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Behavior of the Newtonian potential for ghost-free gravity and singularity free gravity

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In this paper we show that there is a universal prediction for the Newtonian potential for a specific class of infinite derivative, ghost-free, quadratic curvature gravity. We show that in order to make such a theory ghost free at a perturbative level, the Newtonian potential always falls-off as $1/r$ in the infrared limit, while at short distances the potential becomes nonsingular. We provide examples which can potentially test the scale of gravitational nonlocality up to 0.004 eV.

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I. INTRODUCTION

Einstein's theory of general relativity (GR) has passed successfully through innumerable tests from small scales to large scales [1]. One of its predictions, the existence of gravitational waves, has recently been confirmed by the advanced Laser Interferometer Gravitational-Wave Observatory (LIGO), which has observed a transient gravitational-wave (GW) signal and tested the reliability of GR [2]. In all these examples, in the infrared (IR), the theory matches the Newtonian fall of $1/r$ potential. In spite of these great successes, the theory of GR is incomplete in the ultraviolet (UV), the classical solutions of GR exhibit black hole and cosmological type singularities, and at a quantum level the theory is not UV finite. GR definitely requires modifications in the UV; the question is what kinds of corrections in the UV one would expect, which would make the theory well behaved in the classical and the quantum sense, and possibly resolve the short distance singularities.

For a massless graviton, in four dimensions, all the interactions in the UV can in *principle* be captured by incorporating higher derivatives allowed by the diffeomorphism invariance. For instance, it is well known that higher derivatives can ameliorate the UV behavior, i.e. fourth derivative gravity is renormalizable, but at a cost of introducing a ghost term in the spin-2 component of a graviton propagator [3]. Indeed, the presence of ghosts can lead to a destabilizing of the classical vacuum, therefore rendering the theory unpredictable at both the classical and the quantum level.

Recently, the issue of ghosts has been addressed in the context of quadratic gravity—in order to make the theory generally covariant and ghost free at the perturbative level, one would require infinite derivatives [4,5]. Indeed, these *infinite* derivatives would modify the graviton propagator. However, if we capture the roots of these infinite derivatives by the exponential of an entire function, then there will be no new degrees of freedom propagating in spacetime other than the massless transverse and traceless graviton, since such modification of the graviton propagator would not introduce any new pole.

It has been demonstrated that these infinite derivatives with a graviton propagator modified by the exponential of an entire function can indeed soften the quantum UV behavior [6–11]. Furthermore, in a linearized limit, such a prescription also removes the cosmological Big Bang singularity [5,12], and the black hole type singularity in both the static limit [4], and the dynamical context [13]. One intuitive way to understand this is due to the fact that infinite derivatives render the gravitational interactions nonlocal [6,11]. This nonlocality also introduces an inherent new scale in four dimensions, i.e. $M \leq M_p \sim 2.4 \times 10^{18}$ GeV. Furthermore, an intriguing connection can be established between the gravitational entropy [14] and the propagating degrees of freedom in the spacetime. The gravitational entropy for ghost-free, infinite gravity does not get a contribution from the UV, but only from the Einstein-Hilbert action [15], and follows strictly the area law for entropy for a Schwarzschild's black hole.

The aim of this paper is twofold: first we show that for a wide class of infinite derivative theories of gravity which are ghost free, it is possible to recover *not only* the $1/r$ fall of the Newtonian potential in a static limit in the IR, but also to ameliorate the short distance behavior in the UV limit. Second, we wish to put a bound on the scale of nonlocality, i.e. M, from the current table-top experiments from the deviation of Newtonian gravity.

II. QUADRATIC CURVATURE GRAVITATIONAL ACTION

Let us first start by discussing the properties of GR in four dimensions. The linearized GR can be quantized around the Minkowski background, which is described by two massless degrees of freedom. The transverse and traceless components of the graviton propagator in four dimensions can be recast in terms of the spin projector operators, which involves the tensor $\mathcal{P}^{(2)}$, and only one of the scalar components, i.e. $\mathcal{P}_s^{(0)}$ [16]:

$$\Pi(k^2) \sim \frac{1}{k^2}\left(\mathcal{P}^{(2)} - \frac{1}{2}\mathcal{P}_s^{(0)}\right), \tag{1}$$

where k^μ is the 4-momentum vector, and we have suppressed the spacetime indices.

In fact, in [4,17] it has been shown that around the Minkowski background, in four dimensions, the most general quadratic order torsion-free and parity invariant gravitational action which can be made ghost free can be written in terms of the Ricci scalar, R, the symmetric traceless tensor, $S_{\mu\nu} = R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu}$, and $C_{\mu\nu\alpha\beta}$ is the Weyl tensor. It is sufficient to study the quadratic order action—which captures $\mathcal{O}(h^2)$ terms around the Minkowski background, i.e. $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$, where $\bar{g}_{\mu\nu}$ is the Minkowski background, and $h_{\mu\nu}$ are the excitations, in order to find the graviton propagator. The S-tensor vanishes on maximally symmetric backgrounds [Minkowski or (anti–)de Sitter] [17].[1] Therefore the full action can be written as

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + \frac{\lambda}{2}(R\mathcal{F}_1(\Box)R + S_{\mu\nu}\mathcal{F}_2(\Box)S^{\mu\nu} + C_{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box)C^{\mu\nu\lambda\sigma})\right], \tag{2}$$

where M_P^2 is the Planck mass; and λ is a dimensional coupling accounting for the higher curvature modification; and the $\mathcal{F}_i$ are Taylor expandable (i.e. analytic) functions of the covariant d'Alembertian [4], i.e. $\mathcal{F}_i(\Box) = \sum_{n=0} c_{i_n}\Box^n/M^{2n}$, where M is the scale of nonlocality.

The equations of motion of this action have been worked out in [19]. As we shall show now, this class of infinite derivative theory indeed provides a unique platform to study the departure from GR in future table-top experiments [20].

III. UNIVERSALITY OF THE NEWTONIAN POTENTIAL

Physical excitations of this action, Eq. (2), around the Minkowski background have been studied very well. This can be computed by the second variation of the action, using $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$. A quick computation can be made by employing the covariant mode decomposition of the metric [21]:

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} + \bar{\nabla}_\mu A_\nu + \bar{\nabla}_\nu A_\mu + \left(\bar{\nabla}_\mu\bar{\nabla}_\nu - \frac{1}{4}\bar{g}_{\mu\nu}\bar{\Box}\right)B + \frac{1}{4}\bar{g}_{\mu\nu}h, \tag{3}$$

where $\tilde{h}_{\mu\nu}$ is the transverse and traceless spin-2 excitation; A_μ is a transverse vector field; and B, h are two scalar degrees of freedom which mix. Upon linearization around maximally symmetric backgrounds, the vector mode and the double derivative scalar mode vanish identically, and we end up only with $\tilde{h}_{\mu\nu}$ and $\phi = h - \Box B$ [17]. Performing the necessary computations (which are indeed straightforward around Minkowski as all derivatives commute), one gets [17]

$$\begin{aligned}
\delta^2 S(\tilde{h}_{\nu\mu}) &= \int dx^4\sqrt{-\bar{g}}\frac{1}{2}\tilde{h}_{\mu\nu}\bar{\Box}a(\bar{\Box})\tilde{h}^{\mu\nu},\\
a(\bar{\Box}) &= 1 + \frac{\lambda}{M_P^2}\bar{\Box}(\mathcal{F}_2(\bar{\Box}) + 2\mathcal{F}_3(\bar{\Box}))\\
\delta^2 S(\phi) &= -\int dx^4\sqrt{-\bar{g}}\frac{1}{2}\phi\bar{\Box}c(\bar{\Box})\phi,\\
c(\bar{\Box}) &= 1 - \frac{\lambda}{M_P^2}\bar{\Box}\left(6\mathcal{F}_1(\bar{\Box}) + \frac{1}{2}\mathcal{F}_2(\bar{\Box})\right)
\end{aligned} \tag{4}$$

for the tensor component (where the field was rescaled by $M_P/2$ to become canonically normalized), and the scalar component (where the field was rescaled by $M_P\sqrt{3/32}$ to be canonically normalized), respectively.

The full graviton propagator can then be written using a similar method to [16], barring the suppressed indices[2] [17,19,22]:

$$\Pi(k^2) = \frac{\mathcal{P}^{(2)}}{k^2 a(-k^2)} + \frac{\mathcal{P}^{(0)}}{k^2(a(-k^2) - 3c(-k^2))}, \tag{5}$$

where $\mathcal{P}^{(2),(0)}$ are the spin projection operators [16]. Note that the graviton propagator has two unknown functions $a(k^2)$ and $c(k^2)$, where all the information about the infinite derivatives is hiding; see [4,19,22] for an alternative way of deriving the graviton propagator, Eq. (5), and related discussion on form factors. It is possible that $a(\bar{\Box})$ and $c(\bar{\Box})$ are not uniquely defined under field redefinitions [7–9], but this issue is beyond the scope of this paper.

In order to reduce the graviton propagator to that of GR, one method is to assume that $a(\bar{\Box}) = c(\bar{\Box})$. In the IR limit then both $a(k^2 \to 0) = 1$ and $c(k^2 \to 0) = 1$, such that Eq. (5) reduces to Eq. (1). In this limit the theory would match exactly GR's predictions in the IR, but would lead to

[1] The original action was written in terms of $R_{\mu\nu}$ and $R_{\mu\nu\lambda\sigma}$ in [4]. However there is no loss of generality in expressing the action as Eq. (2); see [17]. See also [8,9,18], where ghost conditions have been studied in the context of string theory.

[2] In [16], the authors imposed six projection operators to decompose the spin-2 and spin-0 components of the propagator; here we have employed a slightly different technique to decompose the ten metric degrees of freedom.

modification in the UV. The entire modification can be summarized by one unknown function $a(\bar{\Box})$, which constrains the functions such that (see for instance [19])

$$12\mathcal{F}_1(\bar{\Box}) + 6\mathcal{F}_2(\bar{\Box}) + 4\mathcal{F}_3(\bar{\Box}) = 0.$$

In order that the propagator have no poles except the massless graviton at $k = 0$, we require that $a(\bar{\Box})$ and $(a(\bar{\Box}) - 3c(\bar{\Box}))$ contain no zeros. This way the propagator, Eq. (5), will not contain any extra degrees of freedom propagating in the space-time other than the massless graviton with two helicity states. One possible choice is to assume that $a(\bar{\Box})$ is the exponential of an *entire function*. This choice makes sure that in spite of infinite derivatives, there exist no ghosts at the perturbative level for a quadratic curvature gravity Eq. (2). One such example will be [4,5,7]

$$a(\bar{\Box}) = c(\bar{\Box}) = e^{-\bar{\Box}/M^2}. \tag{6}$$

This choice guarantees that in the UV the theory is softened, as for $k \to \infty$, $a(-k^2) = c(-k^2) = e^{k^2/M^2}$ suppresses the propagator in the UV, i.e. $\Pi(k^2) \to 0$ in Eq. (5), while $k \to 0$ yields the pure 4D GR propagator.

Our aim in this paper will be to generalize this to any entire function $\tau(-k^2)$, such that in the momentum space we have

$$a(-k^2) = c(-k^2) = e^{-\tau(-k^2/M^2)}. \tag{7}$$

The computation of the Newtonian potential, i.e. $\Phi(r)$, for the simplest choice, when $\tau(-k^2/M^2) = -k^2/M^2$ as in Eq. (6), was done already in [5], and the result is

$$\Phi(r) \sim -\frac{\mu}{M_P^2 r}\sqrt{\frac{\pi}{2}}\mathrm{erf}(Mr/2), \tag{8}$$

where μ is the mass of a δ-source. This potential is finite near $r \approx 0$ and decays as $1/r$ at distances above the nonlocality scale, i.e. $r \gg M^{-1}$. The $1/r$ fall of Newtonian gravity has been tested in the laboratory up to 5.6×10^{-5} m [23], which implies that for the scale of nonlocality should be bigger than $M > 0.004$ eV. Indeed, we know very little about the gravitational interaction above this limit. The cornerstone of this computation is the sine Fourier transform

$$f(r) = \int_{-\infty}^{+\infty} \frac{dk}{k} e^{\tau(-k^2)} \sin(kr), \tag{9}$$

where

$$\Phi(r) = -\frac{\mu}{4\pi^2 M_p^2}\frac{f(r)}{r}. \tag{10}$$

When we consider the simplest choice, $\tau = -k^2/M^2$, the function $f(r)$ indeed gives an erf-function.

We now set out to prove that the leading behavior of the potential at small distances, r, away from the source is always given by $\Phi \approx \Phi_0 + \mathcal{O}(r)$, where Φ_0 is constant irrespectively of the form of an entire function $\tau(k^2)$, as long as it does not introduce any extra pole other than the massless graviton.

IV. GENERALIZATIONS OF THE ENTIRE FUNCTION

Note that for an entire function, we can always treat $f(r)$ as a polynomial function. As a warm-up exercise we note that the sine Fourier transformation for

$$\tau = -\frac{k^{2n}}{M^{2n}} \tag{11}$$

gives

$$f(r) = \frac{Mr}{n}\sum_{p=0}^{\infty}(-1)^p \frac{\Gamma(\frac{p}{n} + \frac{1}{2n})}{(2p+1)!}(Mr)^{2p}, \tag{12}$$

using the gamma function $\Gamma(x) \equiv (x-1)!$. The above result is a generalization of [24], where the authors analyzed special cases for $n = 1$, 2, 4. From Fig. 1 we see that the Newtonian potential never blows up at $r = 0$.

An important observation here is that increasing the value of n yields a larger modulation for large r, giving us a clear deviation from predictions of GR at larger distances, and providing us with a glimpse of testing the nonlocality scale M. We can see that by having higher modes we now switch on a new mechanism that can be falsifiable in a near-future experiment.

Tests of the inverse square law assume that departure from the Newtonian potential follows a Yukawa potential, $V(r) = -V_0[1 + \alpha \exp(-r/\lambda)]$. In [23], Adelberger *et al.*

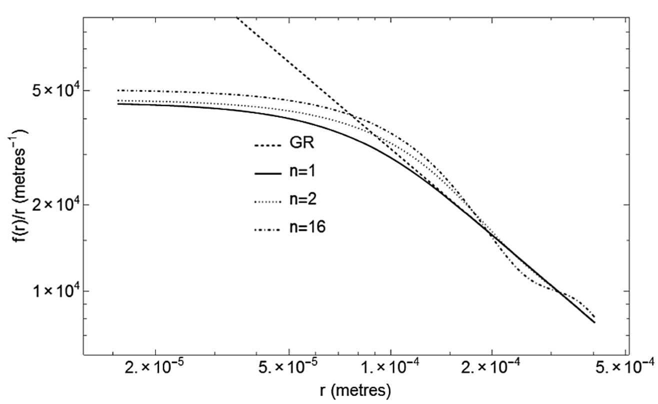

FIG. 1. We plot $f(r)/r$ vs r for different n for Eq. (12), where $n = 1$ corresponds to the error function. Recall that the Newtonian potential $\Phi(r) = -\frac{\mu}{4\pi^2 M_p^2}\frac{f(r)}{r}$. For illustrative purposes, we have taken $M = 4 \times 10^{-3}$ eV.

found in 2007[3] that this potential was ruled out for $\alpha = 1$ down to a length scale of 5.6×10^{-5} m, which means that we can now constrain M for each $\tau(-k^2)$.

For each specific value of n in Eq. (12), we can check for what value of M our potential would be detectable by [23]. The experiment ruled out a Yukawa potential $V(r) = V_0/r(1 + \exp(-r/5.6 \times 10^{-5})$ down to length scales of 5.6×10^{-5} m. Since this Yukawa potential is already ruled out, if a particular value of the scale of nonlocality M provides a larger divergence from GR than this potential, then it can also be ruled out, as otherwise it would have been detected by [23]. Using Eq. (12), this occurs at $M \sim 0.004, 0.02, 0.03, 0.05$ eV for $n = 1, 2, 4, 8$ so we can set these as our lower bounds on the scale of nonlocality.

Clearly this still leaves us with a large hierarchy between $M \geq 0.004$ eV and $M_p \sim 2.4 \times 10^{18}$ GeV, which signifies that indeed very little is known about the gravitational interaction.

Now, let us illustrate the most general situation. When τ is not a monomial, we may represent it as

$$\tau(-k^2) = -\frac{k^2}{M^2} + \rho(k^2). \tag{13}$$

If we expand $e^{\rho(k^2)} = \sum_m \rho_m k^{2m}/M^{2m}$ (clearly $\rho_0 = 1$), we yield the sine Fourier transformation of $e^{\tau}(k^2)$,

$$f(r) = \sum_{m=0}^{\infty} \rho_m (-1)^m \frac{\partial}{\partial \alpha^m} \int \frac{dp}{p} e^{-\alpha \frac{p^2}{M^2}} \sin(pr), \tag{14}$$

which we can calculate either explicitly as

$$f(r) = \sum_{m,p=0}^{\infty} \rho_m (-1)^p \frac{\Gamma(m + p + \frac{1}{2})}{(2p+1)!} (Mr)^{2p+1}, \tag{15}$$

or using Hermitian polynomials $H_m(x)$ as

$$f(r) = \pi \mathrm{erf}\left(\frac{Mr}{2}\right) - 2\sqrt{\pi} e^{-\frac{M^2 r^2}{4}} \sum_{m=1}^{\infty} \rho_m (-1)^m \frac{1}{4^m} H_{2m-1}\left(\frac{Mr}{2}\right). \tag{16}$$

Note that Eq. (16) converges to a constant if ρ_m decreases at least as fast as $\frac{(-1)^m}{m!}$, i.e. $\rho = -k^2/M^2$.

In order to satisfy the low energy requirements of the underlying physics, we require that the function $e^{\tau(-k^2)}$ fall at least as fast as e^{-k^2/M^2} [4]. Any $e^{\tau(-k^2)}$ which does this will also fulfil the convergence condition for Eq. (16), meaning that any physically realistic $a(\Box)$ will give a Newtonian potential which returns to the GR $1/r$ potential in the IR limit.

[3]Although further tests have been carried out, such as in [25], none of these gives a stronger constraint on r for $\alpha = 1$.

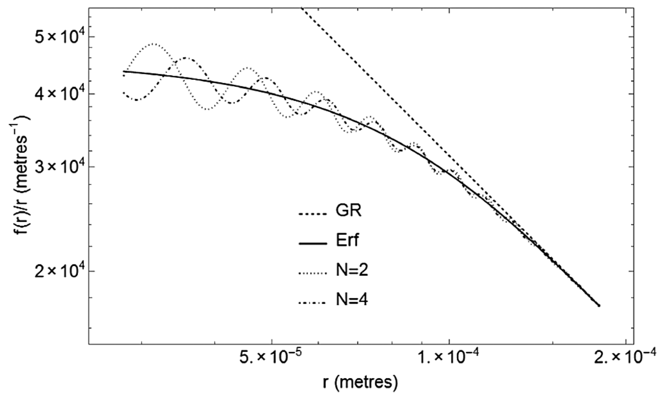

FIG. 2. We have plotted $f(r)/r$ vs r for Eqs. (16) and (18), where we have chosen $a_2 = 4.65 \times 10^{-3}$ and $a_4 = 1.24 \times 10^{-7}$, and for illustrative purposes, we have set $M = 4 \times 10^{-3}$ eV.

Next, in order to graphically show the behavior of Eq. (16) in Fig. 2, we take the next simplest case, where τ is the binomial

$$\tau = -\frac{k^2}{M^2} - a_N \frac{k^{2N}}{M^{2N}}, \tag{17}$$

and the choice of a_N is motivated by the purpose of illustration of the oscillations that occur for $r \approx M^{-1}$. In this case,

$$\rho_m = \frac{(-a_N)^{m/N}}{(m/N)!} \quad \text{for } \frac{m}{N} \in \mathbb{N} \text{and zero otherwise.} \tag{18}$$

V. CONCLUSION

Let us conclude by pointing out that infinite derivative, ghost-free theories of gravity pose a real falsifiable feature compared to GR, which can be tested by measuring the Newtonian potential in near-future experiments. We have shown that there exists a universal class of entire functions for which the theory is ghost free as well as singularity free in the UV, while leaving some tantalizingly small effects in the IR, albeit falling as the $1/r$-fall of the Newtonian potential. The current experimental limit puts the bound on nonlocality to be around $M \sim 0.004$ eV. Indeed, it is intriguing to reiterate that we know very little about gravity and any modification from the Newtonian potential can occur in the gulf of scales spanning some 30 orders of magnitude, i.e. $0.004 \text{ eV} \leq M \leq 10^{18}$ GeV, but this window also provides an opportunity for testing gravity at short distances.

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