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# Revisiting $B \to D\ell\nu$

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We reexamine the determination of $|V_{cb}|$ from $B \to Dl\nu$ in view of recent experimental and theoretical progress, discussing the parameterization of the form factors and studying the role played by the unitarity constraints. Our fit to experimental and lattice results for $B \to D\ell\nu$ gives $|V_{cb}| = 40.49(97)10^{-3}$ and to $R(D) = 0.299(3)$.

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## I. INTRODUCTION

The discrepancy between the determination of $|V_{cb}|$ from inclusive and exclusive semileptonic $B$ decays is a long-standing problem in flavor physics. The Cabibbo-Kobayashi-Maskawa (CKM) element $|V_{cb}|$ plays a central role in the analyses of CKM unitarity [1,2] and in the standard model (SM) prediction of flavor-changing neutral current transitions, where its uncertainty is often the dominant one [3]. Its determination from inclusive decays is based on an operator product expansion which parameterizes the relevant nonperturbative physics in terms of nonperturbative constants that are extracted from experiment; see [4] for a review. A very recent analysis [5] points to

$$|V_{cb}| = (42.00 \pm 0.65) \times 10^{-3}. \tag{1.1}$$

The main channels for the exclusive determination of $|V_{cb}|$ have been so far $B \to D^{(*)}l\nu$, and until recently all analyses have focused on the zero-recoil point, i.e. on maximal $q^2$. Indeed, in the heavy quark limit the relevant form factors are known exactly at zero recoil [6], up to perturbative corrections, and lattice calculations, which are anyway performed at high $q^2$, only need to determine the power-suppressed deviation from that limit. In the $D^*$ case the correction to the heavy quark limit is quadratically suppressed. The downside of zero-recoil analyses, however, is that the decay rates vanish at zero recoil (more rapidly in $B \to Dl\nu$) and that one therefore needs to extrapolate the experimental distributions, a problem which has been thoroughly addressed almost 20 years ago, with the introduction of various model-independent parameterizations [7–9].

To date, the most precise exclusive determination of $|V_{cb}|$ is based on the calculation of the $D^*$ form factor at zero recoil by the FNAL/MILC Collaboration [10] and on the HFAG average [11] of B factory results analyzed in the context of the Caprini-Lellouch-Neubert (CLN) parameterization [9]:

$$|V_{cb}| = (39.04 \pm 0.75) \times 10^{-3}. \tag{1.2}$$

This differs from (1.1) by $3\sigma$, which becomes $2.8\sigma$ once the QED corrections are treated in the same way in both cases. It would be important to have other independent lattice calculations, also in view of indications from heavy quark sum rules that the form factor of [10] is overestimated [12]. Traditionally, the $D$ channel has led to less precise determinations, mostly because of larger experimental errors; see for instance [13].

The discrepancy between (1.1) and (1.2) is unwelcome. In principle, it could signal new physics, as the $B \to D^*$ transition is sensitive only to the axial-vector component of the charged weak current. However, this new physics would require new interactions which seem ruled out by electroweak constraints on the effective $Z\bar{b}b$ vertex [14]. The situation is further complicated by the $3.9\sigma$ discrepancy [11] between the measurement of

$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}\mu\nu)} \tag{1.3}$$

and their SM predictions, which depend on knowledge of the form factors in the whole available $q^2$ range. Different types of new physics could be responsible for this discrepancy; see [15] and references therein for recent discussions.

In this context, any new information on $|V_{cb}|$ and on the semileptonic form factors is of great value. Two new elements have recently made the decays $B \to D\ell\nu$ more interesting in this respect. First, two calculations of the form factors of $B \to D\ell\nu$ *beyond* zero recoil have appeared in 2015 [16,17]. They represent the first unquenched calculations of these form factors performed at different $q^2$ values, which significantly reduces the uncertainty of the extrapolation from the $q^2$ region where most data are taken. Second, a new, more precise Belle measurement has been published [18], which for the first time provides the $q^2$ differential distribution with complete statistical and systematic uncertainties and correlations. As we will show, the combination of these steps forward allows for a competitive extraction of $|V_{cb}|$ and for a very precise determination of the $B \to D$ form factors.

In this paper we revisit the decay $B \to D\ell\nu$ in view of the above developments and upgrade the unitarity bounds using recent three-loop calculations and up-to-date heavy quark masses. In Sec. II we briefly review the model-independent parameterization of the form factors and introduce three well-known parameterizations. We then update and discuss the impact of unitarity bounds, boosting them with the inclusion of higher states. After presenting our inputs in Sec. III, we perform a global fit of the available theoretical and experimental data, which leads to a precise determination of $|V_{cb}|$ and of the form factors. We also employ these form factors in the calculation of $R(D)$ and provide the most precise prediction to date. Section V summarizes our findings.

## II. FORM FACTORS AND THEIR PARAMETERIZATION

The hadronic matrix element governing the $B \to D\ell\nu$ decay is described by two form factors:

$$\langle D(p')|V^\mu|\bar{B}(p)\rangle = f_+(q^2)(p+p')^\mu + f_-(q^2)(p-p')^\mu, \tag{2.1}$$

where $q^2 = (p-p')^2$. The differential rate can be written as

$$\frac{d\Gamma}{dq^2}(B\to Dl\nu_l) = \frac{\eta_{\rm ew}^2 G_F^2 |V_{cb}|^2 m_B \lambda^{1/2}}{192\pi^3}\left(1-\frac{m_l^2}{q^2}\right)^2 \times [c_+^l f_+(q^2)^2 + c_0^l f_0(q^2)^2], \tag{2.2}$$

where $r = m_D/m_B$, $\lambda = (q^2 - m_B^2 - m_D^2)^2 - 4m_B^2 m_D^2$,

$$c_+^l = \frac{\lambda}{m_B^4}\left(1+\frac{m_l^2}{2q^2}\right), \qquad c_0^l = (1-r^2)^2\frac{3m_l^2}{2q^2}, \tag{2.3}$$

and

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_B^2 - m_D^2} f_-(q^2),$$

from which it follows that $f_+(0) = f_0(0)$. In the limit of vanishing lepton mass the $f_0$ contribution becomes irrelevant. In fact, it can be safely neglected except for decays into $\tau$ leptons. The factor $\eta_{\rm ew} = 1 + \alpha/\pi \ln M_Z/m_B \simeq 1.0066$ [19] takes into account the short-distance QED corrections, namely the electromagnetic running of the four-fermion operator from the weak to the $B$ scale, and represents the leading electroweak correction. Unlike Ref. [16] we do not include any Coulomb correction and expect the error due to subleading electroweak corrections to be negligible in comparison with other uncertainties. The knowledge of $f_+$ and $f_0$ in the whole range $m_\mu^2 \le q^2 \le (m_B - m_D)^2$ allows for the calculation of $R(D)$, defined in (1.3).

The proper parameterization of the form factors $f_{+,0}(q^2)$ has been the subject of intense investigation, motivated in particular by the need to extrapolate the information obtained in a restricted $q^2$ region to the whole $q^2$ range. Lattice QCD calculations, for instance, are typically limited to the highest $q^2$ values. Here we consider three different parameterizations, often employed in the literature. The three parameterizations we consider share the same theoretical background, which is summarized in Sec. II A. Additional theoretical input deriving from heavy quark effective theory (HQET) calculations is considered in the CLN case, where the number of parameters is minimal, while Bourrelly-Caprini-Lellouch (BCL) adopts a functional form simpler than in the original Boyd-Grinstein-Lebed (BGL) parameterization.

### A. The BGL parameterization

The BGL parameterization was originally proposed in [7] and further developed in [8,20]. It follows from dispersion relations, analyticity, and crossing symmetry. In the case of semileptonic $B$ decays $q^2$ ranges from $m_l^2$ to $(m_B - m_D)^2$ but the form factors can be continued analytically in the $q^2$ complex plane. They have a cut at $q^2 = (m_B + m_D)^2$ and various poles corresponding to $B_c$ resonances with the appropriate quantum numbers. Adopting the notation of [8], we define

$$t = q^2 = (p-p')^2, \quad t_+ = (m_B+m_D)^2, \quad t_- = (m_B - m_D)^2,$$
$$w = \frac{m_B^2 + m_D^2 - t}{2m_B m_D}, \qquad z(w,\mathcal{N}) = \frac{\sqrt{1+w} - \sqrt{2\mathcal{N}}}{\sqrt{1+w}+\sqrt{2\mathcal{N}}},$$
$$\mathcal{N} = \frac{t_+ - t_0}{t_+ - t_-},$$

where $z(t,t_0)$ maps the $q^2$ plane on a unit disk. The parameter $t_0 < t_+$ is a free parameter which determines the point in the $q^2$ plane to be mapped onto the origin of the $z$ plane by the conformal transformation $q^2 \to z$. The two form factors $f_+$ and $f_0$ are parameterized by

$$f_+(z) = \frac{1}{P_+(z)\phi_+(z,\mathcal{N})}\sum_{n=0}^\infty a_n z^n(w,\mathcal{N}), \tag{2.4}$$

$$f_0(z) = \frac{1}{P_0(z)\phi_0(z,\mathcal{N})}\sum_{n=0}^\infty b_n z^n(w,\mathcal{N}), \tag{2.5}$$

where $P_{+,0}(z)$ are known as *Blaschke factors* and $\phi_{+,0}(z)$ as *outer functions*. They will be introduced shortly. In practice one truncates the series in (2.4) and (2.5) at some maximal power $N$. In our fits we will consider $N = 2, 3, 4$.

The main advantage of the BGL class of parameterizations is that the parameters $a_n$ of Eq. (2.4) are constrained by unitarity conditions

$$\sum_{n=0}^\infty a_n^2 < 1, \qquad \sum_{n=0}^\infty b_n^2 < 1, \tag{2.6}$$

that follow from analyticity, crossing symmetry, and quark-hadron duality. Indeed, it is possible to write dispersion relations for the correlator of two flavor-changing currents evaluated at $q^2 = 0$, where it can be computed reliably in perturbation theory because the heavy quark masses are much larger than the QCD scale. Assuming global quark-hadron duality, the dispersion relations relate integrals of the form factors at $q^2$ values outside the physical range to the perturbative calculation of the correlator. The unitarity bounds of (2.6) then follow; see [21] for a pedagogical introduction.

In the case of $B \to D$ semileptonic decays with massless leptons $z$ can vary between

$$z_{\rm min} = -\frac{\sqrt{\mathcal{N}} - 1}{\sqrt{\mathcal{N}} + 1} \quad \text{and} \quad z_{\rm max} = \frac{1 + r - 2\sqrt{\mathcal{N} r}}{1 + r + 2\sqrt{\mathcal{N} r}}.$$

Choosing $t_0 = t_-$ one has $\mathcal{N} = 1$; the range of variation of $z$ is $0 \leq z \leq \frac{(1-\sqrt{r})^2}{(1+\sqrt{r})^2} \simeq 0.0646$ and the point of zero recoil ($w = 1$) is at $z = 0$. This is the most common choice in the literature. Another convenient choice for $t_0$ is the one which leads to a symmetric $z$ range using the condition $|z_{\rm min}| = |z_{\rm max}|$. This corresponds to $t_0 = t_+ - \mathcal{N}(t_+ - t_-)$ and $\mathcal{N} = \frac{1+r}{2\sqrt{r}}$. With this prescription the maximum physical value of $|z|$ is minimized and $z_{\rm max} \simeq 0.032$. While in principle a smaller range in $z$, combined with (2.6), forces a faster convergence of the $z$ expansion, we have checked that in our case this choice brings no advantage with respect to setting $t_0 = t_-$. This is likely due to the precise lattice QCD constraints we employ. From now on we will only consider the case $t_0 = t_-$ and $z$ will always stand for $z(w, 1)$.

In our case only the transverse and longitudinal parts of the vector current correlator are relevant:

$$\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)\Pi^T(q^2) + \frac{q^\mu q^\nu}{q^2}\Pi^L(q^2) \equiv i\int d^4x e^{iqx}\langle 0|TJ^\mu(x)J^{\dagger\nu}(0)|0\rangle \tag{2.7}$$

with $J^\mu = \bar{c}\gamma^\mu b$. The longitudinal and transverse part correspond to spin 0 and spin 1, respectively. The derivatives

$$\chi^L(q^2) = \frac{\partial \Pi^L}{\partial q^2}, \qquad \chi^T(q^2) = \frac{1}{2}\frac{\partial^2 \Pi^T}{\partial (q^2)^2}$$

satisfy unsubtracted dispersion relations on which the unitarity bounds are built. The value $q^2 = 0$ is sufficiently far from the threshold region and is generally employed.

For the perturbative evaluation of $\chi^{L,T}(0)$ we update Ref. [8] using recent values of the heavy quark masses and the $O(\alpha_s^2)$ corrections computed in [22]. We neglect all contributions of condensates, which have been shown to be negligible. Several precise determinations of the bottom and charm masses have appeared in recent years; see [23] for a review. Here we first use the $\overline{\rm MS}$ values $\bar{m}_b(\bar{m}_b) = 4.163(16)$ GeV, $\bar{m}_c(3\text{ GeV}) = 0.986(13)$ GeV from [24] and $\alpha_s^{(5)}(\bar{m}_b(\bar{m}_b)) = 0.2268(23)$ and get

$$\begin{aligned} \chi^T(0) &= [5.883 + 0.552_{\alpha_s} + 0.050_{\alpha_s^2}] \times 10^{-4}\ \text{GeV}^{-2} \\ &= 6.486(48) \times 10^{-4}\ \text{GeV}^{-2}, \\ \chi^L(0) &= [5.456 + 0.782_{\alpha_s} - 0.034_{\alpha_s^2}] \times 10^{-3} \\ &= 6.204(81) \times 10^{-3}, \end{aligned} \tag{2.8}$$

where the errors reflect only the uncertainties on the input quark masses and we have neglected the small correlation between them. The effect of the $O(\alpha_s^2)$ corrections is less than 1%, but the values differ significantly from those used in [8], $\chi^T(0) = 4.42 \times 10^{-4}$ GeV$^{-2}$ and $\chi^L(0) = 4.07 \times 10^{-3}$, because of the different, more precise inputs. An alternative determination is obtained using the fit to semileptonic moments of [25] and employs the kinetic $b$ mass $m_b^{\rm kin}(1\,\text{GeV}) = 4.553(20)$ GeV, $\bar{m}_c(3\,\text{GeV}) = 0.987(13)$ GeV and $\alpha_s^{(5)}(m_b^{\rm kin}(1\text{ GeV})) = 0.2208(22)$, leading to

$$\begin{aligned} \chi^T(0) &= [4.958 + 1.059_{\alpha_s} + 0.309_{\alpha_s^2}] \times 10^{-4}\ \text{GeV}^{-2} \\ &= 6.326(51) \times 10^{-4}\ \text{GeV}^{-2}, \\ \chi^L(0) &= [5.905 + 0.564_{\alpha_s} - 0.136_{\alpha_s^2}] \times 10^{-3} \\ &= 6.332(74) \times 10^{-3}, \end{aligned} \tag{2.9}$$

where we have taken into account the correlation between $m_b$ and $m_c$ from the fit of [25]. The next-to-next-to-leading-order (NNLO) corrections amount to $+5.1$ and $-2.1\%$, respectively. Because of the off-shell nature of $\chi^{T,L}(0)$, the $O(\alpha_s)$ and $O(\alpha_s^2)$ corrections are more sizable when we employ the kinetic mass for the $b$ quark instead of the $\overline{\rm MS}$ mass; taking into account the theoretical uncertainty due to higher-order corrections, larger in the second case, Eqs. (2.8) and (2.9) are perfectly consistent. In the following, we adopt the more precise values in (2.8) as our reference.

The $1^-$ $B_c$ resonances below the $BD$ pair production threshold contribute as single particles to the unitarity sum. Their effect can be effectively seen as a reduction of $\chi^{T,L}(0)$; see [9]:

$$\tilde{\chi}^T(0) = \chi^T(0) - \sum_{n=1,2}\frac{f_n^2(B_c^*)}{M_n^4(B_c^*)}, \tag{2.10}$$

where $f_n$ are the decay constants and $M_n$ the masses of the $B_c^*$ mesons. The decay constant is strongly suppressed for $0^+$ states and we therefore neglect this contribution to the scalar channel. We likewise do not consider poles above

TABLE I. Relevant $B_c$ masses and decay constants.

| Type | Mass (GeV) | Decay constants (GeV) | Reference |
|---|---|---|---|
| $1^-$ | 6.329(3) | 0.422(13) | [23,26,27] |
| $1^-$ | 6.920(20) | 0.300(30) | [23,26,28] |
| $1^-$ | 7.020 | | [29] |
| $1^-$ | 7.280 | | [30] |
| $0^+$ | 6.716 | | [29] |
| $0^+$ | 7.121 | | [29] |

the $BD$ threshold to avoid double counting. In Table I the relevant $B_c$ masses and decay constants are presented with their sources, among which are recent lattice calculations. The masses will also be used to evaluate the Blaschke factors later on. A conservative 10% uncertainty is assigned to $f_2(B_c^*)$. Since the unitarity bounds emerge from the assumption that a single channel, or a set of channels, saturates the dispersion relation, larger values of $\chi^{T,L}$ constrain the form factors less effectively. Therefore, to be conservative we compute $\tilde{\chi}^T$ using the decay constants reduced by one standard deviation and the mass values increased by one standard deviation. The result in units of $\mathrm{GeV}^{-2}$ is

$$\tilde{\chi}^T(0) = 6.486\times 10^{-4} - \frac{(0.409)^2}{(6.332)^4} - \frac{(0.270)^2}{(6.940)^4} = 5.131\times 10^{-4}. \tag{2.11}$$

Another ingredient in Eqs. (2.4) and (2.5) is the Blaschke factors

$$P_+(z) = \prod_{P_+=1}^{3} \frac{z - z_{P_+}}{1 - z z_{P_+}}, \qquad P_0(z) = \prod_{P_0=1}^{2} \frac{z - z_{P_0}}{1 - z z_{P_0}}, \tag{2.12}$$

where $z_P$ is defined as

$$z_P = \frac{\sqrt{t_+ - m_P^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - m_P^2} + \sqrt{t_+ - t_0}},$$

where $m_P$ represents the location of a $B_c$ narrow resonance. The product is extended to all the $B_c$ resonances below the $BD$ threshold with the appropriate quantum numbers ($1^-$ for $P_+$ and $0^+$ for $P_0$). The Blaschke factors remove the subthreshold poles from the form factors, making the form factors analytic for all $q^2$ values below the cut.

Finally, the outer functions reflect the way in which the form factors enter the dispersive integral, which depends on the helicity amplitude they belong. Their normalization depends on $\chi^{T,L}$ because we want to have the unitarity bounds in the simplest form (2.6). The outer functions $\phi_{+,0}$ are given by

$$\phi_+(z) = k_+ \frac{(1+z)^2\sqrt{1-z}}{[(1+r)(1-z) + 2\sqrt{r}(1+z)]^5}, \qquad k_+ = \frac{8r^2}{m_B}\sqrt{\frac{8n_I}{3\pi\tilde{\chi}^T(0)}} \simeq 12.43, \tag{2.13}$$

$$\phi_0(z) = k_0 \frac{(1-z^2)\sqrt{1-z}}{[(1+r)(1-z) + 2\sqrt{r}(1+z)]^4}, \qquad k_0 = r(1-r^2)\sqrt{\frac{8n_I}{\pi\chi^L(0)}} \simeq 10.11, \tag{2.14}$$

where $n_I$ is a factor that simply counts the number of massless spectator quarks. Although SU(3) breaking appears to be small in form factor calculations, we prefer to be conservative and use $n_I = 2.6$. Replacing (2.12)–(2.14) in Eqs. (2.4) and (2.5) we obtain the BGL parameterizations of $f_+$ and $f_0$. There appears to be some confusion on $k_0$ in the literature (see e.g. [16,18]), possibly generated by the unusual definition of $f_0$ in [8]. We stress that the precise definition and inclusion of both $P_{+,0}$ and $\phi_{+,0}$ is instrumental to using the unitarity bound on the sum of the squared coefficients of the $z$ expansion. Without this bound there is no difference between the $z$ expansion and any other power expansion of the form factors.

### B. CLN parameterization

The CLN parameterization was proposed in [9] and has been extensively used in the literature. It is also based on dispersion relations and unitarity but it additionally exploits HQET to reinforce the unitarity bounds. Indeed, the form factors of the two-meson states contributing to the two-point function are related by heavy quark symmetry and in the heavy quark limit either vanish or are proportional to the Isgur-Wise function [8]. Reference [9] also includes $\mathcal{O}(1/m)$ heavy quark symmetry breaking corrections, computed with input from light-cone sum rules, and leading short-distance corrections to these relations. We will describe in more detail the method in Sec. II D.

The reinforced unitarity bounds allow Ref. [9] to establish approximate relations between the slope and the higher power coefficients of the reference form factor $f_+$ and to provide simplified formulas valid within $\approx 2\%$. For instance, our form factors of interest are expressed in terms of two parameters only:

$$f_+(z) \simeq f_+(0)[1 - 8\rho_1^2 z + (51\rho_1^2 - 10)z^2 - (252\rho_1^2 - 84)z^3)], \tag{2.15}$$

$$\frac{f_0(z)}{f_+(z)} \simeq \left(\frac{2\sqrt{r}}{1+r}\right)^2 \frac{1+w}{2} 1.0036[1 - 0.0068 w_1 + 0.0017 w_1^2 - 0.0013 w_1^3], \tag{2.16}$$

where $w_1 = w - 1$. Notice in particular that the ratio $f_0/f_+$ is fixed by the NLO HQET calculation implemented in [9]. All other $B^{(*)} \to D^{(*)}$ form factors are similarly expressed as $f_+(z)$ times a ratio computed at NLO in HQET. One should bear in mind that the heavy mass expansion here is an expansion in $1/m_c$ and therefore all form factors in this parameterization are subject, in principle, to $\mathcal{O}(1/m_c^2) \sim$ 5%–10% corrections. Indeed, the ratio of the $B \to D^*$ and $B \to D$ form factors at zero recoil is 0.948 at NLO in HQET [9], while the most precise lattice calculations lead to 0.860(14) [10,16]. However, as long as the CLN parameterization is used to describe the shape of a single form factor, like in (2.15), it provides a simple and effective parameterization, unless of course the experimental or theoretical constraints reach the ~1% precision.

### C. The BCL parameterization

The BCL parameterization [31] was proposed to overcome problems that appear due to the truncation of the BGL expansion. When the BGL expansion is truncated at some finite power $N$, the form factor develops an unphysical singularity at the threshold $t_+$ and behaves at large $|q^2|$ in contradiction with perturbative QCD scaling. While these problems are related to the behavior of the form factor at values of $q^2$ much larger than those relevant for $B \to D\ell\nu$ and are therefore likely to be irrelevant in the present context, the BCL parameterization offers a simple alternative to BGL that avoids these potential shortcomings. The two form factors of interest are given by

$$f_+(q^2) = \frac{1}{1 - q^2/M_+^2} \sum_{k=0}^{N} a_k \left[ z^k - (-1)^{k-N-1} \frac{k}{N+1} z^{N+1} \right], \tag{2.17}$$

$$f_0(q^2) = \frac{1}{1 - q^2/M_0^2} \sum_{k=0}^{N} b_k z^k, \tag{2.18}$$

where $M_{+,0}$ are the masses of the two closest $B_c$ resonances in the $+, 0$ channels; see Table I. The $z^{N+1}$ terms in (2.17) ensure the proper behavior of the form factor at $q^2 \approx t_+$. There is no point in introducing additional pole terms for resonances that lie even further. The unitarity bounds we have considered above can be mapped onto the BCL parameters as shown in [31]. As they assume a more complicated form, we will only consider the weak bounds for the BCL parameters.

### D. Strong unitarity constraints

The unitarity bounds of Eq. (2.6) assume that a single hadronic channel, in our case $BD$, saturates the bound; we can label them *weak unitarity bounds*. Of course there are a number of additional two-body channels $(BD^*, B^*D, B^*D^*, \Lambda_b\Lambda_c, \ldots)$ with the right quantum numbers, as well as higher multiplicity channels, that give positive contributions to the absorptive part to the two-point function and can strengthen the unitarity bound on the coefficients of the $BD$ form factors.

In the case of the four states $\bar{B}^{(*)}\bar{D}^{(*)}$, one can use heavy quark symmetry to connect the form factors of the various channels. The implementation of these relations in the unitarity conditions is outlined in [8]: only amplitudes of fixed spin parity enter each dispersion relation, leading to the *strong* unitarity condition

$$\sum_{i=1}^{H} \sum_{n=0}^{\infty} b_{in}^2 \leq 1. \tag{2.19}$$

Here all helicity amplitudes $i = 1 \ldots H$ for processes involving $\bar{B}^{(*)}\bar{D}^{(*)}$ with the right quantum numbers must be included.

We follow here the approach proposed in [8] to derive strong unitarity bounds on the coefficients of $f_+$, but we include short-distance and $1/m$ corrections to the heavy quark limit as done in [9]. One can compute strong constraints for the coefficients of $f_0$ as well, but they would play a marginal role in our analysis. We will therefore use only the weak unitarity constraint for the coefficients of the scalar form factor.

Considering the vector $(1^-)$ form factors for $B^{(*)}D^{(*)}$ states, there are seven helicity amplitudes, $H = 7$. Each form factor $F_i$ can be put in the general form

$$F_i = \tilde{f}_i \sum_n b_{\rm in} z^n, \tag{2.20}$$

where $\tilde{f}_i(z) = 1/P_i(z)\phi_i(z)$ are known functions. If $b_{1n} = a_n$ are the coefficients of $f_+(z)$, it is possible to rearrange the $z$ expansion of each form factor as

$$\sum_n b_{in} z^n = \sum_n a_n z^n c_i(z), \tag{2.21}$$

where $c_i(z) = F_i\tilde{f}_+/(f_+\tilde{f}_i)$ and only the $f_+$ coefficients appear explicitly. In order to use Eq. (2.21) to obtain a unitarity condition involving only the $a_n$, an explicit approximation for the $c_i(z)$ is required; i.e. information on the other form factors is needed. Once the $c_i(z)$ are known, their $z$ expansion allows for the determination of the coefficients $b_{\rm in}$ to be employed in (2.19). Of course, the maximum reached by the index $n$ in (2.19) coincides with the maximum power of $z$ we employ in (2.4), (2.5), namely $N$.

In Ref. [8] the exact heavy quark limit in used in order to fix the functions $c_i(z)$. Here we evaluate them incorporating the $1/m$ heavy quark symmetry breaking and short-distance corrections as done in [9]. In the notation of [8] the form factors involved are $F_i = (f_+, g, \hat{g}, V_{++}, V_{+0}, V_{0+}, V_{00})$. They are related to those in [9] by

$$f_+ = \frac{m_B + m_D}{2\sqrt{m_B m_D}} V_1, \quad g = \frac{V_4}{\sqrt{m_B m_{D^*}}}, \quad \hat{g} = -\frac{V_5}{\sqrt{m_{B^*} m_D}},$$
$$V_{+0} = \frac{V_6}{\sqrt{m_{B^*} m_{D^*}}}, \quad V_{++} = \frac{V_7}{\sqrt{m_{B^*} m_{D^*}}},$$
$$V_{0+} = -\frac{m_{B^*} + m_{D^*}}{\sqrt{2 m_{B^*} m_{D^*}}} V_2, \quad V_{00} = \frac{m_{B^*} + m_{D^*}}{2\sqrt{m_{B^*} m_{D^*}}} V_3.$$

Using these relations and the ratios $V_i/V_1$ computed in [9] at NLO in HQET and given in their Appendix, it is easy to find the relevant $c_i(z)$. Indeed, the ratios $\tilde{f}_+/\tilde{f}_i$ are easily computed from [8]. Depending on the specific form factor either three or four poles have to be subtracted with the Blaschke factors because of the different $t_+$ thresholds.

The strong unitarity bounds are finally obtained using the values in Tables I and II:

(i) $N = 2$
$442.82a_0^2 - 101.619a_0a_1 + 34.947a_1^2 - 127.668a_0a_2 + 33.234a_1a_2 + 16.4754a_2^2 \leq 1.$

(ii) $N = 3$
$1707.54a_0^2 + 1299.57a_0a_1 + 442.82a_1^2 - 356.01a_0a_2 - 101.62a_1a_2 + 34.947a_2^2 - 206.767a_0a_3 - 127.668a_1a_3 + 33.234a_2a_3 + 16.475a_3^2 \leq 1.$

We do not display the longer $N = 4$ expression. These strong unitarity constraints will be used to fit the experimental and lattice results in the BGL parameterization.

We have also considered adding the $\Lambda_b\Lambda_c$ channel to the unitarity bounds, even though heavy quark symmetry does not relate its form factors to those of $BD$ in a direct way. Indeed a lattice QCD calculation of the $\Lambda_b \to \Lambda_c \ell\nu$ form factors has recently appeared [32]. Unfortunately, their precision is not yet sufficient to evaluate their contribution to the unitarity sum in a useful way.

The derivation of the strong unitarity bounds makes essential use of the NLO HQET relations between the form factors, but we have seen that subsubleading $\mathcal{O}(1/m_c^2)$ effects can be sizable. In order to estimate their effect we randomly vary the coefficients $b_{in}$ which appear in (2.19) in such a way that the overall shift in the ratio between form factors with respect to the expressions given in Appendix A of [9] is less than 8%. Equation (2.19) precludes any coefficient $b_{in}$ from being too large. As the $b_{in}$'s in turn depend on the expansion coefficients $a_n$ of $f_+$, the largest $b_{in}$ variations turn out to be incompatible either with the constraints on $a_n$ we have from lattice and experiment or with Eq. (2.19), degrading significantly the quality of the constrained fit, in a way which becomes stronger for higher $N$. Indeed, unitarity is very effective in constraining the higher derivatives terms of the form factors.

We also observe that the HQET expressions for $F_i/f_+$ have a $z$ expansion characterized by a rapid apparent convergence with $O(1)$ coefficients. For instance,

$$\frac{F_2(z)}{f_+(z)} \approx 0.329(1 - 1.14z - 2.38z^2 - 3.61z^3 + \ldots), \tag{2.22}$$

where all the powers of $z$ originate from NLO corrections to the leading HQET result. Limiting ourselves to random variations which satisfy the previous requirement and preserve the order of magnitude (not necessarily the sign) of the $z$-expansion coefficients in $F_i/f_+$, we have verified that (i) employing the HQET relations in the derivation of the strong bounds leads to global fits with nearly optimal $\chi^2$ (i.e only very few and small variations have smaller minimum $\chi^2$); (ii) the fitted value of $|V_{cb}|$ is very (fairly) insensitive to changes in the HQET relations in the $N = 3(4)$ cases, with a marked preference for a slight increase. We will therefore employ the strong unitarity bounds as we have derived them, without assigning any uncertainty. As will be shown in the next section, the difference between results obtained with weak and strong bounds is always minor. However, there is so much missing in the weak bounds that it seems preferable to use our imperfect version of the strong bounds for actual fits.

TABLE II. Mass values employed in the paper.

| Mass | Value (GeV) |
|---|---|
| $m_B$ | 5.27942 |
| $m_D$ | 1.86723 |
| $m_\mu$ | 0.1057 |
| $m_\tau$ | 1.7768 |
| $m_{B^*}$ | 5.325 |
| $m_{D^*}$ | 2.009 |

## III. INPUTS

### A. Experimental data

Most of the $B \to D\ell\nu$ analyses by Aleph, Cleo, *BABAR* and Belle were performed assuming the CLN parameterization and their results are given in terms of $\rho_1^2$ and $\eta_{\mathrm{ew}}\mathcal{G}(1)|V_{cb}|$, where $\mathcal{G}(1) = 2\sqrt{r}/(1+r)f_+(0)$ is the zero-recoil form factor; see [11] for a recent average. In the following we perform a fit to the differential $w$ distribution and therefore use only results provided by Belle and *BABAR*.

The *BABAR* Collaboration published their results on the $w$ distribution in [33]. The $w$ spectrum is divided into ten bins of width 0.06, and the results are expressed in terms of the average $|V_{cb}|\mathcal{G}(w)$ in each bin. Since this entails non-negligible finite bin size effects, we have reexpressed the data in terms of $\Delta\Gamma$ for each bin. As *BABAR* analysis is tied up with the CLN parameterization, only the statistical uncertainties are provided. *BABAR* claims a 3.3% systematic error at small $w$, and we have extended this to the entire $w$ range, with 100% correlation between different bins as recommended to us [34]. An important point is that *BABAR*'s last bin, $1.54 \leq w \leq 1.60$, extends beyond the physical end point $w_{\max} \simeq 1.5905$. This has to be taken properly into account and has a non-negligible effect on the fit.

Concerning Belle, we use the results of a recent analysis [18] of $B \to D\ell\nu$, which provides the $w$ spectrum with full statistical and systematic errors, and correlations. The $w$ range is divided into ten bins as in *BABAR* analysis, but the last one stops at $w_{\max}$. The $B$ and $D$ mass values given in Table II are those employed in Ref. [18] and reflect the relative weight of charged and neutral $B$ mesons in their sample. We assume that these values are suitable for *BABAR* analysis as well and neglect their uncertainty. The experimental data points employed in the fit can be seen in Fig. 1, where they are shown as measurements of $f_+(z)$ after normalizing them to the fitted $|V_{cb}|$.

### B. Lattice QCD calculations

While the experimental results are more precise at large recoil (small $q^2$), lattice QCD calculations of the form factors are performed close to zero recoil. They have made substantial progress in the last two decades and the most recent calculations of $f_{+,0}$ [16,17] have reached a high accuracy both at zero recoil (0.8% and 4%, respectively) and beyond. In fact, they represent the first unquenched calculations of these form factors performed at values of $w$ different from 1, which significantly reduces the uncertainty of the extrapolation from the $w$ region where most data are taken.

The FNAL/MILC Collaboration has presented results for both $f_+(w)$ and $f_0(w)$ in terms of synthetic data points at $w = 1.00$, 1.08 and 1.16 [16], with complete statistical and systematic uncertainties and correlations.

The HPQCD Collaboration [17] instead present their results for $f_+(w)$ and $f_0(w)$ in terms of a BCL parameterization with only the closest $B_c$ pole, whose coefficients are provided with uncertainty and correlations. Since their simulations extend to a maximal value of $z = 0.013$, corresponding to $w \approx 1.11$, we extract from their parameterization synthetic data for $f_{+,0}$ at $w = 1.00$, 1.06 and 1.12. HPQCD $f_{+,0}$ data points have a significantly larger error and are more correlated than those of [16]. While HPQCD individual points are in good agreement with those by FNAL/MILC, there is a mild tension between their $f_+$ slopes at zero recoil: from [17] we find $df_+/dw|_{w=1} = -1.29(11)$, while FNAL has $df_+/dw|_{w=1} = -1.42(4)$, showing a marginal discrepancy that may require further consideration.

We assume no correlation between FNAL/MILC and HPQCD results, although the two calculations both employ some of the MILC gauge ensembles. In fact, the overlap between the sets of ensembles employed by the two collaborations is limited. Moreover, the statistical errors play a minor role in the HPQCD error budget, which is dominated by theoretical systematics related to the use of nonrelativistic QCD. The lattice data points employed in the fit can be seen in Fig. 1.

Because of their limited accuracy, we do not include previous lattice [35,36] and light-cone sum rules [37] results on the form factors. We also do not include the HQE result, partially based on BPS symmetry, $\mathcal{G}(1) = 1.04(2)$ [38].

## IV. RESULTS AND DISCUSSION

### A. Fits to both lattice and experimental data

In this section we report the results of several fits performed with the three parameterizations we have discussed above. To gauge the stability of the fits for different $N$, we compare the $N = 2$, 3, 4 cases. While we normally include all the inputs listed in Secs. III A and III B, we have also performed BGL fits excluding a specific set of inputs: they clarify their role in the fit. The central values and standard deviation of $|V_{cb}|$ and $R(D)$ obtained in each case are reported in Table III together with the minimum $\chi^2$. In the following we discuss the main features of the results and clarify a few technical points.

The first three fits employ the BGL parameterization and all the inputs. In the case $N = 2$ the absolute minimum of $\chi^2$ is always consistent with both weak and strong unitarity bounds. For $N = 3$, 4 the absolute minimum lies outside both the weak and strong unitarity bounds. We therefore look for the constrained minimum imposing the strong unitarity bounds, which modifies slightly the fitted values of $|V_{cb}|$ and $R(D)$ and complicates the error analysis, giving rise to asymmetric uncertainties, which we evaluate using $\Delta\chi^2 = 1$. Weak and strong unitarity constraints lead to very similar results, with $|V_{cb}|$ just $0.05 \times 10^{-3}$ higher in the second case for $N = 4$. For our final result we adopt the $N = 4$ BGL fit, which is the one where the functional form of $f_{+,0}$ has the maximum flexibility.

In all cases the fits have good quality and there is a remarkable stability with respect to the value of $N$, which

TABLE III. Fits using different parameterizations and inputs. See text for explanations.

| Exp data | Lattice data | $N$,par | $10^3 \times \lvert V_{cb}\rvert$ | $\chi^2$/d.o.f. | $R(D)$ |
|---|---|---|---|---|---|
| All | All | 2,BGL | 40.62(98) | 22.1/26 | 0.302(3) |
| All | All | 3,BGL | 40.47(97) | 18.2/24 | 0.299(3) |
| All | All | 4,BGL | **40.49(97)** | 19.0/22 | **0.299(3)** |
| Belle | All | 3,BGL | 40.92(1.12) | 11.6/14 | 0.300(3) |
| *BABAR* | All | 3,BGL | 40.11(1.55) | 12.6/14 | 0.301(4) |
| All | FNAL | 3,BGL | 40.17(1.05) | 10.4/18 | 0.293(4) |
| All | HPQCD | 3,BGL | $40.51^{+1.82}_{-1.71}$ | 10.1/18 | 0.299(7) |
| All | All | CLN | 40.85(95) | 77.1/29 | 0.305(3) |
| All | $f_+$ only | CLN | 40.33(99) | 20.0/23 | 0.305(3) |
| All | All | 2,BCL | 40.49(98) | 18.2/26 | 0.299(3) |
| All | All | 3,BCL | 40.48(96) | 18.2/24 | 0.299(3) |
| All | All | 4,BCL | 40.48(97) | 17.9/22 | 0.299(3) |

TABLE IV. Coefficients of the form factors in the BGL fits to all data.

| $N$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ |
|---|---|---|---|---|---|
| 2 | 0.01566(11) | −0.0342(31) | −0.090(22) | | |
| 3 | 0.01565(11) | −0.0353(31) | $-0.043(^{+21}_{-35})$ | $0.194(^{+19}_{-16})$ | |
| 4 | 0.01564(11) | −0.0353(30) | $-0.044(^{+22}_{-14})$ | $0.111(^{+51}_{-111})$ | $-0.20(^{+20}_{-8})$ |

| $N$ | $b_0$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ |
|---|---|---|---|---|---|
| 2 | 0.07935(58) | −0.205(14) | −0.23(10) | | |
| 3 | 0.07932(58) | $-0.214(^{+15}_{-14})$ | $0.17(^{+10}_{-25})$ | $-0.958(^{+1060}_{-2})$ | |
| 4 | $0.07929(^{+97}_{-93})$ | −0.210(14) | $0.09(^{+12}_{-14})$ | $-0.967(^{+396}_{-11})$ | $0.08(^{+76}_{-71})$ |

could not be achieved without the unitarity bounds. Thanks to the unitarity bounds, the error on $R(D)$ is reduced by 30% in the case of $N = 3$ and by 50% in the case of $N = 4$, while that on $|V_{cb}|$ is only slightly reduced for $N \geq 3$. The coefficients $a_i$, $b_i$ of the form factors $f_{+,0}$ obtained in these three BGL fits are shown in Table IV with their errors. In Table V we provide the correlation matrix for the case $N = 2$, the only case for which it can be properly defined. In the case $N = 4$ the $1\sigma$ uncertainties for $f_{+,0}(z)$ are

$$\delta f_+(z) \simeq 0.00854 + 0.0388z + 0.26z^2,$$
$$\delta f_0(z) \simeq 0.0065 + 0.012z + 1.2z^2. \tag{4.1}$$

These uncertainties are very close to those we obtain from the $N = 2$, 3 fits.

We also present two fits performed with the CLN parameterization. The first one includes all experimental and lattice data and has a very low $p$ value, $3 \times 10^{-6}$. This is due to the fact that in the CLN parameterization the ratio $f_+/f_0$ is fixed to the HQET relation (2.16), which is in striking contrast with the most precise lattice evaluations: Ref. [16] finds $f_+(0)/f_0(0) = 0.753(3)$ at zero recoil, while (2.16) implies 0.775. The two values differ only by 3%, which is of the expected order of magnitude for higher-order corrections to the CLN relation. Clearly, lattice calculations are getting too precise to use CLN without a proper uncertainty. A second CLN fit which excludes all $f_0$ lattice determinations has a good quality, comparable to that of the BGL fits.

TABLE V. Correlation matrix for the $N = 2$ fit. Due to the constraint $f_+ = f_0$ at $q^2 = 0$, the coefficient $b_0$ is given by $4.99a_0 + 0.32a_1 + 0.021a_2 - 0.065b_1 - 0.004b_2$.

| | $a_0$ | $a_1$ | $a_2$ | $b_1$ | $b_2$ |
|---|---|---|---|---|---|
| $a_0$ | 1 | 0.304 | −0.294 | 0.212 | 0.161 |
| $a_1$ | 0.304 | 1 | −0.422 | 0.747 | 0.190 |
| $a_2$ | −0.294 | −0.422 | 1 | −0.034 | 0.148 |
| $b_2$ | 0.212 | 0.747 | −0.034 | 1 | −0.210 |
| $b_2$ | 0.161 | 0.190 | 0.148 | −0.210 | 1 |

With the BCL parameterization we have performed fits to all inputs for $N = 2$, 3, 4, with weak unitarity constraints only. The results are perfectly consistent with those obtained in the BGL parameterization and are very stable for increasing $N$.

Our fits are in good agreement with recent analyses, if one takes into account the different inputs. Belle analysis [18] employs the same lattice data as we do and finds (BGL, $N = 3$) $\eta_{\rm ew}|V_{cb}| = 41.10(1.14) \times 10^{-3}$. Using $\eta_{\rm ew} = 1.0066$ as we do, one gets $|V_{cb}| = 40.83(1.13) \times 10^{-3}$, well compatible with the result of our Belle-only fit. The HPQCD Collaboration reports $|V_{cb}| = 40.2(1.7)(1.3) \times 10^{-3}$ [17] based on *BABAR* data only and using a different $\eta_{\rm ew}$. If we repeat the fit under similar conditions we get $|V_{cb}| = 40.1(2.2) \times 10^{-3}$. The FNAL-MILC Collaboration quotes $|V_{cb}| = 39.6(1.7)(0.2) \times 10^{-3}$ [16], based on *BABAR* data only and using $\eta_{\rm ew} = 1.011(5)$. Under the same hypotheses we get $|V_{cb}| = 39.7(1.7) \times 10^{-3}$. A fit with lattice, *BABAR*, and preliminary Belle data, presented in [39], is again consistent with our results after taking into account the different inputs.

It might be interesting to compare our results with those one obtains using the HFAG average $\eta_{\rm ew}\mathcal{G}(1)|V_{cb}| = 42.65(72)(1.35) \times 10^{-3}$ [11] and the $N = 4$ fit value of $f_+(0)$, corresponding to $\mathcal{G}(1) = 1.0557(78)$. We get $|V_{cb}| = 40.13(1.47) \times 10^{-3}$, which is consistent with but less precise than our final value. This is clearly not surprising because we include new additional information. One should keep in mind that the data averaged by HFAG are the result of CLN extrapolation.

To gauge the impact of nonzero recoil lattice results in the analysis, we perform a fit without all the lattice points at $z \neq 0$: the result is $|V_{cb}| = 39.6(^{1.7}_{2.0}) \times 10^{-3}$ using BGL with $N = 3$. Clearly, nonzero recoil lattice data are very important both for the uncertainty and the central value. If we instead employ CLN, we obtain $|V_{cb}| = 40.0(1.1) \times 10^{-3}$.

The form factors of the $N = 4$ BGL fit are shown in Fig. 1 together with their $1\sigma$ error bands and the lattice input data. We also show bin-average values for $f_+$ obtained from the experimental data, with normalization fixed by the fitted value of $|V_{cb}|$.

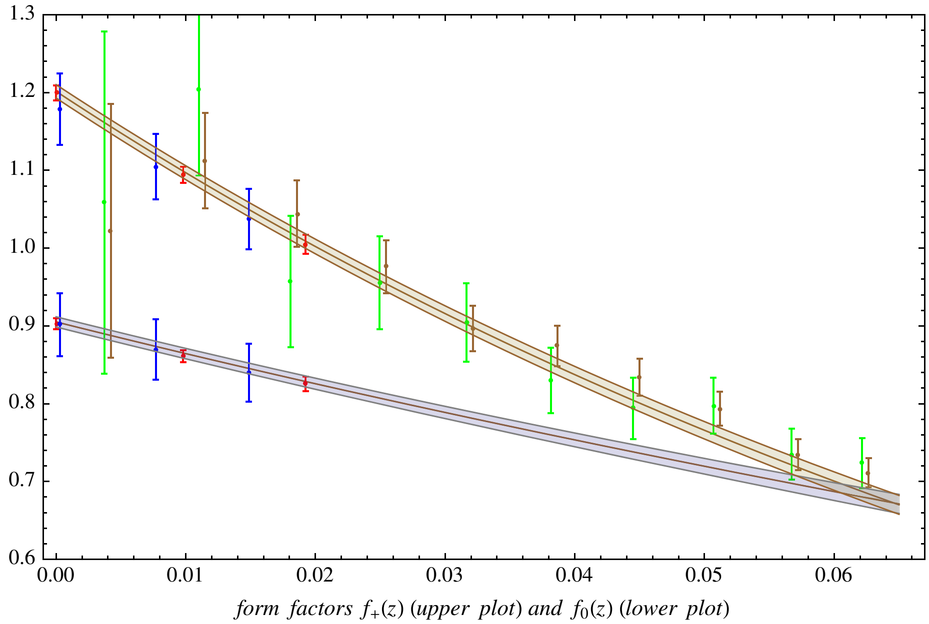


FIG. 1. Form factors in the $N = 4$ fit with data points. FNAL/MILC synthetic data are shown in red, HPQCD in blue, Belle data in brown, and *BABAR* in green.

### B. Fits to lattice results only

We have seen that the difference between using weak and strong unitarity constraints is relatively small in our fits. One might think this is going to change in a case where we do not have data over the whole spectrum and where extrapolation errors become important. To illustrate such a case, we have performed a fit to lattice results only, shown in Fig. 2. Here we include the same $f_+$ and $f_0$ results by the MILC-FNAL and HPQCD Collaborations we have employed in the global fits. The plot shows the $1\sigma$ error band for $f_+$ in the case of the $N = 4$ BGL fit, using strong and weak constraints. The band obtained using strong constraints is up to 25% narrower than the one obtained using the weak constraints. These bands can be compared with similar ones given in Refs. [16,17], keeping in mind that ours is a combined fit. We also note in passing that the implementation of weak unitarity constraints using Gaussian priors, as done for instance in [16], leads to overestimate the width of the band (by up to 30% in the $N = 4$ case with only Fermilab results). In conclusion, while for $N > 2$ it is essential to use unitarity constraints, the gain from using strong rather than weak constraints is not significant in our fits. However, this is not a general statement and the issue should be reconsidered case by case.

## V. SUMMARY

We have reexamined the form factor parameterizations for $B \to D\ell\nu$ in view of recent theoretical and experimental

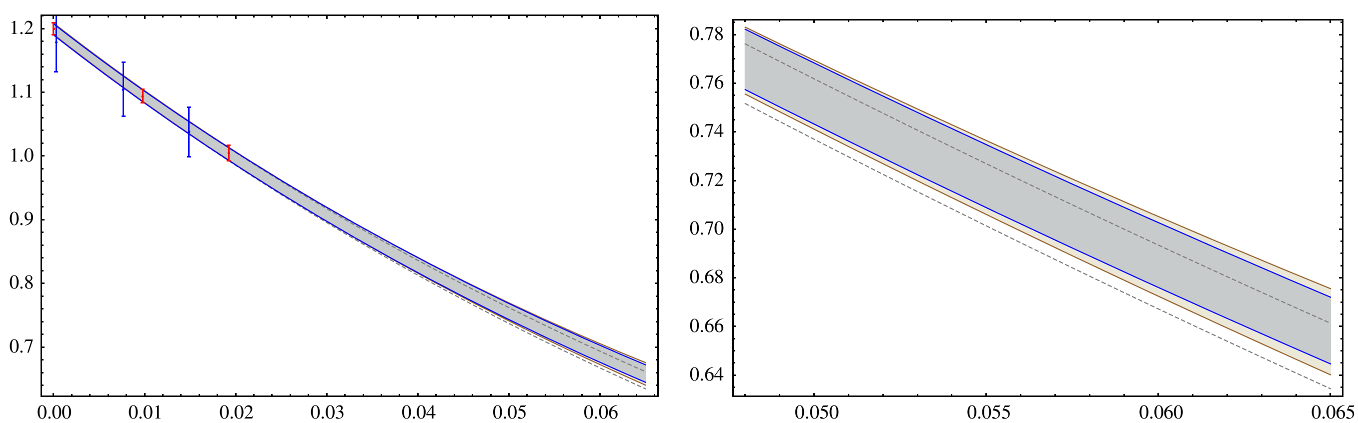


FIG. 2. Form factor $f_+(z)$ in the $N = 4$ BGL fit to lattice data for $f_{+,0}(z)$ with weak (brown band) and strong (gray band) unitarity constraints. The $N = 2$ band (independent of unitarity constraints) is shown in dashed lines for comparison. FNAL/MILC synthetic data are shown in red and HPQCD in blue. On the right, enlarged detail of the tail.

results. After updating the unitarity constraints to $\mathcal{O}(\alpha_s^2)$ with recent quark mass values, we have discussed the strong unitarity bounds, which can improve the precision of the form factors and lead to better determinations of $|V_{cb}|$ and $R(D)$. In our analysis, using strong rather than weak unitarity bounds does not modify significantly the results of the fit to lattice and experimental results. In the future, it might be possible to implement strong unitarity bounds using lattice calculations of different form factors, rather than HQET approximations only, and to perform global fits to experimental and lattice data for different channels (e.g. $B \to D, B \to D^*$, etc., and even $\Lambda_b \to \Lambda_c$).

We have considered the BGL, CLN, and BCL parameterizations; they all yield consistent results for $|V_{cb}|$. However the CLN parameterization, which has played a useful role in the past, may no longer be adequate to cope with the present accuracy of lattice calculations. BGL and BCL are valid alternatives and their fits are almost identical. In both cases, the analysis should be performed with increasing $N$, properly including the unitarity constraints. Our final result for $|V_{cb}|$,

$$|V_{cb}| = 40.49(97) \times 10^{-3}, \tag{5.1}$$

has a 2.4% error and can be improved by more precise lattice calculations and by new measurements of the differential decay rate. It is fair to stress that the level of precision in (5.1) is mostly due to the high precision FNAL-MILC results, which makes it also urgent to have alternative calculations at the same level of accuracy. Concerning experiment, even before Belle-II data are available, the old *BABAR* data could be usefully reexamined using the latest tagging techniques and untying them from the CLN formulas. Our result (5.1) is compatible with but less precise than both the exclusive $V_{cb}$ from $B \to D^* \ell \nu$ in Eq. (1.2) and the inclusive one of Eq. (1.1).

Since the extraction of $|V_{cb}|$ from experimental data for $B \to D^* \ell \nu$ relies on the CLN parameterization, its uncertainty could be underestimated: this decay mode depends on three form factors, whose behavior at $w \neq 1$ has been parameterized using CLN expressions, without accounting for theoretical uncertainties. However, one should keep in mind that the $w$ spectrum of $B \to D^* \ell \nu$ is measured rather precisely over the whole physical region (unlike that of $B \to D \ell \nu$) and that the CLN expressions fit it well. The extrapolation to the zero-recoil point is therefore a small effect which is unlikely to change $|V_{cb}|$ significantly. While the $V_{cb}$ conundrum persists, a new player ($B \to D \ell \nu$) has entered the game.

Our precise fit to both form factors $f_+$ and $f_0$ has allowed us to compute the ratio of tau to muon semileptonic decays $R(D)$ in the SM. Our result is

$$R(D) = 0.299 \pm 0.003,$$

which differs $2\sigma$ from the HFAG average of *BABAR* and Belle measurements [11]

$$R(D)_{\rm exp} = 0.397 \pm 0.040 \pm 0.028.$$

Our SM determination of $R(D)$ is the most precise so far and is in excellent agreement with other recent estimates: the HPQCD Collaboration [17], without recourse to experimental data, reports 0.300(8), while FNAL-MILC [16], using *BABAR* data only, finds 0.299(11). Older analyses such as those of Refs. [40,41] give consistent values.

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