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Can galaxy clusters, type Ia supernovae, and the cosmic microwave background rule out a class of modified gravity theories?

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In this paper we study cosmological signatures of modified gravity theories that can be written as a coupling between an extra scalar field and the electromagnetic part of the usual Lagrangian for the matter fields. In these frameworks, the electromagnetic sector of the theory is affected and variations of fundamental constants, of the cosmic distance-duality relation and of the evolution law of the cosmic microwave background (CMB) radiation, are expected and are related to each other. In order to search these variations we perform jointly analyses with angular diameter distances of galaxy clusters, luminosity distances of type Ia supernovae, and $T_{\mathrm{CMB}}(z)$ measurements. We obtain tight constraints with no significant indication of violation of the standard framework.

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I. INTRODUCTION

Since its publication in 1915, the main theory of gravitation, general relativity (GR), has been put in check, since the observations of galactic velocities in galaxy clusters, the rotational curve of spiral galaxies, and the recent discovery of the accelerated expansion of the Universe [1–6] via observations of Supernovae type Ia in 1998 only can be explained correctly with the addition of new ingredients in nature: the so-called dark matter (DM) and dark energy (DE). DM is a kind of matter that does not interact electromagnetically with other particles of the standard model [7,8]. Actually, DM also has a fundamental role in the evolution of cosmic structures in the GR context. DE is an alternative to explain the accelerated evolution of the Universe [9], that the cosmological constant (CC), which appears naturally in GR, is plagued with several conceptual problems which concerns its nature and origin [10]. In this way, several models of gravity have appeared in literature in order to give alternatives to GR. Among such alternative models, massive gravity theories [11,12], modified Newtonian dynamic (MOND) [13], $f(R)$ and $f(T)$ theories [14], and brane world models [15–21], among others, have been proposed recently in order to accommodate the observations. On the other hand, it is also important to have mechanisms to test whether these theories actually satisfy various observational constraints.

Recently, a wide class of theories of gravity that explicitly breaks the Einstein equivalence principle (EEP) have been considered in the literature and a powerful mechanism to test its signatures in observable constants of nature has been developed by A. Hees *et al.* [22,23]. They consider models which implements the break of the equivalence principle by introducing an additional term into the action, coupling the usual matter fields Ψ to a new scalar field ϕ, which is motivated by scalar-tensor theories of gravity, for instance. The explicit form of the couplings studied by [22,23] are of the type

$$S_m = \sum_i \int d^4x\sqrt{-g}h_i(\phi)\mathcal{L}_i(g_{\mu\nu}, \Psi_i), \qquad (1)$$

where $\mathcal{L}_i$ are the Lagrangians for different matter fields Ψ_i and $h_i(\phi)$ represents nonminimal couplings between ϕ and Ψ_i. When $h_i(\phi) = 1$ we recover the standard GR. Several alternative models can be described by such a coupling. We can cite string dilaton theories [24] at low energies, theories with additional compactified dimensions as Kaluza-Klein [25], models involving axions [26], cosmologies that consider a varying fine structure constant [27], chameleon-field models [28] or $f(R)$ extended gravity theories [29].

The most direct consequence of an interaction of the type (1) concerns its relation to the fine structure constant α of

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the quantum electrodynamics. It is related to the scalar field ϕ by $\alpha \propto h^{-1}(\phi(t))$, such that a time dependence of ϕ will lead to a time variation of the fine structure constant α [27,30]. Actually, all the electromagnetic sectors of the theory also are affected, which implies a nonconservation of the photon number along geodesics, leading to a modification to the expression of the luminosity distance, $D_L(z)$, where z is the redshift, which is the basis for various cosmological estimates and also the violation of the so-called cosmic distance-duality relation (CDDR), $D_L(1+z)^{-2}/D_A = 1$, where D_A is the angular diameter distance [31]. Moreover, also due to the nonconservation of the photon number, a variation of the evolution of the cosmic microwave background (CMB) radiation, affecting its temperature distribution, is expected. Finally, as a consequence of the CMB distribution, we also expect a CMB spectral distortion, which can be parametrized by a non-null chemical potential. In [22] the authors showed that all these effects are closely related to the time evolution of $h(\phi(t))$. By using Gaussian Processes, they also considered $D_A(z)$ measurements of galaxy clusters obtained via their Sunyaev-Zeldovich + z-ray observations (SZE/x-ray technique), $D_L(z)$ of type Ia supernovae, CMB temperature, and absorbers to impose limits on $h(\phi)$. Although the results were not so restrictive, no inconsistency with the standard results was detected.

However, in Ref. [32] it was showed that the SZE/x-ray technique depends strongly on the CDDR validity as well as on the α. These dependencies were used properly in Ref. [33] to search for signatures of the equivalence principle breaking. These authors considered the results from Ref. [22] jointly with galaxy clusters and SNe Ia observations and showed that if the CDDR is not valid, $D_L D_A^{-1}(1+z)^{-2} = \eta$ and $\Delta\alpha/\alpha \neq 1$, the SZE/x-ray technique does not give the true angular diameter distance of galaxy clusters but $D_A^{\rm obs} = D_A \eta^{-3}$. Again, no inconsistency with the standard results was detected by considering two parametrizations to $\eta(z)$, such as $\eta(z) = 1 + \eta_0 z$ and $\eta(z) = 1 + \eta_0 z/(1+z)$. The more restrictive value to η_0 was $\eta_0 = 0.069 \pm 0.106$ at 1σ c.l.

In this paper, we search for signatures of the class of modified gravity theories discussed by Ref. [22] by testing jointly the CDDR and the evolution law of CMB temperature. We consider angular diameter distance samples from galaxy clusters obtained via the SZE/x-ray technique, luminosity distances from SNe Ia, and $T_{\rm CMB}(z)$ measurements. Moreover, four parametrizations for $\eta(z)$ are used. The error bars from our joint analyses are at least 50% smaller than those in Ref. [33]. Our results showed no significant deviation from the standard framework ($\eta_0 = 0$) regardless of the $\eta(z)$ function and galaxy cluster sample used.

This paper is organized as follows: In Sec. II we briefly revise the theory proposed by Ref. [22]. In Sec. III we present our method and the samples used in analyses. In Sec. IV we show the analyses and results and finally, in Sec. V are the main conclusions of this work.

II. CONSEQUENCES OF THE BREAKING OF THE EQUIVALENCE PRINCIPLE

As mentioned before, A. Hees *et al.* [22,23] developed a powerful apparatus to test signatures of models characterized by an interaction term into the action of the form (1) in observable constants of nature. Such kinds of models implement the break of EEP. We cite here briefly three consequences, namely, the temporal variation of the fine structure constant, modification of the CDDR, and variations and distortions on the CMB temperature.

A. Temporal variation of the fine structure constant

Having $\alpha \propto h^{-1}(\phi(t))$ [24,27,34,35], the time variation of the fine structure constant α is associated to the time variation of the coupling of the electromagnetic Lagrangian $h_{\rm EM}(\phi)$ by

$$\frac{\dot{\alpha}}{\alpha} = -\frac{h'_{\rm EM}(\phi)}{h_{\rm EM}(\phi)}\dot{\phi} \tag{2}$$

where the dot corresponds to the temporal derivative and the prime to the derivative with respect to the scalar field ϕ. Writing in terms of the redshift z,

$$\frac{\Delta\alpha}{\alpha} = \frac{\alpha(z) - \alpha_0}{\alpha_0} = \frac{h(\phi_0)}{h(\phi(z))} - 1 = \eta^2(z) - 1, \tag{3}$$

where the subscript 0 stands for the present epoch ($\phi_0 = \phi(z=0)$), we can define the parameter

$$\eta(z) = \sqrt{\frac{h(\phi_0)}{h(\phi(z))}}, \tag{4}$$

which can be directly interpreted as a constraint on the cosmological evolution of the scalar field $\phi(z)$.

B. Modification of the cosmic distance-duality relation

The expression of the luminosity distance D_L is also modified with respect to the general relativity one [36], given by

$$D_L(z) = c(1+z)\sqrt{\frac{h(\phi_0)}{h(\phi(z))}}\int_0^z \frac{dz'}{H(z')}, \tag{5}$$

where c is the speed of the light and $H(z)$ is the Hubble parameter. On the other hand, the angular diameter distance D_A is a purely geometric property that is the same as in general relativity and it is given by [37]

$$D_A(z) = \frac{c}{(1+z)} \int_0^z \frac{dz'}{H(z')}. \tag{6}$$

By comparing with (5) we obtain

$$\frac{D_L(z)}{D_A(z)(1+z)^2} = \sqrt{\frac{h(\phi_0)}{h(\phi(z))}} = \eta(z), \tag{7}$$

which shows that the CDDR can also be related to $\eta(z)$.

C. Modifications of tej CMB temperature

The equations that governs the evolution of the temperature of the CMB are based on the kinetic theory (see it in [38] and [39]), which satisfies the Boltzmann equations of statistical mechanics. However, a nonconservation of the photon number due to a coupling of the form (1) may also alter the evolution of the CMB radiation. There is also a connection between violations of the temperature-redshift relation and variations of the fine structure constant. Furthermore, the coupling (1) also implies that the CMB radiation does not obey the adiabaticity condition [40], whose distortion of the CMB spectrum can be parametrized by a chemical potential μ. The relations for the CMB temperature evolution and the chemical potential as a function of $\eta(z)$ are [22]

$$T(z) = T_0(1+z)[0.88 + 0.12\eta^2(z)], \tag{8}$$

$$\mu = 0.47\left(1 - \frac{1}{\eta^2(z_{\rm CMB})}\right) = 3.92\left(\frac{T(z_{\rm CMB})}{T_0(1+z_{\rm CMB})} - 1\right), \tag{9}$$

which are related each other. It is useful to express the experimental constraints on the evolution of the temperature as a function of the parameter β, denoted by

$$T(z) = T_0(1+z)^{1-\beta}. \tag{10}$$

A. Hees *et al.* [22] have shown that the four cosmological observables, (3), (7), (8), and (9) are directly related to the evolution of the function $h(\phi)$ by

$$\frac{h(\phi_0)}{h(\phi(z))} = \eta^2(z) = \frac{\Delta\alpha(z)}{\alpha} + 1 = 8.33\frac{T(z)}{T_0(1+z)} - 7.33. \tag{11}$$

As it is well known, there are astronomical methods based on the analysis of high-redshift quasar absorption systems to test the $\Delta\alpha/\alpha$ value. The many-multiplet method, which compares the characteristics of different transitions in the same absorption cloud, is the most successful method employed so far to measure possible variations of α. However, very recently, constraints on the variations of the fine structure constant α have been derived directly from cosmological observations such as the SZE and x-ray emission in the galaxy cluster. For example, Ref. [41] proposed a new method using the integrated Comptonization parameter $Y_{\rm SZ}D_A^2$ and its x-ray counterpart Y_X, and the ratio of these two parameters depends on the fine structure constant as $\alpha^{3.5}$. Recently, Holanda *et al.* [33] showed that measurements of the gas mass fraction can also be used to probe a possible time evolution of the fine structure constant. For that purpose, they have showed that observations of the gas mass fraction via x-ray surface brightness and the SZE for the same galaxy cluster are related by

$$f_{\rm SZE} = \phi(z)\eta(z)f_{\rm X\,ray}, \tag{12}$$

where $\phi(z) = \frac{\alpha}{\alpha_0}$. Taking into account a direct relation between the variation of α and the CDDR, Eq. (7), and particularizing the analysis by considering a class of dilaton runaway models in which $\phi(z) = 1 - \gamma \ln(1+z)$, it was found that $\gamma = 0.037 \pm 0.18$ at 1σ c.l., consistent with no variation of α. More recently, a deeper analysis from Ref. [32] of the SZE/x-ray technique showed that measurements of $D_A(z)$ of galaxy clusters using this technique also depends on the fine structure constant. They have shown that if $\alpha = \alpha_0\phi(z)$, current SZE and x-ray observations do not provide the real angular diameter distance but instead

$$D_A^{\rm data}(z) = \phi(z)\eta^2(z)D_A(z). \tag{13}$$

In order to perform their analysis, they have transformed 25 measurements of D_L from current SNe Ia observations into $D_A(z)$, taking into account the direct relation, shown by Hees *et al.* [22], between a variation of α and the CDDR. When combined with 25 measurements of $D_A^{\rm data}(z)$ from galaxy clusters in the range of redshift $0.023 < z < 0.784$, these data sets impose cosmological limits on $\phi(z)$ for a class of dilaton runaway models. So, they have found $\frac{\Delta\alpha}{\alpha} = -(0.042 \pm 0.10)\ln(1+z)$, which is also consistent with no variation of α. On the other hand, in Ref. [33], Eq. (13) has been used and the relation between α and η impose tighter limits on deviation from CDDR than in previous works. By using $\eta(z) = 1 + \eta_0 z$ and $\eta(z) = 1 + \eta_0 z/(1+z)$, the most restrictive value to η_0 is $\eta_0 = 0.069 \pm 0.106$ at 1σ c.l.

III. NEW OBSERVATIONAL CONSTRAINTS

In the present paper we test the same class of modified gravity theories presented in Ref. [22] following Ref. [33]. Nevertheless, we use a different galaxy cluster sample, SNe Ia, and we also include measurements of CMB temperature in order to put tighter constraints from the analyses. In other words, we search for deviations from CDDR validity by

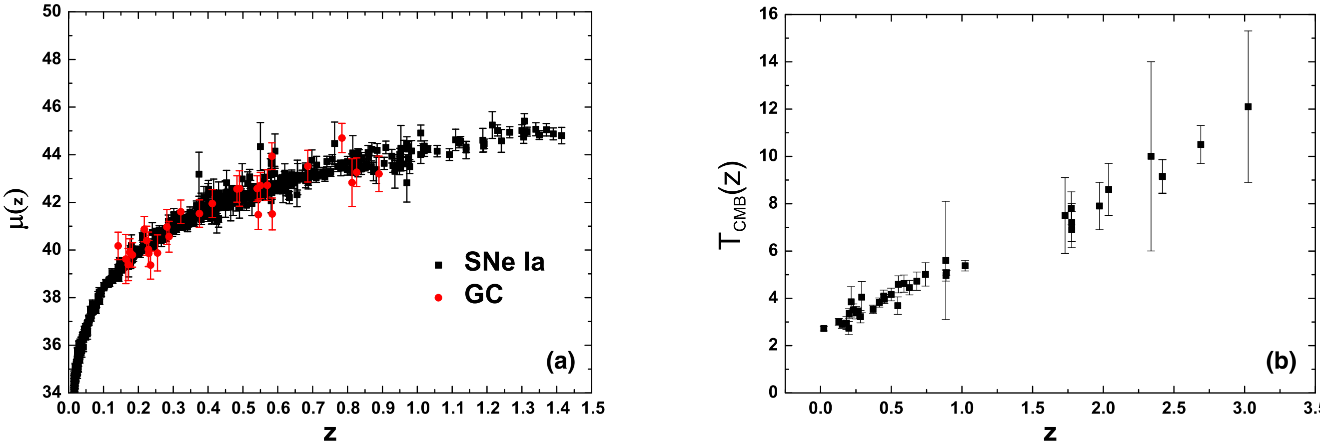

FIG. 1. In Fig. (a) we plot the distance modulus of SNe Ia and also for galaxy clusters (GC) by using the Eq. (16) com $\eta = 1$. In Fig. (b) we plot $T_{\rm CMB}(z)$ data.

using the relations presented in the previous section [see Eq. (11)]. The samples used here are

(i) We consider 29 well-described galaxy clusters by a spherical nonisothermal double β model from an original sample of 38 from Ref. [42]; see Fig. (1a). This model takes into account a possible presence of cooling flow in galaxy cluster cores. We cut off the galaxy clusters that presented questionable reduced χ^2 ($2.43 \leq \chi^2$d.o.f. ≤ 41.62) when described by the hydrostatic equilibrium model. It is important to stress that the frequency used to obtain the SZE signal in the galaxy clusters sample considered was 30 GHz; in this band the effect on the SZE from a variation of $T_{\rm CMB}$ is completely negligible. The best frequency is 150 GHz for negative signals and around 260 GHz for positive signals. Therefore, we do not consider a modified CMB temperature evolution law in the galaxy cluster data.

(ii) The full SNe Ia sample is formed by 580 SNe Ia data compiled in Ref. [43], the so-called Union2.1 compilation; see Fig. 1(a). In order to perform our test we need SNe Ia and galaxy clusters in the identical redshifts. In this way, we consider the 29 angular diameter distance of galaxy clusters from the sample of [42] and, for each i-galaxy cluster, we obtain one distance modulus, $\bar{\mu}$, and its error, $\sigma^2_{\bar{\mu}}$, from all i-SNe Ia with $|z_{\text{cluster}_i} - z_{\text{SNe}_i}| \leq 0.006$. Naturally, this criterion allows us to have some SNe Ia for each galaxy cluster and so we can perform a weighted average with them in order to minimize the scatter observed on the Hubble diagram. Then, we calculate the following weighted average [44] from SNe Ia data:

$$\bar{\mu} = \frac{\sum(\mu_i/\sigma^2_{\mu_i})}{\sum 1/\sigma^2_{\mu_i}}, \qquad \sigma^2_{\bar{\mu}} = \frac{1}{\sum 1/\sigma^2_{\mu_i}}. \tag{14}$$

(iii) The $T_{\rm CMB}(z)$ sample is composed by 38 points; see Fig. 1(b). The data in low redshift are from SZE observations [45] and from observations of spectral lines we have the data at high redshift [46]. In total, this represents 38 observations of the CMB temperature at redshift between 0 and 3. We also use the estimation of the current CMB temperature $T_0 = 2.725 \pm 0.002$ K [47] from the CMB spectrum as estimated from the COBE Collaboration [see Fig. (1b)].

IV. ANALYSES

As already discussed in [33], for the class of theories discussed by Hees *et al.*, the SZE + x-ray measurements of galaxy clusters do not give the true angular diameter distance (ADD), but $D_A^{\rm data} = \eta^4(z)D_A$ [where was used $\phi = \eta^2$ in Eq. (13)]. Moreover, as argued in [48], if one wants to test the CDDR by using $D_L(1+z)^{-2}D_A^{-1} = \eta$ and galaxy clusters via the SZE/X-ray technique, the ADD $D_A(z)$ must be replaced by $D_A(z) = \eta^{-4}D_A^{\rm data}$ (η^{-2} in their case, since variations of α were not considered). In this way, we have access to

$$\frac{D_L}{(1+z)^2 D_A^{\rm data}(z)} = \eta^{-3}(z). \tag{15}$$

By using the equation above, we define the distance modulus of a galaxy cluster data as

$$\mu_{\rm cluster}(\eta, z) = 5\,\lg[\eta^{-3}(z)D_A^{\rm data}(z)(1+z)^2] + 25. \tag{16}$$

We evaluate our statistical analysis by defining the likelihood distribution function, $\mathcal{L} \propto e^{-\chi^2/2}$, where

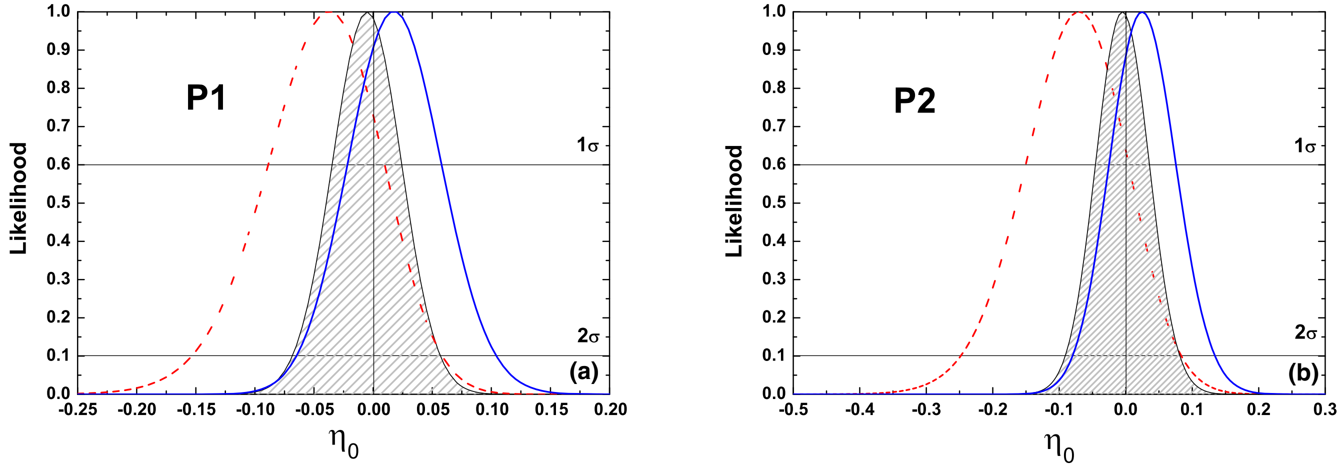

FIG. 2. In both figures, the solid blue and dashed red lines correspond to analyses by using SNe Ia + GC and $T_{\mathrm{CMB}}(z)$, respectively. The dashed area corresponds to the joint analysis [SNe Ia + GC + $T_{\mathrm{CMB}}(z)$]. In panel (a) we plot the results by using the parametrization P1 and in panel (b) by using P2.

$$\chi^2 = \sum_{i=1}^{29} \frac{(\bar{\mu}(z_i) - \mu_{\mathrm{cluster}}(\eta, z_i))^2}{\sigma^2_{\mathrm{obs}}} + \sum_{i=1}^{38} \frac{[T(z_i) - T_{i,\mathrm{obs}}]^2}{\sigma^2_{T_i,\mathrm{obs}}}, \tag{17}$$

with $\sigma^2_{\mathrm{obs}} = \sigma^2_{\bar{\mu}} + \sigma^2_{\mu\mathrm{cluster}}$ and $T(z)$ given by Eq. (8). The sources of statistical uncertainty in the error bars of $D_A^{\mathrm{data}}(z)$ are SZE point sources $\pm 8\%$, x-ray background $\pm 2\%$, Galactic $\mathrm{N}_H \leq \pm 1\%$, $\pm 8\%$ kinetic SZ, and for CMB anisotropy $\leq \pm 2\%$. We have added in quadrature the following systematic errors: SZ calibration $\pm 8\%$, x-ray flux calibration $\pm 5\%$, radio halos $+3\%$, and x-ray temperature calibration $\pm 7.5\%$. Following [43] we added a 0.15 systematic error to SNe Ia data.

In order to explore the dependence of our results with $\eta(z)$ function, we consider four parametrizations, namely,

(i) P1: $\eta(z) = 1 + \eta_0 z$
(ii) P2: $\eta(z) = 1 + \eta_0 z/(1+z)$
(iii) P3: $\eta(z) = (1+z)^{\epsilon}$
(iv) P4: $\eta(z) = 1 + \eta_0 \ln(1+z)$

where η_0 and ϵ are the parameters to be constrained. The limits $\eta_0 = \epsilon = 0$ corresponds to the standard GR results.

Our results are plotted in Figs. (2) and (3) for each parametrization and samples described in Sec. III. Note that in each case the solid (blue) and dashed (red) lines correspond to analyses by using separately CMB temperature and galaxy clusters + SNe Ia data in Eq. (17), respectively. The dashed area are the results from the joint analysis, i.e., the complete Eq. (17) with CMB temperature + galaxy clusters + SNe Ia. In Table I we put our 1σ results from the joint analyses for each

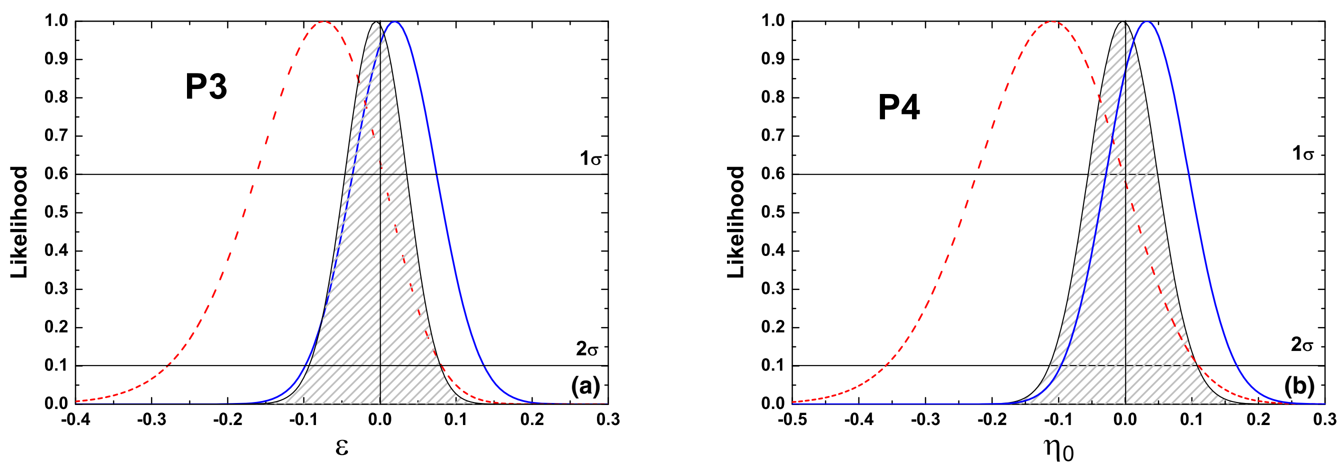

FIG. 3. In both figures, the solid blue and dashed red lines correspond to analyses by using SNe Ia + GC and $T_{\mathrm{CMB}}(z)$, respectively. The dashed area corresponds to the joint analysis [SNe Ia + GC + $T_{\mathrm{CMB}}(z)$]. In panel (a) we plot the results by using the parametrization P3 and in panel (b) by using P4.

TABLE I. A summary of the current constraints on the parameters η_0 for P1, P2, and P4 and ϵ for P3, from angular diameter distance from galaxy clusters and different SNe Ia samples. The symbol * corresponds to the ADD from Ref. [49] and ** corresponds to the angular diameter distance from Ref. [42]. The symbol ‡ corresponds to analyses which do not consider variations of α on the SZE/x-ray technique.

Reference	Data sample	η_0 (P1)	η_0 (P2)	ϵ (P3)	η_0 (P4)
[50]	$ADD^{\ddagger *}$ + SNe Ia	-0.28 ± 0.24	-0.43 ± 0.21	...	...
[50]	$ADD^{\ddagger **}$ + SNe Ia	-0.42 ± 0.14	-0.66 ± 0.16	...	...
[51]	$ADD^{\ddagger *}$ + SNe Ia	-0.07 ± 0.19	-0.11 ± 0.26	...	...
[51]	$ADD^{\ddagger **}$ + SNe Ia	-0.22 ± 0.11	-0.33 ± 0.16	...	...
[52]	$ADD^{\ddagger **}$ + SNe Ia	-0.23 ± 0.12	-0.35 ± 0.18	...	...
[44]	$ADD^{\ddagger *}$ + SNe Ia	-0.047 ± 0.178	-0.083 ± 0.246	...	...
[44]	$ADD^{\ddagger **}$ + SNe Ia	-0.201 ± 0.094	-0.297 ± 0.142	...	...
[53]	$ADD^{\ddagger *}$ + SNe Ia	$0.16^{+0.56}_{-0.39}$	...	...	...
[53]	$ADD^{\ddagger **}$ + SNe Ia	0.02 ± 0.20	...	...	...
[33]	ADD^{*} + SNe Ia	0.069 ± 0.106	$0 \pm .0.135$	...	...
This paper	ADD^{**} + SNe Ia + T_{CMB}	-0.005 ± 0.025	-0.048 ± 0.053	-0.005 ± 0.04	-0.005 ± 0.045
This paper	ADD^{*} + SNe Ia + T_{CMB}	-0.005 ± 0.032	-0.007 ± 0.036	0.015 ± 0.045	0.015 ± 0.047

parametrization and several η_0 values present in literature, obtained by using ADD from galaxy clusters + SNe Ia that did not take into account the effect of a possible α variation on the SZE/x-ray technique. Notoriously, our results present tighter limits on η_0 value than previous analyses. We also present the results obtained by using the galaxy clusters sample from Ref. [49], where the x-ray surface brightness was described by an elliptical isothermal β model in order to compare results. In this case, the galaxy clusters are distributed over the redshift interval $0.023 \leq z \leq 0.784$. It is very important to consider another assumption on the galaxy clusters morphology since the ADD depends on the hypotheses considered. As one may see, our results are in full agreement with each other regardless of the galaxy clusters sample and $\eta(z)$ function used. Moreover, no indication of deviations from standard results is obtained.

V. CONCLUSION

In Ref. [22] a powerful mechanism was developed to test signatures of a wide class of theories of gravity that explicitly break the Einstein equivalence principle. Briefly, they introduced an additional term into the action [see Eq. (1)] coupling the usual matter fields to a scalar additional field, which is motivated by scalar-tensor theories of gravity, for instance. Actually, all the electromagnetic sectors of the theory are affected, leading to deviations of the CDDR validity, $D_L(1+z)^2/D_A = \eta$, variations of fundamental constants, $\Delta\alpha/\alpha$ (where α is the fine structure constant), and of the evolution law of the cosmic microwave background radiation. The distortions of the standard results are related by Eq. (11).

In this paper, we have used ADD of galaxy clusters obtained via their SZE + x-ray surface brightness observations, luminosity distances of SNe Ia, and CMB radiation temperature to search signatures of the class of theories considered. By properly considering these deviations in the data, mainly on the SZE/x-ray technique which depends explicitly on the η and α, we put constraints on four parametrizations of $\eta(z)$ via a joint analysis of data (see the last two lines of Table I). We have obtained tighter constraints on possible deviations from GR than in previous works, and all cases were found to be in full agreement with the standard GR framework, $\eta = 1$. However, the results presented here do not rule out the models under question with high confidence level yet. When larger samples with smaller statistical and systematic uncertainties of x-ray and SZE observations as well as $T_{\mathrm{CMB}}(z)$ measurements and SNe Ia become available, the method proposed here will be able to search deviations from the standard framework with more accuracy.

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