Planck scale effects on the thermodynamics of photon gases

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A particular framework for quantum gravity is the doubly special relativity formalism that introduces a new observer-independent scale (the Planck scale). We resort to the methods of statistical mechanics in this framework to determine how the deformed dispersion relation affects the thermodynamics of a photon gas. The ensuing modifications to the density of states, partition function, pressure, internal energy, entropy, free energy, and specific heat are calculated. These results are compared with the outcome obtained in the Lorentz violating model of Camacho and Marcias [Gen. Relativ. Gravit. **39**, 1175 (2007)]. The two types of models predict different results due to different spacetime structures near the Planck scale. The resulting modifications can be interpreted as a consequence of the deformed Lorentz symmetry present in the particular model we have considered. In the low energy limit, our calculation coincides with the usual results of photon thermodynamics in special relativity theory, in contrast to the study presented in an earlier article [Phys. Rev. D **81**, 085039 (2010)].

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I. INTRODUCTION

A simple paradox confronts us as we seek the quantum theory of gravity. The combination of gravity (G), quantum (h), and relativity (c) gives rise to the Planck length, l_P, or its inverse, the Planck energy E_P [1,2]. These scales mark thresholds beyond which the old description of spacetime breaks down and qualitatively new phenomena are expected to appear. But this proposition obviously opposes the principle of special relativity (SR) theory, where the length of an object varies for separate observers. Thus, an extension of SR theory is needed where along with the velocity of light, another observer-independent quantity, a fundamental length scale, exists. As a result, we must modify SR theory near the high energy (Planck energy). One such modified theory is doubly special relativity (DSR) [3,4], which has drawn a lot of interest as a possible framework of quantum gravity [5,6]. There are mainly two basic principles on which this theory rests: (i) the appearance of a second observer-independent scale [1], which can be the Planck length; and (ii) a naturally emerging noncommutative (NC) spacetime [7,8]. All the models of quantum gravity predict qualitatively different spacetime beyond a certain energy (length) scale, generally considered to be the Planck energy (length). Also, it is now well established [9–11] that a consistent marriage of ideas of quantum mechanics and gravitation need a noncommutative description of spacetime to avoid the paradoxical situation of creation of a black hole for an event that is sufficiently localized in spacetime. So, DSR theory fits the criterion for being a quantum gravity framework in this respect [2,7]. Recently, DSR has also developed for curved space [12].

Amelino-Camelia [4] first proposed a possible solution to this problem. Another model, which is much simpler, was given by Magueijo and Smolin (MS) [1] and is referred to as the MS model in this paper. The stated paradox can be solved if the Lorentz transformations can be modified so as to preserve a single energy or momentum scale. It has been reported that it is possible [8] to build models keeping the principle of relativity for inertial frames intact and to simply modify the laws by which energy and momenta measured by different inertial observers are related to each other. By adding nonlinear terms to the action of the Lorentz transformations on momentum space, one can maintain the relativity of inertial frames. And, then all observers will agree that there is an invariant energy or momentum above which the picture of spacetime as a smooth manifold breaks down. Because there are two constants that are preserved, this theory is named DSR [13]. So, added nonlinear terms to the action of the Lorentz transformation makes it possible to maintain the relativity of inertial frames and to solve the paradox at the same time, but the quadratic invariant of SR is now replaced by a nonlinear invariant, which in turn leads to a modified dispersion relation. In the MS model, the Lorentz algebra is still not deformed, and there are no deformations in the brackets of rotations and momenta [1,8]. Briefly, DSR theory possesses the following simple but strong features: (i) First of all, in DSR the relativity of inertial frames, as proposed by Galileo,

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Newton, and Einstein, is well preserved. (ii) Second, there is an invariant energy scale κ, which is of the order of the Planck scale. (iii) Third, in DSR theories, the notion of absolute locality should be replaced by relative locality as due to the presence of an energy-dependent metric, different observers live in different spacetimes [14].

The well-known dispersion relation (or mass-shell condition) for a particle [1,2] in SR theory is ($\hbar = c = 1$)

$$E^2 = p^2 + m^2, \tag{1}$$

which has to be modified in the MS model as [1]

$$E^2 = p^2 + m^2\left(1 - \frac{E}{\kappa}\right)^2. \tag{2}$$

Here E and p are the energy and the magnitude of the three-momentum of the particle, respectively, while m is the mass of the particle. We will refer to this model as the MS model in this manuscript. A lot of studies have been carried out on this model including analogue gravity [15], noncommutative geometry [16], Bose-Einstein condensate [17], relativistic thermodynamics [10,18,19], cosmology [20] as well as DSR formalism from the conformal group [21]. Recently, a lot of theoretical studies have been done on the thermodynamics of relativistic quantum gas [22] as it plays a crucial role in cosmology [23,24], as well as in condensed matter [25]. A study on the thermodynamics of a photon gas with an invariant energy scale using the MS model has already been reported by Das and Roychowdhury [26]. In their paper, they have constructed the formalism to do such calculations within the MS model. But there is a severe error in their calculation, as they have used Maxwell-Boltzmann statistics while calculating the partition function of photons, but it is well known that photons are integer spin quantum particles [27]. So, they must obey the Bose-Einstein distribution. As a consequence, one must use Bose-Einstein statistics to calculate the thermodynamics of the photon. Because of this serious error, the results obtained by Das and Roychowdhury [26] do not coincide with known results of the thermodynamic quantities of photon gas in the SR theory [28,29]. For instance, their obtained internal energy of photon gas depends upon temperature, T linearly, but it is well known that the internal energy $E \propto T^4$ (Stefan-Boltzman law) [28]. An important point to note is that thermodynamics for photon gas with a different dispersion relation has been studied by Camacho and Marcias [30], where Bose-Einstein statistics have been used as expected. Besides, the thermodynamics of massive bosons and fermions with another different dispersion relation has also been investigated [31]. But both of these two modified dispersion relations appear from a phenomenological point of view, whereas the dispersion relation (2) has a more theoretical motivation.

Because of its very fundamental role in theoretical physics, the Lorentz symmetry has been subjected to some of the highest precision tests [32–34]. It has been advocated by a number of physicists [4,5] that Lorentz invariance (both global and local) is only an approximate symmetry, which is broken at the Planck scale. Camacho and Marcias [30] examined the consequence of Lorentz violation in their Lorentz violating model in a unique but different approach where they introduced a deformed dispersion relation as a fundamental fact for the dynamics of photons and analyzed the effects of this upon the thermodynamics of photon gas. They showed that the breakdown of Lorentz symmetry entails an increase in the number of microstates, and as a consequence a growth of the entropy and other thermodynamic quantities, with respect to the case of SR theory, is observed. So, it will be really intriguing to check the status of the thermodynamic quantities of photons in the MS model, where the relativity of inertial frames, as proposed by Galileo, Newton, and Einstein, is well preserved but at the same time solves the paradox related to the appearance of a second observer-independent scale [1]. The current paper is organized as follows. In Sec. II, we review shortly the nonlinear realization of the Lorentz group, which gives rise to the modified dispersion relation of Eq. (2). In Sec. III, we discuss the density of states and calculate the partition function. In Sec. IV, we go on to study the thermodynamic properties of photon gas using the derived partition function. We do the whole calculation in arbitrary dimensions but especially scrutinize the thermodynamic properties for three-dimensional space. The different relations between the thermodynamic quantities of photons in SR theory, such as the pressure-energy density relation and the entropy-specific heat relation do not remain valid in the Lorentz-violating model of Camacho and Marcias [30]. We carefully check whether these identities are still valid in the MS model, where Lorentz symmetry is still preserved.

II. NONLINEAR REALIZATION OF LORENTZ ALGEBRA AND MODIFIED DISPERSION RELATION

In this section, while working in the MS model [1] we briefly review [8] the nonlinear realization of Lorentz algebra in $(d+1)$-dimensional spacetime. The interested reader can go through [8] for more details. Starting from the familiar (linear) SR Lorentz transformation, the L_{SR} coordinate space variable,

$$\begin{aligned}
X'^0 &= L_{\mathrm{SR}}(X^0) = \gamma(X^0 - \nu X^1),\\
X'^1 &= L_{\mathrm{SR}}(X^1) = \gamma(X^1 - \nu X^0),\\
X'^2 &= L_{\mathrm{SR}}(X^2) = X^2,\\
X'^3 &= L_{\mathrm{SR}}(X^3) = X^3,\\
&\ldots\ldots\ldots\ldots\\
&\ldots\ldots\ldots\ldots\\
X'^d &= L_{\mathrm{SR}}(X^d) = X^d,
\end{aligned} \tag{3}$$

where $\gamma = (1-\nu^2)^{-1/2}$ and the boost is along the X^1 direction with velocity $\nu^i = (\nu, 0, 0, \ldots, 0)$. Continuing in the same way for the momentum space variable we get

$$\begin{aligned}
P'^0 &= L_{\mathrm{SR}}(P^0) = \gamma(P^0 - \nu P^1),\\
P'^1 &= L_{\mathrm{SR}}(P^1) = \gamma(P^1 - \nu P^0),\\
P'^2 &= L_{\mathrm{SR}}(P^2) = P^2,\\
P'^3 &= L_{\mathrm{SR}}(P^3) = P^3,\\
&\ldots\ldots\ldots\ldots\ldots\ldots\\
&\ldots\ldots\ldots\ldots\ldots\ldots\\
P'^d &= L_{\mathrm{SR}}(P^d) = P^d. \qquad (4)
\end{aligned}$$

In Eqs. (2) and (3), (X^μ, P^μ) are the phase space variables that obey normal Poisson bracket algebra, commuting or, more precisely, canonical degrees of freedom. Let us now declare κ-Minkowski phase space elements (x^μ, p^μ), where x^μ and p^μ are the position and momentum space coordinates, respectively, that satisfy noncommutative κ-Minkowski phase space algebra and DSR-Lorentz transformations [8]. Now defining an invertible map F such that [8,10]

$$\begin{aligned}
F(X^\mu) &\to x^\mu,\\
F^{-1}(x^\mu) &\to X^\mu, \qquad (5)
\end{aligned}$$

which in explicit form reads

$$F(X^\mu) = x^\mu\left(1 - \frac{p^0}{\kappa}\right), \qquad (6)$$

$$F^{-1}(x^\mu) = X^\mu\left(1 + \frac{p^0}{\kappa}\right), \qquad (7)$$

$$F(P^\mu) = \frac{p^\mu}{(1 - \frac{p^0}{\kappa})}, \qquad (8)$$

$$F^{-1}(p^\mu) = \frac{P^\mu}{(1 + \frac{P^0}{\kappa})}. \qquad (9)$$

Now the DSR-Lorentz transformation L_{DSR} is formally expressed as

$$x'^\mu = L_{\mathrm{DSR}}(x^\mu) = F\circ L_{\mathrm{SR}}\circ F^{-1}(x^\mu), \qquad (10)$$

$$p'^\mu = L_{\mathrm{DSR}}(p^\mu) = F\circ L_{\mathrm{SR}}\circ F^{-1}(p^\mu). \qquad (11)$$

In the case of x^0,

$$\begin{aligned}
x'^0 &= L_{\mathrm{DSR}}(x^0) = F\circ L_{\mathrm{SR}}\circ F^{-1}(x^0)\\
&= F\circ L_{\mathrm{SR}}\left(X^0\left(1 + \frac{P^0}{\kappa}\right)\right)\\
&= F(\gamma(X^0 - \nu X^1)\left(1 + \frac{\gamma}{\kappa}(P^0 - \nu P^1)\right)\\
&= \gamma\alpha(x^0 - \nu x^1), \qquad (12)
\end{aligned}$$

where

$$\alpha = 1 + \kappa^{-1}((\gamma - 1)P^0 - \gamma\nu P^1). \qquad (13)$$

In the same way we find out that

$$x'^1 = \gamma\alpha(x^1 - \nu x^0), \qquad (14)$$

$$p'^0 = \frac{\gamma}{\alpha}(p^0 - \nu p^1), \qquad (15)$$

$$p'^1 = \frac{\gamma}{\alpha}(p^1 - \nu p^0). \qquad (16)$$

And the transverse component of x^μ and p^μ transforms as

$$x'^i = \alpha x^i, \qquad (17)$$

$$p'^i = \frac{p^i}{\alpha}, \qquad (18)$$

where $i = 2, 3, \ldots, d$. It is very interesting to see how in the present formulation [8], noncommutative effects enter through these generalized (nonlinear) transformation rules. Most importantly, the transverse components also transform due to the nonlinear realization of the Lorentz group, unlike the usual SR transformation [Eq. (3)]. As expected, in the limit $\kappa \to \infty$, the generalized transformation rule coincides with the SR transformation. Therefore, the phase space quantity invariant under the DSR-Lorentz transformation is $\eta_{\mu\nu}p^\mu p^\nu(1 - \frac{p^0}{\kappa})^{-2}$, where $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, \ldots, 1)$, writing this as

$$m^2 = \eta_{\mu\nu}p^\mu p^\nu\left(1 - \frac{p^0}{\kappa}\right)^{-2}. \qquad (19)$$

This yields the well-known dispersion relation due to Magueijo and Smolin in Eq. (2). It is shown in [35] that a modified dispersion relation does not necessarily imply a varying (energy dependent) velocity of light. But there are models [30,31] that admit a varying speed of light. However, in the case of the MS model, for photons ($m = 0$) the dispersion relation (2) is the same as in the SR theory. So, a very important point to notice is that the speed of light c is an invariant quantity in the MS model. Another interesting fact is that the models described in [30] have no finite upper bound on the energy of the photons though they have a momentum upper bound. On the other hand, in the MS model [1,8], though the dispersion relation for the photons is unchanged, there is a finite upper bound on the photon energy that is the Planck energy. But the problem of the addition of momenta is not well established in DSR, so a classical addition law is compatible with the model, but it is not the unique possibility (see, for example, [36]).

III. DENSITY OF STATES AND PARTITION FUNCTION

To study the thermodynamic behavior of photon gas, we first find out the partition function, as it enables us to calculate the thermodynamics. From the modified dispersion relation we find the energy expression for the massless particle as

$$E = pc, \tag{20}$$

where c is the velocity of photons. Considering a d-dimensional box of volume V containing photon gas, we follow the standard procedure as given in [28]. The number of microstates available to the system ($\sum$) in the position range from r to $r + dr$ and in the momentum range from p to $p + dp$ is given by

$$\sum = \frac{1}{h^d} \int \int d^d p d^d r, \tag{21}$$

where h is the phase space volume of a single lattice and $\int \int d^d p d^d r$ is the total volume of the phase space. It should be mentioned that in SR theory the invariant quantities under Lorentz transformation are $\frac{d^d p}{E}$ and $E d^d x$. As a result, Eq. (21) remains invariant under Lorentz transformation. In the case of the κ-Lorentz transformation, we find out that the NC phase space volume transforms as

$$d^d x' d^d p' = \alpha^d \gamma d^d x \frac{\gamma}{\alpha^d} \left(1 - \frac{\nu P^1}{E}\right) d^d p = d^d x d^d p. \tag{22}$$

So, following the way in [28] we find the density of states as

$$g(E) dE = B(V, d) E^{d-1} dE, \tag{23}$$

where

$$B = \frac{2^{1-d} d \pi^{-d/2} V}{\Gamma(\frac{d}{2} + 1) h^d c^d}. \tag{24}$$

Here $\Gamma(j) = \int_0^\infty x^{j-1} e^{-x} dx$. Putting $d = 3$, one can find that Eq. (23) coincides with Ref. [28]. Now in the grand canonical ensemble (GCE) the partition function for massless Bose gas can be written as [28]

$$\log Z = -\sum_E \log(1 - e^{-\beta E}). \tag{25}$$

Here $\beta = \frac{1}{k_B T}$, k_B is the Boltzmann constant, and T is the temperature of the particle. Changing the sum by integral we find out that in SR theory [28]

$$\log Z = -\int_0^\infty g(E) \log(1 - e^{-\beta E}) dE. \tag{26}$$

Here we have used the Bose-Einstein distribution, which is the correct statistics for bosons [28] such as photon [30], but we need to make a modification in Eq. (26) to calculate thermodynamics of photons in DSR theory using the MS model. Because of the presence of an energy upper bound of particles κ in the DSR theory, we have to make a modification in the above expression as below following the spirit of Ref. [26],

$$\log Z = -\int_0^\kappa g(E) \log(1 - e^{-\beta E}) dE. \tag{27}$$

Note that the upper limit of integration is ∞ in Eq. (26), as there is no upper bound of energy in the SR theory but the upper limit of integration is κ in (27). In the MS model that we are considering, the photon dispersion relation is not modified at all as $m = 0$. But still there is modification in the partition function due to the existence of an energy upper bound of particles κ in the theory. In the limit $\kappa \to \infty$, we get back the normal SR theory results.

IV. THERMODYNAMICS OF PHOTON GAS

We have obtained the expression for partition function. In this section we calculate the thermodynamics of photon gas in a Lorentz symmetry conserving the DSR scenario.

A. Free energy

In GCE, the free energy can be evaluated from the partition function,

$$\begin{aligned} F &= -k_B T \log Z \\ &= k_B T \int_0^\kappa \rho(E) \log(1 - e^{-\beta E}) dE \\ &= -\frac{2^{1-d} c^{-d} V \hbar^{-d} \beta^{-(d+1)} \zeta(1+d) \pi^{-\frac{d}{2}}}{\Gamma(\frac{d}{2}+1)} f(\kappa, d), \end{aligned} \tag{28}$$

where $f(\kappa, d) = \Gamma(1 + d) - \Gamma(1 + d, \kappa)$. $\Gamma(j, k)$ is the incomplete gamma function, $\Gamma(j, k) = \int_k^\infty l^{j-1} e^{-l} dl$. We have removed the logarithm through an integration by parts. Most important, the contribution of the observer-independent fundamental energy scale enters through incomplete gamma function.

Taking $\kappa \to \infty$, we can find the SR result in d dimension,

$$F = -\frac{2^{1-d}c^{-d}V\hbar^{-d}\beta^{-(d+1)}\zeta(1+d)\pi^{-d/2}}{\Gamma(\frac{d}{2}+1)}\Gamma(d+1), \quad (29)$$

and putting $d = 3$ we can recover the familiar result for free energy of photon gas [28],

$$F = -\frac{V\pi^2}{45\hbar^3 c^3}(k_B T)^4. \quad (30)$$

We should point out that the free energy as well as other thermodynamic quantities obtained by Das and Roychowdhury [26] is unable to reproduce the known results [28] of photon gas in three-dimensional space. Free energy of photons in three dimensions has temperature dependency as $F \propto -T^4$, but they obtained $F = Nk_B T$, which is not correct but rather the result of classical ideal gas. But this is not surprising as they have used the Maxwell-Boltzmann distribution, which is valid for classical particles only.

B. Internal energy

Another important thermodynamic quantity internal energy E,

$$U = -\frac{\partial}{\partial\beta}\log Z$$
$$= \frac{2^{1-d}c^{-d}V\hbar^{-d}d\beta^{-(d+1)}\zeta(1+d)\pi^{-\frac{d}{2}}}{\Gamma(\frac{d}{2}+1)}f(\kappa, d). \quad (31)$$

Again as κ tends to infinity we retrieve the SR results,

$$U = -\frac{2^{1-d}c^{-d}V\hbar^{-d}\beta^{-(d+1)}d\zeta(1+d)\pi^{-d/2}}{\Gamma(\frac{d}{2}+1)}\Gamma(d+1). \quad (32)$$

In the case of $d = 3$ the above equation coincides with known result as well [28]. In Fig. 1, we plotted the internal energy of photon gas against its temperature for the cases of both the MS model and the SR theory in $d = 3$. It is clearly noticed from the plot that the internal energy grows at a much slower rate in the case of our result than in the SR theory and as temperature increases. This is due to the fact that Lorentz symmetry is further restricted in the MS model. As a result of this, we expect to have a fewer number of microstates and less internal energy in the MS model. Note that in both cases of SR and MS models, internal energy has T^4 dependency but in the MS model, the internal energy is less due to the presence of κ through incomplete gamma function. It should also be mentioned that internal energy in the MS model is related to free energy by

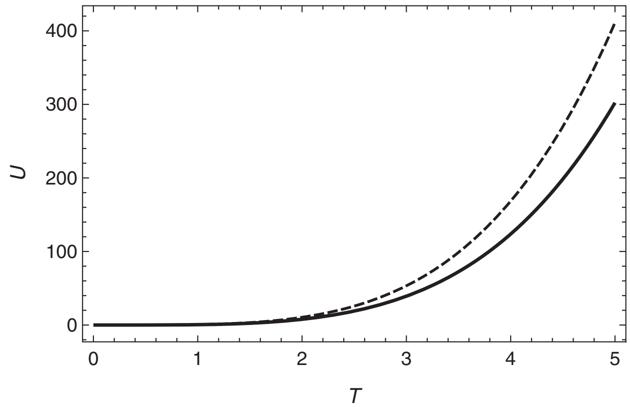

FIG. 1. Plot of internal energy of photon E against temperature T for both the SR theory and the MS model in three-dimensional space; the dashed line corresponds to the SR theory result, and the thick line represents the corresponding quantity in our result. We have used the Planck units, and the corresponding parameters take the following values: $\kappa = 5$, $k_B = 1$, $V = 1$, $h = 1$ in this plot as well as in all other plots in the paper. In this scale, $T = 5$ is the Planck temperature. The dashed line corresponds to the SR theory result, and the thick line represents the quantity in the MS model.

$$F = -\frac{1}{d}U, \quad (33)$$

just as in SR theory [28]. But this relation is not maintained in the Lorentz violating model of Camacho and Marcias [30].

C. Entropy

We can easily calculate the entropy from free energy,

$$S = -\left(\frac{\partial F}{\partial T}\right)_T$$
$$= \frac{2^{1-d}c^{-d}V\hbar^{-d}(d+1)\beta^{-(d+1)}T^{-1}\zeta(1+d)\pi^{-\frac{d}{2}}}{\Gamma(\frac{d}{2}+1)}f(\kappa, d). \quad (34)$$

Again in the limit $\kappa \to \infty$, we find the d-dimensional result for SR theory,

$$S = \frac{2^{1-d}c^{-d}V\hbar^{-d}(d+1)\beta^{-(d+1)}T^{-1}\zeta(1+d)\pi^{-\frac{d}{2}}}{\Gamma(\frac{d}{2}+1)}\Gamma(d+1), \quad (35)$$

which coincides with the known result when $d = 3$ is chosen [28]. In Fig. 2, entropy against temperature for the MS model and normal SR theory are plotted in three-dimensional space. As before, the entropy grows at a much slower rate in the case of our result than in the SR theory, and as temperature increases, the entropy in our considered

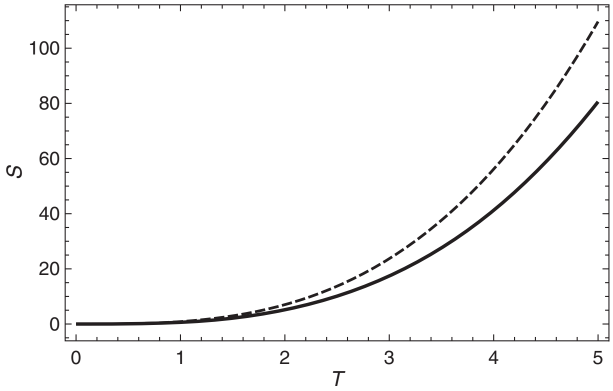

FIG. 2. Plot of entropy of photon S against temperature T for both in the SR theory and in the MS model with the same scaling as Fig. 1. The dashed line corresponds to the SR theory result, and the thick line represents the quantity in the MS model.

model deviates more from the entropy in the SR theory. It is well known that [29] the total number of microstates available to a system is a direct measure of the entropy for that system. Therefore our result merely reflects the fact that due to the existence of an energy upper bound κ, the number of microstates gradually decreases near Planck temperature. But it should be noted that this is not the case in the models with different dispersion relations where Lorentz symmetry is broken [30]. In [30] it is shown that the entropy becomes larger as an unavoidable consequence of this kind of Lorentz violation. But this is not the case in the MS model, as Lorentz symmetry is well preserved here. But nevertheless, in both types of model as $T \to 0$ we find $S \to 0$, indicating the Nernst postulate is always maintained, if the Lorentz symmetry is broken or not.

D. Pressure

The pressure of photon gas,

$$P = \left(\frac{\partial F}{\partial T}\right)_V = \frac{2^{1-d} c^{-d} \hbar^{-d} \beta^{-(d+1)} \zeta(1+d) \pi^{-\frac{d}{2}}}{\Gamma(\frac{d}{2}+1)} f(\kappa, d). \tag{36}$$

In the limit $\kappa \to \infty$, we find the pressure in d-dimensional SR theory is

$$P = \frac{2^{1-d} c^{-d} \hbar^{-d} \beta^{-(d+1)} \zeta(1+d) \pi^{-\frac{d}{2}}}{\Gamma(\frac{d}{2}+1)} \Gamma(d+1). \tag{37}$$

In $d = 3$, we redeem the known result $P = \frac{\pi^2 k_B^4}{45 \hbar^3 c^3} T^4$. The pressure of photon gas has a remarkable contribution in early universe cosmology, as it was well dominated by photons [23,24]. A very well known relation in SR theory between pressure and internal energy in three dimensions is $P = \frac{U}{3V}$. Comparing Eq. (32) and (36) we find out that the same relation is also maintained in the MS model. In the MS model, the d-dimensional relation between pressure and internal energy is

$$U = \frac{P}{d} V. \tag{38}$$

But this relation is not maintained in the other modified dispersion relation [30] due to the breakdown of Lorentz symmetry in their model. As it turns out, the breakdown of Lorentz symmetry manifests as a repulsive interaction. Indeed, the presence of a repulsive interaction (among the particles of a gas) entails the increase of the pressure, compared against the corresponding value for an ideal gas. But we notice the opposite in the MS model in Fig. 1 [37]; i.e., pressure increases in a slower rate with increasing temperature.

E. Specific heat

Specific heat (C_V) is defined as

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{2^{1-d} c^{-d} V \hbar^{-d} d(d+1) T^{-1} \zeta(1+d) \pi^{-\frac{d}{2}}}{\beta^{d+1} \Gamma(\frac{d}{2}+1)} f(\kappa, d). \tag{39}$$

Also, when $\kappa \to \infty$, we recover the d-dimensional result for the specific heat of photons,

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{2^{1-d} c^{-d} V \hbar^{-d} d(d+1) T^{-1} \zeta(1+d) \pi^{-\frac{d}{2}}}{\beta^{d+1} \Gamma(\frac{d}{2}+1)} \Gamma(d+1). \tag{40}$$

The above equation coincides with the known result when $d = 3$. In our calculation, C_V in both the MS model and the SR theory has T^d dependency in d dimension, unlike [26] that has reported a constant value of specific heat in SR theory. This is completely wrong as it is well established that C_V of photon gas has T^3 dependency in three dimensions [28]. The constant specific heat is rather a result of ideal nonrelativistic classical gas. In our study we have noticed from Eqs. (34) and (39) that specific heat is related to entropy as

$$S = \frac{C_V}{d}. \tag{41}$$

This relation is a well-established result in SR [28]. So the above relation along with Fig. 2 dictates that like the other thermodynamic quantities in the MS model, specific heat changes in a slow rate with temperature compared to

SR theory. On the other hand, we find the opposite in the Lorentz violating model of Camacho and Macias [30]. Their interpretation of the breakdown of Lorentz symmetry as the appearance of a repulsive interaction, results in a larger specific heat in SR theory. As C_V is a measurable quantity [30], which in principle could be employed in the experimental quest for violations of Lorentz symmetry, our present calculation is a significant justification of the theoretical status.

V. CONCLUSION

In this paper we successfully calculated the thermodynamics of photon gas in a theory where an observer-independent fundamental energy scale is present. The most important part of the present work is the derivation of the partition function in the MS model in arbitrary dimensions. Because of the deformed dispersion relation, this task becomes highly nontrivial to find the partition function analytically. However, for photons, we find out an exact analytic expression for the partition function, enabling us to calculate thermodynamic quantities such as the free energy, pressure, entropy, internal energy, and specific heat for the MS model, and compare them with the known results of SR theory in three-dimensional space. It should be noted that the influence of the Planck scale enters through incomplete gamma function. As expected, our results match with the known results [28] of SR theory in the limit $\kappa \to \infty$ unlike Ref. [26]. But due to the presence of an invariant energy upper bound in this theory, the microstates can avail energies only up to a finite cutoff, whereas in the SR theory, the microstates can attain energies up to infinity. As a result, the number of the microstates in this MS model is fewer than that in SR theory. This is clear from the result we obtained for entropy (Fig. 2) as entropy indicates the total number of the microstates available. This happens since Lorentz symmetry is not broken but is rather more restricted in the MS model. Just the opposite happens in the model [30], where Lorentz symmetry is not preserved. It is shown in [30] that the number of the microstates available to the corresponding equilibrium state grows, compared to the SR theory. The entropy becomes larger as an unavoidable consequence of this kind of Lorentz violation. Additionally, the breakdown of Lorentz symmetry entails a larger value of pressure, internal energy, or any other thermodynamic quantity [30] compared to the SR results. As noticed, an entirely different scenario is obtained in the current study with the MS model. But it is very intriguing to note that the Nernst postulate, i.e., the third law of thermodynamics, is maintained in both the MS model and the Lorentz violating study of Camacho and Marcias. So in conclusion, in the MS model, the Lorentz algebra is still intact in the presence of the observer-independent fundamental energy scale and yields that the thermodynamic quantities grow slowly against temperature compared to the SR theory, whereas in the Lorentz violating study, they tend to increase more quickly with temperature than in the SR theory. Also some very well established relations [28] among different thermodynamic quantities of photons in SR theories are Eqs. (33), (38), and (41). These equations are valid in the MS model but not in [30]. These are the key differences in the study of photon thermodynamics in Lorentz symmetry violating and Lorentz symmetry obeying models, which will play an important role in examining space-time structure near the Planck scale [38]. It would be interesting if these key differences are also maintained in the case of massive quantum gases.

Since the modification of the dispersion relation has changed the thermodynamics of photons drastically, we need to explicitly examine the thermodynamics of massive quantum gases in the MS model. Since the so-called Bose-Einstein condensation and Fermi degeneracy are purely bosonic and fermionic effects, respectively, we may wonder what happens to this feature if we introduce the generalization to (2) for massive particles. It would certainly change the condensation temperature for Bose gas as well as the Fermi temperature for Fermi gas. The former case is intriguing in the scalar field dark matter model, where the dark matter particle is a spin-0 boson [23,24]. But the latter case is important since the Chandrasekhar mass-radius relation [39] for white dwarfs is a direct consequence of the fermionic statistics. Hence we expect a modification in these studies due to the presence of an observer-independent fundamental energy scale. Besides, one can study the cosmological aspects of the MS model using the Friedmann equations. But, this still remains another open issue to be further studied.

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[1] J. Magueijo and L. Smolin, Phys. Rev. Lett. **88**, 190403 (2002).

[2] J. Magueijo and L. Smolin, Phys. Rev. D **67**, 044017 (2003).

[3] G. Amelino-Camelia, Nature (London) **418**, 34 (2002).

[4] G. Amelino-Camelia, Phys. Lett. B **510**, 255 (2001).

[5] C. Rovelli and L. Smolin, Nucl. Phys. **B442**, 593 (1995); **B456**, 734(E) (1995); G. Amelino-Camelia, Mod. Phys. Lett. A **17**, 899 (2002).

[6] J. Polchinski, arXiv:hep-th/9611050; S. Carlip, Rep. Prog. Phys. **64**, 885 (2001).

[7] J. Kowalski-Glikman and S. Nowak, Phys. Lett. B **539**, 126 (2002); Classical Quantum Gravity **20**, 4799 (2003).

[8] S. Ghosh and P. Pal, Phys. Rev. D **75**, 105021 (2007).

[9] J. Magueijo, Phys. Rev. D **63**, 043502 (2001).

[10] S. Das, S. Ghosh, and D. Roychowdhury, Phys. Rev. D **80**, 125036 (2009).

[11] S. Doplicher, K. Fredenhagen, and J. E. Roberts, Phys. Lett. B **331**, 39 (1994).

[12] A. F. Ali, M. Faizal, and M. M. Khalil, Nucl. Phys. **B894**, 341 (2015); S. H. Hendi and M. Faizal, Phys. Rev. D **92**, 044027 (2015); A. F. Ali, M. Faizal, B. Majumder, and R. Mistry, Int. J. Geom. Methods Mod. Phys. **12**, 1550085 (2015); A. F. Ali, M. Faizal, and M. M. Khalil, J. High Energy Phys. 12 (2014) 159; Phys. Lett. B **743**, 295 (2015); A. Ashour, M. Faizal, A. F. Ali, and F. Hammad, Eur. Phys. J. C **76**, 264 (2016); A. F. Ali, M. Faizal, and B. Majumder, Europhys. Lett. **109**, 20001 (2015).

[13] J. Kowalski-Glikman, Lect. Notes Phys. **669**, 131 (2005); J. Kowalski-Glikman, Phys. Lett. A **299**, 454 (2002).

[14] S. Das, S. Pramanik, and S. Ghosh, SIGMA **10**, 104 (2014).

[15] C. Barcel, S. Liberati, and M. Visser, Living Rev. Relativ. **14**, 3 (2011).

[16] J. Kowalski-Glikman and S. Nowak, Phys. Lett. B **539**, 126 (2002).

[17] F. Briscese, M. Grether, and M. de Llano, Europhys. Lett. **98**, 60001 (2012).

[18] N. Chandra and S. Chatterjee, Phys. Rev. D **85**, 045012 (2012).

[19] X. Zhang, L. Shao, and B.-Q. Ma, Astropart. Phys. **34**, 840 (2011).

[20] J. Magueijo and L. Smolin, Classical Quantum Gravity **21**, 1725 (2004).

[21] A. A. Deriglazov, Phys. Lett. B **603**, 124 (2004).

[22] M. Grether, M. de Llano, and G. A. Baker, Jr., Phys. Rev. Lett. **99**, 200406 (2007).

[23] L. Arturo Urena-Lopez, J. Cosmol. Astropart. Phys. 01 (2009) 014.

[24] S. Das and R. K. Bhaduri, Classical Quantum Gravity **32**, 105003 (2015).

[25] M. M. Faruk and M. M. Rahman, J. Stat. Mech. 03 (2016) 033117.

[26] S. Das and D. Roychowdhury, Phys. Rev. D **81**, 085039 (2010).

[27] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley Publishing Company, Reading, MA, 1995).

[28] R. K. Pathria, *Statistical Mechanics* (Butterworth-Heinemann, Oxford, 1996).

[29] H. B. Callen, *Thermodynamics and an Introduction to Thermostatistics* (Wiley, New York, 1985).

[30] A. Camacho and A. Macius, Gen. Relativ. Gravit. **39**, 1175 (2007).

[31] O. Bertolami and C. A. D. Zarro, Phys. Rev. D **81**, 025005 (2010); M. Gregg and S. A. Major, Int. J. Mod. Phys. D **18**, 971 (2009).

[32] J. Ellis, K. Farakos, N. E. Mavromatos, V. A. Mitsou, and D. V. Nanopoulos, Astrophys. J. **535**, 139 (2000); J. Ellis, N. E. Mavromatos, and D. Nanopoulos, Phys. Rev. D **63**, 124025 (2001).

[33] R. Gambini and J. Pullin, Phys. Rev. D **59**, 124021 (1999).

[34] J. Alfaro, H. A. Morales-Técotl, and L. F. Urrutia, Phys. Rev. Lett. **84**, 2318 (2000).

[35] S. Hossenfelder, Classical Quantum Gravity **23**, 1815 (2006).

[36] J. M. Carmona, J. L. Cortés, and F. Mercati, Phys. Rev. D **86**, 084032 (2012).

[37] As energy density ρ is related to pressure P, by $\rho = \frac{P}{d}$, we did not make another plot for pressure.

[38] J. Alfaro, Phys. Rev. D **72**, 024027 (2005); E. Waxman and J. Bahcall, Phys. Rev. Lett. **78**, 2292 (1997).

[39] P. H. Chavanis, Phys. Rev. D **76**, 023004 (2007).