Looking for bound states and resonances in the $\eta' K\bar{K}$ system

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Motivated by the continuous experimental investigations of $X(1835)$ in three-body decay channels like $\eta'\pi^+\pi^-$, we investigate the $\eta' K\bar{K}$ system with the aim of searching for bound states and/or resonances when the dynamics involved in the $K\bar{K}$ subsystem can form the resonances: $f_0(980)$ in isospin zero or $a_0(980)$ in isospin 1. For this, we solve the Faddeev equations for the three-body system. The input two-body t matrices are obtained by solving Bethe-Salpeter equations in a coupled channel formalism. As a result, no signal of a three-body bound state or resonance is found.

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An observation of a resonancelike structure around 1830 MeV, $X(1835)$, has been reported in several processes, with the most recent finding being in the mass spectrum of $\eta'\pi^+\pi^-$ by the BES Collaboration [1]. The first observation of $X(1835)$ in the $\eta'\pi^+\pi^-$ mass spectrum, in the process $J/\psi \to \gamma\eta'\pi^+\pi^-$, was discussed in Ref. [2], where a Breit-Wigner fit to the data yielded a mass $M = 1833.7 \pm 6.1 \pm 2.7$ MeV and a width $\Gamma = 67.7 \pm 20.3 \pm 7.7$ MeV. The same process is studied with a larger statistics by BESIII in Ref. [1], where, apart from the confirmation of $X(1835)$, the finding of two new states is reported: $X(2120)$ and $X(2370)$. A more recent analysis of the $\eta'\pi^+\pi^-$ data [3], focussed on the energy region of $X(1835)$, shows that a fit to the data in this region requires either the presence of a much broader state ($\Gamma \sim 247$ MeV), distorted by the cusp of $p\bar{p}$, or an interference between a broad and a narrow state. The fit shows that the broad state, in any case, couples strongly to the $p\bar{p}$ system [3]. An enhancement near the $p\bar{p}$ threshold in the BES data has been found in some processes (like $J/\psi \to \gamma p\bar{p}$ and $\psi(2s) \to \gamma p\bar{p}$ [4]) but not in some other processes (like $J/\psi \to \omega p\bar{p}$ [5] and $J/\psi \to \phi p\bar{p}$ [6]). The decay of $\psi(2s)$ has been studied by the CLEO Collaboration also, but the data show no $p\bar{p}$ threshold enhancements in the mass spectra of $\gamma p\bar{p}$, $\pi^0 p\bar{p}$, and $\eta p\bar{p}$ [7]. All these findings have generated a series of discussions on the possibility of the existence of a baryonium or other alternative explanations of the enhancement seen around 1830 MeV [8–19]. A resonancelike structure around 1830 MeV is also found in the mass spectrum of $\eta K\bar{K}$ [20], $\eta\pi^+\pi^-$, where the $K\bar{K}$ is found to come dominantly from $f_0(980)$ in the former case. It is not clear if all the states found around 1830 MeV in different systems are the same, and the origin of this/these state(s) is still an open question. In the present manuscript, we study the possibility of understanding $X(1835)$ as a bound state arising from three pseudoscalar dynamics involving the η' meson.

The dynamics of a system of pseudoscalar mesons is related to the low-energy regime of QCD, which can be described in terms of the chiral perturbation theory (χPT). The latter is an effective field theory based on the fact that the QCD Lagrangian with massless u, d, and s quarks has an $SU(3)_R \times SU(3)_L$ chiral symmetry. This symmetry is spontaneously broken to $SU(3)_V$, giving rise to an octet of Goldstone bosons, which are identified with the octet formed by the pseudoscalar mesons: π, K, and η. These particles become massless in the chiral limit of zero quark masses, $m_{u,d,s} \to 0$. The ninth pseudoscalar, the η' meson, which was found independently, but almost at the same time, by two collaborations [21,22] in 1964, is an interesting hadron; it is closely related to the axial $U_A(1)$ anomaly [23–27]. This fact prevents the η' meson from becoming massless even in the chiral limit. Thus, the η' meson is not included explicitly in the Lagrangian in the conventional χPT.

A way to incorporate η', however, could be inspired by the works of Witten, 't Hooft, and others [24,25], who showed that in the limit of an infinite number of colors ($N_c \to \infty$) of QCD the SU(3) singlet state, η_1, is massless and the global $SU(3)_R \times SU(3)_L$ symmetry is replaced by $U(3)_R \times U(3)_L$. This is because in the large N_c limit the anomaly related to the axial current is $1/N_c$ suppressed. This fact can be used to incorporate η' in an effective field theory based on chiral symmetry, since η_1 becomes the ninth Goldstone boson and can be included in an extended $U(3)_R \times U(3)_L$ chiral Lagrangian (see, for example, Refs. [28–30] for more details). Alternative approaches to including the singlet state in an effective field theory have also been developed [31,32].

Thus, to build a Lagrangian based on chiral symmetry and including at the same time the η' meson, in the spirit of Refs. [24,25,28–32], the physical η and η' fields are introduced as the admixtures of the SU(3) singlet η_1 and octet η_8 states. Indeed, the $\eta - \eta'$ mixing has received a lot of attention in the recent past. Usually, within the mixing scheme, the η and η' mesons are considered as linear combinations of η_1 and η_8 through a mixing angle θ,

$$
\begin{aligned}
|\eta\rangle &= \cos\theta|\eta_8\rangle - \mathrm{sen}\theta|\eta_1\rangle,\\
|\eta'\rangle &= \mathrm{sen}\theta|\eta_8\rangle + \cos\theta|\eta_1\rangle. \qquad (1)
\end{aligned}
$$

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The values obtained for this mixing angle range, typically, from $-13°$ to $-22°$. These values are extracted, just to mention a few examples, from the decays of η and η' to two photons, decays of J/ψ, etc. [33–36]. Considering this mixing angle, the SU(3) matrix containing the Goldstone bosons can be extended to U(3) as

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \frac{2}{\sqrt{6}}\eta' \end{pmatrix}, \tag{2}$$

where the standard $\eta - \eta'$ mixing is considered [Eq. (1) with $\sin\theta = -1/3$, thus $\theta \sim -20°$]. Also, a two-mixing angle scheme has been proposed [37–39] and adopted to explain some decay widths of the η and η' mesons, radiative decays, pseudoscalar decay constants, and other quantities [40,41]. We stick here to the approach with one mixing angle.

Using the matrix in Eq. (2), at leading order in large N_c, the lowest-order Lagrangian describing the interaction between two pseudoscalar mesons is given by [28–30,42]

$$\mathcal{L} = \frac{1}{12f^2}\langle(\partial_\mu\phi\phi - \phi\partial_\mu\phi)^2 + M\phi^4\rangle, \tag{3}$$

with $M = \mathrm{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2)$.

The interaction of the η' meson with other pseudoscalars in the S wave is rather weak, and neither a bound state nor a resonance has been found theoretically due to this dynamics. However, it was shown in Refs. [30,42] that inclusion of η' in the coupled channel analysis is required to reproduce the isospin $I = 1/2$ and $I = 3/2$ S-wave $K\pi$ phase shift up to energies of 1.3 GeV. In fact, a pole around 700 MeV with a width near 600 MeV is found and identified with the κ resonance in Refs. [30,42]. Note, however, that the presence of the $\eta' K$ channel, although being important for the reproduction of the data around 1.3 GeV, is not essential for the understanding of the properties and nature of the κ resonance [43–45].

Contrary to the weakness of the η' interaction with other pseudoscalars, the S-wave interaction of systems like $K\bar{K}$ is known to be strong and generates poles related to the $f_0(980)$ and $a_0(980)$ resonances [43–45]. It is then plausible that in a system like $\eta' K\bar{K}$ the strong attraction in the $K\bar{K}$ system could be enough, together with a weak interaction in the subsystems having an η', to generate a state with a three-body nature. Such a plausibility should not be surprising because the three-body dynamics is more complex and richer than that associated with a two-body system, and states of three-body nature can be found even when the interaction in some subsystems is repulsive. Sometimes, it is possible to generate a three-body state even when the interaction in all the subsystems is not strong enough to form individual two-body bound states or resonances. Such states are called Borromean states [46]. Thus, the interaction between one or two subsystems can be repulsive or weak; however, if the dynamics involved in the remanent subsystem(s) is strong enough to overcome the repulsion/weak attraction, a state of a three-body nature can be formed. This is, indeed, the case of the $KK\bar{K}$, $\phi K\bar{K}$, and $J/\psi K\bar{K}$ systems, and three-body bound states or resonances are found and associated with the $K(1460)$, $\phi(2170)$, and $Y(4260)$ states, respectively [47–49].

The possibility of finding a three-body state in the $\eta' K\bar{K}$ system has actually been studied earlier in Refs. [50,51], but conclusions opposite of each other have been found. While in Ref. [50], when the $\eta' K\bar{K}$ system rearranges as an η' and the $f_0(980)$ resonance, a state is found at 1835 MeV with a width of 70 MeV, no signal of such a state is found in Ref. [51]. The main difference between the two works is the way of dealing with the three-body dynamics. In Ref. [50], for studying the interaction between η' and $f_0(980)$, loops involving these two mesons are introduced and regularized using the dimensional regularization scheme. This implies the introduction of a subtraction constant in the loop function related to the propagation of a meson (η') and a resonance [$f_0(980)$]. In Ref. [51], the formation of states in the $\eta' K\bar{K}$ system is studied within the Faddeev equations in the fixed center approximation approach. In this case, it is assumed that when the η' meson interacts with the $K\bar{K}$ system, which is considered to cluster as the $f_0(980)$ resonance, no changes are produced on the latter. The description of the dynamics in the cluster is introduced through a form factor which depends on the mass and width of the cluster [52].

In this paper, we study the $\eta' K\bar{K}$ system by solving the Faddeev equations with the purpose of looking for possible bound states or/and resonances. We do not assume any cluster formation which cannot be excited in the intermediate scattering states. Such a contribution can be important, as noted in Ref. [53].

The scattering T matrix of the three-body system can be obtained as a sum of the Faddeev partitions [54], T^i, such that

$$T = \sum_{i=1}^{3} T^i. \tag{4}$$

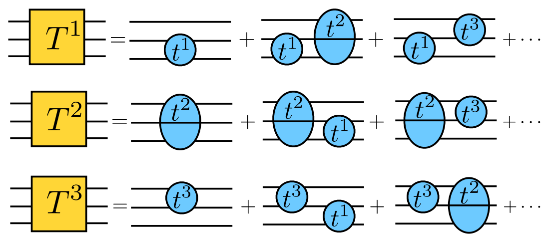

FIG. 1. Schematic representation of the T^i partition. Each horizontal line represents a particle (named as particles 1, 2, 3 from top to bottom). The partition T^1, for example, considers contributions from Feynman diagrams starting from the interaction between particles 2 and 3. The interaction between these two particles is represented through the two-body t^1 matrix.

The formalism used here was developed in Refs. [47–49]. As shown in these latter works, the T^i partitions can be rewritten as (see Fig. 1 for a schematic representation of the Feynman diagrams contributing to each partition)

$$T^i = t^i \delta^3(\vec{k}_i' - \vec{k}_i) + \sum_{j\neq i=1}^{3} T_R^{ij}, \tag{5}$$

where T_R^{ij} satisfy the equations

$$T_R^{ij} = t^i g^{ij} t^j + t^i [G^{iji} T_R^{ji} + G^{ijk} T_R^{jk}] \tag{6}$$

for $i \neq j, j \neq k = 1, 2, 3$. In Eq. (6), the function g^{ij} is the three-body Green's function of the system, which is defined as

$$g^{ij}(\vec{k}_i', \vec{k}_j) = \left(\frac{N_k}{2E_k(\vec{k}_i' + \vec{k}_j)}\right) \times \frac{1}{\sqrt{s} - E_i(\vec{k}_i') - E_j(\vec{k}_j) - E_k(\vec{k}_i' + \vec{k}_j) + i\epsilon}, \tag{7}$$

where $\sqrt{s}$ is the energy in the center of mass of the system, the coefficient N_k is equal to 1 for mesons, and E_l ($l = 1, 2, 3$) is the energy for the particle l.

The G^{ijk} function in Eq. (6) represents a loop function of three particles, and it is written as

$$G^{ijk} = \int \frac{d^3 k''}{(2\pi)^3} \tilde{g}^{ij} \cdot F^{ijk}, \tag{8}$$

with the elements of $\tilde{g}^{ij}$ being

$$\tilde{g}^{ij}(\vec{k}'', s_{lm}) = \frac{N_l}{2E_l(\vec{k}'')} \frac{N_m}{2E_m(\vec{k}'')} \times \frac{1}{\sqrt{s_{lm}} - E_l(\vec{k}'') - E_m(\vec{k}'') + i\epsilon}, \tag{9}$$

for $i \neq l \neq m$, and the F^{ijk} function, with explicit variable dependence, is given by

$$F^{ijk}(\vec{k}'', \vec{k}_j', \vec{k}_k, s_{ru}^{k''}) = t^j(s_{ru}^{k''}) g^{jk}(\vec{k}'', \vec{k}_k) [g^{jk}(\vec{k}_j', \vec{k}_k)]^{-1} [t^j(s_{ru})]^{-1}, \tag{10}$$

for $j \neq r \neq u = 1, 2, 3$. In Eq. (9), $\sqrt{s_{lm}}$ is the invariant mass of the (lm) pair, and it depends on the external variables. The upper index k'' in the invariant mass $s_{ru}^{k''}$ of Eq. (10) indicates its dependence on the loop variable (see Refs. [47–49,55,56] for more details).

The input two-body t matrices of Eq. (6) are obtained by solving the Bethe-Salpeter equation in a coupled channel approach,

$$\begin{aligned} t &= V + V\mathcal{G}t, \\ &= V + \int \frac{d^4 k}{(2\pi)^4} V \frac{1}{[(P-k)^2 - m_1^2 + i\epsilon][k^2 - m_2^2 + i\epsilon]} t, \end{aligned} \tag{11}$$

where the kernel V is determined from the Lagrangian given by Eq. (3).

The $\mathcal{G}$ function in Eq. (11) stands for the two-body loop function; P and k are, respectively, the total 4-momentum of the two-body system and that of the particles in the loop (expressed in the two-body center-of-mass frame), and m_1 and m_2 are the masses of the two particles under consideration.

The first step of our formalism is to solve Eq. (11) for all the two-body subsystems by considering all the relevant coupled channels. In this way, the resonances generated in the two-body subsystems are automatically present in the three-body scattering. As shown in Refs. [44,45,57,58], it is possible to convert the integral Bethe-Salpeter equation [Eq. (11)] into algebraic equations. In this case, the kernel V, and thus, t, can be factorized outside the integral, and Eq. (11) becomes

$$\begin{aligned} t &= [1 - V\mathcal{G}]^{-1} V, \\ &= V + V\mathcal{G}V + V\mathcal{G}V\mathcal{G}V + \ldots, \end{aligned} \tag{12}$$

which sums up the contributions associated with the series of Feynman diagrams shown in Fig. 2. The V present in Eq. (12) is a function of the Mandelstam variables; however, we are only interested in S-wave meson-meson scattering, thus V has to be projected over S waves (for more details, we refer the reader to Refs. [44,45]).

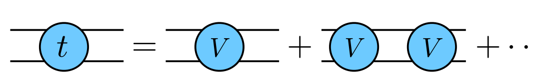

FIG. 2. Schematic representantion of Eq. (12).

The loop function $\mathcal{G}$ in Eq. (12) is regularized using a cutoff or dimensional regularization [44,45]. If the cutoff method is considered, then for a channel r formed by two particles of masses m_{1r} and m_{2r}, in the center-of-mass frame of the two-body system, one has the expression [44]

$$\mathcal{G}_r = \int^{q_{\max}} \frac{d^3q}{(2\pi)^3} I_r(\vec{q}),$$
$$I_r(\vec{q}) = \frac{\omega_{1r}(\vec{q}) + \omega_{2r}(\vec{q})}{2\omega_{1r}(\vec{q})\omega_{2r}(\vec{q})[E^2 - (\omega_{1r}(\vec{q}) + \omega_{2r}(\vec{q}))^2 + i\epsilon]}, \tag{13}$$

with E being the center-of-mass energy of the two-body system, $\omega_i = \sqrt{\vec{q}^2 + m_{ir}^2}$, and $q_{\max}$ a cutoff for the 3-momentum integration.

In the case of the dimensional regularization scheme, the expression found for $\mathcal{G}$ is given by [45]

$$\begin{aligned}\mathcal{G}_r = \frac{1}{16\pi^2}\Big\{ & a_r(\mu) + \ln\frac{m_{1r}^2}{\mu^2} + \frac{m_{2r}^2 - m_{1r}^2 + E^2}{2E^2}\ln\frac{m_{2r}^2}{m_{1r}^2} \\ & + \frac{q_r}{E}[\ln(E^2 - (m_{1r}^2 - m_{2r}^2) + 2q_rE) \\ & + \ln(E^2 + (m_{1r}^2 - m_{2r}^2) + 2q_rE) \\ & - \ln(-E^2 + (m_{1r}^2 - m_{2r}^2) + 2q_rE) \\ & - \ln(-E^2 - (m_{1r}^2 - m_{2r}^2) + 2q_rE)]\Big\},\end{aligned} \tag{14}$$

where q_r is the on-shell center-of-mass momentum, μ is a regularization scale, and $a_r(\mu)$ is a subtraction constant for the channel r. Since a change in μ can be always absorbed into a_r, there is only one independent parameter.

In a fashion similar to Eq. (12), as shown in Refs. [47,56], Eq. (6) is also an algebraic set of six coupled equations. This simplification is a result of the cancellation of the contribution of the off-shell parts of the two-body t matrices in the three-body Faddeev partitions with the contact term(s) of the same topology (the origin of which relies on the Lagrangian used to describe the two-body interaction in the subsystems) [47–49,55,56]. Interestingly, a deduction of cancellations of two-body and three-body forces using a different procedure has recently been reported in Ref. [59]. Because of these cancellations, only the on-shell part of the two-body t matrices is relevant for solving Eq. (6). As a consequence, the T_R^{ij} partitions given in Eq. (6) depend only on the total three-body energy, $\sqrt{s}$, and on the invariant mass of one of the subsystems, which we choose to be the one related to particles 2 and 3 and the invariant mass of which is denoted as $\sqrt{s_{23}}$. The other invariant masses, $\sqrt{s_{12}}$ and $\sqrt{s_{31}}$, can be obtained in terms of $\sqrt{s}$ and $\sqrt{s_{23}}$, as shown in Refs. [48,49].

Using this formalism, we solve Eq. (6) for the $\eta' K\bar{K}$ system. The input two-body $\eta' K$ and $\eta' \bar{K}$ amplitudes are obtained following Ref. [42], where Eq. (12) is solved for the πK, ηK, and $\eta' K$ systems in the S wave and, as a result of this coupled-channel dynamics, the κ resonance is generated. The subtraction constants $a_r(\mu)$ are taken to be, following Ref. [42], -1.383 for channels coupled to isospin $1/2$ ($K\pi$, $K\eta$, and $K\eta'$) and -4.643 for channels coupled to isospin $3/2$ ($K\pi$) for the regularization scale

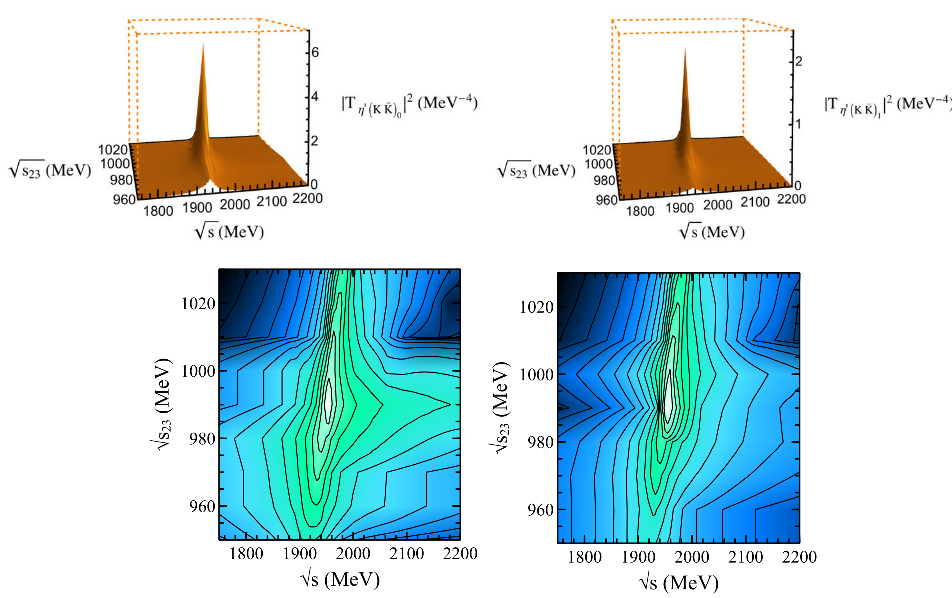

FIG. 3. (Left panel) Modulus squared (top) and contour plot (bottom) of the three-body T matrix for the $\eta' K\bar{K}$ system for total isospin zero; thus, the $K\bar{K}$ subsystem is in isospin zero. (Right panel) Modulus squared (top) and contour plot (bottom) of the three-body T matrix for the $\eta' K\bar{K}$ system for total isospin 1, which implies that the $K\bar{K}$ subsystem is in isospin 1 (right panel). The peak seen in the figures corresponds to the three-body threshold cusp.

value of $\mu = m_K$. These values are obtained in Ref. [42] by fitting the isospin $1/2$ and $3/2$ $K\pi$ phase shifts, and a good reproduction is found up to energies slightly above 1.3 GeV. For the $K\bar{K}$ t matrix, we consider the work of Ref. [44], in which the $\pi\pi$, $K\bar{K}$ system in the S wave is investigated for the isospin-zero configuration and for the isospin-1 case the $K\bar{K}$ and $\pi\eta$ system is considered. The subtraction constants used in the present work are $a_r(\mu) \simeq -1$ for $\mu = 1224$ MeV and for both isospin configurations. These parameters are fixed to reproduce the observed two-body phase shifts and inelasticities for the $K\bar{K}$ system and coupled channels up to energies around 1.2 GeV, as done in Refs. [44,45]. Because of the dynamics involved in these coupled-channel systems, $f_0(600)$ and $f_0(980)$ are found for the isospin-zero case, and $a_0(980)$ is found for the isospin-1 case.

In Fig. 3, we show the plots obtained for the $\eta' K\bar{K}$ T matrix for total isospin zero (left panel) and 1 (right panel) as a function of $\sqrt{s}$ and $\sqrt{s}_{23}$. As can be seen, apart from the threshold enhancement at $(\sqrt{s}, \sqrt{s_{23}}) \simeq (1950, 992)$ MeV in both isospins, no other structure is found, not even for values of $\sqrt{s}_{23}$ around 980 MeV, where the $K\bar{K}$ system in isospin zero forms $f_0(980)$ and in isospin 1 forms $a_0(980)$. A threshold enhancement was also the only effect seen in the study of Ref. [51]. At this point, a question might arise about the stability of our results when the subtraction constants/cutoffs of the loop functions are varied. In the case of the calculation of the two-body t matrices, the subtraction constants used here, as previously mentioned, following Refs. [42] and [44], have been fixed to reproduce relevant data on phase shifts and inelasticities. We have not varied them due to the limited availability of freedom. For the three-body loop functions, Eq. (8), a cutoff of 1000 MeV has been used. We have varied this cutoff in the range 800–1100 MeV, and minor changes in the size of the three-body amplitudes of Fig. 3 are observed. This insensitivity is related to the presence of three-meson propagators in Eq. (8). Thus, our study of the $\eta' K\bar{K}$ system reveals no structure at 1835 MeV, contrary to the finding of Ref. [50], and no structure above the threshold either. Hence, we cannot relate $X(1835)$ and $X(2120)$ with states generated by three-body dynamics. The third X found in Ref. [1], $X(2370)$, is anyways too heavy to be explained as an $\eta' K\bar{K}$ resonance. Apart from $X(1835)$, $X(2120)$, there are some π, η states listed by the Particle Data Group at energies 1800–2100 MeV with large widths, 100–200 MeV: $\eta(1760)$, $\pi(1800)$, $\eta(2225)$. According to the study carried in this work, the dynamics involved in the $\eta' K\bar{K}$ system plays no essential role in understanding the nature of the above-mentioned states. We thus conclude from our work that the origin of $X(1835)$ and $X(2120)$ must be something other than three-pseudoscalar dynamics.

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