Quasi-Yukawa unification and fine-tuning in U(1) extended SSM

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We consider the low scale implications in the $\mathrm{U}(1)'$ extended minimal supersymmetric Standard Model (UMSSM). We restrict the parameter space such that the lightest supersymmetric particle (LSP) is always the lightest neutralino. In addition, we impose quasi-Yukawa unification (QYU) at the grand unification scale (M_{GUT}). QYU strictly requires the ratios among the Yukawa couplings as $y_t/y_b \sim 1.2$, $y_\tau/y_b \sim 1.4$, and $y_t/y_\tau \sim 0.8$. We find that the need for fine-tuning over the fundamental parameter space of QYU is in the acceptable range ($\Delta_{\mathrm{EW}} \leq 10^3$), even if the universal boundary conditions are imposed at M_{GUT}, in contrast to CMSSM and nonuniversal Higgs masses. The UMSSM with universal boundary conditions yields heavy stops ($m_{\tilde{t}} \gtrsim 2.5$ TeV), gluinos ($m_{\tilde{g}} \gtrsim 2$ TeV), and squarks from the first two families ($m_{\tilde{q}} \gtrsim 4$ TeV). Similarly, the stau mass is bounded from below at about 1.5 TeV. Despite this heavy spectrum, we find $\Delta_{\mathrm{EW}} \gtrsim 300$, which is much lower than that needed for the minimal supersymmetric models. In addition, the UMSSM yields a relatively small μ term, and the LSP neutralino is mostly formed by the Higgsinos of mass $\gtrsim$700 GeV. We also obtain bino-like dark matter of mass about 400 GeV. The wino is usually found to be heavier than Higgsinos and binos, but there is a small region where $\mu \sim M_1 \sim M_2 \sim 1$ TeV. We also identify a chargino-neutralino coannihilation channel and A-resonance solutions which reduce the relic abundance of LSP neutralinos down to the ranges compatible with the current WMAP and Planck measurements.

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I. INTRODUCTION

Even if the minimal supersymmetric extension of the Standard Model (MSSM) is compatible with the current experimental measurements for the Higgs boson, recent studies show that realizing a Higgs boson of mass around 125 GeV in minimal models such as constrained MSSM (CMSSM) and models with nonuniversal Higgs masses (NUHM) requires a heavy supersymmetric particle spectrum. The Higgs boson of mass about 125 GeV leads to the stop quark mass in the multi-TeV range [1] or necessitates a large soft supersymmetry breaking (SSB) trilinear term A_t [2]. In addition to the Higgs boson results, the absence of a direct signal in the experiments conducted at the Large Hadron Collider (LHC) has also increased the mass bounds on the supersymmetric particles, especially in the color sector. For instance, the current results exclude the gluino of mass lighter than ~1.8 TeV when $m_{\tilde{g}} \ll m_{\tilde{q}}$ [3], which becomes more severe when $m_{\tilde{g}} \simeq m_{\tilde{q}}$, where $\tilde{q}$ denotes the squarks from the first two families. Even though these bounds are mostly for R-parity conserved CMSSM, they are applicable for a large class of supersymmetric models.

While there are numerous motivations behind the supersymmetry (SUSY) searches, such a heavy spectrum has brought naturalness under scrutiny. It is clear that the recent experimental constraints cannot be satisfied in the natural region identified with $m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1} \lesssim 500$ GeV [4]. Even though it is possible to find $m_{\tilde{t}_1} \ll 500$ GeV [5], $m_{\tilde{t}_2}$ needs to be very heavy because of the necessity of large mixing. Apart from the natural region, one might measure how much fine-tuning is required by considering the Z-boson mass ($M_Z = 91.2$ GeV)

$$\frac{1}{2}M_Z^2 = -\mu^2 + \frac{(m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1}, \quad (1)$$

where μ is the bilinear mixing of the MSSM Higgs doublets, $\tan\beta \equiv \langle H_u \rangle / \langle H_d \rangle$ [the ratio of vacuum expectation values (VEVs)], $\Sigma_{u,d}^{u,d}$ are the radiative effects from the Higgs potential, and $m_{H_{u,d}}^2$ are the SSB mass terms for the Higgs doublets $H_{u,d}$. A recent work [6] has defined the following parameter to quantify the fine-tuning measure:

$$\Delta_{\mathrm{EW}} \equiv \max(C_i)/(M_Z^2/2), \quad (2)$$

where

$$C_i \equiv \begin{cases} C_{H_d} = |m_{H_d}^2/(\tan^2\beta - 1)| \\ C_{H_u} = |m_{H_u}^2 \tan^2\beta/(\tan^2\beta - 1)| \\ C_\mu = |-\mu^2|. \end{cases} \quad (3)$$

The fine-tuning can be interpreted as the presence of some missing mechanisms, and its amount measures the effects of such missing mechanisms. Their effects can be reflected within SUSY models by considering nonuniversality or adding extra sectors to the theory [7]. In this respect, it is interesting to probe the models beyond the MSSM in light of the current experimental results.

Note that in contrast to the natural region characterized by the stop and sbottom masses, the fine-tuning does not depend on these masses directly. From moderate to large $\tan\beta$ values, $\mu^2 \approx -m_{H_u}^2$ is needed in order to obtain the correct Z-boson mass M_Z; hence, the fine-tuning is mostly determined by C_μ, unless μ is so small that the large radiative corrections to m_{H_u} are needed in Eq. (1). Thus, large stop or sbottom masses can still yield an acceptable amount of fine-tuning. We obtain the conclusion that the fundamental parameter spaces of CMSSM and NUHM need to be highly fine-tuned because of the strict universality in the boundary conditions of these models.

In this work we consider the MSSM extended by an additional $\mathrm{U}(1)'$ group (UMSSM) in the simplest form. A general extension of the MSSM by a U(1) group can be realized from an underlying GUT theory involving a gauge group larger than SU(5). For instance, note the following symmetry breaking chain,

$$\begin{aligned} \mathrm{E}(6) \to \mathrm{SO}(10)\times \mathrm{U}(1)_\psi &\to \mathrm{SU}(5)\times\mathrm{U}(1)_\psi\times\mathrm{U}(1)_\chi \\ &\to G_{\mathrm{MSSM}}\times \mathrm{U}(1)', \end{aligned} \tag{4}$$

where $G_{\mathrm{MSSM}} = \mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ is the MSSM gauge group, and $\mathrm{U}(1)'$ can be expressed as a general mixing of $\mathrm{U}(1)_\psi$ and $\mathrm{U}(1)_\chi$ as

$$\mathrm{U}(1)' = \cos\theta_{E_6}\mathrm{U}(1)_\chi + \sin\theta_{E_6}\mathrm{U}(1)_\psi. \tag{5}$$

Emergence of SO(10) and/or SU(5) allows one to imply a set of boundary conditions that can be suited for these groups. For instance, the supersymmetric particle masses can be universal at the grand unified scale (M_{GUT}) in SO(10), while two different mass scales can be imposed to the fields in **5** and **10** representations of SU(5).

In exploring this extension, we briefly aim to analyze the effects from only having another gauge sector, which is not included in the minimal SUSY models, by imposing universal boundary conditions at M_{GUT}. In addition to the boundary conditions imposed on the fundamental parameters, we also restrict the Yukawa sector such that the Yukawa couplings, especially for the third-family matter fields, are determined by the minimal E(6) [or SO(10)] unification scheme. If a model based on the E(6) gauge group [8] is constructed in a minimal fashion in a way that all the matter fields of a family reside in a **27**-dimensional representation and the Higgs fields in another **27**, such a model also proposes unification of the Yukawa couplings (YU), as well as the gauge couplings. This elegant scheme of unification can be maintained if E(6) is broken down to the MSSM gauge group via SO(10) since models based on the SO(10) gauge group reserve YU. Even though it is imposed at M_{GUT}, YU is also strongly effective at the low scale since it requires threshold corrections at the low scale [9]. Relaxing YU to $b-\tau$ YU does not weaken its strength on the low scale implications since y_b still requires large and negative SUSY threshold corrections [10].

Despite its testable predictions at the LHC [9,11], YU instead leads to contradictory mass relations such that $N = U \propto D = L$ and $m_c^0/m_t^0 = m_s^0/m_b^0$, $m_s^0 = m_\mu^0$, and $m_d^0 = m_e^0$. One way to avoid this contradiction and obtain realistic fermion masses and mixing is to propose vectorlike matter multiplets at the GUT scale [12], which are allowed to mix with fermions in 16-plet representations of SO(10). This approach is also equivalent to introducing nonrenormalizable couplings along with nonzero VEVs of a nonsinglet SO(10) field [13]. Another way is to extend the Higgs sector with an assumption that the MSSM Higgs doublets are superpositions of fields from different SO(10) representations [14].

Even though YU for the third family can be consistently maintained under assumptions that the extra fields negligibly interact with the third family and the MSSM Higgs doublets solely reside in a 10-dimensional representation of SO(10), these two approaches, in general, break YU in SO(10). On the other hand, if one can formulate the asymptotic relation among the Yukawa couplings, then the contributions can be restricted such that the quasi-YU (QYU) can be maintained. For instance, it was shown in Ref. [15] that in the presence of Higgs fields from $H'(15,1,3)$, in addition to those from $h(1,2,2)$ of the Pati-Salam model [16], Yukawa couplings at M_{GUT} can be expressed as

$$y_t : y_b : y_\tau = |1+C| : |1-C| : |1+3C|. \tag{6}$$

The gauge group of the Pati-Salam model, $G_{\mathrm{PS}} = \mathrm{SU}(4)_c\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$, is the maximal subgroup of SO(10), and hence these extra Higgs fields can be employed in SO(10) GUT models. The parameter C denotes the contributions to Yukawa couplings from the extra Higgs fields, and restricting these contributions as $C \le 0.2$, Eq. (6) refers to the QYU condition. Note that C can be either positive or negative, but it is possible to restrict it to positive values without loss of generality by adjusting the phase of the representations H' and h. QYU yield significantly different low scale phenomenology [17] than the exact YU. In addition, QYU can provide an interesting scenario in respect to the fine-tuning since a better fine-tuning prefers that the ratios of Yukawa couplings are different from unity [18], when the universal boundary conditions are imposed at M_{GUT}.

In this work we analyze the fine-tuning requirements in UMSSM with the QYU condition imposed at the GUT scale. The outline of the paper is the following. We will briefly describe the UMSSM in Sec. II. After summarizing our scanning procedure and the experimental constraints we employ in our analysis in Sec. III, we present our results in the fundamental parameter space of QYU in Sec. IV. The mass spectrum of the supersymmetric particles and dark matter (DM) implications are considered in Sec. V. Finally, we summarize and conclude with our results in Sec. VI.

II. MODEL DESCRIPTION

In this section we briefly summarize the E(6) based supersymmetric $\mathrm{U}(1)'$ models whose symmetry breaking patterns and resultant gauge group are given in Eq. (4) (for a detailed consideration, see [19,20]). If the matter fields reside in a 27-plet representation of E_6, its decomposition yields additional vectorlike families denoted by Δ and $\bar{\Delta}$ [20]. The superpotential in such models can be given as

$$W = Y_u \hat{Q}\hat{H}_u\hat{U}^c + Y_d\hat{Q}\hat{H}_d\hat{D}^c + Y_e\hat{L}\hat{H}_d\hat{E}^c + h_s\hat{S}\hat{H}_d\hat{H}_u + h_\Delta\hat{S}\,\hat{\Delta}\,\hat{\bar{\Delta}}, \tag{7}$$

where $\hat{Q}$ and $\hat{L}$ denote the left-handed chiral superfields for the quarks and leptons, while $\hat{U}^c$, $\hat{D}^c$ and $\hat{E}^c$ stand for the right-handed chiral superfields of u-type quarks, d-type quarks and leptons, respectively. H_u and H_d MSSM Higgs doublets and $Y_{u,d,e}$ are their Yukawa couplings to the matter fields. Finally, $\hat{S}$ denotes a chiral superfield, which does not exist in the MSSM. This field is a singlet under the MSSM group, and its VEV is responsible for the breaking of $\mathrm{U}(1)'$ symmetry. In addition, $\hat{S}$ is also responsible for the masses of the vectorlike families Δ and $\bar{\Delta}$. The invariance under $\mathrm{U}(1)'$ requires an appropriate charge assignment for the MSSM fields. Table I displays the charge configurations for the $\mathrm{U}(1)_\psi$ and $\mathrm{U}(1)_\chi$ models. Note that Eq. (5) allows an infinite number of different charge configurations depending on θ_{E_6}.

Equation (7) is almost the same as the superpotential in the MSSM except for the last term. As is well known, a bilinear mixing of the MSSM Higgs doublets is introduced with the term $\mu\hat{H}_u\hat{H}_d$ in the MSSM, and the μ-term plays an essential role in the electroweak symmetry breaking (EWSB). However, in the MSSM, the μ-term preserves the SUSY, and hence it can be at any scale, despite its connection with the EWSB. This is the so-called μ-problem in the MSSM. On the other hand, if an extra $\mathrm{U}(1)'$ group, under which the MSSM fields have nontrivial charges, is introduced, the invariance principle forbids us to introduce such terms like $\mu\hat{H}_u\hat{H}_d$ since H_u and H_d are charged under $\mathrm{U}(1)'$, and their charges do not have to cancel each other. Rather, another term can be introduced such as $h_s\hat{S}\hat{H}_d\hat{H}_u$, where S is a dynamical field, and its nonzero VEV breaks the extra $\mathrm{U}(1)'$ symmetry while also inducing a bilinear mixing between H_u and H_d with $\mu \equiv h_s\langle S\rangle$. In this picture the μ-term can be related to the $\mathrm{U}(1)'$ breaking scale, and it can be generated dynamically.

TABLE I. Charge assignments for the fields in several models.

Model	$\hat{Q}$	$\hat{U}^c$	$\hat{D}^c$	$\hat{L}$	$\hat{E}^c$	$\hat{H}_d$	$\hat{H}_u$	$\hat{S}$	Δ	$\bar{\Delta}$
$2\sqrt{6}\mathrm{U}(1)_\psi$	1	1	1	1	1	−2	−2	4	−2	−2
$2\sqrt{10}\mathrm{U}(1)_\chi$	−1	−1	3	3	−1	−2	2	0	2	−2

Before proceeding, one of the important tasks for $\mathrm{U}(1)'$ models is to deal with the anomalies and make sure that the model under consideration is anomaly-free. Several attempts have been made [21] by either adding exotics or imposing nonuniversal charges to the families. The charge assignments, given in Table I, correspond to the universal charge configurations for the families; with the vectorlike fields Δ and $\bar{\Delta}$, they provide an anomaly-free configuration while preserving the gauge coupling unification. The Yukawa coupling $h_{\mathcal{D}}$ can be large [22], and hence these vectorlike fields are expected to be heavy consistently with the experimental bounds. Since these particles only interact with the field S as seen in Eq. (7), they can contribute to the sparticle spectrum through higher loop diagrams; as they are heavy, these contributions can be neglected. Even if these particles are heavy, they can still contribute to the proton decay. In this case, one can consider the UMSSM along with SO(10) which forbids baryon and lepton number violating processes [21]. In addition, these vectorlike particles change the β-functions of the MSSM gauge couplings to $(b_1, b_2, b_3) = (\frac{48}{5}, 4, 0)$ [23]. Finally, we should note the existence of right-handed neutrinos. We neglect the contributions from the right-handed neutrinos since these contributions are suppressed due to the smallness of the established neutrino masses [24], unless the inverse seesaw mechanism is imposed [25].

In addition to the MSSM particle content, the UMSSM yields two more particles at the low scale, one of which is the gaugino associated with the gauge fields of $\mathrm{U}(1)'$, and the other is the supersymmetric partner of the MSSM singlet S. Since these two particles have no electric charge, they mix with the MSSM neutralinos after EWSB, which enriches the dark matter implications in the UMSSM [26]. EWSB also yields a mixing $Z - Z'$, where Z' is the gauge boson associated with $\mathrm{U}(1)'$. Hence, one can expect some effects from the interference of Z', but since the mass bound on Z' is strict, these effects are highly suppressed by its heavy mass. Finally, the content of the charged sector of the MSSM remains the same, but h_s and $\langle S\rangle$ are effective in this sector since they generate the μ-term effectively, which also determines the mass of Higgsinos.

A minimal E(6) model, in which the matter fields reside in a **27**-plet and the MSSM Higgs fields in $(\mathbf{27}_L + \mathbf{27}^*_L)$, also proposes YU via $y\ \mathbf{27}_i\mathbf{27}_j\mathbf{27}_H$ in the superpotential. The discussion on the contradictory mass relations in the fermion sector can be handled by extending the Higgs sector with **351**- and $\overline{\mathbf{351}}$-plets within the $E(6)$ framework [27]. In our work we assume the minimal layout for the E(6) model. However, the Higgs fields emerging from $(\mathbf{27}_L + \mathbf{27}_L^*)$ can also break SO(10) to the Pati-Salam model [28], which is based on the gauge group $G_{\rm PS} \equiv {\rm SU}(4)_c \times {\rm SU}(2)_L \times {\rm SU}(2)_R$. In such a framework the Yukawa sector may also include interactions between the matter fields and the Higgs fields from the $H'(15,1,3)$ representation of $G_{\rm PS}$. If one assumes that $G_{\rm PS}$ breaks into the MSSM gauge group at about the GUT scale, the known Yukawa couplings can be stated as given in Eq. (6) at $M_{\rm GUT}$. Note that emergence of $G_{\rm PS}$ in the breaking chain allows nonuniversal gaugino masses at the GUT scale such that [29]

$$M_1 = \frac{3}{5}M_2 + \frac{2}{5}M_3. \tag{8}$$

In this case M_3 can be varied over the parameter space as a free parameter; hence, the tension from the heavy gluino mass bound can be significantly relaxed, which yields drastic improvement in regard to the fine-tuning. However, as stated above, we restrict ourselves to the universal boundary conditions, and we impose only one SSB mass term for all three gauginos.

Even though there are an infinite number of different charge assignments accordingly to Eq. (5), we restrict ourselves to the $\mathrm{U}(1)_\psi$ model (i.e., $\cos\theta_{E_6} = 0$ and $\sin\theta_{E_6} = 1$) and consider the YU range predicted by SO(10) GUT ($y_t \approx y_b \approx y_\tau \sim 0.6$) for the Yukawa couplings of the MSSM matter families. The deviation from YU in the case of QYU can be analyzed by considering a parameter defined as

$$R = \frac{\max(C_1, C_2, C_3)}{\min(C_1, C_2, C_3)} \tag{9}$$

with

$$C_1 = \left|\frac{y_t - y_b}{y_t + y_b}\right|, \quad C_2 = \left|\frac{y_\tau - y_t}{3y_t - y_\tau}\right|, \quad C_3 = \left|\frac{y_\tau - y_b}{3y_b + y_\tau}\right| \tag{10}$$

where $y_{t,b,\tau}$ are Yukawa couplings at $M_{\rm GUT}$, and $C_{1,2,3}$ denote the contributions to these couplings. The consistency with QYU requires $C_1 = C_2 = C_3$, i.e., $R = 1$. However, Yukawa couplings can receive some contributions from the interference of S, Z' [30] and even exotics at $M_{\rm GUT}$, as well as unknown threshold corrections from the symmetry breaking. Even though these contributions can be neglected, we allow at most 10% uncertainty in R to count for such contributions. Hence, a solution compatible with QYU satisfies $R \le 1.1$ as well as $|C| \le 0.2$.

III. SCANNING PROCEDURE AND EXPERIMENTAL CONSTRAINTS

We have employed the SPheno 3.3.3 package [31] obtained with SARAH 4.5.8 [32]. In this package the weak scale values of the gauge and Yukawa couplings present in the UMSSM are evolved to the unification scale $M_{\rm GUT}$ via the renormalization group equations (RGEs). $M_{\rm GUT}$ is determined by the requirement of the gauge coupling unification through their RGE evolutions. Note that we do not strictly enforce the unification condition $g_1 = g_2 = g_3$ at $M_{\rm GUT}$ since a few-percent deviation from the unification can be assigned to unknown GUT-scale threshold corrections [33]. With the boundary conditions given at $M_{\rm GUT}$, all the SSB parameters, along with the gauge and Yukawa couplings, are evolved back to the weak scale. Note that the gauge coupling associated with the $B-L$ symmetry is determined by the unification condition at the GUT scale by imposing $g_1 = g_2 = g' \approx g_3$, where g' is the gauge coupling associated with the $\mathrm{U}'(1)$ gauge group.

We have performed random scans over the following parameter space:

$$\begin{aligned}
&0 \le m_0 \le 5\ (\mathrm{TeV}) \quad 0 \le M_{1/2} \le 5\ (\mathrm{TeV}) \\
&35 \le \tan\beta \le 60 \quad -3 \le A_0/m_0 \le 3 \\
&-1 \le A_{h_s} \le 15\ (\mathrm{TeV}) \quad 1 \le v_s \le 25\ (\mathrm{TeV})
\end{aligned} \tag{11}$$

where m_0 is the universal SSB mass term for all the scalar fields including H_u, H_d, S fields, and similarly $M_{1/2}$ is the universal SSB mass term for the gaugino fields including one associated with the $\mathrm{U}(1)'$ gauge group. Note that $\tan\beta = \langle v_u\rangle/\langle v_d\rangle$ is the ratio of VEVs of the MSSM Higgs doublets, and A_0 is the SSB trilinear scalar interaction term. Similarly, A_{h_s} is the SSB interaction between the S and $H_{u,d}$ fields, which is varied free from A_0 in our scans. Finally, v_s denotes the VEV of S fields, which indicates the $\mathrm{U}(1)'$ breaking scale. Recall that the μ-term of the MSSM is dynamically generated such that $\mu = h_s v_s$. Its sign is assigned as a free parameter in the MSSM since the radiative electroweak symmetry breaking (REWSB) condition can determine its value but not its sign. On the other hand, in the UMSSM it is forced to be positive by h_s and v_s. Finally, we set the top quark mass to its central value ($m_t = 173.3$ GeV) [34]. Note that the sparticle spectrum is not too sensitive in 1σ or 2σ variations in the top quark mass [35], but it can shift the Higgs boson mass by 1–2 GeV [36].

In our scan over the parameter space of the UMSSM, the ranges of m_0 and $M_{1/2}$ are restricted as ≤ 5 TeV based on previous studies on the fine-tuning. The fine-tuning condition usually requires $m_0, M_{1/2} \lesssim 2$ TeV [6]. In our setup we allow these parameters to lie in a range up to 5 TeV in order to see how the regions with low fine-tuning can be enlarged when an extra U(1) sector is present. In addition, the range for the trilinear scalar couplings, A_0 and A_{h_s}, is adjusted by the requirement to avoid color and/or charge breaking minima [37]. Among these parameters $\tan\beta$ is bounded at 35 from below. Even though the general UMSSM framework can be consistent with the current experimental bounds including the Higgs boson mass, Yukawa unification instead requires high $\tan\beta$ values. This requirement can be understood by considering the mass ratio of the top and bottom quarks as follows:

$$\frac{m_t}{m_b} \approx \frac{y_t}{y_b}\tan\beta. \tag{12}$$

If one imposes the exact Yukawa unification at the GUT scale as $y_t = y_b$, then $\tan\beta$ should be larger than about 40 in order to have the correct masses for the top and bottom quarks ($m_t/m_b \sim 40$). If there is a slight deviation from the exact Yukawa unification as in the case of quasi-Yukawa unification, consistent solutions are realized in the region with $\tan\beta \gtrsim 55$ [17]. We vary $\tan\beta$ in the range between 35 and 60 to focus on the region which is compatible with Yukawa unification.

The requirement of REWSB [38] puts an important theoretical constraint on the parameter space. Another important constraint comes from the relic abundance of the stable charged particles [39], which excludes the regions where charged SUSY particles such as stau and stop become the lightest supersymmetric particle (LSP). In our scans we allow only the solutions for which one of the neutralinos is the LSP and the REWSB condition is satisfied.

In scanning the parameter space, we use our interface, which employs the Metropolis-Hasting algorithm described in [40]. After collecting the data we impose the mass bounds on all the sparticles [41] and the constraints from the rare B-decays such as $B_s \to \mu^+\mu^-$ [42], $B_s \to X_s\gamma$ [43], and $B_u \to \tau\nu_\tau$ [44]. In addition, we impose the WMAP [45] and Planck [46] bounds on the relic abundance of the neutralino LSP within 5σ uncertainty. These experimental constraints can be summarized as follows:

$$\begin{aligned}
m_h &= 123\text{–}127~\text{GeV}\\
m_{\tilde{g}} &\geq 1.8~\text{TeV}\\
M_{Z'} &\geq 2.5~\text{TeV}\\
0.8\times10^{-9} \leq \mathrm{BR}(B_s \to \mu^+\mu^-) &\leq 6.2\times10^{-9}~(2\sigma)\\
2.99\times10^{-4} \leq \mathrm{BR}(B \to X_s\gamma) &\leq 3.87\times10^{-4}~(2\sigma)\\
0.15 \leq \frac{\mathrm{BR}(B_u\to\tau\nu_\tau)_{\mathrm{MSSM}}}{\mathrm{BR}(B_u\to\tau\nu_\tau)_{\mathrm{SM}}} &\leq 2.41~(3\sigma)\\
0.0913 \leq \Omega_{\mathrm{CDM}}h^2 &\leq 0.1363~(5\sigma)(\mathrm{WMAP})\\
0.1089 \leq \Omega_{\mathrm{CDM}}h^2 &\leq 0.1309~(5\sigma)(\mathrm{Planck}).
\end{aligned} \tag{13}$$

We have emphasized the bounds on the Higgs boson [47] and the gluino [3] because they have drastically changed since the LEP era. Even though the mass bound on Z' can be lowered through detailed analyses [48], we require our solutions to yield heavy Z' since it is not directly related to our considerations. One of the stringent bounds listed above comes from the rare B-meson decay into a muon pair since the supersymmetric contribution to this process is proportional to $(\tan\beta)^6/m_A^4$. We have considered the high $\tan\beta$ region in the fundamental parameter space as given in Eq. (11), and m_A needs to be large to suppress the supersymmetric contribution to $\mathrm{BR}(B_s \to \mu^+\mu^-)$. In addition, the WMAP and Planck bounds on the dark matter density are also highly effective in shaping the parameter space since the relic abundance of the neutralino LSP is usually high over the fundamental parameter space. One needs to identify some coannihilation channels in order to have solutions compatible with the WMAP and Planck bounds. However, if such mechanisms exist, the Boltzmann equation for Ωh^2 becomes highly nonlinear, and it can be solved only numerically in a reasonable approximation. In addition, the form of the Boltzmann equation's solution is exponential; hence, even a small uncertainty in the relevant parameters used in the solution causes a large uncertainty in the calculation of Ωh^2. Considering such uncertainties in our calculation, we have assumed the WMAP and Planck results to yield very similar DM phenomenology. To take into account the uncertainties in the calculation as much, we applied the WMAP bound within a 5σ model as given in Eq. (13), along with the other constraints, since its range is slightly larger than the Planck bound. The DM observables in our scan are calculated by micrOMEGAs [49] obtained by SARAH [32]. Finally, we impose the fine-tuning condition as $\Delta_{\mathrm{EW}} \leq 10^3$.

IV. FUNDAMENTAL PARAMETER SPACE OF QYU

We present our results for the fundamental parameter space in light of the experimental constraints mentioned in the previous section. Figure 1 illustrates the QYU parameter space in correlation with the usual CMSSM fundamental parameters in the $C - m_0$, $C - M_{1/2}$, $C - A_0/m_0$, and $C - \tan\beta$ planes. All points are consistent with REWSB and the neutralino LSP. Green points satisfy the mass bounds and the constraints from the rare B-decays. Orange points form a subset of the green ones, and they are compatible with the WMAP bound on the relic abundance of the neutralino LSP within 5σ. Finally, the blue points are a subset of the orange ones, which are consistent with the QYU and fine-tuning condition. The dashed lines indicate $C = 0.2$. As seen from $C - m_0$, QYU requires a universal scalar mass parameter larger than 2 TeV, as in the case of the MSSM with nonuniversal gauginos imposed at M_{GUT}; with the current experimental bounds, m_0 is expected to be

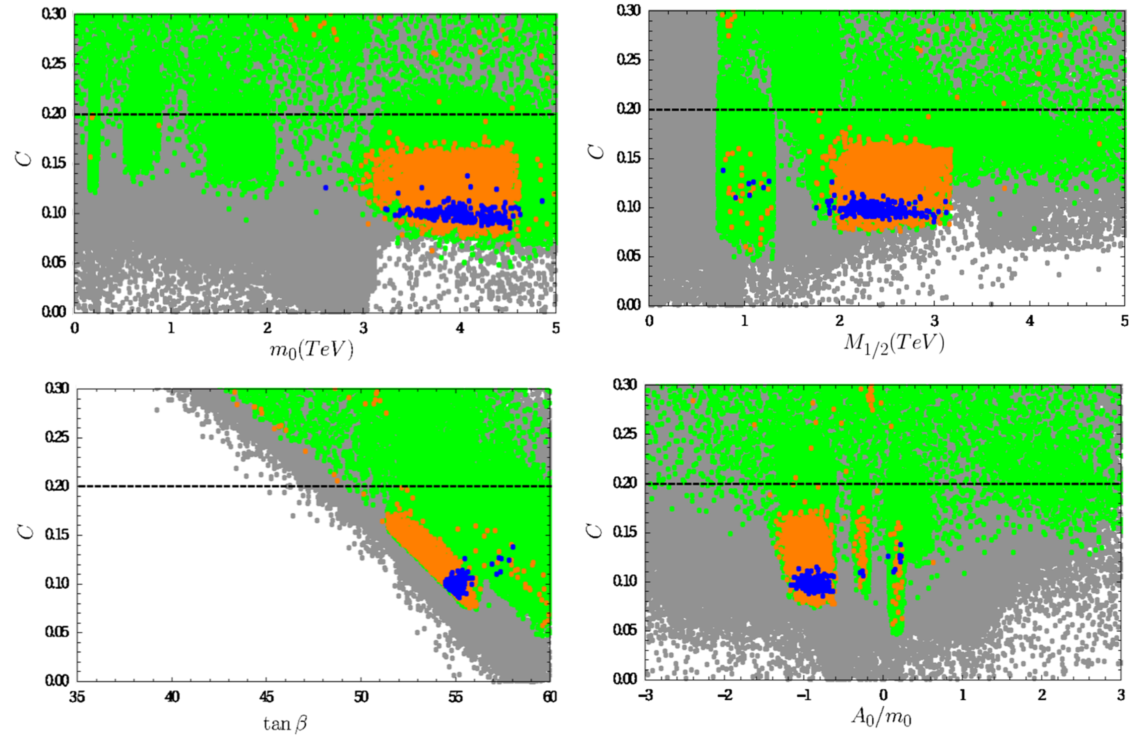

FIG. 1. Plots in the $C - m_0$, $C - M_{1/2}$, $C - A_0/m_0$, and $C - \tan\beta$ planes. All points are consistent with the REWSB and neutralino LSP. Green points satisfy the mass bounds and the constraints from the rare B-decays. Orange points form a subset of the green ones, and they are compatible with the WMAP bound on the relic abundance of the neutralino LSP within 5σ. Finally, the blue points are a subset of the orange ones, which are consistent with the QYU and fine-tuning condition. The dashed lines indicate $C = 0.2$.

much larger in the CMSSM framework. Similarly, the $C - M_{1/2}$ plane indicates that $M_{1/2}$ can only be as light as 800 GeV. This bound is not strictly imposed by the QYU condition; instead the heavy gluino mass bound requires heavy $M_{1/2}$, when the universal gaugino masses are imposed. The QYU condition mostly restricts the $\tan\beta$ parameter to values larger than about 54 as seen from the $C - \tan\beta$ plane, which happens in the MSSM. Finally, A_0 values are mostly found in the negative region, while it is possible to realize QYU with small positive A_0/m_0 values.

In addition to the fundamental parameters of the CMSSM, Fig. 2 displays the results in the UMSSM parameters with plots in the $C - h_s$ and $C - v_s$ planes. The color-coding is the same as Fig. 1. The $C - h_s$ plane shows that the QYU solutions accumulate mostly in the region with $0.1 \lesssim h_s \lesssim 0.2$, while it can be enlarged to about 0.3 with good statistics. On the other hand, the region with $h_s \gtrsim 0.4$ is excluded by the current experimental constraints (green). The plane $C - v_s$ shows that the lowest scale for the $U(1)'$ breaking is about 5 TeV. This breaking scale is restricted to $\lesssim$10 TeV by QYU and the fine-tuning condition (blue).

Since the breaking scale along with h_s generates the μ-term, it is worth considering how large a μ-term can be realized in the UMSSM. Figure 3 represents our results with plots of the $C - \mu$, $C - \Delta_{\rm EW}$, $m_0 - \Delta_{\rm EW}$, and $M_{1/2} - \Delta_{\rm EW}$ planes. The color-coding is the same as Fig. 1, without the fine-tuning condition. The $C - \mu$ plane shows that the alignment between v_s and h_s allows the range $\mu \in \sim$800–1500 GeV, which yields low fine-tuning ($\Delta_{\rm EW} \gtrsim 300$) compatible with the QYU condition as seen from the $C - \Delta_{\rm EW}$ plane. Such a low fine-tuning can be achieved even when $m_0 \gtrsim 3$ TeV and $M_{1/2} \gtrsim 2$ TeV, as shown in the bottom panels of Fig. 3.

Figure 4 displays the ratios of the Yukawa couplings with plots in the $y_t/y_b - \Delta_{\rm EW}$, $y_\tau/y_b - \Delta_{\rm EW}$, $y_t/y_\tau - \Delta_{\rm EW}$, and $\Delta_{\rm EW} - \tan\beta$ planes. The color-coding is the same as Fig. 1, without the fine-tuning condition. QYU requires certain ratios among the Yukawa couplings. Even though y_t/y_b can lie from 1.1 to about 2, QYU instead restricts this ratio to $y_t/y_b \sim 1.2$. Similarly, it restricts $y_\tau/y_b \sim 1.4$ and $y_t/y_\tau \sim 0.8$, as seen from the $y_\tau/y_b - \Delta_{\rm EW}$ and $y_t/y_\tau - \Delta_{\rm EW}$ planes. These ratios hold for any value of the fine-tuning parameter. Finally, $\Delta_{\rm EW} - \tan\beta$ indicates that $\tan\beta$ can be as high as 58 without disturbing the Yukawa coupling ratios or raising the amount of fine-tuning.

V. SPARTICLE SPECTRUM

This section presents the sparticle spectrum compatible with QYU. We start with the color sector, as well as staus

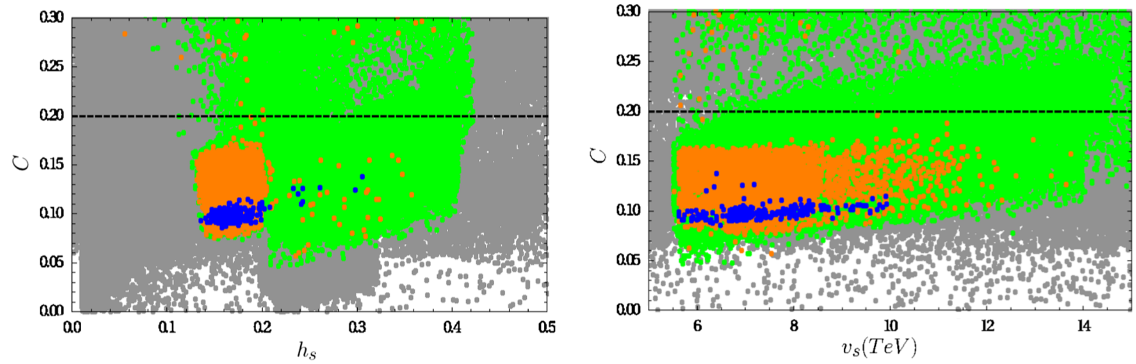

FIG. 2. Plots in the $C - h_s$ and $C - v_s$ planes. The color-coding is the same as Fig. 1.

and m_A, as shown in Fig. 5 with plots in the $m_{\tilde{g}} - m_{\tilde{t}_1}$, $m_{\tilde{q}} - m_{\tilde{g}}$, $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$, and $m_A - \tan\beta$ planes. The color-coding is the same as Fig. 1. As seen from the $m_{\tilde{g}} - m_{\tilde{t}_1}$ plane, the gluino can be as light as about 2 TeV, while the region with $m_{\tilde{t}_1} \lesssim 2.5$ TeV is not compatible with the QYU condition. Even though it is possible to realize the Higgs boson of mass about 125 GeV with light stops in the UMSSM framework, such light stop solutions are mostly excluded by the heavy gluino mass spectrum. Similarly, the squarks from the first two families are required to be heavier than about 3 TeV, as seen from the $m_{\tilde{q}} - m_{\tilde{g}}$ plane. The $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$ plane shows that even though one can realize the stau mass to be almost degenerate with the LSP neutralino consistently with the WMAP bound (orange), the QYU, together with the fine-tuning condition, requires $m_{\tilde{\tau}_1} \gtrsim 1.5$ TeV. The last panel of Fig. 5 shows m_A in correlation with the $\tan\beta$ parameter. The results in the $m_A - \tan\beta$ plane show that the A boson can be as light as 400 GeV, compatible with QYU, despite the high $\tan\beta$ values. The exclusion limit set on m_A does not allow the solutions with $m_A \lesssim 900$ GeV in the MSSM, when $\tan\beta$ is large [50]. Even though there could be new decay channels that can lose this bound, the UMSSM proposes rather heavy new particles such as the MSSM-singlet field S and vectorlike fields $\Delta, \bar{\Delta}$, and this exclusion limit remains more or less the same as in the MSSM. The solutions with $m_A \gtrsim 900$ for high $\tan\beta$ values can be excluded further, but

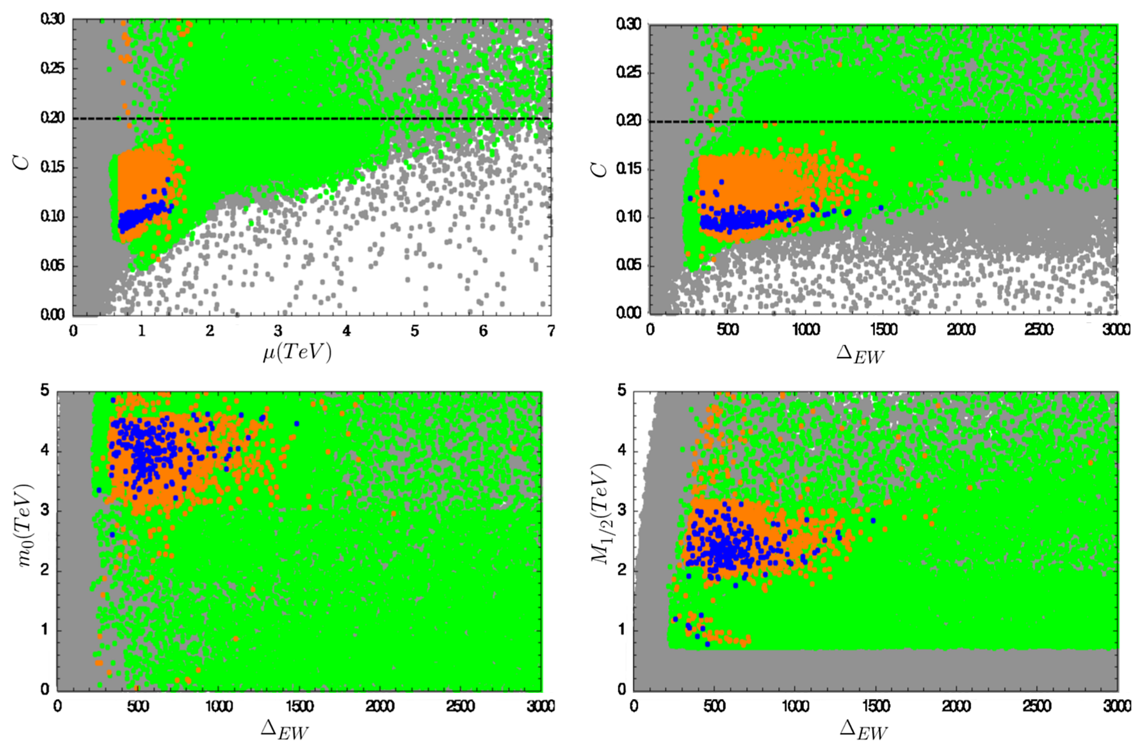

FIG. 3. Plots in the $C - \mu$ and $C - \Delta_{\rm EW}$, $m_0 - \Delta_{\rm EW}$, and $M_{1/2} - \Delta_{\rm EW}$ planes. The color-coding is the same as Fig. 1, without the fine-tuning condition.

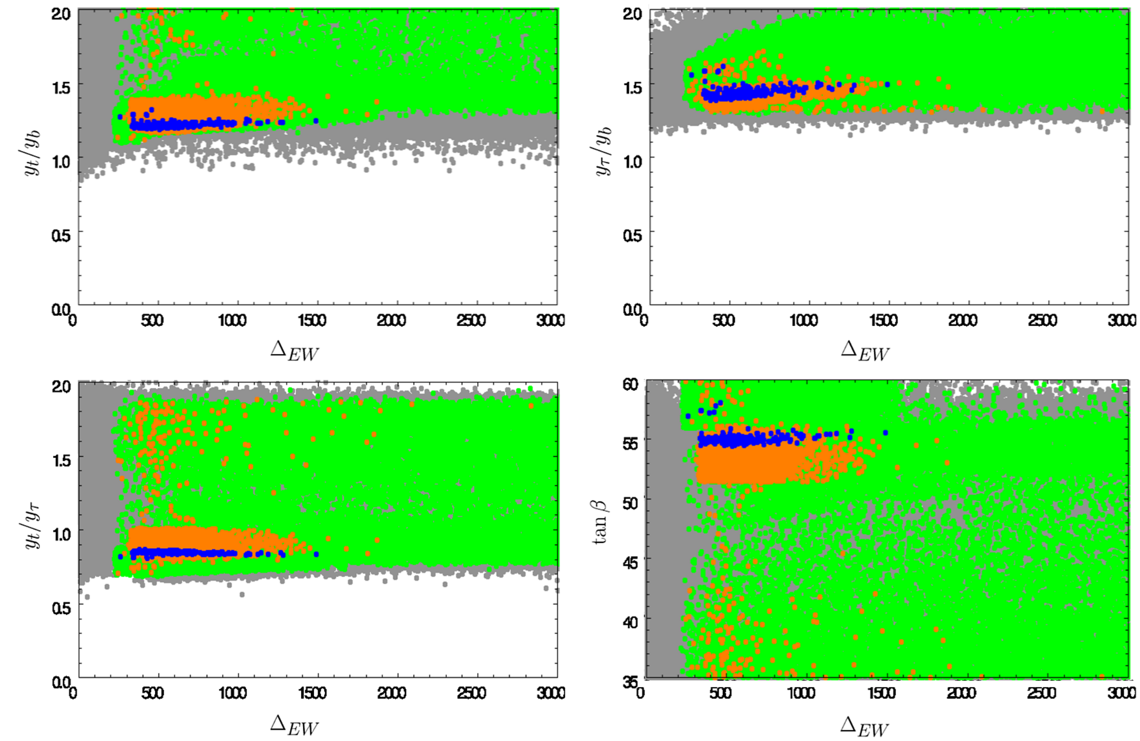

FIG. 4. Plots in the $y_t/y_b - \Delta_{\mathrm{EW}}$, $y_\tau/y_b - \Delta_{\mathrm{EW}}$, $y_t/y_\tau - \Delta_{\mathrm{EW}}$, and $\Delta_{\mathrm{EW}} - \tan\beta$ planes. The color-coding is the same as Fig. 1, without the fine-tuning condition.

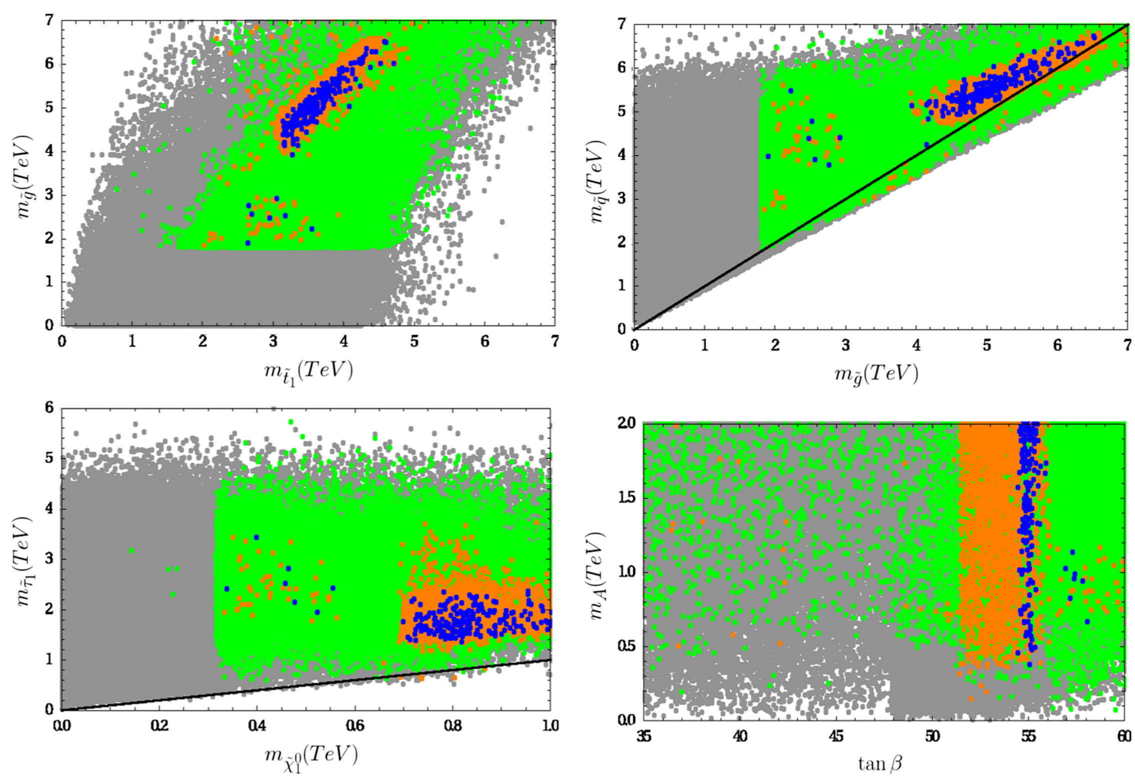

FIG. 5. Plots in the $m_{\tilde{g}} - m_{\tilde{t}_1}$, $m_{\tilde{q}} - m_{\tilde{g}}$, $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$, and $m_A - \tan\beta$ planes. The color-coding is the same as Fig. 1.

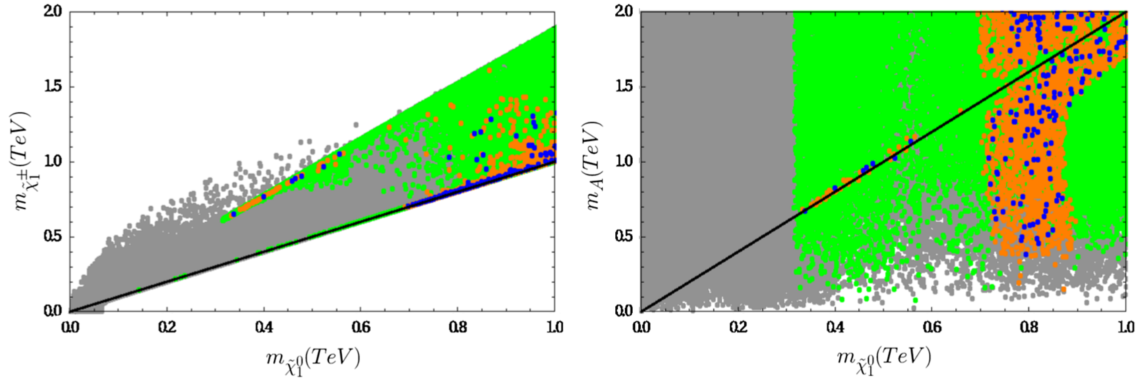

FIG. 6. Plots in the $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0}$ and $m_A - m_{\tilde{\chi}_1^0}$ planes. The color-coding is the same as Fig. 1. The diagonal line indicates the regions with $m_{\tilde{\chi}_1^\pm} = m_{\tilde{\chi}_1^0}$ in the left panel and $m_A = 2m_{\tilde{\chi}_1^0}$ in the right panel.

there are still a significant number of solutions with heavy A bosons, compatible with QYU, and they can escape from this bound.

Figure 6 displays the sparticle spectrum in the $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0}$ and $m_A - m_{\tilde{\chi}_1^0}$ planes. The color-coding is the same as Fig. 1. The diagonal line indicates the regions with $m_{\tilde{\chi}_1^\pm} = m_{\tilde{\chi}_1^0}$ in the left panel and $m_A = 2m_{\tilde{\chi}_1^0}$ in the right panel. The $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0}$ case shows that the chargino and LSP neutralino are mostly degenerate in mass in the region where $m_{\tilde{\chi}_1^0} \gtrsim 700$ GeV. This region may indicate the Higgssino DM, and the degeneracy can arise from the that of two Higgsinos. These solutions favor the chargino-neutralino coannihilation processes which reduce the relic abundance of the LSP neutralino such that the solutions can be consistent with the WMAP bound. This region also yields A-resonance solutions, as seen from the $m_A - m_{\tilde{\chi}_1^0}$ plane. It is also possible to realize lighter LSP neutralino solutions ($m_{\tilde{\chi}_1^0} \gtrsim 400$ GeV). There is no mass degeneracy between the LSP neutralino and chargino in this region. Hence, one can conclude that in the light LSP neutralino region, the LSP neutralino is binolike, and the WMAP bound on the relic abundance of the LSP neutralino is satisfied through A-resonance solutions, in which two neutralinos annihilate into an A boson. As mentioned in the discussion of Fig. 5, the solutions with $m_A \lesssim 1$ TeV can be excluded by the exclusion limit set with respect to $\tan\beta$. However, the UMSSM still proposes A-resonance solutions for $m_A \gtrsim 1$ TeV in reducing the LSP neutralino relic abundance to the current bounds from the WMAP and Planck experiments.

The LSP neutralino composition can be seen better from the $\mu - M_1$ and $\mu - M_2$ planes shown in Fig. 7. The color-coding is the same as Fig. 1. The diagonal line indicates the region where $\mu = M_1$ ($\mu = M_2$) in the left (right) plane. The $\mu - M_1$ plane shows that the μ-parameter is smaller than M_1 over most of the parameter space. The LSP neutralino is formed by the Higgsinos in this region. Such solutions also yield high scattering cross sections at the nuclei used in the direct detection experiments since the interactions between quarks in nuclei and the LSP neutralino happen via Yukawa interactions. The Higgsinos and bino are almost degenerate in mass in the region around the diagonal line, and this region indicates bino-Higgsino mixing in formation of the LSP neutralino. It is also possible to realize binolike DM, as represented with the solutions below the diagonal line where $M_1 \leq \mu$. One can

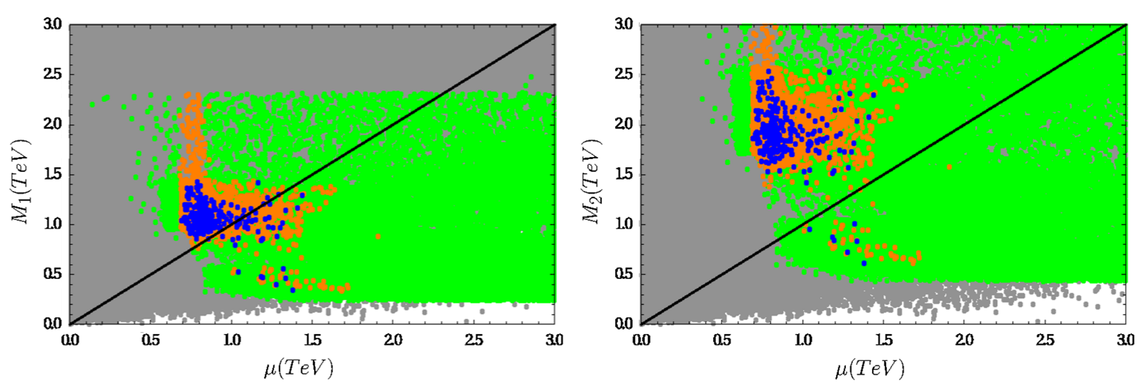

FIG. 7. Plots in the $\mu - M_1$ and $\mu - M_2$ planes. The color-coding is the same as Fig. 1. The diagonal line indicates the region where $\mu = M_1$ ($\mu = M_2$) in the left (right) plane.

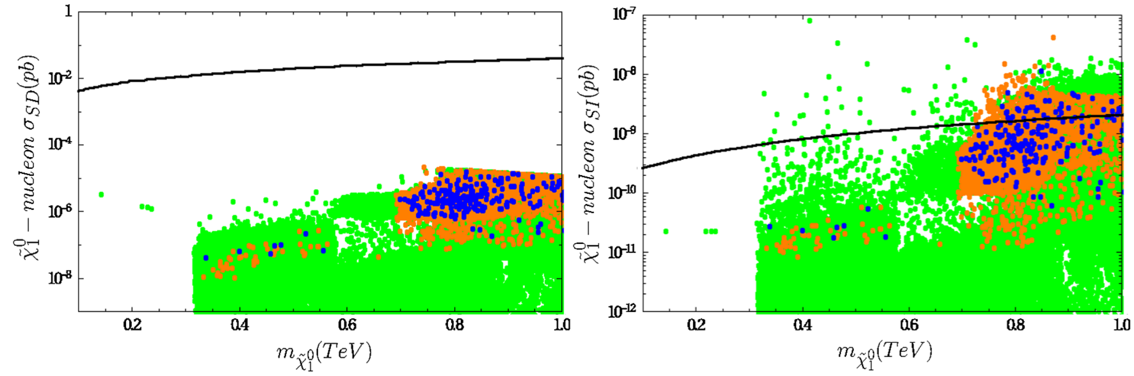

FIG. 8. Latest LUX results for both spin-dependent [51] (left) and spin-independent [52] (right) dark matter scattering cross sections over the dark matter implications of the UMSSM. The color-coding is the same as Fig. 1. The solid line represents the latest LUX results.

also check if it is possible to have a wino mixture in the formation of the LSP neutralino. The $\mu - M_2$ plane shows that the wino is usually heavier than μ and hence M_1. However, there could be some solutions at about $\mu \sim 1$ TeV, for which the Higgsinos and wino are nearly degenerate in mass. Comparing with solutions shown in the $\mu - M_1$ plane, M_1 is seen to be at about 1 TeV for this solution; i.e., $\mu \sim M_1 \sim M_2$, and the wino mixture in the formation of the LSP neutralino becomes as significant as the bino and Higgsinos.

Based on the discussion above about the LSP composition, another strict constraint can come from the direct detection experiments which are performed to measure the dark matter scattering at nuclei. In the supersymmetric models with universal boundary conditions such as CMSSM, the dark matter is mostly formed by the bino, and it yields very low scattering cross sections; hence, it can escape from the direct detection results. However, as shown in Fig. 7, the UMSSM yields the low scale results with $\mu \lesssim M_1$, M_2. Such results yield the dark matter mostly formed by the Higgsinos, and in these cases the scattering cross section is rather large since the LSP is scattered by nuclei through the Yukawa interactions with the quarks. Figure 8 represents the latest LUX results for both spin-dependent [51] (left) and spin-independent [52] (right) dark matter scattering cross sections over the dark matter implications of the UMSSM. The color-coding is the same as Fig. 1. The solid line represents the latest LUX results. Figure 8 reveals that the scattering cross section is mostly large, and it is consistent with having Higgsino-like dark matter at the low scale. Even though it is large, as we can see from the left panel of Fig. 8, the results for the spin-dependent cross section are way below the exclusion limit. On the other hand, the right panel represents the impact from the latest LUX results. Some of the solutions (blue above the exclusion limit) are already excluded. On the other hand, the UMSSM yields many solutions just below the exclusion limit. In this context these solutions can be expected to be tested in near future experiments with increasing sensitivity to the dark matter scattering.

VI. CONCLUSION

We explore the low scale implications in the UMSSM. We restrict the parameter space such that the LSP is always the lightest neutralino. In addition, we impose QYU at the grand unification scale ($M_{\rm GUT}$). The fundamental parameters of the UMSSM are found to be in a large range such as $m_0 \gtrsim 3$ TeV, $M_{1/2} \gtrsim 800$ GeV. The $\tan\beta$ parameter is mostly restricted to the region where $\tan\beta \geq 54$ by the QYU condition. Also, QYU strictly requires the ratios among the Yukawa couplings as $y_t/y_b \sim 1.2$, $y_\tau/y_b \sim 1.4$, and $y_t/y_\tau \sim 0.8$. In addition, the breaking of the $U(1)'$ group takes place at energy scales from about 5 TeV to 10 TeV.

We find that the need for fine-tuning over the fundamental parameter space of QYU is in the acceptable range, even if the universal boundary conditions are imposed at $M_{\rm GUT}$, in contrast to CMSSM and NUHM. Such a setup yields heavy stops ($m_{\tilde{t}} \gtrsim 2.5$ TeV), gluinos ($m_{\tilde{g}} \gtrsim 2$ TeV), and squarks from the first two families ($m_{\tilde{q}} \gtrsim 4$ TeV). Similarly, the stau mass is bounded from below at about 1.5 TeV. Despite this heavy spectrum, we find $\Delta_{\rm EW} \gtrsim 300$, which is much lower than that needed for the minimal supersymmetric models. In addition, the UMSSM yields a relatively small μ-term, and the LSP neutralino is mostly formed by the Higgsinos of mass $\gtrsim$700 GeV. We also obtain binolike DM with mass of about 400 GeV. The wino is usually found to be heavier than the Higgsinos and binos, but there is a small region where $\mu \sim M_1 \sim M_2 \sim 1$ TeV. We also identify a chargino-neutralino coannihilation channel and A-resonance solutions which reduce the relic abundance of the LSP neutralino down to the ranges compatible with the current WMAP and Planck measurements. Finally, we consider the constraints on UMSSM implications about the dark matter with respect to the latest LUX results for the direct detection. Even though it is severe when the dark matter is light ($m_{\tilde{\chi}_1^0} \lesssim 100$ GeV), it still has a considerable impact at the heavy mass scales. Some low scale solutions are already excluded, but the UMSSM provides many solutions which are allowed by the current results and testable in near future experiments with increasing sensitivity to the dark matter scattering.

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