}\Psi_{\alpha(2s-1)\beta(2)}+\text{c.c.})$$
$$+\frac{4(2s-1)}{(2s+1)^2}\Psi^{\alpha(2s-2)\beta}\partial^\gamma{}_\beta\bar{\Psi}_{\alpha(2s-2)\gamma}+\frac{s-1}{s}(\bar{\Psi}^{\alpha(2s-3)}\partial^{\beta(2)}\Psi_{\alpha(2s-3)\beta(2)}+\text{c.c.})-\frac{2s-3}{2s-1}\frac{s-1}{s}\bar{\Psi}^{\alpha(2s-4)\beta}\partial^\gamma{}_\beta\Psi_{\gamma\alpha(2s-4)}\bigg\},\tag{C14}$$

which proves to be invariant under gauge transformations of the form

$$\delta\Psi_{\alpha(2s+1)}=\partial_{(\alpha_1\alpha_2}\xi_{\alpha_3\ldots\alpha_{2s+1})},\tag{C15a}$$

$$\delta\Psi_{\alpha(2s-1)}=\partial^\beta{}_{(\alpha_1}\xi_{\alpha_2\ldots\alpha_{2s-1})\beta},\tag{C15b}$$

$$\delta\Psi_{\alpha(2s-3)}=\partial^{\beta\gamma}\xi_{\alpha(2s-3)\beta\gamma},\tag{C15c}$$

with $\xi_{\alpha(2s-1)}$ a complex gauge parameter.

Note that the action (C14) involving the complex fields $\{\Psi_{\alpha(2s+1)},\Psi_{\alpha(2s-1)},\Psi_{\alpha(2s-3)}\}$ is a sum of two actions involving real fields. More precisely, let us define the action

$$S^J_{s+\frac{1}{2}}\coloneqq\frac{\mathrm{i}}{2}\left(-\frac{1}{2}\right)^s\int\mathrm{d}^3x\bigg\{\psi^{J\alpha(2s)\beta}\partial^\gamma{}_\beta\psi^J_{\alpha(2s)\gamma}+2\frac{2s-1}{2s+1}\psi^{J\alpha(2s-1)}\partial^{\beta(2)}\psi^J_{\alpha(2s-1)\beta(2)}+\frac{4(2s-1)}{(2s+1)^2}\psi^{J\alpha(2s-2)\beta}\partial^\gamma{}_\beta\psi^J_{\alpha(2s-2)\gamma}$$
$$+\frac{s-1}{s}\psi^{J\alpha(2s-3)}\partial^{\beta(2)}\psi^J_{\alpha(2s-3)\beta(2)}-\frac{2s-3}{2s-1}\frac{s-1}{s}\psi^{J\alpha(2s-4)\beta}\partial^\gamma{}_\beta\psi^J_{\gamma\alpha(2s-4)}\bigg\},\tag{C16}$$

which is invariant under the following gauge symmetry:

$$\delta\psi^J_{\alpha(2s+1)}=\partial_{(\alpha_1\alpha_2}\xi_{\alpha_3\ldots\alpha_{2s+1})},\tag{C17a}$$

$$\delta\psi^J_{\alpha(2s-1)}=\partial^\beta{}_{(\alpha_1}\xi_{\alpha_2\ldots\alpha_{2s-1})\beta},\tag{C17b}$$

$$\delta\psi^J_{\alpha(2s-3)}=\partial^{\beta\gamma}\xi_{\alpha(2s-3)\beta\gamma},\tag{C17c}$$

with real gauge parameter $\xi_{\alpha(2s-1)}$. Here the label $J=\underline{1},\underline{2}$ denotes two identical sets of real fermionic fields, $\{\psi^J_{\alpha(2s+1)},\psi^J_{\alpha(2s-1)},\psi^J_{\alpha(2s-3)}\}$. Each action (C16) is equivalent to the massless spin-$(s+\frac{1}{2})$ model (B14), and the precise identification is as follows:

$$\psi^J_{\alpha(2s+1)}=\psi_{\alpha(2s+1)},\qquad \psi^J_{\alpha(2s-1)}=\frac{2s+1}{2s-1}\psi_{\alpha(2s-1)},$$
$$\psi^J_{\alpha(2s-3)}=\psi_{\alpha(2s-3)}.\tag{C18}$$

It follows then that

$$\tilde{S}_{\rm ferm}=S^{\underline{1}}_{s+\frac{1}{2}}+S^{\underline{2}}_{s+\frac{1}{2}},\tag{C19}$$

with the complex fields Ψ related to the real ones ψ^J by the rule

$$\sqrt{2}\Psi_{\alpha(2s+1)}=\psi^{\underline{1}}_{\alpha(2s+1)}+\mathrm{i}\psi^{\underline{2}}_{\alpha(2s+1)},\tag{C20a}$$

$$\sqrt{2}\Psi_{\alpha(2s-1)}=\psi^{\underline{1}}_{\alpha(2s-1)}-\mathrm{i}\psi^{\underline{2}}_{\alpha(2s-1)},\tag{C20b}$$

$$\sqrt{2}\Psi_{\alpha(2s-3)}=\psi^{\underline{1}}_{\alpha(2s-3)}-\mathrm{i}\psi^{\underline{2}}_{\alpha(2s-3)}.\tag{C20c}$$

APPENDIX D: PROPERTIES OF THE SUPERCONFORMAL FIELD STRENGTH

In this appendix we prove (i) invariance of the field strength (6.22) under the gauge transformation (6.21); and (ii) the Bianchi identies (6.24).

1. Gauge invariance

The superconformal field strength (6.22) is constructed from the superfields

$$X^J_{\alpha(n)}\coloneqq\Delta\Box^J\partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-2J}}{}^{\beta_{n-2J}}H_{\alpha_{n-2J+1}\ldots\alpha_n)\beta(n-2J)},\tag{D1a}$$

$$Z^J_{\alpha(n)} := \Delta^2 \Box^J \partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-1-2J}}{}^{\beta_{n-1-2J}} H_{\alpha_{n-2J}\ldots\alpha_n)\beta(n-1-2J)}. \tag{D1b}$$

which form a basis for the field strength. In order to prove the gauge invariance of the field strength, it suffices to vary (D1) and then impose the condition of gauge invariance. The condition of gauge invariance generates a recursion relation which is solved to give the binomial coefficients appearing in (6.22). Alternatively, one can directly show that $W_{\alpha(n)}$ is gauge invariant as follows. First we compute the variations of the basis superfields (D1). The results are

$$\begin{aligned}\delta_L X^J_{\alpha(n)} &:= \Delta \Box^J \partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-2J}}{}^{\beta_{n-2J}} \delta H_{\alpha_{n-2J+1}\ldots\alpha_n)\beta(n-2J)}\\
&= \Delta\left\{\frac{J}{n/2}\Box^J \partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-2J}}{}^{\beta_{n-2J}} \bar D_{\alpha_{n-2J+1}} L_{\ldots\alpha_n)\beta(n-2J)} + \frac{n/2-J}{n/2}\Box^J \partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-2J}}{}^{\beta_{n-2J}} \bar D_{|\beta_1} L_{\beta_2\ldots\beta_{n-2J}|\alpha_{n-2J+1}\ldots\alpha_n)}\right\}\\
&= \mathrm{i}\left\{\frac{J}{n}\Box^J \partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-2J}}{}^{\beta_{n-2J}} D_{\alpha_{n-2J+1}} \bar D^2 L_{\ldots\alpha_n)\beta(n-2J)} + \frac{n-2J}{2n}\Box^J \partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-2J}}{}^{\beta_{n-2J}} D_{|\beta_1} \bar D^2 L_{\beta_2\ldots\beta_{n-2J}|\alpha_{n-2J+1}\ldots\alpha_n)}\right\},\\
&\quad 0\le J\le \lfloor n/2\rfloor;\end{aligned} \tag{D2a}$$

$$\begin{aligned}\delta_L Z^J_{\alpha(n)} &:= \Delta^2 \Box^J \partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-1-2J}}{}^{\beta_{n-1-2J}} \delta H_{\alpha_{n-2J}\ldots\alpha_n)\beta(n-1-2J)}\\
&= \Delta\left\{-\frac{n-(2J+1)}{n/2}\Box^{J+1} \partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-2J-2}}{}^{\beta_{n-2J-2}} \bar D_{\alpha_{n-2J-1}} L_{\alpha_{n-2J}\ldots\alpha_n)\beta(n-2J-2)}\right.\\
&\quad\left. -\frac{2J+1}{n/2}\Box^J \partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-2J}}{}^{\beta_{n-2J}} \bar D_{|\beta_1} L_{\beta_2\ldots\beta_{n-2J}|\alpha_{n-2J+1}\ldots\alpha_n)}\right\}\\
&= \mathrm{i}\left\{-\frac{n-(2J+1)}{n}\Box^{J+1} \partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-2J-2}}{}^{\beta_{n-2J-2}} D_{\alpha_{n-2J-1}} \bar D^2 L_{\alpha_{n-2J}\ldots\alpha_n)\beta(n-2J-2)}\right.\\
&\quad\left. -\frac{2J+1}{n}\Box^J \partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-2J}}{}^{\beta_{n-2J}} D_{|\beta_1} \bar D^2 L_{\beta_2\ldots\beta_{n-2J}|\alpha_{n-2J+1}\ldots\alpha_n)}\right\},\ 0\le J\le \lfloor n/2\rfloor -1.\end{aligned} \tag{D2b}$$

Note that the generalized binomial coefficient $\binom{n}{k}$ has the following property,

$$\binom{n}{k} = 0, \qquad k>n, \tag{D3}$$

which allows us to take the sum from $J=0$ to $\lfloor n\rfloor 2$ for both basis fields in the field strength expression. Therefore, the variation of the field strength (6.22) is given by

$$\begin{aligned}2^n\delta W_{\alpha(n)} &= \frac12\sum_{J=0}^{\lfloor n/2\rfloor}\left\{\binom{n}{2J}\delta X^J_{\alpha(n)} + \frac12\binom{n}{2J+1}\delta Z^J_{\alpha(n)}\right\}\\
&= \mathrm{i}\sum_{J=0}^{\lfloor n/2\rfloor}\left\{\binom{n}{2J}\frac{J}{n}\Box^J \partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-2J}}{}^{\beta_{n-2J}} D_{\alpha_{n-2J+1}}\bar D^2 L_{\ldots\alpha_n)\beta(n-2J)} - \binom{n}{2J+1}\frac{n-2J-1}{2n}\right.\\
&\quad\times\Box^{J+1}\partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-2J-2}}{}^{\beta_{n-2J-2}} D_{\alpha_{n-2J-1}}\bar D^2 L_{\alpha_{n-2J}\ldots\alpha_n)\beta(n-2J-2)}\\
&\quad+\binom{n}{2J}\frac{n-2J}{2n}\Box^J \partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-2J}}{}^{\beta_{n-2J}} D_{|\beta_1}\bar D^2 L_{\beta_2\ldots\beta_{n-2J}|\alpha_{n-2J+1}\ldots\alpha_n)}\\
&\quad\left.-\binom{n}{2J+1}\frac{2J+1}{2n}\Box^J \partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-2J}}{}^{\beta_{n-2J}} D_{|\beta_1}\bar D^2 L_{\beta_2\ldots\beta_{n-2J}|\alpha_{n-2J+1}\ldots\alpha_n)}\right\}\\
&= \sum_{J=0}^{\lfloor n/2-1\rfloor}\frac12\left\{\left(\binom{n}{2J+2}\frac{2J+2}{n} - \binom{n}{2J+1}\frac{n-2J-1}{n}\right)\Box^{J+1}\partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-2J-2}}{}^{\beta_{n-2J-2}} D_{\alpha_{n-2J-1}}\bar D^2 L_{\alpha_{n-2J}\ldots\alpha_n)\beta(n-2J-2)}\right\}\\
&\quad+\sum_{J=0}^{\lfloor n/2\rfloor-1}\frac12\left\{\left(\binom{n}{2J}\frac{n-2J}{n} - \binom{n}{2J+1}\frac{2J+1}{2n}\right)\times\Box^J \partial_{(\alpha_1}{}^{\beta_1}\partial_{\alpha_2}{}^{\beta_2}\ldots\partial_{\alpha_{n-2J}}{}^{\beta_{n-2J}} D_{|\beta_1}\bar D^2 L_{\beta_2\ldots\beta_{n-2J}|\alpha_{n-2J+1}\ldots\alpha_n)}\right\} = 0,\end{aligned} \tag{D4}$$

since for any positive integer n the following identities hold:

$$\binom{n}{2J+2}\frac{2J+2}{n}-\binom{n}{2J+1}\frac{n-2J-1}{n}=0,\quad \forall J; \tag{D5a}$$

$$\binom{n}{2J}\frac{n-2J}{n}-\binom{n}{2J+1}\frac{2J+1}{n}=0,\qquad \forall J. \tag{D5b}$$

2. Bianchi identities

We now prove that the field strength (6.22) obeys the Bianchi identities (6.24) This amounts to computing the following relations:

$$\begin{aligned}D^{\gamma}X^J_{\gamma\alpha(n-1)} &= -\mathrm{i}\frac{n-2J}{4n}\Box^J D^2\bar{D}^{\alpha_n}\partial_{\alpha_n}{}^{\beta_{n-2J}}\partial_{(\alpha_1}{}^{\beta_1}\ldots\partial_{\alpha_{n-2J-1}}{}^{\beta_{n-2J-1}}H_{\ldots\alpha_{n-1})\beta(n-2J)}\\ &\quad -\mathrm{i}\frac{J}{2n}\Box^J D^2\bar{D}^{\gamma}\partial_{(\alpha_1}{}^{\beta_1}\ldots\partial_{\alpha_{n-2J}}{}^{\beta_{n-2J}}H_{\ldots\alpha_{n-1})\gamma\beta(n-2J)},\end{aligned} \tag{D6a}$$

$$\begin{aligned}D^{\gamma}Z^J_{\gamma\alpha(n-1)} &= +\mathrm{i}\frac{n-2J-1}{4n}\Box^{J+1}D^2\bar{D}^{\gamma}\partial_{(\alpha_1}{}^{\beta_1}\ldots\partial_{\alpha_{n-2J-2}}{}^{\beta_{n-2J-2}}H_{\ldots\alpha_{n-1})\gamma\beta(n-2J-2)}\\ &\quad +\mathrm{i}\frac{2J+1}{4n}\Box^J D^2\bar{D}^{\alpha_n}\partial_{\alpha_n}{}^{\beta_{n-2J}}\partial_{(\alpha_1}{}^{\beta_1}\ldots\partial_{\alpha_{n-2J-1}}{}^{\beta_{n-2J}}H_{\ldots\alpha_{n-1})\beta(n-2J)},\end{aligned} \tag{D6b}$$

and then using them to evaluate

$$2^{n-1}D^{\beta}W_{\alpha(n-1)\beta}=\sum_{J=0}^{\lfloor n/2\rfloor}\binom{n}{2J}D^{\beta}X^J_{\alpha(n-1)\beta}+\sum_{J=0}^{\lfloor n/2\rfloor}\binom{n}{2J+1}D^{\beta}Z^J_{\alpha(n-1)\beta}. \tag{D7}$$

This leads to essentially the same calculation as (D4), grouping the two independent types of structures that appear and showing that the coefficients of each type of structure vanish. In particular, we arrive again at the relations (D5).

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