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Vertex operators for irregular conformal blocks: Supersymmetric case

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We construct supersymmetric irregular vertex operators of arbitrary rank, appearing in the colliding limit of primary fields. We find that the structure of the supersymmetric irregular vertices differs significantly from the bosonic case: upon supersymmetrization, the irregular operators are no longer the eigenstates of positive Virasoro and W_N generators but block diagonalize them. We relate the block-diagonal structure of the irregular vertices to contributions of the Ramond sector to the colliding limit.

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I. INTRODUCTION

Primary vertex operators in two-dimensional conformal field theory are the objects playing an important role in the Alday-Gaiotto-Tachikawa conjecture [1], connecting regular Liouville conformal blocks to Nekrasov's partition function [2] for $N = 2$ supersymmetric gauge theories in four dimensions. These theories, however, admit a special case with nontrivial IR fixed point, described by Argyres-Douglas theory [3,4]. This class of theories does not allow marginal deformations and is described in terms of the colliding limit of the primary vertex operators. The operator of rank q, obtained from the colliding limit [5,6], generates an irregular state of rank q when applied to the vacuum. The irregular states of rank q are annihilated by positive Virasoro generators L_k with $k > 2q$ and are simultaneous eigenstates of L_k with $q \leq k \leq 2q$. These irregular states are called Gaiotto states [7] or Whittaker states [8]. In the generalizations of Liouville theory to Toda field theories, the irregular states also possess W_N symmetries, being simultaneous eigenstates of positive W_N generators. For example, in the simplest case of two fields the irregular states are eigenvectors of positive W_3 generators $(W_3)_k$ with $2q \leq k \leq 3q$. For higher numbers of components, higher W_N symmetries are switched on as well, and the irregular states are the eigenstates of $(W_N)_k$ with $N(q-1) \leq k \leq Nq$.

In our previous paper [9] we have performed explicit construction of the irregular vertex operators describing these states. In general, the irregular vertex operators describing the states of rank q for n-component Toda field $\vec{\phi} = (\phi_1, \ldots \phi_n)$ are given by

$$U_{n,q} = e^{\sum_{k=0}^{q} \vec{\alpha}_k \partial^k \vec{\phi}}. \tag{1.1}$$

The Virasoro eigenvalues of $U_{n,q}$ are bilinear in the components of $\vec{\alpha}$, while the W_N eigenvalues are polynomials of degree N in α (see also [10–12] related reference).

The objects of the type (1.1) generalize regular vertex operators in Toda theories and in bosonic string theory and are in fact of interest far beyond their relevance to gauge theories in four dimensions. First of all, such objects have to appear naturally in AdS/CFT correspondence, being the AdS string duals of composite operators on the CFT side. For example, the operators of the type $T^q \sim T_{m_1 n_1} \ldots T_{m_q n_q}$ (with T_{mn} being the energy-momentum tensor in super Yang-Mills theory) have to correspond to the colliding limit of q gravitons, or the irregular vertex of rank q in AdS string theory. At the same time, the objects of this type should be dual to higher spin fields with mixed symmetries in AdS higher spin gravity. Unlike regular vertex operators, these objects are not in Becchi-Rouet-Stora-Tyutin cohomology and are off shell. Nevertheless, they are completely meaningful in the string field theory (SFT) context. On the other hand, the world sheet correlators of $U_{n,q}$ can be understood as generating functions for the higher spin interactions in string theory since the derivatives $H^{m_1 \ldots m_s} \frac{\partial}{\partial^{m_1}_{\alpha_{n_1}}} \cdots \frac{\partial}{\partial^{m_s}_{\alpha_{n_s}}} U_{n,q}|_{\vec{\alpha}_1 \ldots \vec{\alpha}_n = 0}$ describe on-shell higher spin vertex operators with momentum $\vec{\alpha}_0$ and masses $m \sim (\sum_{j=1}^{s} n_j)^{\frac{1}{2}}$, making the irregular vertex operators (1.1) natural candidates for SFT solutions describing nonperturbative higher spin backgrounds.

Since the irregular states are typically of relevance to models with supersymmetries, it is naturally of interest to find the supersymmetric extensions of (1.1). This turns out to be a major challenge in the conventional matrix model approach and, so far, very little progress has been made in describing supersymmetrizing the colliding limit. The purpose of this work is to elaborate on this problem

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using the superspace extension of the vertex operator formalism (1.1). The results that we find are somewhat surprising. It turns out that the states, emerging in the supersymmetric colliding limit, no longer diagonalize Virasoro generators. Instead, the positive Virasoro generators turn out to be block diagonal in their basis, with the multiplicities related to the ranks of the blocks. Such a situation is familiar in logarithmic conformal field theories (LCFT) where the Hamiltonian L_0 is also well known to be a Jordan matrix. The Hamiltonian being a Jordan block leads to appearance of the logarithmic partners and modified conformal symmetry constraints on correlation functions involving these partners. Solving these constraints leads to emergence of logarithms in the correlators. The supersymmetric extension of the vertices (1.1) thus, in a sense, may give the irregular analogues of logarithmic operators in LCFT in the interacting case, although we will not elaborate on this question in the present paper.

The rest of the paper is organized as follows. In the Sec. II we perform the supersymmetrization of the operators (1.1) using the superspace approach. Next, we study the properties of the supersymmetric irregular vertices under Virasoro transformations. In the Sec. III we discuss the reasons behind the appearance of the block-diagonal structure for the irregular vertices, with the positive super Virasoro generators having the form of Jordan blocks in their basis. We relate this structure to contributions of operators from the Ramond sector to the colliding limit, with the multiplicities of eigenvalues of the Jordan blocks connected to various possible structures of these contributions.

II. SUPERCONFORMAL PROPERTIES OF IRREGULAR VERTICES

The vertex operators (1.1) can be supersymmetrized in a straightforward and manifest way by replacing the fields by the superfields and covariantizing the derivatives:

$$\begin{aligned} \vec{\phi}(z) &\to \vec{\phi}(z,\theta) = \vec{\phi}(z) + \theta\vec{\psi} \\ \partial \equiv \partial_z &\to D = \theta\partial_z + \partial_\theta \end{aligned} \tag{2.1}$$

and expanding in θ. Using

$$\begin{aligned} D^{2k}\vec{\phi}(z,\theta) &= \partial^k\vec{\phi} + \theta\partial^k\vec{\psi} \\ D^{2k+1}\vec{\phi}(z,\theta) &= \partial^k\vec{\psi} + \theta\partial^{k+1}\vec{\phi} \\ k &= 0,1,\ldots \end{aligned} \tag{2.2}$$

one easily finds

$$\begin{aligned} W_{q+\frac{1}{2}} &= e^{\sum_{n=0}^{2q+1}\vec{\alpha}_n D_\theta^n\vec{\phi}(z,\theta)} = e^{\sum_{n=0}^{q}\vec{\alpha}_{2n}\partial^n\vec{\phi}} \\ &\times\prod_{k=0}^{q}(1+\vec{\alpha}_{2k+1}\partial^k\vec{\psi}) \\ &\times\left(1+\theta\left(\sum_{n=0}^{q}\vec{\alpha}_{2n}\partial^n\vec{\psi} - \vec{\alpha}_{2n+1}\partial^{n+1}\vec{\phi}\right)\right) \\ W_q &= e^{\sum_{n=0}^{2q}\vec{\alpha}_n D_\theta^n\vec{\phi}(z,\theta)} = e^{\sum_{n=0}^{q}\vec{\alpha}_{2n}\partial^n\vec{\phi}}\prod_{k=0}^{q-1}(1+\vec{\alpha}_{2k+1}\partial^k\vec{\psi}) \\ &\times\left(1+\theta\left(\alpha_{2q}\partial^q\psi + \sum_{n=0}^{q-1}\vec{\alpha}_{2n}\partial^n\vec{\psi} - \vec{\alpha}_{2n+1}\partial^{n+1}\vec{\phi}\right)\right) \end{aligned} \tag{2.3}$$

for the odd and even ranks respectively. The $\vec{\alpha}_k$ coefficients are Grassmann even for even k's and Grassmann odd for the odd k's. Clearly, the supersymmetrization reduces the total rank of the operators by half. In particular, $W_{\frac{1}{2}}$ is just a combination of regular vertex operators with different conformal dimensions:

$$W_{\frac{1}{2}} = e^{\vec{\alpha}\vec{\phi}}(1+\vec{\beta}\vec{\psi} + \theta(\vec{\alpha}\vec{\psi} - \vec{\beta}\partial\vec{\phi})) \tag{2.4}$$

or simply

$$U_{\frac{1}{2}} = \int d\theta W_1 = e^{\vec{\alpha}\vec{\phi}}(\vec{\alpha}\vec{\psi} - \vec{\beta}\partial\vec{\phi}) \tag{2.5}$$

upon θ-integration. Unlike the irregular bosonic vertices, the operators (2.4) and (2.5) are not by themselves the eigenvectors of positive super Virasoro generators. For example, $W_{\frac{1}{2}}$ is annihilated by L_n for $n \geq 2$, however the action of L_1 on $W_{\frac{1}{2}}$ gives

$$\begin{aligned} L_1 W_{\frac{1}{2}} &= \theta e^{\vec{\alpha}\vec{\phi}}\vec{\beta}\vec{\psi} \equiv \theta\tilde{W}_{\frac{1}{2}} \\ L_1^2 W_{\frac{1}{2}} &= 0 \end{aligned} \tag{2.6}$$

implying that L_1 acts on $(W_{\frac{1}{2}}, \tilde{W}_{\frac{1}{2}})$ as a block-diagonal Jordan matrix with $(W_{\frac{1}{2}}, \tilde{W}_{\frac{1}{2}})$ having eigenvalue zero and multiplicity 2 (for brevity and simplicity, here and elsewhere we set the background charge q to zero; the generalization to $q \neq 0$ is, however, straightforward). Next, consider the case of W_1. In components, one has

$$W_1 = e^{\vec{\alpha}\vec{\phi}+\vec{\gamma}\partial\vec{\phi}}(1+\vec{\beta}\vec{\psi})(1+\theta(\vec{\alpha}\vec{\psi} - \vec{\beta}\partial\vec{\phi} + \vec{\gamma}\partial\vec{\psi})). \tag{2.7}$$

This operator block diagonalizes L_1 and L_2. Acting with L_2, one obtains

$$L_2 W_1 = -\frac{1}{2}\gamma^2 W_1 + \frac{1}{2}\theta\vec{\beta}\vec{\gamma}\, e^{\vec{\alpha}\vec{\phi}+\vec{\gamma}\partial\vec{\phi}} \equiv -\frac{1}{2}\gamma^2 W_1 + \tilde{W}_1^{(1)}$$
$$L_2 \tilde{W_1^{(1)}} = -\frac{1}{2}\gamma^2 \tilde{W_1^{(1)}} \tag{2.8}$$

implying the block-diagonal structure of L_2 acting on W_1 with eigenvalue $-\frac{1}{2}\gamma^2$ and multiplicity 2. Acting with L_1, one obtains

$$L_1 W_1 = -\vec{\alpha}\vec{\gamma}\, W_1 + (\vec{\gamma}\vec{\psi} + \theta\vec{\alpha}\vec{\beta}) e^{\vec{\alpha}\vec{\phi}+\vec{\gamma}\partial\vec{\phi}} \equiv -\vec{\alpha}\vec{\gamma}\, W_1 + \tilde{W}_1^{(2)}$$
$$L_1 \tilde{W}_1^{(2)} = -\vec{\alpha}\vec{\gamma}\, \tilde{W}_1^{(2)} \tag{2.9}$$

implying the block-diagonal structure of L_1 acting on W_1 with eigenvalue $-\vec{\alpha}\vec{\gamma}$ and multiplicity 2. For the $W_{\frac{3}{2}}$ case, the structure becomes more diverse.

The operator in components is

$$W_{\frac{3}{2}} = e^{\vec{\alpha}\vec{\phi}+\vec{\gamma}\partial\vec{\phi}}(1+\vec{\beta}\vec{\psi})(1+\vec{\lambda}\partial\vec{\psi}) \times (1+\theta(\vec{\alpha}\vec{\psi}+\vec{\beta}\partial\vec{\phi}+\vec{\gamma}\partial\vec{\psi}+\vec{\lambda}\partial^2\vec{\phi})). \tag{2.10}$$

It block diagonalizes the Virasoro generators L_1, L_2 and L_3 with multiplicities 4,3 and 2 according to

$$L_1 W_{\frac{3}{2}} = -\vec{\alpha}\vec{\gamma}\, W_{\frac{3}{2}} + \tilde{W}_{\frac{3}{2}}^{1,1}$$
$$L_1 \tilde{W}_{\frac{3}{2}}^{1,1} = -\vec{\alpha}\vec{\gamma}\, \tilde{W}_{\frac{3}{2}}^{1,1} + \tilde{W}_{\frac{3}{2}}^{1,2}$$
$$L_1 \tilde{W}_{\frac{3}{2}}^{1,2} = -\vec{\alpha}\vec{\gamma}\, \tilde{W}_{\frac{3}{2}}^{1,2} + \tilde{W}_{\frac{3}{2}}^{1,3}$$
$$L_1 \tilde{W}_{\frac{3}{2}}^{1,3} = -\vec{\alpha}\vec{\gamma}\, \tilde{W}_{\frac{3}{2}}^{1,3} \tag{2.11}$$

for L_1,

$$L_2 W_{\frac{3}{2}} = -\frac{1}{2}\gamma^2 W_{\frac{3}{2}} + \tilde{W}_{\frac{3}{2}}^{2,1}$$
$$L_2 \tilde{W}_{\frac{3}{2}}^{2,1} = -\frac{1}{2}\gamma^2 \tilde{W}_{\frac{3}{2}}^{2,1} + \tilde{W}_{\frac{3}{2}}^{2,2}$$
$$L_2 \tilde{W}_{\frac{3}{2}}^{2,2} = -\frac{1}{2}\gamma^2 \tilde{W}_{\frac{3}{2}}^{2,2} \tag{2.12}$$

for L_2 and

$$L_3 W_{\frac{3}{2}} = \tilde{W}_{\frac{3}{2}}^{3,1}$$
$$L_3 \tilde{W}_{\frac{3}{2}}^{3,1} = 0 \tag{2.13}$$

for L_3 with

$$\tilde{W}_{\frac{3}{2}}^{1,1} = e^{\vec{\alpha}\vec{\phi}+\vec{\gamma}\partial\vec{\phi}} \times \{(1+\vec{\beta}\vec{\psi})(1+\vec{\lambda}\partial\vec{\psi})(\vec{\alpha}\vec{\beta}-2\vec{\lambda}\partial\vec{\phi}-\vec{\gamma}\vec{\psi})\theta + (1+(\vec{\beta}+\vec{\lambda})\vec{\psi})(1+\theta(\vec{\alpha}\vec{\psi}+\vec{\beta}\partial\vec{\phi}+\vec{\gamma}\partial\vec{\psi}+\vec{\lambda}\partial^2\vec{\phi}))\}$$
$$\tilde{W}_{\frac{3}{2}}^{1,2} = 2e^{\vec{\alpha}\vec{\phi}+\vec{\gamma}\partial\vec{\phi}} \times \{(1+\vec{\beta}\vec{\psi})(1+\vec{\lambda}\partial\vec{\psi})\vec{\alpha}\vec{\lambda} + (1+(\vec{\beta}+\vec{\lambda})\vec{\psi})(\vec{\alpha}\vec{\beta}-\vec{\lambda}\partial\vec{\phi}-\vec{\gamma}\vec{\psi})\}\theta$$
$$\tilde{W}_{\frac{3}{2}}^{1,3} = 4e^{\vec{\alpha}\vec{\phi}+\vec{\gamma}\partial\vec{\phi}}(1+(\vec{\beta}+\vec{\lambda})\vec{\psi})\vec{\alpha}\vec{\lambda}\,\theta$$
$$\tilde{W}_{\frac{3}{2}}^{2,1} = e^{\vec{\alpha}\vec{\phi}+\vec{\gamma}\partial\vec{\phi}} \times \Big\{(1+\vec{\beta}\vec{\psi})(1+\vec{\lambda}\partial\vec{\psi})(\vec{\gamma}\vec{\beta}+2\vec{\alpha}\vec{\lambda})\theta - \frac{1}{2}\vec{\beta}\vec{\lambda}(1+\theta(\vec{\alpha}\vec{\psi}+\vec{\beta}\partial\vec{\phi}\vec{\gamma}\,\partial\vec{\psi}\vec{\lambda}\,\partial^2\vec{\phi}))$$
$$+\frac{1}{2}((1+\vec{\lambda}\partial\vec{\psi})\vec{\beta}\vec{\gamma}-(1+\vec{\beta}\vec{\psi})\vec{\alpha}\vec{\lambda})\theta\Big\}$$
$$\tilde{W}_{\frac{3}{2}}^{2,2} = -e^{\vec{\alpha}\vec{\phi}+\vec{\gamma}\partial\vec{\phi}}(\vec{\beta}\vec{\gamma}+2\vec{\alpha}\vec{\lambda})\vec{\beta}\vec{\lambda}\,\theta$$
$$\tilde{W}_{\frac{3}{2}}^{3,1} = -2e^{\vec{\alpha}\vec{\phi}+\vec{\gamma}\partial\vec{\phi}}\vec{\gamma}\vec{\lambda}\,\theta. \tag{2.14}$$

Accordingly, the Jordan block constraints for $W_{\frac{3}{2}}$ are

$$(L_1+\vec{\alpha}\vec{\gamma})^4 W_{\frac{3}{2}} = 0$$
$$\left(L_2+\frac{1}{2}\gamma^2\right)^3 W_{\frac{3}{2}} = 0$$
$$L_3^2 W_{\frac{3}{2}} = 0. \tag{2.15}$$

It is straightforward to check that the same pattern holds for the higher q's as well. At half integer orders, the operators $W_{q+\frac{1}{2}}$ block diagonalize L_n with $q \le n \le 2q+1$ with the eigenvalues given by

$$\lambda_n = -\frac{1}{2}\sum_{0\le k_1;k_2\le q;k_1+k_2=n} k_1!k_2!\vec{\alpha}_{k_1}\vec{\alpha}_{k_2} \tag{2.16}$$

for $n \le 2q$ and $\lambda_{2q+1} = 0$. The Jordan block conditions are

$$(L_n-\lambda_n)^{m(n,q)} W_{q+\frac{1}{2}} = 0 \tag{2.17}$$

with the multiplicities ranging from

$$m(2q+1,q) = 1+\left[\frac{2}{q}+\frac{q+1}{2}\right] \tag{2.18}$$

for the highest Virasoro operator L_{2q+1} to $m(q,q) \geq [\frac{q}{4} + \frac{5}{8} + \frac{13}{8q}]$ for the lowest Virasoro generator L_q. Likewise, at the integer orders the W_q operators block diagonalize the Virasoro generators from L_q to L_{2q} with the similar eigenvalues λ_n (2.16) and with the multiplicities ranging from

$$m(2q,q) = \left[\frac{q+1}{2q}\right] + q \tag{2.19}$$

for L_q and $m(q,q) \geq \frac{q}{2} + [\frac{q+1}{4q}]$ for L_{2q}. The analysis of W_q behavior under the supersymmetry transformations by the supercurrent modes is performed analogously. The irregular vertices block diagonalize the supercurrent modes from $G_{q-\frac{1}{2}}$ up to $G_{2q-\frac{1}{2}}$ for integer ranks and $G_{2q+\frac{1}{2}}$ for half-integer ranks. This concludes the analysis of the CFT properties of supersymmetric irregular vertex operators.

III. COLLIDING LIMIT AND BLOCK-DIAGONAL STRUCTURE

In the bosonic case, the irregular vertex operators with the structure $W_q = e^{\sum_n=0^q \vec{\alpha}_n \partial^n \vec{\phi}}$ appear in the colliding limit of q regular vertex operators as a result of simultaneous normal ordering which structure is rather complicated. Namely, for N colliding operators sitting at points $z_1, \ldots, z_N$ one has for N vertices at $z_1, \ldots z_N$ around z_1

$$\begin{aligned} W_N &= \lim_{z_2,\ldots z_N \to z_1} e^{\alpha_1\phi}(z_1)\ldots e^{\alpha_N}(z_N) \\ &= \prod_{p=2}^{N} (z_{k1})^{-\alpha_p(\alpha_1+\ldots\alpha_{p-1})} \\ &\quad\times \sum_{n_1,\ldots,n_{N-1}} \sum_{k_1,\ldots k_{N-2}} \sum_{q_1\ldots,q_{N-2}} (z_{21})^{n_1}(z_{31})^{n_2-k_1}\ldots \\ &\quad\times (z_{N1})^{n_{N-1}-k_{N-2}} \prod_{j=1}^{N-1} \lambda_{\{n,k,q\}} :B_{\alpha_{j+1}}^{(n_j-q_j(k_1,\ldots k_{N-1}))} \\ &\quad\times e^{(\alpha_1+\cdots+\alpha_N)\phi}:(z_1) \end{aligned} \tag{3.1}$$

with the q-numbers satisfying

$$\sum_{j=1}^{N-2} k_j = \sum_{j=1}^{N-2} q_j \tag{3.2}$$

and $\lambda_{\{n,k,q\}}$ are some constants. Here $z_{ij} = z_i - z_j$ and $B_\alpha^{(n)}$ are the normalized Bell polynomials in the derivatives of ϕ defined as [13]

$$B_\alpha^{(n)} = \sum_{p=1}^{n} \alpha^p \sum_{n|k_1\ldots k_p} \frac{\partial^{k_1}\phi\ldots\partial^{k_p}\phi}{k_1!q_{k_1}!\ldots k_p!q_{k_p}!}. \tag{3.3}$$

Here the sum is taken over the ordered length p partitions of n ($1 \leq p \leq n$): $n = k_1 + \cdots + k_p$; $k_1 \leq k_2 \cdots \leq k_p$ and q_{k_j} is the multiplicity of an element k_j in the partition. The irregular operators, appearing as a result of the summation, diagonalize positive Virasoro generators, with the derivatives of fields in the exponents appearing as a result of summing over all orders of z_{ij}, involving the exponents multiplied by Bell polynomials of various degrees. The Bell polynomials (times exponents) form a basis which is somewhat inconvenient to describe the colliding limit, therefore, the relation between complicated summation (3.1) and a relatively simple structure of the resulting irregular vertex operator (1.1) is not straightforward to obtain. Nevertheless, the colliding limit may be useful to illustrate the difference between the bosonic case (where the Virasoro operators are diagonal) and the supersymmetric case, where they become the Jordan blocks (with the eigenvalues, inherited from the irregular bosonic vertices, except for the half-integer case, where an extra block-diagonal generator L_{2q+1} with the zero eigenvalue appears as well). The important difference between these two cases is related to the fact that the supersymmetric irregular vertex operators involve the colliding limit of the regular vertices from the Ramond sector involving $\sigma_\pm$, satisfying

$$\begin{aligned} \sigma_+(z)\sigma_-(w) &\sim (z-w)^{\frac{3}{8}}\psi \\ \sigma_+(z)\sigma_+(w) &\sim (z-w)^{-\frac{1}{8}} \\ \sigma_-(z)\sigma_-(w) &\sim (z-w)^{-\frac{1}{8}}. \end{aligned} \tag{3.4}$$

The simplest supersymmetric vertex, $W_{\frac{1}{2}}$, appears as a result of a collision of spin operator multiplied by exponent of super Liouville field with the spin operator without exponents:

$$\lim_{z\to w} :\sigma_\pm e^{\alpha\phi}(z)::\sigma_\pm:(w) \tag{3.5}$$

(or equivalently, OPE expansion and normal ordering around the midpoint may be considered). In such a collision, the exponent does not develop irregularity (appearance of derivatives of ϕ) because all the Bell polynomials B_α appearing in the collision will multiply by the exponent with the same α, producing total derivatives of $e^{\alpha\phi}$ that will sum up to the exponent at the location of the normal ordering. The higher ranks, such as W_1 or W_2, admit more possibilities in the colliding limit. For example, one can either collide $e^{\alpha\phi}\sigma_\pm$ at z_1 with $e^{\beta\phi}\sigma_\pm$ at z_2, or collide the same two exponents with extra insertions of the spin operators $\sigma_\pm$, say, at z_3 and z_4 with the collision structure $\sigma_\pm e^{\alpha\phi} - \sigma_\pm e^{\beta\phi} - \sigma_\pm - \sigma_\pm$. In the absence of supersymmetry, the insertion number of free σ's can be arbitrary; in the supersymmetric case the number is limited. It is precisely this "ambiguity" that stands behind the appearance of multiplicities of eigenvalues and, as a result, the super Virasoro generators become the Jordan blocks.

IV. CONCLUSION

In this paper we have constructed the supersymmetric extension of irregular vertex operators, which correlation functions describe irregular supersymmetric conformal blocks. We found that the superconformal structure of the irregular vertices differs significantly from the bosonic case, with the supersymmetric irregular operators block diagonalizing the positive Virasoro generators. The block-diagonal form of the positive Virasoro generators in the supersymmetric case is related to contributions of the operators from the Ramond sector to the colliding limit. Although in this paper we only considered the Virasoro symmetry in details, it is straightforward to check that, in case of the multiple (Toda) fields, the same block-diagonal structure emerges when the irregular vertices are transformed by the positive modes of the W_N-currents. It will be important to understand the impact of the structures, observed in this work, in the language of the matrix model approach such as in [5,12,14–16] and their relevance to Seiberg-Witten curves in related supersymmetric gauge theories in four dimensions. As we also mentioned, the irregular vertex operators appear to be natural objects in background-independent string field theory, being natural blocks for solutions describing both nonperturbative higher-spin backgrounds, as well as cosmological backgrounds, such as the noncommutative rolling tachyons. We hope to be able to address these questions soon in the future papers.

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