Rare meson decays with three pairs of quasidegenerate heavy neutrinos

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We consider a low-scale seesaw scenario where the masses of its heavy Majorana neutrinos are arranged in a pattern of three quasidegenerate pairs, in the range of $\mathcal{O}(1\text{–}6\ \text{GeV})$. Since they can violate lepton number by two units, they contribute to rare decays of D_s and B_c mesons, providing the conditions for maximal CP violation. We find that new phenomenology is possible depending on how many of on-shell pairs mediate these decays. In particular, we present new constraints on muon-heavy neutrino mixing parameters.

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I. INTRODUCTION

Recent experiments have shown that neutrinos have nonzero masses [1,2], and that the mixing pattern between mass and flavor states is explained by the Maki-Nakagawa-Sakata (MNS) matrix, U_{MNS} [3]. Given the requirement that the matrix which diagonalizes the whole neutrino mass matrix is unitary, current uncertainties in the elements of U_{MNS} allow a small range for mixing between standard model (SM) flavor states and new sterile ones [4–7], which would imply a tiny interaction of the latter with SM particles. Likewise, as the θ_{13} angle of U_{MNS} is nonzero [8,9] the possibility that the light neutrino sector could violate the CP symmetry remains open; certainly, this is not enough to explain baryogenesis via leptogenesis. Additionally, depending on whether the neutrino nature is Dirac or Majorana we would have one (Dirac) or three (Majorana) CP violation phases. [10]. It has been proposed [11–18] that the detection of rare decays of mesons of the type $M \to \ell_1 \ell_2 M'$ (with M, M' being mesons, whereas ℓ_1, ℓ_2 are charged leptons)—which exhibit (i) asymmetry between modes which are charge conjugates of each other (CP asymmetry), and (ii) lepton number violation (LNV) and/or lepton flavor violation—would reveal, respectively, the presence of such phase(s) and that neutrinos are in fact Majorana particles. Concerning processes with $\Delta L = 2$, the rare meson decays (RMDs) studied in this paper play a different role than neutrinoless double-beta decay, allowing one to extend the neutrino mass range from $\lesssim$100 MeV to a few GeV. In this line, it is known that the CP asymmetry of such processes is maximized when two quasidegenerate heavy neutrinos (QDHν) participate as an on-shell intermediate state [17–20], producing a resonance as their masses become almost degenerate. The proposed framework with such QDHν is the type I seesaw mechanism (S1) [21–24], defined by the addition of one SM-fermion singlet (right-handed neutrinos, ν_{Ri}) per generation, resulting in a neutrino mass matrix given, in the basis $\nu = (\nu_L^c, \nu_R)$, by

$$M_\nu = \begin{pmatrix} 0 & Yv \\ (Yv)^T & M_R \end{pmatrix}. \tag{1}$$

Here, $v = 246$ GeV is the vacuum expectation value (VEV) of the Higgs Field, Y is a 3×3 Yukawa coupling matrix, and M_R is a 3×3 mass matrix corresponding to a Majorana mass term. Since the neutrino mass sector of S1 gives a mass matrix [Eq. (1)] contracted with a basis made up of charge conjugates (ν^c and ν), we say that the whole mass matrix is a Majorana-type matrix, and its M_R term is the source of explicit LNV. Diagonalization of Eq. (1) provides both light and heavy mass states: the former have masses given by the eigenvalues of $m_\ell^{\text{SS}} \sim (Yv)^2/M_R$, whereas the latter have masses $M_h \sim M_R$. The only restriction over Y and M_R is that they have to reproduce the magnitude of light neutrinos masses, $(Yv)^2/M_R \sim 0.1$ eV. In particular, there is a minimal extension to the SM—called the νMSM [25–28]—whose main features are that (i) the masses of SM neutrinos are due to a small Yukawa coupling $Y \sim 10^{-8}$ and $M_R \gtrsim 10^2$ MeV, (ii) one of the heavy neutrinos, whose mass is in the $\mathcal{O}(10)$ keV range, becomes a candidate for dark matter, and (iii) the masses of the other two heavy states (which lie in the range $M_h \gtrsim 100$ MeV) are close enough to produce the above-mentioned effect for RMD. Recently the CERN-SPS Collaboration [29] has proposed searching for heavy neutral leptons from the rare decay of B, B_c, K, and, preferentially, D and D_s mesons.

From an experimental perspective, the drawbacks of seesaw mechanisms (type I, as well as types II [24,30–33] and III [34]) is that they require values of M that are very large in order to reproduce $m_\ell \sim \mathcal{O}(0.1)$ eV. In fact, assuming that Y lies in the range of the Yukawa coupling for SM fermions (i.e., $Y \sim 10^{-6}\text{–}10^0$) we obtain that $M \sim 10^3\text{–}10^{15}$ GeV, and, consequently, the mixing between SM flavor states and heavy neutrino mass states is $\frac{Yv}{M} \sim 10^{-6}\text{–}10^{-11}$, so any manifestation of such heavy neutrinos is out of reach of current reactors. In order to avoid this problem, low-scale seesaw (LSS) models add not

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one but two right-handed neutrinos per family to the SM ($\nu_{R\alpha}$ and S_α). Typically, ν_R are the same as those of S1 (with the same SM charges) and are used to construct the SM-like Yukawa term $Y_D\overline{L}\Phi\nu_R$ [Y_D plays the same role as Y in Eq. (1)]. Likewise, S are SM singlets which—with the addition of a SM-singlet scalar field χ—allow for the Yukawa term $Y_\chi\overline{\nu_R^c}S\chi$. Of course, all of these new terms have to be invariant under any new gauge group that we want to impose. Then, with the addition of the SM singlets ν_R, S, and χ we can reproduce (after the scalar fields acquire a VEV) a neutrino mass matrix, written in the basis (ν_L^c, ν_R, S), given by

$$M_\nu = \begin{pmatrix} 0 & Y_D v & 0 \\ (Y_D v)^T & 0 & M_\chi \\ 0 & M_\chi^T & 0 \end{pmatrix}, \qquad (2)$$

where $M_\chi = Y_\chi\langle\chi\rangle$, whose diagonalization yields exactly massless active neutrinos. It is possible to introduce the blocks $(M_\nu)_{33} = \mu$ and $(M_\nu)_{13} = \varepsilon$, after which active neutrinos acquire masses given by $m_\ell^{\rm IS} \sim (Y_D v)^2 \mu M_\chi^{-2}$ (inverse seesaw, IS) and $m_\ell^{\rm LS} \sim (Y_D v)\varepsilon M_\chi^{-1}$ (linear seesaw, LS), respectively. In these regimes the smallness of the neutrino masses is not due to a large M_χ in the denominator of m_ℓ, but rather to a small μ or ε in the numerator, allowing (incidentally) the scale of new physics M_χ to not need to be as large as in S1 [when $M_\chi \gtrsim 10^1(Y_D v)$ the heavy neutrino states have masses in the range of $m_N \sim \sqrt{(Y_D v)^2 + M_\chi^2}$]. For instance, values as small as 100 eV for μ or ϵ allow $M_\chi \sim 1$–10 TeV, which is a reachable scale in the short and middle terms. The IS mechanism was originally proposed as a superstring SO(10) model whose symmetry is broken by the VEV of a scalar field, producing the mass term for S [35]. On the other hand, the LS mechanism was proposed as a model invariant under the gauge group $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{\rm B-L}$, where the scalar and fermionic fields correspond to irreducible high-dimensional representations of a supersymmetric SO(10) theory [36]. Also, in Ref. [37] we find an extension of the SM which adds [besides the right-handed neutrinos (ν_R, S) and the scalar singlet χ] a scalar doublet H. In the scalar potential of this model only the term $\lambda(\Phi^\dagger H\chi^2 + {\rm H.c.})$ gives a relation between the U(1) charges of all these fields, so it is only when $\lambda \to 0$ that the potential is invariant under any transformation $(\Phi, H, \chi) \to (e^{\imath\alpha_1}\Phi, e^{\imath\alpha_2}H, e^{\imath\alpha_3}\chi)$, where α_j are independent global phases. Therefore, we can propose λ to be naturally small, in the 't Hooft sense, because its cancellation increases the symmetries of the Lagrangian. Likewise, as the charges under the U(1) gauge groups of this model have been defined in order to allow for the Yukawa term $Y_H\bar{L}HS$, the spontaneous symmetry breaking $|H| \to v_H$ yields the mass term $\varepsilon\overline{\nu_L}S$, where $\varepsilon = (M_\nu)_{13} = Y_H v_H$ is, consequently, naturally small.

The key to obtaining two QDHν in the νMSM lies in the fact that when its masses are exactly equal (and, by construction, the keV mass vanishes) the model presents a global U(1) symmetry; so, by slightly breaking this symmetry we obtain both the keV mass and the quasidegeneration between the states with $M \gtrsim 100$ MeV [28]. Furthermore, as this quasidegeneracy comes from the removal of a symmetry in the Lagrangian, its smallness is protected from radiative corrections (we say they are naturally small, in the 't Hooft sense [38]). On the other hand, the mechanism for obtaining an enhancement in the CP asymmetry between a process P and its charge conjugate P^c is, essentially, that their amplitude is the sum of two diagrams, each of which is mediated by Majorana neutrinos with masses M_1 and M_2. Then, assuming that (i) such QDHν are on-shell, and (ii) their interactions with SM particles are very weak (i.e., the narrow-width approximation, NWA), we obtain both the amplitude of RMDs and the CP asymmetry is maximized when $\Delta m_N = m_{N_2} - m_{N_1} \sim \Gamma_N$, where Γ_N is the decay width of the heavy neutrino N [18,19] (Γ_N is a soft function of m_N [39]).

In this paper we propose that LSS scenarios—i.e., those obtained when we add blocks $(M_\nu)_{33} = \mu$ or $(M_\nu)_{13} = (M_\nu)_{31}^T = \varepsilon$ to Eq. (2)—can provide not one but three pairs of QDHν, which could enhance the branching ratio (BR) of RMD of mesons going to two charged leptons and another meson, and, eventually, the CP asymmetry between modes which are charge conjugates of each other. For this purpose, in principle, all of the intermediate heavy neutrinos must be on shell (i.e., $m_M - m_{\ell 1} > m_N > m_{M'} + m_{\ell 2}$), so their masses have to be in the range of $\mathcal{O}(100)$ MeV for K decay, and in the range of $\mathcal{O}(1)$ GeV for B and D decays. Since LSS models propose that masses of heavy neutrinos can be as large as ~TeV, we regard scenarios where at least one pair of QDHν lie in the range of $\mathcal{O}(10^{-2} - 1)$ TeV, so they could contribute to processes that are testable at the LHC.

The program of this paper is the following. In Sec. II we explain how to get three pairs of QDHν in a LSS scenario. In Secs. III and IV we present the formalism for meson decays mediated by three pairs of quasidegenerate heavy neutrinos and its corresponding results for B, B_c, D, and D_s cases. Finally, in Sec. V we present our conclusions.

II. PROPOSAL

We consider the LSS extension of the SM consisting of two families of sterile neutrinos $\{\nu_{Ri}\}$ and $\{S_i\}$ (with $i = e$, μ, τ) [35,40,41], which yields, in principle, the blocks of Eq. (2) (up to here, active neutrinos remain massless). Then, by generating the term $\frac{1}{2}\mu_{ij}\overline{S_i^c}S_j$ (μ term) or $\varepsilon_{ij}\overline{\nu_{Li}}S_j$ (ε term), we obtain masses for active neutrinos according to inverse and linear seesaw regimes, respectively. With this, we express the neutrino mass sector in either the flavor basis (ν_L^c, ν_R, S) or mass basis (ν_ℓ, N_1, N_2) according to

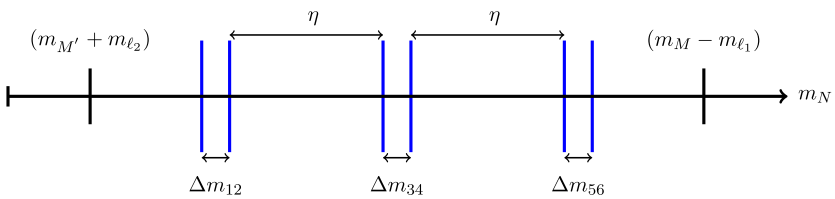


FIG. 1. Schematic representation of the pair distribution in the on-shell mass range for Scenario III.

$$\begin{aligned}\mathcal{L}^{\nu}_{\rm mass} &= m_D\overline{\nu_L}\nu_R + M\overline{\nu^c_R}S + [\mu\,\text{term or}\,\varepsilon\,\text{term}] + \text{H.c.}\\ &= m_\ell\overline{\nu^c_\ell}\nu_\ell + m_1\overline{N^c_1}N_1 + m_2\overline{N^c_2}N_2\\ &= m_\ell\overline{\nu^c_\ell}\nu_\ell + m_1(\overline{N^c_1}N_1 + \overline{N^c_2}N_2) + \Delta\overline{N^c_2}N_2, \qquad (3)\end{aligned}$$

where $m_D \propto vY$ and $\Delta = m_2 - m_1$ (all of them are 3×3 matrices). Now, following the reasoning of Ref. [28], we can fix $\Delta = 0$, obtaining three pairs of exactly degenerate heavy neutrinos (their masses are given by the eigenvalues of $m_1 \simeq M$, i.e., we have three pairs of exactly degenerate heavy neutrinos); after this, we produce a tiny term $\Delta'\overline{N^c_2}N_2$, with $\Delta' \ll m_1$, which causes the spectrum in the $N_{1,2}$ states to become quasidegenerate. Later, in Sec. III, we show that the assumption of weakly interacting heavy neutrinos ($\Gamma_N \ll m_N$), together with the requirement of maximum CP violation, is enough to justify small values for Δ. Then, we ask about the symmetry that we have lost in the transition from the $\Delta = 0$ case to the one in Eq. (3). In fact, noting that the second term in the third line of Eq. (3) can be written as $m_1\overline{\Psi^c}\Psi$, where $\Psi = N_1 + N^c_2$ (they have the same absolute eigenvalues), we can establish that the states ν_ℓ and $\Psi = N_1 + N^c_2$ have definite charges $(q_\ell, q_\Psi) = (0, \neq 0)$ under a certain U(1) group, so the operation

$$\begin{aligned}\nu_\ell &\to e^{\imath q_\ell \alpha}\nu_\ell,\\ \Psi &\to e^{\imath q_\Psi \alpha}\Psi \qquad (4)\end{aligned}$$

(where α is some global phase) leaves $\mathcal{L}^{\nu}_{\rm mass}|_{\Delta=0}$ invariant. Thus, the inclusion of $\Delta'\overline{\Psi^c}\Psi$, with $\Delta' \ll M$, slightly spoils this symmetry (it is clear that $q_{N_2} = -q_\Psi$). Besides, the fact that $\Delta' = 0$ restores a (global) symmetry in the Lagrangian [which is due to Eq. (4) in Eq. (3)] is enough to expect Δ' to be naturally small, in the 't Hooft sense [38]. This means that any correlation function which does not conserve the charges q of Eq. (4) should be proportional to Δ', and thus the running coupling of Δ' is necessarily proportional to itself. In other words, if we start with a small Δ' at some given energy, the corresponding β function (which is also proportional to Δ') implies that it remains small at another energy. (This is the same as in the correction to fermion masses: the mass term itself breaks the chiral symmetry in the Lagrangian, and therefore the anomalous dimension is proportional to m.)

Therefore, because we know how to get QDHν, we shall consider three possible scenarios, depending on how many of them can mediate as on-shell particles in the above-mentioned RMD. For this purpose we require that the masses m_{N_i} of all the intermediate neutrinos lie in the range [18]

$$m_{M'} + m_{\ell_2} \leq m_{N_i} \leq m_M - m_{\ell_1}, \qquad (5)$$

where ℓ_i are the final charged leptons. Scenario I includes only one on-shell pair and does not offer new phenomenology (at the GeV neutrino-mass scale) with respect to that proposed in the νMSM model [25,26], where the masses of quasidegenerate neutrinos are in the few-GeV range, which was studied in Ref. [27] (see Refs. [17–19] for predictions about RMD). Scenario II has two pairs of QDHν in the on-shell range. It is important to mention that, because their masses are sufficiently large, both Scenarios I and II offer opportunities for searching for sterile neutrinos in collider experiments such as the LHC [42–46] (particularly for neutrino masses on the order of TeV); however, it has been shown (with some caveats) that sterile neutrinos heavier than the LHC mass scale do not cause leptogenesis in cases where LNV is observed at the LHC [47] (see Ref. [48], Sec. 6, for a helpful discussion). Finally, Scenario III takes into account all three pairs in the on-shell mass range (see Fig. 1). For simplicity, we shall assume that the mass gap η between different pairs satisfies $\eta/m_N \sim \Delta m_\ell/m_\ell$, or $\eta \sim 0.1 m_N$, where m_ℓ is the mass of light neutrinos (i.e., heavy neutrino pairs are as degenerate as light ones). In Sec. IV we show results of RMD within the assumptions of each of these scenarios. As mentioned, future experiments such as SHiP [29] will be meson factories and could explore intermediate particle masses, from ≈106 MeV to ≈6 GeV, depending on the initial and final states.

III. MESON DECAYS MEDIATED BY THREE PAIRS OF QDHν

Now we describe the RMD process $M^+ \to \ell^+_1\ell^+_2 M'^-$, where M and M' are pseudoscalar mesons: $M = D_s, B_c$ and $M' = \pi, K, D_s$. The most relevant contribution to this decay is shown in Fig. 2, and occurs via exchanges of on-shell neutrinos N_j. The contributions mediated by off-shell neutrinos and processes including loops (t-channel) are strongly

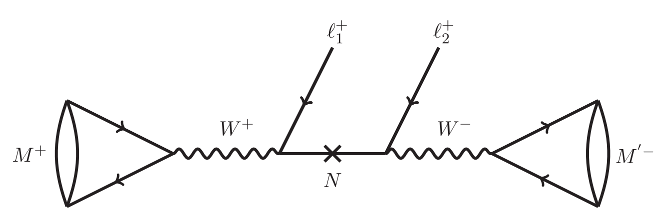


FIG. 2. The s-channel of the lepton-number-violating decay $M^+ \to \ell^+_1\ell^+_2 M'^-$.

suppressed [16,49]. Therefore, we focus on the on-shell mass region [Eq. (5)] and tree-level processes (s-channel).

As we can see in Fig. 2, the process violates the lepton number by two units; in consequence, the intermediate neutrinos (N_j) must be Majorana fermions.

In order to fix notation, we consider that states $\{N_j, N_k\}$ are neutrinos with masses $m_1, \ldots, m_{N_h}$ ($N_h = 6$), where the quasidegenerate pairs are 12, 34, and 56, whereas the states with arbitrary differences ($\sim 10^{1,2}$ MeV) are 13, 14, 15, 16, 23, 24, 25, 26, 35, 36, 45, and 46. With this, let [20]

$$\mathcal{M}_i = -G_F^2 V_{q_u q_d} V_{q'_u q'_d} f_M f_{M'} \frac{B_{\ell_1 N_i} B_{\ell_2 N_i} m_{N_i}}{p_{N_i}^2 - m_{N_i}^2 + \imath m_{N_i} \Gamma_{N_i}} \times \bar{u}(l_2) \not{p}_{M'} \not{p}_M (1 - \gamma_5) v(l_1) \tag{6}$$

be the amplitude for the process $M^+ \to \ell_1^+ \ell_2^+ M'^-$ ntermediated by the eigenstate N_i, with mass m_{N_i}, which enters the charged current through the mixing $B_{\ell_i N_j} = \sum_\alpha (V_{i\alpha}^{\mathrm{lep}})^* U_{\alpha j}$ [where $V_{i\alpha}^{\mathrm{lep}}$ ($U_{\beta j}$) is the matrix element which relates the ith (jth) charged lepton (neutrino) mass state with the αth (βth) flavor one]. Here we consider that the $B_{\ell N}$ elements includes all the CP-violating phases [50]. Further, p_M, $p_{M'}$ are the momenta of mesons M, M' and l_1, l_2 are the momenta of charged leptons ℓ_1, ℓ_2, whereas f_M, $f_{M'}$ are the meson decay constants, and $V_{\alpha\beta}$ corresponds to the Cabibbo-Kobayashi-Maskawa (CKM) element (for instance, if M is a kaon K^+, $V_{q_u q_d} = V_{us}$). Thus, the squared amplitude probability for this process is given by

$$|\mathcal{M}|^2 = \sum_{a,b=1}^{N_h} \mathcal{M}_a^\dagger \mathcal{M}_b = \sum_{i=1}^{N_h} |\mathcal{M}_i|^2 + \sum_{\substack{j,k>j \\ \text{adpairs}}} (\mathcal{M}_j^\dagger \mathcal{M}_k + \mathcal{M}_k^\dagger \mathcal{M}_j) + \sum_{\substack{i=1,3,5 \\ i'=i+1}} (\mathcal{M}_i^\dagger \mathcal{M}_{i'} + \mathcal{M}_{i'}^\dagger \mathcal{M}_i)$$
$$= \sum_{i=1}^{N_h} |\mathcal{M}_i|^2 + 2 \sum_{\substack{j,k>j \\ \text{adpairs}}} \mathrm{Re}[\mathcal{M}_j^\dagger \mathcal{M}_k] + 2 \sum_{\substack{i=1,3,5 \\ i'=i+1}} \mathrm{Re}[\mathcal{M}_i^\dagger \mathcal{M}_{i'}], \tag{7}$$

where "ad pairs" refers to neutrino pairs which have arbitrarily different masses (13,14,15,$\cdots$). Given the fact that heavy neutrinos are weakly interacting particles it is useful to implement the NWA,

$$\frac{m_N}{(p_N^2 - m_N^2)^2 + (m_N \Gamma_N)^2} \to \pi \frac{\delta(p_N^2 - m_N^2)}{\Gamma_N}, \tag{8}$$

where Γ_N is the total decay width of the intermediate on-shell neutrinos, which can be approximated in the following way:

$$\Gamma_N \approx \mathcal{K}_j^{Ma} \frac{G_F^2 M_{N_j}^5}{96\pi^3}. \tag{9}$$

Here

$$\mathcal{K}_j^{Ma} \equiv \mathcal{K}_j^{Ma}(M_{N_j}) = \mathcal{N}_{eN_j}^{Ma} |B_{eN_j}|^2 + \mathcal{N}_{\mu N_j}^{Ma} |B_{\mu N_j}|^2 + \mathcal{N}_{\tau N_j}^{Ma} |B_{\tau N_j}|^2, \quad (j = 1, \ldots, n), \tag{10}$$

where $\mathcal{N}_{\ell N_j}^{Ma}$ are the effective mixing coefficients presented in Fig 3.

In the mass range of our interest ($\sim$1–6 GeV), and taking into account the upper limits over the mixing elements ($|B_{eN_j}|^2$, $|B_{\mu N_j}|^2$, and $|B_{\tau N_j}|^2$) presented in Ref. [39], the parameter $\mathcal{K}_j^{Ma}$ can take values between 10^{-4}–10^{-6}, and consequently $\Gamma_N \sim 10^{-14}$–10^{-16} GeV (i.e., heavy neutrinos are weakly interacting particles). In order to obtain an analytical expression for the terms of Eq. (7), and by extending the treatment of the decay width for only one pair of QDHν, in Refs. [18,19] we find the corresponding decay width for three pairs:

$$\Gamma_{\mathrm{RMD}} = \frac{1}{2!} (2 - \delta_{\ell_1 \ell_2}) \frac{1}{2 M_M (2\pi)^6} \int d_3 |\mathcal{M}|^2$$
$$= 2(2 - \delta_{\ell_1 \ell_2}) \Bigg[\sum_{i=1}^{6} |B_{\ell_1 N_i}|^2 |B_{\ell_2 N_i}|^2 \tilde{\Gamma}_M^{(ii)} + \sum_{\substack{j,k>j \\ \text{adpairs}}} 2 |B_{\ell_1 N_j}| |B_{\ell_1 N_k}| |B_{\ell_2 N_j}| |B_{\ell_2 N_k}| \tilde{\Gamma}_M^{(jj)} \cos\theta_{jk} \delta_{jk}$$
$$+ 2 |B_{\ell_1 N_1}| |B_{\ell_1 N_2}| |B_{\ell_2 N_1}| |B_{\ell_2 N_2}| \tilde{\Gamma}_M^{(11)} \cos\theta_{12} \delta_{12} + 2 |B_{\ell_1 N_3}| |B_{\ell_1 N_4}| |B_{\ell_2 N_3}| |B_{\ell_2 N_4}| \tilde{\Gamma}_M^{(33)} \cos\theta_{34} \delta_{34}$$
$$+ 2 |B_{\ell_1 N_5}| |B_{\ell_1 N_6}| |B_{\ell_2 N_5}| |B_{\ell_2 N_6}| \tilde{\Gamma}_M^{(55)} \cos\theta_{56} \delta_{56} \Bigg]. \tag{11}$$

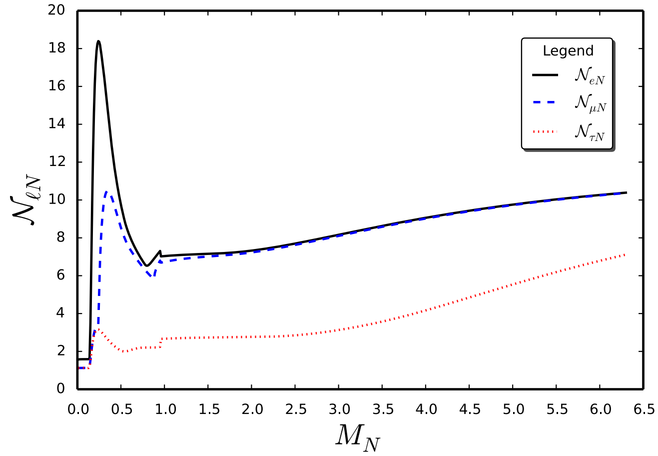


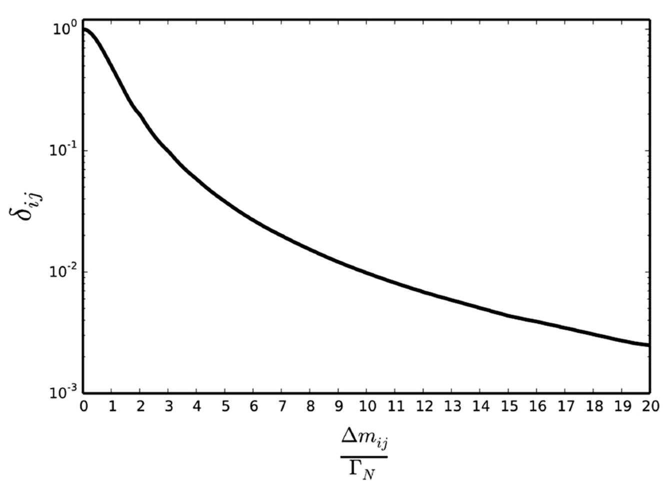
FIG. 3. Effective mixing coefficients for Majorana neutrinos.

Here d_3 is the number of states available per unit of energy in the final state (three-body phase space) where the factor $(2-\delta_{\ell_1\ell_2})$ refers to the symmetry factor of the amplitude, the factor 2 in front of the latter is due to the contribution of the crossed channel $(\ell_1 \leftrightarrow \ell_2)$, θ_{jk} represents the phase difference $\theta_{jk} = (\phi_{1j} + \phi_{2j} - \phi_{1k} - \phi_{2k})$ related to the heavy-light neutrino mixing elements by means of $B_{\ell_j N_k} \equiv |B_{\ell_j N_k}| e^{i\phi_{jk}}$ (where j, $k = 1$, 2; see Ref. [19]), and δ_{jk} measures the effect of N_k-N_j overlap and is given by $\delta_{jk} = \frac{\mathrm{Re}\tilde{\Gamma}_M^{(jk)}}{\tilde{\Gamma}_M^{(jj)}}$; their values are obtained via numerical calculations implemented independently in PYTHON and FORTRAN using the VEGAS algorithm [51], and are presented in Fig. 4. It is worth mentioning that δ_{jk} only depend on the mass difference Δm_{jk}. Finally,

$$\tilde{\Gamma}_M^{(jj)} = \frac{K_M^2 m_M^5}{128\pi^2} \frac{m_{N_j}}{\Gamma_{N_j}} \lambda^{1/2}(1, x_j, x_{\ell_1}) \times \lambda^{1/2}\left(1, \frac{x'}{x_j}, \frac{x_{\ell_2}}{x_j}\right) \times Q(x_j; x_{\ell_1}, x_{\ell_2}, x') \quad (j = 1, \ldots, 6) \tag{12}$$

is the normalized decay width of each sterile neutrino [19] (x_j, x_{ℓ_j}, the functions λ and Q coming from the integration in d_3, and K_M are detailed in the Appendix).

The main difference with the case with only one pair of QDHν (Scenario I) is the presence, in Eq. (11), of all the interference terms except the one corresponding to $N_1 N_2$. As we shall see in Sec. IV, these contributions will increase the branching ratios for RMD, allowing strict restrictions over the couplings $B_{\ell N}$. Besides, in Eq. (12) we use the fact that the interference term $\tilde{\Gamma}_M^{(jk)}$ between heavy neutrinos is proportional to $\tilde{\Gamma}_M^{(jk)} \propto \tilde{\Gamma}_M^{(jj)} \delta_{jk} \propto \frac{m_N}{\Gamma_N} \delta_{jk}$. On the other hand, the numerical integrations over the squared amplitude of Eq. (7) show that only the adjacent pairs (12, 34, and 56) contribute, whereas all the other interferences (13, 14, …, 46) are strongly suppressed due to the fact that $\Delta m_{13}, \Delta m_{14}, \ldots, \Delta m_{46} \sim \eta \sim 0.1 m_N \ggg \Gamma_N$, and we see in Fig. 4 that $\delta_{ij} \to 0$ when $\Delta m_{ij} \ggg \Gamma_N$. Therefore, the decay width of the pseudoscalar meson [Eq. (11)] depends on the neutrino masses m_N, the matrix elements $B_{\ell N}$, and indirectly on the degeneracy level $y_{jk} \equiv \frac{\Delta m_{jk}}{\Gamma_{N_j}}$ [18]. It is important to note that the relation between Δm_{jk} and y_{jk} is independent of the already assumed NWA; besides, this y_{jk} enters only indirectly into Eq. (12), through the overlaps δ_{jk}. The latter are manifested implicitly in the parameter Γ_{N_j} present in Eq. (12). Previous studies [18,19,49,52] have shown that $\Delta m_{jk} = \Gamma_N$ is the best choice for measurable CP violation and feasible baryogenesis via leptogenesis [53–55]. From now on we shall choose $\delta_{jk} = 0.5$ $(\Delta m_{jk} = \Gamma_N)$ in order to leave an open door for leptogenesis. In addition, we must take into account the acceptance factor, which is defined as the probability of the on-shell neutrino N_j to decay inside the detector of length L,



$\frac{\Delta m_{ij}}{\Gamma_N}$	δ_{ij}	*error*
0.0	$1.00 \cdot 10^{0}$	$6.17 \cdot 10^{-4}$
0.2	$9.62 \cdot 10^{-1}$	$6.22 \cdot 10^{-4}$
0.4	$8.63 \cdot 10^{-1}$	$5.21 \cdot 10^{-4}$
0.6	$7.35 \cdot 10^{-1}$	$4.78 \cdot 10^{-4}$
0.8	$6.10 \cdot 10^{-1}$	$4.21 \cdot 10^{-4}$
1.0	$5.00 \cdot 10^{-1}$	$4.21 \cdot 10^{-4}$
3.0	$9.99 \cdot 10^{-2}$	$3.74 \cdot 10^{-4}$
5.0	$3.83 \cdot 10^{-2}$	$2.71 \cdot 10^{-4}$
7.0	$2.00 \cdot 10^{-2}$	$2.05 \cdot 10^{-4}$
9.0	$1.21 \cdot 10^{-2}$	$1.69 \cdot 10^{-4}$
11.0	$8.14 \cdot 10^{-3}$	$1.35 \cdot 10^{-4}$
13.0	$5.87 \cdot 10^{-3}$	$1.26 \cdot 10^{-4}$
15.0	$4.36 \cdot 10^{-3}$	$1.11 \cdot 10^{-4}$
17.0	$3.46 \cdot 10^{-3}$	$1.03 \cdot 10^{-4}$
19.0	$2.71 \cdot 10^{-3}$	$8.63 \cdot 10^{-5}$

FIG. 4. Left: δ_{ij} parameter which measures the $N_i - N_j$ overlap. Right: Some values of δ_{ij} and their respective errors.

$$P_{N_j} \approx \frac{L}{\gamma_{N_j}\tau_{N_j}\beta_{N_j}} \approx \frac{L\Gamma_{N_j}}{\gamma_{N_j}}, \qquad (13)$$

where γ_{N_j} is the Lorentz time dilation factor in the lab system (~2). Consequently, the effective branching ratio (EBR) is

$$\mathrm{Br}^{\mathrm{eff}}(M) = P_{N_j}\mathrm{Br}(M) = P_{N_j}\frac{\Gamma_{\mathrm{RMD}}}{\Gamma(M^{\pm} \to \mathrm{all})}. \qquad (14)$$

IV. RESULTS

Now we apply what we know about the decay of mesons mediated by three pairs of on-shell QDHν to the processes $D_s^+ \to \mu^+\mu^+\pi^-$, $D_s^+ \to \mu^+\mu^+K^-$, $B_c^+ \to \mu^+\mu^+\pi^-$, and $B_c^+ \to \mu^+\mu^+D_s^-$. As we mentioned in the previous section, we can deal with three possible scenarios, depending on how many pairs of QDHν can mediate as on-shell particles in the RMD, which depends on whether their masses lie in the range of Eq. (5). For simplicity, we shall assume that mass gaps between different pairs η satisfy $\eta/m_N \sim \Delta m_\ell/m_\ell$, or $\eta \sim 0.1m_N$, where m_ℓ represents the masses of active neutrinos. Then, the masses of QDHν are labeled as $m_{N_1} = m_N - \eta$, $m_{N_3} = m_N$, and $m_{N_5} = m_N + \eta$, where just the second one will be our independent variable for phenomenological purposes. In consequence, the masses of the heavy neutrinos $(N_1, N_2; N_3, N_4; N_5, N_6)$ are given, respectively, by $(m_{N_1}, m_{N_1} + \Delta m_{12}; m_{N_3}, m_{N_3} + \Delta m_{34}; m_{N_5}, m_{N_5} + \Delta m_{56})$. In Fig. 5 we show the EBR per unit of coupling $|B_{\ell N}|^4$

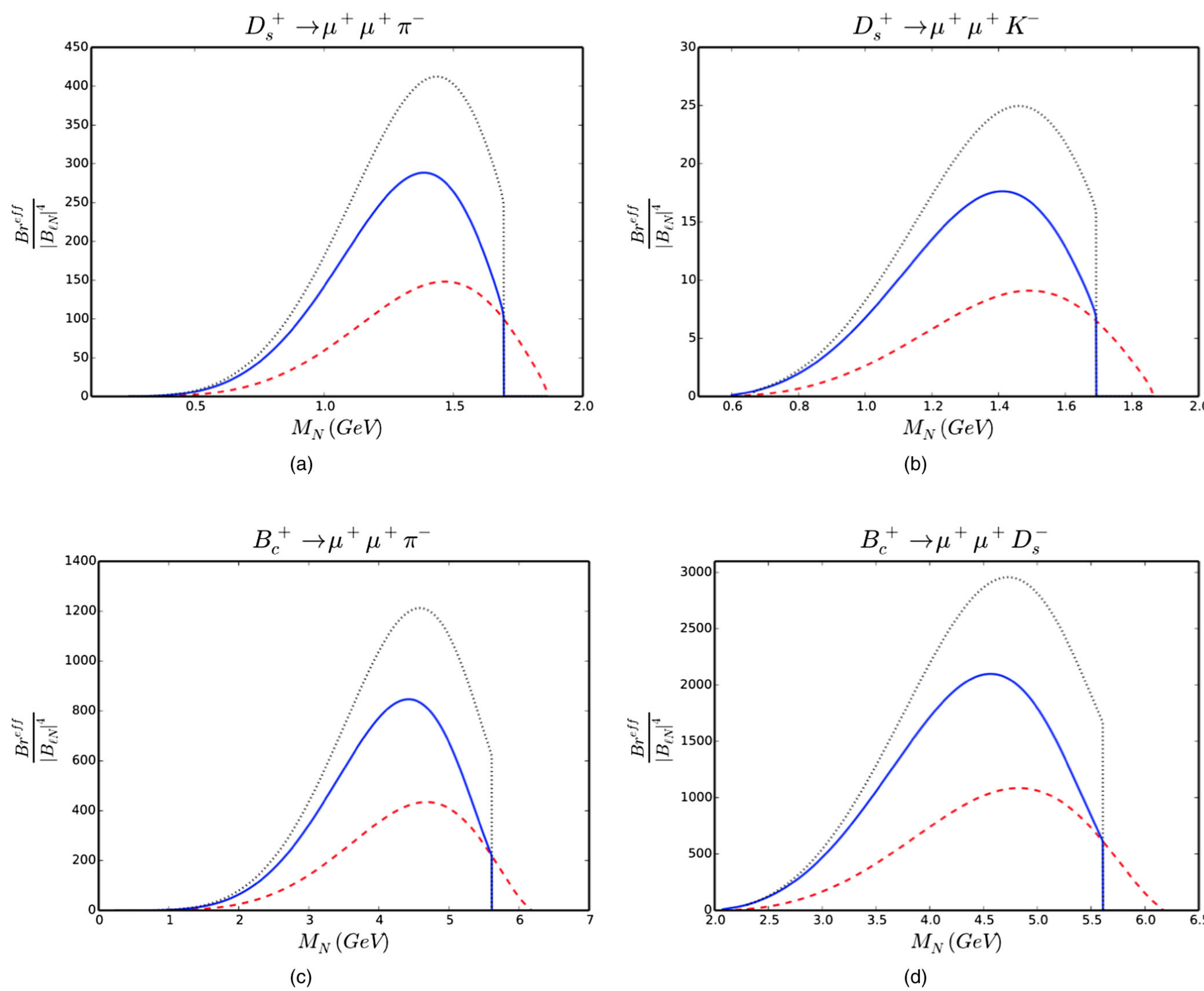


FIG. 5. Effective branching ratio divided by $|B_{\ell N}|^4$, for the processes (a) $D_s^+ \to \mu^+\mu^+\pi^-$, (b) $D_s^+ \to \mu^+\mu^+K^-$, (c) $B_C^+ \to \mu^+\mu^+\pi^-$, and (d) $B_C^+ \to \mu^+\mu^+D_s^-$, as a function of sterile neutrino mass, for $L = 1$[m], $\gamma_N = 2$, and $\eta = 0.1m_N$. The dashed line represents the values for Scenario I, the solid line represents the value for Scenario II, and the dotted line represents the values for Scenario III. We regard the cases with CP violation ($\delta_{ij} = 0.5$) and $\cos\theta_{jk} = \frac{1}{\sqrt{2}}$.

[Eq. (14)] for different meson decays ($M = D_s, B_c$ and $M' = D_s, \pi, K$) assuming $|B_{\mu N_i}| = |B_{\ell N}|$ for $i = 1, \ldots, 6$ (all equal), as a function of such $m_{N_3} = m_N$, regarding all three scenarios for different initial and final states. In all of them we see that the inclusion of two or three pairs of QDHν (Scenario II or III, respectively) results in an increase of the EBR in comparison with the case with only one such pair [18,19]. In Fig. 6 we show the ratio between the EBR calculated with three pairs of QDHν (EBR3) and the one with only one pair (EBR1) for the decays of Fig. 5. In fact, we see that even when m_N lies in the range of Eq. (5) (i.e., [0.25–1.76] GeV and [0.60–1.76] GeV for D_s, [0.25–6.30] GeV and [1.98–6.30] GeV for B_c), the actual ranges for the plots of Scenario III in Figs. 5–6 are the ones for which

$$\frac{m_{M'} + m_{\ell_2}}{1 - f} \leq m_N \leq \frac{m_M - m_{\ell_1}}{1 + f}, \qquad (15)$$

where $f = \eta/m_N \sim 0.1$, because we demand that all three pairs contribute to the EBR, and, then to their respective ratios with EBR1 (otherwise we are in Scenario I or II, which are not the goal of this work). This is the reason why the decays of D_s and B_c exhibit an abrupt cut at $m_N \simeq 1.7$ GeV and $m_N \simeq 5.7$ GeV, respectively. Besides, in Fig. 6 we see that predictions for EBR3 are between 3 and 4 times greater than EBR1. It is interesting to note that, even when these ratios are almost constant in the allowed range for m_N, they have a significant increase (cusp) near the extremes. To see why this happens, in Fig. 6 we show the corresponding ratios for different values of η (the

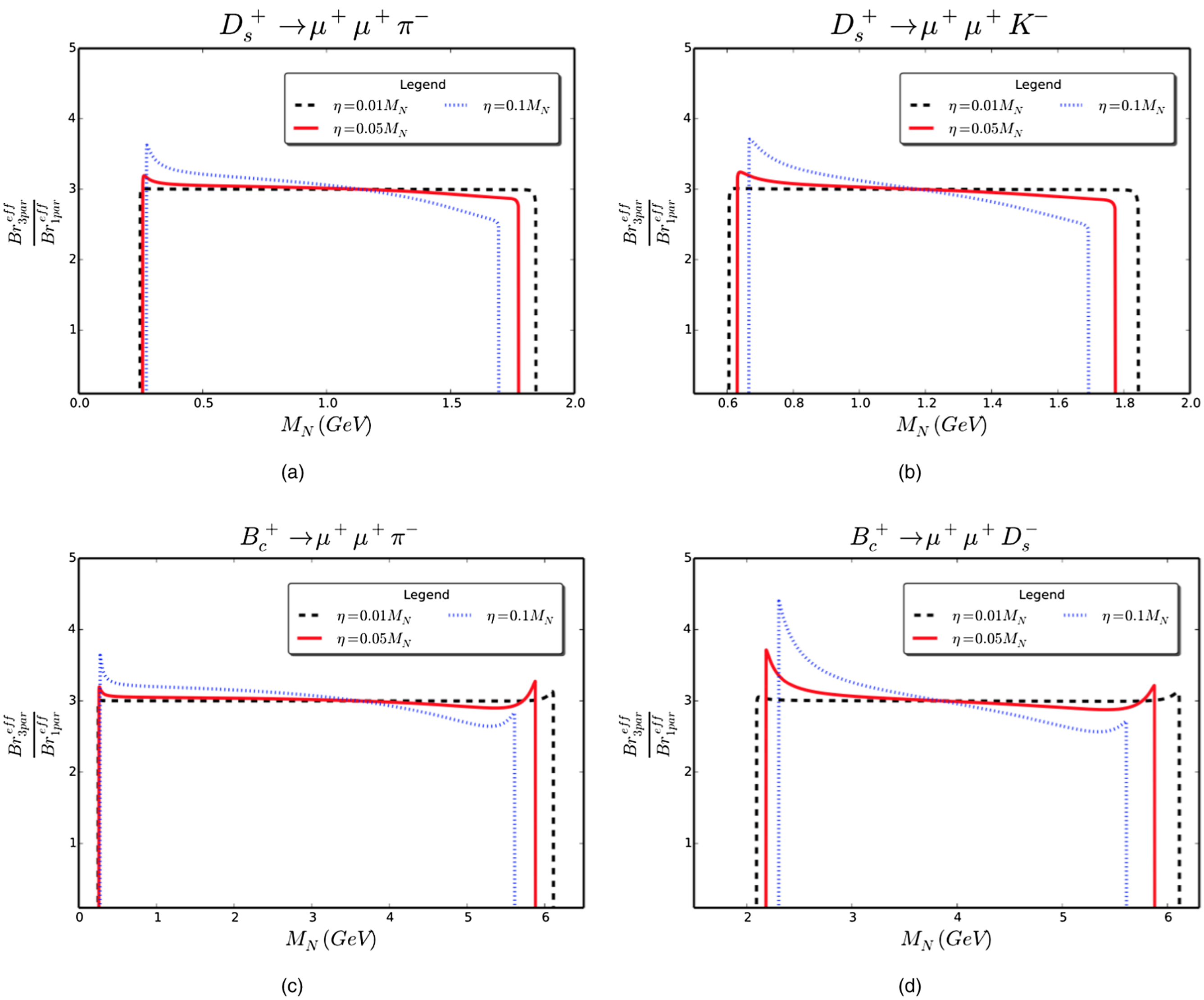


FIG. 6. Quotients of the effective branching ratios for different values of η using one pair and three pairs of QDHν, for the processes (a) $D_s^+ \to \mu^+\mu^+\pi^-$ and (b) $D_s^+ \to \mu^+\mu^+K^-$, (c) $B_c^+ \to \mu^+\mu^+\pi^-$, and (d) $B_c^+ \to \mu^+\mu^+D_s^-$, as a function of sterile neutrino mass. We used $L = 1$[m] and $\gamma_N = 2$.

degeneracy among QDHν), and we note that we get a smaller increase as we reduce η. This is easy to understand in light of Figs. 1 and 5: (i) as $m_N = m_{N_3} \simeq m_{N_4}$, the EBR for values of small (large) m_N always get contributions from one pair with masses around $m \sim m_N + \eta$ ($\sim m_N - \eta$), so (ii) only when η is sufficiently small do all the pairs lie in the extreme zone, given a total EBR corresponding only to extreme masses; (iii) otherwise, when η is large an EBR labeled with an extreme mass contains contributions from masses closer to the middle region of Eq. (15), which clearly yields greater values of the EBR. It is worth mentioning that when we ignore the η effect (i.e., making $\eta \ll m_N$), the EBR3 is just amplified by a factor of 3 with respect to EBR1 (hence the shape of the dashed lines in the plots of Fig. 6). [This is because all the mass dependence from phase space in Eq. (12) is the same for each intermediate heavy neutrino.] Finally, we note that the η effect produces an increase or a decrease of these ratios when m_N is, respectively, smaller or larger than a certain m_N (a function of the masses of external particles). This can be understood by looking at Fig. 5, where the peak of EBR3 always occurs for a mass smaller than the mass for which EBR1 has its maximum; therefore, comparing the slopes of EBR1 and EBR3 after their respective maxima, we see that the latter decreases faster than the former, contributing to the decrease of the ratio in comparison with the dashed curves of Fig. 6. Also, we note that the interference terms in Eq. (11) do not seem to manifest in the ratios of Fig. 6. This is due to the fact that—as we have one such interference in the denominator and three in the numerator—they mutually cancel, with only the above-mentioned factor of 3 remaining. It is important to point out

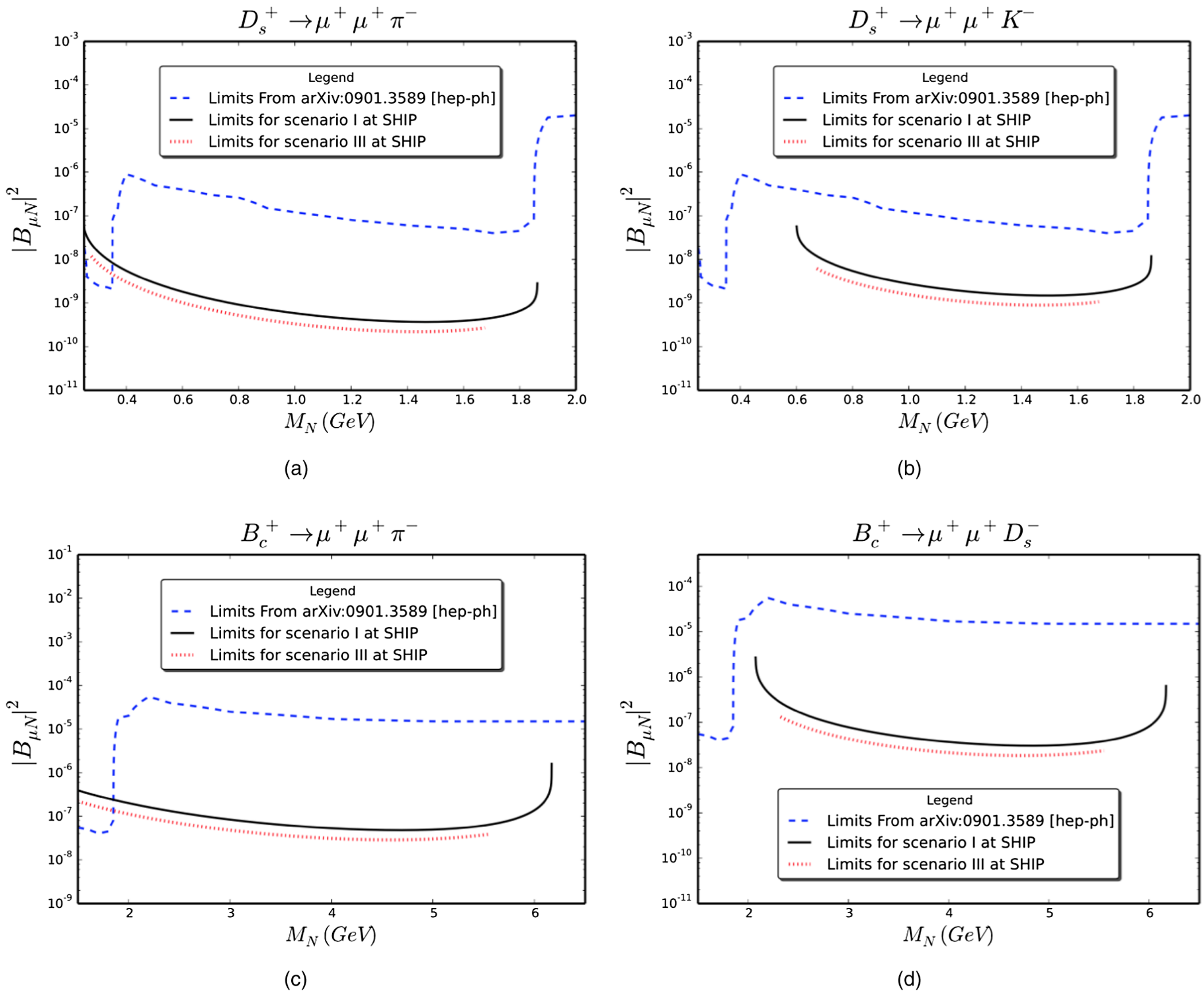


FIG. 7. Limits for $|B_{\mu N}|^2$ from Ref. [39] vs the ones we get using one pair and three pairs of QDHν for the processes (a) $D_s^+ \to \mu^+\mu^+\pi^-$, (b) $D_s^+ \to \mu^+\mu^+K^-$, (c) $B_c^+ \to \mu^+\mu^+\pi^-$, and (d) $B_c^+ \to \mu^+\mu^+D_s^-$, based on expected luminosities for mesons. As before, we used $L = 1[\mathrm{m}]$, $\gamma_N = 2$, and $\eta = 0.1 m_N$.

that our choice of $\theta_{ij} = \pi/4$ in Figs. 5–6 is a necessary condition to simultaneously maximize CP violation ($y_{jk} = 1$, i.e., $\delta_{jk} = 0.5$) and the decay width presented in Eq. (11). It is worth mentioning that the exact point of maximal CP violation implies $\delta_{jk} = 0.5$ and simultaneously $\cos\theta_{ij} = 0$ [18,19].

Finally, in Fig. 7 we show a comparison between the current upper limits for $|B_{\mu N}|^2$ provided by Ref. [39] (based on type I seesaw model) and the ones obtainable from the predictions of our Scenarios I and III, again under the assumption that $|B_{\mu N}| \sim |B_{\ell N}|$; for an extra discussion about heavy-light neutrino mixing see Refs. [56–58]. This was done by demanding that the number of predicted events for RMD was $N_{\rm RMD} = (|B_{\mu N}|^4 f_k) \times N_{\rm mes} \geq 1$, where $N_{\rm mes}$ is the production rate of mesons per year at SHiP,[1] $N_{D_s} \simeq 5.0 \times 10^{16}$ and $N_{B_c} \sim 10^{12}$, and f_k is the factor that includes all the kinematics due to each scenario (in fact, the η effect is present). Therefore, the plots of Scenarios 1 and 3 indicate the minimum value of $|B_{\mu N}|^2$ capable of producing one event of RMD. Even when predictions for EBR3 allow smaller limits for $|B_{\mu N}|$, their differences with respect to the ones for EBR1 are dominated only by the factor $\sqrt{3}$ coming from the three pairs of QDHν, which is even less notable in a logarithmic plot. Now, the fact that these limits are so close implies that it will be difficult to decide which underlying seesaw scenario is the origin of these RMDs.

V. CONCLUSION

We studied the rare decays of mesons ($D_s^+ \to \mu^+\mu^+\pi^-$, $D_s^+ \to \mu^+\mu^+K^-$, $B_c^+ \to \mu^+\mu^+\pi^-$, and $B_c^+ \to \mu^+\mu^+D_s^-$) as they can produce six on-shell heavy neutrinos with masses in the range of ~1–6 GeV. For this purpose, we worked in the context of a low-scale seesaw model constructed with the SM field ν_L and two extra neutrinos ν_R and S, where the mass of light neutrinos (m_ℓ) is obtained by the introduction of a small parameter in the neutrino mass matrix (μ or ε for the inverse or linear seesaw regime, respectively), allowing that the large scale of the model (M), the same as the heavy neutrino masses (m_N), lies in the above-mentioned range. In order to reproduce the conditions we find in the literature (those leading to maximum CP violation and feasible baryogenesis through leptogenesis) we promoted an argument, based on naturalness, which produces a heavy neutrino mass spectrum with three pairs of quasidegenerate neutrinos (Fig. 1), where the differences between adjacent masses satisfy $m_{N_2} - m_{N_1} \simeq \Gamma_{N_1}$, $m_{N_4} - m_{N_3} \simeq \Gamma_{N_3}$, and $m_{N_6} - m_{N_5} \simeq \Gamma_{N_5}$, where $\Gamma_{N_i} \sim 10^{-20}$ GeV are the total decay widths of N_i's. In other words, we assumed that heavy neutrinos are particles interacting weakly with SM physics. Likewise, we fixed the difference among pairs of heavy neutrinos, $\eta \sim 0.1 m_N$, such that these pairs have similar relative mass patterns as the active neutrinos. In our calculations we simplified many numerical details concerning the effective branching ratios, making all the couplings between the heavy neutrinos and muons equal, $B_{\mu N_i} = B_{\mu N_j}$. We enhanced CP violation effects by choosing the conditions $y_{jk} = 1$, implying that the overlap parameters δ_{jk} between neutrino resonances become appreciable ($\delta_{jk} = 0.5$). For definiteness, we chose the CP-violating phase differences $\phi_i - \phi_j$ such that $\cos(\phi_i - \phi_j) = 1/\sqrt{2}$ when the overlap between wave functions of heavy neutrinos N_i and N_j is $\delta_{ij} = 0.5$. Since the masses of the on-shell heavy neutrinos needed to be in a determined kinematic range related with the masses of the external particles of the decays, we considered three possible scenarios depending on how many pairs this range actually contains, and we obtained a consistent increase in the EBR of RMD as we increased the number of QDHν. In particular, we concluded that the inclusion of two new pairs of QDHν essentially triples the EBR of the RMD decay width in comparison with the case with only one pair. Besides, we worked with an effective range for neutrino masses in order to consider all three pairs, and we found that the ratio between EBR3 and EBR1 was not exactly 3, but there was a small variation due to the fact that these pairs were separated by an amount $\eta \leq 0.1 m_N$; this effect vanishes as $\eta \to 0$. The approximate tripling of the EBR we found is consistent with the fact that the mass factor coming from the phase-space integral is approximately the same, independently of the number of intermediate on-shell neutrino pairs. On the other hand, RMD detection—together with the maximization of CP asymmetry (and hence the necessity of QDHν)—.is not necessarily attributable only to scenarios like the νMSM, but also to low-scale seesaw mechanisms. Furthermore, the latter needs a smaller coupling between charged leptons and sterile neutrinos than the former (Fig. 7). Even when our RMDs need neutrinos with masses around a few GeV, it is interesting to note that the off-shell range neutrinos of Scenarios I and II could display new phenomenology, given that their masses lie in the appropriate range [59]. Therefore, there is a phenomenological distinction between this proposal and the one, for instance, of the νMSM: in fact, Scenarios I and II simultaneously provide QDHν pairs of neutrinos which contribute to the EBR of RMD and heavier neutrinos (not necessarily QDHν) whose phenomenology is testable at the LHC. As a consequence, if experiments find both RMD in a manner compatible with QDHν and phenomenology of heavier neutrinos at, say, the 100 GeV scale, it could be a signal in favor of the LSS mechanism rather than the type I seesaw mechanism. Finally, it is worth mentioning that, for instance, our Scenario II provides a couple of quasidegenerate neutrinos

[1] M. Drewes (TU Munich) and N. Serra (Zurich U) (private communication).

whose masses are still free to be set in the appropriate range in order to contribute to neutrinoless double-beta decay; some work has already been done in this context [60–63].

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APPENDIX: KINEMATIC FUNCTIONS

The kinematic functions shown in Eq. (12), coming from the phase-space integration, are given by the expressions

$$\begin{aligned}
\lambda(y_1,y_2,y_3) &= y_1^2+y_2^2+y_3^2-2y_1y_2-2y_2y_3-2y_3y_1,\\
Q(x;x_{\ell_1},x_{\ell_2},x') &= \left\{\frac{1}{2}(x-x_{\ell_1})(x-x_{\ell_2})(1-x-x_{\ell_1})\left(1-\frac{x'}{x}+\frac{x_{\ell_2}}{x}\right)\right.\\
&\quad+[-x_{\ell_1}x_{\ell_2}(1+x'+2x-x_{\ell_1}-x_{\ell_2})-x_{\ell_1}^2(x-x')+x_{\ell_2}^2(1-x)\\
&\quad\left.+x_{\ell_1}(1+x)(x-x')-x_{\ell_2}(1-x)(x+x')]\right\}\\
&= \frac{1}{2}[(1-x)x+x_{\ell_1}(1+2x-x_{\ell_1})]\left[x-x'-2x_{\ell_2}-\frac{x_{\ell_2}}{x}(x'-x_{\ell_2})\right],
\end{aligned} \tag{A1}$$

where

$$x_j=\frac{M_{N_j}^2}{M_M^2},\qquad x_{\ell_s}=\frac{M_{\ell_s}^2}{M_M^2},\qquad x'=\frac{M_{M'}^2}{M_M^2},\qquad (j=1,2;\ell_s=\ell_1,\ell_2).$$

Since the valence quark content of M^+ and M'^- is $q_u\bar{q}_d$ and $q'_u\bar{q}'_d$, respectively, the constants involved in the normalized decay widths of Eq. (12) are

$$K_M=-G_F^2V_{q_uq_d}V_{q'_uq'_d}f_Mf_{M'}\quad\text{with}\quad K_M=(K_M)^*,$$

where f_M and $f_{M'}$ are the meson decay constants of M^+ and M'^-, whereas $V_{q_uq_d}$ and $V_{q'_uq'_d}$ are its CKM elements.

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