PHYSICAL REVIEW D **94,** 103514 (2016)

Asymptotically safe gravity and nonsingular inflationary big bang with vacuum birth

Georgios Kofinas[1,*] and Vasilios Zarikas[2,3,†]

[1] *Research Group of Geometry, Dynamical Systems and Cosmology, Department of Information and Communication Systems Engineering, University of the Aegean, Karlovassi 83200, Samos, Greece*
[2] *Central Greece University of Applied Sciences, Department of Electrical Engineering, 35100 Lamia, Greece*
[3] *Nazarbayev University, School of Engineering, Astana 010000, Republic of Kazakhstan*

(Received 4 July 2016; published 15 November 2016)

General nonsingular accelerating cosmological solutions for an initial cosmic period of pure vacuum birth era are derived. This vacuum era is described by a varying cosmological "constant" suggested by the renormalization group flow of asymptotic safety scenario near the ultraviolet fixed point. In this scenario, a natural exit from inflation to the standard decelerating cosmology occurs when the energy scale lowers and the cosmological constant becomes insignificant. In the following period where matter is also present, cosmological solutions with characteristics similar to the vacuum case are generated. Remarkably the set of equations allows for particle production and entropy generation. Alternatively, in the case of nonzero bulk viscosity, entropy production and reheating is found. As for the equations of motion, they modify Einstein equations by adding covariant kinetic terms of the cosmological constant which respect the Bianchi identities. An advance of the proposed framework is that it ensures a consistent description of both a quantum vacuum birth of the universe and a subsequent cosmic era in the presence of matter.

DOI: 10.1103/PhysRevD.94.103514

I. INTRODUCTION

An interesting framework for the discovery of a theory of everything is the renormalization group approaches to quantum gravity [1] that encapsulate perturbative and nonperturbative field theoretic techniques and functional renormalization group flow investigations. A concrete and minimal scheme for quantum gravity that includes no inconsistencies is the asymptotic safety (AS) program, or otherwise called quantum Einstein gravity [2]. It is a model that keeps the same fields and symmetries from general relativity and it was first proposed as an idea by Weinberg [3]. The key issue is the existence of a non-Gaussian fixed point (NGFP) of the renormalization group (RG) flow for gravity. Due to this NGFP that determines the behavior of the theory at the UV, all measured quantities are free from nonphysical divergences.

The asymptotic safety scenario is based on the mathematical technique of the functional renormalization group equation for gravity [4], which enables the detailed analysis of the gravitational RG flow at a nonperturbative level [5,6]. It was possible to prove that the scaling behavior of the dimensionful Newton's constant is antiscreened at high energies [7], a behavior that leads to the NGFP which is necessary for asymptotic safety. Further studies include matter and more gravitational operators in the action [8–13]. The key ingredient of the theory is the gravitational effective average action Γ_k. This keeps only the effect of the quantum fluctuations with momenta $p^2 > k^2$, thus Γ_k expresses an approximate description of physics at the momentum scale $p^2 \approx k^2$. The truncated RG flow equations leave two running couplings (with respect to energy), the gravitational constant $G(k)$ and the (positive) cosmological constant $\Lambda(k)$. Near the non-Gaussian UV fixed point the coupling G is known to approach a zero value, while on the other hand, the coupling Λ goes to infinity.

The description with the help of the effective average action and the functional RG flow enables also the development of phenomenological investigations in the context of the asymptotic safety proposal. Various investigations of "RG improved" black holes first appeared in [14]. Other works extended the studies to the Vaidya metric [15] and to the modified Kerr metric [16]. The thermodynamic properties of these black holes were described in [17]. Black hole solutions from the inclusion of higher derivative terms in the effective average action were presented in [18]. Other works analyzing black holes with RG improvements have been proposed in [19–25].

Of particular interest are investigations of the nature of the microscopic structure of the asymptotically safe quantum spacetime [26–29]. It seems that quantum corrections at high energies (near the nontrivial UV fixed points) modify drastically the classical picture since fractal dimensionality seems to appear.

An important phenomenological topic that can also test the properties and the new point of view of AS gravity is the study of RG improved cosmologies, which first appeared

*gkofinas@aegean.gr
†vzarikas@teilam.gr

in [30], and was further studied in [31–37] (see also review [38]). Along these lines of research it was possible to propose a solution of the cosmic entropy issue [39]. Another interesting outcome is that RG improved cosmologies admit exponential or power-law inflationary solutions [40].

The scope of this work is to investigate in the context of AS gravity the evolution of the universe at high energies. First, we work on the assumption that cosmos had a quantum vacuum birth, first speculated in [41]. Furthermore, the consequent period in the presence of matter is analyzed. Matter is expected to appear due to energy transfer from vacuum to matter fields as the expansion proceeds. In summary, we hypothesize that the universe starts in a vacuum state that is characterized by an energy dependent cosmological constant $\Lambda(k)$. Subsequently, Einstein equations include a nonzero matter energy momentum tensor with an energy dependent Newton's constant. Both Λ and G respect the energy dependence that is predicted in the context of AS at the NGFP. It will be shown that the matter solutions predict particle production and entropy generation or negative viscous pressure associated with entropy production and reheating.

It is common in AS literature, in order to improve existing solutions of Einstein equations to set $\Lambda(k)$ and $G(k)$ as functions of energy k. The simple input of $\Lambda(k)$ and $G(k)$ into the classical vacuum equations results in violation of the Bianchi identities, while this same input into a classical solution creates a metric which is not a solution of a well-defined theory. In [42], the formalism of obtaining RG improved solutions that respect Bianchi identities is presented at the action level, while in [43] an alternative and mathematically more solvable approach was developed at the level of equations of motion. In [43] the formalism includes the appropriate covariant kinetic terms that support an arbitrary source field $\Lambda(k)$ without any symmetry assumption. Here, we extend the formalism presented in [43], beyond the vacuum case, to also include matter. The present study provides novel quantum gravity inspired modified Einstein equations capable of describing both absence of matter cases and configurations where vacuum and matter contributions are realized. This new scheme proposes a consistent way to respect Bianchi identities in both alternatives.

Consequently, an important question is how to relate the RG scale parameter k to cosmological time/proper length in order for the differential equations to make sense. The first works [30] have chosen in cosmology the RG scale inversely proportional to cosmological time, and subsequently, the more favorable connection with the Hubble scale was investigated. In other works the RG scale is linked with the fourth root of the energy density [44], the cosmological event/particle horizons [45], or curvature invariants like Ricci scalar [46–48].

The novel scheme that is encapsulated in the presented new quantum inspired equations of motion exhibits various new interesting features. The modified Einstein equations, together with the modified energy-momentum conservation, suggest a constraint on the allowed/compatible matter content, and either set a constraint or not on the allowed functional dependence $k(L)$ between the energy scale k and the geometrical scale/length L that is connected to the expansion of the universe. Remarkably both alternatives result in interesting consequences. When $k(L)$ is left free, it is possible to get entropy generation from particle production and nonsingular accelerating solutions, while when the energy dependence $k(L)$ is restricted, similar cosmologies with bulk viscosity, entropy generation and reheating arise. One should notice that the presented modified Einstein equations in the spirit of AS program is an effective description of gravity near the NGFP and they do not describe the low energy cosmic expansion.

The organization of the paper is as follows. In Sec. II we solve at high energies near the NGFP the consistent RG improved equations that govern the homogeneous and isotropic universe with energy dependent cosmological constant for different choices of the energy-length scaling. In Sec. III we present the modified equations which contain both vacuum energy and matter and are consistent with the vacuum equations. The full space of solutions is found with either particle production or bulk viscosity. In Sec. IV a discussion of the inflation and the thermodynamics of the universe is presented. Finally, we conclude in Sec. V.

II. VACUUM COSMOLOGICAL SOLUTIONS

In [43] consistent modified Einstein equations have been presented which describe how a classical spacetime is affected/shaped in the presence of a quantum vacuum. The quantum vacuum is modeled through a nonzero cosmological constant term which is energy dependent according to the AS program. Several interesting spherically symmetric solutions have also been derived there, with some of them exhibiting nonsingular behavior. In this section we solve the same vacuum field equations but for the case of a homogenous and isotropic metric. Vacuum solutions with the cosmological constant as the only source are of particular importance. The reason is that the birth of our universe from a vacuum fluctuation is a favored scenario in various quantum gravity inspired cosmological models. An extension of the vacuum field equations appears in the next sections, capable to describe consistently the cosmic evolution including both a vacuum and a matter content, using a positive cosmological "constant" $\Lambda(k)$ and a gravitational Newton's constant $G(k)$ in the spirit of AS. Thus, the same set of equations will be able to describe both cosmological eras, namely an initial quantum vacuum birth and a subsequent period with nonzero vacuum and matter contributions.

The proposed modified vacuum Einstein equations can be seen as a general Λ varying model of modified gravity. It is not related to a specific energy dependent RG running law of the coupling Λ and that is why it can be useful to describe all types of running laws of Λ. Usually in the AS literature RG improved Einstein equations (or solutions thereof) are taken to give an effective description of physics at a characteristic energy scale k. The same is true for our equations. Since our vacuum equations contain no matter, they are perfect for the theoretical description of a semi-classical analysis of AS spacetime near the center of black holes or in the big bang trans-Planckian regime. Thus, the produced vacuum cosmological solutions do not contradict with other proposed AS inspired cosmologies that appear in the literature; they might be seen that describe the very initial period of the universe before the period described by other asymptotic safe gravity cosmologies that appear in the literature.

Let us begin modeling the quantum vacuum dominated initial cosmic era. The mathematical description of this era is based on the modified vacuum Einstein equations derived in [43]

$$G_{\mu\nu} = -\bar{\Lambda}e^{\psi}g_{\mu\nu} - \frac{1}{2}\psi_{;\mu}\psi_{;\nu} - \frac{1}{4}g_{\mu\nu}\psi^{;\rho}\psi_{;\rho} + \psi_{;\mu;\nu} - g_{\mu\nu}\Box\psi. \tag{2.1}$$

The field $\psi(x)$ is related to the cosmological constant through $\Lambda = \bar{\Lambda}e^{\psi}$, where $\bar{\Lambda}$ is an arbitrary constant reference value. Equations (2.1) form a minimal extension of Einstein equations containing first and second derivatives of ψ. They are by construction identically covariantly conserved for any $\psi(x)$, so the Bianchi identities are satisfied. Since $\psi(x)$ does not have its own equation of motion, it can be determined externally, e.g. as implied by the AS scenario. Indeed, geometry independent RG flow equations predict the running of both $\Lambda(k)$, $G(k)$ at high energies near the NGFP, where the cosmological constant/coupling is given by [49]

$$\Lambda = \lambda_* k^2 \tag{2.2}$$

with $\lambda_* > 0$ a dimensionless constant.

We will be interested in the present work in the investigation of cosmology at very high energies, so that Eq. (2.2) can be applied. The spatially homogeneous and isotropic cosmological metric is

$$ds^2 = -n(t)^2dt^2 + a(t)^2\left[\frac{dr^2}{1-\kappa r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{2.3}$$

where $n(t)$ is the lapse function and $\kappa = -1, 0, 1$ characterizes the spatial curvature. The coupling $\Lambda(x)$ carries the same symmetries, so it is $\Lambda = \Lambda(t)$. For the metric (2.3) the nonvanishing components of the Einstein tensor G^{μ}_{ν} are

$$G^t_t = -3\left(H^2 + \frac{\kappa}{a^2}\right)$$
$$G^i_j = -\left(\frac{2}{n}\dot{H} + 3H^2 + \frac{\kappa}{a^2}\right)\delta^i_j, \tag{2.4}$$

where the indices i, j refer to the spatial coordinates, $H = \frac{\dot{a}}{na}$ is the Hubble parameter and a dot denotes a differentiation with respect to t. It is also possible to evaluate

$$\psi^{;\rho}\psi_{;\rho} = -\frac{\dot{\psi}^2}{n^2}, \qquad \Box\psi = -\frac{1}{n}\left(\frac{\dot{\psi}}{n}\right)^{\cdot} - 3H\frac{\dot{\psi}}{n} \tag{2.5}$$

and the nonvanishing components of $\psi_{;\mu;\nu}$ by

$$\psi_{;t;t} = n\left(\frac{\dot{\psi}}{n}\right)^{\cdot}, \qquad \psi_{;r;r} = -\frac{Ha^2}{1-\kappa r^2}\frac{\dot{\psi}}{n},$$
$$\psi_{;\theta;\theta} = -Ha^2r^2\frac{\dot{\psi}}{n}, \qquad \psi_{;\phi;\phi} = -Ha^2r^2\sin^2\theta\frac{\dot{\psi}}{n}. \tag{2.6}$$

Therefore, the two components of (2.1) are

$$3\left(H^2 + \frac{\kappa}{a^2}\right) = \bar{\Lambda}e^{\psi} - 3H\frac{\dot{\psi}}{n} - \frac{3\dot{\psi}^2}{4n^2} \tag{2.7}$$

$$\frac{2}{n}\dot{H} + 3H^2 + \frac{\kappa}{a^2} = \bar{\Lambda}e^{\psi} - 2H\frac{\dot{\psi}}{n} - \frac{\dot{\psi}^2}{4n^2} - \frac{1}{n}\left(\frac{\dot{\psi}}{n}\right)^{\cdot}. \tag{2.8}$$

Equations (2.7) and (2.8) are satisfied by construction for any $\psi(t)$. Equation (2.8) is not independent, since by differentiating Eq. (2.7) with respect to t and using (2.7) itself, we get (2.8) multiplied by $1 + \frac{\dot{\psi}}{2nH}$. Hereafter, in the following vacuum solutions we consider t to be the cosmic time and take $n = 1$. In order to proceed with the solution of (2.7) we have to determine ψ as a function of e.g. t, H, a, using Eq. (2.2). This will come by the selection of a scaling that associates the energy of RG scale k to a characteristic time or length of the solution.

It is worth mentioning that k as a function of cosmic time starts with an infinite or a very high value and should decrease as the universe departs from the NGFP. However, it is possible that during the cosmic evolution, still in the proximity of the NGFP, $k(t)$ may increase for some era.

A. Scaling $k \propto 1/t$

In cosmological models of asymptotically safe gravity it is common to use as a reasonable scaling the following expressions [30] and [31]:

$$k = \frac{\xi}{t}, \tag{2.9}$$

where $\xi > 0$ is a dimensionless parameter and time t is considered positive valued. We are interested in understanding the cosmological behavior in a regime where time takes sufficiently small values so that k takes its high energy values. From (2.9) obviously k decreases with time. The relation of ψ with time is $e^{\psi} = \lambda_* \xi^2/(\bar{\Lambda} t^2)$ and the Friedmann equation (2.7) becomes

$$H^2 + \frac{\kappa}{a^2} = \frac{\lambda_* \xi^2 - 3}{3t^2} + \frac{2H}{t}. \tag{2.10}$$

Equation (2.10) is invariant under the transformation $t \to \lambda t$, $a \to \lambda a$, therefore defining

$$z = \frac{a}{t}, \tag{2.11}$$

we find the equation

$$\dot{z}^2 = \frac{\omega^2 z^2 - \kappa}{t^2}, \tag{2.12}$$

where

$$\omega = \sqrt{\frac{\lambda_* \xi^2}{3}}. \tag{2.13}$$

For $\kappa = 0$, the solution is

$$a(t) = c t^{1 \pm \omega}, \tag{2.14}$$

where $c > 0$ is the integration constant. For the upper branch, or for the lower branch with $\omega < 1$, the solutions are expanding starting for $t = 0$ at zero scale factor. These solutions have divergent Ricci scalar R at $t = 0$, therefore they are typical singular solutions. Note that the upper branch describes a power law inflation which becomes stronger as $\omega \gg 1$. Especially the lower branch for $\omega = \frac{1}{2}$ has $R = 0$ for $t = 0$, but the divergencies appear in higher curvature invariants. For the lower branch with $\omega > 1$ the solution is contracting starting for $t = 0$ at infinite scale factor. An interesting general comment can be made at this point. In the AS scenario, an inflationary period is always followed by a natural exit from inflation and this occurs when the energy scale becomes smaller than the inflation scale. Then, the quantum modifications of $\Lambda(k)$ start to become insignificant and its value is rapidly decreasing, leading to the standard decelerating cosmology.

For $\kappa = +1$ it should be from (2.12) $a > \omega^{-1} t$. The solution of (2.12) gives

$$a(t) = \frac{t}{2\omega}(c t^{\pm\omega} + c^{-1} t^{\mp\omega}), \tag{2.15}$$

where $c > 0$ is the integration constant, under the constraint $a < c\omega^{-1} t^{1\pm\omega}$, i.e. $c t^{\pm\omega} > 1$. Due to this constraint it is seen that the upper branch describes a nonsingular (finite curvature invariants) expanding universe with $a_{\min} = \omega^{-1} c^{-\frac{1}{\omega}}$, $t_{\min} = c^{-\frac{1}{\omega}}$, where the maximum value $k_{\max} = \xi c^{\frac{1}{\omega}}$ can be made as large as desired choosing c sufficiently large. Moreover, it is seen that this solution has $\ddot{a} > 0$ which means that it is accelerating. For the lower branch with $\omega < 1$, the solution is expanding starting for $t = 0$ at zero scale factor; at $t = 0$ there is a singularity with divergent Ricci scalar and close to $t = 0$ it is to leading order $a \approx \frac{c}{2\omega} t^{1-\omega}$. Moreover, this universe enters from a decelerating to an accelerating phase. The lower branch for $\omega > 1$ describes a universe which starts from infinite volume, collapses, and at a finite scale factor bounces to an expanding universe which has an end; at the bounce it is $t \sim c^{\frac{1}{\omega}}$, $a \sim c^{\frac{1}{\omega}}$ and the energy scale $k \sim \xi c^{-\frac{1}{\omega}}$ can be as large as desired choosing c sufficiently small. This branch is also accelerating. The lower branch with $\omega = 1$ describes also a nonsingular expanding universe.

Finally, for $\kappa = -1$, the solution is

$$a(t) = \frac{t}{2\omega}(c t^{\pm\omega} - c^{-1} t^{\mp\omega}), \tag{2.16}$$

where $c > 0$ is the integration constant. Since $a > 0$ it should be $c t^{\pm\omega} > 1$. For the upper branch the solution is expanding starting for $t = c^{-\frac{1}{\omega}}$ at zero scale factor where there is a curvature singularity; this solution is accelerating. For the lower branch with $\omega < 1$ the solution initially expands from a singularity starting for $t = 0$ at zero scale factor and finally bounces and collapses again to a singular zero volume; this solution is decelerating. For the lower branch with $\omega > 1$ the solution is contracting starting for $t = 0$ at infinite scale factor and results to a singular big crunch.

We resume with the most interesting general solutions of the scaling $k \propto 1/t$. For the spatially flat 3-space topology a strong power law inflation can occur close to the initial singularity. For the positively curved case all solutions have accelerating phases which can support an inflationary epoch; they either avoid the initial big bang singularity or they possess a big bang or during a collapsing phase avoid the big crunch towards expansion. For the negatively curved topology a singular accelerating cosmology can appear.

For the alternative scaling $k = \frac{\xi}{a(t)}$, where k is inversely proportional to the proper distance at fixed t, Eq. (2.7) is satisfied for any $a(t)$ given that $\kappa = 1 = \frac{\lambda_* \xi^2}{3}$, so it does not provide useful information. This is also an outcome of other AS cosmology studies found in the literature [30,31].

B. Scaling $k \propto H(t)$

In order to investigate the time evolution of the cosmic scale factor using the full effective action $\Gamma(g_{\mu\nu})$, we can use the fact that the Hubble parameter appears as a mass in propagators. Therefore, a sensible approximation is to disregard the contributions of quantum fluctuations with wavelengths greater than H^{-1} since they are suppressed. This leads to use as a connection of energy scale to the length scale a relation of the form $k \sim H(t)$ [31]. Thus, we assume here

$$k = \xi H(t), \tag{2.17}$$

where the dimensionless parameter ξ is $\xi > 0$ for $H > 0$ and $\xi < 0$ for $H < 0$. It is obvious that a bouncing solution is not possible in this case. Then, $e^{\psi} = \lambda_* \xi^2/(\bar{\Lambda} H^2)$ and the Friedmann equation (2.7) becomes

$$(1-\omega^2)H^2 + 2\dot{H} + \frac{\dot{H}^2}{H^2} + \frac{\kappa}{a^2} = 0, \tag{2.18}$$

where

$$\omega = \sqrt{\frac{\lambda_* \xi^2}{3}}. \tag{2.19}$$

Equation (2.18) is written as

$$a^2 \ddot{a}^2 - \omega^2 \dot{a}^4 + \kappa \dot{a}^2 = 0. \tag{2.20}$$

Setting

$$u = \dot{a}, \tag{2.21}$$

Eq. (2.20) takes the form

$$a\frac{du}{da} = \pm\sqrt{\omega^2 u^2 - \kappa}. \tag{2.22}$$

If $\kappa = 1$ it should be $|u| > \omega^{-1}$.

For $\kappa = 0$ the solution of (2.22) is

$$u = c a^{\pm\omega}, \tag{2.23}$$

with c an integration constant, and thus

$$a(t) = [c(1 \mp \omega)(t - t_0)]^{\frac{1}{1\mp\omega}}, \tag{2.24}$$

where t_0 is an integration constant. The constant c should be positive for expanding solutions. The upper branch with $\omega < 1$ or the lower branch describe typical power law singular expanding solutions. However, this upper branch is inflationary and the inflation can become very strong if $\omega \to 1$. Moreover, in both cases it can be seen that k decreases with time. The upper branch with $\omega > 1$ describes a collapsing universe which asymptotically goes to zero scale factor with finite however curvature invariants.

Especially for the upper branch with $\omega = 1$ we get the nonsingular de Sitter solution $a \propto e^{ct}$ which describes a typical inflationary period with the main advance, as referred previously, that a natural exit occurs.

For $\kappa = +1$ the solution of (2.22) is

$$u = \frac{1}{2\omega}(c a^{\pm\omega} + c^{-1} a^{\mp\omega}), \tag{2.25}$$

where $c > 0$ is the integration constant. Since $u > 0$, there are only expanding solutions and it is $\xi > 0$. For the upper branch it is $a > c^{-\frac{1}{\omega}}$ which means that the solution avoids the zero scale factor regime. It is obvious from equations (2.17), (2.21), and (2.25) that this solution has a finite $k_{\max}$. Moreover, it can be shown that the energy scale k decreases with time. For the lower branch it is $a < c^{\frac{1}{\omega}}$.

Using (2.25), the Ricci scalar as a function of the scale factor can be found to be

$$R = \frac{3}{2\omega^2 a^2}[c^2(1 \pm \omega)a^{\pm 2\omega} + c^{-2}(1 \mp \omega)a^{\mp 2\omega} + 2(1 + 2\omega^2)]. \tag{2.26}$$

Therefore, the upper branch has finite scalar curvature and leads to a nonsingular cosmology, while the lower branch has a curvature singularity at $a = 0$. From (2.22) it is seen that the upper branch is accelerating and the lower decelerating.

Integrating (2.25),[1] the dependence on time can be obtained as

$$t - t_0 = \frac{2c\omega}{1 \pm \omega} a^{1\pm\omega} f(-c^2 a^{\pm 2\omega}), \tag{2.27}$$

where t_0 is an integration constant which can be absorbed into a redefinition of t. Here the function $f(x)$ satisfies the hypergeometric differential equation

$$x(1-x)\frac{d^2 f(x)}{dx^2} + [\alpha + 2 - (\alpha + 3)x]\frac{df(x)}{dx} - (\alpha + 1)f(x) = 0, \tag{2.28}$$

where $\alpha = \frac{\pm 1 - \omega}{2\omega}$. Equation (2.28) has (for α not an integer) two independent solutions,[2] one is $x^{-\alpha-1}$ and the other ${}_2F_1(1, \alpha + 1; \alpha + 2; x)$. The first solution just contributes to the constant t_0, thus

$$t - t_0 = \sigma \frac{2c\omega}{1 \pm \omega} a^{1\pm\omega} {}_2F_1\left(1, \frac{\omega \pm 1}{2\omega}; \frac{3\omega \pm 1}{2\omega}; -c^2 a^{\pm 2\omega}\right), \tag{2.29}$$

where σ is a proportionality constant to be determined from some limiting process where the time integral can be computed exactly and it arises because the hypergeometric equation is homogeneous.

For the upper branch, since $a > c^{-\frac{1}{\omega}}$, in the limit $a \to \infty$ it can be found the behavior $t - t_0' \approx \frac{2c^{-1}\omega}{1-\omega} a^{1-\omega}$. On the other hand, the hypergeometric function in (2.29) is expressed[3] as ${}_2F_1(1, \alpha + 1; \alpha + 2; -x) = \frac{\alpha+1}{\alpha}\frac{1}{x}{}_2F_1(1, -\alpha; 1 - \alpha; -\frac{1}{x}) + \frac{\Gamma(\alpha+2)\Gamma(-\alpha)}{x^{\alpha+1}}$, therefore for $x \to +\infty$ it is ${}_2F_1(1, \alpha+1; \alpha+2; -x) \approx \frac{\alpha+1}{\alpha}\frac{1}{x} + \frac{\Gamma(\alpha+2)\Gamma(-\alpha)}{x^{\alpha+1}}$. Comparing the two asymptotic expressions it is found $\sigma = 1$ and (2.29) becomes

[1]The differential equation (2.25) or (2.32) $2\omega\dot{a} = c a^{\pm\omega} + \kappa c^{-1} a^{\mp\omega}$ is integrated through the transformation $x = c^2 a^{\pm 2\omega}$ to $t - t_0 = \pm c^{\mp\frac{1}{\omega}} \int \frac{x^\alpha}{x+\kappa} dx$, thus $t - t_0 = \pm c^{\mp\frac{1}{\omega}} \frac{x^{\alpha+1}}{\alpha+1} f(-\kappa x)$, where $f(x)$ satisfies Eq. (2.28).

[2]See [50], p. 71, formula 8.

[3]See [51], p. 559, formula 15.3.7.

$$t-t_0=\frac{2c\omega}{1+\omega}a^{1+\omega}{}_2F_1\left(1,\frac{\omega+1}{2\omega};\frac{3\omega+1}{2\omega};-c^2a^{2\omega}\right). \tag{2.30}$$

For $\omega<1$ this solution expresses a nonsingular universe starting at a finite t and expanding to infinity at infinite proper time, while for $\omega>1$ it is again a nonsingular universe expanding to infinity in finite proper time, so it develops a big rip (of course this big rip is not true since the validity of the approximation terminates after some time).

For the lower branch, since $a<c^{\frac{1}{\omega}}$, in the limit $a\to 0$ it can be found the behavior $t-t_0'\approx\frac{2c^{-1}\omega}{1+\omega}a^{1+\omega}$. Similarly as before, approximating the hypergeometric function in the neighborhood of $a=0$, we find $\sigma=1$, thus (2.29) becomes

$$t-t_0=\frac{2c\omega}{1-\omega}a^{1-\omega}{}_2F_1\left(1,\frac{\omega-1}{2\omega};\frac{3\omega-1}{2\omega};-c^2a^{-2\omega}\right). \tag{2.31}$$

This solution represents a singular expanding cosmology.

For $\omega=1$ the above analysis does not work since α becomes an integer. In this case the analytic solution of (2.25) is simpler, $a=c^{\mp1}\sqrt{e^{c^{\pm1}(t-t_0)}-1}$. The upper branch is an expanding nonsingular solution and the lower an expanding singular one.

For $\kappa=-1$ the solution of (2.22) is

$$u=\frac{1}{2\omega}(ca^{\pm\omega}-c^{-1}a^{\mp\omega}), \tag{2.32}$$

where $c>0$ is the integration constant. In this case there is no bound for a. Integrating (2.32) the dependence on time can be obtained as

$$t-t_0=\frac{2c\omega}{1\pm\omega}a^{1\pm\omega}f(c^2a^{\pm2\omega}), \tag{2.33}$$

where $f(x)$ satisfies again the differential equation (2.28). Similarly to the case $\kappa=1$ it arises for α noninteger

$$t-t_0=\sigma\frac{2c\omega}{1\pm\omega}a^{1\pm\omega}{}_2F_1\left(1,\frac{\omega\pm1}{2\omega};\frac{3\omega\pm1}{2\omega};c^2a^{\pm2\omega}\right). \tag{2.34}$$

There are no expanding solutions in this case, since for $H>0$ it is from (2.32) that $ca^{\pm\omega}>1$, but then, the argument of the hypergeometric function in (2.34) is larger than 1 and the hypergeometric function gets complex values. Concerning the collapsing solutions, for the upper branch it is $a<c^{-\frac{1}{\omega}}$ and from the limiting behavior for $a\approx 0$ it arises $\sigma=-1$, thus the collapsing solution starts with the finite initial scale factor with $t\to-\infty$ and results to $a=0$ at finite time. Concerning the collapsing solutions of the lower branch it is $a>c^{\frac{1}{\omega}}$ and from the limiting behavior for $a\to\infty$ it arises $\sigma=-1$. Thus for $\omega>1$ the collapsing solution starts with the infinite initial scale factor at finite time and results to the finite scale factor at infinite t. For $\omega<1$ the solution starts with an infinite scale factor at $t=-\infty$ and results to finite a with $t=+\infty$.

We summarize with the most interesting solutions of the scaling $k\propto H$. For the spatially flat 3-space topology a strong power law inflation can happen close to the initial singularity. For the positively curved case there are general expanding solutions which are nonsingular and accelerating, so they can support inflation.

III. COSMOLOGICAL MATTER SOLUTIONS

In this section we add, beyond the spacetime-dependent cosmological constant $\Lambda(x)$, some extra matter carrying an energy-momentum tensor $T_{\mu\nu}$. The full gravitational equation is now obtained by supplementing the vacuum equations (2.1) with $T_{\mu\nu}$ in the following minimal way:

$$G_{\mu\nu}=-\bar{\Lambda}e^{\psi}g_{\mu\nu}-\frac{1}{2}\psi_{;\mu}\psi_{;\nu}-\frac{1}{4}g_{\mu\nu}\psi^{;\rho}\psi_{;\rho}+\psi_{;\mu;\nu}-g_{\mu\nu}\Box\psi+8\pi GT_{\mu\nu}. \tag{3.1}$$

The Newton's constant G will also be assumed to be spacetime dependent, $G(x)$, while again the field $\psi(x)$ is defined through $\Lambda=\bar{\Lambda}e^{\psi}$ with $\bar{\Lambda}$ an arbitrary constant reference value. In the scenario of asymptotic safety both Λ, G are supposed to be determined uniquely as functions of the energy scale from the RG flow equations, so it is $\Lambda(k)$, $G(k)$, but at present we construct a formulation where Λ, G can be kept arbitrary functions. Later, we will employ for the early-times cosmological period the energy dependent couplings predicted at the NGFP of AS scenario to be

$$\Lambda=\lambda_*k^2,\qquad G=\frac{g_*}{k^2}, \tag{3.2}$$

where $\lambda_*, g_*>0$ are dimensionless constants (the above scalings are also consistent with dimensional analysis without the introduction of a new energy scale). Since in the absence of $T_{\mu\nu}$, Eq. (3.1) is identically covariantly conserved for any $\psi(x)$, the following conservation equation for $T_{\mu\nu}$ holds:

$$(GT_{\mu\nu})^{;\nu}=0, \tag{3.3}$$

which provides an interaction between $T_{\mu\nu}$ and G. This way, Eq. (3.1) is meaningful either in the absence or in the presence of a matter content. The main advance of our theory is indeed that it encapsulates the vacuum case, something that has not appeared so far in the literature of varying constants or AS gravities.

An alternative option, assumed often in the literature, other than Eq. (3.3), would be to ignore the ψ-kinetic terms in (3.1), and then, the Bianchi identities would imply another conservation equation for $T_{\mu\nu}$ containing both Λ, G and their derivatives. Another option, also met in the literature, would be, besides ignoring again the ψ-kinetic terms, to assume the exact conservation of matter $T_{\mu\nu}^{;\nu}=0$, and then, Λ, G cannot be picked arbitrarily and are usually incompatible with AS relation $G\sim\Lambda^{-1}$. However, both of these two approaches are not satisfactory since in the absence of matter the system would be inconsistent. Considering a variation of an action principle, equations containing different than (3.1) ψ-kinetic terms, as well as G-kinetic terms, could in principle arise, however, such a scheme cannot be explicitly implemented for a general case matter content $T_{\mu\nu}$. Finally, in the literature [52,53] another approach considers specific form for the metric (cosmological, spherically symmetric, etc.), solves some RG-like differential equations together with the Einstein equations and finds different than (3.2) functional dependences $\Lambda(k), G(k)$. In the scheme that we follow we have the advantage that we use the functions (3.2) suggested by the background independent [4] running of RG equations.

As in the previous section with the vacuum solutions, we also here assume a spatially homogeneous and isotropic metric for the cosmic spacetime of the form (2.3). Since the external fields $\Lambda(x), G(x)$ carry the same symmetries, they will be of the form $\Lambda(t), G(t)$. We consider a diagonal energy-momentum tensor T^{μ}_{ν}, so we take as matter content a nonperfect fluid with energy density ρ, thermodynamic pressure p and a nonequilibrium part π [54–57]. Due to the working symmetries of isotropy and homogeneity shear viscosity and energy fluxes are disregarded. The energy momentum tensor is

$$T^{\mu\nu}=\rho u^{\mu}u^{\nu}+(p+\pi)(g^{\mu\nu}+u^{\mu}u^{\nu}) \tag{3.4}$$

with u^{μ} the fluid 4-velocity. The extra pressure π can either be associated to a pressure due to particle production/destruction or to a bulk viscous pressure. In the next two subsections these two cases will be discussed together with the assumption (3.2) of AS at the very early high energy universe. The term π could indeed be important during the transition phase that connects the quantum vacuum stage of the universe to the subsequent era with nonzero matter density.

The two independent components of (3.1) are

$$3\left(H^2+\frac{\kappa}{a^2}\right)=\bar{\Lambda}e^{\psi}-3H\frac{\dot{\psi}}{n}-\frac{3\dot{\psi}^2}{4n^2}+8\pi G\rho \tag{3.5}$$

$$\frac{2}{n}\dot{H}+3H^2+\frac{\kappa}{a^2}=\bar{\Lambda}e^{\psi}-2H\frac{\dot{\psi}}{n}-\frac{\dot{\psi}^2}{4n^2}-\frac{1}{n}\left(\frac{\dot{\psi}}{n}\right)^{\cdot}-8\pi GP, \tag{3.6}$$

where the total effective pressure is $P=p+\pi$. The conservation equation (3.3) takes the form

$$\dot{\rho}+3nH(\rho+P)+\rho\frac{\dot{G}}{G}=0. \tag{3.7}$$

The system of equations (3.5) and (3.6) is satisfied by construction for any $\psi(t)$. However, the effective pressure P is constrained to be $P=-\frac{1}{3}\rho$. Indeed, differentiating (3.5) with respect to t and using (3.7) to substitute $\dot{\rho}$ and also (3.5) itself we find

$$\left(1+\frac{\dot{\psi}}{2nH}\right)\left[\frac{2}{n}\dot{H}+3H^2+\frac{\kappa}{a^2}-\bar{\Lambda}e^{\psi}+2H\frac{\dot{\psi}}{n}+\frac{\dot{\psi}^2}{4n^2}+\frac{1}{n}\left(\frac{\dot{\psi}}{n}\right)^{\cdot}+8\pi GP\right]=4\pi G\frac{\dot{\psi}}{nH}\left(P+\frac{1}{3}\rho\right). \tag{3.8}$$

From (3.6) and (3.8) it arises the constraint

$$p+\frac{1}{3}\rho+\pi=0. \tag{3.9}$$

Therefore, given this consistency condition, Eq. (3.6) is redundant due to the time reparametrization and can be omitted. So, we remain with Eqs. (3.5), (3.7), and (3.9) and the gauge $n=1$ will be adopted.

We find that the constraint (3.9) is an interesting issue of the proposed framework, providing restrictions on the acceptable forms of equations of state. Although it may look as a disadvantage at first sight, however such constraints could be desired in modified gravities and orient us to the specification of the physical content of the theory. It is well known that in the context of conventional quantum field theory, renormalizability poses constraints in the allowed fields or interactions. Physics is still unknown in the trans-Planckian regime where our solutions are supposed to be valid and it may be that this constraint equation provides an insight about this regime through handable equations. There may also be a concern how one will be able, following our framework, to retrieve classical Friedmann-Robertson-Walker (FRW) cosmology, since Eq. (3.9) restricts some equations of states of matter that appear afterwards. The answer to this puzzle is that the proposed framework is valid only near the NGFP and not afterwards at lower energies. It is expected that the transition from the trans-Planckian regime to the classical regime will be explained through a quantum process of the type of decoherence on the ensemble of superimposed quantum spacetimes instead of a set of modified Einstein equations. Thus, such a transition requires to be modeled with a set of quantum field equations of motion. Anyway, the asymptotic safety scenario guarantees that later on we recover pure general relativity (GR) equations without modification. Our equations do not describe neither the passage regime nor the FRW period. We note that one should always keep in mind that at large k the existence of a NGFP is more a mathematical issue, as it provides a starting point for a well-defined quantum-gravitational path integral. The fundamental Lagrangian and the underlying physical processes are not yet known. It is expected that a quantum gravity interpretation will be fully resolved/understood when it will be possible to analyze how the classical spacetime at low energies arises from the quantum ensemble of $g_{\mu\nu}$ states in a meaningful way, allowing a measurement theory in the fully quantum regime where no classical time and clocks exist. The present work proposes a new framework to obtain modified Einstein equations with varying Λ, G and finds nonsingular spacetimes assuming that Λ, G have the behavior proposed by AS near the NGFP; such classical nonsingular solutions should contribute only at the quantum-gravitational path integral.

Note that for simplicity we have not included G-kinetic terms in (3.1). In a more complete treatment, however, such terms should be added. If we parametrize G by $G = \bar{G}e^{\chi}$, an extra energy-momentum tensor $\vartheta_{\mu\nu}^{(G)}$ of G would be added on the right-hand side of (3.1), constructed out of χ and its first and second derivatives,

$$\vartheta_{\mu\nu}^{(G)} = A(\chi)\chi_{;\mu}\chi_{;\nu} + B(\chi)g_{\mu\nu}\chi^{;\rho}\chi_{;\rho} + C(\chi)\chi_{;\mu;\nu} + E(\chi)g_{\mu\nu}\Box\chi + F(\chi)g_{\mu\nu}. \tag{3.10}$$

The Bianchi identities imply $\theta_{\mu\nu}^{(G);\mu} = 0$, which provides the following equations, following the process appeared in [43]

$$A = C' - \frac{1}{2}C^2, \qquad B = -C' - \frac{1}{4}C^2, \qquad E = -C, \qquad F = 0, \tag{3.11}$$

where a prime means differentiation with respect to χ. The various ψ-kinetic terms in (3.1) arise from the demand that their total covariant derivative cancels against the cosmological constant term $\Lambda g_{\mu\nu}$ and this is enough in order to determine their form uniquely. The important point is that the covariant cancellation of the χ-kinetic terms occurs against zero, and therefore, a new arbitrary field C arises. At the cosmological level that we elaborate it is $\chi = \chi(t)$, $C = C(t)$ and $C' = \dot{C}/\dot{\chi}$. Then, the consistency of Eqs. (3.5) and (3.6) will provide a differential equation of second order for $C(t)$. Indeed, the presence of the χ terms in the field equations will add extra terms on the right-hand sides of (3.5) and (3.6). Each such term contains products of mth time derivatives of C with nth time derivatives of χ (m, $n = 0$, 1, 2), where let us denote such products as $C^{(m)}\chi^{(n)}$ for convenience. The conservation equation (3.7) remains the same. Following the same process which led to (3.8), instead of the constraint (3.9), we will obtain a differential equation of second order for $C(t)$. This equation will still contain $P + \frac{1}{3}\rho$ as one term, while all the others will be $C^{(m)}\chi^{(n)}$ terms. If we choose the initial conditions $C(0) \simeq 0$, $\dot{C}(0) \simeq 0$ for the differential equation, then in a short time interval around $t = 0$, Eq. (3.9) will arise approximatively. At the same time, in this interval the tensor $\theta_{\mu\nu}^{(G)}$ will be approximately zero and Eq. (3.1) will arise. The result is that our inclusion of only the Λ-kinetic terms, implying the constraint (3.9), simplifies the analysis without mixing up with extra integration constants, and moreover is a consistent option at early times.

Let us finish with a few comments. First, a nonconservation equation of the form (3.7) implies an energy transfer between the energy density ρ and the gravitational coupling G. Consistently with the scaling (3.2), it is $\frac{\dot{G}}{G} = -2\frac{\dot{k}}{k} = -\frac{\dot{\Lambda}}{\Lambda}$, and it will be verified from the following matter solutions that in most cases $k(t)$ decreases. Thus, Λ also decreases with time and there is an energy transfer from ρ to G. In one matter solution it is found that $k(t)$ increases and the opposite behavior of a transfer from G to ρ occurs. Moreover, the above equation can lead to entropy production and reheating, as will be discussed in Sec. IV. Second, a constraint of the form (3.9) does not appear in other studies of AS inspired cosmologies and it implies an extra negative pressure π (whenever $\rho + 3p > 0$). Then, from Eq. (3.7) it turns out that the decay rate of the energy density ρ becomes smaller compared to the free dilution. One way to interpret this negative pressure is due to particle production and matter creation and another way is due to bulk viscosity (both mechanisms can be present simultaneously, but this situation will not be considered here due to complexity). Third, in the case of particle production, π is solely given by Eq. (3.9) and then, Eqs. (3.5) and (3.7) provide the solution assuming the AS scaling (3.2) as well as an energy-length scaling (this case will be examined in Sec. III A). The subcase $\pi = 0$ has the same treatment and provides the cosmic stringlike equation of state $p = -\frac{1}{3}\rho$, which for a standard dilution of ρ is also the equation of state for the curvature term; this equation of state also arises out of dimensional arguments assuming that the mass M of a spherical region obeys in the early universe a Machian expression where $G_N M$ is proportional to the radius r [58]. In the case of bulk viscosity, π is additionally given by another expression, and Eqs. (3.5), (3.7), and (3.9) provide the solution given the scaling properties (3.2), but with the difference that now the energy-length scale is determined by these equations (this case will be examined in Sec. III B).

At the NGFP (3.2), the conservation equation (3.7), together with the constraint (3.9), is written as

$$\dot{\rho} = 2\rho\left(\frac{\dot{k}}{k} - H\right). \tag{3.12}$$

Thus, depending on the sign of $\dot{k}$, the energy density ρ can either decrease or increase. Usually k, ρ decrease with time, but since in the present work it is proposed that the vacuum solutions probably describe the initial stage of the universe, it can be allowed to have k temporarily increasing in a subsequent stage of matter solutions.

A. Particle production

As explained above, the model at hand possesses naturally a nonequilibrium pressure π given in terms of the energy density ρ and pressure p by the expression (3.9),

$$\pi = -\left(\frac{1}{3}\rho + p\right). \tag{3.13}$$

This pressure turns out to be negative (as long as $\rho + 3p > 0$). Equation (3.13) is also written as

$$\pi = -\frac{\rho + p}{3H}\frac{\dot{N}}{N} = -\frac{\rho + p}{\mathrm{n}}\frac{dN}{dV}, \tag{3.14}$$

where the ratio of the change of the number N of particles in the proper comoving volume $V \propto a^3$ is

$$\frac{\dot{N}}{N} = \frac{\rho + 3p}{\rho + p}H \tag{3.15}$$

and $\mathrm{n} = \frac{N}{V}$ is the particle number density. Due to the second equation in (3.14), the conservation equation (3.7) is written as

$$d(\rho V) + p dV - \frac{\rho + p}{\mathrm{n}}dN + \frac{\rho V}{G}dG = 0. \tag{3.16}$$

The third term in this equation expresses the presence of matter creation in the context of open systems [59], with the important difference that here the form of this creation is predicted by the theory itself, as given by Eq. (3.15). Therefore, one way to interpret the supplementary pressure π of Eq. (3.13) is that it corresponds to particle production. Equation (3.16) expresses the thermodynamical energy conservation of an open system in the case of adiabatic transformation ($dQ = 0$) and the "heat" exchanged by the system in our case is due not only to the change of the number of particles but also to the change of the gravitational constant G. Equation (3.15) is of a special form among the various models in the literature parametrizing the particle change rate in the case of isentropic particle production as $\frac{\dot{N}}{N} = 3\beta H_*(\frac{H}{H_*})^\alpha$ [60,61], where α, β are $\mathcal{O}(1)$ dimensionless constants and H_* is a reference value, e.g. the present Hubble rate. So, for $p = w\rho$, in our case it is $\alpha = 1$ and $\beta = \frac{1+3w}{3(1+w)}$ (for example, for a reasonable equation of state in the early universe that of relativistic matter $p = \frac{1}{3}\rho$ it is $\pi = -\frac{2}{3}\rho$). Therefore, it is remarkable that the very same modified Einstein equations suggest a transfer of energy from the gravitational field to matter through particle production.

Integration of the nonconservation equation (3.7), using (3.13), gives

$$\rho = \frac{\rho_o}{Ga^2}, \tag{3.17}$$

where $\rho_o > 0$ is an integration constant (note that no particular w has been chosen). Finally, the Friedmann equation (3.5) is written as

$$H^2 + \frac{\kappa}{a^2} = \frac{\bar{\Lambda}}{3}e^\psi - H\dot{\psi} - \frac{1}{4}\dot{\psi}^2 + \frac{8\pi}{3}\frac{\rho_o}{a^2}. \tag{3.18}$$

Note that G has disappeared in (3.18) and no particular form for $G(k)$ has been assumed. However, according to AS, the forms $G = \frac{g_*}{\xi^2}t^2$ for the scaling $k = \frac{\xi}{t}$, or $G = \frac{g_*}{\xi^2}H^{-2}$ for $k = \xi H$ are needed for the determination of ρ in (3.17). Setting

$$\mu = \kappa - \frac{8\pi\rho_o}{3}, \tag{3.19}$$

Eq. (3.18) is written as

$$H^2 + \frac{\mu}{a^2} = \frac{\bar{\Lambda}}{3}e^\psi - H\dot{\psi} - \frac{1}{4}\dot{\psi}^2. \tag{3.20}$$

If $\mu = 0$, which means $\kappa = 1$ and $\rho_o = \frac{3}{8\pi}$, Eq. (3.20) is identical with the vacuum equation (2.7) with $\kappa_v = 0$ (we denote by κ_v the curvature index of the vacuum case); thus the solutions in this case coincide with the vacuum solutions of the previous section with $\kappa_v = 0$. Therefore, in this case there are strong power law inflationary solutions close to the initial singularity.

If $\mu \neq 0$, Eq. (3.20) takes the form

$$\frac{1}{a^2}\left(\frac{da}{dt'}\right)^2 + \frac{\mathrm{sgn}(\mu)}{a^2} = \frac{\bar{\Lambda}'}{3}e^\psi - \frac{1}{a}\frac{da}{dt'}\frac{d\psi}{dt'} - \frac{1}{4}\left(\frac{d\psi}{dt'}\right)^2, \tag{3.21}$$

where $t' = \sqrt{|\mu|}t$, $\bar{\Lambda}' = \bar{\Lambda}/|\mu|$ and $\mathrm{sgn}(\mu)$ denotes the sign of μ. Equation (3.21) coincides with the vacuum equation (2.7) given that $\mathrm{sgn}(\mu) = \kappa_v \neq 0$ and t' is replaced by t. So, for $\kappa = 1$, $\rho_o < \frac{3}{8\pi}$, the solutions coincide with the vacuum solutions with $\kappa_v = 1$, just rescaling time. Therefore, in this case there are accelerating (inflationary) solutions which either avoid the big bang singularity, or possess a big bang, or during a collapsing phase avoid the big crunch towards expansion. Note from Eq. (3.17) that when $a \neq 0$ and $k_{\mathrm{max}} < \infty$, as happens with the nonsingular vacuum solutions found previously, the energy density ρ remains finite. For $\kappa = 1$, $\rho_o > \frac{3}{8\pi}$ or for $\kappa \leq 0$ with any ρ_o, the solutions coincide with the vacuum solutions with $\kappa_v = -1$, just rescaling time. In this case a singular accelerating cosmology can occur. The property of a decreasing $k(t)$ shown for the vacuum expanding solutions is also transferred to the associated matter solutions discussed here. As for the energy density ρ, it decreases with time due to (3.12).

To summarize with the most interesting matter solutions with particle production, they refer to the positively curved case and have power law inflation or are nonsingular and accelerating.

B. Bulk viscosity

Since detailed physics in the proximity of the NGFP is still unknown, it is worth exploring the possibility that the negative nonequilibrium pressure π of Eq. (3.9) is due to nonzero bulk viscosity through dissipative processes. In this case the bulk viscous pressure π has the form

$$\pi = -\zeta u^{u}{}_{;\mu} = -3\zeta H, \tag{3.22}$$

where ζ is the bulk viscosity coefficient, which will be assumed here to be constant. Bulk pressures could be the consequence of the process where different matter components cool with the expansion of the universe with different rates and the system moves away from equilibrium. The expression (3.22) arises in some limit in the context of the second-order theory of nonequilibrium thermodynamics [62]. If we assume $\rho + 3p > 0$, for an expanding phase of the universe the expression (3.22) is consistent with the constraint (3.9) given that $\zeta > 0$. This means that there are no contracting parts in a solution at all, and all solutions are expanding. A reasonable equation of state in the early universe is that of relativistic matter $p = \frac{1}{3}\rho$. However, this restriction is not essential for the following analysis, so we assume a general barotropic fluid with $p = w\rho$ and $1 + 3w > 0$. Then, from Eqs. (3.9) and (3.22) it arises a direct connection between the energy density and the Hubble parameter

$$\rho = \frac{9}{1+3w}\zeta H. \tag{3.23}$$

Integration of the nonconservation equation (3.7) gives

$$\rho = \frac{\rho_o}{Ga^2}, \tag{3.24}$$

where $\rho_o > 0$ is an integration constant. The combination of (3.23) and (3.24) gives

$$H = \frac{(1+3w)\rho_o}{9\zeta}\frac{1}{Ga^2}. \tag{3.25}$$

Close to the NGFP defined by (3.2) it arises from (3.25) that

$$e^{\psi} = \nu H a^2, \qquad \nu = \frac{9\zeta g_* \lambda_*}{(1+3w)\rho_o \bar{\Lambda}} > 0. \tag{3.26}$$

Finally, the Friedmann equation (3.5) gives due to (3.24)

$$H^2 + \frac{\kappa}{a^2} = \frac{\bar{\Lambda}}{3}e^{\psi} - H\dot{\psi} - \frac{1}{4}\dot{\psi}^2 + \frac{8\pi}{3}\frac{\rho_o}{a^2}. \tag{3.27}$$

Plugging (3.26) into (3.27) and converting the time derivatives to a-derivatives we get the equation

$$a^2\left(\frac{dH}{da}\right)^2 + 8aH\frac{dH}{da} + 16H^2 - \frac{4\nu\bar{\Lambda}}{3}a^2 H + 4\left(\kappa - \frac{8\pi\rho_o}{3}\right)\frac{1}{a^2} = 0. \tag{3.28}$$

Setting

$$z = a^4 H > 0, \qquad x = a^3, \tag{3.29}$$

Eq. (3.28) gets the form

$$\frac{dz}{dx} = \pm\sqrt{\frac{4\nu\bar{\Lambda}}{27}z + \frac{4}{9}\left(\frac{8\pi\rho_o}{3} - \kappa\right)}, \tag{3.30}$$

where the square root has to be positive. Integration of (3.30) gives

$$H = \beta a^2 \pm \frac{c}{a} + \frac{\gamma}{a^4}, \tag{3.31}$$

where c is integration constant and

$$\beta = \frac{\nu\bar{\Lambda}}{27} > 0, \qquad \gamma = \frac{27}{4\nu\bar{\Lambda}}\left[c^2 - \frac{4}{9}\left(\frac{8\pi\rho_0}{3} - \kappa\right)\right], \tag{3.32}$$

under the constraints $c \pm 2\beta a^3 > 0$, $\beta a^6 \pm ca^3 + \gamma > 0$. For $y = c \pm 2\beta a^3$, the first of these constraints becomes $y > 0$ and the second $y^2 > c^2 - 4\beta\gamma = \frac{4}{9}(\frac{8\pi\rho_o}{3} - \kappa)$.

(i) If $c^2 - 4\beta\gamma < 0 \Leftrightarrow \kappa = 1, \rho_o < \frac{3}{8\pi}$, the only constraint is $c \pm 2\beta a^3 > 0$ and there are three cases: (i) for the upper branch with $c < 0$ it is $a > (\frac{|c|}{2\beta})^{1/3}$, (ii) for the upper branch with $c > 0$ there is no bound on a, and (iii) for the lower branch it is $c > 0$ and $a < (\frac{c}{2\beta})^{1/3}$.

(ii) If $c^2 - 4\beta\gamma > 0 \Leftrightarrow \kappa \leq 0$, or $\kappa = 1$, $\rho_o > \frac{3}{8\pi}$, the only constraint is $c \pm 2\beta a^3 > \sqrt{c^2 - 4\beta\gamma}$. For the upper branch there are two cases: (i) if $c < 0$, or if $c > 0$, $\gamma < 0$ it is $a > [\frac{1}{2\beta}(\sqrt{c^2 - 4\beta\gamma} - c)]^{1/3}$ and (ii) if $c > 0$, $\gamma > 0$ there is no bound on a. For the lower branch it has to be $c > 0$, $\gamma > 0$ and $a < [\frac{1}{2\beta}(c - \sqrt{c^2 - 4\beta\gamma})]^{1/3}$.

From Eqs. (3.23) and (3.31) it is obvious that the energy density is finite for the solutions which avoid the zero scale factor, so the universe avoids the infinite density singularity. Moreover, from Eq. (3.31) we can calculate the Ricci scalar, which takes the form

$$\frac{R}{6} = 4\beta^2 a^4 \pm 5\beta ca + \frac{2\beta\gamma + c^2 + \kappa}{a^2} \mp \frac{\gamma c}{a^5} - \frac{2\gamma^2}{a^8}. \tag{3.33}$$

Therefore, the solutions which avoid an infinite density singularity avoid also a curvature singularity. Concerning the acceleration it is found similarly

$$\frac{\ddot{a}}{3a^2} = \beta(\beta a^3 \pm c) - \frac{\gamma}{a^9}(\gamma \pm ca^3). \tag{3.34}$$

For case (i) above with $c^2 - 4\beta\gamma < 0$ the universe at its minimum scale factor starts decelerating and enters into acceleration. For case (i) with $c^2 - 4\beta\gamma > 0$ the universe at its minimum scale factor starts with zero acceleration, and immediately after, it accelerates.

We summarize saying that there are branches of solutions for any spatial topology which are expanding, nonsingular and accelerating.

The dependence of the scale factor with time can be found integrating Eq. (3.31),

$$t - t_0 = \int \frac{da}{\beta a^3 \pm c + \gamma a^{-3}} \tag{3.35}$$

$$= \frac{1}{3\beta}\int \frac{(u+\sigma)^{\frac{1}{3}}}{u^2-\tau} du, \tag{3.36}$$

where t_0 is integration constant and

$$u = a^3 \pm \frac{c}{2\beta}, \qquad \sigma = \mp \frac{c}{2\beta}, \qquad \tau = \frac{c^2 - 4\beta\gamma}{4\beta^2}. \tag{3.37}$$

In the case $c^2 - 4\beta\gamma > 0$, this integral can be performed analytically in closed form. First, it is

$$6\beta\sqrt{\tau}(t-t_0) = \theta^{\frac{1}{3}} \int \frac{v^{\frac{1}{3}}}{v-\epsilon} dv - \tilde{\theta}^{\frac{1}{3}} \int \frac{\tilde{v}^{\frac{1}{3}}}{\tilde{v}-\tilde{\epsilon}} d\tilde{v}, \tag{3.38}$$

where $v = \theta^{-1}a^3$, $\tilde{v} = \tilde{\theta}^{-1}a^3$, $\theta = |\sigma + \sqrt{\tau}|$, $\tilde{\theta} = |\sigma - \sqrt{\tau}|$, $\epsilon = \mathrm{sgn}(\sigma + \sqrt{\tau})$, $\tilde{\epsilon} = \mathrm{sgn}(\sigma - \sqrt{\tau})$. We write

$$2\epsilon \int \frac{v^{\frac{1}{3}}}{v-\epsilon} dv = \int \frac{1 + 2\epsilon v^{\frac{1}{3}}}{v-\epsilon} dv - \int \frac{1}{v-\epsilon} dv$$

$$= \int \frac{1}{v^{\frac{1}{3}} - \epsilon} dv - \int \frac{v^{\frac{1}{3}}}{v^{\frac{2}{3}} + \epsilon v^{\frac{1}{3}} + 1} dv - \int \frac{1}{v-\epsilon} dv \tag{3.39}$$

$$= 3\int \frac{q^2}{q-\epsilon} dq - 3\int \frac{q^3}{q^2 + \epsilon q + 1} dq - \ln|v-\epsilon|, \qquad q = v^{\frac{1}{3}} = \theta^{-\frac{1}{3}} a \tag{3.40}$$

$$= 6\epsilon v^{\frac{1}{3}} + \ln\frac{|v^{\frac{1}{3}} - \epsilon|^3}{|v-\epsilon|} - 2\sqrt{3}\epsilon \arctan\frac{2v^{\frac{1}{3}} + \epsilon}{\sqrt{3}}. \tag{3.41}$$

Finally,

$$6\beta\sqrt{\tau}(t-t_0) = \frac{1}{2}\ln\left[\left(\frac{|\theta^{-\frac{1}{3}}a - \epsilon|^3}{|\theta^{-1}a^3 - \epsilon|}\right)^{\epsilon\theta^{1/3}} \left(\frac{|\tilde{\theta}^{-1}a^3 - \tilde{\epsilon}|}{|\tilde{\theta}^{-\frac{1}{3}}a - \tilde{\epsilon}|^3}\right)^{\tilde{\epsilon}\tilde{\theta}^{1/3}}\right] - \sqrt{3}\left(\theta^{\frac{1}{3}}\arctan\frac{2\theta^{-\frac{1}{3}}a + \epsilon}{\sqrt{3}} - \tilde{\theta}^{\frac{1}{3}}\arctan\frac{2\tilde{\theta}^{-\frac{1}{3}}a + \tilde{\epsilon}}{\sqrt{3}}\right). \tag{3.42}$$

Note that in the present case of bulk viscosity, it was nowhere assumed some energy-length scaling. Actually $k(t)$ is determined from Eq. (3.26) as follows:

$$k = \sqrt{\frac{9\zeta g_*}{(1+3w)\rho_o}} a\sqrt{H}. \tag{3.43}$$

For small scale factors with $\gamma > 0$, due to (3.31), Eq. (3.43) implies $k \sim \frac{1}{a}$, so k scales inversely proportional to the proper distance at fixed time. It is worth emphasizing that the physical characteristics of the fluid determine $k(t)$. This property is reasonable since different matter content should necessarily result to different scaling laws due to concrete physical reasons. Indeed, in reality the details of the "thermodynamic" (or the essential relevant parameters in case of nonequilibrium evolution) properties of the statistical ensemble of quantum particles should determine how "strong" the relation is between the measure of mean energy k and the geometrical cosmological measure of the "distance." For the solution with $c^2 - 4\beta\gamma < 0$ which possesses a minimum scale factor, it can be shown that the scale $k(t)$ decreases in a region near the minimum (and also ρ decreases). However, for larger values of a the function $k(t)$ increases. For the nonsingular solution with $c^2 - 4\beta\gamma > 0$ the function $k(t)$ is found to increase near the minimum scale factor (and also ρ increases). Both cases with increased k can be interpreted as intermediate stages in the cosmic evolution.

IV. INFLATION, REHEATING AND ENTROPY GENERATION

Here, a short discussion about the inflationary period and the possible subsequent reheating and entropy production will be given. Contrary to the aim of several other works concerning global cosmological solutions in the AS program, the focus here is the cosmic period near the NGFP regime. This high energy regime is of particular importance for the possibility of an inflationary period. Fortunately, this is also a regime where the behavior of Λ and G is known much better and there are geometry independent methods handling the running of these couplings. The solutions found in the previous sections possess accelerating phases either in the vacuum or the matter sector. The cosmic scenario we are going to analyze is based on the assumption that the universe first starts in a pure vacuum (perhaps creation of the universe from a vacuum fluctuation), where the relevant equations of motion contain only a vacuum contribution. Subsequently, it enters a period where matter starts to become more important (still inside the NGFP regime) that ends when $k \approx m_{pl}$. At this energy scale there is a transition towards a third stage of conventional FRW universe with negligible Λ and constant G. The derived solutions appearing in the preceding sections can model both the first and the second stage of cosmic evolution. The second stage can be modeled either by solutions that suggest, as it will be seen, particle production with entropy generation or by solutions with bulk viscosity associated with entropy production and possible reheating. Both of these matter solutions are described by equations which are consistent with the vacuum case equations.

A. First stage: Inflationary cosmogenesis

An acceptable approximation [40] to describe the RG improved UV early cosmological history is to work separately at the different three stages using the developed solutions. It remains to the details of a full RG running to prove that the derived classical cosmological solutions (used to describe the first two stages of cosmic evolution), inspired by the energy scaling of the couplings Λ and G, are indeed fair approximations of the quantum average spacetime that describes the early universe.

Some long-standing qualitative arguments speculate that due to Heisenberg's uncertainty principle, the universe was created from "empty" spacetime. A small true vacuum bubble/void of expanding vacuum space can be created probabilistically by quantum fluctuations of a metastable false vacuum through a first/second order phase transition. If this initial bubble/void cannot expand rapidly, it will disappear soon. In case this initial baby universe expands rapidly to a large enough size, the universe can then be created irreversibly. This baby universe created probabilistically by quantum vacuum fluctuations starts with a finite volume. Thus, it is expected that the energy scale k may not initiate from infinity and the corresponding $\Lambda(k)$ is finite.

In more detail we consider a time interval $t_0 < t < t_1$, where t_0 is the initial time of quantum birth and t_1 is the transition time to the second stage of matter appearance. From the derived vacuum solutions, we will pick the simple power-law spatially flat solution (2.14) to model this era,

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{1+\omega}, \quad \text{for } t \geq t_0, \tag{4.1}$$

where a_0 is the initial scale factor at t_0. The larger the value of ω, the stronger the inflation is. For a structure of comoving length Δx, the corresponding physical (proper) length at any t is $L(t) = a(t)\Delta x$. Due to Eq. (4.1) it is

$$L(t) = \left(\frac{t}{t_1}\right)^{1+\omega} L(t_1). \tag{4.2}$$

Now, the Hubble radius $\ell_H(t) \equiv \frac{1}{H(t)}$ is given by

$$\ell_H(t) = \frac{t}{1+\omega}. \tag{4.3}$$

In order to study when $L(t)$ crosses the Hubble radius $\ell_H(t)$ we evaluate their ratio:

$$\frac{L(t)}{\ell_H(t)} = \left(\frac{t}{t_1}\right)^{\omega} \frac{L(t_1)}{\ell_H(t_1)}. \tag{4.4}$$

It is obvious that the proper length of a part of the universe increases fast enough to cross the Hubble radius. The desired 60 e-folds can be easily achieved for moderate values of ω. Indeed, let us assume for simplicity that all the required 60 e-foldings are achieved during the first cosmological stage, although it is possible to have a second inflationary period with different characteristics during the second stage. Then, at $t = t_1$ we need $L(t_1)$ to be e^{60} times the Hubble radius $\ell_H(t_1)$ and we get

$$\frac{L(t)}{\ell_H(t)} = e^{60}\left(\frac{t}{t_1}\right)^{\omega}. \tag{4.5}$$

The time when L crosses the Hubble radius happens for $t = t_{cr}$ with $L(t_{cr}) = \ell_H(t_{cr})$, and Eq. (4.5) becomes

$$t_{cr} = t_1 e^{-\frac{60}{\omega}}. \tag{4.6}$$

It is obvious from the above equation that for moderate values of ω, the time t_{cr} can be much shorter than the transition time t_1.

B. Second stage: Heat transfer and entropy production

Subsequently, a second cosmic period holds for $t_1 < t < t_2$, where t_2 is the transition to the FRW universe. Here, apart from the vacuum contribution there is also matter. The study of the matter solutions derived previously reveals the existence of either deceleration or inflationary eras. Since now matter is present, it is essential to analyze the thermodynamics of the universe.

1. Entropy production through particle production

In the case of particle production, thermodynamics of open systems, as applied to cosmology, takes into account both matter and entropy creation on a macroscopical level. This consideration generalizes the standard thermodynamics in cosmology, since beyond ρ and p, the particle density n also enters naturally. If $U = \rho V$ is the internal energy in a proper comoving volume V with corresponding entropy S and temperature T, the entropy change dS is given by

$$\begin{aligned} TdS &= d(\rho V) + pdV - \mu dN = \frac{\rho + p}{\mathrm{n}} dN - \mu dN - \frac{\rho V}{G} dG \\ &= T\frac{s}{\mathrm{n}} dN - \frac{\rho V}{G} dG. \end{aligned} \tag{4.7}$$

The second equation arises do to (3.16) and the third equation arises due to that the chemical potential μ is given by the Euler's equation $\mu\mathrm{n} = \rho + p - Ts$, where $s = \frac{S}{V}$ is the entropy per unit volume. As long as the right-hand side of Eq. (4.7) is positive, the second law of thermodynamics is satisfied, $dS > 0$. Using (3.15), Eq. (4.7) reduces to a differential equation for the entropy S:

$$\dot{S} = \frac{\rho + 3p}{\rho + p} HS - \frac{\rho V}{T}\frac{\dot{G}}{G}. \tag{4.8}$$

It is not an easy issue [40] to succeed at the same time entropy and particle production and in the present work we have managed this. Assuming a radiation equation of state, $w = \frac{1}{3}$, the Boltzmann law $\rho = \sigma_B T^4$ holds, and Eq. (4.8), due to (3.17), takes the form

$$\frac{dS}{d\alpha} = \frac{3}{2}S + \frac{4}{3}v\sigma_B^{\frac{1}{4}}\rho_o^{\frac{3}{4}}e^{\frac{3\alpha}{2}}\frac{dG^{-\frac{3}{4}}}{d\alpha}, \tag{4.9}$$

where $\alpha = \ln a$ and $V = va^3$ with v being the comoving volume. Integration of (4.9) gives the solution

$$S = va^{\frac{3}{2}}\left(c + \frac{4\sigma_B^{\frac{1}{4}}\rho_o^{\frac{3}{4}}}{3G^{\frac{3}{4}}}\right), \tag{4.10}$$

where c is the integration constant. Using the NGFP scaling (3.2) of G, we find

$$S = va^{\frac{3}{2}}(c + \nu k^{\frac{3}{2}}), \tag{4.11}$$

where $\nu = \frac{4\sigma_B^{1/4}\rho_o^{3/4}}{3g_*^{3/4}}$. From (4.11) it arises

$$\dot{S} = \frac{3}{2}va^{\frac{3}{2}}\left[cH + \nu k^{\frac{3}{2}}\left(H + \frac{\dot{k}}{k}\right)\right]. \tag{4.12}$$

For the case of particle production we adopted two energy-length scalings. For the first one, $k = \frac{\xi}{t}$, it is

$$\dot{S} = \frac{3}{2}va^{\frac{3}{2}}\left[cH + \nu k^{\frac{3}{2}}\left(H - \frac{1}{t}\right)\right]. \tag{4.13}$$

It can be easily seen that all the corresponding interesting matter solutions with particle production for any spatial topology κ have $H - \frac{1}{t} > 0$ in the expanding phase (this property applies also for the nonsingular solution found). So, for $c \geq 0$ there is a natural entropy production. For the second scaling, $k = \xi H$, it is

$$\dot{S} = \frac{3}{2}va^{\frac{3}{2}}\left(cH + \nu k^{\frac{3}{2}}\frac{\ddot{a}}{aH}\right). \tag{4.14}$$

Therefore, if $c \geq 0$, whenever there is acceleration, at the same time there is an entropy production. Now, the accelerating solutions found previously, with either $\kappa = 0$ or $\kappa = 1$, share this property (the nonsingular solution is included).

2. *Entropy production through bulk viscosity*

In the case of bulk viscosity, one can use the standard thermodynamic relation of closed systems $dU + pdV = TdS$, from where it arises immediately

$$\frac{T}{V}\dot{S} = \dot{\rho} + 3H(\rho + p). \tag{4.15}$$

Making use of the conservation equation (3.7) it turns out

$$\frac{T}{V}\dot{S} = -\rho\frac{\dot{G}}{G} - 3H\pi. \tag{4.16}$$

In conventional FRW cosmology where the right-hand side of Eq. (4.16) is zero, it can be concluded that the entropy of a comoving volume remains the same as the universe expands, $\dot{S} = 0$. In our model, in the NGFP regime it is $\frac{\dot{G}}{G} = -2\frac{\dot{k}}{k}$, and since usually k drops with the expansion, in order to have increasing entropy we need $\pi < 0$ which is offered by the mechanism of bulk viscosity. Moreover, in the study of bulk viscosity, we found previously one case where $k(t)$ increases, and thus, $S(t)$ also increases.

Equation (4.16), making use of (3.9) with $w = \frac{1}{3}$, takes the form

$$\frac{T}{V}\dot{S} = \rho\left(2H - \frac{\dot{G}}{G}\right) = 2\rho\left(H + \frac{\dot{k}}{k}\right). \tag{4.17}$$

Finally, Eq. (4.17), using (3.43), gives

$$\frac{T}{V}\dot{S} = \rho\left(\frac{\dot{H}}{H} + 4H\right) = \frac{\rho}{H}\left(\frac{\ddot{a}}{a} + 3H^2\right). \tag{4.18}$$

It is possible to prove that all the upper branch solutions (3.31) are associated with entropy increase (the nonsingular solution is included).

3. *Reheating*

Now, we are going to discuss the evolution of the temperature. If we consider a radiation equation of state, the Boltzmann law of radiation is $\rho = \sigma_B T^4$, and it follows from (3.12) that

$$\dot{T} = \frac{T}{2}\left(\frac{\dot{k}}{k} - H\right). \tag{4.19}$$

Therefore, in order to have reheating, $\dot{T} > 0$, the time derivative $\dot{k}$ has to be sufficiently positive. For the particle production this is not true, since we have found that $k(t)$ is permanently decreasing; in this case reheating could be realized by other means, perhaps with the occurrence of possible phase transitions. For the bulk viscosity, Eq. (4.19) takes the form

$$\dot{T} = \frac{T}{4H}\dot{H}. \tag{4.20}$$

Thus, in order to have a temperature raise, a superacceleration, $\dot{H} > 0$, should occur. For the nonsingular solutions it can be shown that, depending on the parameters, the universe can have $\dot{T} > 0$ already from its minimum scale factor onwards, or the temperature raise can appear later.

In summary, the first period cosmic evolution can be described by vacuum AS modified equations with Λ present. This period of cosmic genesis is associated with strong inflation. At the second period the universe evolution is described by modified equations which include matter and is able to solve the cosmological entropy problem. In the case of bulk viscosity there is also heat transfer from vacuum to the matter sector. Finally, the third stage of cosmic evolution, which is not described by the set of modified Einstein equations presented here, happens when k departs from m_{pl}. At this point another semiclassical description of the spacetime applies where both G, Λ are almost constant. The framework of the AS program ensures that for lower energies the conventional FRW universe is recovered with negligible Λ.

V. DISCUSSION AND CONCLUSIONS

General branches of new cosmological solutions have been obtained in the context of asymptotic safe gravity (quantum Einstein gravity) at high energies close to the NGFP. The derived solutions arise from a new consistent system of modified Einstein equations. The framework handles two different cosmic periods. First a possible quantum birth from a vacuum state and second the addition of a matter component at high energies. However, the presented framework has to be modified to treat physics away from the trans-Planckian regime.

In the first cosmic era the source is an energy-dependent cosmological constant that scales at high energies as the AS scenario suggests near the NGFP, i.e. $\Lambda(k) \propto k^2$. This scaling is also the unique one which is consistent with dimensional analysis without the introduction of a new energy scale. The cosmological constant becomes time dependent under the assumption of an energy-length scaling. The modified Einstein equations are uniquely defined and arise by adding appropriate covariant kinetic terms of Λ in order to ensure the satisfaction of the Bianchi identities. The importance of the presented vacuum solutions, consistent with a quantum birth, lies on the fact that they provide inflationary expansion and at the same time completely remove the initial singularity in all scale factor, energy density and curvature invariants. Exit from inflation is a natural output of AS scenario and occurs when the energy scale becomes lower, and then, $\Lambda(k)$ becomes insignificant and standard decelerating cosmology arises.

In the second cosmic era the inclusion of matter close to the NGFP was possible to be modeled generalizing the vacuum equations of motion. An energy exchange arises between the matter and the varying gravitational constant $G(k) \propto k^{-2}$. A negative nonequilibrium pressure beyond the thermodynamic one is also an outcome and can be attributed to either a particle production or to a mechanism of bulk viscosity. In both cases, there are general solutions which are inflationary (with different characteristics than those arising during the first period) and nonsingular, and such behaviors can be found for any spatial topology. The barotropic equation of state is not particularly significant. In the case of bulk viscosity the relation between the energy scale and the time is implied by the theory itself. The most interesting feature of the matter solutions is that they suggest either particle production with entropy generation or bulk viscosity with entropy production and reheating.

Since the presented solutions cover two consecutive cosmic eras, both close to the NGFP, various phenomenological investigations are worth being investigated. It would certainly be interesting also to study with the help of RG flow equations the transition between the two eras. Extending/generalizing appropriately the present framework of modified Einstein equations and the associated energy conservation, it may also be possible to describe the sub-Planckian cosmic evolution with emerging high energy corrections to the conventional expansion rate that could explain baryogenesis [63], or dark energy [64].

Let us close with a few general comments. First, note that physical predictions, e.g. on the early universe, should actually depend on universal quantities, like the critical exponent at a fixed point, but not the fixed-point values themselves. This means that all the presented cosmological solutions (vacuum and matter) are phenomenological in the same sense as the standard model of Weinberg-Salam. Although the formalism and all the derived solutions are general and do not depend on specific values of the parameters g_* and λ_*, at the end, the analysis of the results depends on these parameters through inequalities in the parameter space and not through fine-tuning. For example, the nonsingular or the inflationary behavior are not properties that arise from specific values of the parameters. Only when the asymptotic safety program will be able to provide the final Lagrangian and critical exponents [65], precision conclusions about the characteristics of the inflation will be possible. Note, however, that the final picture regarding the understanding of the gravitational field would be much more conceptually different than the case of the rest fermion/bosonic content [65]. The reason is that we have to answer how measurements are performed, something that results in radical new physical requirements. Thus, the presented solutions can be regarded as useful phenomenological models of metrics that would probably describe a quantum gravity inspired prototype model of the average spacetime near the NGFP, or possibly describe some state spacetimes of the quantum ensemble. Nevertheless, it is natural and not problematic (in the context of AS) to expect that near the NGFP there is a quantum superposition of *nonsingular* spacetimes.

ACKNOWLEDGMENTS

V. Zarikas acknowledges the hospitality of Nazarbayev University.

[1] http://www.perimeterinstitute.ca/video-library/collection/renormalization-group-approaches-quantum-gravity.

[2] M. Niedermaier and M. Reuter, The asymptotic safety scenario in quantum gravity, Living Rev. Relativ. **9**, 5 (2006); M. Reuter and F. Saueressig, Functional renormalization group equations, asymptotic safety, and quantum Einstein gravity, arXiv:0708.1317; R. Percacci, in Approaches to quantum gravity, arXiv:0709.3851; O. Lauscher and M. Reuter, Asymptotic safety in quantum Einstein gravity: Nonperturbative renormalizability and fractal spacetime structure, arXiv:hep-th/0511260; M. Reuter and F. Saueressig, Quantum Einstein gravity, New J. Phys. **14**, 055022 (2012); B. Koch and F. Saueressig, Black holes within asymptotic safety, Int. J. Mod. Phys. A **29**, 1430011 (2014); A. Bonanno, Astrophysical implications of the asymptotic safety scenario in quantum gravity, *Proc. Sci.*, CLAQG08 (2011) 008 [arXiv:0911.2727]; M. Niedermaier, The asymptotic safety scenario in quantum gravity: An introduction, Classical Quantum Gravity **24**, R171 (2007).

[3] S. Weinberg, What is quantum field theory, and what did we think it is?, arXiv:hep-th/9702027; Effective field theory, past and future, *Proc. Sci.*, CD09 (2009) 001 [arXiv:0908.1964].

[4] M. Reuter, Nonperturbative evolution equation for quantum gravity, Phys. Rev. D **57**, 971 (1998).

[5] C. Wetterich, Exact evolution equation for the effective potential, Phys. Lett. B **301**, 90 (1993); T. R. Morris, The exact renormalization group and approximate solutions, Int. J. Mod. Phys. A **09**, 2411 (1994); D. Dou and R. Percacci, The running gravitational couplings, Classical Quantum Gravity **15**, 3449 (1998); M. Reuter and F. Saueressig, Renormalization group flow of quantum gravity in the Einstein-Hilbert truncation, Phys. Rev. D **65**, 065016 (2002); O. Lauscher and M. Reuter, Flow equation of quantum Einstein gravity in a higher derivative truncation, Phys. Rev. D **66**, 025026 (2002); W. Souma, Nontrivial ultraviolet fixed point in quantum gravity, Prog. Theor. Phys. **102**, 181 (1999); A. Bonanno and M. Reuter, Proper time flow equation for gravity, J. High Energy Phys. 02 (2005) 035; R. Percacci and D. Perini, Asymptotic safety of gravity coupled to matter, Phys. Rev. D **68**, 044018 (2003).

[6] D. F. Litim, Fixed Points of Quantum Gravity, Phys. Rev. Lett. **92**, 201301 (2004); P. Fischer and D. F. Litim, Fixed points of quantum gravity in extra dimensions, Phys. Lett. B **638**, 497 (2006); J. E. Daum and M. Reuter, Effective potential of the conformal factor: Gravitational average action and dynamical triangulations, Adv. Sci. Lett. **2**, 255 (2009); D. Benedetti, K. Groh, P. F. Machado, and F. Saueressig, The universal RG machine, J. High Energy Phys. 06 (2011) 079.

[7] M. Reuter and H. Weyer, The role of background independence for asymptotic safety in quantum Einstein gravity, Gen. Relativ. Gravit. **41**, 983 (2009); Background independence and asymptotic safety in conformally reduced gravity, Phys. Rev. D **79**, 105005 (2009); P. F. Machado and R. Percacci, Conformally reduced quantum gravity revisited, Phys. Rev. D **80**, 024020 (2009); E. Manrique and M. Reuter, Bare versus effective fixed point action in asymptotic safety: The reconstruction problem, *Proc. Sci.*, CLAQG08 (2011) 001 [arXiv:0905.4220].

[8] A. Codello and R. Percacci, Fixed Points of Higher Derivative Gravity, Phys. Rev. Lett. **97**, 221301 (2006).

[9] A. Bonanno, An effective action for asymptotically safe gravity, Phys. Rev. D **85**, 081503 (2012).

[10] D. Benedetti, P. F. Machado, and F. Saueressig, Four-derivative interactions in asymptotically safe gravity, AIP Conf. Proc. **1196**, 44 (2009).

[11] M. Niedermaier, Gravitational fixed points and asymptotic safety from perturbation theory, Nucl. Phys. **B833**, 226 (2010).

[12] A. Codello, R. Percacci, and C. Rahmede, Ultraviolet properties of f(R)-gravity, Int. J. Mod. Phys. A **23**, 143 (2008); Investigating the ultraviolet properties of gravity with a Wilsonian renormalization group equation, Ann. Phys. (Amsterdam) **324**, 414 (2009).

[13] P. F. Machado and F. Saueressig, On the renormalization group flow of f(R)-gravity, Phys. Rev. D **77**, 124045 (2008).

[14] A. Bonanno and M. Reuter, Quantum gravity effects near the null black hole singularity, Phys. Rev. D **60**, 084011 (1999); Renormalization group improved black hole spacetimes, Phys. Rev. D **62**, 043008 (2000).

[15] A. Bonanno and M. Reuter, Spacetime structure of an evaporating black hole in quantum gravity, Phys. Rev. D **73**, 083005 (2006).

[16] M. Reuter and E. Tuiran, The eleventh Marcel Grossmann meeting, arXiv:hep-th/0612037; Quantum gravity effects in the Kerr spacetime, Phys. Rev. D **83**, 044041 (2011).

[17] K. Falls and D. F. Litim, Black hole thermodynamics under the microscope, Phys. Rev. D **89**, 084002 (2014).

[18] Y.-F. Cai and D. A. Easson, Black holes in an asymptotically safe gravity theory with higher derivatives, J. Cosmol. Astropart. Phys. 09 (2010) 002.

[19] D. Becker and M. Reuter, Running boundary actions, asymptotic safety, and black hole thermodynamics, J. High Energy Phys. 07 (2012) 172; 13th Marcel Grossmann meeting on recent developments in theoretical and experimental general relativity, astrophysics, and relativistic field theories (MG13),arXiv:1212.4274.

[20] B. F. L. Ward, Planck scale remnants in resummed quantum gravity, Acta Phys. Pol. B **37**, 1967 (2006).

[21] S. Basu and D. Mattingly, Asymptotic safety, asymptotic darkness, and the hoop conjecture in the extreme UV, Phys. Rev. D **82**, 124017 (2010).

[22] R. Casadio, S. D. H. Hsu, and B. Mirza, Asymptotic safety, singularities, and gravitational collapse, Phys. Lett. B **695**, 317 (2011).

[23] R. Torres, Singularity-free gravitational collapse and asymptotic safety, Phys. Lett. B **733**, 21 (2014); R. Torres and F. Fayos, Singularity free gravitational collapse in an effective dynamical quantum spacetime, Phys. Lett. B **733**, 169 (2014).

[24] K. Falls, D. F. Litim, and A. Raghuraman, Black holes and asymptotically safe gravity, Int. J. Mod. Phys. A **27**, 1250019 (2012).

[25] B. Koch and F. Saueressig, Structural aspects of asymptotically safe black holes, Classical Quantum Gravity **31**, 015006 (2013); B. Koch, C. Contreras, P. Rioseco, and F. Saueressig, Black holes and running couplings: A comparison of two complementary approaches, Springer Proc. Phys. **170**, 263 (2016).

[26] O. Lauscher and M. Reuter, Fractal spacetime structure in asymptotically safe gravity, J. High Energy Phys. 10 (2005) 050.

[27] M. Reuter and F. Saueressig, Fractal space-times under the microscope: A renormalization group view on Monte Carlo data, J. High Energy Phys. 12 (2011) 012.

[28] S. Rechenberger and F. Saueressig, The R^2 phase diagram of QEG and its spectral dimension, Phys. Rev. D **86**, 024018 (2012).

[29] G. Calcagni, A. Eichhorn, and F. Saueressig, Probing the quantum nature of spacetime by diffusion, Phys. Rev. D **87**, 124028 (2013).

[30] A. Bonanno and M. Reuter, Cosmology of the Planck era from a renormalization group for quantum gravity, Phys. Rev. D **65**, 043508 (2002); Cosmology with self-adjusting vacuum energy density from a renormalization group fixed point, Phys. Lett. B **527**, 9 (2002).

[31] M. Reuter and F. Saueressig, From big bang to asymptotic de Sitter: Complete cosmologies in a quantum gravity framework, J. Cosmol. Astropart. Phys. 09 (2005) 012.

[32] E. Bentivegna, A. Bonanno, and M. Reuter, Confronting the IR fixed point cosmology with high redshift supernova data, J. Cosmol. Astropart. Phys. 01 (2004) 001.

[33] A. Bonanno, G. Esposito, C. Rubano, and P. Scudellaro, Noether symmetry approach in pure gravity with variable G and Lambda, Classical Quantum Gravity **24**, 1443 (2007).

[34] S. Weinberg, Asymptotically safe inflation, Phys. Rev. D **81**, 083535 (2010).

[35] A. Bonanno and M. Reuter, Cosmological perturbations in renormalization group derived cosmologies, Int. J. Mod. Phys. D **13**, 107 (2004); M. Reuter and H. Weyer, Quantum gravity at astrophysical distances?, J. Cosmol. Astropart. Phys. 12 (2004) 001; Running Newton constant, improved gravitational actions, and galaxy rotation curves, Phys. Rev. D **70**, 124028 (2004).

[36] M. Reuter and H. Weyer, On the possibility of quantum gravity effects at astrophysical scales, Int. J. Mod. Phys. D **15**, 2011 (2006); B. F. L. Ward, Planck scale cosmology in resummed quantum gravity, Mod. Phys. Lett. A **23**, 3299 (2008); A. Bonanno, A. Contillo, and R. Percacci, Inflationary solutions in asymptotically safe f(R) theories, Classical Quantum Gravity **28**, 145026 (2011).

[37] S.-H. H. Tye and J. Xu, Comment on asymptotically safe inflation, Phys. Rev. D **82**, 127302 (2010); B. F. L. Ward, Planck scale cosmology and asymptotic safety in resummed quantum gravity, *Proc. Sci.*, ICHEP2010 (2010) 477 [arXiv:1012.2680]; R. J. Yang, Asymptotically safe phantom cosmology, Eur. Phys. J. C **72**, 1948 (2012); Y. F. Cai and D. A. Easson, Asymptotically safe gravity as a scalar-tensor theory and its cosmological implications, Phys. Rev. D **84**, 103502 (2011); A. Contillo, M. Hindmarsh, and C. Rahmede, Renormalization group improvement of scalar field inflation, Phys. Rev. D **85**, 043501 (2012).

[38] M. Reuter and F. Saueressig, Asymptotic safety, fractals, and cosmology, Lect. Notes Phys. **863**, 185 (2013).

[39] A. Bonanno and M. Reuter, Primordial entropy production and Lambda-driven inflation from quantum Einstein gravity, J. Phys. Conf. Ser. **140**, 012008 (2008).

[40] A. Bonanno and M. Reuter, Entropy signature of the running cosmological constant, J. Cosmol. Astropart. Phys. 08 (2007) 024.

[41] E. P. Tryon, Is the universe a vacuum fluctuation, Nature (London) **246**, 396 (1973).

[42] M. Reuter and H. Weyer, Renormalization group improved gravitational actions: A Brans-Dicke approach, Phys. Rev. D **69**, 104022 (2004).

[43] G. Kofinas and V. Zarikas, Avoidance of singularities in asymptotically safe quantum Einstein gravity, J. Cosmol. Astropart. Phys. 10 (2015) 069.

[44] B. Guberina, R. Horvat, and H. Stefancic, Renormalization group running of the cosmological constant and the fate of the universe, Phys. Rev. D **67**, 083001 (2003).

[45] F. Bauer, The running of the cosmological and the Newton constant controlled by the cosmological event horizon, Classical Quantum Gravity **22**, 3533 (2005).

[46] A. V. Frolov and J. Q. Guo, Small cosmological constant from running gravitational coupling, arXiv:1101.4995.

[47] M. Hindmarsh and I. D. Saltas, f(R) gravity from the renormalization group, Phys. Rev. D **86**, 064029 (2012).

[48] E. J. Copeland, C. Rahmede, and I. D. Saltas, Asymptotically safe Starobinsky inflation, Phys. Rev. D **91**, 103530 (2015).

[49] E. Manrique, M. Reuter, and F. Saueressig, Bimetric renormalization group flows in quantum Einstein gravity, Ann. Phys. (Amsterdam) **326**, 463 (2011).

[50] A. Erdeleyi, *Higher Transcendental Functions* (McGraw-Hill Book Company, Inc., New York, 1953), Vol. 1.

[51] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables* (Dover Books on Mathematics, New York, 1965).

[52] M. Hindmarsh, D. Litim, and C. Rahmede, Asymptotically safe cosmology, J. Cosmol. Astropart. Phys. 07 (2011) 019.

[53] C. Ahn, C. Kim, and E. V. Linder, From asymptotic safety to dark energy, Phys. Lett. B **704**, 10 (2011).

[54] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (Wiley, New York, 1972).

[55] S. Sen and A. A. Sen, Late time acceleration in Brans-Dicke cosmology, Phys. Rev. D **63**, 124006 (2001).

[56] W. Zimdahl, D. J. Schwarz, A. B. Balakin, and D. Pavon, Cosmic anti-friction and accelerated expansion, Phys. Rev. D **64**, 063501 (2001).

[57] S. Floerchinger, N. Tetradis, and U. A. Wiedemann, Accelerating Cosmological Expansion from Shear and Bulk Viscosity, Phys. Rev. Lett. **114**, 091301 (2015).

[58] M. S. Berman and L. A. Trevisan, On the creation of the Universe out of nothing, Int. J. Mod. Phys. D **19**, 1309 (2010); M. S. Berman and R. M. Marinho, Jr., Astrophys. Space Sci. **278**, 369 (2001).

[59] I. Prigogine, J. Geheniau, E. Gunzig, and P. Nardone, Thermodynamics and cosmology, Gen. Relativ. Gravit. **21**, 767 (1989).

[60] M. P. Freaza, R. S. de Souza, and I. Waga, Cosmic acceleration and matter creation, Phys. Rev. D **66**, 103502 (2002).

[61] R. O. Ramos, M. V. d. Santos, and I. Waga, Matter creation and cosmic acceleration, Phys. Rev. D **89**, 083524 (2014).

[62] W. Israel and J. M. Stewart, Transient relativistic thermodynamics and kinetic theory, Ann. Phys. (N.Y.) **118**, 341 (1979).

[63] G. Aliferis, G. Kofinas, and V. Zarikas, Efficient electroweak baryogenesis by black holes, Phys. Rev. D **91**, 045002 (2015).

[64] G. Kofinas and V. Zarikas, A solution of the coincidence problem based on the recent galactic core black hole energy density increase, Eur. Phys. J. C **73**, 2379 (2013).

[65] J. Braun, H. Gies, and D. D. Scherer, Asymptotic safety: A simple example, Phys. Rev. D **83**, 085012 (2011).