New class of de Sitter vacua in string theory compactifications

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String theory contains few known working examples of de Sitter vacua, four-dimensional universes with a positive cosmological constant. A notorious obstacle is the stabilization of a large number—sometimes hundreds—of moduli fields that characterize the compact dimensions. We study the stability of a class of supersymmetric moduli (the complex structure moduli and dilaton in type-IIB flux compactifications) in the regime where the volume of the compact space is large but not exponentially large. We show that, if the number of moduli is very large, random matrix theory provides a new stability condition, a lower bound on the volume. We find a new class of stable vacua where the mass spectrum of these supersymmetric moduli is gapped, without requiring a large mass hierarchy between moduli sectors or any fine-tuning of the superpotential. We provide the first explicit example of this class of vacua in the $\mathbb{P}^4_{[1,1,1,6,9]}$ model. A distinguishing feature is that all fermions in the supersymmetric sector are lighter than the gravitino.

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I. INTRODUCTION

It has been known for decades that string theory has low-energy solutions describing a four-dimensional universe with negative or zero cosmological constant, with the extra six dimensions "compactified" (for a review see [1–4]). From the four-dimensional point of view the compactified space is described by a set of fields called the moduli which describe, roughly, the size and shape of the extra dimensions. A much harder question is whether string theory can describe a four-dimensional universe with broken supersymmetry and a positive cosmological constant, a so-called *de Sitter vacuum* (dS), with a meta-stable compactification. In type-IIB string theory this question has been answered positively in a few scenarios, the best studied being the KKLT [5] constructions, first introduced by Kachru, Kallosh, Linde and Trivedi, large volume scenarios (LVS) [6,7] and the so-called *Kähler uplifted vacua* [8–14]. The effective low-energy theories describing these models typically involve hundreds of moduli fields, which can be divided into two classes: *Kähler moduli* and *complex structure moduli*. In addition we also have the *dilaton*, whose expectation value determines the string coupling constant. The interactions among all these fields are given by a complicated scalar potential, which makes a detailed perturbative stability analysis of these vacua unfeasible except in very simplified scenarios. In type-IIB flux compactifications, at the classical level, the scalar potential is induced by the presence of background fluxes (higher dimensional generalizations of electromagnetic fields) on the compact space [15]. Due to a Dirac condition these fluxes need to be quantized, and are therefore characterized by a set of integers. This leading contribution of the scalar potential depends only on the dilaton and the complex structure moduli (for short, the *complex structure sector*), and therefore it is necessary to take into account quantum effects in order to fix the remaining Kähler moduli.

To make the problem more tractable, it is often assumed that the background fluxes provide an effective stabilization mechanism for the complex structure sector, and it is not considered any further. The consistency of this approach has been checked for KKLT vacua [16–21] and large volume scenarios [22,23]. Here we discuss this matter for Kähler uplifted dS vacua.

In the large volume regime of type-IIB flux compactifications, both for LVS and Kähler uplifted dS vacua, the stabilization of the Kähler moduli is a result of the competition between the leading nonperturbative and α' (radiative) quantum corrections.[1] For these corrections to be under control it is necessary that the volume of the compactification, which belongs to the Kähler sector, has a large expectation value compared to the string length. A large compactification volume is also essential for the consistency of the 4-dimensional supergravity description of these models, and in particular for the Kaluza-Klein (KK) scale to be large compared to the supersymmetry

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[1] See [24–26] for discussions on the effect of string loop (g_s) corrections in these models.

breaking scale [7,23]. In LVS the vacua obtained in this way have a negative cosmological constant (anti–de Sitter), and thus additional interactions are needed to make the cosmological constant positive. Kähler uplifted vacua are particularly interesting because the dS vacuum is achieved without the need for extra ingredients (matter or branes), just with an appropriate tuning of the parameters. The downside of the latter models is that the volume is fixed only at moderately large values, narrowing the regime of validity of the effective field theory.

II. STABILITY OF THE COMPLEX STRUCTURE SECTOR

An underlying assumption of many constructions based on the scenarios above is that, with the right choice of fluxes, the complex structure sector can be stabilized at a supersymmetric configuration where the masses of fermions and scalars are much larger than the relevant cosmological energy scales. In that case, this sector can be safely integrated out, and then the attention is focused on the stabilization of the lighter Kähler moduli, which is much trickier. While this is a reasonable starting point, we will argue that, at least in Kähler uplifted scenarios, this assumption becomes untenable as the number of complex structure moduli increases. We will show that this observation leads to further constraints on the parameter space of the model which are more restrictive than those derived from the consistency of the effective field theory. Moreover, for a very large number of moduli—typical numbers are in excess of $\mathcal{O}(100)$—a new class of stable vacua emerges, in which the fermions of the complex structure sector are all *lighter* than the gravitino.

In LVS and Kähler uplifted vacua, the potential that stabilizes the moduli is a small deformation of the tree-level potential, with quantum corrections suppressed by the volume $\mathcal{V}$ of the compact Calabi-Yau space [7,10]

$$V = V_{\text{tree-level}} + m_{3/2}^2 \times \mathcal{O}(\hat{\xi}/\mathcal{V}). \tag{1}$$

Here the parameter $\hat{\xi}/\mathcal{V}$ characterizes the magnitude of the leading quantum corrections, and we have written explicitly its dependence on the volume for clarity (see [10,25,26]). The tree-level potential is positive semidefinite and is of the "no-scale" type: it is flat for the Kähler moduli leaving undetermined the expectation values of these fields, and in particular the overall volume $\mathcal{V}$ and gravitino mass $m_{3/2}$. The dilaton and complex structure moduli are stabilized at a supersymmetric configuration that is determined by the fluxes and by the geometric and topological properties of the compactified space [15]. This configuration defines a Minkowski vacuum where, in general, supersymmetry is broken by the Kähler moduli.

An important point is that, if we ignore quantum corrections, there is a relation between the masses of the fermions m_λ (with λ running through the complex structure moduli and dilaton) and the squared masses of the scalars in the complex structure sector [27–30] $\mu^2_{\pm\lambda}$:

$$\mu^2_{\pm\lambda}|_{\text{tree-level}} = (m_{3/2} \pm m_\lambda)^2. \tag{2}$$

At tree level, there are no instabilities in the supersymmetric sector, since the potential is non-negative and the no-scale vacuum is Minkowski. But note that, for every fermion in the supersymmetric sector with the same mass as the gravitino, there is a massless scalar in the tree-level spectrum. The sign of the $m_{3/2}^2 \times \mathcal{O}(\hat{\xi}/\mathcal{V})$ quantum corrections is unknown so these massless scalars are not protected and can become perturbatively unstable[2] (tachyonic, $\mu^2 < 0$). The same is true for sufficiently light scalars, to which we turn next, but first we need to characterize the spectrum of fermion masses.

III. THE MODEL

The tree-level fermion mass spectrum of the complex structure sector is determined by the geometry of the internal space and by the configuration of background fluxes. However, the high complexity of these theories makes a detailed calculation impractical in generic compactifications, so instead, we will follow a statistical approach. Intuitively, it is clear that, as the number of complex structure moduli increases, so does the probability that there are fermions with tree-level masses close to the gravitino mass, and with it the expected percentage of very light scalars that are susceptible of becoming tachyonic by the effect of $\hat{\xi}/\mathcal{V}$ corrections. This intuition can be made quantitative in the framework of *random matrix theory* (RMT) [31–37], and was confirmed in great detail in [29].

The idea is to promote to random variables the entries of the fermion mass matrix and then to characterize the spectrum of these matrices using standard techniques from RMT [33–35]. The universality theorems in RMT ensure that the result depends only mildly on the (unknown) distribution of the couplings for sufficiently large matrices [38,39], and therefore is insensitive to the details of the compactification.[3] Assuming that all complex structure moduli can be treated on equal footing, i.e., statistical isotropy in field space, the appropriate ensemble to represent the fermion mass matrix is the Altland-Zirnbauer C*I* matrix ensemble [33–35]. Proceeding in this way, and using the relation (2), the authors of [29] constructed a random matrix model to characterize the tree-level scalar mass spectrum of the complex structure sector in type-IIB flux compactifications. In the limit

[2]The form of the tree-level potential prevents these instabilities from being runaway directions. Thus a different vacuum may be found nearby, but there is no guarantee that it will be dS.

[3]See [37] for a recent discussion on the applicability of random matrix theory to study flux compactifications.

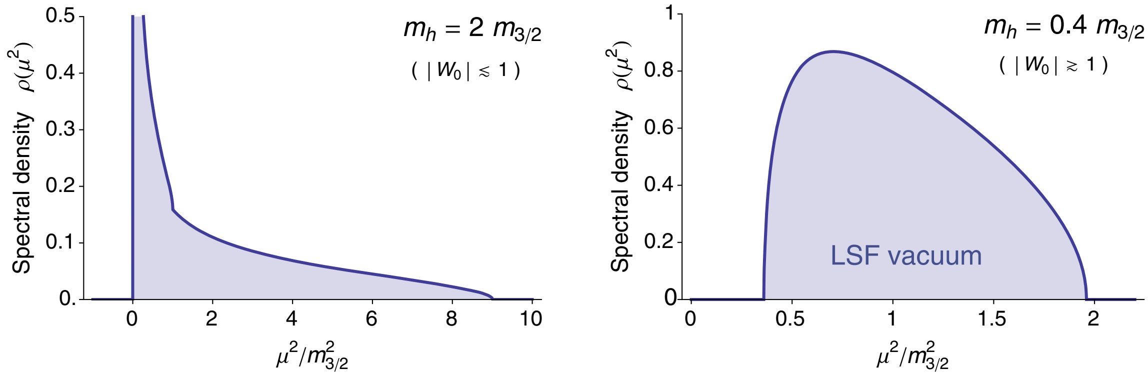

FIG. 1. Scalar mass spectrum, (3), of the complex structure sector at tree level with a large number of fields, $N \to \infty$. The spectrum is always tachyon-free, but when the heaviest fermion is heavier than the gravitino, $m_h > m_{3/2}$ (left), the spectral density diverges as $\rho(\mu^2) \sim 1/\mu$ near $\mu = 0$. By contrast, if the heaviest fermion is lighter than the gravitino, $m_h < m_{3/2}$ (right), the stability of the configuration is protected by a gap in the mass spectrum of size $\mu^2_{\text{min}} = (m_{3/2} - m_h)^2$.

where the number of (complex) fields is large, $N \to \infty$, the spectral density $\rho(\mu^2)$ for the tree-level scalar masses converges with order one probability to a particularly simple form

$$\rho(\mu^2) = \frac{2N m_{3/2}^2}{\pi m_h^2 \mu}\left[\Theta(m_h^2 - (m_{3/2}+\mu)^2)\sqrt{m_h^2 - (m_{3/2}+\mu)^2} + \Theta(m_h^2 - (m_{3/2}-\mu)^2)\sqrt{m_h^2 - (m_{3/2}-\mu)^2}\right], \qquad (3)$$

where $\Theta(x)$ is the Heaviside theta function. It is important to stress that the spectral density (3) is just the *most likely scalar mass spectrum* predicted by the random matrix theory model. Thus, it is possible to find vacua with a different mass spectrum, but they occur with an exponentially suppressed probability [40–42].

IV. A NEW CLASS OF VACUA

The spectral density (3) depends on two free parameters m_h and $m_{3/2}$, which represent the mass scale of the fermions in the complex structure sector and the gravitino mass, respectively. To be precise, the parameter m_h is defined as the expectation value of the largest fermion mass $m_h \equiv \mathbb{E}[m_{\text{max}}]$, and is related to the flux energy scale, $m_h \sim M_p/\mathcal{V}$, where M_p stands for the Planck mass. The gravitino mass is determined by the volume and the expectation value of the flux superpotential W_0, $m_{3/2} = M_p|W_0|/\mathcal{V}$ [7]. Figure 1 shows the typical tree-level spectrum (3) of the complex structure sector. Notice the accumulation of very light scalars in the case when the heaviest fermion is heavier than the gravitino, ($m_h > m_{3/2}$). By contrast, if the heaviest fermion in the complex structure sector is lighter than the gravitino, ($m_h < m_{3/2}$), the scalar density develops a mass gap. In the latter regime it is also possible to find *atypical vacua*, i.e. deviations from (3), where the smallest scalar mass is comparable in size to the quantum corrections, however the fraction of such vacua is exponentially suppressed [29]

$$\mathbb{P}\left(\mu^2_{\text{min}} < \frac{\hat{\xi}}{\mathcal{V}}\right) \sim e^{-\frac{4}{3}N x^{\frac{3}{2}}},$$

$$x = \left(1 - \sqrt{\frac{\hat{\xi}}{\mathcal{V}}}\right)^2 \frac{m_{3/2}^2}{m_h^2} - 1. \qquad (4)$$

The conclusion is that, for very large numbers of moduli, $N \sim \mathcal{O}(100)$, requiring the de Sitter vacua to be free from tachyonic instabilities in the supersymmetric sector favors vacua with all fermions lighter than the gravitino. We will denote these stable de Sitter configurations "LSF vacua," which stands for "light(er) supersymmetric fermions." Note that lighter than the gravitino does not necessarily mean light; the actual fermion masses can easily be in the grand unification scale as long as the gravitino is even heavier.[4]

V. COMPARISON WITH KKLT AND LVS REGIMES

The KKLT scenario corresponds to fine-tuning the fluxes so that the complex structure moduli have large masses compared with the supersymmetry breaking scale, which is set by the gravitino mass, that is $m_h \gg m_{3/2}$ ($|W_0| \ll 1$) [16,17]. In this regime the stability of this sector is guaranteed since the tree-level masses are large compared to the contributions induced by quantum effects. In LVS and in Kähler uplifted vacua the absence of fine-tuning of W_0 implies that the fermions typically have masses

[4]If $m_h \sim \mathcal{O}(m_{3/2})$, the smallest fermion mass is of order $m_{3/2}/N$ [29,35]. In Kähler uplifted vacua $m_{3/2}$ is typically of the order of the grand unification (GUT) scale [9], leading to $m_{\text{min}} \sim M_{\text{GUT}} \times 10^{-2}$.

comparable (but not necessarily close) to the gravitino mass, so that generically we have $\mu^2_{\pm\lambda} \sim \mathcal{O}(m^2_{3/2})$ [1,22,43]. The corrections to the tree-level spectrum can still be consistently neglected in this setting as long as the volume of the compactification is exponentially large, and thus the corrections are tiny, $\hat{\xi}/\mathcal{V} \sim 10^{-10}$. For Kähler uplifted vacua this is no longer true. Since the volume is not exponentially large, typically we have $\hat{\xi}/\mathcal{V} \sim 10^{-2}$–$10^{-4}$ [9,10,12], implying that the corrections in (1) could in principle induce tachyonic instabilities if some of the complex structure moduli are sufficiently light at tree level,

$$\mu^2_{\pm\lambda}|_{\mathrm{tree-level}} \lesssim m^2_{3/2} \times \mathcal{O}(10^{-2}\text{–}10^{-4}). \tag{5}$$

Figure 2 shows the percentage of scalar moduli estimated using (3) that are light enough to be destabilized by the quantum corrections, for a range of values of $\hat{\xi}/\mathcal{V}$. Note that, for moderately large volumes $\hat{\xi}/\mathcal{V} \sim 0.01$, this fraction can rise up to a 6%–7%, with the maximum occurring at $m_h \approx \sqrt{2} m_{3/2}$.

Requiring that the number of light fields, N_{light}, is less than one irrespective of the details of the stabilization of the complex structure sector, i.e. regardless of the value of the mass parameter m_h, we find a bound for the size of the α' corrections

$$\max\{N_{\mathrm{light}}\} \approx \frac{4N}{\pi}\sqrt{\frac{\hat{\xi}}{\mathcal{V}}} \ll 1 \Rightarrow \frac{\hat{\xi}}{\mathcal{V}} \ll \frac{\pi^2}{16N^2}. \tag{6}$$

Note that in a generic compactification with hundreds of complex structure moduli this bound is much more

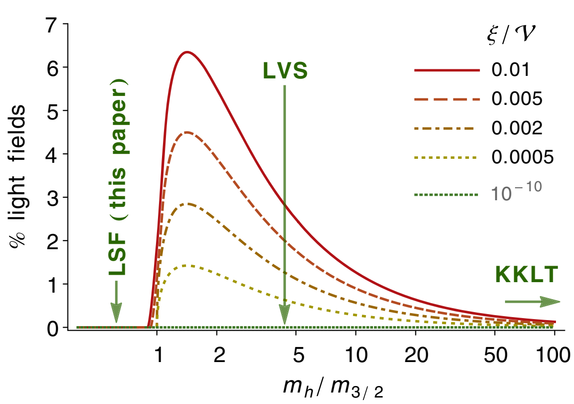

FIG. 2. Percentage of (real) scalars in the complex structure sector with tree-level masses smaller than the size of the leading quantum corrections, $\mu^2 \le m^2_{3/2} \cdot \hat{\xi}/\mathcal{V}$. The horizontal axis represents the typical mass scale in this sector, m_h. The spectrum of perturbations of the LSF vacua ($m_h < m_{3/2}$), contains no light scalar modes at tree level. Stability is also ensured if there is a large hierarchy between the masses of the supersymmetric complex structure sector and the supersymmetry breaking scale, $m_h \gg m_{3/2}$ (KKLT regime), or an exponentially large volume, $\hat{\xi}/\mathcal{V} \sim 10^{-10}$ (LVS).

restrictive than just requiring the α' corrections to be small, $\hat{\xi}/\mathcal{V} \ll 1$. This remark is particularly relevant for dS solutions and inflationary models built with the method of Kähler uplifting that do not satisfy the constraint (6) (see examples in [9,10,13,14,44,45]), as this signals the possible presence of tachyonic instabilities. Other models which could be affected by the same issue are those based on LVS vacua where the volume is only moderately large [46–49]. In all these constructions one could still search for atypical vacua where all fields in the complex structure sector are much heavier than the gravitino, as in KKLT scenarios. However the probability of such vacua is exponentially suppressed as, without fine-tuning W_0, the parameters satisfy $m_h \sim m_{3/2}$, and thus [29,35,50]

$$\mathbb{P}(\mu^2_{\mathrm{min}} \geq m^2_{3/2}) \sim e^{-\frac{8m^2_{3/2}}{m^2_h}N^2} \ll 1. \tag{7}$$

By contrast, LSF vacua, where all fermions are lighter than the gravitino, occur with probability of order one when $m_h \lesssim m_{3/2}$ ($|W_0| \gtrsim 1$), and thus they are a more natural configuration to stabilize the complex structure sector in this regime. In Fig. 2 it can be seen that when LSF vacua become dominant, the typical spectrum contains no light fields, a direct consequence of the appearance of the mass gap. Other scenarios which satisfy constraint (6) are [11,14,47,51].

VI. DISCUSSION

Having established that LSF vacua are stable, the next question is how to find them. Reference [52] provides a systematic way of looking for LSF vacua by looking in the vicinity of configurations in which all fermions in the supersymmetric sector are *massless*. The massless fermion limit (MFL) is not always realized at a physical vacuum, because the massless condition may require noninteger values of the fluxes that are not actually realized. However, it provides the "lamppost" near which actual stable vacua may be found. Following this procedure we have found a fully stabilized Kähler uplifted dS vacuum in the Calabi-Yau $\mathbb{P}^4_{[1,1,1,6,9]}$, with the complex structure sector fixed at a LSF vacuum. The details of this example can be found in the Appendix.

This brings us to another important point. The explicit examples of Kähler uplifted vacua constructed to date [9,12,13] have been found in models consistent with the supersymmetric truncation of a large sector of the moduli fields [12,53–59]. This can be achieved by considering special points of the moduli space, for instance fixed points of global symmetries of the moduli space metric, where the majority of the fields can be fixed at a supersymmetric configuration. By this procedure it is possible to obtain a reduced theory involving, in addition to the Kähler moduli, a small fraction of the complex structure fields and the dilaton so that a detailed stability analysis is possible. In particular, in the examples discussed in [9,12] the complex structure moduli surviving the truncation were fixed at vacua with large supersymmetric masses, i.e. $m_\lambda \gg m_{3/2}$, that is, imposing a large hierarchy between the masses of these fields and the supersymmetry breaking scale. This method ensures the stability of the field configuration in the reduced theory, however it cannot guarantee that the truncated fields are fixed at minima of the potential and, for this reason, neither does it guarantee the consistency of this reduction. It is therefore crucial to understand under what conditions it is possible to ensure the stability of the full set of moduli fields, *including the truncated ones*.

In paper [52] it is also shown that, when the fraction of complex structure fields surviving the truncation are stabilized at the MFL of a critical point, then *all* the complex structure fields (including the truncated ones) and the dilaton have a mass equal to $m_{3/2}$ at tree level, i.e. the full sector is also at the MFL of the vacuum.

Given that it is not feasible to check the stability of hundreds of supersymmetric moduli—-except perhaps in very special cases—we would like to suggest a compromise: apply the usual analytic and numerical techniques to check stability of the surviving low-energy sector (typically, the Kähler moduli and the complex moduli that sit at points of enhanced symmetry) and supplement these with the use of random matrix theory techniques to assess the stability of the truncated moduli that do not appear in the low-energy description. Here we made use of the random matrix theory model presented in [29] to characterize the mass spectrum of the complex structure sector. Our conclusion—in line with our previous work in [27–29]—is that, in compactifications where the number of complex structure moduli is very large, there is a class of stable flux configurations, not previously considered, in which all fermions of the supersymmetric sector—-including truncated ones—are lighter than the gravitino.

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APPENDIX: EXPLICIT EXAMPLE IN THE $\mathbb{P}^4_{[1,1,1,6,9]}$ MODEL

In the present section we provide an explicit example of the class of vacua presented in the main body of the article. In particular we present a fully stabilized Kähler uplifted dS vacuum where the complex structure sector displays a light fermion spectrum, $m_\lambda < m_{3/2}$, that is, where all the scalar fields in the complex structure sector have masses of the order of the gravitino mass, $m_{3/2}$. This vacuum was found using the strategy described in the main text. We consider the compactification of the weak string coupling regime of type-IIB string theory in the Calabi-Yau $X_3 = \mathbb{P}^4_{[1,1,1,6,9]}$ in the presence of background fluxes and $D7$-branes [15]. When the supersymmetry breaking scale is much lower than the KK scale, the corresponding low-energy action admits a description in terms of $\mathcal{N} = 1$ four-dimensional supergravity. For simplicity we will only discuss the dynamics of the $h^{1,1} = 2$ Kähler moduli fields T^i, $i = 1, 2$, and the $h^{1,2} = 272$ complex structure moduli z^a, $a = 1, \ldots, 272$, and the axio-dilaton, τ [7,54,60]. Thus, we ignore further degrees of freedom, such as D-brane positions or matter fields. In this setting, the effective four-dimensional action can be characterized in terms of a Kähler potential K, and a superpotential, W. At leading order in α' and nonperturbative corrections, and neglecting the effect of warping due to the fluxes, they read[5] (see [7])

$$K = -2\log\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right) - \log[-\mathrm{i}(\tau - \bar{\tau})] - \log \mathrm{i}\int_{X_3} \Omega\wedge\bar{\Omega},$$
$$W = \frac{1}{\sqrt{4\pi}}\left(W_{\mathrm{flux}} + \sum_i A_i \mathrm{e}^{-a_i T_i}\right), \tag{A1}$$

where we have set the Planck mass to unity. The couplings of the complex structure sector can be written conveniently in terms of the period vector $\Pi^T = (Z^I, F_I)$, defined as

$$\Pi = \begin{pmatrix} \int_{A^I} \Omega \\ \int_{B_I} \Omega \end{pmatrix}, \tag{A2}$$

where (A^I, B_I) with $I = 0, \ldots, h^{1,2}$ denotes an integral and symplectic homology basis of $H_3(X_3, \mathbb{Z})$, satisfying

$$A^I \cap B_J = \delta^I_J, \qquad A^I \cap A^J = B_I \cap B_J = 0. \tag{A3}$$

[5] As in [7], the volume $\mathcal{V}$ of X_3 (expressed in units of $l_s = 2\pi\sqrt{\alpha'}$) and the Kähler moduli T^i are measured in the Einstein frame, and the fields are taken to be dimensionless.

In particular, in terms of the period vector, the Kähler potential for the complex structure sector $K_{c.s.}$ reads

$$e^{-K_{cs}} = i\int_M \Omega\wedge\bar{\Omega} = i\Pi^\dagger\cdot\Sigma\cdot\Pi, \tag{A4}$$

where Σ denotes the symplectic matrix

$$\Sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{A5}$$

Thus, Z^I represent the homogeneous coordinates on the complex structure moduli space. The corresponding inhomogeneous coordinates define the complex structure moduli fields $z^a = Z^a/Z^0$ with $a = 1, \ldots, h^{1,2}$, and $Z^0 \neq 0$. Moreover, the periods F_I can be expressed as the gradients of a holomorphic function $F(Z^I)$, the so-called *prepotential*, $F_I = \partial F/\partial Z^I$. In (A1) the Kähler potential for the Kähler moduli sector is expressed in terms of $\mathcal{V}$, the volume of the Calabi-Yau

$$\mathcal{V} = \gamma((T_1+\bar{T}_1)^{3/2} - (T_2+\bar{T}_2)^{3/2}), \tag{A6}$$

where $\gamma = 1/36$, and the dilaton dependent α' corrections are given by [61]

$$\hat{\xi}(\tau,\bar{\tau}) = -\frac{\zeta(3)}{4\sqrt{2}(2\pi)^3}\chi(-i(\tau-\bar{\tau}))^{3/2}, \tag{A7}$$

with $\chi = 2(h^{2,1} - h^{1,1})$, and $\zeta(3) \approx 1.202$. In the presence of RR (F_3) and NS-NS (H_3) fluxes wrapping nontrivial three cycles of X_3, the superpotential receives a contribution of the form [62]

$$W_{\text{flux}} = l_s^{-2}\int_{X_3}(F_3 - \tau H_3)\wedge\Omega = (f-\tau h)^T\cdot\Sigma\cdot\Pi(z^a), \tag{A8}$$

where l_s denotes the string scale and

$$f = \begin{pmatrix} \int_{A^I} F_3 \\ \int_{B_I} F_3 \end{pmatrix}, \qquad h = \begin{pmatrix} \int_{A^I} H_3 \\ \int_{B_I} H_3 \end{pmatrix} \tag{A9}$$

stand for the vectors of integrally quantized fluxes. The extra contributions to the superpotential in (A1) correspond to the nonperturbative corrections induced by the gaugino condensation on the stacks of $D7$-branes. In [10] it was shown that when the Calabi-Yau volume has a swiss-cheese structure similar to (A6) it is possible to derive semi-analytical formulas for the range of parameters allowing for the existence of Kähler uplifted de Sitter vacua. Using these results we have fixed the constants A_i and $a_i = 2\pi/N_i$ to the values

$$A_1 = 0.272, \qquad A_2 = 0.250, \qquad N_1 = 400, \qquad N_2 = 3, \tag{A10}$$

where N_i denote the ranks of the condensing gauge groups. The large value for the rank of the condensing group, N_1, is a common feature of Kähler uplifted vacua, and it is necessary for the compactification volume to be sufficiently large to ensure the consistency of the 4-dimensional effective theory [10,12]. The rank N_1 can be reduced if the Calabi-Yau volume has only an approximately swiss-cheese structure, as in [12], or by considering more complicated models where the $\mathcal{N} = 1$ scalar potential involves a D-term contribution [46,47].

The complex structure moduli space of the $\mathbb{P}^4_{[1,1,1,6,9]}$ model is symmetric with respect to the group $\mathcal{G} = \mathbb{Z}_6\times\mathbb{Z}_{18}$, which acts nontrivially on 270 of the moduli. Provided we restrict to flux configurations invariant under this symmetry, it is possible to fix the non-invariant fields at a supersymmetric configuration, leaving a reduced theory involving only two complex structure moduli [12,56,57]. The prepotential $F(Z^I)$ characterizing the moduli space geometry in the reduced theory was calculated in [60]. In the large complex structure limit the prepotential takes the form

$$F(Z^I) = \xi(Z^0)^2 - \frac{1}{4}Z^0Z^1 - \frac{3}{2}Z^0Z^2 - \frac{9}{4}(Z^1)^2 - \frac{3}{2}Z^1Z^2 + \frac{1}{2Z^0}(3(Z^1)^3 + 3(Z^1)^2Z^2 + Z^1(Z^2)^2) + F_{\text{inst}}, \tag{A11}$$

where $\xi = \frac{i\zeta(3)\chi}{2(2\pi)^3} \approx -1.308i$. The term $F_{\text{inst}}(z^1,z^2)$ is the contribution from instantons to the prepotential [60] which is exponentially small in the large complex structure limit, i.e. when $\text{Im}(z^1), \text{Im}(z^2) \gtrsim 1$, and thus we will neglect it in our calculations. We turn on fluxes only on the $\mathcal{G}$-invariant cycles

$$f_A = (1,0,0), \qquad f_B = (0,14,4), \qquad h_A = (0,1,1), \qquad h_B = (-1,4,1), \tag{A12}$$

where $f^T = (f_A, f_B)$, $h^T = (h_A, h_B)$, and for clarity we have omitted the components on noninvariant cycles. The allowed choices of flux configurations are subject to the tadpole cancellation condition, which requires the total $D3$-brane charge, including the contribution from $D3$-branes N_{D3}, to cancel some negative charge, L, induced by the $D7$ and $O7$ (orientifold) planes,

$$N_{\text{flux}} + N_{D3} \leq L. \tag{A13}$$

The $D3$ charge induced by the fluxes is determined by the expression

TABLE I. Vacuum expectation values of the moduli fields (dimensionless). We also show the expectation value of the volume $\mathcal{V}$ (in Einstein frame) measured in units of l_s, and of the scalar potential V in Planck units.

	Tree level	Fully stabilized vacuum
$\langle\tau\rangle$	$-0.142 + 1.033i$	$-0.143 + 1.066i$
$\langle z^1\rangle$	$0.036 + 0.929i$	$0.035 + 0.920i$
$\langle z^2\rangle$	$0.160 + 1.663i$	$0.157 + 1.650i$
$\langle T_1\rangle$	⋯	$180.13 + 295.97i$
$\langle T_2\rangle$	⋯	$1.759 + 2.220i$
$\langle\mathcal{V}\rangle$	⋯	$189.76 l_s^6$
$\langle V\rangle$	0	$4.06\times10^{-10} M_p^4$

$$N_{\rm flux} = \frac{1}{l_s^4}\int_M H_{(3)}\wedge F_{(3)} = h^T\cdot\Sigma\cdot f. \qquad \text{(A14)}$$

Thus the flux configuration we specified above (A12) induces a D3 charge $N_{\rm flux} = 19 < \mathcal{O}(100)$, and thus it can satisfy the tadpole conditions in this model [12,54].

The scalar potential can be derived from (A1) using the standard $\mathcal{N} = 1$ supergravity formula

$$V = e^K(K^{\alpha\bar\beta}D_\alpha W D_{\bar\beta}\bar W - 3|W|^2), \qquad \text{(A15)}$$

where $K^{\alpha\bar\beta}$ is the inverse of the moduli space metric $K_{\alpha\bar\beta} = \partial_\alpha\partial_{\bar\beta}K$. We have also denoted by $D_\alpha W = \partial_\alpha W + (\partial_\alpha K)W$ the Kähler covariant derivative of the superpotential. At tree level, that is, without including the α' corrections and the nonperturbative contribution to the superpotential, it is possible to find a supersymmetric configuration of the complex structure sector (see Table I) where the tree-level fermion masses are all smaller than the gravitino mass, and given by

$$\frac{m_1}{m_{3/2}} = 0.366, \quad \frac{m_2}{m_{3/2}} = 0.139, \quad \frac{m_3}{m_{3/2}} = 0.064. \qquad \text{(A16)}$$

TABLE II. Masses of the canonically normalized scalar fields in units of $m_{3/2}$. In particular $\mu^2_{\pm a}$, $a = 1, 2, 3$, correspond to the masses squared in the complex structure and dilaton sector. For completeness we also display the gravitino mass and the Kaluza-Klein scale in Planck units.

	Tree level	Fully stabilized vacuum
$\mu^2_{\pm 1}$	(1.866,0.402)	(1.547,0.569)
$\mu^2_{\pm 2}$	(1.297,0.741)	(1.225,0.789)
$\mu^2_{\pm 3}$	(1.133,0.876)	(1.080,0.9135)
$\mu^2_{\pm T_1}$	(0,0)	(0.02381,0.00872)
$\mu^2_{\pm T_2}$	(0,0)	(43.158,38.308)
$m^2_{3/2}$	⋯	$4.12\times10^{-5} M_p^2$
m^2_{kk}	⋯	$2.86\times10^{-3} M_p^2$

TABLE III. Consistency checks of the construction: instanton contributions to the prepotential, size of the α' corrections, size of the nonperturbative contributions to the superpotential, and magnitude of the supersymmetry breaking scale relative to the KK scale.

Quantity	Relative magnitude
$\lvert F_{\rm inst}\rvert/\lvert F\rvert$	6.3×10^{-3}
$\langle\hat\xi/\mathcal{V}\rangle$	7.6×10^{-3}
$\langle A_1 e^{-a_1 {\rm Re}(T_1)}/\lvert W_{\rm flux}\rvert\rangle$	1.5×10^{-4}
$\langle A_2 e^{-a_2 {\rm Re}(T_2)}/\lvert W_{\rm flux}\rvert\rangle$	3.9×10^{-4}
$m_{3/2}/m_{kk}$	0.12

Note that this vacuum is precisely an example of the LSF vacua introduced in the main text. Further details regarding this field configuration at tree level are listed in Tables II and III, where in particular it can be checked that the vacuum is consistent with the assumptions of weak string coupling $g_s = 1/{\rm Im}\langle\tau\rangle \lesssim 1$, and neglecting the instanton contributions to the prepotential $|F_{\rm inst}|/|F| \ll 1$. Upon including the α' and nonperturbative corrections the Kähler moduli are stabilized at the configuration also displayed in Table I, where the expectation value of the overall volume is fixed to $\langle\mathcal{V}\rangle = 189.76$. Table I also shows the corrected values for the expectation values of the complex structure fields, and Table II contains the full scalar mass spectrum at the final vacuum. It is worth noting that the expectation value of the scalar potential can be tuned to be arbitrarily close to zero choosing conveniently the constants A_1 and A_2. We have also computed various quantities relevant to check the consistency of the construction (see Table III). For instance, the gravitino mass is shown to be well below the Kaluza-Klein scale [7] $m_{3/2}/m_{kk} < 1$, as required for the four-dimensional supergravity description to be valid. It can also be checked that the α' corrections and the nonperturbative contributions to the superpotential are actually small.

Our results prove explicitly the perturbative stability of the Kähler moduli sector, the complex structure moduli surviving the truncation z^1, z^2, and the dilaton τ, in the final vacuum. It is worth noting that the perturbative stability of the remaining 270 truncated moduli has never been addressed before in the literature. The arguments outlined in the main text indicate that stabilizing moduli at LSF vacua can provide the means to guarantee the perturbative stability of all the moduli, including the 270 truncated ones. Indeed, as will be proved rigorously in [52], provided the surviving moduli in the complex structure sector are stabilized with sufficiently small fermion masses, i.e. close to the MFL, all the moduli in the complex structure sector will remain perturbatively stable in the final de Sitter vacuum.

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