Dissociation of heavy quarkonium in hot QCD medium in a quasiparticle model

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Following a recent work on the effective description of the equations of state for hot QCD obtained from a hard thermal loop expression for the gluon self-energy, in terms of the quasigluons and quasiquarks and antiquarks with respective effective fugacities, the dissociation process of heavy quarkonium in hot QCD medium has been investigated. This has been done by investigating the medium modification to a heavy quark potential. The medium-modified potential has a quite different form (a long-range Coulomb tail in addition to the usual Yukawa term) in contrast to the usual picture of Debye screening. The flavor dependence binding energies of the heavy quarkonia states and the dissociation temperature have been obtained by employing the Debye mass for pure gluonic and full QCD case computed employing the quasiparticle picture. Thus, estimated dissociation patterns of the charmonium and bottomonium states, considering Debye mass from different approaches in the pure gluonic case and full QCD, have shown good agreement with the other potential model studies.

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I. INTRODUCTION

The problem of dissociation of bound states in a hot QCD medium is of great importance in heavy ion collisions as it provides evidence for the creation of the quark-gluon plasma there [1]. Matsui and Satz [2] proposed J/ψ suppression caused by the Debye screening by the quark-gluon plasma (QGP) as an important signature to reaffirm its formation in heavy ion collisions. The physical understanding of the quarkonium dissociation within a deconfined medium has undergone some definite refinements in the last couple of years [3–7]. Since the heavy quark and antiquark in a quarkonia state are bound together by almost static (off-shell) gluons, the issue of their dissociation boils down to how the gluon self-energy behaves at high temperatures. It has been noticed that the gluon self-energy has both real and imaginary parts [8]. Note that the real part leads to the Debye screening, while the imaginary part leads to Landau damping and gives rise the thermal width to the quarkonia.

The fate of quarkonia at zero temperature can be understood in terms of nonrelativistic potential models (as the velocity of the quarks in the bound state is small, $v \ll 1$) [9] using the Cornell potential [10]. Further, the physics of the fate of a given quarkonium state in the QGP medium, is encoded in its spectral function [11,12]. Therefore, following the temperature behavior of the spectral function, theoretical insight into the quarkonium properties at finite temperature can be made. There are mainly two lines of theoretical approaches to determine quarkonium spectral functions *viz.* the potential models [13–16] which have been widely used to study quarkonia states (their applicability at finite temperature is still under scrutiny), and the lattice QCD studies [17,18] which provide the reliable way to determine spectral functions, but the results suffer from discretization effects and statistical errors and, thus, are still inconclusive. These two approaches show poor matching as far as their predictions are concerned. None of these two approaches leads towards a complete framework to study the properties of quarkonia states at finite temperature. However, some degree of qualitative agreement was still achieved for the S-wave correlators. In contrast, the finding was somehow ambiguous for the P-wave correlators. Additionally, the temperature dependence of the potential model was even qualitatively different from the lattice one. Refinement in the computations of the spectral functions has recently been done (including the zero modes in both the S and P channels) [19,20]. It has been observed that these contributions cure most of the previously observed discrepancies with lattice calculations. This supports the fact that the employment of potential models at finite temperature can serve as an important tool to complement lattice studies. The potential model can actually be derived directly from QCD as an effective field theory (potential nonrelativistic QCD-pNRQCD) by integrating out modes above the scales m_Q and then $m_Q v$, respectively [21–23].

Note that the potential models have served as useful approaches while exploring the physics of heavy quarkonia since the discovery of J/ψ [21,24]. It indeed provides a useful way to examine quarkonium binding energies, quarkonium wave functions, reaction rates, transition rates,

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and decay widths. It further allows the extrapolation to the region of high temperatures by expressing screening effects reflecting on the temperature dependence of the potential. The effects of the dynamics of quarks on the stability of quarkonia can be studied by using potential models extracted from thermodynamic quantities that are computed in full QCD. At high temperatures, the deconfined phase of QCD exhibits screening of static color-electric fields [25]; it is, therefore, expected that the screening will lead to the dissociation of quarkonium states. After the success at zero temperature while predicting hadronic mass spectra, the potential model descriptions have also been applied to understand quarkonium properties at finite temperature.

Note that the production of J/ψ and Υ mesons in hadronic reactions occurs, in part, via the production of higher excited $c\bar{c}$ (or $b\bar{b}$) states and their decay into respective ground states. Since the lifetime of different quarkonium states is much larger than the typical lifetime of the medium produced in nucleus-nucleus collisions, their decay occurs almost completely outside the produced medium [26,27]. This is crucial due to the fact that the produced medium can be probed not only by the ground state quarkonium but also by different excited quarkonium states. Since different quarkonia states have different sizes and binding energies, one expects that higher excited states will dissolve at smaller temperature as compared to the smaller and more tightly bound ground states. These facts may lead to a sequential suppression pattern in the J/ψ and Υ yield in nucleus-nucleus collision as the function of the energy density. The potential model in this context could be helpful in predicting the binding energies of various quarkonia states by setting up and solving appropriate Schrödinger equation in the hot QCD medium. The first step towards this is to model an appropriate medium-dependent interquark interaction potential at finite temperature. The dissociation of heavy quarkonium derived by the presence of screening of static color fields in hot QCD medium has long been proposed as a signature of a deconfined medium and QGP formation [2]. Since then, this has been an area of active research [28–34]. However, a precise definition of the dissociation temperature is still elusive and is a matter of intense theoretical and phenomenological investigations either from the perspective of lattice spectral function studies [33,35–40] or potential inspired models [41–44] or effective quarkonia field theories [45]. The heavy quarks and antiquarks such as $c\bar{c}$ are bound together by almost static gluons [8,46,47]. Therefore, the gluon self-energy in the static limit can be helpful in understanding the fate of such states in the hot QCD medium.

While modeling the medium-modified potential, the nonperturbative effects coming from the nonzero string tension between the quark-antiquark pair in the QGP phase are not an unreasonable consideration. This is simply due to the fact that the hadronic to the QGP transition is a crossover. Therefore, the string tension will not vanish abruptly at or closer to T_c. One should certainly study its effect on the behavior of quarkonia even above the deconfinement temperature. This fact has been exploited in the recent past in Refs. [31,48], where a medium-modified form of the heavy quark potential has been obtained by correcting the full Cornell potential, not only its Coulomb part alone, as is usually done in the literature, with a dielectric function encoding the effects of the deconfined medium. The medium-modified potential, thus obtained, has a long-range Coulomb tail with an (reduced) effective charge [31] along with the usual Debye-screened form employed in most of the literature. We subsequently used this form to determine the binding energies and the dissociation temperatures of the ground and the first excited states of the charmonium and bottomonium spectra.

In the present paper, we shall consider an isotropic QGP medium which is described in terms of quasiparticle degrees of freedom based on a recently proposed quasiparticle model for hot QCD equations of state based on improved perturbative techniques at weak coupling [49,50]. We further implement a similar description for the lattice-QCD-based equations of state [51]. We first obtain the medium-modified heavy quark potential (both real and imaginary parts) and estimate the dissociation temperatures for two- and three-flavor hot QCD medium. As an intermediate step, the binding energies of the different quarkonia states and their respective thermal widths have been obtained in the hot QCD/QGP medium. Our predictions have been found to be consistent with the results obtained from other approaches.

The manuscript is organized as follows. The real part of the heavy-quark potential is discussed in Sec. II along with the Debye mass obtained from a quasiparticle model of the hot QCD equation of state along with binding energies of various quarkonia bound states by solving the Schrödinger equation (numerically). In Sec. III, computations on the imaginary part of the potential and, thereby, the thermal width of the quarkonium have been presented. Section IV deals with results and discussions. Finally, the conclusions and future prospects for the work are presented in Sec. V.

II. HEAVY-QUARK POTENTIAL

The interaction potential between a heavy quark and antiquark gets modified in the presence of a medium. The static interquark potential plays a vital role in understanding the fate of quark-antiquark bound states in the hot QCD/QGP medium. These aspects have been well studied in the literature and, in this direction, several excellent reviews exist [52,53] that cover potential model-based phenomenology as well as the lattice-QCD-based approaches. In all these studies, the potential in the deconfined phase is of the Yukawa form (screening Coulomb). The prime assumption is that the melting of the string between the quark-antiquark pairs in the deconfined phase is motivated by the fact that there is a phase transition from a hadronic matter to a QGP phase. In the present analysis, we incorporate the modification to both the Coulomb part and confining part in the deconfined medium [31,54]. This is based on the fact that the transition between the hadronic to the QGP phase is a crossover as shown by the recent lattice studies [55]. In the case of finite-temperature QCD, we here employ the ansatz that the medium modification enters in the Fourier transform of heavy quark potential $V(k)$ as [31]

$$\tilde{V}(k) = \frac{V(k)}{\epsilon(k)}, \tag{1}$$

where $\epsilon(k)$ is the dielectric permittivity which is obtained from the static limit of the longitudinal part of the gluon self-energy [56]:

$$\epsilon(k) = \left(1 + \frac{\Pi_L(0,k,T)}{k^2}\right) \equiv \left(1 + \frac{m_D^2}{k^2}\right). \tag{2}$$

In our case, $V(k)$ in Eq. (1) is the Fourier transform (FT) of the Cornell potential (to compute the FT, we need to introduce a modulator of the form $\exp(-\gamma r)$ and finally let the γ tend to zero), which is obtained as

$$\mathbf{V}(k) = -\sqrt{(2/\pi)}\frac{\alpha}{k^2} - \frac{4\sigma}{\sqrt{2\pi}k^4}. \tag{3}$$

Next, substituting Eq. (2) and Eq. (3) into Eq. (1) and evaluating the inverse FT, we obtain the r dependence of the medium-modified potential [48,57]:

$$\mathbf{V}(r,T) = \left(\frac{2\sigma}{m_D^2} - \alpha\right)\frac{\exp(-m_D r)}{r} - \frac{2\sigma}{m_D^2 r} + \frac{2\sigma}{m_D} - \alpha m_D. \tag{4}$$

Interestingly, this potential has a long-range Coulombic tail in addition to the standard Yukawa term. The constant terms are introduced to yield the correct limit of $V(r,T)$ as $T \to 0$ (it reduces to the Cornell form). Note that such terms could appear naturally while performing the basic computations of real-time static potential in hot QCD [58] and from the real and imaginary time correlators in a thermal QCD medium [59]. The three -dimensional form is motivated from the fact that, at finite temperature, the flux tube structure may expand in more than one dimension [60]. In the limiting case $r \gg 1/m_D$, the dominant terms in the potential are the long-range Coulombic tail and αm_D. The potential will look like

$$V(r,T) \sim -\frac{2\sigma}{m_D^2 r} - \alpha m_D \tag{5}$$

and can be tackled analytically while solving for the binding energies and the dissociation temperatures for the ground and first excited states of $c\bar{c}$ and $b\bar{b}$. In general, one is required to set the Schrödinger equation with the full potential and solve it numerically for the binding energy. Here, we consider the full potential and estimate the binding energies and the dissociation temperatures for heavy quarkonia. We analyze the spatial dependence of the heavy quark potential later and compare it against the other known forms of the potentials in the forthcoming sections. To that end, we employ the Debye mass computed from the effective fugacity quasiparticle model (EQPM) [49,50] and compare all the predictions with the Debye mass obtained in HTL and lattice QCD computations. Let us now proceed to discuss the EQPM and Debye mass below.

A. The Debye mass from a quasiparticle picture of hot QCD

The Debye mass, m_D, in QCD is generically nonperturbative and gauge invariant [61] unlike QED. The Debye mass in leading order in QCD coupling at high temperature has been long known and is perturbative in nature [62]. In a work in the past, Rebhan [63] defined m_D by seeing the relevant pole of the static quark propagator instead of the zero momentum limit of the time-time component of the gluon self-energy. The m_D thus obtained is seen to be gauge independent. This follows from the fact that the pole of the self-energy is independent of choice of gauge. In their work, Braaten and Nieto [64] calculated the m_D for the QGP at high temperature to the next-to-leading-order (NLO) in QCD coupling from the correlator of two Polyakov loops (this agrees to the HTL result [63]). Arnold and Yaffe [61] pointed out that the contribution of $O(g^2T)$ to the Debye mass in QCD needs the knowledge of the nonperturbative physics of confinement of magnetic charge. They further argued that a perturbative definition of the Debye mass as a pole of gluon propagator no longer holds. Importantly, in lattice QCD, the definition of m_D itself, encounters difficulty due to the fact that unlike QED the electric field correlators are not gauge invariant in QCD [65]. To circumvent this problem, the approaches based on effective theories obtained by dimensional reduction [66], spatial correlation functions of gauge-invariant meson correlators [67], and the behavior of the color singlet free energies [68] have been proposed. In a very recent attempt by Burnier and Rothkopf [65], a gauge-invariant mass has been defined from a complex static in medium heavy-quark potential obtained from lattice QCD.

To capture all the interaction effects present in hot QCD equations of state in terms of noninteracting quasipartons (quasigluons and quasiquarks), several attempts have been made. These quasipartons are nothing but the thermal excitations of the interacting quarks and gluons. We can categerize them as (i) effective mass models [69,70], (ii) effective mass models with Polykov loop [71], (iii) models based on PNJL and NJL [72], and (iv) effective fugacity models [49,50]. In QCD, the quasiparticle model is a phenomenological model which is widely used to describe the nonideal behavior of QGP near the phase transition point. The system of interacting massless quarks and gluons can be effectively described as an ideal gas of "massive" noninteracting quasiparticles in quasiparticle model. The mass of these quasiparticles is temperature dependent and arises because of the interactions of quarks and gluons with the surrounding matter in the medium. These quasiparticles retain the quantum numbers of the real particles, i.e., the quarks and gluons [73].

Here, we consider the quasiparticle description [49,50] of $O(g^5)$ hot QCD [61,74] and $O(g^6 \ln(1/g)$ hot QCD EoSs [75], we call them EoS1 and EoS2, respectively. We further consider the lattice QCD EoS [76] in terms of its quasiparticle description, we denote it as LEoS. Although there are more recent lattice results with improved lattice actions and more refined lattice [77,78], but to update the current model requires pure glue results for the trace anomaly with the same lattice set-up. Therefore, such attempts are beyond the scope of the present work. We intend to explore these possibilities in near future.

The equilibrium distribution function is written in the form given below:

$$f_{g,q} = \frac{z_{g,q}\exp(-\beta p)}{(1 \mp z_{g,q}\exp(-\beta p))}. \tag{6}$$

where g stands for quasigluons, and q stands for quasiquarks. z_g is the quasigluon effective fugacity and z_q is quasiquark effective fugacity. These distribution functions are isotropic in nature. These fugacities should not be confused with any conservation law (number conservation) and have merely been introduced to encode all the interaction effects at high temperature QCD. Both z_g and z_q have a very complicated temperature dependence and asymptotically reach to the ideal value unity [50]. The temperature dependence z_g and z_q fits well to the form given below,

$$z_{g,q} = a_{q,g}\exp\left(-\frac{b_{g,q}}{x^2} - \frac{c_{g,q}}{x^4} - \frac{d_{g,q}}{x^6}\right). \tag{7}$$

(Here $x = T/T_c$ and a, b and c and d are fitting parameters), for both EoS1 and EoS2.

The Debye mass, m_D is defined in terms of the equilibrium (isotropic) distribution function as,

$$m_D^2 \equiv -g^2 \int \frac{d^3\bar{\bar{p}}}{(2\pi)^3} \frac{df_{\rm eq}(\bar{p})}{d\bar{p}}. \tag{8}$$

where, $f_{\rm eq}$ is taken to be a combination of ideal Bose-Einstein and Fermi-Dirac distribution functions as [79], and is given by:

$$f_{\rm eq} = 2N_c f_g(\vec{p}) + 2N_f(f_q(\vec{p}) + f_{\bar{q}}(\vec{p})). \tag{9}$$

Since, we are dealing with the QGP system with vanishing baryon density, therefore, $f_q = f_{\bar{q}}$ (here, f_g and f_q are the quasiparton thermal distributions given in Eq. (6)). This combination of $f_{\rm eq}$ leads to the leading order HTL expression ($m_D^2 = g^2(T)T^2(N_c/3 + N_f/6)$) for the Debye mass in hot QCD. Here, N_c denotes the number of colors and N_f the number of flavors.

Now, considering quasiparton distributions, we obtain, m_D in the pure gluonic case:

$$m_D^2 = g^2(T)T^2\left(\frac{N_c}{3} \times \frac{6\text{PolyLog}[2, z_g]}{\pi^2}\right) \tag{10}$$

and full QCD:

$$m_D^2 = g^2(T)T^2\left[\left(\frac{N_c}{3} \times \frac{6\text{PolyLog}[2, z_g]}{\pi^2}\right) + \left(\frac{N_f}{6} \times \frac{-12\text{PolyLog}[2, -z_q]}{\pi^2}\right)\right]. \tag{11}$$

Here, $g(T)$ is the QCD running coupling constant, $N_c = 3$ ($SU(3)$) and N_f is the number of flavor, the function PolyLog$[2, z]$ having form, PolyLog$[2, z] = \sum_{k=1}^{\infty} \frac{z^k}{k^2}$. We get same expressions from the chromo-electric response functions in [80] for the interacting QGP.

The medium-modified m_D in terms of effective fugacities can be understood by relating it with the charge renormalization in the medium. This could be done by defining the effective charges for the quasigluons and quarks as Q_g and Q_q. These effective charges are given by the equations:

$$Q_g^2 = g^2(T)\frac{6\text{PolyLog}[2, z_g]}{\pi^2}$$
$$Q_q^2 = g^2(T)\frac{-12\text{PolyLog}[2, -z_q]}{\pi^2}. \tag{12}$$

Now the expressions for the Debye mass can be rewritten as

$$m_D^2 = \begin{cases} Q_g^2 T^2 \frac{N_c}{3} & \text{for pure gauge,} \\ T^2\left(\frac{N_c}{3}Q_g^2\right) + \left(\frac{N_f}{6}Q_q^2\right) & \text{for full QCD} \end{cases} \tag{13}$$

Here, $\{Q_g^2, Q_q^2\} \leq g^2(T)$ since it acquire the ideal value $g^2(T)$ asymptotically. As mentioned earlier, the effective fugacities, z_g and z_q are obtained for EoS1, EoS 2 and LEoS. The Debye mass with LEoS using our quasiparticle model is seen closer to that for EoS1 and EoS as compared to other cases. It is farthest as compared lattice Debye mass as the factor of 1.4 in the definition of the lattice Debye mass cannot be reproduced by perturbative or improved perturbative QCD or transport theory.

The temperature dependence of the quasiparticle Debye mass m_D^{QP} in pure and full QCD with $N_f = 2, 3$ is depicted in Fig. 1 and Fig. 2, compared with the LO and NLO in HTL and lattice parametrized Debye masses which are denoted as m_D^{LO} and m_D^L, respectively. These various Debye masses have the following mathematical expressions,

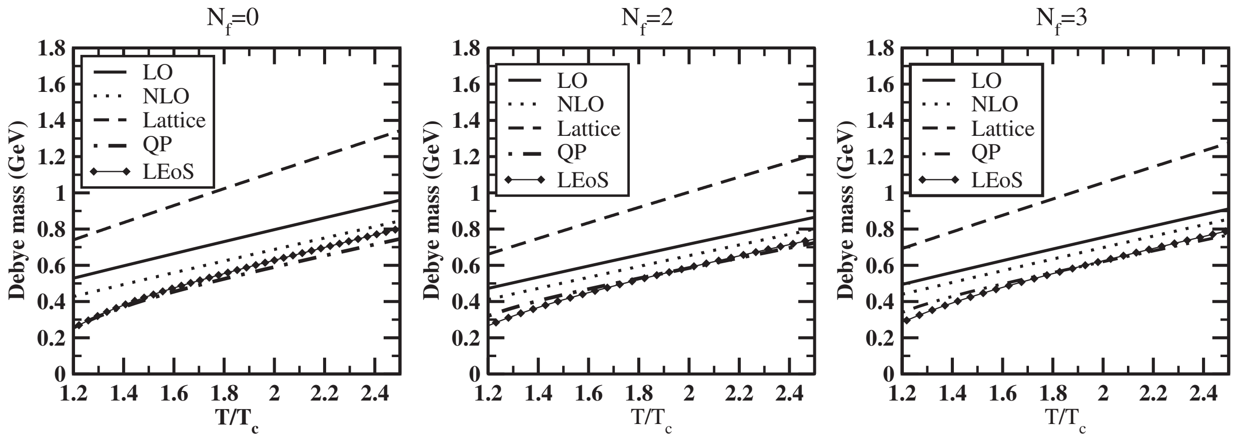

FIG. 1. Debye mass versus temperature (T/T_c) for quasiparticle (QP), lattice EoS, lattice parametrized, next-to-leading-order and leading-order cases, when we used the fugacity EoS 1. Left panel represents the pure gluonic case, middle and right panel represents two-flavor and three-flavor, respectively.

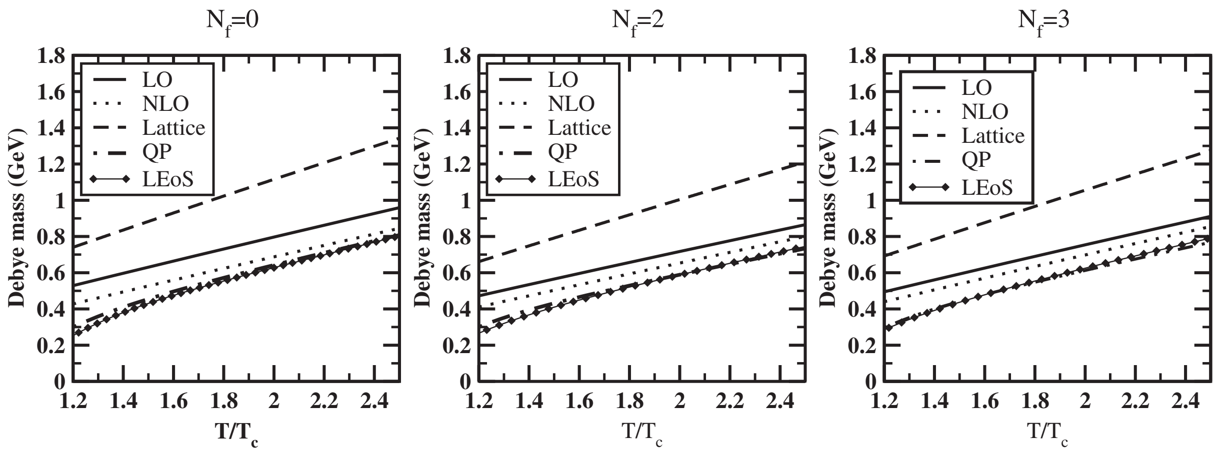

FIG. 2. Debye mass verse temperature (T/T_c) for quasiparticle (QP), lattice EoS, lattice parametrized, and leading order (LO) cases, when we used the fugacity EoS 2. Left panel represents the pure gluonic case, middle and right panel represents two-flavor and three-flavor, respectively.

$$
\begin{aligned}
m_D^{LO} &= g(T)T\sqrt{\frac{N_c}{3}+\frac{N_f}{6}},\\
m_D^{\mathrm{NLO}} &= m_D^{LO} + \frac{N_c g^2(T)T}{4\pi}\ln\left(\frac{m_D^{LO}}{g^2(T)T}\right),\\
m_D^{L} &= 1.4 g(T)T,\\
m_D^{QP} &= g(T)T\left[\frac{2N_c}{3\pi^2}\mathrm{PolyLog}[2,z_g]\right.\\
&\left.\quad -\frac{2N_f}{\pi^2}\mathrm{PolyLog}[2,-z_q]\right]^{\frac{1}{2}}. \qquad (14)
\end{aligned}
$$

For $g(T)$, we employ two expression for the running coupling in finite temperature QCD [81]. Clearly, m_D^{QP} is lowest among all other cases for the whole range of temperature considered here. The m_D^{LO} is higher and m_D^{NLO}, and m_D^{L} is largest among them for the whole range of temperature. From its temperature dependence in Eq. (14), it is straightforward to see that it will approach to the m_D^{LO} asymptotically ($z_{g,q} \to 1$). These observations are holding true for all ($N_f = 0, 2, 3$) cases and for the EoS1 and ESO2.

B. Heavy quark potential and quankonia Binding energies with EQPM

1. The Heavy-quark Potential

The heavy-quark potential given in Eq. (4) is shown as a function of rT for fixed T/T_c for pure gluonic, $N_f = 2$ and $N_f = 3$ cases in Fig. 3 (for EoS1) and Fig. 4 (EoS2) The expressions for the m_D has been taken from Eq. (14) and employed in the expression for the potential in Eq. (4). As expected the potential as a function of rT is lowest with the m_D^{L} and highest for the m_D^{QP} for the fixed T for the entire range of rT (this just follows from the temperature

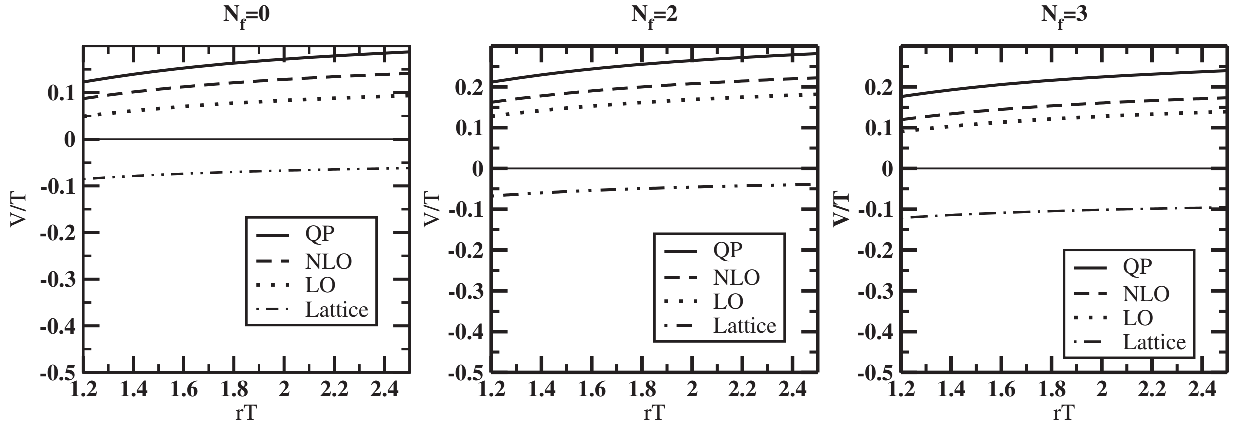

FIG. 3. The behavior of $V(r,T)/T$ as a function of rT for a fixed $(T/T_c = 3.32)$ for EoS1. The left panel represents the pure gluonic, and the middle and right panes represent two-flavor and three-flavor, respectively.

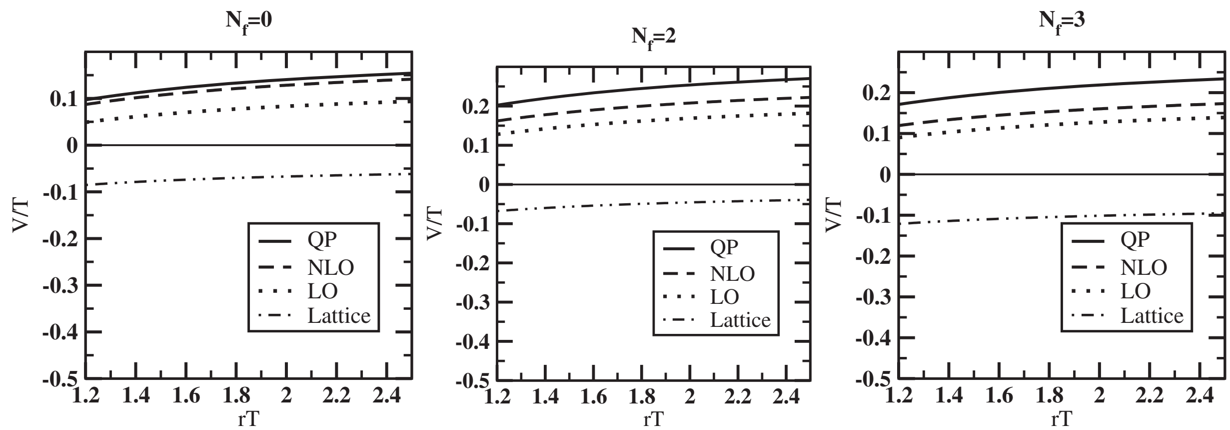

FIG. 4. The behaviour of $V(r,T)/T$ as a function of rT for a fixed $(T/T_c = 3.32)$ for EoS2. The left panel represents the pure gluonic, and the middle and right panels represent two-flavor and three-flavor, respectively.

dependence of the Debye mass i. e., higher the Debye mass higher the screening). The similar observations are seen for $N_f = 2$ and 3 and for both EoS1 and EoS2.

2. *The Binding Energies of J/Ψ and Υ*

To obtain the binding energy (BE) with heavy quark potential in Eq. (4), we need to solve the Schrödinger equation numerically with the full medium dependent complex potential [22]. Clearly, the binding energy will have both real and the imaginary parts. One can take the intersection point of real and imaginary parts of the binding energies while plotting their temperature dependences to define the dissociation temperature of quarkonia state under consideration. Another approach to look at the quarkonia dissociation is to first compute the thermal width of the given quarkonia from the imaginary part of the potential and equate it with the twice of the binding energies (real part). We follow the latter approach to estimate the dissociation temperatures. Therefore, we shall mostly concentrate on the real part of the binding energies and thermal width of the quarkonia.

In the limiting case discussed earlier, the real part of the medium-modified potential resembles the hydrogen atom problem [2]. The solution of the Schrödinger equation gives the eigenvalues for the ground states and the first excited states in charmonium (J/ψ, ψ' etc.) and bottomonium (Υ, Υ' *etc.*) spectra,

$$E_n = -\frac{1}{n^2}\frac{m_Q \sigma^2}{m_D^4}, \tag{15}$$

where m_Q is the mass of the heavy quark.

In the present case, we solve the Schrödinger equation with full potential and obtain the binding energies. The temperature dependence of the binding energies is shown in Figs. 5–8. For our analysis here, we consider J/Ψ, Ψ' binding energies with EoS 1 and EoS 2 as a function of temperature in Fig. 5 and Fig. 7, respectively. The Υ and Υ' binding energies as a function of temperature are shown in Fig. 6 and Fig. 8, respectively.

We have also plotted the LEoS estimates for BEs of various quarkonia states based on the quasiparticle understanding along with predictions for EoS1 and

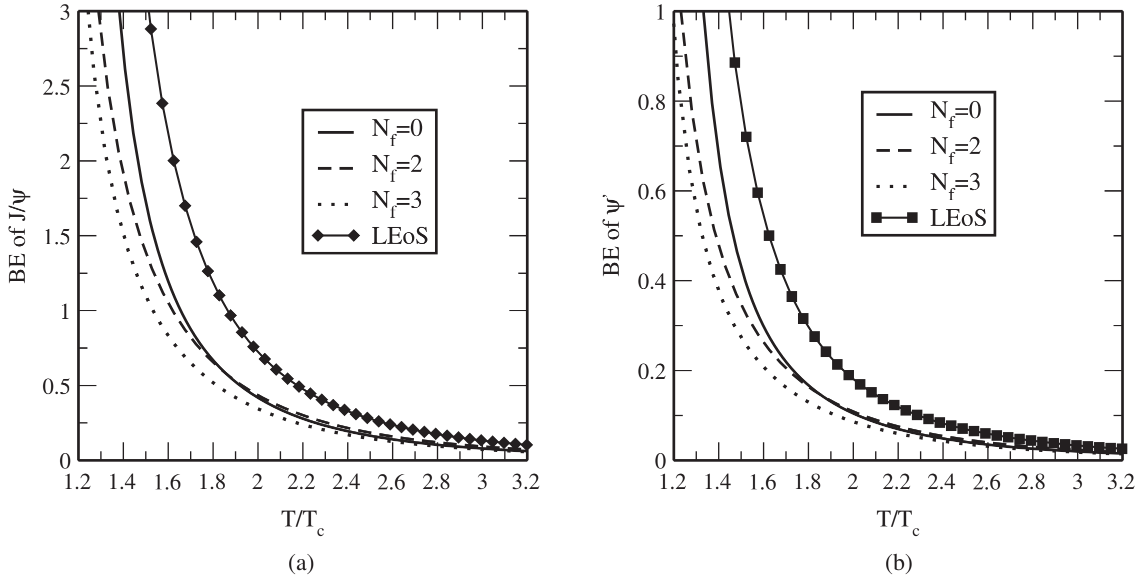

FIG. 5. Dependence of binding energy(in GeV) of (a) J/ψ and (b) ψ' on temperature T/T_c with fugacity equation of state EoS1.

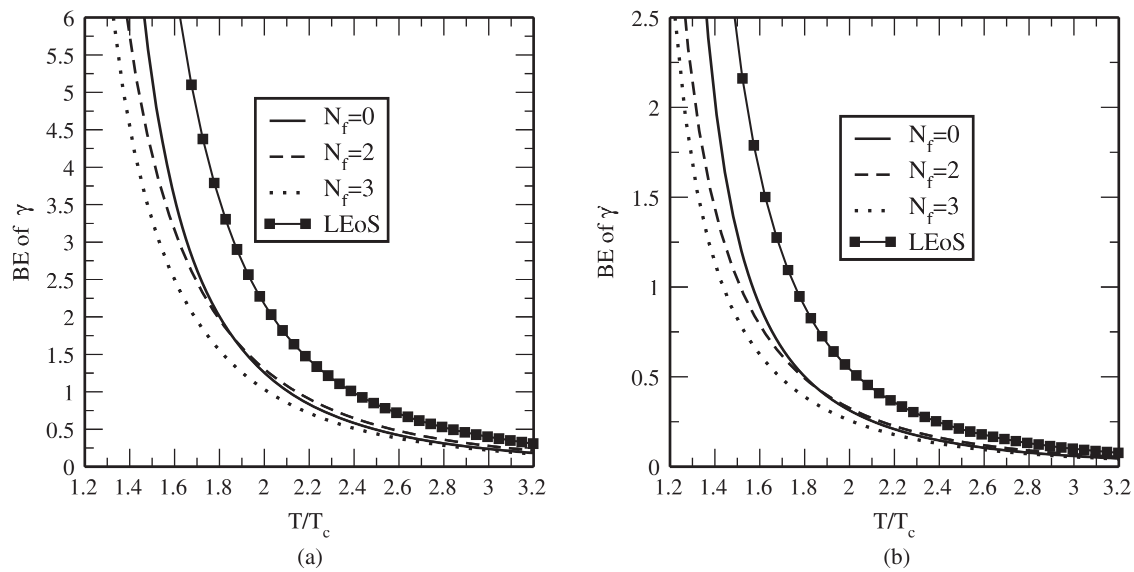

FIG. 6. Dependence of binding energy(in GeV) of (a) Υ and (b) Υ' on temperature T/T_c with fugacity equation of state EoS1.

EoS2. The BE in this case is largest as compared to $N_f = 0$, 2, 3 using EoS1 and EoS2 for the considered range of temperature. This observation is seen to be valid not only for J/ψ, Ψ' but also for the Υ and Υ' states. In each of the cases, the behavior is shown for $N_f = 0$, 2, and 3. There are some interesting observations that could be made while having a closer look at the temperature dependence of the binding energies in each case. Comparing the J/Ψ and Ψ' cases, we see that the binding energy is approaching to zero sharply in the later case. This roughly implies that the latter state will dissolve before the former one. The same statement could me made for Υ and Υ' states i. e., the former will dissociate later in temperature as compared to the latter state. We shall see that these observations are indeed true while we estimate the dissociation temperature for these states later. Interesting, for the three cases ($N_f = 0$, 2, 3) with either EoS 1 and EoS 2, these predictions for the dissociation temperatures come out true.

Let us now proceed to the computation of the dissociation temperatures for the above-mentioned quarkonia bound states. To that end, we need to compute the imaginary part of the heavy-quark potential and thus estimate the thermal width.

III. THE COMPLEX INTERQUARK POTENTIAL

Here we discuss how to obtain the complex interquark potential. The real part of the potential will be same as

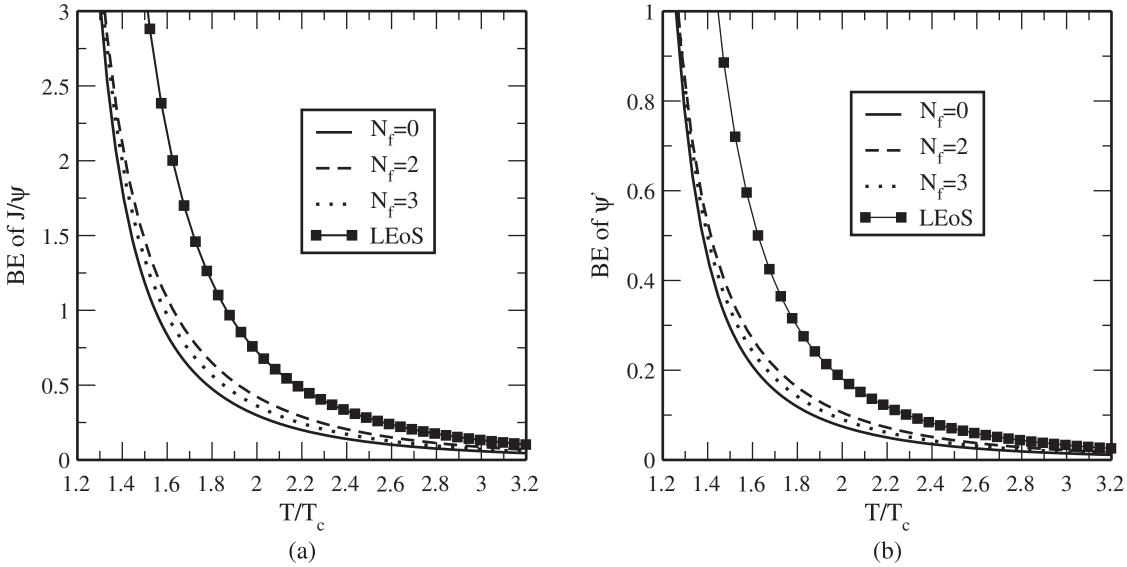

FIG. 7. Dependence of binding energy(in GeV) of (a) J/ψ and (b) ψ' on temperature T/T_c with fugacity equation of state EoS2.

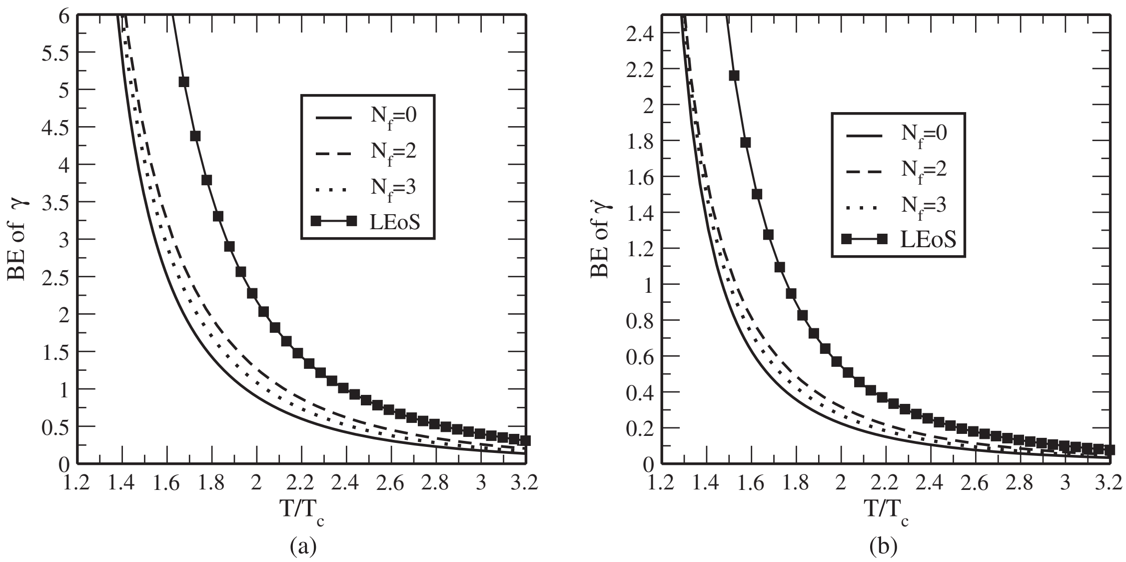

FIG. 8. Dependence of binding energy(in GeV) of (a) Υ and (b) Υ' on temperature T/T_c with fugacity equation of state EoS2.

Eq. (4). We follow the similar procedure to obtain the imaginary part of the potential as discussed below. To obtain the imaginary part of the interquark potential, we first need to obtain the imaginary part of the symmetric self energy in the static limit. This can be done by obtaining the imaginary part of the HTL propagator which represents the inelastic scattering of an off-shell gluon to a thermal gluon [34,59,82,83]. The imaginary part of the potential plays crucial role in weakening the bound state peak or transforming it to mere threshold enhancement and eventually in dissociating it (finite width (Γ) for the resonance peak in the spectral function, is estimated from the imaginary part of the potential which, in turn, determines the dissociation temperatures for the respective quarkonia). This sets the dissociation criterion, i. e., it is expected to occur while the (twice) binding energy becomes equals the width $\sim\Gamma$ [29,84]. The equality will do the quantitative determination of the dissociation temperature.

To obtain the imaginary part of the potential in the QGP medium, the temporal component of the symmetric propagator in the static limit has been considered as [8],

$$\mathrm{Im}D^{00}_{F(\mathrm{iso})}(0,k) = \frac{-2\pi T m_D^2}{k(k^2+m_D^2)^2}. \tag{16}$$

The same expression Eq. (16) could also be obtained for partons with space-like momenta ($\omega^2 < k^2$) from the retarded (advanced) self energy [85] using the relation [8,59]:

$$\ln\frac{\omega + k \pm i\epsilon}{\omega - k \pm i\epsilon} = \ln\left|\frac{\omega + k}{\omega - k}\right| \mp i\pi\theta(k^2 - \omega^2). \quad (17)$$

The imaginary part of the symmetric propagator Eq. (16) leasds to the imaginary part of the dielectric function in the QGP medium as:

$$\frac{1}{\epsilon(k)} = -\pi T m_D^2 \frac{k^2}{k(k^2 + m_D^2)^2}. \quad (18)$$

Afterwards, the imaginary part of the in medium potential is easy to obtain owing the definition of the potential Eq. (1) as mentioned in [86]:

$$\begin{aligned}\mathrm{Im}V(r,T) &= -\int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}}(e^{i\mathbf{k}\cdot\mathbf{r}} - 1) \\ &\quad\times \left(-\sqrt{\frac{2}{\pi}}\frac{\alpha}{k^2} - \frac{4\sigma}{\sqrt{2\pi}k^4}\right)\frac{-\pi T m_D^2 k}{(k^2 + m_D^2)^2} \\ &\equiv \mathrm{Im}V_1(r,T) + \mathrm{Im}V_2(r,T),\end{aligned} \quad (19)$$

where $\mathrm{Im}V_1(r,T)$ and $\mathrm{Im}V_2(r,T)$ are the imaginary parts of the potential due to the medium modification to the short-distance and long-distance terms, respectively:

$$\begin{aligned}\mathrm{Im}V_1(r,T) &= -\frac{\alpha}{2\pi^2}\int d^3\mathbf{k}(e^{i\mathbf{k}\cdot\mathbf{r}} - 1) \\ &\quad\times\left[\frac{\pi T m_D^2}{k(k^2 + m_D^2)^2}\right],\end{aligned} \quad (20)$$

$$\begin{aligned}\mathrm{Im}V_2(r,T) &= -\frac{4\sigma}{(2\pi)^2}\int\frac{d^3\mathbf{k}}{(2\pi)^{3/2}}(e^{i\mathbf{k}\cdot\mathbf{r}} - 1) \\ &\quad\times\frac{1}{k^2}\left[\frac{\pi T m_D^2}{k(k^2 + m_D^2)^2}\right].\end{aligned} \quad (21)$$

After performing the integration, the contribution due to the short-distance term to imaginary part becomes (with $z = k/m_D$)

$$\begin{aligned}\mathrm{Im}V_1(\mathbf{r},T) &= 2\alpha T\int_0^\infty \frac{z dz}{(z^2+1)^2}\left(1 - \frac{\sin z\hat{r}}{z\hat{r}}\right) \\ &\equiv \alpha T\phi_0(\hat{r}),\end{aligned} \quad (22)$$

and the contribution with the nonzero string tension becomes

$$\begin{aligned}\mathrm{Im}V_2(r,T) &= \frac{4\sigma T}{m_D^2}\int_0^\infty\frac{dz}{z(z^2+1)^2}\left(1 - \frac{\sin z\hat{r}}{z\hat{r}}\right) \\ &\equiv \frac{2\sigma T}{m_D^2}\psi_0(\hat{r}),\end{aligned} \quad (23)$$

where the functions, $\phi_0(\hat{r})$ and $\psi_0(\hat{r})$ at leading-order in $\hat{r}$ are

$$\phi_0(\hat{r}) = \left(-\frac{\hat{r}^2}{9}(-4 + 3\gamma_E + 3\log\hat{r})\right). \quad (24)$$

$$\psi_0(\hat{r}) = \frac{\hat{r}^2}{6} + \left(\frac{-107 + 60\gamma_E + 60\log(\hat{r})}{3600}\right)\hat{r}^4 + O(\hat{r}^5). \quad (25)$$

In the short-distance limit ($\hat{r} \ll 1$), both the contributions, at the leading logarithmic order, reduce to

$$\mathrm{Im}V_1(r,T) = \alpha T\frac{\hat{r}^2}{3}\log\left(\frac{1}{\hat{r}}\right), \quad (26)$$

$$\mathrm{Im}V_2(r,T) = -\frac{2\sigma T}{m_D^2}\frac{\hat{r}^4}{60}\log\left(\frac{1}{\hat{r}}\right). \quad (27)$$

Therefore, the sum of Coulomb and string tension dependent terms leads to the imaginary part of the potential:

$$\mathrm{Im}V(r,T) = T\left(\frac{\alpha\hat{r}^2}{3} - \frac{\sigma\hat{r}^4}{30m_D^2}\right)\log\left(\frac{1}{\hat{r}}\right). \quad (28)$$

One thus immediately observes that for small distances the imaginary part vanishes and its magnitude is smaller as compared to the case with only the Coulombic term [85]. The effect of nonperturbative contribution coming from the string terms, thus, reduces the width of the resonances in thermal medium. The imaginary part of the potential above, provides an estimate for the width (Γ) for a resonance state. The width Γ can be computed in first-order perturbation, while folding the imaginary part of the potential with the unperturbed (1S) Coulomb wave function as:

$$\Gamma = \left(1 + \frac{3\sigma}{\alpha m_Q^2}\right)\frac{4T}{\alpha}\frac{m_D^2}{m_Q^2}\log\frac{\alpha m_Q}{2m_D}. \quad (29)$$

It is possible to solve the integral for the functions $\phi_0(\hat{r})$ and $\psi_0(\hat{r})$ in the right hand side of the Eq. (23) exactly. The compact mathematical expressions are presented in the Appendix. The behavior of these functions as a function of $\hat{r}$ is depicted in Figs. 9 and 10 where we have compared the small $\hat{r}$ behavior in Eq. (27) with approximate result in Eqs. (24) and (25) along with results for larger $\hat{r}$. Clearly the approximation works fantastically well of $\hat{r} < 1$ for $\phi_0(\hat{r})$ and better for $\psi_0(\hat{r})$. The behavior at large $\hat{r}$ is crucial to understand the fate of higher (excited) states of quarkonia. The analytic estimate for $\psi_0(\hat{r})$ based on the expression quoted in the appendix is well behaved until $\hat{r} \leq 16$–17. For $\hat{r} > 17$ the functions $\psi_0(\hat{r})$ show large fluctuations that grow rapidly for larger $\hat{r}$. Therefore, in that region, we perhaps can not utilize it for phenomenological purposes.

A. The dissociation temperatures for heavy quarkonia

There are two criteria for the dissociation of quarkonia bound state in the QGP medium that are under consideration here. The first one is the dissociation of a given quarkonia bound state by the thermal effects alone. On the other hand, the second criterion is based on the dissolution of a given quarkonia state while its thermal width is overcomed by the twice of the real part of the binding

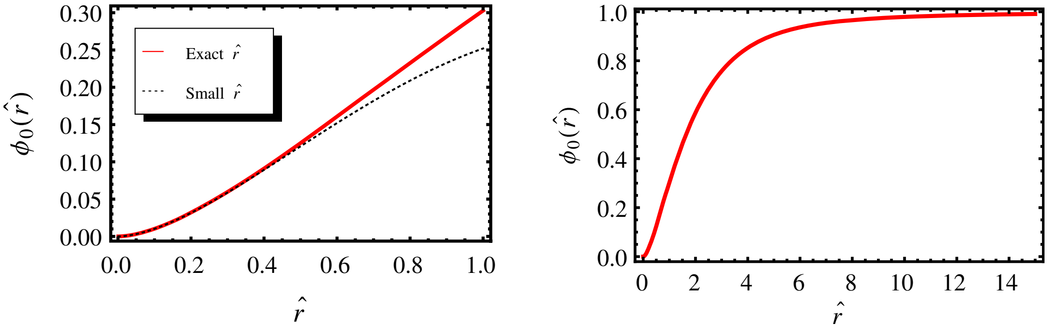

FIG. 9. Dependence of $\phi_0(\hat{r})$ on $\hat{r}$. The lower panel shows the comparison between the small $\hat{r}$ approximation and exact one.

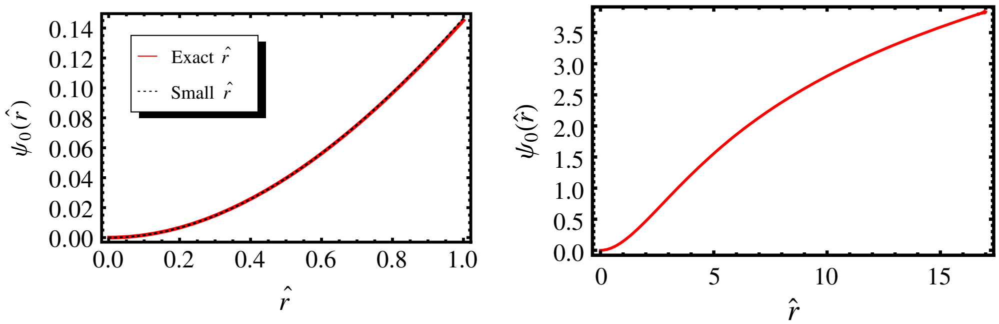

FIG. 10. Dependence of $\psi_0(\hat{r})$ on $\hat{r}$. The figure in the left shows the comparison between the small $\hat{r}$ approximation and exact results. The right figure depicts the exact results until $\hat{r} = 17$ beyond which fluctuations start showing up and grow rapidly.

energy. We shall employ both of them one by one below and present the comparison of the quantitative estimates of the dissociation temperatures.

1. Dissociation by thermal effects

Dissociation of a quarkonia bound state in a thermal QGP medium will occur whenever the binding energy (BE), E_B of the said state will fall below the mean thermal energy of a quasiparton. In such situations the thermal effect can dissociate the quakonia bound state.

To obtain the lower bound of the dissociation temperatures of the various quarkonia states, the (relativistic) thermal energy of the partons will be $3T$. On the other hand, the upper bound of the dissociation temperature (T_D) is obtained by considering the mean thermal energy to be T. The dissociation is supposed to occur whenever,

$$E_B(T_D) = 3T_D \quad \textbf{or} \quad T_D. \tag{30}$$

While solving for the E_B, the string tension (σ) is taken as 0.184 GeV^2, and critical temperatures (T_c) are considered as 270 MeV, 203 MeV and 197 MeV for pure, 2-flavor and 3-flavor QCD at high temperature for both the equations of state. The binding energies are shown as a function of temperature in earlier plots. The dissociation temperatures for the ground state and the first excited state of $c\bar{c}$ (J/Ψ and Ψ') and $b\bar{b}$ sates (Υ and Υ') are presented in Table I and III while considering two different criteria of quarkonia dissociation.

2. Overcoming thermal width of the resonance by the binding energy

Whenever the thermal width, Γ of the a given quarkonium is as large as twice the binding energy (real part) the given quarkonia state will dissolve [86].

TABLE I. Lower(upper) bound on the dissociation temperature (T_D) for the quarkonia states (in units of T_c) for using fugacity parameters of EoS 1.

State	Pure QCD	$N_f = 2$	$N_f = 3$
J/ψ	1.6(1.9)	1.6(2.1)	1.5(2.0)
ψ'	1.3(1.5)	1.3(1.6)	1.3(1.5)
Υ	1.9(2.4)	2.1(2.6)	2.0(2.5)
Υ'	1.5(1.8)	1.6(1.9)	1.5(1.9)

TABLE II. Lower(upper) bound on the dissociation temperature (T_D) for the quarkonia states (in units of T_c) for using fugacity parameters of EoS 2.

State	Pure QCD	$N_f = 2$	$N_f = 3$
J/ψ	1.5(1.8)	1.7(2.0)	1.6(1.9)
ψ'	1.2(1.4)	1.3(1.6)	1.3(1.6)
Υ	1.8(2.2)	2.0(2.6)	2.0(2.5)
Υ'	1.4(1.7)	1.6(1.9)	1.6(1.9)

TABLE III. The dissociation temperature (T_D) for the quarkonia states (in units of T_c) for using fugacity parameters of EoS 1, when thermal width = 2 BE.

State	Pure QCD	$N_f = 2$	$N_f = 3$
J/ψ	1.8	2.0	1.9
ψ'	1.6	1.8	1.8
Υ	2.6	2.8	2.2
Υ'	2.1	2.2	2.1

We applied the criteria for the $c\bar{c}$ bound states (J/ψ and ψ') and $b\bar{b}$ bound state (Υ and Υ'). The quantitative estimates for the respective dissociation temperatures are enlisted in Tables II and IV.

Let us now analyze the quantitative estimates for J/ψ and ψ' dissociation temperatures for EoS1 equating the thermal width with the twice of the BE. The J/ψ state is seen to dissociate at $T = 1.8T_c$ for $N_f = 0$, $T = 2.0T_c$ for $N_f = 2$ and for $N_f = 3$ at $T = 1.9T_c$. On the Ψ' is seen to dissociate at $T = 1.6T_c$ for $N_f = 0$, $T = 1.8T_c$ for $N_f = 2$ and for $N_f = 3$ at $T = 1.8T_c$. On the other hand, for EoS2, J/ψ is seen to dissociate at $T = 1.7T_c$ for $N_f = 0$, $T = 1.9T_c$ for $N_f = 2$ and for $N_f = 3$ at $T = 1.9T_c$. Ψ' is seen to dissociate at $T = 1.5T_c$ for $N_f = 0$, $T = 1.7T_c$ for $N_f = 2$ and for $N_f = 3$ at $T = 1.7T_c$. As stated earlier (on the basis of temperature dependence of the BE) Ψ' is seen to dissociate at lower temperatures as compared to J/ψ for both the equations of state.

Similarly, for Υ and Υ' dissociation temperatures are recorded in Table III and Table IV. The Υ state is seen to dissociate at $T = 2.6T_c$ for $N_f = 0$, $T = 2.8T_c$ for $N_f = 2$ and for $N_f = 3$ at $T = 2.2T_c$ while employing EoS1 through the quasiparticle picture. On the other hand, Υ' is seen to dissociate at $T = 2.1T_c$ for $N_f = 0$, $T = 2.2T_c$ for $N_f = 2$ and for $N_f = 3$ at $T = 2.1T_c$ for the same EoS. With EoS2, Υ is seen to dissociate at $T = 2.5T_c$ for $N_f = 0$, $T = 2.7T_c$ for $N_f = 2$ and for $N_f = 3$ at $T = 2.6T_c$ and Υ' is seen to dissociate at $T = 2.0T_c$ for $N_f = 0$, $T = 2.2T_c$ for $N_f = 2$ and for $N_f = 3$ at $T = 2.1T_c$. Again, we can see (on the basis of temperature dependence of the BE) that Υ is seen to dissociate at higher temperatures as compared to Υ' for both the equations of state.

The estimates for various quarkonia states under consideration with LEoS are quoted in Table V. The first row records the estimates for the case while the quarkonia dissociation has been led by the average thermal energy of the $q/\bar{q}$. On the other hand the second row captures estimates while the BEs are overcomed by the thermal width of quarkonia due to complex nature of the potential (interquark). The upper bound obtained in row1 are closer to those with the latter criterion. On comparing the estimate only slightly different.

TABLE IV. The dissociation temperature (T_D) for the quarkonia states (in units of T_c) for using fugacity parameters of EoS 2, when thermal width = 2 BE.

State	Pure QCD	$N_f = 2$	$N_f = 3$
J/ψ	1.7	1.9	1.9
ψ'	1.5	1.7	1.7
Υ	2.5	2.7	2.6
Υ'	2.0	2.2	2.1

TABLE V. Lower (upper) bound on the dissociation temperature (T_D) for the quarkonia states for 2 + 1 flavour (in units of T_c) case while using the fugacity parameters of the LEoS (second row). The third row records the estimates with second criterion of the dissociation (2 BE ≡ thermal width).

State	J/ψ	ψ'	Υ	Υ'
LEoS	1.9(2.3)	1.5(1.8)	2.3(2.8)	1.8(2.1)
LEoS	2.1	1.8	3.1	2.6

Comparing the numbers for the T_D for various quarkonia states, quoted in Table I and Table III, we observe that the quantitative estimates in Table III are quite closer to the upper bound(NR) criteria. Note that the former estimates are based on the dissolution of a given quarkonia state by the mean thermal energy of the quasipartons in the hot QCD/QGP medium, the latter one is based on equating the thermal width to the real part of the binding energy (twice). Similar observations are obtained while comparing the estimates from Table II and Table IV. Interestingly, the numbers regarding the dissociation temperatures obtained by employing EoS1 and EoS2 with the latter criterion of quarkonia dissociation are not very different from each other.

IV. RESULTS AND DISCUSSION

The hot QCD equations of state corresponding to interactions up to $O(g^5)$ and $O(g^6(\ln 1/g))$ in the improved perturbative QCD can significantly impact the fate of quarkonia in the QGP medium. The medium-modified form of the heavy quark potential in which the medium modification causes the Debye screening of color charges has been obtained by employing the Debye mass obtained by utilizing the quasiparticle understanding of these equations of state. This, in turn, leads to the temperature-dependent binding energies for the J/ψ and ψ'. The binding energies are seen to decrease less sharply for the pure gluonic case in comparison to the full QCD medium. Similar patterns have been observed for the case of Υ and Υ' states.

To estimate the dissociation temperature, we consider two criteria *viz.* the dissociation by mean thermal energy of the quasiparticles in the QGP medium and the binding energy overcoming the thermal width of the quakonia bound state. The upper and lower bound within the first criterion were obtained by thermal energy T and $3T$, respectively. In numbers for the dissociation temperatures from both the criteria are seen to be consistent with the recent predictions from the recent quarkonium spectral function studies using a potential model. The effects of realistic EoS for the QGP have significant impact on the binding energies and the dissociation temperatures for the various quarkonia states.

V. CONCLUSION AND OUTLOOK

In conclusion, we have studied the quarkonia dissociation in QGP in the isotropic case employing quasiparton equilibrium distribution functions obtained from $O(g^5)$ and $O(g^6(\ln 1/g))$ hot QCD equations of state and LEoS and medium modification to a heavy quark potential. We have found that medium modification causes a dynamical screening of color charge which, in turn, leads to a temperature-dependent binding energy. We have systematically studied the temperature dependence of binding energy for the ground and first excited states of charmonium and bottomonium spectra in pure gluonic and full QCD medium. We have then determined the dissociation of heavy quarkonium in hot QCD medium by employing the medium modification to a heavy quark potential and explore how the pattern changes for the pure gluonic case and full QCD in the Debye mass.

We intend to look for extensions of the present work in the case of hydrodynamically expanding viscous QGP medium. Another interesting direction would be to couple the analysis to the physics of momentum anisotropy and instabilities in the early stages of the heavy-ion collisions and the impact on the physics of heavy quarkonia dissociation and yields in heavy-ion collisions.

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APPENDIX: IMAGINARY PART OF INTERQUARK POTENTIAL

It is possible to solve the integral in Eq. (22) and Eq. (23) for real $\hat{r}$. The expressions for the functions $\phi^0(\hat{r})$ and $\psi^0(\hat{r})$ are obtained as

$$\phi_0(\hat{r}) = 1 - \sqrt{\pi} G^{2,1}_{1,3}\left(\frac{\hat{r}^2}{4}\middle| \begin{matrix} 0 \\ 0, 1, -\frac{1}{2} \end{matrix}\right),$$

$$\begin{aligned}\psi_0(\hat{r}) = \frac{1}{2|\hat{r}|}(&-6|\hat{r}| + 4|\hat{r}|\gamma_E + 4\hat{r}\mathrm{Log}[|\hat{r}|] \\ &+ (Ci(-i|\hat{r}|) + Ci(i|\hat{r}|)) \\ &\times [|\hat{r}|\cosh(|\hat{r}|) - 3\sinh(|\hat{r}|)] \\ &+ 2Shi(|\hat{r}|)[3\cosh(|\hat{r}|) - \hat{r}|\sinh(|\hat{r}|)]). \end{aligned} \tag{A1}$$

Here, G is the *MeijerG* function and

$$\begin{aligned} Ci(z) &= \mathrm{CosIntegral}(z) = -\int_z^\infty \frac{\cos(t)}{t} dt, \\ Shi(z) &= \mathrm{SinhIntegral}(z) = \int_0^z \frac{\sinh(t)}{t} dt. \end{aligned} \tag{A2}$$

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