PHYSICAL REVIEW D **94,** 096003 (2016)

Studies of Wigner-Weyl solution and external magnetic field in an NJL model

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(Received 21 July 2016; published 7 November 2016)

In this paper, we explore the dynamical chiral symmetry breaking by employing a two-flavor Nambu–Jona-Lasinio (NJL) model with constant external magnetic field. After changing the coupling strength of the NJL model, we found that the Wigner-Weyl solution and Nambu-Goldstone solution of the gap equation could coexist. Even though the gap equation only has Nambu-Goldstone solution at zero temperature, the Wigner-Weyl solution may appear when magnetic field strength and temperature are nonzero. For the Nambu-Goldstone solution, magnetic field and temperature have opposite impact on the chiral dynamical mass. In the chiral limit, the magnetic field dependence of chiral dynamical mass reveals the existence of inverse magnetic catalysis for the Wigner-Weyl solution. However, the two phases have different responses to the magnetic field and temperature in the chiral limit but the same beyond chiral limit. Furthermore, the order of the transition from the Nambu-Goldstone phase to Wigner-Weyl phase depends on the choice of model parameters. We have also calculated the susceptibilities of dynamical mass with respect to the temperature.

DOI: 10.1103/PhysRevD.94.096003

I. INTRODUCTION

It is believed that temperature and magnetic field play important roles in the study of phase transition and chiral symmetry restoration of strongly interacting matter [1–4]. Quantum chromodynamics (QCD) is now commonly accepted as the basic theory for strong interaction, even in the study of small-momentum-transfer processes where the coupling constant becomes so strong. The Nambu–Jona-Lasinio (NJL) model was originally a theory of interacting nucleons. The idea is that the mass gap in the Dirac spectrum of the nucleon is analogous to the energy gap of a superconductor in BCS theory [5–7]. It was reinterpreted as a quark model after the development of QCD. So the NJL model has its advantages in the study of the phase transition of strong interacting matter [8,9].

The strength of the four-fermion interaction in the NJL model is represented by the dimensional coupling constant G. Usually it is given by fitting the pion decay constant and pion mass in the two-flavor NJL model, while researchers might take different values of G, especially within different regularization. However, the strength of the coupling constant G may affect the spontaneous chiral symmetry breaking. The chiral symmetry can be preserved with a small G while it is broken in a magnetic field environment. As a QCD inspired model, the coupling constant of the NJL model may look like the same role of the strength constants of four-fermion interaction in the weak theory. And the coupling constant may have dependence on the quark and gluon condensate [10–13]. However, it is not available at present to calculate this dependence from the first principle of QCD. To study the confinement phase, effective coupling depending on the Polyakov loop $G(\phi)$ is discussed in the renormalization group method [14–16]. The coupling has different values in confinement and deconfinement phases, which makes the chiral and deconfinement crossovers almost coincide in the Polyakov-loop extended NJL model [17]. Besides, a modified coupling constant was introduced as $G \to G1 + G2\langle\bar{\psi}\psi\rangle$ [18], which is inspired by the nonperturbative gluon propagator from a QCD sum rule result [13], which manifests the difference of the Nambu-Goldstone and Wigner-Weyl phases, and the $G2$ weighs the influence of the quark propagator to the gluon propagator. The Nambu-Goldstone phase is usually considered as the only solution of the quark gap equation beyond the chiral limit; however, as investigated in Refs. [19–29], the gap equation may have a Wigner-Weyl solution for nonzero current quark mass, and in this case a coupling constant reflecting the difference of the two phases is necessary. Actually, the multisolutions also exist in the case of finite temperature and quark number chemical potential for nonzero current quark mass and the critical point appears. Further, this newly defined coupling might make the chiral phase transition from crossover to first order in a magnetic field free environment [18].

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The influence of magnetic field on the dynamical chiral symmetry breaking, known as "magnetic catalysis," has been studied widely [30–36] in recent years. In the model with constant coupling the magnetic field always enhances the chiral dynamical mass while the temperature weakens it; for every value of magnetic field strength there is a critical temperature at which the broken phase transits to the symmetric phase, and the results with a constant coupling show that there is no inverse magnetic catalysis (IMC). The IMC effect was discovered from lattice QCD simulations [37] and some explanations were proposed, one of which requires a modification to the coupling, especially in the effective model [38–42]. Recently the lattice results were used to study the thermomagnetic dependence of coupling $G(B,T)$ [43,44]. In Fig. 2 of Ref. [37], the IMC appears at high temperature and strong field strength. For a condensate dependent coupling, the coupling is deduced as the temperature increasing, which has a behavior similar to the fitted formula in Ref. [43]. Besides, the model without magnetic field in Ref. [18] shows that as the $G1$ starts from a small value and increases, the first order phase transition converts to a crossover. So it is worthwhile to study the coexistence of the two phases and their dynamical chiral symmetry breaking as the magnetic field is introduced.

In the following we give the two-flavor NJL Lagrangian with an external magnetic field in Sec. II A. By introducing the condensate dependent coupling, the gap equations with different parameters are compared. Our main results are shown in Secs. II B–II D. In these subsections, we study the coexistence of the Nambu-Goldstone and Wigner-Weyl phases for zero and nonzero current quark masses. We show here that the inverse magnetic catalysis appears in the Wigner-Weyl phase. Also a comparison with the lattice QCD results is given. Further, susceptibilities, which are related to the critical coefficients, are calculated. The phase transitions and the critical temperatures as functions of magnetic field and coupling are also discussed. At the end we give a brief summary.

II. MODEL AND RESULTS

We use the NJL model with an external magnetic field in this work. Usually, the Fock-Schwinger proper time method is applied to compute the fermion propagator when temperature and chemical potential are considered [45–47]. If the chemical potential corresponding to the particle number density is not included, another new method develops as magnetic field is polarized. Without the chemical potential the final formulas are more concise. We just consider this in the following analysis.

A. The Nambu–Jona-Lasinio model

The Lagrangian of the two-flavor NJL model with an external magnetic field A is [47–50]

$$L = \sum_{i=u,d} \bar{\psi}_i (i\partial\!\!\!/ + eq_i A\!\!\!/)\psi_i - \frac{G}{2N_c}\left\{\left(\sum\nolimits_{i=u,d}\bar{\psi}_i\psi_i\right)^2 + \left(\sum\nolimits_{i,j=u,d}\bar{\psi}_i i\gamma^5 \vec{\tau}_{ij}\psi_j\right)^2\right\}. \quad (1)$$

It is easy to check that this Lagrangian has the global chiral symmetry while the flavor symmetry is broken, and is invariant under the transformation $\psi_i \to \exp\{i\theta \frac{\tau^3_{ij}}{2}\gamma_5\}\psi_j$. In a convenient way, the Lagrangian is rewritten in the bosonized form

$$L = \bar{\psi}(i\partial\!\!\!/ + eA\!\!\!/ \otimes Q - \sigma - i\gamma^5 \otimes \vec{\pi}\cdot\vec{\tau})\psi - \frac{N_c}{2G}\Sigma^2 \quad (2)$$

with the correspondences $\sigma \sim -\langle\bar{\psi}\psi\rangle$ and $\vec{\pi} \sim -\langle\bar{\psi}i\gamma^5\vec{\tau}\psi\rangle$. G is the coupling constant and independent of the energy scale. N_c is the number of colors and the other quantities are

$$Q = \begin{pmatrix} q_u & 0 \\ 0 & q_d \end{pmatrix}, \qquad q_u = \frac{2}{3}$$
$$q_d = -\frac{1}{3}, \qquad q_f = q_u, q_d, \quad (3)$$

$$\Sigma^2 = \sigma^2 + \pi^2, \qquad \pi^2 = \sum_{i=1}^{3}\pi_i^2 \quad (4)$$

$$\vec{\tau} = (\tau^1, \tau^2, \tau^3), \qquad \tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$
$$\tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (5)$$

To give a constant external magnetic field along the third direction of space, A can be defined as

$$(A_0, A_1, A_2, A_3) = \left(0, \frac{B}{2}x^2, -\frac{B}{2}x^1, 0\right). \quad (6)$$

The general gap equation can be easy deduced by a partial derivation from the free energy

$$F = \frac{N_c}{2G}(\sigma^2 + \pi^2) + N_c i\mathrm{Tr}\,\ln[i\partial\!\!\!/ + eA\!\!\!/ \otimes Q - (\sigma + i\gamma^5 \otimes \vec{\pi}\cdot\vec{\tau})], \quad (7)$$

where the trace is to be taken in spin, flavor, and momentum space, with the assumption that the ground state does not break the local $U(1)_{\mathrm{EM}}$ symmetry, only the isospin singlet, i.e., the σ state remains. So we only consider the gap equation of σ,

$$\frac{\sigma}{G}\int d^4x = i\mathrm{Tr}(\hat{S}), \tag{8}$$

with

$$\hat{S} = \frac{1}{\not{p} + eA \otimes Q - \sigma}. \tag{9}$$

By using the method developed in Ref. [36] to compute the propagator, the gap equations at 0 and finite temperature are written respectively as

$$\frac{4\pi^2}{G} = N_f \int_{\frac{1}{\Lambda^2}}^{+\infty} \frac{e^{-\sigma^2 s}}{s^2} ds + \sum_f |q_f| eB \int_0^{+\infty} \frac{e^{-\sigma^2 s}}{s} \times \left[\coth(|q_f|eBs) - \frac{1}{|q_f|eBs}\right] ds \tag{10}$$

and

$$\frac{4\pi^2}{G} = 4\sqrt{\pi} N_f T \int_{\frac{1}{\Lambda^2}}^{+\infty} \left(\sum_{n=0}^{+\infty} e^{-\omega_n^2 s}\right) \frac{e^{-\sigma^2 s}}{s\sqrt{s}} ds + 4\sqrt{\pi} T \sum_f |q_f| eB \int_0^{+\infty} \left(\sum_{n=0}^{+\infty} e^{-\omega_n^2 s}\right) \times \frac{e^{-\sigma^2 s}}{\sqrt{s}} \left[\coth(|q_f|eBs) - \frac{1}{|q_f|eBs}\right] ds. \tag{11}$$

The above two equations are the results in the chiral limit with $\omega_n = (2n+1)\pi T, n \in \{\ldots -2, -1, 0, 1, 2\ldots\}$. The values of cutoff and coupling are determined by fitting the pion mass and pion decay constant as $\Lambda = 0.99$ GeV, $G = 25.4$ GeV^{-2}. The gap equation with current quark mass m and finite temperature is

$$\frac{4\pi^2}{G}\sigma = 4\sqrt{\pi} N_f T(\sigma + m) \int_{\frac{1}{\Lambda^2}}^{+\infty} \left(\sum_{n=0}^{+\infty} e^{-\omega_n^2 s}\right) \frac{e^{-(\sigma+m)^2 s}}{s\sqrt{s}} ds + 4\sqrt{\pi} T(\sigma + m) \sum_f |q_f| eB \int_0^{+\infty} \left(\sum_{n=0}^{+\infty} e^{-\omega_n^2 s}\right) \times \frac{e^{-(\sigma+m)^2 s}}{\sqrt{s}} \left[\coth(|q_f|eBs) - \frac{1}{|q_f|eBs}\right] ds. \tag{12}$$

B. Evolution of chiral dynamical mass with magnetic field and temperature

With a constant coupling, the gap equation only has one solution for a given temperature, while if we replace the coupling constant G by

$$G(\sigma) = g_1 + g_2\sigma, \tag{13}$$

another solution with smaller dynamical mass σ appears; see Fig. 1 for $g_1 < 25.4$ GeV^{-2}. As the G and Λ are given

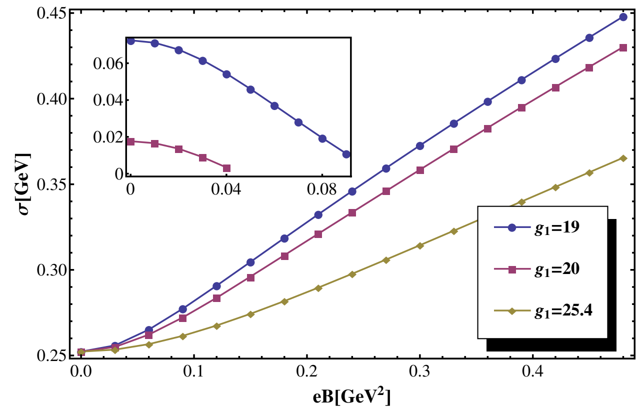

FIG. 1. The eB dependence of dynamical mass σ when $T = 0$. For $g_1 = 25.4$ GeV^{-2}, there is only one solution.

to fit the experimental value, the gap equation of Eq. (10) gives $\sigma = 0.25$ GeV when $T = 0$ and $eB = 0$. The values of g_2 are fitted to the point ($G = 25.4$ GeV^{-2}, $\sigma = 0.25$ GeV) for a given g_1. For nonzero T and B, there are no experimental values as references and the σ value can be adjusted by the couplings, so we still take g_1 and g_2 to fit the point ($G = 25.4$ GeV^{-2}, $\sigma = 0.25$ GeV) for nonzero T and B. The effective quark mass M can be determined as usual via the self-consistent gap equation

$$M = m - \frac{G}{N_c}\langle\bar{\psi}\psi\rangle, \tag{14}$$

where m is the current mass of the quark. As $\sigma \sim -\langle\bar{\psi}\psi\rangle$, the two solutions of σ are identified as the Wigner-Weyl phase and Nambu-Goldstone phase.

For larger g_1, only the Nambu-Goldstone phase exists in zero temperature, while for a relatively small value of g_1 (smaller than 20.3 GeV^{-2}, when $eB = 0.02$ GeV^2), see Figs. 1 and 2, the gap equation has two solutions. The solution with smaller σ goes to 0 quickly as the magnetic field strength increases, which means the spontaneous chiral symmetry restores in some critical value of eB. We identify it as the Wigner-Weyl solution. The other solution with a relatively large σ is the Nambu-Goldstone solution. As we can see from Fig. 1, the two solutions for dynamical mass show inverse trends of magnetic dependence. So the modification of the coupling constant makes the two phases coexistent and the Wigner-Weyl solution manifests an inverse magnetic catalysis.

In the above, we have replaced the G by $G(\sigma)$ in those gap equations as in Ref. [18]. If we make this replacement in the free energy, we must multiply $(G + g_1)/2G$ in the left of those gap equations. Since we consider the case in which g_1 does not very largely deviate from G and the additional multiplier may cause nonphysical solutions, we get rid of it in the calculation.

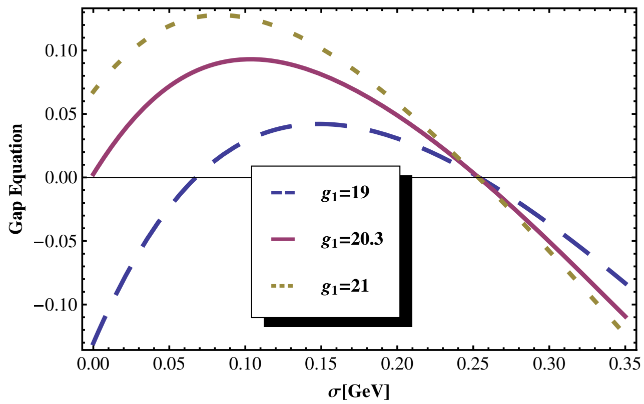

FIG. 2. The difference of the left and right in the gap equation Eq. (10) when $T = 0$ and $eB = 0.02\ \mathrm{GeV}^2$.

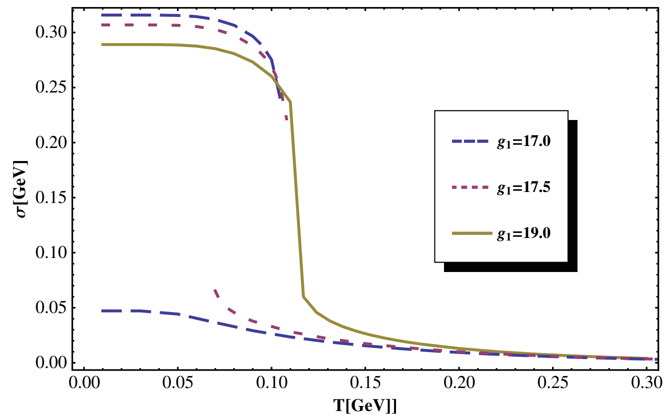

FIG. 4. The temperature dependence of dynamical mass σ when $eB = 0.01\ \mathrm{GeV}^2$ and $m = 5.5$ MeV for different g_1.

In the chiral limit, the Wigner-Weyl solution is the trivial solution of the gap equation, which is easy to see from Eq. (12). When $T = 0$ and $B = 0$, the gap equation has solution of $\sigma = 0$ as $m = 0$. For nonzero temperature, a plot of $g_1 = 19\ \mathrm{GeV}^{-2}$ is shown in Fig. 3. If we still identify the smaller solution as the Wigner-Weyl phase then the two phases are separated at relatively low temperature and reflect two different types of magnetic catalysis. As the temperature is high enough the two phases converge with each other. In the model with a constant coupling G, only the Nambu-Goldstone solution exists [36]. Dynamical mass σ starts from 0 and increases to finite value for high temperature as the magnetic field strength exceeds a specific quantity. For constant coupling, such a convergence of Nambu-Goldstone and Wigner-Weyl solutions only appears beyond the chiral limit as temperature increases.

The temperature dependence of the two phases for nonzero current mass and magnetic field strength is shown in Fig. 4. The picture is similar to Figs. (5)–(7) in Ref. [18] where the magnetic field strength is 0. The two phases are separated for small g_1 and smoothly linked for a larger one. The behaviors of the two solutions are also similar to that of the normal NJL model when chemical potential is high enough. The order of the phase transition is from crossover to first order when chemical potential exceeds some critical value. Here, as the temperature increases, the order of the chiral phase transition depends on our choice of the g_1.

In the above discussions we see that the multisolution may appear in the chiral limit even when temperature and magnetic field strengths are 0. As a comparison with the results from a model of constant G, we only study the case when $g_1 \geq 21\,\mathrm{GeV}^{-2}$. In this case there is only the Nambu-Goldstone phase when $T = 0$ and $B = 0$. To study the magnetic catalysis, we analyze the temperature-dependent gap equation Eq. (11). Besides, the results from $g_1 \geq 25.4\ \mathrm{GeV}^{-2}$ are analogous with the results from the model with constant coupling, so we do not discuss this situation further.

In Fig. 5, we show the solutions for $\sigma = 0$. For $g_1 = 25.4\ \mathrm{GeV}^{-2}$ and $g_2 = 0$, the left-up plane is identified as the chiral symmetric phase and the right-down plane is identified as the broken phase. To reach the symmetric phase, a critical temperature exists and increases as the magnetic field increases. As we reduce the g_1 value, the critical temperature increases. For smaller g_1 the σ value is

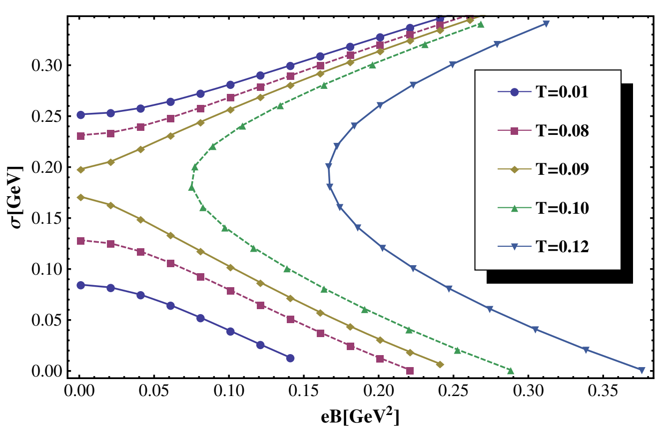

FIG. 3. The eB dependence of dynamical mass σ for different temperature when $g_1 = 19\ \mathrm{GeV}^{-2}$ and $m = 0$.

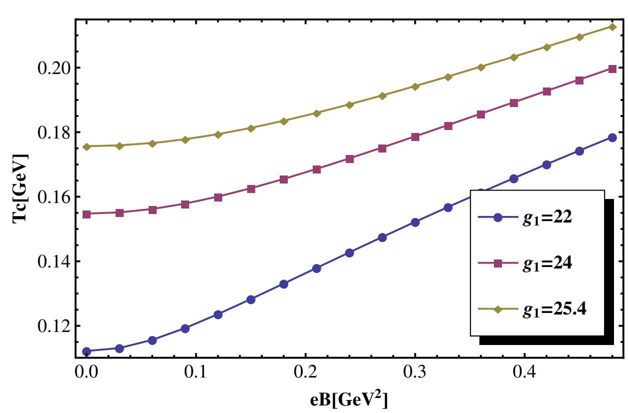

FIG. 5. The eB dependence of critical temperature T_c for different g_1 when $\sigma = 0$ and $m = 0$.

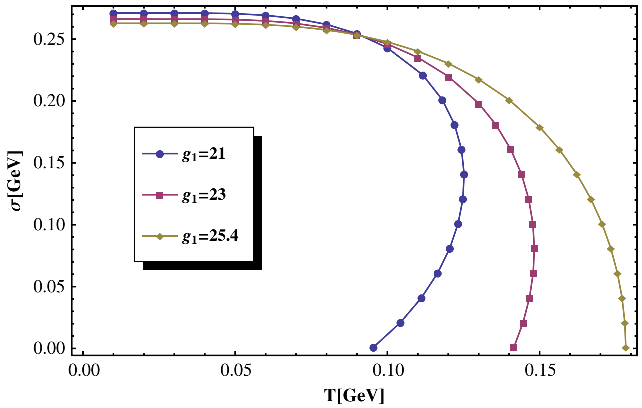

FIG. 6. The temperature dependence of dynamical mass σ for different g_1 when $eB = 0.1\ \mathrm{GeV}^2$ and $m = 0$.

not monotonously decreasing as showing in Fig. 6 when $g_1 < 25.4\ \mathrm{GeV}^{-2}$. This is similar to Fig. 3, where a crossover from the Nambu-Goldstone phase to Wigner-Weyl phase exists at high temperature. The two phases have different susceptibility related to the temperature and different susceptibility related to the magnetic field. Because the parameter we choose is confined to have only one solution at zero temperature and magnetic field strength, it is reasonable to have multisolutions at finite temperature and magnetic field. This result is obviously different from model with constant coupling G.

For fixed temperature, see Figs. 7 and 8, the Wigner-Weyl solution appears again at some ranges of magnetic field. The curves show the magnetic catalysis effect for the Nambu-Goldstone solution and the inverse one for the Wigner-Weyl solution. When the magnetic field is strong enough, there is only the Nambu-Goldstone solution again. For very low temperature, there is only the Nambu-Goldstone solution too; see Fig. 9.

For $m \neq 0$ and $g_1 \geq 21$, plots are shown in Figs. 10 and 11. Three g_1 values are chosen to give only one solution at zero temperature and magnetic field strength. As the

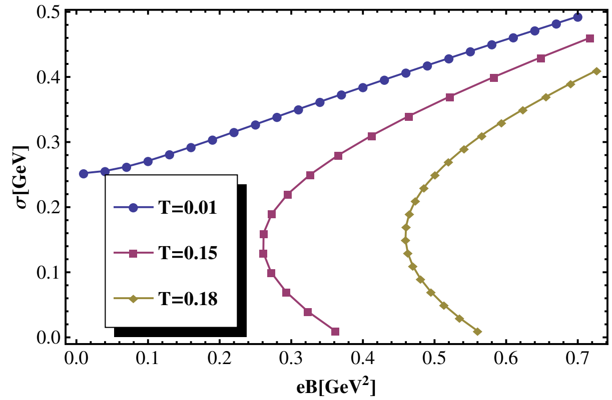

FIG. 8. The eB dependence of dynamical mass σ for different temperatures when $g_1 = 21\ \mathrm{GeV}^{-2}$ and $m = 0$.

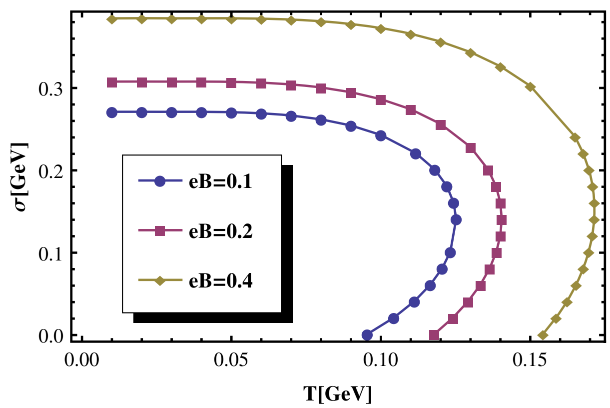

FIG. 9. The temperature dependence of dynamical mass σ for different eB when $g_1 = 21\ \mathrm{GeV}^{-2}$ and $m = 0$.

quarks get a nonzero mass, the Nambu-Goldstone phase and Wigner-Weyl phase no longer coexist. Figure 11 also shows that there is no inverse magnetic catalysis. The transition for the Nambu-Goldstone phase to the Wigner-Weyl phase is now a crossover. Figure 11 for temperature

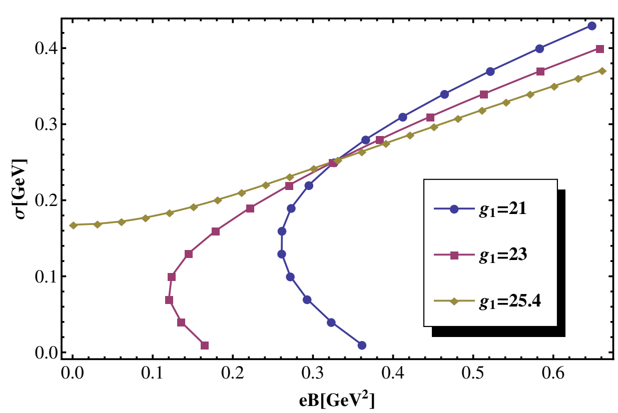

FIG. 7. The eB dependence of dynamical mass σ for different g_1 when $T = 0.15$ GeV and $m = 0$.

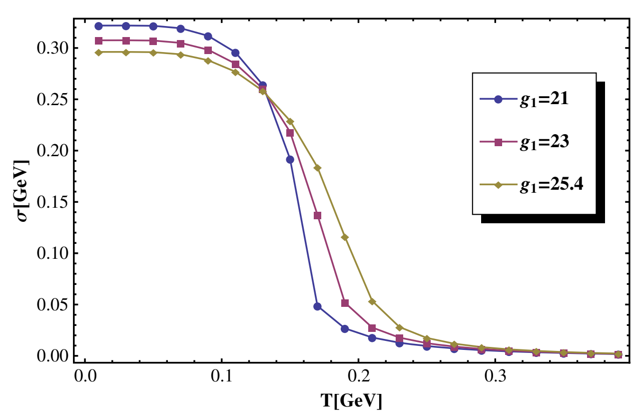

FIG. 10. The temperature dependence of dynamical mass σ for different g_1 when $eB = 0.2$ GeV and $m = 5.5$ MeV.

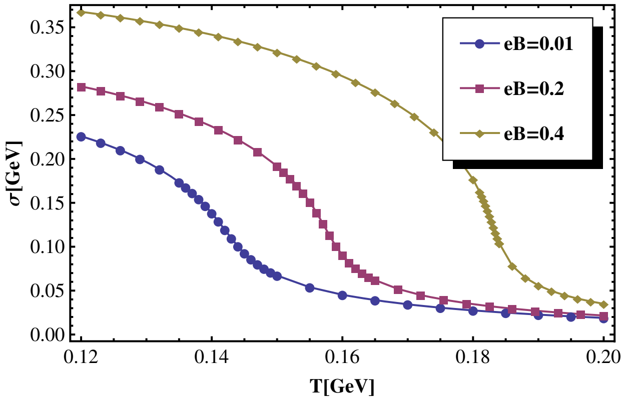

FIG. 11. The temperature dependence of dynamical mass σ for different eB when $g_1 = 21\ \mathrm{GeV}^{-2}$ and $m = 5.5$ MeV.

from 0.12 to 0.2 GeV shows well the crossover points and their reaction against the magnetic field. For a better understanding of this we show the susceptibilities to the temperature in Sec. II D.

C. Comparison with lattice results

As for nonzero current quark mass, the IMC effect does not exist in the model with the condensate dependent coupling as g_1 is larger than 21 GeV^{-2}. It is worthwhile to take the lattice QCD result as an input to fix the g_1 value whenever it is possible. In Fig. 12, the $g_1 = 22.3\ \mathrm{GeV}^{-2}$ is the best fit when $B = 0$ (to fit it, four parameters and an exponential function are used in Ref. [43]). Although we only have one free parameter in the linear condensate dependent coupling, it still has excellent fit with the lattice result at temperature smaller than 0.15 GeV. The condensate is deduced to 0 quickly as temperature is larger than 0.15 GeV in the lattice computation, which means the dynamical chiral symmetry is restored for relatively low temperature. If we only consider the condensate influence to the effective coupling, higher order parameters are needed so as to give a better fit to the lattice result at high temperature. When fixed the g_1 to 22.3 GeV^{-2}

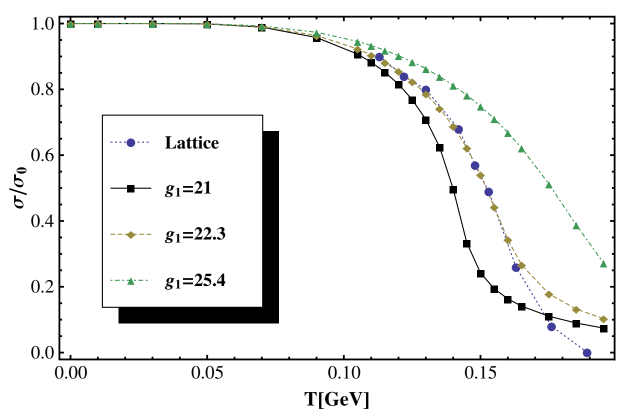

FIG. 12. Comparison to lattice result [37] for $\Delta(\Sigma_u + \Sigma_d)/2$ at $eB = 0$ for different g_1 values when $m = 5.5$ MeV.

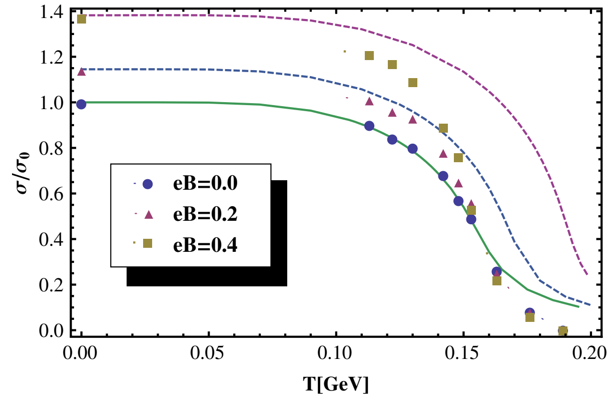

FIG. 13. Comparison to lattice results as g_1 fixed to 22.3 GeV^{-2} for different magnetic field strength when $m = 5.5$ MeV.

comparisons to lattice results for nonzero magnetic field is shown in Fig. 13. The lines are separating from each other and lines of the larger eB always lie above, which mean there is no IMC. For strong magnetic field our results are larger than the results from lattice QCD at high temperature but near zero temperature the deviations are not very obvious. Since the condensate σ is a function of temperature and magnetic field, i.e., $\sigma = \sigma(B, T)$, the comparisons for nonzero magnetic field hint that higher order amending may give better results as given by the thermomagnetic dependence of $G(B, T)$.

D. The susceptibility related to temperature

The susceptibilities related to magnetic field and temperature are useful for the study of phase transitions [51–53]. As there is no chemical potential, they are defined as the partial derivative of eB and T, which can be read out from figures of the last subsection. Also the susceptibilities can be derived directly from the gap equations. Here, we consider the susceptibilities related to temperature only, which is defined as

$$\chi_T = -\frac{\partial \sigma}{\partial T}. \tag{15}$$

From the gap equation Eq. (12), with G replaced by $G(\sigma) = g_1 + g_2\sigma$ we can get

$$\chi_T = (1 + \chi_c)\left[\frac{G(\sigma)}{4\pi^2}(\sigma + m) f_T - \frac{\sigma}{T}\right]\left(\frac{1}{1 - \Delta}\right), \tag{16}$$

with

$$\chi_c = \frac{1 - \Delta}{\frac{m}{\sigma + m} + \frac{G(\sigma)}{4\pi^2}(\sigma + m)^2 f_m - \Delta} - 1, \tag{17}$$

$$\Delta = \frac{g_2 \sigma}{G(\sigma)}, \tag{18}$$

$$f_m(\sigma, m, T, eB) = 8\sqrt{\pi}N_f T\int_{\frac{1}{\Lambda^2}}^{+\infty}\left(\sum_{n=0}^{+\infty}e^{-\omega_n^2 s}\right)\frac{e^{-(\sigma+m)^2 s}}{\sqrt{s}}ds + 8\sqrt{\pi}T\sum_f |q_f|eB\int_0^{+\infty}\left(\sum_{n=0}^{+\infty}e^{-\omega_n^2 s}\right) \times e^{-(\sigma+m)^2 s}\sqrt{s}\left[\coth(|q_f|eBs) - \frac{1}{|q_f|eBs}\right]ds, \qquad (19)$$

and

$$f_T(\sigma, m, T, eB) = 8\pi^{\frac{5}{2}}N_f T^2\int_{\frac{1}{\Lambda^2}}^{+\infty}\left(\sum_{n=0}^{+\infty}(2n+1)^2 e^{-\omega_n^2 s}\right)\frac{e^{-(\sigma+m)^2 s}}{\sqrt{s}}ds + 8\pi^{\frac{5}{2}}T^2\sum_f |q_f|eB\int_0^{+\infty}\left(\sum_{n=0}^{+\infty}(2n+1)^2 e^{-\omega_n^2 s}\right) \times e^{-(\sigma+m)^2 s}\sqrt{s}\left[\coth(|q_f|eBs) - \frac{1}{|q_f|eBs}\right]ds. \qquad (20)$$

In Ref. [36] with a constant coupling G, the susceptibility shows a crossover in the critical temperature. It is also the case for $G(\sigma)$. The temperature dependance of dynamical mass is shown in Figs. 10 and 11 of the last section. By introducing the current mass m, the dynamical mass does not decrease to 0 near the critical temperature. In Fig. 14, for fixed eB and g_1, there is a crossover point. These points give the critical temperature for the phase transition. The critical temperature increases as the magnetic field strength or g_1 increases, which means to switch to the Wigner-Weyl phase, we must increase the temperature in a strong magnetic field environment. This manifests the opposite effect from magnetic field and temperature on dynamical mass. In Ref. [36] with constant coupling, the crossover points are located around $T = 0.2$ GeV for $eB = 0.01 \sim 0.39$ GeV^{-2}. In our cases, the points are located separately from 0.14 to 0.18 GeV. So the phase transition temperature is lowered by including a σ dependent coupling and the magnetic field improves the transition from the Nambu-Goldstone phase to the Wigner-Weyl phase.

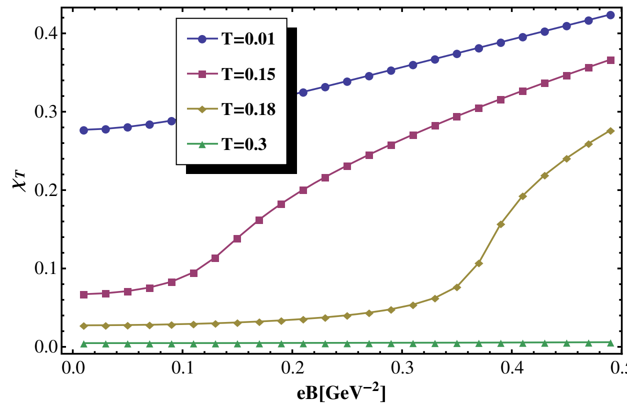

FIG. 15. The eB dependence of χ_T for different T when $g_1 = 21$ GeV^{-2} and $m = 5.5$ MeV.

For fixed temperature, the eB dependence of the susceptibilities is shown in Fig. 15. There exists no inverse magnetic catalysis for small current mass. And for a relatively large temperature the chiral dynamical mass almost does not react to the magnetic field.

As for $m = 0$, the slopes of the transit points are infinite; see Fig. 9. The related susceptibilities defined in Eq. (15) give positive infinity as the temperature increasing and then change to negative; see Figs. 16 and 17. So we refer to the critical coefficients.

The critical coefficient related to temperature is defined by the relation

$$\chi_T \sim t^{-\gamma_T} \qquad (21)$$

or

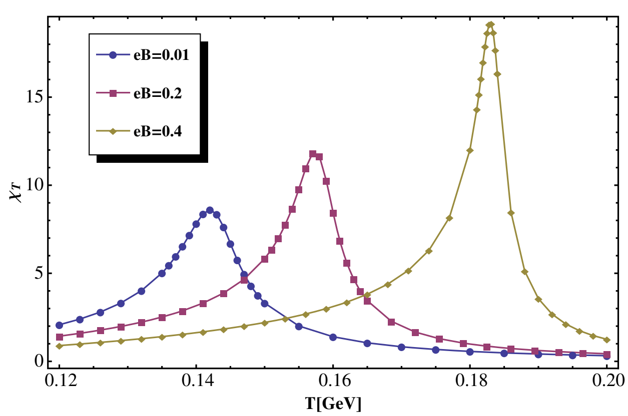

FIG. 14. The temperature dependence of χ_T for different eB when $g_1 = 21$ GeV^{-2} and $m = 5.5$ MeV.

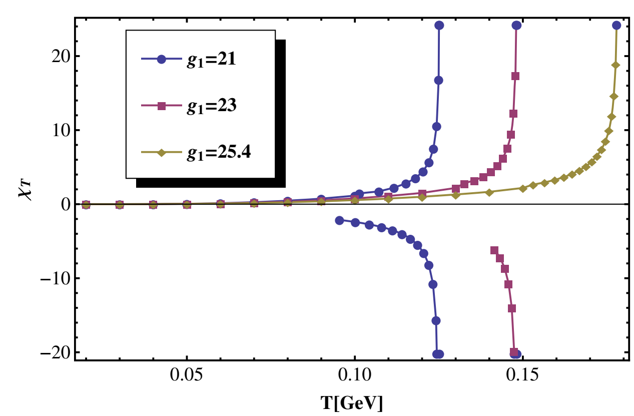

FIG. 16. The temperature dependence of χ_T for different g_1 when $m = 0$.

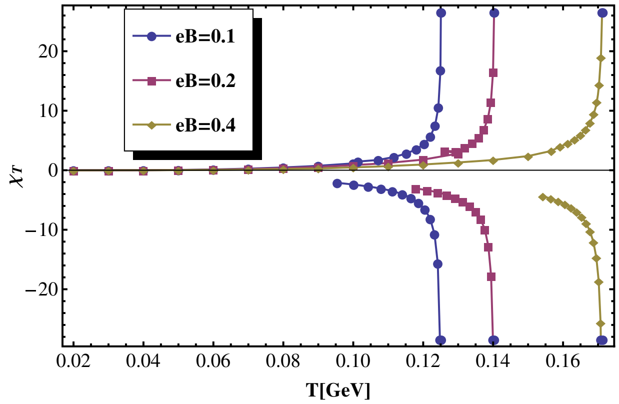

FIG. 17. The temperature dependence of χ_T for different eB when $m = 0$.

$$ln\chi_T = -\gamma_T lnt + c, \tag{22}$$

with γ_T being the critical coefficient and the t defined as

$$t = 1 - \frac{T}{T_c}. \tag{23}$$

In Fig. 18, the slope of the curve is the value of $-\gamma_T$. It shows that the $-\gamma_T$ is almost a constant near the critical temperature, i.e., the slope of the small side of $ln(t)$. The functions with linear fit of the curves close to the critical temperature are

$$\begin{aligned} eB = 0.01:&\ -0.58x - 0.52 \\ 0.2:&\ -0.52x - 0.01 \\ 0.4:&\ -0.49x + 0.21. \end{aligned} \tag{24}$$

The critical coefficients are easy to read out from Eq. (24). They are almost around 0.5, which is a little smaller than the results in Ref. [36] with a constant coupling but close to the mean field results; see Table III of Ref. [25].

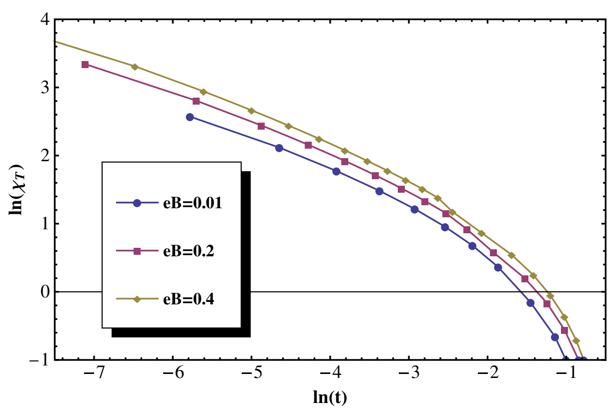

FIG. 18. The critical coefficient relation of Eq. (22) when $m = 0$.

III. SUMMARY

We have studied the magnetic catalysis and chiral phase transition with a modified two-flavor NJL model. In most NJL and Polyakov-loop extented NJL models with constant coupling, the Nambu-Goldstone solution and Wigner-Weyl solution cannot exist at the same time. The Wigner-Weyl solution only exists at high temperature or chemical potentials. In the magnetic environment, the dynamical mass always increases as the magnetic strength increases. In this paper, the constant coupling is modified to condensate dependent coupling $G(\sigma)$; then the Wigner-Weyl phase and Nambu-Goldstone phase may coexist in some cases beyond and in the chiral limit. The coupling may also have dependance on temperature and the others, but it is still impossible to calculate such dependence from the first principle of QCD. So we adopt a form deduced from the QCD sum rule. We take the chiral dynamical mass as the order parameter. The Wigner-Weyl phase and Nambu-Goldstone phase are the solutions of the gap equation that are deduced by a method different from the Schwinger proper time approach. We find that the Nambu-Goldstone phase shows magnetic catalysis while the Wigner-Weyl phase shows the inverse one. Even if we do not allow the Wigner-Weyl phase to appear in the zero temperature, the Wigner-Weyl solution still exists at finite temperature. And the start point of the temperature depends on the strength of the magnetic field. The transition from the Nambu-Goldstone phase to the Wigner-Weyl phase in the chiral limit is first order, which is indicated by the temperature-related susceptibility. And the critical coefficients we obtained agreed with other calculations. In the case with small current quark mass, we can still have the two phases coexist, which is a first order transition. But as we choose the parameter to give only one solution of the gap equation in low temperature, the two phases do not coexist and the transition from the Nambu-Goldstone phase to the Wigner-Weyl phase is a crossover. Further, the crossover points in the model with $G(\sigma)$ are less and more widely separated than points in the model with constant G. The comparisons with the lattice result may fix the model parameter that does not allow the Wigner-Weyl phase to appear. The NJL model with modified coupling gives good fit to the lattice results of weak magnetic field at low temperature but no IMC effect at high temperature and strong magnetic field. So higher order condensate modification to the coupling may be plausible if we do not obviously include the magnetic field and temperature in the effective coupling. In the end, the modification here to the NJL model shows many differences to the normal model, so it is worthwhile to take such modification into the other effective model. Especially, we hope the combination of this modification and the Polyakov-loop-dependent coupling may give us some new insight into the issue of coincidence of chiral symmetry restoration and the confinement-to-deconfinement transition.

ACKNOWLEDGMENTS

Q.-W. W. thanks Professor Xiao-Fu Lü for helpful discussions on the gap equation and renormalization group method. This work is supported in part by the National Natural Science Foundation of China (under Grants No. 11047020, No. 11404157, No. 11275097, No. 11475085, and No. 11535005), the China Postdoctoral Science Foundation (under Grant No. 2015M581765), and the Jiangsu Planned Projects for Postdoctoral Research Funds (under Grant No. 1402006C).

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