

Dark matter from $f(R,T)$ gravityRaziyeh Zaregonbadi,^{*} Mehrdad Farhoudi,[†] and Nematollah Riazi[‡]*Department of Physics, Shahid Beheshti University, G.C., Evin, Tehran 19839, Iran*

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We consider the $f(R, T)$ modified theory of gravity, in which the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar and the trace of the energy-momentum tensor of the matter, in order to investigate the dark-matter effects on the galaxy scale. We obtain the metric components for a spherically symmetric and static spacetime in the vicinity of general relativity solutions. However, we concentrate on a specific model of the theory where the matter is minimally coupled to the geometry, and derive the metric components in the galactic halo. Then, we fix the components by the rotational velocities of the galaxies for the model, and show that the mass corresponding to the interaction term (which appears in the Einstein modified field equation) leads to a flat rotation curve in the halo of galaxies. In addition, for the proposed model, the light-deflection angle has been derived and drawn using some observed data.

DOI: [10.1103/PhysRevD.94.084052](https://doi.org/10.1103/PhysRevD.94.084052)**I. INTRODUCTION**

The issue of dark matter has been a long-outstanding problem in cosmology and galactic astronomy. The observational aspects, such as the behavior of the galactic rotation curves and the mass discrepancy in clusters of galaxies, have necessarily led to the consideration of the existence of dark matter [1]. According to the Newtonian gravity, the galactic rotation curves require the velocity of a star or an interstellar cloud, rotating in the disk, to linearly increase within the bulge and to drop off as the square root of $1/r$ in the outer parts. In contrast, the observed rotation curves of spiral galaxies show that even though the rotational velocities increase from the center of the galaxy, they then attain approximately constant values in the outer range of the baryonic matter's disk (up to several luminous radii,¹ see, e.g., Ref. [3]). Such evidence indicates the possible existence of some new invisible matter distributed inside and (mostly) around galaxies, which is known as dark matter [3,4].

In this respect, it is well known that the mass of a cluster can be estimated in two ways. The total baryonic mass, M_b , is estimated by considering the total sum of all observable mass members. However, by taking into account the dynamical motions of galaxies, the virial theorem provides an estimated mass, M , for a cluster. The comparison between the total baryonic mass (obtained from the observed data) with the virial mass of a cluster shows that their ratio is $M/M_b \approx 20\text{--}30$. This fact is usually explained by postulating the existence of dark matter [1,5]. In addition, the curvature of the spacetime near any

gravitating mass (including the dark matter), deflects passing rays of light and distorts the images of the background galaxies. Indeed, gravity acts as a lens to bend the light from a more distant source (such as a quasar) around a massive object (such as a cluster of galaxies) lying between the source and the observer, in accordance with general relativity (GR) [6]. Such an effect was first observed in 1919, during a solar eclipse in front of the Hyades star cluster, whose stars appeared to move as they passed behind the sun [7]. However, this concept was improved in 1937, when Zwicky suggested that the ultimate measurement of the cluster masses would emerge from gravitational lensing [8]; this has indeed become the most successful probe of the dark sector. Nowadays, the measurements of such effects provide constraints on the mean density of dark matter [9].

There are several candidates for dark matter. One possible categorization tells us that dark matter can be either baryonic or nonbaryonic. The baryonic sector is made of baryons (protons and neutrons) that make up stars, interstellar matter, and planets. The main baryonic candidates are the massive astronomical compact halo objects (MACHOs) that include brown dwarf stars and black holes. The nonbaryonic candidates are basically elementary particles that have nonstandard properties. Among the nonbaryonic candidates, one can point to the axions, which are considered as a solution to the strong- CP problem. However, the largest class is that of the weakly interacting massive particle (WIMP), which can be various types of unknown particles [10]. The most popular of these WIMPs is the neutralino from supersymmetry. The WIMP interaction cross section with normal baryonic matter is extremely small, but nonzero; thus, their direct detection is a weak possibility. Heavy neutrinos may also be considered as possible candidates for dark matter [10].

Moreover, accelerator and reactor experiments do not yet support the particle scenarios in which dark matter

^{*}r_zare@sbu.ac.ir[†]m-farhoudi@sbu.ac.ir[‡]n_riazi@sbu.ac.ir¹However, beyond the range of nearly flat rotational velocity, there is some observational evidence showing the decay of the galactic rotation curves, see, e.g., Ref. [2].

emerges. To deal with the question of dark matter, a great number of alternative efforts have been concentrated on various modifications to the Einstein field equations (see, e.g., Refs. [11–18]). For instance, theories of $f(R)$ -modified gravity (as the simplest family of the higher-order gravities), which are based on replacing the scalar curvature R in the Einstein-Hilbert action with an arbitrary differentiable function of it, have had some successes in explaining the accelerated expansion of the Universe [19–22], and have accounted for dark-matter-like effects [23–27].

In this work, we propose to explain the effect related to dark matter by another type of modified gravity theory, namely, $f(R, T)$ gravity. The theory of $f(R, T)$ gravity generalizes theories of gravity by *a priori* incorporation of the trace of dustlike matter in addition to the Ricci scalar into the Lagrangian (see, e.g., Refs. [28–37]). That is, the Lagrangian of $f(R, T)$ gravity model depends on a source term, which itself represents the variation of the energy-momentum tensor with respect to the metric. In other words, the appearance of the matter with an unusual coupling with the geometry may have a relation with the geometrical curvature induction of the matter in addition to the pure geometry in spacetime. Hence, we investigate whether the interaction between the matter and geometry can explain the observational data in the galaxy halo instead of considering an additional mysterious mass as dark matter.

The work is organized as follows. The field equations in $f(R, T)$ gravity, in particular when the matter is minimally coupled to the curvature in a specific form, are presented in Sec. II, where we also specify the metric components for this type of theory in a spherical galactic halo. In Sec. III, we fix the metric components while using the tangential velocity of a galaxy. Then, we study the propagation of the light in this type of theory for a typical galaxy in Sec. IV. Finally, in Sec. V we present the conclusions. Through the work, we use the sign convention $(-, +, +, +)$ and geometrical units with $c = 1$.

II. MODIFIED FIELD EQUATIONS OF $f(R, T)$ GRAVITY

In this section, we derive the field equations and some corresponding dynamical parameters of $f(R, T)$ -modified gravity in four-dimensional spacetime. The action is simply written in the form

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} f(R, T) + L_m \right], \quad (1)$$

where $\kappa \equiv 8\pi G$, g is the determinant of the metric, and L_m is the matter Lagrangian density. The energy-momentum tensor is defined as

$$T_{\mu\nu}^{[m]} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_m)}{\delta g^{\mu\nu}}, \quad (2)$$

where the lowercase Greek indices run from 0 to 3 and the index m stands for the baryonic matter. The variation of the action with respect to the metric tensor gives the field equations

$$f_R R_{\mu\nu} - \frac{f}{2} g_{\mu\nu} + (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) f_R = (\kappa + f_T) T_{\mu\nu}^{[m]}, \quad (3)$$

where $\square \equiv \nabla_\mu \nabla^\mu$ and we have defined the following functions for the derivative of the function $f(R, T)$ with respect to its arguments, i.e.,

$$f_R \equiv \frac{\partial f(R, T)}{\partial R} \quad \text{and} \quad f_T \equiv \frac{\partial f(R, T)}{\partial T}. \quad (4)$$

It is usually more instructive to write the field equations in the form of the Einstein equations with an effective energy-momentum tensor as

$$G_{\mu\nu} = \frac{\kappa}{f_R} (T_{\mu\nu}^{[m]} + T_{\mu\nu}^{[\text{int}]}) = \frac{\kappa}{f_R} T_{\mu\nu}^{[\text{eff}]}, \quad (5)$$

where $T_{\mu\nu}^{[\text{eff}]} \equiv T_{\mu\nu}^{[m]} + T_{\mu\nu}^{[\text{int}]}$. The interaction energy-momentum tensor has been defined as

$$T_{\mu\nu}^{[\text{int}]} \equiv \frac{1}{\kappa} \left[f_T T_{\mu\nu}^{[m]} + \frac{1}{2} (f - R f_R) g_{\mu\nu} + (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) f_R \right], \quad (6)$$

which may be interpreted as a fluid composed of the interaction between the matter and the curvature terms.

In the following subsections, we first derive the field equations for a spherically symmetric system. Then, we study a particular minimal coupling model in $f(R, T)$ gravity to investigate the dark-matter effect on the galactic scale, wherein we also obtain the metric components for this model.

A. Field equations for a spherically symmetric system

Let us now consider an isolated system that is described by a static and spherically symmetric metric

$$ds^2 = -e^{a(r)} dt^2 + e^{b(r)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2. \quad (7)$$

Thus, the energy-momentum tensor of the matter can be described by an effective density, $\rho^{[\text{eff}]}$, and an effective anisotropic pressure with radial, $p_r^{[\text{eff}]}$, and tangential, $p_\perp^{[\text{eff}]}$, components [38]. Henceforth, the field equations become

$$G_t^t = -\frac{1}{r^2} + \frac{1}{r^2 e^b} - \frac{b'}{r e^b} = -\frac{\kappa}{f_R} (\rho^{[m]} + \rho^{[\text{int}]}) \quad (8)$$

$$G_r^r = -\frac{1}{r^2} + \frac{1}{r^2 e^b} + \frac{a'}{r e^b} = \frac{\kappa}{f_R} p_r^{[\text{int}]}, \quad (9)$$

and

$$G_\theta^\theta = G_\phi^\phi = \frac{1}{4e^b} (2a'' + a'^2 - a'b') + \frac{a' - b'}{2re^b} = \frac{\kappa}{f_R} p_\perp^{[\text{int}]}. \quad (10)$$

From definition (6), we get the energy density and the radial pressure for the interaction term as

$$\rho^{[\text{int}]} = -T_t^{t[\text{int}]} = \frac{1}{\kappa} \left[f_T \rho^{[m]} + \frac{1}{2} (R f_R - f) - \frac{a' f'_R}{2e^b} + \square f_R \right], \quad (11)$$

$$p_r^{[\text{int}]} = T_r^{r[\text{int}]} = \frac{1}{\kappa} \left[\frac{1}{2} (f - R f_R) - \frac{1}{e^b} \left(\frac{b'}{2} f'_R - f''_R \right) - \square f_R \right], \quad (12)$$

$$p_\perp^{[\text{int}]} = T_\theta^{\theta[\text{int}]} = T_\phi^{\phi[\text{int}]} = \frac{1}{\kappa} \left[\frac{1}{2} (f - R f_R) + \frac{f'_R}{re^b} - \square f_R \right], \quad (13)$$

where prime is derivative with respect to the r coordinate. In these relations, $\square f_R$ is

$$\square f_R = \frac{1}{e^b} \left[\left(\frac{a' - b'}{2} \right) f'_R + \frac{2}{r} f'_R + f''_R \right]. \quad (14)$$

To obtain the components of the metric for this type of modified gravity, we first derive a useful relation from metric (7), namely,

$$\frac{R_{tt}}{e^a} + \frac{R_{rr}}{e^b} = \frac{a' + b'}{re^b}, \quad (15)$$

as well as another one, from the field equations (3), as

$$\frac{R_{tt}}{e^a} + \frac{R_{rr}}{e^b} = \frac{1}{f_R} \left[(\kappa + f_T) \rho^{[m]} - \frac{f'_R}{2e^b} (a' + b') + \frac{f''_R}{e^b} \right]. \quad (16)$$

Replacing relations (11) and (12) into the right-hand side of relation (16), and then inserting relation (16) into [17] gives

$$\frac{a' + b'}{re^b} = \frac{\kappa}{f_R} (\rho^{[m]} + \rho^{[\text{int}]} + p_r^{[\text{int}]}). \quad (17)$$

On the other hand, our purpose is to obtain solutions that differ from the classical GR only slightly. In this respect, if the combination $a' + b'$ is a well-behaved differential expression, it should have a solution of the form $e^{a(r)} e^{b(r)} = A(r)$; however, in order to remain in the vicinity of GR, the function $A(r)$ would slightly be different from 1. For instance, one can consider [39]

$$A(r) = (r/s)^\beta \Rightarrow a' + b' = \frac{\beta}{r}, \quad (18)$$

where β is a dimensionless parameter and s is the length scale of the system; however, if $\beta \ll 1$, one gets $A(r) \approx 1 + \beta \ln(r/s)$ and, thus, the desired proposal will be fulfilled. By inserting relation (18) into (17), we can specify the components of the metric as

$$e^{b(r)} = \frac{\beta f_R}{\kappa r^2 (\rho^{[m]} + \rho^{[\text{int}]} + p_r^{[\text{int}]})} \quad (19)$$

and

$$e^{a(r)} = \left(\frac{r}{s} \right)^\beta e^{-b(r)}. \quad (20)$$

B. Minimal coupling model

Among different types of modified $f(R, T)$ gravity inspired from our previous work [35], we consider a model where the matter, in a simple manner, is minimally coupled to the geometry as

$$f(R, T) = R - \alpha(-T)^n, \quad (21)$$

where $T = -\rho^{[m]}$, and (positive) α and n are constants, wherein n is a power which determines the strength of the effect of the matter. Furthermore, it is worth mentioning that beyond the radius of the galactic halo, if there is a true vacuum status (i.e., $T_{\mu\nu}^{[m]} = 0$), then the model will obviously reduce to GR. For this model, we have

$$f_R = 1 \quad \text{and} \quad f_T = \alpha n (\rho^{[m]})^{n-1}. \quad (22)$$

Hence, as the interaction energy density, relation (11), gives

$$\kappa \rho^{[\text{int}]} = \alpha \left(n + \frac{1}{2} \right) (\rho^{[m]})^n, \quad (23)$$

a plausible requirement $\rho^{[\text{int}]} \geq 0$, with positive α , implies $n \geq -1/2$. However, in order to have a solution for the vacuum case, the power of the matter density should be equal or more than zero, i.e., $n \geq 0$, where $n = 0$ corresponds to GR theory. Also, from relations (11) and (12), we get

$$\kappa (\rho^{[\text{int}]} + p_r^{[\text{int}]}) = \alpha n (\rho^{[m]})^n. \quad (24)$$

In the galactic halo, where the density of the baryonic matter is very low, we assume that the mass assigned to the density of the interaction between the matter and geometry can explain the observations with no need to introduce a mysterious dark matter. In fact, with $0 < n < 1$, relation (23) still yields $\rho^{[m]} \ll \rho^{[\text{int}]}$ in this region. Indeed, where there is very little matter, but as far as $T^{[m]}$ is not exactly zero (although $T^{[m]} \rightarrow 0$), the model can, in principle, maintain the flatness of the rotation curves.

Thus, by assuming $\rho^{[m]} \ll \rho^{[int]}$ in the galactic halo and inserting relation (24) into relation (19), we can rewrite the component of the metric as

$$e^{b(r)} = \frac{\beta}{\alpha n r^2 (\rho^{[m]})^n}. \quad (25)$$

In addition, although the baryonic matter in the form of stars, interstellar gas, and dust in spiral galaxies is mostly concentrated in the flattened disk, it is instructive to suppose that, in the galactic halo, the density distribution of the baryonic matter is approximated by a spherically symmetric model and rapidly decreases with radius according to the power-law profile²

$$\rho^{[m]} = \rho_0 r^{-\gamma}, \quad (26)$$

where³ $\gamma > 2$ and ρ_0 is a constant that, without loss of generality, we set equal to 1. Also, we take the length scale of the system to be equal to the radius of the baryonic matter dominated, i.e., $s = r_B$. Hence, from relation (25), we get

$$e^{b(r)} = \frac{\beta}{\alpha n} r^{\gamma n - 2} \quad (27)$$

and, consequently, from relation (20),

$$e^{a(r)} = \frac{\alpha n}{\beta r_B^\beta} r^{\beta + 2 - \gamma n}. \quad (28)$$

The interaction density, relation (23), reads

$$\kappa \rho^{[int]} = \alpha \left(n + \frac{1}{2} \right) r^{-\gamma n}. \quad (29)$$

In the next section, we attempt to fix the metric components (27) and (28) for a galactic model with the flat rotation curve.

III. GALACTIC ROTATION CURVES

The observational data shows that the rotational velocity increases linearly within the bulge and approaches a constant value of about 200–500 km/s, as one moves away from the core up to several luminous radii [1,3]. In this section, we consider a test particle that moves in a timelike geodesic orbit in a static and spherically symmetric system in the plane (without loss of generality) $\theta = \pi/2$. Hence, the corresponding geodesic equation for the r coordinate is

²Note that, we are not building a fully realistic model, hence, we have restricted ourselves to a spherically symmetric approximation, as many other authors, see, e.g., Refs. [40,41].

³Note that, according to the Newtonian gravity, one has to have a constant rotational velocity in the halo of the galaxy, i.e., $M \propto r \Rightarrow \rho^{[m]} \propto r^{-2}$.

$$\frac{d^2 r}{d\tau^2} + \frac{a' e^{a(r)}}{2 e^{b(r)}} \left(\frac{dt}{d\tau} \right)^2 + \frac{b'}{2} \left(\frac{dr}{d\tau} \right)^2 - \frac{r}{e^b} \left(\frac{d\phi}{d\tau} \right)^2 = 0, \quad (30)$$

where τ is the affine parameter along the geodesic. When a test particle moves along its geodesic in a gravitational field in a spherically symmetric spacetime, the momenta P_0 and P_3 are conserved [42,43], namely,

$$E = e^{a(r)} \left(\frac{dt}{d\tau} \right) = \text{const} \quad \text{and} \quad J = r^2 \left(\frac{d\phi}{d\tau} \right) = \text{const}, \quad (31)$$

where E is the energy and J is the ϕ coordinate of the angular momentum of the test particle.

Now, let us study the motion associated with the stable circular orbits, i.e., $dr/d\tau = 0$. In this case, we can rewrite Eq. (30) as

$$\frac{a' E^2}{2 e^{a(r)}} = \frac{J^2}{r^3}. \quad (32)$$

In the weak-field approximation, for an inertial observer far from the source, one can measure the circular orbital speed as [44,45]

$$v = \frac{r d\phi}{\sqrt{e^{a(r)}} dt}. \quad (33)$$

While using relation (31), one can rewrite relation (33) in terms of the conserved quantities as

$$v = \frac{\sqrt{e^{a(r)}} J}{r E}, \quad (34)$$

that, by inserting Eq. (32), leads to

$$v^2 = \frac{r a'}{2}. \quad (35)$$

In this regard, for the specified minimal coupling model, from relation (28) in the galactic halo, we have

$$a' = \frac{\beta + 2 - \gamma n}{r}. \quad (36)$$

Hence, for the model, the tangential velocity reads

$$v^2 = \frac{\beta + 2 - \gamma n}{2}, \quad (37)$$

where the value of the tangential velocity of test particles in the circular stable orbits around galactic halo is in the range of the observed flat rotation curves, i.e., approximately 200–500 km/s [1]. Thus, by considering the range of the tangential velocity v in a typical spiral galaxy,⁴ it is known that $v^2 \approx \mathcal{O}(10^{-6})$. On the other hand, in order to remain closed to GR, as mentioned after relation (18), the β parameter must be much smaller than 1. In fact, the

⁴Note that v is measured in units of c .

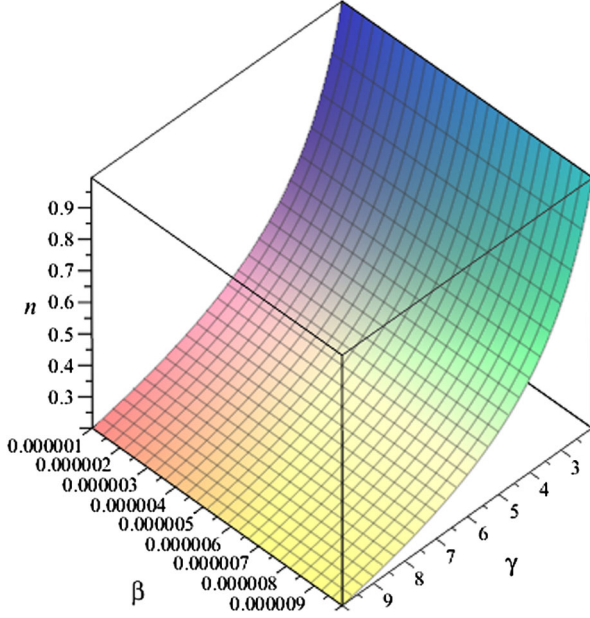


FIG. 1. The plot of relation (37) with $\gamma > 2$, $v = 300$ km/s, and $10^{-6} < \beta < 10^{-5}$.

observed data from the galactic rotation curves indicates that the β parameter is approximately equal to the second power of tangential velocity,⁵ i.e., $\beta \approx v^2$ [39]. We have plotted relation (37) in Fig. 1 to obtain the allowed values of the n parameter in the specified minimal coupling model of $f(R, T)$ gravity.

Figure 1 dictates that $\gamma n \approx 2$, where, in turn, relation (29) gives the interaction density as

$$\rho^{[\text{int}]} \propto r^{-2}. \quad (38)$$

In this case, the metric components (27) and (28) are

$$e^{b(r)} = \frac{\beta}{\alpha n} = \text{const} \quad (39)$$

and

$$e^{a(r)} = \frac{\alpha n}{\beta r_B^\beta} r^\beta, \quad (40)$$

that, as has been set, yield $e^{a(r)} e^{b(r)} = (r/r_B)^\beta \approx 1 + \beta \ln(r/r_B)$ for very small β . On the other side, by integrating Eq. (8), we get

$$\begin{aligned} e^{-b(r)} &= 1 - \frac{2G}{r} \int 4\pi(\rho^{[m]} + \rho^{[\text{int}]}) r'^2 dr' \\ &= 1 - \frac{2G}{r} (M^{[m]} + M^{[\text{int}]}). \end{aligned} \quad (41)$$

⁵In Ref. [39], it has been shown that the parameter β should depend on the mass of the gravitating body, because any localized matter manifests no characteristic other than its mass when is sensed from far distances.

As it is plausibly assumed that in the inner parts of the galaxy (up to $r = r_B$) the baryonic matter is dominant and that, in this radius, the mass of the baryonic matter has a fixed value, at the boundary of the baryonic mass with dark matter, the metric component $e^{-b(r)}$ in relation (41) is, therefore, a constant. Hence, Eq. (41) reads

$$e^{-b(r_B)} \approx 1 - \frac{2G}{r_B} M^{[m]}|_{r=r_B}. \quad (42)$$

Furthermore, in the galaxy halo (i.e., in the range $r_B < r < r_D$, where r_D is the radius wherein the halo terminates⁶), the metric component (41) reduces to

$$e^{-b(r)} \approx 1 - \frac{2G}{r} M^{[\text{int}]}. \quad (43)$$

Now, due to the continuity of the metric components in the flat rotation curve boundary of a galaxy, one can set relation (42) equal to relation (43)—while considering the constancy of relation (39)—to achieve

$$M^{[\text{int}]} = \frac{r}{r_B} M^{[m]}|_{r=r_B}. \quad (44)$$

That is, $M^{[\text{int}]}$ varies linearly with r , which is consistent with $\rho^{[\text{int}]} \propto r^{-2}$ behavior. Moreover, we can obtain relation (41) as a constant in the dark-matter-dominated area.

Another link with the observation is the effect of dark matter on the light-deflection angle, which is considered in the next section for the assumed $f(R, T)$ model.

IV. LIGHT-DEFLECTION ANGLE

This section investigates the light-deflection angle as an effect of dark matter for the specified minimal coupling model. The deflection angle $\Delta\phi$ is given by

$$\Delta\phi = 2|\phi(r_0) - \phi(\infty)| - \pi, \quad (45)$$

where r_0 is the radius of the closest approach to the center of the galaxy. The geodesic equation, Eq. (30), for a photon reduces to [42]

$$\phi(r_0) - \phi(\infty) = \int_{r_0}^{\infty} e^{\frac{b(r)}{2}} \left[e^{a(r_0) - a(r)} \left(\frac{r}{r_0} \right)^2 - 1 \right]^{-\frac{1}{2}} \frac{dr}{r}. \quad (46)$$

Using relations (39) and (40), we can rewrite relation (46) as

⁶That is, the distance up to which the tangential velocity remains constant is considered as the radius of the dark matter, i.e., up to where the rotation curves are flat.

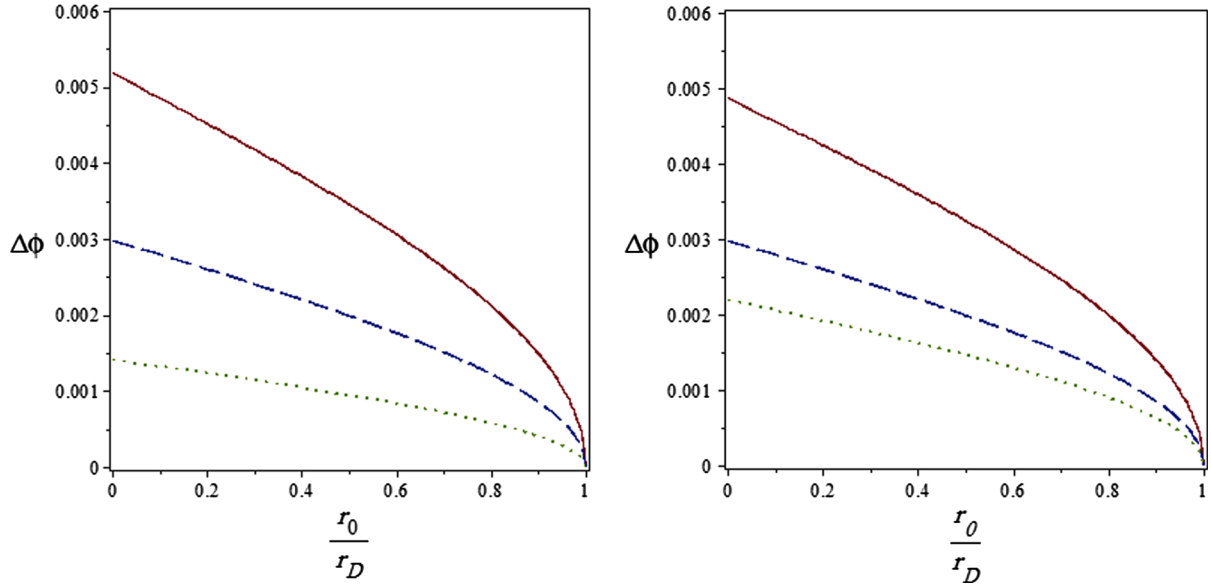


FIG. 2. The panels illustrate the light-deflection angle [Eq. (48)] for (left) the galaxies NGC 5533 (solid line), NGC 4138 (dashed line), and UGC 6818 (dotted line) with $n = 1/2$ and the β parameter as mentioned in the text and (right) the NGC 4138 galaxy with $n = 1/5$ (solid line), $n = 1/2$ (dashed line), and $n = 20/21$ (dotted line), and all the curves with $\alpha = 1$.

$$\begin{aligned} \phi(r_0) - \phi(\infty) = & \sqrt{\frac{\beta}{\alpha n}} \int_{r_0}^{r_D} \left[\left(\frac{r_0}{r} \right)^{\beta-2} - 1 \right]^{-\frac{1}{2}} \frac{dr}{r} \\ & + \int_{r_D}^{\infty} \frac{1}{\sqrt{1 - \frac{2GM}{r}}} \left[\frac{1 - \frac{2GM}{r_0}}{1 - \frac{2GM}{r}} \left(\frac{r}{r_0} \right)^2 - 1 \right]^{-\frac{1}{2}} \frac{dr}{r}, \end{aligned} \quad (47)$$

where, in the first integral term, we have considered r_0 in the region of the flat rotation curves, i.e., $r_B \leq r_0 < r_D$.

One can now exactly integrate the first term in relation (47), which corresponds to the dark-matter-dominated area (in the region of the flat rotation curve). The second integral term in relation (47) relates to the exterior region of the dark-matter halo, wherein we integrate it with the Schwarzschild metric while using the Robertson expansion [42]. Hence, we achieve

$$\begin{aligned} \Delta\phi = & 2 \left| \sqrt{\frac{\beta}{\alpha n}} \left(\frac{2}{2-\beta} \right) \arctan \left(\sqrt{\left(\frac{r_0}{r_D} \right)^{\beta-2} - 1} \right) \right. \\ & + \arcsin \left(\frac{r_0}{r_D} \right) \\ & \left. + \frac{GM}{r_0} \left[2 - \sqrt{1 - \left(\frac{r_0}{r_D} \right)^2} - \sqrt{\frac{r_D - r_0}{r_D + r_0}} \right] \right| - \pi. \end{aligned} \quad (48)$$

The variation of the light-deflection angle as a function of r_0/r_D in the galactic halo has been plotted in Fig. 2 (left), with the data from the NGC 5533 galaxy with $\beta = 1.4 \times 10^{-6}$, the NGC 4138 galaxy with $\beta = 0.48 \times 10^{-6}$ and the UGC 6818 galaxy with $\beta = 0.12 \times 10^{-6}$, and all with $n = 1/2$ and $\alpha = 1$. For the value of r_0 in the

second and third terms of relation (48), which have emerged from the second integral term of relation (47), we have plausibly considered $r_0 \approx r_D$ in the relevant region, see, e.g., Ref. [39]. However, when $r_0 = r_D$, Eq. (48) yields $\Delta\phi = 4GM/r_D$ as GR, which is consistent with the observation in this range. Note that the value $n = 1/2$ corresponds to a particular minimal coupling model that satisfies the Bianchi identity for the Einstein tensor on the cosmological scale, see, e.g., Refs. [33,35]. Figure 2 (left) indicates that the deflection angle can be appreciable as $r_0 \ll r_D$. Moreover, we have plotted the light-deflection angle for the NGC 4138 galaxy with $\alpha = 1$, but different values of n , in Fig. 2 (right). The figure shows that by decreasing the value of n , i.e., by increasing γ , the deflection angle increases for each fixed value of r_0/r_D .

Let us now compare the obtained deflection angle from the minimal coupling model of $f(R, T)$ gravity with some other dark-matter models.

In this regard, first consider a commonly used model of the galactic dark-matter halo with the matter density

$$\rho(r) = \rho_0 e^{-(r/r_c)^2}, \quad (49)$$

where ρ_0 is the halo core density and r_c is the radius of core. This model is a generalization of the pseudoisothermal dark-matter model [46]. In the weak-field limit, the light-deflection angle in this model is given by [47]

$$\Delta\phi = \frac{4GM(r)}{r} = \frac{4G}{r} \int_0^r 4\pi\rho(r')r'^2 dr', \quad (50)$$

where $M(r)$ is the effective mass of the dark matter inside the radius r . Throughout the galactic halo, inserting the matter density (49) into Eq. (50) gives

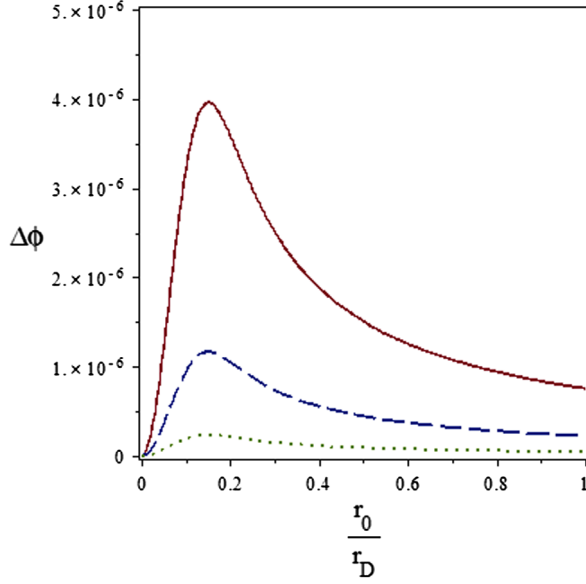


FIG. 3. The curves illustrate the light-deflection angle [Eq. (52)] for galaxies NGC 5533 (solid line), NGC 4138 (dashed line), and UGC 6818 (dotted line) with the parameters mentioned in the text.

$$\Delta\phi = \frac{16\pi G\rho_0}{r_0} \int_0^{r_0} r'^2 e^{-(r'/r_c)^2} dr'. \quad (51)$$

Now, assuming the halo cutoff r_D to be $r_D = 10r_c$, where $\rho(r_D) = 0$, then Eq. (51) leads to

$$\Delta\phi = 16\pi G\rho_0 \left(\frac{r_D}{10}\right)^2 \left[-\frac{1}{2} e^{-(r_0/r_D)^2} + \frac{\sqrt{\pi}}{4} \left(\frac{r_D}{r_0}\right) \text{erf}\left(\frac{r_0}{r_D}\right) \right]. \quad (52)$$

This variation of the light-deflection angle, relation (52), has been plotted in Fig. 3, together with the data of the NGC 5533 galaxy with $r_D = 72$ kpc, the NGC 4138 galaxy with $r_D = 13$ kpc, and the UGC 6818 galaxy with $r_D = 6$ kpc, and all with $\rho_0 = 10^{-14}$ kg/m³ (these data are from Refs. [39,47]). Figure 3 illustrates that the deflection angle in this model increases up to r_c and then starts to decline in the dark-matter halo.

The comparison of the deflection angle of our model with the brane- $f(R)$ gravity model (proposed in Ref. [27])

indicates that both models have the same behavior; i.e., the deflection angle reduces in the dark-matter halo with the radius.

V. CONCLUSIONS

In this work, we have considered $f(R, T)$ -modified theory of gravity to explain the dark-matter effects in spiral galaxies as inferred from the flat rotation curves. The additional term in the field equations that is acquired in this type of theory can be interpreted as an alternative for the dark-matter effects in the galaxy. We have obtained the metric components for a spherically symmetric and static spacetime in the vicinity of general relativity solutions. However, we have concentrated on a particular minimal coupling model in this theory, and have derived the metric components in the galactic halo. Then, we have fixed the components by the rotational velocities of galaxies for the model. Finally, we have obtained that the mass corresponding to the interaction term (which appears in the Einstein modified field equation) varies linearly with the radius; hence, the interaction mass can cause a flat rotation curve in the halo of galaxies.

In addition, we have obtained the light-deflection angle, using the metric components derived for the model, in the galactic halo. The diagrams for the light-deflection angle consistently indicate that a galaxy with larger mass has a bigger deflection angle. For a galaxy with a specified mass, we have plotted the light-deflection angle for different values of n (a power that shows the strength of the effect of the trace of the energy-momentum tensor of the matter in the specified minimal coupling model). The figures indicate that by decreasing the value of n , the light-deflection angle increases. This can be explained by noting that the density of the baryonic matter, for smaller values of n , rapidly decreases in the halo. Indeed, this is due to the fact that the density of the baryonic matter, for smaller values of n , is more concentrated in the region $r_0 \ll r_D$ compared to the density of the interaction term.

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