

CP-violating baryon oscillationsDavid McKeen^{*} and Ann E. Nelson[†]*Department of Physics, University of Washington, Seattle, Washington 98195, USA*

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We enumerate the conditions necessary for CP violation to be manifest in $n - \bar{n}$ oscillations and build a simple model that can give rise to such effects. We discuss a possible connection between neutron oscillations and dark matter, provided the mass of the latter lies between $m_p - m_e$ and $m_p + m_e$. We apply our results to a possible baryogenesis scenario involving CP violation in the oscillations of the Ξ^0 .

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I. INTRODUCTION

Given the evidence for an initial period of inflation in the early Universe, the asymmetry between matter and antimatter that we observe implies that baryon number (B) conservation must be violated [1]. Furthermore, while the standard model (SM) accidentally conserves B and lepton number (L) at the renormalizable level, both are broken by nonperturbative effects [2], with only the combination $B - L$ conserved (however, such anomalous processes cannot give rise to proton decay as they conserve $B \bmod 3$). If neutrino masses are of the Majorana type, as in the theoretically attractive seesaw mechanism, they provide evidence for $B - L$ violation, via a $\Delta L = 2$ operator, providing further motivation to consider B violation.

The two phenomenological hallmarks of B violation are neutron-antineutron oscillation [3] and proton decay. Some recent reviews of $n - \bar{n}$ oscillation are given in Ref. [4]. While proton decay is much more strongly constrained by experiment than neutron-antineutron oscillation, the former requires that B is violated by one unit while the latter requires violation by two units. Therefore, in theories of baryon number violation that only admit $\Delta B = 2$ operators, $n - \bar{n}$ oscillations can occur at a potentially observable rate without leading to proton decay (for recent work in this direction, see, e.g., Refs. [5,6]). For this reason, studying $n - \bar{n}$ oscillation is a critical component of understanding the origin of B violation.

The basic formulas governing neutron oscillation were first derived some time ago [7]. If B is not conserved, there can be a nonzero transition amplitude between the (flavor eigenstates) n and \bar{n} that we denote as δ . There is a splitting, Δm , between mass eigenstates that are a linear combination of n and \bar{n} , due to δ as well as from other interactions, e.g., with a background magnetic field. Typically $\Delta m \gg \delta$. Using a basis in which the spin axis aligns with the background magnetic field, the system is effectively two independent two-state systems, with the component of

angular momentum which is aligned with the magnetic field being conserved during the transition. For time scales short enough to neglect the possibility of neutron decay or when the neutron is stabilized by being in a stable nucleus, the probability for a state that is a neutron at $t = 0$ to oscillate to an antineutron is

$$P_{n \rightarrow \bar{n}} = \frac{2\delta^2}{\Delta m^2} (1 - \cos \Delta m t). \quad (1)$$

In a typical experimentally relevant situation where $\Delta m t \ll 1$, this becomes $P_{n \rightarrow \bar{n}} \simeq \delta^2 t^2 \equiv (t/\tau_{n\bar{n}})^2$, which defines the oscillation lifetime. Neutron oscillations in nuclei lead to nuclear decays which presently set the best observational limit on $\tau_{n\bar{n}}$. The lower bound on the ^{16}O lifetime from Super-Kamiokande of 1.9×10^{32} yr [8] at 90% C.L. corresponds [9] to $\tau_{n\bar{n}} > 3.5 \times 10^8$ s, or $\delta < 1.9 \times 10^{-33}$ GeV.

In this paper we reconsider neutral baryon oscillations in the presence of CP -violating and baryon-number-violating new physics, and discuss the conditions under which CP violation can be exhibited in the oscillations. We also consider whether a dark matter particle could be observed in neutron decays and whether such decays could contribute to observable CP violation and/or baryogenesis. We find that while decays of oscillating neutrons are not likely to exhibit a significant amount of CP violation, it is possible that oscillations of neutral baryons containing heavier flavors could perhaps even be enough to create the baryon asymmetry of the Universe. We briefly outline a baryogenesis scenario involving CP violation in oscillations of baryons containing heavy flavor.

II. CP VIOLATION IN NEUTRAL FERMION OSCILLATIONS

Because only states with the same spin can mix (in a basis where the spin aligns with any external magnetic field), a two-state Hamiltonian H suffices to describe oscillations of spin-1/2 particles. Here we use the $n - \bar{n}$ system as an example, but our results can be applied to any neutral spin-1/2 particle. In vacuum H is given by

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$$H = \begin{pmatrix} m_n - \frac{i}{2}\Gamma_n & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & m_n - \frac{i}{2}\Gamma_n \end{pmatrix}, \quad (2)$$

$$m_n = m_n^*, \quad \Gamma_n = \Gamma_n^*. \quad (3)$$

Details of our formalism and how to derive M_{12} and Γ_{12} from a more fundamental theory follow from the treatment of Refs. [10,11] and are recapped in the Appendix. While some authors [12] have recently claimed that CP violation is required for neutron oscillations, it is always possible to reparametrize the system and use a definition of CP so that the mass matrix is not CP violating [13,14]. CP violation in interference between mixing and decays is possible, leading to the $n \rightarrow \bar{n}$ oscillation probability in vacuum differing from that for $\bar{n} \rightarrow n$,

$$\frac{P_{|n\rangle \rightarrow |\bar{n}\rangle}}{P_{|\bar{n}\rangle \rightarrow |n\rangle}} - 1 = \frac{2\Im(M_{12}\Gamma_{12}^*)}{|M_{12}|^2 - |\Gamma_{12}|^2/4 - \Im(M_{12}\Gamma_{12}^*)}. \quad (4)$$

Generally, $|M_{12}| \gg |\Gamma_{12}|$ so that

$$\frac{P_{|n\rangle \rightarrow |\bar{n}\rangle}}{P_{|\bar{n}\rangle \rightarrow |n\rangle}} - 1 \simeq \frac{2|\Gamma_{12}|}{|M_{12}|} \sin\beta, \quad (5)$$

with $\beta \equiv \arg M_{12}\Gamma_{12}^*$ a reparametrization-invariant CP -violating phase. We now examine the characteristics necessary for a model that gives $n - \bar{n}$ oscillations at a rate not too far from the current upper bounds while also generating CP violation that is not vanishingly tiny.

Models that violate baryon number only by two units allow for $n - \bar{n}$ oscillations without generating proton decay, which is subject to extremely strong constraints (see, e.g., Ref. [5]). In such a model, if lepton number is not violated, Γ_{12} for the neutron system can be generated by operators that allow for the decays

$$n \rightarrow \bar{p}e^+\nu_e, \quad \bar{n} \rightarrow pe^-\bar{\nu}_e \quad (6)$$

to proceed directly. However, the operators that generate these decays are dimension 12, and it is difficult for them to result in a value of Γ_{12} that is not exceedingly small compared to 10^{-33} GeV. We are therefore led to consider new states that are lighter than the neutron.

As a motivation to consider such states, let us assume that baryon number is only absolutely conserved mod 2, so that we have a conserved \mathbb{Z}_2 symmetry which is a subgroup of baryon number. This \mathbb{Z}_2 symmetry could be used to guarantee the stability of a dark matter Majorana fermion χ , provided that χ is lighter than $m_p + m_e$. Stability of the proton requires that χ be heavier than $m_p - m_e$.¹ A slightly stronger lower bound of $m_\chi > 937.9$ MeV comes from requiring that ${}^9\text{Be}$ remain stable and not decay via the

¹The stability of dark matter due to its mass being in this range in a model to explain baryogenesis was considered in Ref. [15].

reaction ${}^9\text{Be} \rightarrow {}^8\text{Be} + \chi$. We have checked that this is the strongest bound that comes from requiring that all stable nuclides are kinematically forbidden from decaying to χ or χe^+ . We then consider the decays of both the neutron and antineutron into $\chi + \gamma$ in order to generate Γ_{12} . Justifying why χ should conveniently lie in this very narrow mass range is not the point of this paper, but it could be argued for using anthropic reasoning, or perhaps χ could be a baryon of a sector mirror to ours [16], with its own gauge interactions. It is also possible that χ only couples to heavy flavors but has a mass such that it can only decay into light flavors, with 2 or more weak interactions required for it to decay, in which case the constraint on its mass can be weakened.

As an example of how to generate an interaction that would allow decays of the neutron or antineutron into $\chi + \gamma$, consider adding a scalar diquark ϕ that is a color antifundamental and carries hypercharge 1/3. Then it can couple to quarks through

$$\mathcal{L} \supset g\phi\bar{u}\bar{d} + \text{H.c.}, \quad (7)$$

where \bar{u} and \bar{d} are the (left-handed) up and down $SU(2)$ singlet quarks. We couple ϕ and \bar{d} to the Majorana fermion χ ,

$$\mathcal{L} \supset y\phi^*\bar{d}\chi + \text{H.c.} \quad (8)$$

These interactions generate a transition dipole operator, through diagrams like the one shown in Fig. 1, which appears as a term in the effective Lagrangian involving the neutron field as

$$\mathcal{L}_{\text{eff}} \supset \mu\bar{\chi}\Sigma_{\mu\nu}nF^{\mu\nu} + \text{H.c.}, \quad (9)$$

with

$$\mu \sim e\kappa \times \frac{gy}{m_\phi^2 m_n^2}, \quad (10)$$

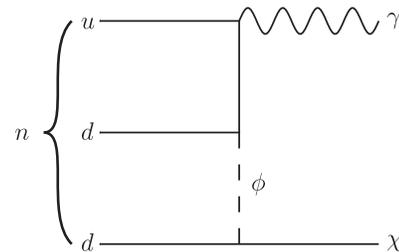


FIG. 1. Diagram that leads to the decay $n \rightarrow \chi\gamma$ as well as to an absorptive portion to the $n - \bar{n}$ transition amplitude, Γ_{12} , given a Majorana mass for χ . Diagrams where the photon is attached to the scalar ϕ are suppressed by the large ϕ mass. Removing the photon gives the diagram responsible for $n - \chi$ (and $\bar{n} - \chi$) transitions.

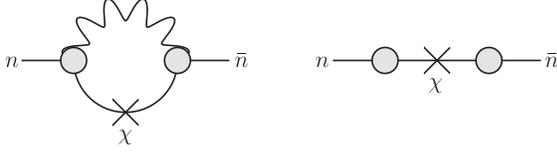


FIG. 2. Left: Diagram responsible for Γ_{12} through intermediate on-shell χ and γ . Right: Diagram responsible for M_{12} . The crosses represent χ mass insertions, and the blobs are the higher dimensional operators responsible for $n\text{-}\chi$ and $\bar{n}\text{-}\chi$ transitions, as seen in Fig. 1.

where $\kappa \sim 10^{-2} \text{ GeV}^3$ comes from evaluating the hadronic matrix element $\langle n | (udd)^2 | n \rangle$ (for a detailed computation of this matrix element using lattice QCD, see [17]). As shown in the left diagram of Fig. 2, this coupling gives a contribution to Γ_{12} that is

$$\Gamma_{12} = \frac{\mu^2 m_n^2 m_\chi}{16\pi} \left(1 - \frac{m_\chi^2}{m_n^2}\right)^3 \quad (11)$$

$$\sim 10^{-47} \text{ GeV} \left(\frac{10^8 \text{ GeV}}{m_\phi/\sqrt{gY}}\right)^4 \left(\frac{\Delta M}{1 \text{ MeV}}\right)^3, \quad (12)$$

where $\Delta M \equiv m_n - m_\chi \ll m_n$. There is also a contribution to M_{12} from an off-shell intermediate χ exchange, seen in the right diagram of Fig. 2, that is larger than Γ_{12} ,

$$M_{12} \sim \left(\frac{\kappa g y}{m_\phi^2}\right)^2 \frac{m_\chi}{m_n^2 - m_\chi^2} \quad (13)$$

$$\sim 10^{-33} \text{ GeV} \left(\frac{10^8 \text{ GeV}}{m_\phi/\sqrt{gY}}\right)^4 \left(\frac{1 \text{ MeV}}{\Delta M}\right). \quad (14)$$

For CP violation to be present, there must exist another contribution to M_{12} , since this necessarily has the same phase as Γ_{12} . This can be done any number of ways; e.g., we can add another Majorana fermion χ' with similar couplings as χ but different phases in the couplings. Without fine-tuning the two separate contributions to M_{12} against each other, we would then generically expect the level of CP violation in the neutron-antineutron system to be tiny,

$$\frac{P_{|n\rangle \rightarrow |\bar{n}\rangle}}{P_{|\bar{n}\rangle \rightarrow |n\rangle}} - 1 \propto \frac{|\Gamma_{12}|}{|M_{12}|} \lesssim 10^{-14} \left(\frac{\Delta M}{1 \text{ MeV}}\right)^4. \quad (15)$$

One may wonder whether there is a contribution to the neutron EDM that comes from attaching a photon to diagrams with external neutrons and a $\chi\text{-}\gamma$ loop. Since the amplitudes responsible for an EDM must have $\Delta B = 0$ (as we argued above, and as also done in Ref. [14],) potential contributions to the EDM from these diagrams must not contain a B -violating χ mass insertion. However, the lack of a factor of m_χ in such diagrams (in contrast to

M_{12} and Γ_{12}) means that they do not contain a physical CP -violating phase and thus do not contribute to the neutron EDM.

There is a contribution to the neutron EDM from a d quark (chromo)EDM that arises through a $\chi\text{-}\phi$ loop. However, since we assume that the coupling of ϕ to quark doublets is absent, this requires a light quark mass insertion. Therefore, ϕ masses in the 10–100 TeV range are safe, even with $g \sim \mathcal{O}(1)$. For a detailed discussion of experimental limits on similar models, see [5,18].

While a significant amount of CP violation in $n\text{-}\bar{n}$ oscillations does not appear promising, this analysis suggests that oscillations of neutral baryons which are less directly constrained by nuclear stability can exhibit larger CP violation. One might consider $\Lambda - \bar{\Lambda}$ oscillations [19], but $\Delta B = 2$ operators with $\Delta S = 2$ are highly constrained from decays of dinucleons into two kaons. Other neutral baryons are more promising. Consider oscillations of Ξ^0 and $\bar{\Xi}^0$. Like neutron oscillation, this process involves operators that have $\Delta B = 2$, but also requires $\Delta S = 4$. Since four kaons are more massive than two nucleons, such operators are subject to less stringent constraints than those with $\Delta S = 0$ or $\Delta S = 2$. In particular, they are not subject to strong limits from dinucleon decays. It is necessary to avoid generating a similar size for the much more constrained baryon-number-violating $\Delta S = 0, 1, 2$ and $\Delta S = 3$ [20] operators. Provided that only $SU(2)$ singlet strange quarks are involved in the $\Delta S = 4$ operators, then the radiative generation of $\Delta S = 0, 1$ and $\Delta S = 2$ operators is suppressed by two more weak loop factors as well as light quark mass insertions. However, a $\Delta S = 3$ operator may be generated via a single weak loop, greatly constraining the size of the operator which allows oscillations of the Ξ^0 . Oscillations of baryons containing heavier flavors are less constrained. For instance, oscillations of the Ξ_b^0 were considered by Kuzmin in 1996 [21], and an oscillation rate which is comparable to the decay rate was shown to be compatible with nuclear stability bounds. Therefore, if ϕ couples dominantly to heavy flavor $SU(2)$ singlet quarks, it is conceivable that the strongest limits on ϕ come from collider searches, and m_ϕ/\sqrt{gY} could be a few hundred GeV or less. A detailed study of the various flavor combinations for neutral baryon oscillations is underway in Ref. [22] and has shown that there are several baryons whose oscillation rates could be comparable to their decay rates without leading to excessive dinucleon decay. In addition, if there exists a neutral χ fermion which both the baryon and antibaryon can decay into, the larger mass splitting between χ and the heavy baryon allows for sizable $|\Gamma_{12}/M_{12}|$. Given a mechanism for producing neutral heavy flavored baryons out of equilibrium in the early Universe, such as via the late decay of some heavier χ particle as in Ref. [11], then CPV in their oscillations and decays is a potential new mechanism for baryogenesis.

III. CONCLUSIONS

There is firm evidence from the matter-antimatter asymmetry of the Universe that B is violated. Although proton decay searches are very sensitive probes of B violation, they are less sensitive to theories that only allow for $\Delta B = 2$ amplitudes. Searches for $n - \bar{n}$ oscillations directly test these theories. Much more rapid oscillations of heavy flavor baryons is allowed, and this possibility has not yet been experimentally explored.

In this paper we have enumerated the requirements for physical CP violation in neutral baryon oscillations. For the first time, a simple model that would give CP violation in neutral baryon oscillations was built. The model connects the symmetry which suppresses proton decay to the stabilization of a dark matter candidate with mass around the mass of the proton. The amount of CP violation in neutron oscillation in this model is still very small. Heavy flavor baryon oscillations could exhibit significantly more CP violation, and, given a mechanism (such as the one used in [11]) to produce them in the early Universe, after the QCD hadronization transition but before nucleosynthesis, CP violation in heavy baryon oscillations and decays could be the origin of the matter-antimatter asymmetry. This possibility is currently under detailed study in Ref. [22].

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APPENDIX GENERAL CASE OF OSCILLATIONS IN EXTERNAL FIELDS

Here we perform a detailed analysis of neutron oscillation in the generic case where the spin quantization axis does not align with the magnetic field, deriving the non-relativistic Hamiltonian from the effective Lagrangian describing the neutron-antineutron system. Our aim is to clarify some statements in the literature and make our formalism more explicit. We show that a simple two-state description of this phenomenon is sufficient² (as previous studies [7] had used). Similar work has been done recently [13,14]; however, as we consider some new, previously

²Electromagnetic and CPV effects were recently studied by Gardner and Jafari in Ref. [23] and CP violation by Berezhiani and Vainshtein [12]. Our results are not in agreement with those published results, although Gardner and Yan have updated and extended their analysis [14].

unconsidered possibilities, such as exotic neutron decays and the possibility of observable CP violation in neutron oscillations, for completeness we include our results here.

1. Deriving the Hamiltonian

Our starting point is the four-component neutron field n . It carries a charge $B = +1$ under a global $U(1)_B$ of baryon number. We write the most general Lagrangian density for describing a free neutron as

$$\mathcal{L} = \bar{n}\gamma^\mu\partial_\mu n + \mathcal{L}_B + \mathcal{L}_{\cancel{B}}, \quad (\text{A1})$$

where the bar denotes Dirac conjugation, $\bar{n} = n^\dagger\gamma^0$. We have separated the kinetic term from the bilinear terms that conserve baryon number, \mathcal{L}_B , and those that violate baryon number, $\mathcal{L}_{\cancel{B}}$. The baryon-preserving terms are

$$-\mathcal{L}_B = \bar{n}(m_n P_L + m_n^* P_R)n, \quad (\text{A2})$$

where $P_{L,R} = (1 \mp \gamma^5)/2$ project out the left and right chiralities of the four-component spinors. To construct the bilinears that break baryon number, we use the charge conjugated field n^c , which carries $B = -1$,

$$-\mathcal{L}_{\cancel{B}} = \bar{n}^c(\delta_1 P_L + \delta_2^* P_R)n + \bar{n}(\delta_2 P_L + \delta_1^* P_R)n^c. \quad (\text{A3})$$

It can be useful to express the four-component spinors in the chiral basis in terms of left-handed, two-component Weyl spinors, ξ and η , which carry $B = +1$ and -1 , respectively,

$$n = \begin{pmatrix} \xi \\ \eta^\dagger \end{pmatrix}, \quad n^c = \begin{pmatrix} \eta \\ \xi^\dagger \end{pmatrix}. \quad (\text{A4})$$

Note that we suppress the spinor indices for clarity. For a thorough description of two-component spinor techniques, see [24]. In terms of these fields,

$$-\mathcal{L}_B = m_n \eta \xi + \text{H.c.} \quad (\text{A5})$$

and

$$-\mathcal{L}_{\cancel{B}} = \delta_1 \xi \xi + \delta_2 \eta \eta + \text{H.c.} \quad (\text{A6})$$

We have defined charge conjugation so that

$$n \xrightarrow{C} n^c, \quad \xi \leftrightarrow \eta. \quad (\text{A7})$$

Thus, C leaves \mathcal{L}_B unchanged for any m_n , and it leaves $\mathcal{L}_{\cancel{B}}$ unchanged if $\delta_1 = \delta_2$.

A parity transformation P flips helicities and can be implemented by

$$n \xrightarrow{P} \gamma^0 n, \quad \xi \leftrightarrow \eta^\dagger. \quad (\text{A8})$$

\mathcal{L}_B is therefore P invariant if m_n is real, while $\mathcal{L}_{\bar{B}}$ is parity invariant if $\delta_1 = \delta_2^*$.

Given the C and P transformations above, a combined CP transformation (which is equivalent to time reversal T since we are dealing with a local, Lorentz-invariant theory) takes

$$n \xrightarrow{CP} \gamma^0 n^c, \quad \xi \xrightarrow{CP} \xi^\dagger, \quad \eta \xrightarrow{CP} \eta^\dagger. \quad (\text{A9})$$

CP is then conserved by \mathcal{L}_B and $\mathcal{L}_{\bar{B}}$ if m_n, δ_1, δ_2 are all real.

To conserve CP , if $U(1)_B$ is violated, this apparently requires removing three phases to make m_n, δ_1, δ_2 all real, while we only have two fields at our disposal to rephase, ξ and η . Equivalently, in the four-component language, we can make a chiral transformation $n \rightarrow e^{i\alpha\gamma^5} n$ to make m_n real and then $n \rightarrow e^{i\alpha'} n$ to remove the phase of δ_1 or δ_2 , but not both. Thus, at first glance it appears that if $U(1)_B$ is broken, it is not possible to remove CP violation. However, if $U(1)_B$ is only violated by $\delta_{1,2}$, then we can take a linear combination $n \rightarrow \cos\theta n + \sin\theta n^c$, $n^c \rightarrow \cos\theta n^c - \sin\theta n$, redefining $U(1)_B$, so that δ_1 or δ_2 is removed. Thus, CP need not be violated by the mass terms since there are then only two phases to remove. As long as we can redefine CP to the product of the original CP and any purely internal transformation, such that the new CP is a symmetry, there is no physically observable CP violation.

We also incorporate interactions with the electromagnetic field,

$$\mathcal{L}_{\text{dipole}} = \frac{1}{2} F_{\mu\nu} \bar{n} \Sigma^{\mu\nu} (a P_L + a^* P_R) n \quad (\text{A10})$$

$$= a F_{\mu\nu} \eta \sigma^{\mu\nu} \xi + \text{H.c.}, \quad (\text{A11})$$

where $\Sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$, and $\sigma^{\mu\nu} = (i/4)(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu)$, $\sigma^\mu = (1, \boldsymbol{\sigma})$, $\bar{\sigma}^\mu = (1, -\boldsymbol{\sigma})$ with $\boldsymbol{\sigma}$ the Pauli matrices. Here, $a = \mu_n - id_n$ with μ_n, d_n the neutron's magnetic (electric) dipole moment. We do not consider baryon-number-violating interactions with the electromagnetic field of the form $\bar{n}^c \Sigma^{\mu\nu} n \supset \xi \sigma^{\mu\nu} \xi, \eta \sigma^{\mu\nu} \eta$ since these vanish identically due to Fermi statistics.³

To construct the Hamiltonian, we first introduce operators that create (and annihilate) neutron and antineutron states,

³In Ref. [23], it was speculated that such interactions could develop through higher dimensional operators due to the composite nature of the neutron, as is the case for the neutron charge radius; however, that work has been updated with different conclusions [25]. In the language of effective field theory, a charge radius can develop because gauge invariance forbids writing the operator $\bar{n} \gamma^\mu A_\mu n$ (since the neutron is neutral) but allows $\bar{n} \gamma^\mu \partial^\nu F_{\mu\nu} n / \Lambda^2$, with Λ the compositeness scale of the neutron. However, operators like $\bar{n}^c \Sigma^{\mu\nu} F_{\mu\rho}^\rho (g_{\nu\rho} + \partial_\nu \partial_\rho / \Lambda^2 + \dots) n$ couple two identical fermions to total angular momentum $J = 1$. This configuration is symmetric under the interchange of the identical fermion fields; therefore, these operators vanish.

$$|n; \mathbf{p}, s\rangle = a_p^{s\dagger} |0\rangle, \quad |\bar{n}; \mathbf{p}, s\rangle = b_p^{s\dagger} |0\rangle. \quad (\text{A12})$$

The algebra satisfied by the creation and annihilation operators is

$$\{a_p^s, a_k^{r\dagger}\} = \{b_p^s, b_k^{r\dagger}\} = (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{k}) \delta^{sr}, \quad (\text{A13})$$

with all other anticommutators zero. We decompose the Weyl spinors in terms of these as

$$\xi_\alpha = \sum_s \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E_p}} [x_{p\alpha}^s a_p^s e^{-ip\cdot x} + y_{p\alpha}^s b_p^{s\dagger} e^{ip\cdot x}], \quad (\text{A14})$$

$$\eta_\alpha = \sum_s \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E_p}} [x_{p\alpha}^s b_p^s e^{-ip\cdot x} + y_{p\alpha}^s a_p^{s\dagger} e^{ip\cdot x}], \quad (\text{A15})$$

where $E_p = \sqrt{\mathbf{p}^2 + m_n^2}$ if we assume that baryon number is nearly conserved, $\delta_{1,2} \ll m_n$. Here, x and y are solutions to the Dirac equation, and they carry a spinor index. They can be expressed in terms of spin eigenstates ω^s , such that $\omega^{s\dagger} \omega^r = \delta^{sr}$, as

$$x_{p\alpha}^s = \sqrt{p \cdot \bar{\sigma}} \omega^s, \quad y_{p\alpha}^s = 2s \sqrt{p \cdot \bar{\sigma}} \omega^{-s}, \quad (\text{A16})$$

$$x_p^{s\alpha} = -2s \omega^{-s\dagger} \sqrt{p \cdot \bar{\sigma}}, \quad y_p^{s\alpha} = \omega^{s\dagger} \sqrt{p \cdot \bar{\sigma}}. \quad (\text{A17})$$

The matrix elements of the Hamiltonian, $H_{ij}^{ss'}$ ($i, j = n, \bar{n}; s, s' = \pm 1/2$), for the neutron-antineutron system at rest, can be computed through

$$-\langle i; \mathbf{p}, s | \int d^3x \mathcal{L} | j; \mathbf{p}', s' \rangle |_{\mathbf{p} \rightarrow 0} \quad (\text{A18})$$

$$= (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{p}') H_{ij}^{ss'}. \quad (\text{A19})$$

To incorporate decays of the neutron and antineutron (as is necessary when discussing CP), we decompose the Hamiltonian into a dispersive portion M and an absorptive part Γ , generated by on-shell intermediate states,

$$H = M - \frac{i}{2} \Gamma. \quad (\text{A20})$$

In the basis ($|n, +\rangle, |n, -\rangle, |\bar{n}, +\rangle, |\bar{n}, -\rangle$), where the spin quantization axis is taken as the \hat{z} direction, the dispersive portion of the Hamiltonian reads⁴

⁴Our Hamiltonian has $\langle n; + | H | \bar{n}; + \rangle = \langle n; - | H | \bar{n}; - \rangle$, while the one in Ref. [23] has $\langle n; + | H | \bar{n}; + \rangle = -\langle n; - | H | \bar{n}; - \rangle$ which violates Lorentz invariance. As pointed out in Ref. [12], this error leads to incorrect eigenvalues of the Hamiltonian in Ref. [23].

$$M = \begin{pmatrix} \text{Re}(m_n) \times \mathbb{1} - \mathbf{H} \cdot \boldsymbol{\sigma} & M_{12} \times \mathbb{1} \\ M_{12}^* \times \mathbb{1} & \text{Re}(m_n) \times \mathbb{1} + \mathbf{H} \cdot \boldsymbol{\sigma} \end{pmatrix}, \quad (\text{A21})$$

where $\mathbf{H} \equiv \mu_n \mathbf{B} - d_n \mathbf{E}$, $\mathbb{1}$ is the 2×2 unit matrix, and $M_{12} \equiv \delta_1^* + \delta_2$. While we have retained it in this expression for completeness, in what follows we ignore the (known to be tiny) electric dipole moment of the neutron and keep only the magnetic dipole moment. Because the dispersive portion of the Hamiltonian comes from on-shell intermediate states and we are assuming Lorentz invariance, Γ must be trivial in spin. Thus, in this basis, it is given by

$$\Gamma = \begin{pmatrix} \Gamma_n \times \mathbb{1} & \Gamma_{12} \times \mathbb{1} \\ \Gamma_{12}^* \times \mathbb{1} & \Gamma_n \times \mathbb{1} \end{pmatrix}, \quad (\text{A22})$$

where $\Gamma_n = 1/885.6 \text{ s} = 7.4 \times 10^{-28} \text{ GeV}$ is the free neutron beta decay rate and Γ_{12} represents any common final state that both the neutron and antineutron may decay into (which necessarily requires physics beyond the SM).

A CP transformation on H is implemented by taking the Hermitian conjugate of M and Γ separately. Given the form of these matrices, we see that CP violation requires a

$$U_B H U_B^\dagger = \begin{pmatrix} m_n - \mu_n B_z - \frac{i}{2} \Gamma_n & H_{12} & 0 & 0 \\ H_{21} & m_n + \mu_n B_z - \frac{i}{2} \Gamma_n & 0 & 0 \\ 0 & 0 & m_n + \mu_n B_z - \frac{i}{2} \Gamma_n & H_{12} \\ 0 & 0 & H_{12} & m_n - \mu_n B_z - \frac{i}{2} \Gamma_n \end{pmatrix}, \quad (\text{A25})$$

defining $H_{12} \equiv M_{12} - (i/2)\Gamma_{12}$, $H_{21} \equiv M_{12}^* - (i/2)\Gamma_{12}^*$. Thus, we see that the system described by the 4×4 Hamiltonian given by Eqs. (A21) and (A22) is actually two separate, identical two-state systems—this transformation aligned the spin quantization axis along the direction of

nontrivial phase difference between $M_{12} = \delta_1^* + \delta_2$ and Γ_{12} .

The Hamiltonian in this basis can be broken up into 2×2 blocks according to its effect on baryon number,

$$H \sim \begin{pmatrix} \Delta B = 0 & \Delta B = 2 \\ \Delta B = -2 & \Delta B = 0 \end{pmatrix}. \quad (\text{A23})$$

The $\Delta B = 0$ blocks mix spins (if there is an external electromagnetic field) while, because of Fermi statistics, the $\Delta B = \pm 2$ blocks do not. Because of this, even in the presence of (constant) electromagnetic fields, a 2×2 Hamiltonian is sufficient to describe the neutron-antineutron system. To see that, we make a unitary transformation of the Hamiltonian using the matrix

$$U_B = \begin{pmatrix} c_\theta & e^{-i\alpha} s_\theta & 0 & 0 \\ 0 & 0 & c_\theta & e^{-i\alpha} s_\theta \\ -e^{i\alpha} s_\theta & c_\theta & 0 & 0 \\ 0 & 0 & -e^{i\alpha} s_\theta & c_\theta \end{pmatrix}, \quad (\text{A24})$$

with $\tan 2\theta = \sqrt{B_x^2 + B_y^2}/B_z$ and $\tan \alpha = B_y/B_x$. Operating on H with this matrix gives

the magnetic field. This justifies previous analyses of $n-\bar{n}$ transitions that used a two-state Hamiltonian [7], despite the presence of electromagnetic fields that could have required the consideration of neutron and antineutron spin.⁵

To diagonalize H we make use of the matrix U ,

$$U = \begin{pmatrix} H_{12}^{-1/2} c_1 c_\theta & H_{12}^{-1/2} c_1 e^{-i\alpha} s_\theta & -H_{21}^{-1/2} s_1 c_\theta & -H_{21}^{-1/2} s_1 e^{-i\alpha} s_\theta \\ -H_{12}^{-1/2} c_1 e^{i\alpha} s_\theta & H_{12}^{-1/2} c_1 c_\theta & -H_{21}^{-1/2} s_1 e^{i\alpha} s_\theta & H_{21}^{-1/2} s_1 c_\theta \\ H_{12}^{-1/2} s_2 c_\theta & H_{12}^{-1/2} s_2 e^{-i\alpha} s_\theta & H_{21}^{-1/2} c_2 c_\theta & H_{21}^{-1/2} c_2 e^{-i\alpha} s_\theta \\ H_{12}^{-1/2} s_2 e^{i\alpha} s_\theta & -H_{12}^{-1/2} s_2 c_\theta & -H_{21}^{-1/2} c_2 e^{i\alpha} s_\theta & H_{21}^{-1/2} c_2 c_\theta \end{pmatrix}, \quad (\text{A26})$$

which gives

⁵The time evolution is more complicated if the direction of the external field is time dependent, as considered in Ref. [23], as the matrix which block diagonalizes the Hamiltonian is then time dependent. However, the neutron-antineutron mixing angle is always suppressed by an external magnetic field. We do not consider time-varying fields here.

$$UHU^{-1} = \begin{pmatrix} m_n - \Delta/2 - \frac{i}{2}\Gamma_n & & & \\ & m_n + \Delta/2 - \frac{i}{2}\Gamma_n & & \\ & & m_n + \Delta/2 - \frac{i}{2}\Gamma_n & \\ & & & m_n - \Delta/2 - \frac{i}{2}\Gamma_n \end{pmatrix}. \quad (\text{A27})$$

In these expressions, we have taken m_n real and defined $\Delta \equiv 2\sqrt{\mu_n^2 B^2 + H_{12}H_{21}}$, and we use s_θ and c_θ to denote $\sin \theta$ and $\cos \theta$, with θ as given above. We have also defined $c_{1,2} = N_{1,2}\sqrt{1+z}$, $s_{1,2} = N_{1,2}\sqrt{1-z}$ with $z = 2\mu_n B/\Delta$ and $N_{1,2}^2 = |H_{12}(1 \pm z)| + |H_{21}(1 \mp z)|$.

2. Transition probabilities

The transition probabilities are given in terms of the matrix U by

$$P_{i \rightarrow j} = |U_{ki}U_{jk}^{-1}e^{-iM_k t}|^2, \quad (\text{A28})$$

with M_k the mass of eigenstate k . As mentioned, because the $\Delta S = \pm 2$ amplitudes do not mix spins, the probability for a neutron to oscillate into an antineutron of the opposite spin (or vice versa) is computed to be zero,

$$P_{|n;+\rangle \rightarrow |\bar{n};-\rangle} = P_{|\bar{n};+\rangle \rightarrow |n;-\rangle} = 0. \quad (\text{A29})$$

We find the probability as a function of time for a neutron at $t = 0$ to transition to an antineutron of the same spin (and omitting superfluous spin indices) to be

$$P_{|n\rangle \rightarrow |\bar{n}\rangle} = \frac{2|H_{21}|^2}{|\Delta|^2} \left(\cosh \frac{\Delta\Gamma t}{2} - \cos \Delta m t \right) e^{-\Gamma_n t}, \quad (\text{A30})$$

and that for an antineutron at $t = 0$ to oscillate to a neutron, it is

$$P_{|\bar{n}\rangle \rightarrow |n\rangle} = \frac{2|H_{12}|^2}{|\Delta|^2} \left(\cosh \frac{\Delta\Gamma t}{2} - \cos \Delta m t \right) e^{-\Gamma_n t}. \quad (\text{A31})$$

We note here that, for $\Gamma_{12} = 0$, our expressions for the oscillation probabilities in Eqs. (A30) and (A31) agree with standard formulas that have appeared in the literature [7] and, in particular, are suppressed when $\mu_n B \gg |H_{12}|, |H_{21}|$. The current experimental limit on the ^{16}O lifetime translates to $|H_{12,21}| \lesssim 10^{-33}$ GeV.

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