Anomalous mass dimension in multiflavor QCD

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Models of strongly interacting theories with a large mass anomalous dimension (γ_m) provide an interesting possibility for the dynamical origin of the electroweak symmetry breaking. A laboratory for these models is QCD with many flavors, which may present a nontrivial fixed point associated to a conformal region. Studies based on conformal field theories and on Schwinger-Dyson equations have suggested the existence of bounds on the mass anomalous dimension at the fixed points of these models. In this note we discuss γ_m values of multiflavor QCD exhibiting a nontrivial fixed point and affected by relevant four-fermion interactions.

Models of strongly interacting theories with a large mass anomalous dimension are important for extensions of the standard model in the context of walking chiral symmetry breaking dynamics [1,2]. These theories present a critical coupling constant for the onset of chiral symmetry breaking, related to a nontrivial fixed point associated to a conformal region.

Lattice QCD simulations provide a powerful tool to investigate the presence of a large anomalous dimension in the multiflavor case. However, when the conformal behavior is approached as the number of flavor (N_f) is increased the simulations demand larger and larger lattice volumes. The enormous task to obtain limits on the anomalous dimension of multiflavor QCD can be verified in Ref. [3] and references therein.

Earlier determinations of these anomalous dimensions include a study based on Schwinger-Dyson equations (SDE) where it was verified that $\gamma_m \approx 1$ and is not strongly affected by high order corrections [4]. Another strong bound based on the unitarity of conformal field theories gives $\gamma_m \leq 2$ [5]. Recent studies based on conformal bootstrap in $SU(N_f)_V$ symmetric conformal field theories suggest $\gamma_m < 1.31$ for $N_f = 8$ [6] and $\gamma_m \leq 1.29$ for $N_f = 12$ [7]. However, the existence of a conformal fixed point in the $N_f = 12$ model is still controversial [8].

It was observed for massless QED, when the coupling constant reaches a certain critical value, that the chiral symmetry is broken by the vacuum [9]. Later it was verified that at this critical coupling four-fermion interactions acquire an anomalous dimension such that the theory becomes renormalizable [10,11]. The importance of a large

anomalous dimension in what are now known as gauged Nambu-Jona-Lasinio models became clear in the works of Refs. [12–17]. In these models two coupling constants enter into action: the gauge coupling (λ) and the four-fermion one (g), and there is a critical line described by a combination of these couplings where the chiral symmetry is broken. At this critical line the dynamical fermion mass behaves as a slowly decreasing function with the momentum [18,19], and not much different from what is expected in a theory with bare masses.

The problem of mass generation with an enhanced fermionic condensate along the critical line associated to a slowly running coupling, i.e., near a nontrivial fixed point, in non-Abelian gauge theories began to be studied in Refs. [20–22]. In Ref. [22] the fixed point was obtained from the two-loop β function for an $SU(N)$ theory with fermions in the fundamental representation. One analysis of this problem in the case of other groups and fermionic representations can be seen in Ref. [23]. These studies began the search for strongly interacting theories with a critical behavior, like that of the gauged Nambu-Jona-Lasinio model, associated to a nontrivial fixed point and consequently slowly running coupling constant, possibly in the Banks and Zaks scenario [24] of a perturbative value for the critical coupling. However in these cases the critical line starts depending on the number of fermions and on the dimension of the fermionic representations. At this point we should mention that one of the contributions of this work, compared with the previous studies, is that we expand the resolution of the conformal window near the critical line expected to exist in QCD, because we are computing the anomalous mass dimension also in the case of a nonperturbative fixed point.

In this article we do not provide a stronger bound on the anomalous dimension than the ones discussed in the

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previous paragraphs. We consider a different approach where a dynamical fermion mass has a hard asymptotic behavior associated to a nontrivial fixed point and relevant four-fermion interaction. This is also expected due to the coalescence of the SDE solutions [25]. The theory is fully described by the critical coupling g_c, where $\beta(g_c) \approx 0$, and the effect of the four-fermion interaction together with this critical gauge coupling leads us near the critical line; this is translated in the fact that we necessarily have a hard fermionic self-energy.

The anomalous dimension governs the renormalization effects on the fermion condensate at one large scale Λ as

$$\langle \bar{\psi}\psi \rangle_\Lambda = \exp\left[\int_\mu^\Lambda \frac{d\kappa}{\kappa} \gamma_m(\alpha(\kappa))\right] \langle \bar{\psi}\psi \rangle_\mu. \qquad (1)$$

Assuming that the theory has a slowly (or almost constant) running coupling constant in the interval μ to Λ [26], we have the following scaling relation,

$$\langle \bar{\psi}\psi \rangle_\Lambda \approx \eta \left(\frac{\Lambda}{\mu}\right)^{\gamma_m} \mu^3, \qquad (2)$$

where η is a constant and μ is the characteristic scale of the theory. A large γ_m modifies the asymptotic dynamics and alleviates some of the problems suffered by composite models of dynamical electroweak symmetry breaking [26].

The fermion condensate as a function of the fermion self-energy ($\Sigma(p^2)$) can be written as

$$\langle \bar{\psi}\psi \rangle_\Lambda = -\frac{N_R}{4\pi^2} \int_0^{\Lambda^2} dx \frac{x\Sigma(x)}{x + \Sigma^2(x)}, \qquad (3)$$

where N_R is the dimension of the fermionic representation. We can compute $\langle \bar{\psi}\psi \rangle_\Lambda$ with Eq. (3) and one specific expression for $\Sigma(p^2)$ and compare it with the infrared condensate value in order to obtain a value for γ_m.

It is known that the infrared behavior of the fermionic self-energy solution is a constant up to the scale μ that roughly defines the infrared dynamically generated mass. Therefore, apart from constants that may be assimilated in the infrared mass scale μ we have

$$\langle \bar{\psi}\psi \rangle_\mu \propto \mu^3. \qquad (4)$$

This is what is expected in standard QCD regardless of whether the self-energy is a soft one like

$$\Sigma^{(1)}(p^2 \to \infty) \sim \frac{\mu^3}{p^2}, \qquad (5)$$

which is the known behavior predicted by the operator product expansion [27], or has a harder behavior than this one. Equation (4) is just reflecting the infrared condensate behavior.

If the theory is affected by relevant four-fermion interactions the self-energies may vary up to the hardest behavior

with the momentum, as shown in Refs. [18,19], and are equal to

$$\Sigma^{(0)}(p^2) \sim \mu[1 + bg^2(\mu^2) \ln(p^2/\mu^2)]^{-\delta}, \qquad (6)$$

where

$$\delta = 3c/16\pi^2 b, \qquad (7)$$

and c is the quadratic Casimir operator given by

$$c = \frac{1}{2}[C_2(R_1) + C_2(R_2) - C_2(R_3)], \qquad (8)$$

where $C_2(R_i)$ are the Casimir operators for fermions in the representations R_1 and R_2 that form a composite boson in the representation R_3. In the QCD case, for quarks in the fundamental representation $R = F$, $c = [(N^2 - 1)/2N]$ and b is the coefficient of the g^3 term in the renormalization group β function and is given by $b = (33 - 2N_f)/48\pi^2$.

Equation (6) may be a legitimate nonperturbative self-energy solution driven by confinement when the gluons have a dynamically generated mass, as discussed in Ref. [28], or the solution when the theory is dominated by the four-fermion interactions [18,19] that also can be the result of coalescence of the SDE solutions [25]. The critical coupling for this occurrence is the same one considered in Ref. [4], determined earlier by the authors of Refs. [29–35] and equal to

$$\alpha_c = \frac{\pi}{3c}. \qquad (9)$$

Note that effects like dynamical gluon mass generation or confinement have not been taken into account in Refs. [29–35]; however, even in these cases we should expect a similar coupling constant critical value for the onset of chiral symmetry breaking [28].

Notice that Eq. (6) is the solution of the renormalization group equation, in Landau gauge, given by

$$\left(\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g}\right)\Sigma = \frac{3c}{8\pi^2} g^2\Sigma, \qquad (10)$$

and normalized such that $\Sigma(p^2 = 0) = \mu$. If we go to a higher order in the coupling constant we would expect that the coefficient $bg^2(\mu^2)$ appearing in Eq. (6) would be traded by higher order terms of the β function. It is interesting to note that Eq. (10) can be a generalization of the solution shown in Eq. (6), because it could be solved considering higher order terms in the β function.

Let us now consider the fermion condensate given by Eq. (3) calculated at the high energy scale Λ and $\Sigma(p^2)$ given by Eq. (6). It is straightforward to show that the leading contribution to the QCD fermion condensate is

$$\langle \bar{\psi}\psi \rangle_\Lambda \propto \frac{3}{4\pi^2}\Lambda^2\mu\left[1 + bg^2(\mu)\ln\frac{\Lambda^2}{\mu^2}\right]^{-\delta}. \qquad (11)$$

This is the extreme behavior for the condensate. With this result and Eq. (4) we obtain

$$\frac{\langle\bar\psi\psi\rangle_\Lambda}{\langle\bar\psi\psi\rangle_\mu} = \frac{\Lambda^2}{\mu^2}\left[1 + bg^2(\mu)\ln\frac{\Lambda^2}{\mu^2}\right]^{-\delta}. \qquad (12)$$

However,

$$\langle\bar\psi\psi\rangle_\Lambda = Z_m^{-1}\langle\bar\psi\psi\rangle_\mu, \qquad (13)$$

and the renormalization constant Z_m is related to the anomalous dimension as

$$\gamma_m \equiv \mu\frac{\partial}{\partial\mu}\ln Z_m, \qquad (14)$$

leading, with the help of Eq. (12) to

$$\gamma_m(\Lambda^2) = 2 - 2\delta b\frac{g^2(\mu^2)}{[1 + bg^2(\mu^2)\ln\frac{\Lambda^2}{\mu^2}]}. \qquad (15)$$

The anomalous mass dimension of Eq. (15) was calculated using the fermionic condensate Eq. (3) and not directly from the gap equation as usual. Furthermore, we use the hard self-energy [Eq. (6)] and not the soft one giving by Eq. (5).

If we deal with a conventional theory (or standard QCD), the asymptotic freedom condition ($\alpha(\Lambda^2) \to 0$) applied to Eq. (15) leads to $\gamma_m = 2$, where we recover the known result of Ref. [5]. However, this is not the case we are interested in, which is the one of a large number of flavors, where the theory is dominated by four-fermion interactions and we have an almost conformal theory, i.e., $\beta(g) \approx 0$ (or strictly 0), and we cannot use the leading order coupling constant as considered in Eq. (15).

Before making a detailed calculation of Eq. (15) taking into account a possible conformal behavior of the coupling constant, we can recall that at the same time the chiral symmetry is broken and nontrivial self-energies like that of Eq. (6) are generated; composite pseudoscalar Goldstone bosons (the pions) and scalar massive bosons (the σ) are formed, whose wave functions (respectively Φ_{BS}^P and Φ_{BS}^S) are solutions of the Bethe-Salpeter (BS) equation related to the fermionic self-energy as

$$\Sigma(p^2) \approx \Phi_{BS}^P(p,q)|_{q\to0} \approx \Phi_{BS}^S(p,q)|_{q^2=4m_{dyn}^2}. \qquad (16)$$

These BS wave functions are subjected to a normalization condition [36,37]. One analysis of this condition applied to the pseudoscalar and scalar boson wave functions given by Eq. (6) can be found respectively in Refs. [38] and [39]. We do not enter into details of the calculation but just present the final constraint, which is

$$\delta > \frac{1}{2}. \qquad (17)$$

In the QCD case Eq. (17) implies $N_f > 5$. Gauge theories, with a not too large number of fermions, can feature a nontrivial fixed point in or near the so-called "conformal window" [40]. Recently, the authors in Refs. [41,42] demonstrated using lattice simulations, with the conformal window for the $SU(3)$ gauge theory lying in the range $8 < N_{fc} < 12$. In this region we indeed have a slowly running coupling ($\beta \approx 0$), where the limit of Eq. (17) is valid. Therefore in the subsequent calculations the limit imposed by Eq. (17) is always be taken into account, leading to a large number of fermions.

We now return to Eq. (15). This equation is a consequence of Eq. (6); it involves a determination of the coupling constant $g^2(\mu^2)$ in the large N_f limit, in one region where $\beta(g) \approx 0$, when the theory is dominated by a fixed point and four-fermion interactions are relevant. However even if Eq. (6) is a consequence of four-fermion interactions it is only dependent on the leading coefficient (b) of the β function, and all information about the walking behavior should be present in the coupling constant. This means that this coupling, in order to disclose such behavior, should be computed perturbatively beyond leading order or within some nonperturbative approach. Both possibilities are discussed in the following.

We first consider the perturbative approach and follow the same steps of Ref. [22], with a walking two-loop β function given by

$$\beta(\alpha_\mu) = -\alpha_\mu(\beta_0\alpha_\mu + \beta_1\alpha_\mu^2), \qquad (18)$$

where $(\beta_0\alpha_\mu + \beta_1\alpha_\mu^2) \ll 1$ in order to warrant a walking behavior, as well the quantity being positive in order to assure asymptotic freedom. We thus have the following approximation for the coupling constant,

$$\alpha(p) \approx \alpha_\mu\left(1 - \frac{\beta(\alpha_\mu)}{\alpha_\mu}\ln\frac{p}{\mu}\right), \qquad (19)$$

which, if inserted into Eq. (15), entails

$$\gamma_m(\Lambda^2) \approx 2 - \gamma_m + \frac{1}{2}\frac{\beta[\alpha(\mu)]}{\alpha_c}\ln\frac{\Lambda^2}{\mu^2}, \qquad (20)$$

where $\gamma_m = \frac{1}{2}\frac{\alpha(\mu^2)}{\alpha_c}$.

Equation (20) is justified when the coupling constant is given by Eq. (18), but small differences appear as we go to more loops in the β function. Within this approximation and assuming $\beta(\alpha_\mu = \alpha_c) \approx 0$ we obtain $1 \lesssim \gamma_m(\Lambda) < 2$, which is the known range for this quantity [1,2]. Note that the complete independence of γ_m on the scale happens only in the presence of a fixed point; if this is not the case its perturbative determination will also be dependent, above the two-loop order of the β function, on the renormalization scheme.

It is interesting to comment on the behavior of Eq. (15) in the case where we indeed have a fixed point. We can indicate the value of the coupling constant as α^*

TABLE I. Anomalous dimensions evaluated at the fixed points obtained from $\beta(\alpha(\mu)) = 0$ at four loops in the range ($8 \lesssim N_f \lesssim 12$), where α^* is the coupling constant value at the fixed point for each N_f.

N_f	α^*	γ^*	γ_m
8	1.138	0.724	1.275
9	0.952	0.608	1.394
10	0.445	0.283	1.717
11	0.259	0.165	1.835
12	0.176	0.112	1.887

TABLE II. Anomalous dimensions evaluated at the fixed points obtained from the nonperturbative (DGM) $\beta(g)$ function [58], in the range ($8 \lesssim N_f \lesssim 12$), where α^* is the coupling constant value at the fixed point for each N_f.

N_f	α^*	γ^*	γ_m
8	0.640	0.407	1.593
9	0.700	0.446	1.554
10	0.780	0.496	1.504
12	1.060	0.676	1.324

and consider $\beta(\alpha_\mu = \alpha^*) = 0$, and now the anomalous dimension is described by

$$\gamma_m \approx 2 - \frac{1}{2}\frac{\alpha^*}{\alpha_c} \approx 2 - \gamma^*, \tag{21}$$

where $\gamma^* \equiv \frac{1}{2}(\alpha^*/\alpha_c)$. We first consider the α^* as the coupling constant determined perturbatively in the Banks and Zaks scenario (BZ) [23,40,43], where

$$\alpha^* = -4\pi \frac{11N - 4N_f d(R)}{34N^2 - 2N_f d(R)[10N + 6C_2(R)]}, \tag{22}$$

and $d(R)$ is the dimension of the representation R of the group $SU(N)$. Assuming the QCD conformal window in the range $8 \lesssim N_f \lesssim 12$, for $N_f \lesssim 12$ and α^* given by Eq. (22) we obtain an upper limit of $\gamma_m \lesssim 1.5$.

In Table I we show the results obtained for (γ_m), $\alpha*$ and γ^*, for QCD assuming $\beta(\alpha(\mu)) = 0$ at four loops, as a function of N_f in the $\overline{MS}$ scheme [44] in the range ($8 \lesssim N_f \lesssim 12$).

As a second determination of γ_m we consider the case where the fixed point is a result of the dynamical generation of gluon masses (DGM) [45–48]. As a consequence of dynamical gluon mass generation ($m_g(k^2)$) by the QCD vacuum, the theory acquires a nontrivial infrared (IR) fixed point [49]. The running coupling constant becomes IR finite at moderately small values [50] and we have the following nonperturbative β function [51–53],

$$\beta_{\mathrm{DGM}} = -bg^3\left(1 - \frac{4m_g^2}{\Lambda^2}e^{-\frac{1}{bg^2}}\right), \tag{23}$$

where b is the coefficient of the g^3 term in the renormalization group β function and, for simplicity, the running behavior of the dynamical gluon mass was neglected. The variation of m_g with N_f was determined in lattice simulations [54] and with Schwinger-Dyson equations [55] for a small number of flavors. These results were extrapolated for a large number of flavors as in Refs. [56–58], and we use the following parametrization: $m_g^{-1} = m_{g_0}^{-1}.e^{0.05942N_f}$

with $m_{g_0} = 440$ MeV [58]. The values of α^* for $N_f = 8, 9, 10$, and 12 can be obtained from Table II of Ref. [58] and the anomalous dimensions that can be obtained in this approach are shown in Table II.

In this note we discussed the anomalous mass dimension of multiflavor QCD. The calculation is performed in the case in which the theory has a nontrivial fixed point or critical line and the theory is dominated by four-fermion interactions leading naturally to a hard asymptotic quark self-energy [18,19], what may also happen due to the coalescence of the SDE solutions [25] or to a confinement mechanism of chiral symmetry breaking [28].

In the context of the simple BZ scenario at the two-loop level and within the conformal window with $N_f \lesssim 12$ we obtain $\gamma_m \lesssim 1.5$. Similar results are obtained in the case of fixed points obtained from $\beta(\alpha^*) = 0$ at four loops, for $N_f = 8 - 10$ and in the DGM nonperturbative case. The data set presented in Tables I and II points to the existence of a limit on the mass anomalous dimension compatible with $\gamma_m \approx (1.3–1.5)$. Therefore, we obtained new estimates of the mass anomalous dimension of multiflavor QCD in the presence of critical coupling constants associated to perturbative and nonperturbative fixed points. In order to compare with previous studies we can say that in this work we expanded the resolution of the conformal window near the critical line expected to exist in QCD, see Ref. [44], once we computed the mass anomalous dimension also in the case of a nonperturbative fixed point. We hope that these γ_m values may be confronted with the ones that can be obtained with QCD simulations and conformal bootstrap methods.

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