PHYSICAL REVIEW D **94,** 096002 (2016)

$f_0(980)$ production in $D_s^+ \to \pi^+\pi^+\pi^-$ and $D_s^+ \to \pi^+K^+K^-$ decays

J. M. Dias,* F. S. Navarra,† and M. Nielsen‡
Instituto de Física, Universidade de São Paulo, C.P. 66318, 05389-970 São Paulo, São Paulo, Brazil

E. Oset§
Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC, Institutos de Investigación de Paterna, Aptdo. 22085, 46071 Valencia, Spain
(Received 17 May 2016; published 4 November 2016)

We study the $D_s^+ \to \pi^+\pi^+\pi^-$ and $D_s^+ \to \pi^+K^+K^-$ decays adopting a mechanism in which the D_s^+ meson decays weakly into a π^+ and a $q\bar{q}$ component, which hadronizes into two pseudoscalar mesons. The final state interaction between these two pseudoscalar mesons is taken into account by using the chiral unitary approach in coupled channels, which gives rise to the $f_0(980)$ resonance. Hence, we obtain the invariant mass distributions of the pairs $\pi^+\pi^-$ and K^+K^- after the decay of that resonance and compare our theoretical amplitudes with those available from the experimental data. Our results are in a fair agreement with the shape of these data, within large experimental uncertainty, and a $f_0(980)$ signal is seen in both the $\pi^+\pi^-$ and K^+K^- distributions. Predictions for the relative size of $\pi^+\pi^-$ and K^+K^- distributions are made.

DOI: 10.1103/PhysRevD.94.096002

I. INTRODUCTION

The analysis of heavy meson weak decays measured in B-factories and at the LHC has been very important for the study of new hadronic states and ultimately for the understanding of hadron dynamics. In these reactions, the weak decay leads to hadronic states, in general composed of two or three hadrons, which undergo "final state interactions" (FSI), through which they form the final particles. The FSI is very complex and can influence all the conclusions concerning new states and even provide the strength of CP violation [1]. In this work we consider the $D_s^+ \to \pi^+\pi^+\pi^-$ and $D_s^+ \to \pi^+K^+K^-$ decays and we study the effect of FSI on the measured invariant mass spectra. These decays have been studied by several experimental groups [2–9] and they have been considered excellent tools to study FSI. Their distinctive feature is the fact that they are Cabibbo favored. From the experimental data we know the branching fractions [10]:

$$\frac{\Gamma(D_s^+ \to \pi^+\pi^+\pi^-)}{\Gamma_{\text{total}}} = (1.09 \pm 0.05) \times 10^{-2}; \qquad (1)$$

$$\frac{\Gamma(D_s^+ \to \pi^+K^+K^-)}{\Gamma_{\text{total}}} = (5.39 \pm 0.21) \times 10^{-2}. \qquad (2)$$

The corresponding ratio $\Gamma(D_s^+ \to \pi^+\pi^+\pi^-))/(\Gamma(D_s^+ \to \pi^+K^+K^-) \simeq 0.2$ is in agreement with the value $0.265 \pm 0.041 \pm 0.031$ found in a previous estimate [6].

While the differences between these numbers require some quantitative analysis, the qualitative relation between these decay rates can be easily understood when we look at the Cabibbo favored decay diagram in Fig. 1, which is also helicity and color favored, where the $\bar{d}u$ makes up a π^+ and one has an extra $s\bar{s}$ pair. The final $s\bar{s}$ pair hadronizes by creating extra $\bar{q}q$ pairs, which lead to $K\bar{K}$ or $\eta\eta$ but not $\pi\pi$. The final state $\pi^+K^+K^-$ can be produced directly and through rescattering ($K^0\bar{K}^0$ or $\eta\eta \to K^+K^-$). In contrast, the final state $\pi^+\pi^+\pi^-$ can only be produced through rescattering (K^+K^- or $K^0\bar{K}^0$ or $\eta\eta \to \pi^+\pi^-$).

Since the original $s\bar{s}$ pair produced in the Cabibbo favored D_s^+ weak decay, shown in Fig. 1, has isospin zero, all the hadrons produced in the hadronization process, like the $K\bar{K}$ or $\pi\pi$ final states, have also isospin zero. This means that only isospin zero resonances, like f_0, can contribute. In the case of the K^+K^- final state, one can also have the contribution of the ϕ meson (K^+K^- in P

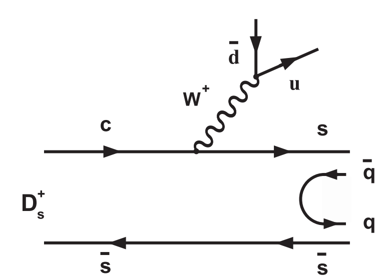

FIG. 1. Schematic representation of the hadronization of the $s\bar{s}$ pair in the Cabibbo favored D_s^+ weak decay, with the external π^+ emission. The inserted $\bar{q}q$ pair represents the isoscalar combination $\bar{u}u + \bar{d}d + \bar{s}s$.

*jdias@if.usp.br
†navarra@if.usp.br
‡mnielsen@if.usp.br
§oset@ific.uv.es

wave). However, the ρ meson will not appear in the $D_s^+ \to \pi^+\pi^+\pi^-$ decay since the ρ has isospin 1. This is of course for the dominant mechanism chosen, but one could expect a small contribution from subleading terms. In this work we shall study the processes depicted in Fig. 1, looking for the $f_0(980)$ signal in the spectra of the invariant masses $m_{\pi^+\pi^-}$ and $m_{K^+K^-}$.

II. FORMALISM

In order to produce a pair of mesons, the $s\bar{s}$ pair shown in Fig. 1 has to hadronize into two mesons. To do that, an extra $\bar{q}q$ pair with the quantum numbers of the vacuum, $\bar{u}u + \bar{d}d + \bar{s}s$, is added to the already existing quark pair. In order to find out the meson-meson components in the hadronized $s\bar{s}$ pair we define the $q\bar{q}$ matrix M [11]:

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}, \tag{3}$$

which has the property

$$M \cdot M = M \times (\bar{u}u + \bar{d}d + \bar{s}s). \tag{4}$$

The next step consists of writing the matrix M in terms of mesons. Using the standard $\eta - \eta'$ mixing [12], the matrix M corresponds to [13]

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}. \tag{5}$$

Therefore, in terms of two pseudoscalars we have the correspondence:

$$s\bar{s}(\bar{u}u + \bar{d}d + \bar{s}s) \equiv (\phi \cdot \phi)_{33} = K^-K^+ + \bar{K}^0K^0 + \frac{1}{3}\eta\eta, \tag{6}$$

where we have neglected the η' contribution since the mass of η' is too large to be relevant here. These are the states which are produced in the first step, prior to FSI. Once a pair of mesons is created they start to interact and the final K^+K^- or $\pi^+\pi^-$ mesons can be formed as a result of complex two-body interactions with coupled channels described by the Bethe-Salpeter equation. First steps in this direction were given in [14] in the $\gamma\gamma \to$ meson-meson reaction, proving the accuracy of the method.

In the decay represented in Fig. 1 the π^+ is treated as a spectator and the $s - \bar{s}$ pair may hadronize into K^+K^-, as shown above and, after rescattering, it can produce $\pi^+\pi^-$ and also K^+K^-. The π^+ that we consider as a spectator can also interact with the π^- of the $\pi^+\pi^-$ pair. Yet, investigation of the Dalitz plot indicates that the strength of this interaction is shared in a wide region between 530 MeV and 1700 MeV and thus its contribution in the narrow region of the $f_0(980)$ of the other pair is negligible.

The D_s^+ decay width into a π^+ and two mesons will be labeled $\Gamma_{P^+P^-}$, where P^+P^- refers to the two pseudoscalar final mesons: K^+K^- or $\pi^+\pi^-$. The differential decay width, as a function of the invariant mass of the pair P^+P^- is then given by

$$\frac{d\Gamma_{P^+P^-}}{dM_{\rm inv}} = \frac{1}{(2\pi)^3}\frac{p_\pi \tilde{p}_P}{4M_{D_s}^2}|T_{P^+P^-}|^2, \tag{7}$$

where

$$p_\pi = \frac{\lambda^{1/2}(M_{D_s}^2, m_\pi^2, M_{\rm inv}^2)}{2M_{D_s}}, \tag{8}$$

$$\tilde{p}_P = \frac{\lambda^{1/2}(M_{\rm inv}^2, m_{P^+}^2, m_{P^-}^2)}{2M_{\rm inv}}, \tag{9}$$

and

$$\lambda(x^2, y^2, z^2) = x^4 + y^4 + z^4 - 2x^2y^2 - 2x^2z^2 - 2y^2z^2. \tag{10}$$

In the above formula $m_{P^+} = m_{K^+}$ or m_{π^+} and $m_{P^-} = m_{K^-}$ or m_{π^-}. The amplitudes in Eq. (7) are given by

$$T_{K^+K^-} = V_0\bigg(1 + G_{K^+K^-}t_{K^+K^-\to K^+K^-} + G_{K^0\bar{K}^0}t_{K^0\bar{K}^0\to K^+K^-} + \frac{2}{3}\frac{1}{2}G_{\eta\eta}\tilde{t}_{\eta\eta\to K^+K^-}\bigg), \tag{11}$$

with $\tilde{t}_{\eta\eta\to K^+K^-} = \sqrt{2}t_{\eta\eta\to K^+K^-}$ and

$$T_{\pi^+\pi^-} = V_0\bigg(G_{K^+K^-}t_{K^+K^-\to\pi^+\pi^-} + G_{K^0\bar{K}^0}t_{K^0\bar{K}^0\to\pi^+\pi^-} + \frac{2}{3}\frac{1}{2}G_{\eta\eta}\tilde{t}_{\eta\eta\to\pi^+\pi^-}\bigg), \tag{12}$$

with $\tilde{t}_{\eta\eta\to\pi^+\pi^-} = \sqrt{2}t_{\eta\eta\to\pi^+\pi^-}$. The function G_l is the loop function given by

$$G_l(s) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{(p-q)^2 - m_1^2 + i\varepsilon}\frac{1}{q^2 - m_2^2 + i\varepsilon}, \tag{13}$$

with p being the total four-momentum of the P^+P^- system and, hence, the Mandelstam invariant s is $s = p^2 = M_{\rm inv}^2$. The masses m_1 and m_2 are the masses of the mesons in the loop for the l-channel. The factors 2 and $1/2$ in Eqs. (11) and (12) come from the two combinations to create the $\eta\eta$ state from two η fields and the reduction of $1/2$ from the loop, all that due to the identity of the two η particles. The factor $\sqrt{2}$ relating $\tilde{t}$ and t for the $\eta\eta$ channel has the same root in the identity of these two particles because for convenience, in the chiral unitary approach the amplitudes t are evaluated with the unitary normalization, in this case $|\eta\eta\rangle/\sqrt{2}$ for the $\eta\eta$ state.

The method used here to hadronize the $s\bar{s}$ component and implement final state interaction of the resulting meson pair has an early precedent in the study of the $J/\psi \to \phi\pi\pi(K\bar{K})$ decays in [15], where a relationship between the $s\bar{s}$ and nonstrange form factors and the meson-meson interaction was established. A different reformulation of the problem, closer to the one followed here, is given in [16,17].

In our calculations the integral on q^0 in Eq. (13) is performed exactly analytically and a cutoff, $|\vec{q}_{\rm max}| = 600$ MeV$/c$, is introduced in the integral on $\vec{q}$. The elements of the scattering matrix $t_{i\to j}$ are the solutions of the Bethe-Salpeter equation. Namely, we obtain these elements by solving a coupled-channel scattering equation in an algebraic form

$$t_{i\to j}(s) = V_{ij}(s) + \sum_{l=1}^{5} V_{il}(s)G_l(s)t_{l\to j}(s), \tag{14}$$

where each value assumed by the i, j, and l indices in the range from 1 to 5 indicates the channels: 1 for $\pi^+\pi^-$, 2 for $\pi^0\pi^0$, 3 for K^+K^-, 4 for $K^0\bar{K}^0$ and 5 for $\eta\eta$. V is the interaction kernel which corresponds to the tree-level transition amplitudes obtained from phenomenological Lagrangians developed in Ref. [18], complemented with the inclusion of the matrix elements for the $\eta\eta$ channels given in [19]. This cutoff of 600 MeV/c, different from the one used in [18], is needed to reproduce experimental amplitudes when the $\eta\eta$ channel is introduced explicitly [20,21]. We have

$$\begin{aligned}
&V_{11} = -\frac{1}{2f^2}s, \qquad V_{12} = -\frac{1}{\sqrt{2}f^2}(s - m_\pi^2),\\
&V_{13} = -\frac{1}{4f^2}s, \qquad V_{14} = -\frac{1}{4f^2}s,\\
&V_{15} = -\frac{1}{3\sqrt{2}f^2}m_\pi^2, \qquad V_{22} = -\frac{1}{2f^2}m_\pi^2,\\
&V_{23} = -\frac{1}{4\sqrt{2}f^2}s, \qquad V_{24} = -\frac{1}{4\sqrt{2}f^2}s,\\
&V_{25} = -\frac{1}{6f^2}m_\pi^2, \qquad V_{33} = -\frac{1}{2f^2}s,\\
&V_{34} = -\frac{1}{4f^2}s, \qquad V_{35} = -\frac{1}{12\sqrt{2}f^2}(9s - 6m_\eta^2 - 2m_\pi^2),\\
&V_{44} = -\frac{1}{2f^2}s, \qquad V_{45} = -\frac{1}{12\sqrt{2}f^2}(9s - 6m_\eta^2 - 2m_\pi^2),\\
&V_{55} = -\frac{1}{18f^2}(16m_K^2 - 7m_\pi^2),
\end{aligned} \tag{15}$$

where f represents the pion decay constant; $f = f_\pi = 93$ MeV; and m_π, m_K, and m_η are the averaged masses of pion, kaon, and η mesons, respectively.

The large overlap of the $f_0(980)$ and small one for the $f_0(500)$, with the $s\bar{s}$ components, was emphasized in [22], where by using the linear σ model and a mixing of strange and nonstrange $q\bar{q}$ components, a qualitative description of the $\phi \to \pi^0\pi^0\gamma$, $f_0(980) \to \gamma\gamma$, $J/\psi \to \omega\pi\pi$ was given. Previous work using chiral Lagrangians for the meson-meson interactions, together with unitarization in coupled channels, has produced precise quantitative descriptions of these and many other reactions. The $\phi \to \pi^0\pi^0\gamma$ reaction was studied in [23–25]. The $J/\psi \to \omega\pi\pi$ and $J/\psi \to \phi\pi\pi$ reactions were studied in [15–17], obtaining a quantitative description of the spectra, and explaining why the $f_0(500)$ is seen in the first reaction and the $f_0(980)$ in the second one. The $f_0(500)$ and $f_0(980)$ coupling to $\gamma\gamma$ was studied in [14,18,26]. A review on these and related issues using the chiral unitary approach is given in [27]. In the present work we have used the chiral unitary approach of [18], which has proved to be a precise tool to account for strong interactions at low energies [27].

III. RESULTS

The numerical results for the amplitude squared $|T_{K^+K^-}|^2$ as a function of the K^+K^- invariant mass, as obtained from Eq. (11), are shown in Fig. 2. In this figure we also show the experimental data for the s-wave contribution for the $K\bar{K}$ mass distribution extracted from Ref. [7]. We adjust the V_0 parameter in Eq. (11) to approximately fit the data. The theoretical curve represents essentially $|t^{I=0}_{K\bar{K}\to K\bar{K}}|^2$, which is dominated by the $f_0(980)$ pole in that region. Indeed, since (recall that $|K^-\rangle = -|1/2 - 1/2\rangle$)

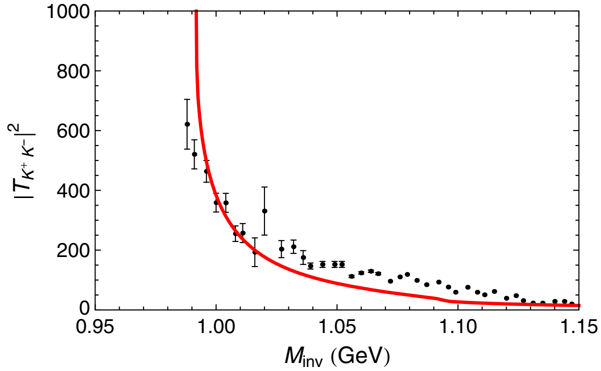

FIG. 2. $|T_{K^+K^-}|^2$ as a function of the K^+K^- invariant mass as obtained from Eq. (11) (solid line). The experimental data are taken from [7].

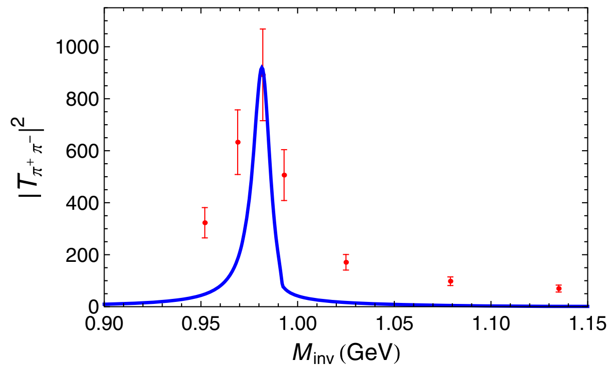

FIG. 3. $|T_{\pi^+\pi^-}|^2$ as a function of the $\pi^+\pi^-$ invariant mass as obtained from Eq. (12) (solid line). The experimental data are taken from [8].

$$|K\bar{K}, I=0, I_3=0\rangle = -\frac{1}{\sqrt{2}}(K^+K^- + K^0\bar{K}^0),$$

$$|K^+K^-\rangle = -\frac{1}{\sqrt{2}}(|K\bar{K}, I=1\rangle + |K\bar{K}, I=0\rangle), \tag{16}$$

then

$$t_{K^+K^-\to K^+K^-} + t_{K^0\bar{K}^0\to K^+K^-} = t^{I=0}_{K\bar{K}\to K\bar{K}}, \tag{17}$$

and from the Bethe-Salpeter equation, ignoring $\eta\eta \to K^+K^-$, we have

$$\frac{T_{K^+K^-}}{V_0} \equiv 1 + G t^{I=0}_{K\bar{K}\to K\bar{K}} \simeq \frac{V^{I=0} + V^{I=0} G t^{I=0}}{V^{I=0}} = \frac{t^{I=0}_{K\bar{K}\to K\bar{K}}}{V^{I=0}}, \tag{18}$$

where $V^{I=0}$ is the $K\bar{K} \to K\bar{K}$ potential in $I=0$. The sign $\simeq$ is used because we also ignore the $\pi\pi$ channel in that equation, which plays a minor role around the $f_0(980)$ region. Thus, Eq. (11) is roughly proportional to $T_{K^+K^-}$, which reflects the $f_0(980)$ resonance in this region.

We have chosen to reproduce the data around 1 GeV and the agreement looks fair above this energy, but clear discrepancies are seen for smaller values of $M_{\rm inv}$. The discrepancies with the data are unavoidable because in [7] a mass of 922 MeV and width of 240 MeV were obtained for the $f_0(980)$, while our calculations provide results in good agreement with the PDG average. The PDG results are $M = (980 \pm 20)$ MeV, $\Gamma = (40\text{–}100)$ MeV. From the $B_s \to J/\psi\pi\pi$ reaction one obtains similar results $M = (972 \pm 20)$ MeV, $\Gamma \approx 50$ MeV. The clear discrepancies of [7] with the standard results should be enough motivation to look again at this reaction with more detail.

In Fig. 3 we show $|T_{\pi^+\pi^-}|^2$ as a function of the invariant mass of the pair of pions $\pi^+\pi^-$ obtained from Eq. (12), and the experimental data for the s-wave contribution for the $\pi\pi$ mass distribution extracted from [8]. The normalization of the K^+K^- production rates divided by the phase space of [7] and the normalization of the $\pi^+\pi^-$ production divided by the phase space of [8] are not the same. In Ref. [7] the distributions are superposed (see Fig. 6 of that reference) to show that their "profile" around the $f_0(980)$ is the same. We can normalize the value of V_0 to these $\pi^+\pi^-$ data. At the $f_0(980)$ peak position our theoretical curve agrees with the data, by construction, but for lower and higher values of the $\pi^+\pi^-$ invariant mass the experimental distribution is broader than the theoretical calculation. One reason for that could be the fact that in the experimental data they found an s-wave contribution from the $f_0(1370)$ and $f_0(1500)$ resonances, which are not included in this calculation.

It is worth emphasizing that our calculations do not take into account sources of background, which would come, as we discussed earlier, from the consideration of the interaction of the spectator pion with the other pions. Moreover, we should note that in Ref. [8] the authors have bins of about 15 MeV or more, which are used to construct the few $\pi^+\pi^-$ experimental points of the mass distribution. Thus, in order to have a better comparison with data, we do two things. First, we integrate our mass distribution over the same bins as experiment, dividing by the size of the bins. Second, we add a background to our results. This background is a chosen constant in $M_{\rm inv}$, such as to get a fair reproduction of the last three experimental points. The result is shown in Fig. 4. Now, the agreement with the data looks better than in Fig. 3, but still our distribution seems a bit narrower than the experimental one. It is also worth mentioning that for this latter observable, the theoretical work of [21] also misses some strength on the sides of the $f_0(980)$ resonance with respect to the experimental data of Ref. [28]. One approach based on the use of the $s\bar{s}$ pion form factor, obtained with an Omnes representation constructed from experimental pion-pion phase shifts, fills up this region [29]. An alternative approach that could be tested in these reactions is the one used in Ref. [30] using light cone sum rules to evaluate form factors, together with unitarization of the final meson pairs. It is also worth

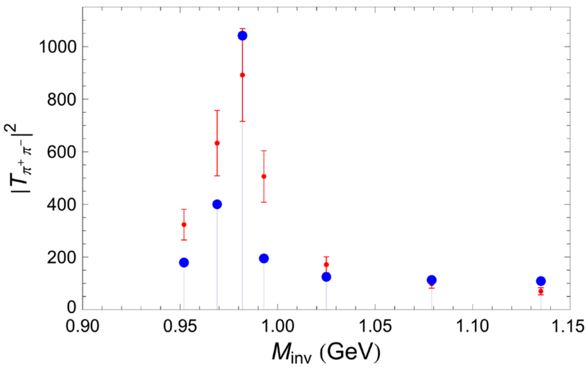

FIG. 4. $|T_{\pi^+\pi^-}|^2$ as a function of the $\pi^+\pi^-$ invariant mass. Here, the theoretical results (thick circles without error bars) are folded in order to have the same size of the experimental bins, which is 25 MeV. The experimental data are taken from [8].

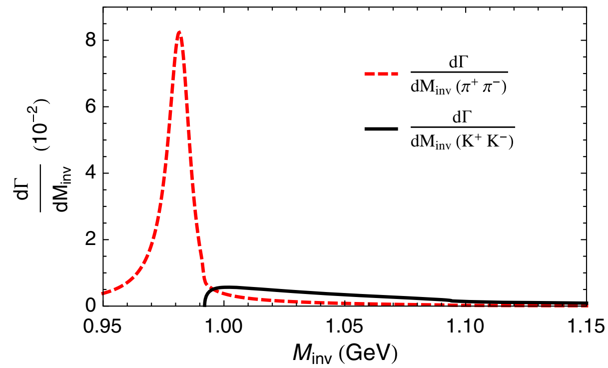

FIG. 5. The $\pi^+\pi^-$ and K^+K^- invariant mass distributions for the $D_s^+ \to \pi^+\pi^-\pi^+$ (dashed line) and $D_s^+ \to \pi^+K^-K^+$ (solid line) with arbitrary normalization, respectively.

mentioning that if we extrapolate the $\pi^+\pi^-$ distribution to lower invariant masses we do not find a trace of the $f_0(500)$. This feature is also noted in Ref. [8] and it was also the case in the $B_s^0 \to J/\psi\pi^+\pi^-$ experiment [28], as well as in the theoretical descriptions in Refs. [21,29,31].

So far we have discussed only the shapes of the $|T_{\pi^+\pi^-}|^2$ and $|T_{K^+K^-}|^2$ amplitudes and now we wish to make predictions for the relative strength of the rates of the two reactions. We can use Eq. (7) in order to predict the $\pi^+\pi^-$ and K^+K^- distributions, as illustrated in Fig. 5. According to this figure, we see the $f_0(980)$ signal in the spectra of the invariant mass $m_{\pi^+\pi^-}$, as indicated by the dashed curve. On the other hand, the K^+K^- distribution gets strength from the underlying $f_0(980)$ resonance close to the K^+K^- threshold. It would be most useful to determine experimentally the strength of these two distributions to compare with these predictions, which, up to a global common normalization factor, are predictions of the chiral unitary approach with no free parameters.

One should stress once more that the predictions are limited to the region close to $f_0(980)$. In principle one should study dynamics involving three meson interactions [32], but, as discussed earlier, the wide range of invariant masses of the spectator π^+ with any of those producing the $f_0(980)$ dilutes its contribution into a background. As for the $K\bar{K}$ distribution in Fig. 5 one should also note that, if one goes to higher invariant masses, the method used here would have to be complemented with extra channels that are for instance discussed in [33,34].

The results of Fig. 5 might look to be in conflict with the ratio obtained from the data in Eqs. (1) and (2). We mentioned in the Introduction that from these one finds the $\pi^+K^+K^-$ rate to be about five times larger than that of $\pi^+\pi^+\pi^-$. The results of Fig. 4 in the range of M_{inv} of the figure are opposite and the $\pi^+\pi^+\pi^-$ strength is bigger than the one of K^+K^-. The discrepancy is only apparent because the rates of Eq. (1) and (2) extend to the entire range of invariant masses and for any possible partial wave. We only consider s-wave, which can be disentangled in an experimental analysis. For instance, the $D_s^+ \to \pi^+K^+K^-$ decay gets a large contribution, from $D_s^+ \to \pi^+\phi$ ($\phi \to K^+K^-$), with a branching ratio of 2.27×10^{-2} [10], which we do not consider, and there are contributions from higher mass resonances that couple to $K\bar{K}$. Our predictions are limited to low values of $M_{\rm inv}$ close to the $K\bar{K}$ threshold, and exclude the P-wave ϕ production followed by $\phi \to K^+K^-$.

IV. A DISCUSSION ON THE TETRAQUARK PICTURE AND THE PRESENT REACTION

Concerning the $f_0(980)$ and other light scalar mesons, $f_0(500)$, $a_0(980)$, and $\kappa(800)$, there has been much discussion about their nature as $q\bar{q}$, tetraquark, molecules, dynamically generated states, etc. [35]. The consensus that the scientific community seems to have reached is that they are not ordinary, $q\bar{q}$, mesons (see extensive information on the subject in the report [36]). There has been more discussion on whether they are tetraquarks or they appear to be dynamically generated from the meson-meson interaction—the picture we have adopted here, and which we implement using the chiral unitary approach.

The tetraquark picture for mesons developed in [37] has been extensively used in the literature concerning the scalar mesons [38–42]. The most common configuration is given for the $f_0(500)$ and $f_0(980)$ by

$$f_0(500) = [ud][\bar{u}\,\bar{d}],$$
$$f_0(980) = \frac{1}{\sqrt{2}}([su][\bar{s}\,\bar{u}] + [sd][\bar{s}\,\bar{d}]), \tag{19}$$

by means of which one finds a qualitative description of the masses of these mesons. There are also problems since the $f_0(980)$ does not couple to $\pi\pi$ in the picture of Eq. (19) and the coupling $f_0\pi\pi$ is too small compared with experiment even if some configuration mixing is considered [39]. This means that in those pictures one would get a very small $D_s^+ \to \pi^+\pi^+\pi^-$ rate compared to $D_s^+ \to \pi^+K^+K^-$ with respect to our predictions in Fig. 5. In some refinements to the basic model, new elements are introduced to solve one or another problem related to phenomenology. In [39] the authors include instanton components to fix the $\pi\pi$ coupling problem. In [40] $q\bar{q}$-gluonium components are introduced to address the problem of $\pi\pi \to \pi\pi$, $\pi\pi \to K\bar{K}$ and $\gamma\gamma \to \pi\pi$ scattering. In [41] in order to reproduce the data of the $\phi \to \pi^0\pi^0\gamma$ reaction, the tetraquark picture is also invoked, but the $f_0(980)$ is claimed to be largely made of the $[sd][\bar{s}\,\bar{d}]$ component. Some basic features of spectra can be related to the fact that there are four quarks, independent of the particular rearrangements [38]. What seems to be missing in this approach is a unique picture that describes all processes where these mesons appear, instead of invoking different dynamical aspects for each one of them.

In this respect it is interesting to mention that, using the picture of Eq. (19) for the tetraquarks, it was shown in [42] that it was not possible to reconcile the ratios of decay rates to $f_0(500)$ and $f_0(980)$ seen in the $B^0 \to J/\psi\pi^+\pi^-$ and $B_s^0 \to J/\psi\pi^+\pi^-$ decays, which have a large signal for the $f_0(980)$ in $B_s^0 \to J/\psi\pi^+\pi^-$ decay and practically no $f_0(500)$, while the reverse situation is found in $B^0 \to J/\psi\pi^+\pi^-$.[1]

There is also one feature that cannot escape this discussion. In physical processes involving these resonances one looks for $\pi\pi$ or $K\bar{K}$ in the final states. Independently of the dynamics generating the resonances, the $\pi\pi$ or $K\bar{K}$ will undergo final state interaction, scattering and making transitions among them, something that is not normally accounted for in the tetraquark pictures. Also some reactions have large contributions from tree level production of pairs of mesons, which can revert into $K\bar{K}$ or $\pi\pi$ at the end through rescattering, and this dynamics escapes the description of the process in terms of tetraquarks alone.

Accepting that some of the dynamics on the tetraquarks models is well founded, our approach is different and does not necessarily contradict it. Our approach starts accepting that QCD dynamics at low energies is governed by the effective chiral Lagrangians [43]. From these Lagrangians we construct the leading terms of the meson-meson interaction and then, using a unitary chiral approach in coupled channels, we generate the full meson-meson amplitudes. In s-wave and $I = 0$ these amplitudes contain poles which correspond to the $f_0(500)$ and $f_0(980)$ resonances. In $I = 1$ the $\eta\pi$, $K\bar{K}$ amplitudes generate the $a_1(980)$ resonance and the πK and ηK give rise to the $\kappa(800)$. All this is obtained with only one parameter which is needed to regularize the loops. Hence, the approach contains the scalar mesons and the scattering amplitudes needed to face different problems where the resonances are produced. It is most rewarding to see that the problems mentioned above, that required the introduction of different elements in the tetraquark pictures, are well described in this unified picture. In this sense, the $\phi \to \pi^0\pi^0\gamma$, $\pi^+\pi^-\gamma$, $\pi^0\eta\gamma$ reactions are described within this picture in [23]. The couplings of the $f_0(500)$, $f_0(980)$ to $\pi\pi$, $K\bar{K}$ are obtained in [18] and [44] in agreement with phenomenology. The $\gamma\gamma \to \pi\pi$ reaction is also addressed successfully within this picture in [14] and the puzzle addressed in [42] concerning the $B^0 \to J/\psi\pi\pi$ and $B_s^0 \to J/\psi\pi\pi$ reactions was properly described in this picture in [21]. These are only a few examples of cases where the chiral unitary approach proves most suited to describe the physical processes where the scalar resonances are produced. A more complete description can be obtained in the review papers [27] and [45].

[1]In [42] the results were found compatible with a $q\bar{q}$ picture, but the overwhelming evidence against it from the discussions in [36] do not make this coincidence a case in favor of the $q\bar{q}$ picture for the scalars.

V. CONCLUSIONS

In this paper we addressed the study of the D_s^+ decays into $\pi^+\pi^+\pi^-$ and $\pi^+K^+K^-$ mesons. The $\pi^+\pi^-$ and K^+K^- meson pairs in the final state were allowed to undergo interactions in coupled channels and lead to the $f_0(980)$ resonance production. We adopted a mechanism which involves the D_s^+ weak decays into a π^+ and a $q\bar{q}$ component, that is Cabibbo and also color favored. Upon hadronization of the $q\bar{q}$ component into a pair of two pseudoscalar mesons, the final state interaction between them is taken into account by using the chiral unitary theory where $f_0(980)$ emerges as a dynamically generated resonance, which then decays into $\pi^+\pi^-$ and also into K^+K^- mesons. In order to do that, we solved the Bethe-Salpeter equation in coupled channels. We observe that our curves for the $|T_{\pi^+\pi^-}|^2$ and $|T_{K^+K^-}|^2$ amplitudes, obtained as a function of the $\pi^+\pi^-$ and K^+K^- invariant masses, respectively, have a shape in fair agreement with the data reported in Refs. [7,8], with the unavoidable discrepancies for $|T_{K^+K^-}|^2$ at low masses because of the small mass of the $f_0(980)$ obtained in [7] of 922 MeV. To the best of our knowledge, in the present work these data are for the first time addressed from the theoretical point of view.

We could also determine the shape and strength of the $\pi^+\pi^-$ or K^+K^- mass distributions in those two reactions, which, up to a common global normalization constant, are a prediction of the theoretical approach with no further parameters.

These decays provide an important scenario to test the predictions of the chiral unitary theory as well as the nature of the $f_0(980)$ resonance, since the latter emerges from this approach after taking into account the interaction between two pseudoscalar mesons, which generates dynamically the low lying scalar mesons. So far only shapes for these reactions have been established experimentally. The measurement of the relative strength of these two mass distributions would be most welcome to contrast them with the theoretical predictions.

Another interesting issue would be to study the $\pi^+\pi^0\eta$ decay mode. This would generate the $a_0(980)$ and one could address again the issue of the $f_0(980)$ and $a_0(980)$ mixing [46]. This would be obtained in our formalism by taking different masses of the charged and neutral kaons in the loop function G for $K\bar{K}$, as done in [47].

ACKNOWLEDGMENTS

We thank Ignacio Bediaga, Alberto Correa dos Reis and A. M. Torres for useful discussions and comments. We acknowledge the support by FAPESP, CNPq and by Open Partnership Joint Projects of JSPS Bilateral Joint Research Projects. This work is partly supported by the Spanish Ministerio de Economia y Competitividad and European FEDER funds under Contracts No. FIS2011-28853-C02-01 and No. FIS2011-28853-C02-02, and the Generalitat Valenciana in the program Prometeo II-2014/068.

[1] J. H. Alvarenga Nogueira, I. Bediaga, A. B. R. Cavalcante, T. Frederico, and O. Loureno, Phys. Rev. D **92**, 054010 (2015).
[2] P. U. E. Onyisi *et al.* (CLEO Collaboration), Phys. Rev. D **88**, 032009 (2013).
[3] A. Zupanc *et al.* (Belle Collaboration), J. High Energy Phys. 09 (2013) 139.
[4] J. P. Alexander *et al.* (CLEO Collaboration), Phys. Rev. D **79**, 052001 (2009).
[5] J. P. Alexander *et al.* (CLEO Collaboration), Phys. Rev. Lett. **100**, 161804 (2008).
[6] P. L. Frabetti *et al.* (E687 Collaboration), Phys. Lett. B **407**, 79 (1997).
[7] P. del Amo Sanchez *et al.* (*BABAR* Collaboration), Phys. Rev. D **83**, 052001 (2011).
[8] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **79**, 032003 (2009).
[9] E. M. Aitala *et al.* (E791 Collaboration), Phys. Rev. Lett. **86**, 770 (2001).
[10] K. A. Olive *et al.* (Particle Data Group Collaboration), Chin. Phys. C **38**, 090001 (2014).
[11] A. Martinez Torres, L. S. Geng, L. R. Dai, B. X. Sun, E. Oset, and B. S. Zou, Phys. Lett. B **680**, 310 (2009).
[12] A. Bramon, A. Grau, and G. Pancheri, Phys. Lett. B **283**, 416 (1992).
[13] D. Gamermann, E. Oset, and B. S. Zou, Eur. Phys. J. A **41**, 85 (2009).
[14] J. A. Oller and E. Oset, Nucl. Phys. **A629**, 739 (1998).
[15] U. G. Meissner and J. A. Oller, Nucl. Phys. **A679**, 671 (2001).
[16] L. Roca, J. E. Palomar, E. Oset, and H. C. Chiang, Nucl. Phys. **A744**, 127 (2004).
[17] T. A. Lahde and U. G. Meissner, Phys. Rev. D **74**, 034021 (2006).
[18] J. A. Oller and E. Oset, Nucl. Phys. **A620**, 438 (1997); **A652**, 407 (1999).
[19] D. Gamermann, E. Oset, D. Strottman, and M. J. Vicente Vacas, Phys. Rev. D **76**, 074016 (2007).
[20] J. J. Xie, L. R. Dai, and E. Oset, Phys. Lett. B **742**, 363 (2015).
[21] W. H. Liang and E. Oset, Phys. Lett. B **737**, 70 (2014).
[22] R. Delbourgo, D. s. Liu, and M. D. Scadron, Phys. Lett. B **446**, 332 (1999).
[23] E. Marco, S. Hirenzaki, E. Oset, and H. Toki, Phys. Lett. B **470**, 20 (1999).
[24] J. E. Palomar, L. Roca, E. Oset, and M. J. Vicente Vacas, Nucl. Phys. **A729**, 743 (2003).
[25] Y. S. Kalashnikova, A. E. Kudryavtsev, A. V. Nefediev, C. Hanhart, and J. Haidenbauer, Eur. Phys. J. A **24**, 437 (2005).
[26] J. A. Oller and L. Roca, Eur. Phys. J. A **37**, 15 (2008).
[27] J. A. Oller, E. Oset, and A. Ramos, Prog. Part. Nucl. Phys. **45**, 157 (2000).
[28] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. D **89**, 092006 (2014).
[29] J. T. Daub, C. Hanhart, and B. Kubis, J. High Energy Phys. 02 (2016) 009.
[30] U. G. Meissner and W. Wang, Phys. Lett. B **730**, 336 (2014).
[31] Y. J. Shi and W. Wang, Phys. Rev. D **92**, 074038 (2015).
[32] A. M. Torres, K. P. Khemchandani, L. S. Geng, M. Napsupiale, and E. Oset, Phys. Rev. D **78**, 074031 (2008).
[33] M. Albaladejo and J. A. Oller, Phys. Rev. Lett. **101**, 252002 (2008).
[34] L. S. Geng and E. Oset, Phys. Rev. D **79**, 074009 (2009).
[35] E. Klempt and A. Zaitsev, Phys. Rep. **454**, 1 (2007).
[36] J. R. Pelaez, arXiv:1510.00653.
[37] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977); **15**, 281 (1977).
[38] A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D **79**, 074014 (2009).
[39] G. 't Hooft, G. Isidori, L. Maiani, A. D. Polosa, and V. Riquer, Phys. Lett. B **662**, 424 (2008).
[40] G. Mennessier, S. Narison, and X.-G. Wang, Phys. Lett. B **696**, 40 (2011).
[41] N. N. Achasov and A. V. Kiselev, Phys. Rev. D **83**, 054008 (2011).
[42] S. Stone and L. Zhang, Phys. Rev. Lett. **111**, 062001 (2013).
[43] S. Weinberg, Physica A (Amsterdam) **96A**, 327 (1979).
[44] J. A. Oller and E. Oset, Phys. Rev. D **60**, 074023 (1999).
[45] E. Oset *et al.*, Int. J. Geom. Methods Mod. Phys. **E25**, 1630001 (2016).
[46] N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Phys. Lett. **88B**, 367 (1979); J. J. Wu, Q. Zhao, and B. S. Zou,

Phys. Rev. D **75**, 114012 (2007); C. Hanhart, B. Kubis, and J. R. Pelaez, Phys. Rev. D **76**, 074028 (2007); J. J. Wu and B. S. Zou, Phys. Rev. D **78**, 074017 (2008); M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **83**, 032003 (2011); L. Roca, Phys. Rev. D **88**, 014045 (2013); V. E. Tarasov, W. J. Briscoe, W. Gradl, A. E. Kudryavtsev, and I. I. Strakovsky, Phys. Rev. C **88**, 035207 (2013); T. Sekihara and S. Kumano, Phys. Rev. D **92**, 034010 (2015).

[47] F. Aceti, W. H. Liang, E. Oset, J. J. Wu, and B. S. Zou, Phys. Rev. D **86**, 114007 (2012).