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Black hole collapse and democratic models

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We study the evolution of black hole entropy and temperature in collapse scenarios in asymptotically anti–de Sitter spacetime, finding three generic lessons. First, entropy evolution is extensive. Second, at large times, entropy and temperature ring with twice the frequency of the lowest quasinormal mode. Third, the entropy oscillations saturate black hole area theorems in general relativity. The first two features are characteristic of entanglement dynamics in "democratic" models. Solely based on general relativity and the Bekenstein-Hawking entropy formula, our results point to democratic models as microscopic theories of black holes. The third feature can be taken as a prediction for microscopic models of black hole physics.

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I. INTRODUCTION

One of the most influential results in black hole physics is the discovery of black hole entropy [1,2]:

$$S_{\mathrm{BH}} = \frac{A}{4}, \tag{1}$$

where A is the area of the event horizon in natural units. This relation is believed to be a fundamental equation in quantum gravity, and a huge amount of effort has been dedicated to understanding its origin and nature. Approaches might be divided into two classes. The first class comprises those effective approaches which relate (1) to some entropy of quantum fields, such as counting the thermal entropy of near horizon degrees of freedom [3], or computing different notions of entanglement entropy (total or renormalized) of quantum fields [4–7]. The second class comprise those fundamental approaches relating (1) to some type of entropy in an exact microscopic description of quantum gravity, such as microstate counting [8–10], or entanglement entropy in AdS/CFT [11]. It is interesting to observe that the effective approaches do not use any notion of quantum gravity, just quantum mechanics and classical general relativity, while the second approach obviously rests in the knowledge of the microscopic theory.

A natural question arises as to which are the bridges connecting the effective approaches with the microscopic ones. In this vein, it is interesting to ask for novel generic implications of (1) within potential theories of quantum gravity. In this article, we give evidence that by considering black hole entropy in time-dependent scenarios, one can obtain valuable information about the microscopic structure of the theory.

Conceptually, the problem is that of entropy production in thermalization processes. Entropy production is a macroscopic quantity directly related to the microscopic interaction structure. The reason is that entropy is always generated by some coarse graining, defined as a practical inability of measuring certain types of information (correlators). During a thermalization process, deterministic evolution distributes information evenly over all correlators [12,13], increasing the entropy of the coarse-grained description of the system. Entropy growth is then directly related to the ability of the system to create correlations between the coarse-grained variables and the rest, and these correlations are controlled by the microscopic interaction structure.

Technically, since black hole entropy is a geometric quantity (1), the problem is that of studying scenarios with dynamical geometry. Considering general relativity coupled to a scalar field, we choose an initial state containing a black hole with entropy $S_{\mathrm{i}} = A_{\mathrm{i}}/4$ and a scalar field profile containing enough energy to backreact on the geometry. As time evolves, the scalar field collapses towards the black hole, increasing its entropy to $S_{\mathrm{f}} = A_{\mathrm{f}}/4$. We will examine concrete examples of spacetimes with different dimensions and scalar fields with different masses; see Sec. II. We will arrive at three generic lessons, contained in Eqs. (7), (10), and (12).

In Sec. III we argue that those lessons contain information about the microscopic interaction structure of black hole dynamics. By reviewing Ref. [14], we will show that the dynamics of black hole entropy perfectly matches the dynamics of entanglement entropy in democratic models. By democratic models we mean the strongest type of nonlocal models, in which every degree of freedom interacts with every other degree of freedom. The importance of such nonlocal physics for black holes was first pointed out in [15].

Our results fit well in the context of large-N matrix models and the AdS/CFT conjecture [16–18]. They provide further evidence of the claims presented in [14] concerning both the entanglement dynamics of large-N matrix models and the fast scrambling conjecture [15]. For the same reasons our results nicely connect with the model of black hole dynamics proposed in [10], and therefore they might have implications for strange metals. Finally, from a different perspective, we also expect our results to contribute to the understanding of the connection between geometry and entanglement (see [6,7,11,19–21] and references therein).

II. ENTROPY PRODUCTION IN BLACK HOLES

To study entropy production in black hole collapse scenarios we consider Einstein gravity coupled to a scalar field. The action reads

$$I = \frac{1}{8\pi}\int d^{d+1}x\sqrt{-g}\left(R + \Lambda - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2\right), \quad (2)$$

where $\Lambda = d(d-1)$ is the cosmological constant and the scalar has mass m. This means we will be looking at asymptotically anti–de Sitter (AdS) solutions. This choice is taken to make direct contact with the AdS/CFT correspondence [18]. In the discussion we comment on what would change in the asymptotically flat case.

The equations of motion are given by

$$0 = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{1}{2}g_{\mu\nu}\Lambda + \frac{1}{2}g_{\mu\nu}\left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2\right) - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi, \quad (3a)$$

$$0 = \partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) - \sqrt{-g}m^2\phi. \quad (3b)$$

In what follows we take the system to be homogeneous and isotropic at all times. Any such spacetime can be described with the ansatz

$$ds^2 = -f(r,t)dt^2 + 2dtdr + \Sigma(r,t)^2 d\vec{x}^2_{d-1}, \quad (4a)$$

$$\phi = \phi(r,t), \quad (4b)$$

which we choose to simplify the numerical solution later.

The equilibrium solution is a planar AdS-Schwarzschild black hole with temperature T_f and entropy S_f, and with vanishing scalar $\phi = 0$. Out of equilibrium, the geometry is asymptotically AdS and the scalar behaves as

$$\phi(r,t) = \frac{\phi_0(t)}{r^\Delta} + \cdots, \quad (5)$$

where $m^2 \equiv \Delta(d-\Delta)$ and dots denote higher order corrections in $1/r$.

To compute entropy production we assume that (1) generalizes to time-dependent scenarios [22]. Later on, we give some evidence of the validity of this assumption. Notice that time dependence brings a troubling ambiguity: we can consider many possibilities for the area appearing in (1) which all coincide in the equilibrium case. In this article, we will study two interesting cases, namely, the event horizon area and the area of the apparent horizon [23]. This is the area of an outermost trapped surface [26] and it can be defined locally in time (a feature which seems necessary if we want to equate it to some entanglement entropy at the boundary). Below we will study the time evolution of both choices. Although the main features will be shared by both of them, we find more compelling results for the apparent horizon area. Aside from entropy, we will also associate a temperature to the surface gravity of both horizons; see [27].

A. Extensivity of black hole entropy evolution

From (4), the area of a surface $\mathcal{A}$ at fixed t and r is given by

$$\mathrm{Area}\,(\mathcal{A}) = \int_{\mathcal{A}} d^{d-1}x\sqrt{-\gamma}, \quad (6)$$

with $\sqrt{-\gamma} = \Sigma(r,t)^{d-1}$ the determinant of the spatial part of the metric. Quite strikingly, just from the assumption that the entropy is given by the area of the horizon (apparent or event), it follows directly that given any two disconnected horizon patches $\mathcal{A}$ and $\mathcal{B}$:

$$S_{\mathcal{A}\cup\mathcal{B}}(t) = S_{\mathcal{A}}(t) + S_{\mathcal{B}}(t). \quad (7)$$

Black hole entropy is thus extensive at all times. This implies that the mutual information between different horizon patches vanishes at all times:

$$I_{\mathcal{A}\cup\mathcal{B}}(t) \equiv S_{\mathcal{A}}(t) + S_{\mathcal{B}}(t) - S_{\mathcal{A}\cup\mathcal{B}}(t) = 0. \quad (8)$$

Both previous relations are trivial observations from the point of view of gravity, but we will argue in Sec. III that they are nontrivial from a putative microscopic point of view. Notice that even though we will focus on a homogeneous system, the previous equations (7) and (8) hold generally.

An immediate consequence of (7) is that the characteristic time scale for the stabilization of the entropy evolution of a certain horizon patch does not depend on the size of the chosen patch. By entropy stabilization we mean the time by which the entropy evolution enters the plateau regime, where near equilibrium physics hold. We study the physics of the plateau in the next section.

B. Quasinormal ringing of geometric quantities

Given the extensive behavior of black hole entropy evolution (7), we would like to find the characteristic time scale of near equilibrium relaxation as well as the specific law governing the evolution of geometric quantities at the plateau.

First, we recall that close to equilibrium the scalar field is well described by a sum of quasinormal modes. These are solutions to the linearized equations of motion with ingoing boundary conditions at the event horizon and vanishing Dirichlet conditions at the boundary [28]. These solutions behave as damped harmonic oscillators,

$$\phi(r,t) = Ae^{-\omega_I t}(\cos(\omega_R t+\delta)\phi_I(r) + \sin(\omega_R t+\delta)\phi_R(r)). \tag{9}$$

There is a discrete spectrum of these modes, with higher modes decaying more quickly, so that at late times only the lowest $\omega = \omega_R + i\omega_I$ contributes. In our setup, the only quasinormal modes that can be excited with the ansatz (4) are those of ϕ, which we obtain by solving the generalized eigenvalue problem associated to Eq. (3b) [29,30].

Following methods developed in [31], one can study the backreaction produced by (9) on the geometry. We first expand the scalar and geometry around the equilibrium, $\phi = (\phi_0 = 0) + \epsilon\delta\phi^{(1)} + \epsilon^2\delta\phi^{(2)} + \cdots$ and similar for f and Σ. Plugging this into the equations of motion (3), at first order in ϵ we obtain the quasinormal mode equation, there is no backreaction at this order. At second order, $\delta f^{(2)}$ and $\delta\Sigma^{(2)}$ are induced by $(\delta\phi^{(1)})^2$. This implies that at late times, when $\delta\phi^{(1)}$ is the lowest quasinormal mode (9), the geometry will decay to equilibrium with frequency $2\omega_I$, twice as fast as the scalar.

Inspired by this and by the results of [14], we consider the following ansatz for the black hole entropy at the plateau:

$$\delta S(t) \equiv S_f - S(t) = Ae^{-2\omega_I t}(\cos(2\omega_R t + \delta) + B), \tag{10}$$

where again ω is the lowest quasinormal mode of the scalar field. Here A and δ parametrize the initial amplitude and phase. The parameter B, which we term the *damping shift*, suppresses ($B > 1$) or enhances ($B < 1$) the oscillations around the decaying exponential. We consider the same ansatz for the temperature and apply both to the apparent and event horizon.

Notice that entropy evolution is special in one regard. By Hawking's area theorem [32,33] [34], we have

$$S'(t) \geq 0, \tag{11}$$

implying the following constraint on the damping shift:

$$B \geq B_{\min} \equiv \frac{\sqrt{\omega_I^2 + \omega_R^2}}{\omega_I}. \tag{12}$$

We stress that in the following we do not assume Eqs. (11) and (12). This is merely what we expect and indeed what we will confirm.

To determine whether the ansatz (10) provides a good description of the time evolution, and to find the parameters in the ansatz, we must solve the full nonlinear equations of motion (3). We do this numerically following the methods of [36]. This approach requires a choice of an initial profile for the scalar, which we take to be $\phi(t=0,r) = T/r^\Delta$, but we stress that our results do not depend on this choice. During the evolution, we keep track of the constraint equation, which is not used to obtain the solution but

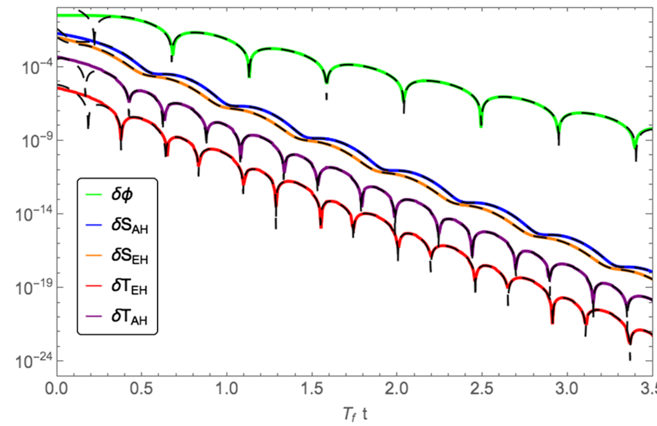

FIG. 1. Logarithmic plot of the near equilibrium evolution of the geometry in response to a small scalar perturbation, for $d = 4$ and $\Delta = 3$. Shown, from top to bottom, are the scalar field (green), the areas of the apparent and event horizon (blue, orange), and the temperatures (surface gravities) of the event and apparent horizon (red, purple). The δ always refers to the final value minus the current value, and the last two quantities are shifted down by 10^{-1} and 10^{-3}, respectively, for display purposes. Through each of these quantities a fit of the form of Eq. (10) is plotted as a dashed line.

should be satisfied automatically provided it is solved at the boundary, and thus provides a good check on the numerics. We make sure this constraint remains well below any of the other quantities we present, in all cases considered.

The results are shown in Fig 1, where we show the evolution of the entropy and temperature derived from both event and apparent horizons for $d = 4$ and $\Delta = 3$. Dashed lines are fits of the form (10) to each of these quantities, showing that this ansatz accurately describes the evolution. Actually we find that the full geometry, $\delta f(r,t)$ and $\delta\Sigma(r,t)$, is described by the ansatz of Eq. (10), where now the parameters A, B, δ depend on r.

By studying different initial profiles we see that the parameters A and δ depend on the initial conditions, but the damping shift B does not. We repeat the same process for a range of different dimensions and masses, obtaining in each case a picture which is qualitatively the same as Fig 1. In Table I the damping shift B is shown for each quantity in all the studied cases.

Remarkably, the evolution of the apparent horizon area saturates the bound of Eq. (11) at each period of oscillation, as can be seen qualitatively from the blue line in Fig 1 and quantitatively for each case by the equality $B_{S_{AH}} = B_{\min}$ from Table I. The oscillations around the decaying exponential are maximal and saturate the area theorem in general relativity. Notice that the saturation associated with the apparent horizon cannot be inferred by the area theorems themselves. These theorems are inequalities and, indeed, in the case of the event horizon we find no saturation, the growth of entropy being strictly greater than zero at all times.

TABLE I. Fit results of the damping shift B of Eq. (10) for the entropy and temperature as defined by the apparent horizon and event horizon, for a range of different dimensions d and scaling dimensions Δ.

(d, Δ)	$\omega/\pi T$	$B_{\rm min}$	$B_{\rm SAH}$	$B_{\rm SEH}$	$B_{\rm TAH}$	$B_{\rm TEH}$
(3, 2)	$1.6372 + 2.0444i$	1.281134	1.281134	1.4547	0.464	0.464
(3, 3)	$2.4659 + 3.5518i$	1.217376	1.217377	1.3845	0.532	0.532
(4, 3)	$2.1988 + 1.7595i$	1.600513	1.60051	2.0465	0.194	0.236
(4, 4)	$3.1195 + 2.7467i$	1.513227	1.51323	1.97	0.196	0.285
(5, 3)	$1.6165 + 0.8419i$	2.164801	2.1648	2.880	0.0314	0.640
(5, 4)	$2.4574 + 1.4855i$	1.933039	1.9330	2.718	0.0108	0.905
(5, 5)	$3.3087 + 2.1547i$	1.832485	1.8325	2.7	0.00994	1.10

On the other hand, the oscillations of the temperature are generally enhanced with respect to the scalar field, i.e., $B_{\rm T} < 1$. Here again, the temperature of the apparent horizon oscillates more strongly than that of the event horizon, with the difference becoming much more pronounced for larger dimensions.

Finally, we would like to note that by adding a constant source for the scalar, backreaction occurs at first order. This changes the factor $2\omega_{\rm I}$ in (10) to $\omega_{\rm I}$, but we check that the apparent horizon still saturates the bound (11), the damping shift being given by (12).

Summarizing, we arrived at three generic lessons. First, the evolution of black hole entropy is extensive (7). This implies that the evolution of any subsystem enters the near equilibrium regime in a time scale which is independent of the system size, just dependent on the lowest quasinormal mode. Second, the evolution at the plateau is accurately described by the ansatz (10). Third, when measured by the apparent horizon, black hole entropy evolution saturates area theorems, allowing us to extract an analytical formula for the damping shift B appearing in (10).

III. ENTROPY EVOLUTION IN DEMOCRATIC MODELS

In the previous section we studied the evolution of black hole entropy in general relativity, arriving at three generic lessons. As for the static Bekenstein-Hawking formula (1), we expect these lessons to contain information about the microscopic structure of quantum gravity.

The first question we want to ask is: what entropy is the horizon area computing in time-dependent scenarios? A natural proposal is the following:

$$\Delta S(t) \equiv S(t) - S_i = \Delta S_{\rm BH_{in}}(t) + \Delta S_\varphi(t), \qquad (13)$$

where $\Delta S_{\rm BH_{in}}(t)$ accounts for the change in entropy of the degrees of freedom conforming the initial black hole and $\Delta S_\varphi(t)$ accounts for the change in entropy of the microscopic degrees of freedom supporting the scalar field. Notice that the previous relation must hold at stationarity, and it is certainly natural to assume it in the time-dependent scenario. Given that entropy in unitary quantum mechanical scenarios emerges as entanglement entropy [37–40], the variation $\Delta S_\varphi(t)$ should account for the entanglement entropy between the scalar field and the degrees of freedom supporting the initial black hole. Writing $\delta S_{\rm BH_{in}}(t) \equiv \Delta S_{\rm BH_{in}}(t_f) - \Delta S_{\rm BH_{in}}(t)$ and $\delta S_\varphi(t) \equiv \Delta S_\varphi(t_f) - \Delta S_\varphi(t)$, relation (13) can be written as

$$\delta S(t) = S_{\rm f} - S(t) = \delta S_{\rm BH_{in}}(t) + \delta S_\varphi(t). \qquad (14)$$

Since both terms in the right-hand side are positive, $\delta S(t)$ is an upper bound for both terms separately. Therefore, black hole entropy production should bound the approach of the entanglement entropy between the scalar field and the black hole:

$$\delta S(t) > \delta S_\varphi(t). \qquad (15)$$

Before continuing let us remark that this interpretation and result nicely connects with the derivation of the Bousso bound using entanglement techniques [7]. It also connects with the recent proposal connecting vacuum entanglement with the Einstein equations [6]. But notice that here we are considering entropy variations in collapse scenarios, using the full nonlinear Einstein theory. Also, our objective will be to provide a tentative connection of such macroscopic results with some putative microscopic quantum dynamics. We also want to remark that computing the entropy difference is a necessary step towards computing the relative entropy [41] between the thermal state and the time evolving state. Notice that given that known modular Hamiltonians in conformal field theory (CFT) and AdS/CFT are integrals of the local energy-momentum tensor, and that our process conserves the energy density at the boundary, it might be the case that the entropy difference gives directly the relative entropy, an interesting path which we leave for future work, and which connects with the questions raised in [43].

Coming back to the bound (15), notice that the deviation from stationarity of black hole entropy is controlled by the time scales associated to the scalar field, and not by any internal properties. This observation suggests that the classical gravitational description misses $\delta S_{\rm BH_{in}}(t)$, providing

$$\delta S(t) = Ae^{-2\omega_I t}(\cos(2\omega_R t + \delta) + B) = \delta S_\varphi(t). \quad (16)$$

It seems that to get $\delta S_{\mathrm{BH_{in}}}(t)$ we have to resort to the microscopic theory, an interesting observation worth exploring further.

To avoid possible confusion, we remark that our results are not in contradiction with the known evolution of spatial entanglement entropy in AdS/CFT; see [44–46]. Here we are not considering "spatial" entanglement entropy of a putative dual field theory. The claim is that we are bounding the entanglement between the initially out of equilibrium fields and the black hole.

The second question we would like to ask is the following: what is the class of models satisfying such entropy dynamics? A specific class of many body quantum theories displaying the scaling behavior (7) and quasinormal decay (10) for the dynamics of their entanglement entropies has recently been found in [14]. This is the class of democratic theories, in which every oscillator interacts with every other oscillator. For these theories the entanglement entropy evolution of a set A of oscillators, labeled by $i = 1, \ldots, M$ was found to satisfy

$$S_A(t) = \sum_{i=1}^{M} S_i(t), \quad (17)$$

where

$$S_i(t) = (n_i(t) + 1)\log(n_i(t) + 1) - n_i(t)\log n_i(t), \quad (18)$$

and $n_i = \langle a_i^\dagger a_i \rangle$ is the average occupation number of the ith oscillator. The previous result is valid up to subleading corrections in the thermodynamic limit. It mirrors (7) and it again implies that mutual information vanishes in the thermodynamic limit at all times as in (8). Besides, on general grounds the behavior of the occupation number is that of the square of the corresponding oscillator, since we need to construct a product of creation and annihilation operators. Therefore, it was argued in [14] that n_i would ring at the plateau with twice the quasinormal frequency of the lowest quasinormal mode of subset A:

$$n_i(t) \simeq n_i^\beta + Ae^{-2\omega_I t}(\cos(2\omega_R t + \delta) + 1), \quad (19)$$

implying an analogous behavior for the entropy evolution. The behavior found in [14] for democratic systems perfectly matches the black hole results (10) and (16) at late times. But the black hole results add new physics, predicting a damping shift given by (12).

IV. DISCUSSION

In the first part of this article we studied black hole entropy production. Since black hole entropy is a geometric quantity (1) we considered scenarios with dynamical geometry. We found three general lessons:

(i) Black hole entropy evolution is extensive (7). The characteristic time scale for entering the near equilibrium regime is independent of the system size.
(ii) At the entropy plateau, black hole entropy and temperature ring with twice the frequency of the lowest quasinormal mode (10).
(iii) For the apparent horizon area, the damping shift B_{SAH} is the maximal one compatible with area theorems in general relativity (12).

These universal macroscopic lessons are interesting in their own right.

In the second part of the article, we provided a coherent microscopic interpretation of the first two macroscopic results. First, we argued on general grounds that the deviation from stationarity of black hole entropy should bound the deviation from stationarity of the entanglement between the out of equilibrium modes associated to the classical scalar field and the initial black hole degrees of freedom. Second, we presented a class of systems with such highly nontrivial entanglement dynamics. This is the class of democratic systems; see Sec. III and [14]. Given this connection, the fact that the damping shift B_{SAH} saturates area theorems in general relativity is an unexpected prediction from black hole physics to the physics of democratic systems, and might turn into a prediction for the physics of strange metals [10].

We end this article with some remarks. The first is that these results have been obtained from a purely macroscopic perspective. We have not invoked any specific microscopic theory of quantum gravity, such as string theory [47]. It is tempting to conclude that any putative theory of quantum gravity should show such democratic behavior, whether at a microscopic or at an emergent level. On the other hand, string theory complies with such requirements [15]. Indeed, within matrix models and AdS/CFT [16–18] our results fit remarkably well. In this context, the deviation from stationarity of black hole entropy bounds the entanglement dynamics between different subsectors of the large-N matrix model. In the field theory, this type of entanglement dynamics were studied in [14], where the behavior (10) was found. This unexpected agreement between both computations provides further evidence of the claims made in [14] concerning the fast dynamics of entanglement growth in large-N matrix models, and deserves further consideration. For the same reasons, our results might also be interesting for the microscopic approach to black hole dynamics presented in [10].

While in this article we focused on homogeneous, isotropic, scalar-driven evolution of an asymptotically AdS black brane, we would like to comment on several generalizations. First, the case of a massless scalar corresponds exactly to that of an anisotropy in the metric, generalizing away from just scalars, and from isotropy. As to homogeneity, while this was an assumption, this is not a strict requirement, since the entropy evolution (10) applies to late times it is sufficient that any present inhomogeneous mode dies out faster than the homogeneous part. Finally regarding the asymptotics, we focused on an asymptotically AdS black hole, but we note that the extensivity (7) applies also to the asymptotically flat case, and so does the factor 2 in (10) [48]. Whether or not the saturation of the damping shift generalizes is an open question. We did confirm, although not presented here, that in the anisotropic asymptotically Lifshitz setup of [49] the damping shift is also saturated.

We also want to remark that the observed damping shift B deserves further consideration. It would be interesting to obtain analytical control over it in the case of other geometric quantities like the temperature and also obtain a microscopic understanding of the mechanism that gives rise to the saturation (12).

Another interesting application of our results concerns the dynamics of spatial entanglement entropy in AdS/CFT [45,46]. Given that at sufficiently large times the minimal area surface has a contribution coming from the area of the event horizon, we expect to see the quasinormal oscillations in such a quantity as well. These oscillations were missed in those references due to the specific nature of the Vaidya metric, and we expect to see them in more realistic collapse scenarios.

Lastly, we hope that the relation between horizon area, quasinormal ringing, and entanglement dynamics uncovered in this article will contribute to a better understanding of the puzzle of deriving geometry from the properties of entanglement (see [11,19–21,35] and references therein), and also to the connection between entanglement and the Bousso bound [7]. Potentially, it might also be of interest to approaches of emergent general relativity based on entropy densities of holographic screens [6,50].

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[22] Although this might seem for some readers a trivial assumption, the authors are not aware of any proof or clear argument that this is indeed the case from a microscopic point of view, and will therefore leave it as an assumption.

[23] There is an ambiguity in the definition of the apparent horizon in that it depends on how the spacetime is foliated by spacelike hypersurfaces. A lot of work has been done recently to remove this ambiguity by defining different, closely related, horizons; see, e.g., [24,25]. Here we take the practical approach, common in the holographic thermalization literature, of simply picking a foliation [slices of constant t as defined in (4)].

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