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We suggest that holography can be formulated in terms of the information capacity of the Stückelberg degrees of freedom that maintain gauge invariance of the theory in the presence of an information boundary. These Stückelbergs act as qubits that account for a certain fraction of quantum information. Their information capacity is measured by the ratio of the inverse Stückelberg energy gap to the size of the system. Systems with the smallest gap are maximally holographic. For massless gauge systems this information measure is universally equal to the inverse coupling evaluated at the systems' length scale. In this language it becomes very transparent why the Stückelberg information capacity of black holes saturates the Bekenstein bound and accounts for the entire information of the system. The physical reason is that the strength of quantum interaction is bounded from below by the gravitational coupling, which scales as area. Observing the striking similarity between the scalings of the energy gap of the boundary Stückelberg modes and the Bogoliubov modes of critical many-body systems, we establish a connection between holography and quantum criticality through the correspondence between these modes.

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I. STÜCKELBERG QUBITS

The scaling of Bekenstein entropy [1] as the black hole area in Planck units, $S_{\text{BH}} = (R/L_p)^2$, suggests that the information is stored in some boundary qubit degrees of freedom, one per each Planck-area-sized pixel. This approach is usually called the principle of black hole holography [2]. Without committing to the literal validity of such a picture, it is nevertheless necessary to understand the fundamental reason behind the area law. One microscopic explanation [3,4] (see [5–7] for further discussions) is that the area law is a reflection of the scaling of the quantum gravitational coupling, $\alpha(R)$, of gravitons of wavelength R . Notice that the most basic property of the inverse gravitational coupling is to “run” with the length-scale R as the *area*,

$$\frac{1}{\alpha(R)} = \frac{R^2}{L_p^2}. \quad (1)$$

Moreover note that the above relation between the quantum gravitational coupling and the area holds in an arbitrary number of dimensions. Thus, in the picture of [3,4], the black hole entropy is fundamentally defined by the inverse graviton coupling

$$S_{\text{BH}} = \frac{1}{\alpha(R)}, \quad (2)$$

and the area law is a direct consequence of (1). The microscopic reason behind (2) according to [4,5] is the quantum criticality of the black hole state, which is characterized by the appearance of $N = \alpha^{-1}(R)$ collective Bogoliubov-Goldstone modes with energy gap $\Delta = \alpha(R) \frac{\hbar}{R}$. The corresponding entropy is obviously given by (2).

Hence, in this picture holography is an emergent phenomenon that results from the *quantum criticality* of the underlying gravitational system, for which both the number N as well as the energy gap Δ of the qubits is determined by the gravitational coupling (1). As we shall see, we shall discover a connection, strikingly similar to (2), between the information capacity and the inverse coupling of the system, from a completely different approach. This allows us to draw an interesting conclusion about the nature of the information qubits of holographic systems, in which gauge redundancy and quantum criticality appear as two sides of the same coin.

In order to formulate this alternative approach, in the present paper we would like to ask whether there exists a simple way to understand the area law from a fundamental gauge principle, without the knowledge of the microscopic picture of the black hole interior.

We shall be interested in systems that contain subsystems separated by surfaces that act as boundaries from the information point of view. Consider two regions of the space, A and B , separated by a closed surface S . Let us

assume that due to some dynamical reason the retrieval of information from region B into A is highly suppressed or simply impossible. We shall say that for an observer in region A (we shall call her Alice) the surface S acts as an *information boundary*. The suppression of the information-retrieval probability can be, for example, due to an energy barrier that prevents some particle species crossing over from B to A (see examples below). One natural example that fits within our definition of the information boundary is a black hole horizon. In this case the region B is identified with the interior of the black hole and Alice is an external observer.

The general question is the following: How can Alice characterize the information-storage capacity of the hidden region B ? One known possibility is that Alice traces over the degrees of freedom in B and ends up with a density matrix. Naturally, in such a treatment an entanglement entropy is attributed to region B . The problem with this information measure is that it is based on complete ignorance about region B . Consequently, the resulting entanglement entropy, instead of giving a real information measure, gives only an upper bound on the information-storage capacity of region B . In reality, the true information capacity may be much smaller. For black holes, the entanglement entropy gives the same scaling with area as the Bekenstein entropy. However, this fact does not constitute a satisfactory microscopic explanation of the black hole entropy, but rather a reflection of a simple coincidence, namely that a black hole happens to saturate the Bekenstein entropy bound. Correspondingly, the two entropies agree in scaling. Therefore, in our approach we shall not rely on entanglement entropy.

The question we would like to ask is whether there is a more efficient measure of information-storage capacity that Alice could derive without the detailed knowledge of the microscopic properties of system B .

In the present paper we would like to point out that in systems in which the information carriers are some sort of gauge degrees of freedom, the information-storage capacity of the interior region can be understood in terms of the energy gap of the boundary Stückelberg degrees of freedom that are necessary for maintaining the gauge redundancy of Alice's description in the presence of the information boundary S .

In order to outline the general argument, let us assume that Alice deals with a bulk gauge theory of a gauge field h with a Lagrangian density $L(h)$, that under gauge transformation shifts by a total derivative $\delta L = d\Omega$. In the presence of the information boundary S , this generates a surface term $\int_S \Omega$, where the integral is taken over the world volume of the boundary surface S . By gauge redundancy, such a term must be canceled by a shift of some Stückelberg field, η .

Thus, the effective theory (from the information perspective) that Alice deals with is a bulk gauge theory plus the world-volume action of surface S with the Stückelberg degrees of freedom,

$$S = \int_A L(h) + \int_S \eta, \quad (3)$$

such that the variation of η is $\delta\eta = -\Omega$.

It is important to stress here the key point of the former argument. Alice needs to have in her region *manifest* gauge invariance. The necessity of imposing this condition on her physics forces her to decorate what for her is acting as an information boundary with new physical degrees of freedom. In other words, she is deriving part of the actual dynamics of the information boundary by imposing a manifest gauge invariance in the region that is, in information terms, accessible to her.

The new degree of freedom η provides the natural qubits that store (part of the) information about region B . The elementary qubits can be labeled either by the world-volume coordinates of the boundary, or equivalently by the world-volume momentum modes up to some maximal momentum. For example, for a spherical boundary these can be labeled by the usual spherical harmonics l, m . Obviously, the total number of elementary qubits scales as the *area* of the boundary S . Thus, the quantum information can be stored in the occupation numbers of different world-volume momentum modes. For example, in the spherical case the quantum information can be included in a set of occupation numbers $n_{m,l}$ labeled by the spherical harmonics m, l .

The capacity of information storage is measured by the energy gap of the qubit η . Let this gap be Δ . Let us also assume that the size of region B is R . Then, a useful measure of the information-storage capacity is the following quantity,

$$\mathcal{I} \equiv \frac{\hbar}{R\Delta}. \quad (4)$$

The physical meaning of the quantity \mathcal{I} is the following. In a generic system of size R , with weakly interacting degrees of freedom, we can store a single bit of information at the energy expense $\sim \hbar/R$. Indeed, this energy roughly measures the spacing between the ground state and the first excited energy level in such systems. Thus, the quantity \mathcal{I} gives us a measure of how costly it is, relative to this energy, to store a single bit of information in the Stückelberg degree of freedom η . If $\mathcal{I} \gg 1$ the system is an efficient storer of information. In the opposite case, $\mathcal{I} \ll 1$, the information-storage capacity of the system is very poor.¹

Notice that the quantity \mathcal{I} measures the part of the entropy that is stored in the Stückelberg modes. This is

¹There exists a different natural interpretation of \mathcal{I} that Alice may immediately think about. Since Alice knows the size R of region B , she can infer the quantum uncertainty in the energy stored in this region as $\frac{\hbar}{R}$. However, for her this uncertainty is *resolved* into \mathcal{I} Stückelberg boundary states per each qubit given by η . Hence, she can already infer that \mathcal{I} measures a part of the *degeneracy* of the ground state, whatever may be the underlying reason from the microscopic physics point of view, describing region B .

simple to understand. In order to evaluate this part of the entropy, we have to compute the number of Stückelberg states that populate the energy levels within the energy gap \hbar/R and then take the log of this number. Restricting ourselves by considering the first excited level for each qubit, the state with N excited qubits has energy $E = N\Delta$. Restricting this energy to be $E < \hbar/R$, we easily get that the maximal allowed number of excited qubits is $N_{\max} = \mathcal{I}$. The corresponding number of states obviously is $\sim 2^{\mathcal{I}}$ and the resulting contribution to the entropy is \mathcal{I} .

We can derive a more general expression relating the entropy to the number of Stückelberg modes N and the maximal number of excited modes \mathcal{I} . Through simple combinatorics, we obtain

$$e^S = \sum_n^{\mathcal{I}} \binom{N+n-1}{n} = \frac{\Gamma(1+N+\mathcal{I})}{\Gamma(1+\mathcal{I})\Gamma(1+N)}, \quad (5)$$

where the sum is to count all states with occupation $n \leq \mathcal{I}$. For large N and \mathcal{I} , the leading order expression for general N and Δ is

$$S \approx \mathcal{I} \left(1 + \log \frac{N}{\mathcal{I}} \right). \quad (6)$$

How big is Δ ? On very general physical grounds, we can estimate it in the following way. Let m be the characteristic mass scale of the theory in the boundary region. For example, if the mass gap is generated by fluctuations of some massive particles composing the boundary, the scale m will be set by the mass of these degrees of freedom. In the case of no preexisting mass gap in the theory, the natural scale will be given by the inverse size of the system, $m = \hbar/R$. Let α be the quantum coupling of the fluctuating degrees of freedom. Then the expected energy gap for the Stückelberg modes is

$$\Delta_{\text{Stückelberg}} = \alpha(m)m, \quad (7)$$

where $\alpha(m)$ is the coupling evaluated at the energy scale m (or equivalently at the length scale \hbar/m). Thus, the information capacity is

$$\mathcal{I} \equiv \frac{1}{\alpha(m)} \frac{\hbar}{Rm}. \quad (8)$$

On the other hand, if the gauge theory in question is massless, then we must take $m = \hbar/R$ and the Stückelberg qubit energy gap becomes

$$\Delta_{\text{Stückelberg}} = \alpha(R) \frac{\hbar}{R}. \quad (9)$$

The information measure is therefore given by

$$\mathcal{I} = \frac{1}{\alpha(R)}. \quad (10)$$

Thus, for a gapless gauge theory that can dynamically produce the information boundary, the information capacity is measured by the inverse quantum coupling, exactly in the same way as this was happening, according to (2), for Bekenstein entropy. In other words, applying (10) to the black hole horizon, and taking into account that the gravitational coupling at the scale R is given by (1), yields

$$\mathcal{I}_{\text{BH}} = \frac{R^2}{L_P^2}. \quad (11)$$

For a black hole, the number of Stückelberg modes N and the information measure \mathcal{I} coincide. Hence we recover from Eq. (6) the well-known area law,

$$S = \frac{R^2}{L_P^2} = S_{\text{BH}}. \quad (12)$$

Let us summarize our main message. In any gauge theory, Alice can characterize the part of the quantum information hidden in region B in terms of the Stückelberg qubits. For an arbitrary gapless gauge system, this information is measured by (10). But, for a generic system, it accounts only for a part of the hidden information. For black holes, due to the scaling of the gravitational quantum coupling $\alpha(R)$, the amount of information that can be accounted for by Stückelberg qubits saturates the bound and thus accounts for the entire information of the hidden region.

Finally, an interesting connection emerges between the Stückelberg formulation presented here and the previous picture of holography developed in [3,4]. As explained above, in the latter picture, holography results from the many-body description of the black hole interior in terms of a critical graviton condensate delivering Bogoliubov modes with tiny energy gaps. In the present paper we understand holography in the gauge-redundancy language of an external observer in terms of a very similar energy gap of the Stückelberg modes. This coincidence suggests that we are dealing with two languages describing the same physics. The striking similarity between (10) and (2) indicates a possible fundamental connection between holography and quantum criticality. This connection maps the Stückelberg qubits of Alice's description onto the Bogoliubov modes of the many-body quantum critical state of [3,4].

In the rest of the paper we shall elaborate on these ideas.

II. SPIN-2 CASE

We shall now discuss our idea in more detail. The first part of our argument is generic for any bounded system with gauge redundancy. Let us therefore illustrate it for linearized gravity. The nonlinearities can be taken into account, but they do not add anything to the essence of the phenomenon.

Consider the action of linearized Einstein gravity,

$$S_E = \int d^4x L(h) = \int d^4x h^{\mu\nu} \mathcal{E}h_{\mu\nu}, \quad (13)$$

where $\mathcal{E}h_{\mu\nu} \equiv \square h_{\mu\nu} - \eta_{\mu\nu} \square h - \partial_\mu \partial^\alpha h_{\alpha\nu} - \partial_\nu \partial^\alpha h_{\alpha\mu} + \partial_\mu \partial_\nu h + \eta_{\mu\nu} \partial^\alpha \partial^\beta h_{\alpha\beta}$ is the linearized Einstein tensor and $h \equiv h^\alpha_\alpha$. This system exhibits a gauge redundancy under the following transformation,

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \quad (14)$$

where $\xi_\mu(x)$ is a transformation parameter vector. Under this transformation, the Lagrangian density shifts by a total derivative,

$$\delta L = \partial_\mu \Omega^\mu \quad \text{where } \Omega_\mu \equiv 2\xi^\nu \mathcal{E}h_{\mu\nu}. \quad (15)$$

Let us now introduce a boundary. The role of it can be played by an arbitrary closed two-surface described by the target space coordinates X^μ , $\mu = 0, 1, 2, 3$ and the world-volume coordinates y^a , $a = 0, 1, 2$. At the moment, the precise origin of the boundary is not important for us. For instance, it can represent a dynamical solution of the theory, such as, e.g., a black hole horizon or a brane bubble. Alternatively, it can be imposed by hand as a mathematical surface. In either case, what is important for us is that the boundary separates the system into two subsystems. We now wish to describe the gauge redundancy from the point of view of the exterior subsystem.

Alice sees a gauge theory on a space with boundary. In this description, naively, under the gauge shift (14) the action changes by a boundary term

$$\delta S = \int dX^\mu \wedge dX^\nu \wedge dX^\alpha \epsilon_{\mu\nu\alpha\beta} \Omega^\beta. \quad (16)$$

However, since we started by a manifestly gauge-redundant theory, the same redundancy must hold in the correct description of the subsystem. In particular, this is obvious in cases in which the boundary represents a solution of the full gauge-invariant theory.

The only way to accommodate gauge invariance is to allow the boundary to host a new degree of freedom, η_μ , that acts as a Stückelberg field for maintaining the original gauge invariance. The action of this new degree of freedom is fixed from the above condition and has the following form,

$$S_\eta = \int dX^\mu \wedge dX^\nu \wedge dX^\alpha \epsilon_{\mu\nu\alpha\beta} \eta^\beta. \quad (17)$$

Obviously, the combined action,

$$S = S_E + S_\eta, \quad (18)$$

is invariant under (14) provided η_μ shifts as

$$\eta_\mu \rightarrow \eta_\mu - \Omega_\mu, \quad (19)$$

with Ω_μ given by (15). Notice that η_μ is a fully legitimate degree of freedom that transforms as a scalar from the point of view of the boundary world-volume theory. The very existence of this Stückelberg degree of freedom follows solely from the requirement of symmetry under small diffeomorphisms. However, what promotes them into the physical carriers of information is the interaction.

Let us at this point emphasize that the Stückelberg modes encode physical degrees of freedom, although they originate from a local transformation. To see this clearly, consider the theory prior to the introduction of Stückelberg fields. Here, only gauge transformations that vanish at the boundary are redundancies. On the other hand, ‘‘large’’ gauge transformations are not redundancies and cannot be used to eliminate degrees of freedom. Thus, there exist configurations of the gauge field that appear to be pure gauge, but cannot be eliminated through a gauge transformation. These are precisely those configurations that are nontrivial at the boundary. Now, we can reformulate the theory in such a way that all gauge transformations are redundancies, even those that are large. In this case, the degree of freedom count of the gauge field works just as if the boundary were absent. However, this requires a new field to be introduced that shifts under the large gauge transformations. This is precisely our Stückelberg field.

We thus have two complementary descriptions of the same physical situation. In one case, there are modes of the gauge field that look like they are pure gauge, but are not since they are nontrivial at the boundary. They are thus physical. In the other case, even large gauge transformations become redundancies and the corresponding modes of the new gauge field are unphysical. However, here the physical degrees of freedom are carried by the Stückelberg fields.

We can make this point even more transparent by considering a well-known example of the Stückelberg trick, Proca’s theory of a massive vector field:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} m^2 A_\mu^2. \quad (20)$$

Prior to the introduction of the Stückelberg field, the theory is not invariant under the usual gauge shifts of the vector, $A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$. The longitudinal mode cannot be eliminated through a gauge transformation and is physical. The presence of an additional third degree of freedom as compared to Maxwell’s theory is of course a physical consequence of the mass.

We can reformulate the theory by introducing a gauge-variant vector and scalar field,

$$A_\mu = \tilde{A}_\mu + \partial_\mu \chi. \quad (21)$$

The action Eq. (20) is now invariant under local transformations of the form

$$A_\mu \rightarrow A_\mu + \partial_\mu \Lambda, \quad \chi \rightarrow \chi - \Lambda. \quad (22)$$

Consequently, in this formulation, the longitudinal mode of the new vector \tilde{A}_μ can be eliminated, for example by imposing a Coulomb-like gauge condition on \tilde{A}_μ , $\nabla_i A_i = 0$. However, with this choice, one pays the price of the additional scalar field χ , which then carries the additional degree of freedom. Thus the modes encoded by the Stückelberg fields correspond to physical degrees of freedom.

Classically the Stückelberg modes are exactly gapless and can be labeled by world-volume coordinates of the boundary theory. This means that the number of Stückelberg qubits scales as the area of the boundary. This certainly rings a bell, and makes it tempting to identify the Stückelberg fields as the holographic degrees of freedom for a black hole. However, the same coincidence also raises the question: Why are not all information boundaries holographic? We shall address this question in the rest of the paper.

III. QUANTUM GENERATION OF ENERGY GAP AND INFORMATION COUNTING

In our derivation of boundary Stückelberg degrees of freedom, we have never used black hole properties. Our argument solely relies on gauge invariance and is therefore generic for an arbitrary boundary. So, what is special about the black hole horizon? As we shall explain now, the question is closely related to the question of generation of an energy gap in boundary Stückelberg modes. As discussed above, this gap defines the information capacity (4) of the Stückelberg modes, which in general can account for a certain fraction of the total information capacity of the bounded inner system. Black holes turn out to be the systems with the smallest Stückelberg energy gap, which is just enough to saturate the bound on information. With such a small gap, black hole horizon Stückelbergs can account for the entire information of the system. This is why black holes are holographic.

The source of the energy gap is quantum fluctuations. The crucial point is that in a quantum theory, we must allow the boundary to fluctuate. For example, for a closed brane bubble, the fluctuations are due to the degrees of freedom that compose it. Similarly, for any finite mass black hole there is at least one model-independent source of horizon fluctuation: the backreaction from Hawking evaporation. Once the boundary fluctuations are taken into account, they create bilinear (and higher) terms in the effective Stückelberg action. Such terms generate the energy gap in Stückelberg qubits. On very general grounds, the energy gap can be estimated as (7), where $\alpha(R)$ is the characteristic quantum coupling of the fluctuating degrees of freedom, evaluated at the scale R , and m is their characteristic energy scale. In a massless theory it is given by the inverse size of

the system, $m = \hbar/R$. Or, if the theory has an intrinsic mass gap in the spectrum, m is set by the latter.

In order to see more formally why the gap is generated, let us rewrite the action (17) in a four-dimensional bulk language

$$\mathcal{S}_\eta = \int d^4x \eta_\mu(x) J^\mu(x), \quad (23)$$

where J^μ is the Hodge dual, $J^\mu \equiv \epsilon^{\mu\nu\alpha\beta} J_{\nu\alpha\beta}$, of the boundary current,

$$J^{\nu\alpha\beta}(x) \equiv \int dX^\nu \wedge dX^\alpha \wedge dX^\beta \delta^4(x^\mu - X^\mu). \quad (24)$$

Due to virtual quantum processes the current-current correlator is nonvanishing (once interactions are taken into account) and induces a bilinear term in the action,

$$\eta_\mu \eta_\nu \langle J^\mu J^\nu \rangle. \quad (25)$$

Of course, the bilinear term will be generated for the full gauge-invariant combination,

$$(L(h) + \partial^\mu \eta_\mu)^2. \quad (26)$$

Thus, as is usual, the Stückelberg field acquires an energy gap via a Higgs-type mechanism. The coefficient of this term can only vanish in the limit in which the fluctuations of the boundary can be ignored. The higher the ability of the boundary to fluctuate, the bigger the generated mass gap is. It is now clear what limits the capacity of information storage (4). It is the energy gap, Δ , in the Stückelberg qubit. If Δ is large, the storage of information in qubits is very costly, and the corresponding system is far from being holographic. This is the reason why most of the systems with information boundaries are not holographic.

We are now ready to understand, at least qualitatively, why the black hole horizon is special among all possible information boundaries. We shall show this using two different arguments.

The first argument comes from a straightforward application of (7) to the black hole case. If we assume that the quantum fluctuations are due to gravitons of wavelength R , we have to take for $\alpha(R)$ the expression (1) and $m = \hbar/R$. Substituting both into (7), we get

$$\Delta_{\text{BH}} = \frac{\hbar L_P^2}{R^3}. \quad (27)$$

Remarkably, as already discussed above, this gives an information measure (4) which, as shown at the end of Sec. I, reproduces Bekenstein entropy.

An alternative argument that leads us to the same result is based on estimating the strength of quantum fluctuations using the measure of backreaction from the Hawking radiation. For this, first recall that the quantum fluctuations

of the horizon vanish in the semiclassical limit, in which $L_P \rightarrow 0$ and the black hole size R is kept finite. In this limit the black hole horizon becomes *infinitely rigid* and is insensitive to any type of quantum backreaction. In the case of finite L_P , the backreaction on the horizon from Hawking radiation can easily be estimated to be of order L_P^2/R^2 , by taking into account the relative change of the black hole mass, δM_{BH} , or its temperature, due to each Hawking emission,

$$\frac{\delta M_{\text{BH}}}{M_{\text{BH}}} = -\alpha(R). \quad (28)$$

As it is clear, this backreaction parameter coincides with the coupling of gravitons of wavelength R discussed above, so we have denoted both by the same symbol $\alpha(R)$. Thus, the parameter $\alpha(R)$ is the measure of the black hole quantumness. It is natural to expect that the quantum-mechanically generated energy gap in Stückelberg qubits must be suppressed by this additional factor relative to a gap expected for an ordinary quantum system of size R , which would be \hbar/R . This gives a simple alternative estimate for the quantum-generated energy gap of the Stückelberg qubit,

$$\Delta_{\text{BH}} = \alpha(R) \frac{\hbar}{R}, \quad (29)$$

which agrees with (27).

Hence, both estimates give us the same energy gap of Stückelberg modes, which leads to the expressions (10) and (12) for the quantity \mathcal{I} that matches the Bekenstein entropy of a black hole of radius R .

From (7) it is easy to understand why there are no systems of size R with the energy gap in the Stückelberg fields smaller than the black hole horizon. Indeed, no matter how weakly the quantum constituents are interacting by other forces, they must interact gravitationally. Correspondingly, putting aside accidental cancellations, the gravitational coupling α sets the lowest bound on the quantum interaction strength and subsequently on the generated energy gap.²

For a larger energy gap Δ , the entropy of the bounded inner system would be much less than the black hole entropy. This allows us to understand the high capacity of black hole information storage in terms of the lowest mass gap in Stückelberg fields.

IV. EXAMPLES WITH LARGE ENERGY GAP FOR THE BOUNDARY STÜCKELBERG MODE

In order to get a broader view, let us now consider other gauge systems with dynamically generated information boundaries. In these examples, the vacuum structure of the theory prevents information from being carried out of or

²In this light, it would be interesting to see if the energy gap can be maintained smaller due to cancellations, e.g., by supersymmetry.

into a region of space, which precisely corresponds to our definition of an information boundary. Moreover, we can directly track the generation of an energy gap for the Stückelberg modes, which in turn sets the information storage capacity of the boundary. As we shall see, the information measure \mathcal{I} in these systems is very small, due to a large gap which goes hand in hand with the generation of the boundary. We shall consider two examples, which in a certain sense are dual to each other.

A. Classical energy gap: Information boundary from the Meissner effect

The first example is given by a slightly deformed Higgs model that in addition to the usual Higgs vacuum, with a massive photon, admits a second vacuum in which the photon is massless and the $U(1)$ -gauge theory is in the Coulomb phase. The simple model is given by the following Lagrangian,

$$L_\Phi = |D_\mu \Phi|^2 - F_{\mu\nu} F^{\mu\nu} - \lambda^2 \frac{|\Phi|^2}{v^2} (|\Phi|^2 - v^2)^2, \quad (30)$$

where $\Phi(x) = |\Phi(x)|e^{i\theta(x)}$ is a complex scalar field, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the Maxwellian field strength and $D_\mu \equiv \partial_\mu - igA_\mu$ with g being the gauge coupling. The two physically inequivalent vacua are $\langle |\Phi| \rangle = 0$ and $\langle |\Phi| \rangle = v$. In the first vacuum, the propagating fields are the massless photon and a complex scalar of mass $m_\Phi = \hbar\lambda v$, whereas in the second one we have a massive photon with the mass $m_A = \hbar gv$ and a real Higgs scalar with the mass $m_\Phi = \hbar\lambda v$. In both vacua the total number of propagating degrees of freedom is equal to four.

This theory also admits configurations in which the two vacua coexist and are separated by a domain wall across which the Higgs expectation value interpolates from 0 to v . Without loss of generality we can place the wall at $z = 0$. The thickness of the wall is set by the Compton wavelength of the Higgs particle $\delta = (\lambda v)^{-1}$. At energies $\ll m_\Phi$ we can integrate out the thickness of the wall and the effective theory of the photon field becomes

$$L_{\text{eff}} = -F_{\mu\nu} F^{\mu\nu} + \hbar^{-2} m^2(z) \left(A_\mu - \frac{1}{g} \partial_\mu \theta \right)^2, \quad (31)$$

where the function $m^2(z)$ can be approximated as a step function $m^2(z < 0) = 0, m^2(z > 0) = m_A^2$. At energies $\ll m_A$ the photon cannot penetrate the Higgs region and the effective theory of the photon is a Coulomb domain bounded by the wall. The eaten-up Goldstone θ plays the role of the Stückelberg field that is maintaining the gauge redundancy, $A_\mu \rightarrow A_\mu + \frac{1}{g} \partial_\mu \omega, \theta \rightarrow \theta + \omega$, despite the presence of the boundary. However, there is a mass gap present already at the classical level. In fact, this allows an observer to conclude that the other side of the wall is in the Higgs phase.

Let us take for a region B the interior of a Higgs vacuum bubble of radius R , embedded into the Coulomb vacuum (region A).³ Then region B , which is in the Higgs phase, at low energies is bounded by the sphere. The Stückelberg field is the eaten-up Goldstone that maintains the gauge redundancy throughout the space. But there is a large energy gap. The information measure I is always very small, unless $m_A < \hbar/R$. The latter choice however would mean that effectively there is no information boundary for the photon. In such a regime, we need to consider the information stored in the Goldstone mode θ separately.

The latter statement becomes obvious by taking an extreme case when we switch off the gauge coupling completely, $g = 0$, but keep v finite. Then there is a massless Goldstone mode localized within the bubble and the information can be stored in its zero momentum mode. However, in this limit the bubble is transparent for the photon and no information boundary exists for the gauge field.

B. Quantum gap: Boundary from dual Meissner effect

Let us consider a situation in which the information boundary is dynamically formed by a magnetic monopole condensate. This can be achieved by employing the massless gauge field localization mechanism of [8], which is based on interpolation between the confining and Coulomb phases.

We can achieve this situation by modification of the model (30) by embedding the $U(1)$ into a gauge $SU(2)$ -symmetry and promoting the Higgs field into a real triplet representation Φ^a , $a = 1, 2, 3$. The Lagrangian now becomes

$$L_{SU(2)} = |D_\mu \Phi^a|^2 - F_{\mu\nu}^a F^{a\mu\nu} - \lambda^2 \frac{\Phi^a \Phi^a}{v^2} (\Phi^b \Phi^b - v^2)^2, \quad (32)$$

where D_μ is the usual $SU(2)$ -covariant derivative and $F_{\mu\nu}^a$ is a non-Abelian field strength. We have chosen the Higgs potential as in [9]. Now, the two degenerate vacua are $\langle \Phi^a \rangle = 0$ and $\langle \Phi^a \rangle = \delta_3^a v$. In the first one, the theory is in the $SU(2)$ -phase. In the second one, the $SU(2)$ -group is Higgsed down to a $U(1)$ subgroup, which is in the Coulomb phase. Again, we can consider a configuration in which the two phases are separated by a domain wall placed at $z = 0$. Notice that, unlike the previous example,

³If the two vacua are exactly degenerate, as it is the case for the Lagrangian (30), the finite size bubble cannot be static and will tend to collapse because of the tension of the bubble wall. This is not a big complication for our purposes, if we are interested in information-storage properties over short time intervals. Moreover, we can make the bubble of a given radius static by balancing the tension force by a pressure difference created via a small difference between the energy densities of the two vacua. An alternative way is to stabilize the bubble by introducing some charged particles that are localized within the bubble wall.

here at the classical level the wall is transparent for the $U(1)$ -photon. However, in the quantum theory the story changes dramatically. The $SU(2)$ -phase becomes confining and develops a mass gap given by the QCD scale Λ . The photon traveling across the wall can only penetrate the $SU(2)$ -domain in the form of a glueball of mass Λ . Correspondingly, at energies below Λ the effective theory of the $U(1)$ -photon is a gauge theory with a boundary. The gauge invariance is again maintained by a Stückelberg field, the role of which is played by the phase of the monopole condensate that gives mass to a magnetic photon (see [9,10] for detailed discussion of different aspects of such a picture). But, the very same physics that generates the monopole condensate also generates the mass gap due to the confinement of the electric charges.

In order to make the example more transparent let us place Alice in the $SU(2)$ -domain. Imagine that Alice is observing the bubble of the $U(1)$ -vacuum. Let us take the size of the bubble R to be much larger than the QCD length, $R \gg \hbar/\Lambda$. Inside this bubble information can be stored in the form of the low-energy photon quanta. If these quanta have energies $\ll \Lambda$, the information stored in them cannot penetrate into the $SU(2)$ -domain, where the energy gap is Λ .⁴

Thus, the domain wall separating the $U(1)$ -Coulomb and $SU(2)$ -confining phases acts as an information boundary for Alice. However, the barrier in this case is rather peculiar. Indeed, the $U(1)$ factor is nowhere Higgsed throughout the space and consequently there is no potential energy barrier for the photon at the level of the fundamental Lagrangian of the form (31). What prevents the photon to escape out of the bubble is not a charge condensate, but rather a condensate of magnetic monopoles. However, the magnetic monopole condensate, unlike the charge condensate, is *not* generating any fundamental mass term for a photon, but rather is making it *nonpropagating*. The mass gap generated for the Stückelberg field satisfies the general relation of the type (8), with the only caveat that the role of m is played by Λ and the role of $\alpha(m)$ is played by the QCD coupling $\alpha_{SU(2)}(\Lambda)$ evaluated at the scale Λ , which is of order one.

Thus, in both considered examples the mass gap in the Stückelberg field is generated simultaneously with the generation of the boundary. In both cases the information boundary is bounding the regions of condensed charges, which are either electric or magnetic. In both cases, the Stückelberg field is the phase of the condensate and in both cases the generation of the mass gap in this degree of

⁴Recently, a coherent state picture of photons in a ball was studied in [11]. It would be interesting to see if the $SU(2)$ -confining region discussed here provides the boundary conditions imposed in this analysis and also to see if the critical transition can be modeled in the many-body photon language, when the ball reaches the size of $SU(2)$ QCD length, $R \sim \hbar/\Lambda$.

freedom is inseparable from the generation of the information boundary.⁵

The difference is that in the electric condensate case, this phenomenon can take place already at the classical level, whereas in the case of magnetic charges the entire effect is quantum.

V. RELATION BETWEEN HOLOGRAPHY AND QUANTUM CRITICALITY

According to (9), in massless gauge systems with an information boundary the energy gap in Stückelberg qubits is suppressed by the coupling α , relative to the typical energy gap, which would be expected to be $\sim\hbar/R$. This expression suggests an interesting underlying relation with quantum criticality. According to the studies of [4,5], systems at the quantum critical point are very efficient storers of information, due to the appearance of qubits with an energy gap that exhibits a suppression very similar to (9). In particular, this is a property of a gas of bosons with attractive interaction strength α . In this case, the criticality is reached when the occupation number is equal to α^{-1} . This system has been studied in a series of papers [4,5] from a quantum information perspective, as a prototype model for black holes. By now it is well established that the quantum critical point is populated by nearly gapless qubits. These qubits can be described in different languages. In the many-body language they can be described as Bogoliubov modes of the critical Bose gas. Alternatively, they can be characterized as the Goldstone modes of a nonlinearly realized symmetry of the condensate [5]. Based on this connection, the black hole portrait of [3–5] suggests that black hole information is carried by the Bogoliubov-Goldstone qubits of the critical graviton condensate, with the energy gap given by

$$\Delta_{\text{Bogoliubov}} = \alpha(R) \frac{\hbar}{R}. \quad (33)$$

The approach developed in the present paper suggests yet another description of these modes, which naturally leads us to a potential link between quantum criticality and holography. Namely, the striking similarity between the energy gaps (9) and (33) for the Stückelberg modes in a gauge description and Bogoliubov modes in a many-body description of black holes suggests that we are dealing with two sides of the same coin and that the holographic systems microscopically represent the systems at a quantum critical point. The modes that by Alice are seen as Stückelberg degrees of freedom, are, in the microscopic

⁵In all these examples for information measure \mathcal{I} we get (8) where m is the mass gap of Alice's physics. Obviously this mass gap must be bigger than $\frac{\hbar}{R}$ to allow Alice to know the actual existence of a hidden region of size R , so in these cases \mathcal{I} is trivially bounded by (10).

many-body description of the black hole, represented as the Bogoliubov-Goldstone modes of the critical graviton condensate. It is tempting to generalize this correspondence to other types of holographic systems. This relation can also work in the opposite direction, suggesting that critical systems must exhibit some sort of holography in the sense presented here. In this respect it would be interesting to investigate the observation of equivalence, in the large N -limit, of the ground states of a one-dimensional N -particle Bose gas on a ring and a gauge theory on a two-sphere [12].

The connection between the Stückelberg formulation of holography and quantum criticality gives an exciting possibility of experimentally studying yet another black hole property in critical systems that can be manufactured in table-top labs, in the spirit of [4,5].

VI. CONCLUSIONS

In this note we have suggested that some key features of holography can be understood in terms of the basic principles of gauge redundancy. The maintenance of this redundancy requires the appearance of Stückelberg degrees of freedom at the information boundary. These modes then act as qubits that store a fraction or the entire information of the inaccessible region. The amount of information stored by Stückelberg qubits is determined by their energy gap Δ . Even if absent classically, this gap is generated by quantum effects and is given by (7). Correspondingly, we have introduced a measure of information capacity \mathcal{I} given by (8), which quantifies how energetically cheap is the excitation of Stückelberg qubits relative to the usual energy gap of a system of the same size, typically expected to be \hbar/R . For gauge theories without an intrinsic mass gap in the particle spectrum, the quantities Δ and \mathcal{I} are given by (9) and (10) respectively,

$$\Delta_{\text{Stückelberg}} = \alpha(R) \frac{\hbar}{R}, \quad \mathcal{I} = \frac{1}{\alpha(R)}, \quad (34)$$

which we have copied here for the reader's convenience. For the gravitational coupling $\alpha(R)$, which scales as (1), the above expression for \mathcal{I} coincides with the Bekenstein entropy. The expression (34) makes it clear why among all possible information boundaries the black hole horizon can store a maximal amount of information. The reason is simply that gravity sets the lower bound on how weakly the quantum modes can interact. Even if particles in the theory interact via additional hypothetical forces much weaker than gravity, they must still couple gravitationally. Correspondingly, black holes house Stückelberg modes with the smallest possible gap in nature. In this respect, it would be interesting to study what happens for extremal black holes or other systems (e.g., supersymmetric ones) in which gravitational attraction of the constituents can be compensated by the repulsion due to other forces. One may

expect that the Stückelbergs for such systems remain gapless.

We thus conclude that black holes carry Stückelberg hair, which is closely analogous to the Bogoliubov–Goldstone hair of the quantum critical picture of [3–6]. Both types of hair become infinite but unresolvable in the classical limit, because in this limit $\alpha(R)$ vanishes and \mathcal{I} diverges.

Finally, taking into account the striking similarities in suppressions of their energy gaps, we have suggested a connection between the Stückelbergs and Bogoliubov modes of the critical graviton condensate. We have proposed that holographic systems are secretly systems at a quantum critical point. Generalizing this hypothesis in the opposite direction, the usual critical systems that can be obtained in table-top labs, such as critical Bose-Einstein condensates of cold atoms, must also exhibit some notion

of holography, which can open up very interesting experimental prospects.

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